Measuring and Improving Network Robustness: A Chilean Case Study

Fernando G. Morales, Marcia H. M. Paiva, and Javier A. Bustos-Jiménez

Abstract— **Robustness is a desired feature for any backbone network. As a minimum requirement to be considered robust, a network must remain connected after any single node or link failure, so that each node is able to communicate with all other nodes. It turns out that some national optical backbones do not satisfy this condition, the Chilean Internet backbone being an example. To solve this problem, it is needed the addition of link(s) to the network, which can usually be done in different ways, leaving room to do that while minimizing other metrics of interest. This letter discusses metrics to evaluate the robustness of such networks (specifically, edge betweenness centrality, the number of link cutsets, and node Wiener impact) and proposes a variable neighborhood search heuristic to improve it by adding a few well-placed links. As a case of study, results are presented for the Chilean Internet backbone, considering three and four extra links.**

Index Terms— **Optical backbone networks, physical topology, graph theory, network robustness.**

I. Introduction

CHILE is a country highly susceptible to natural disasters: earthquakes, tsunamis, volcanic eruptions, and landslides are just some examples of them. A shared characteristic of such disasters is that they cause damage to large areas. These disasters not only directly impact the population, but also Internet networks.

Telephony has been structured so that it is impossible to operate in case of congestion, which always occur in consequence of emergencies. Instead, the Internet is designed to be a resilient network, prepared to correct partial failures and to continue operations while there are alternative paths available. On the other hand, citizenship, government and companies have become increasingly more dependent on Internet infrastructure and the enormous penetration of social networks makes critical infrastructure for emergencies. The Internet should be available and operational 100% of the time, even during a great disaster, so people are able to communicate, calm and organize. Internet was designed to withstand these difficulties, but there are limitations. A clear example is the earthquake in Chile on February 27, 2010, where national connectivity failed and most of the Web requests were made to servers outside the country, saturating the international link [1].

Network robustness plays an important role in many areas, and have been attracting attention from researchers coming from diverse science fields such as mathematics, physics, biology and many more [2]. Some relevant issues in this field are how to define and measure robustness in different applications [3], [4], how to improve robustness [5], and how to prevent network breakdowns [6]. In communication engineering, it is also concerned with the design of communication systems that can still work in case of unexpected disruptions or failures [7].

In optical backbone networks, in particular, analyzing physical topology robustness helps us to understand how and why could these networks fail [8]. The way optical fibers are distributed along these networks to communicate influences the network performance in many ways, such as communication speed. In general, if redundancy is added in these networks, it could improve not only the network speed, but also its robustness. Thus, when a node or a link fails, the network will still remain connected.

Unfortunately, there are some high risk scenarios where a high level of robustness cannot be achieved. Such is the case of the country of Chile, with its long and thin geography, susceptible to many disasters like earthquakes (notice that the territory is right next to the Nazca tectonic plate), avalanches and even car accidents. In such scenarios communication becomes more crucial than ever, thus lowering the risks associated to disasters is an important task to complete.

This letter contributes to solve this problem, by proposing a simple and efficient method to add a few well placed links to the network in such a way that it will not break in case of a single node or link failure, and it will optimize the desired robustness measure. In [9] and references therein, different solutions have been proposed to similar problems. In particular, the redesign method proposed in [9] focus on minimizing costs and requires the use of a commercial software. The present approach stands out for its simplicity. It explores, in the set of equal cost solutions, those that maximize robustness. Moreover, it does not require the use of commercial software.

Our method is applied to a real-world optical backbone network, the Chilean internet backbone, to find and compare infrastructure improvements by adding a restricted number of links. As far as the authors knowledge, this is the first attempt to study the Chilean backbone internet robustness.

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II. Preliminaries

A. Graph Theory

The physical network topology can be represented as a *graph* $G = G(V, E)$, that is a pair of two sets: the set of *vertices*, which represent the network nodes, and the set of *edges* E, which represent bidirectional links.

The *degree* of a vertex $d(v)$ is the number of edges incident to it, that is, the number of edges that have v as an endpoint.

A *path* is a sequence of edges that lead from a vertex u to a vertex v. The *path length* is the number of edges contained in a path. Given a pair of vertices u and v, there may well be several paths in a given graph to connect them; the length of the shortest of those paths, in number of edges (or hops), is called the *distance* $D(u, v)$.

A graph G is said to be *connected* if the distance between each pair of nodes $u, v \in G$ is finite. Otherwise, G is called *disconnected*.

Let G be a connected graph. The transmission of a vertex $v \in V(G)$ is defined as [10]:

$$T(v) = \sum_{u \in V} D(u, v), \quad (1)$$

whereas the Wiener index of G is given by [11]:

$$W(G) = \sum_{u \in V} \sum_{v \in V, v < u} D(u, v) \quad (2)$$

Important concepts to identify network vulnerable elements are bridges and cut-points. An edge e of a connected graph G is called a *bridge* if the removal of e from G leads to a disconnected graph. Analogously, a vertex v of G is a *cut-point* if the removal of v from G leads to a disconnected graph. A graph without cut-points (and consequently bridgeless) is called biconnected.

B. Chilean Backbone Network

Chilean Internet backbone is a joint effort of multiple stack-holders such as government institutions (SUBTEL), private ISPs and universities. The latter is a key point in infrastructure given that in the 90s the main objective of Internet was to communicate universities all along the country, and the infrastructure (fiber) has remain intact from those days until today.

To mimic the Chilean backbone network we use the Chilean's National University network (Red Universitaria Nacional: REUNA [12]). Figure 1 shows a simplified version of REUNA network, where parallel links were considered as single links. This consideration is important for topological analysis, specially when parallel links are likely to fail together in case of natural disasters.

From Fig. 1, it can be noticed that REUNA presents many vulnerable elements, i.e., nodes and links whose removal lead to a disconnected network: seven of 18 nodes (about 40%) are cut-points, and 11 of 19 links (about 60%) are bridges. Therefore, REUNA is not biconnected and presents a low level of robustness. Other ways to evaluate REUNA topology are discussed in next sections.

III. Proposed Method

A. Measuring Robustness

Being that network measurement varies depending on what context one assess robustness on, the metrics to be used must be carefully chosen for each application.

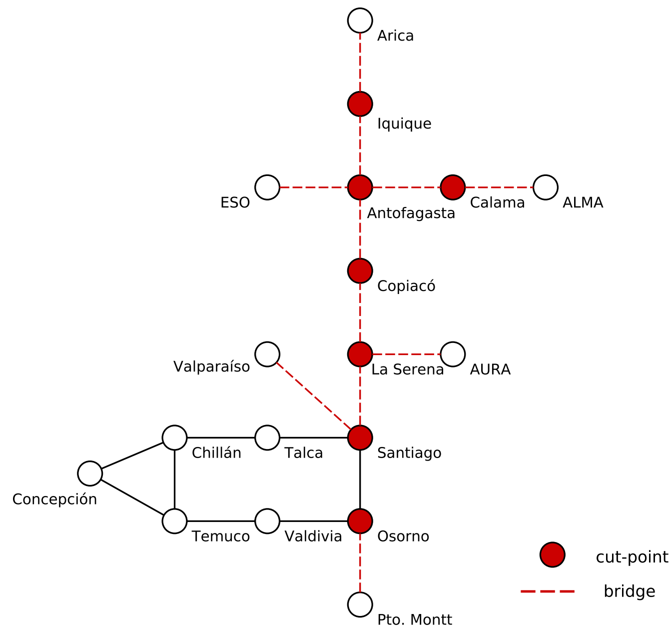

Fig. 1. REUNA topology (adapted from [12]). The most vulnerable elements of this network are marked in red. Notice that the network becomes disconnected after removing any of them.

Most of the modern (and often more convoluted) robustness measures have been proposed to deal with large and strongly connected networks, and are not able to quantify in a sensitive and granular way robustness aspects of optical backbone networks, which usually are small and weakly connected.

Besides the number of bridges and cut-points, key measures for analyzing optical backbone networks are the edge betweenness centrality, and the number of link cutsets [8], [13]. Also, the node Wiener impact [13] applies to biconnected backbone networks. These metrics are defined as follows.

- Edge betweenness centrality $EBC(e)$ [14]: For each edge e, $EBC(e)$ is proportional to the number of shortest paths that pass through e. It measures the importance of that edge in terms of path length quality.
- Relative number of cutsets of size i, CS_i [8], [15]: A cutset of size i in a connected graph G is a set of i edges whose removal from G results in a disconnected graph. In particular, cutsets of size 1 are bridges. CS_i is defined as the ratio of the number of cutsets of size i, from all possible edge sets of size i.
- Node Wiener impact $NWI(v)$ [13]: Let G be a biconnected graph. The Wiener impact of a vertex $v \in V(G)$ is defined as:

$$NWI(v) = W(G - v) - W(G) + T(v), \quad (3)$$

where $G - v$ refers to the graph obtained by removing v from G, T represents the transmission of a vertex, and W represents the Wiener index of a graph. The Wiener impact of v measures how much the total distances of the graph (in number of hops) are affected by removing v.

Algorithm 1 Variable Neighborhood Search

```
procedure VNS(S, t_max)
    S* ← S                              ▷ Best known solution
    t ← 1
    while some_criteria do
        S' ← Perturb(S*, t)
        S' ← LocalSearch(S')
        if S' better than S* then
            S* ← S'
            t ← 1
        else
            t ← t + 1
            if t > t_max then
                t ← 1
    return S*
```

B. Improving Robustness

To achieve some minimum level of robustness, a backbone network must not break in case of a single node or link failure, i.e., it must be biconnected. If such condition is not satisfied, as it is the case of REUNA network, link(s) must be added to the topology until achieve that.

The minimum number of edges l required for a given network to become biconnected (and an efficient algorithm to do that) can be found in [16]. Since these l additional links can usually be placed in different ways, there is room to do that while giving priority to optimizing a metric of interest. Moreover, besides the minimum number of links needed to get a biconnected network, any extra link added can potentially improve network robustness.

Next, we propose an algorithm to solve the problem of adding a restricted number of links to a given topology, in such a way that it becomes biconnected and minimizes each metric of interest among the ones discussed in Sec. III-A. The main goal is to find the best solution in the solutions set. In general, this is a NP-hard problem, and its mathematical formulation is fully described in [17].

1) Variable Neighborhood Search Meta-Heuristic: Our implemented algorithm uses the Variable Neighborhood Search meta-heuristic (VNS) [17], used in many fields to solve optimization problems. The basic algorithm works by periodically perturbing and (locally) searching for solutions, always keeping the best solution. A pseudo code is presented in Algorithm 1.

In our implementation, the initial solution is the original network more l additional links placed in such a way to form a biconnected network. A neighborhood of that initial solution is any biconnected network obtained by moving one of these l links from one node pair to a pair of non-adjacent nodes.

In the $LocalSearch$ sub-routine, such a link is moved until no improvement is possible in the measure being optimized. This solution is called local optima.

In each iteration, in order to save computational effort and explore better solutions (so the algorithm does not get stuck in local optima), the best known solution is perturbed ($Perturb$) by applying the transformation t successive times.

TABLE I

All Possible Solutions Obtained by Adding to REUNA Three Extra Links to Get a Biconnected Backbone

Solution ID	Extra links
REUNA+3(a)	(Valparaíso, ESO), (Arica, Pto. Montt), (AURA, ALMA)
REUNA+3(b)	(Valparaíso, ALMA), (Arica, Pto. Montt), (AURA, ESO)
REUNA+3(c)	(Valparaíso, ALMA), (Arica, AURA), (Pto. Montt, ESO)
REUNA+3(d)	(Valparaíso, Arica), (ALMA, AURA), (Pto. Montt, ESO)
REUNA+3(e)	(Valparaíso, ESO), (Arica, AURA), (Pto. Montt, ALMA)
REUNA+3(f)	(Valparaíso, Arica), (ESO, AURA), (Pto. Montt, ALMA)

TABLE II

Metric Values for REUNA, and Its Optimized Versions with 3 and 4 Extra Links (See Table I, and Fig. 3)

	$CS_1[\%]$	$CS_2[\%]$	$CS_3[\%]$	$max\{EBC\}$	$max\{NWI\}$
REUNA	57.89	90.06	100.00	81.00	∞
REUNA+3(a)	0	9.96	31.04	43.89	182
REUNA+3(b)	0	9.96	31.17	43.00	193
REUNA+3(c)	0	9.96	30.78	43.17	161
REUNA+3(d)	0	9.96	30.78	43.17	161
REUNA+3(e)	0	9.96	31.04	43.89	182
REUNA+3(f)	0	9.96	31.17	43.00	193
REUNA+4	0	6.32	19.93	34.33	63

That is, the $Perturb$ sub-routine moves one of those l links t successive times, always keeping biconnectivity, in order to get out of local optima.

To find the best solution, we aim to minimize each metric of interest, namely, maximum node Wiener impact ($max\{NWI\} = max_v\{NWI(v)\}$), maximum edge betweenness centrality ($max\{EBC\} = max_e\{EBC(e)\}$) and relative number of cutsets of size 2 (CS_2). We set $i = 2$ taking into consideration the computational costs involved.

IV. Results and Discussion

Due to the presence of brigdes and cut-points, as shown in Section II-B, REUNA network does not satisfy the minimum level of robustness required for backbones. Thus, link(s) must be added to REUNA until it achieve biconnectivity. In REUNA, the minimum number of extra links needed is three, connecting the three node pairs of unitary degree ($d(v) = 1$). There are six possible ways to connect these node pairs to form biconnected graphs, as shown in Table I.

Table II presents the values of the selected metrics for each possible solution, and also for the original topology. As one can see, these values were significantly reduced for all solutions.

To assess how much REUNA may be upgraded using four instead of three extra links, our VNS implementation was applied for minimizing each of the selected metrics. The results are shown in Fig. 2. To enable a quality evaluation of the VNS's results, Fig. 2 also shows brute force results.

Notice in Fig. 2 that a solution achieving global minimum for one metric will not necessarily be the same solution with global minimum of another metric. The same metrics values (and also the same solutions) were obtained by minimizing CS_2 and $max\{EBC\}$, which is probably due to the algorithm being stuck in a local optima. The largest gains occurred when minimizing the node Wiener impact, i.e., minimizing $max\{NWI\}$ leads to better values for all other metrics, compared to the solutions minimizing CS_2 and $max\{EBC\}$.

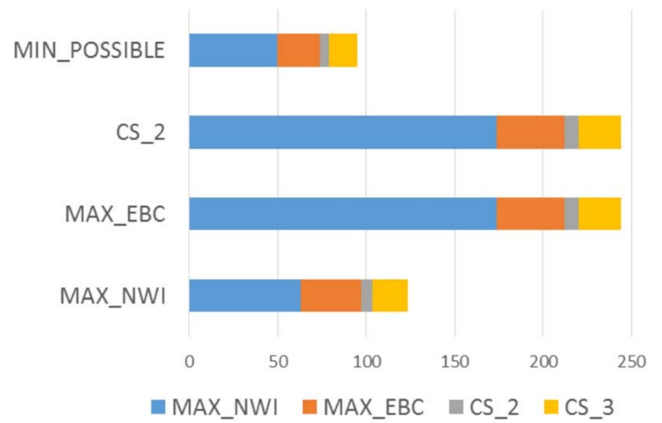

Fig. 2. Each solution performance according to each selected metric. Minimum possible (MIN_POSSIBLE) refers to the global minimum for each metric, found by brute force (not applicable for large networks).

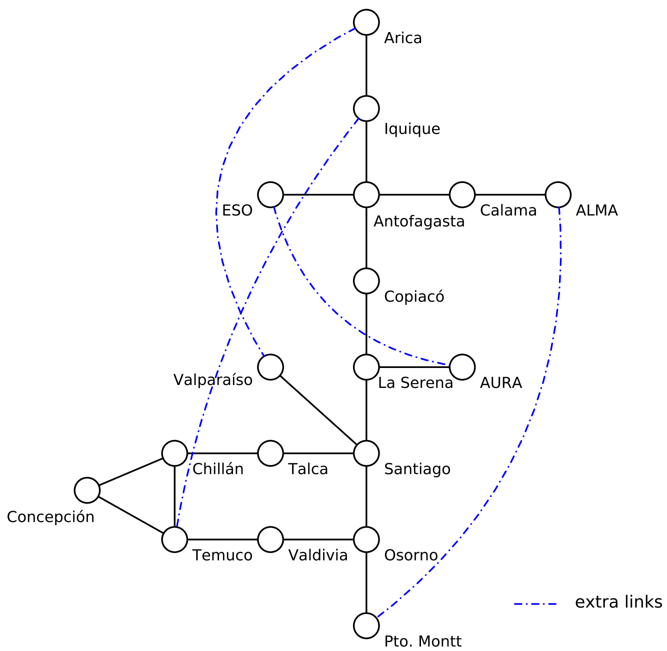

Fig. 3. Best solution found with 4 additional links, minimizing the maximum node Wiener impact.

Figure 3 presents the best solution found, that is denoted as REUNA+4. It is interesting to notice that this solution is close to solution REUNA+3(f) shown in Table I, which is one of the worst ones in the first set of solutions. The values of the selected metrics for REUNA+4 are presented in Table II. As one can see, at the cost of adding one extra link to REUNA+3(f), a significant improvement is achieved on each robustness metric. For instance, a reduction of 67% is observed for the maximum node Wiener impact.

V. Conclusion

In this work we proposed a Variable Neighborhood Search heuristic to improve backbone networks by adding a few well placed links. The heuristic optimizes a metric of interest, ensuring as a minimum robustness requirement the network remains connected after any single node or link failure. Three metrics were selected for the tests: edge betweenness centrality, number of link cutsets and node Wiener impact, and the best results were obtained by optimizing the last one.

As a case of study, results are presented for REUNA, a real-world Chilean internet backbone, considering three and four extra links. In REUNA, the addition of three links is required to achieve biconnectivity, whereas a fourth link leads to a significant improvement on all analyzed metrics. For instance, a reduction of 67% was obtained for the maximum node Wiener impact.

For future work, we aim to consider a trade-off between robustness and cost, taking into account geographical distances and other infrastructure network costs.

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