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Change of support using non-additive variables with Gibbs Sampler: Application to metallurgical recovery of sulphide ores

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ABSTRACT

Flotation tests at laboratory scale describe the metallurgical behavior of the minerals that will be processed in the operational plant. This material is generally composed of ore and gangue minerals. These tests are usually scarce, expensive and sampled in large supports. This research proposes a methodology for the geostatistical modelling of metallurgical recovery, covering the change of support problems through additive auxiliary variables. The methodology consists of simulating these auxiliary variables using a Gibbs Sampler in order to infer the behavior of samples with smaller supports. This allows downscaling a large sample measurement into smaller ones, reproducing the variability at different scales considering the physical restrictions of additivity balance of the metallurgical recovery process. As a consequence, it is possible to apply conventional multivariate geostatistical tools to data at different supports, such as multivariable exploratory analysis, calculation of cross-variograms, multivariate estimations, among others. The methodology was tested using a drillhole database from an ore deposit, modelling recovery at a smaller support than that of the metallurgical tests. The support allowed for the use of the geochemical database, to consistently model the metal content in the feed and in the concentrate, in order to obtain a valid recovery model. Results show that downscaling the composite size reduces smoothing in the final model.

1. Introduction

The sample support of drillholes (whether geochemical grades, geological logging, metallurgical testing, etc.) is often different. In the case of geochemical grades, the drillholes are composited to a constant length to perform conventional statistical and geostatistical analyses, for example exploratory data analysis, variogram analysis, estimation or simulation (eg. (Chiles and Delfiner, 2012), (Deutsch and Journel, 1998), (Goovaerts, 1997), (Isaaks and Srivastava, 1989)). In the case of metallurgical variables, compositing methods can cause problems of statistical bias given the non-additive nature of these variables (Carrasco et al., 2008). In the particular case of the metallurgical recovery in the flotation process, it is calculated as the ratio between the mass of metal in the concentrate and the mass of metal in the feed (Mular and Barratt, 2002). The metal of the concentrate and feed are additive variables from a statistical point of view (mass properties), but the recovery is not (Carrasco et al., 2008).

Flotation tests are usually performed to describe the behavior of a mineral in the metallurgical process plant. The metallurgical response will depend mainly on two factors: the geological properties of the

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[2] Contribution: Reviewed and improved the methodology, extensively reviewed the paper and contributed to the writing of the article.
[3] Contribution: Worked on implementation of the methodology.
[4] Contribution: Provided insight about mineral processing and reviewed the paper.
[5] Contribution: Provided insight about the geological aspects and reviewed the paper.
[6] Contribution: Contributed with the implementation and reviewed the paper.

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material and the operational parameters. There are different studies where the relations between geology and the metallurgical response are observed (Garrido et al., 2016), (Hu et al., 2009), (Hunt et al., 2011), (Lund and Lamberg, 2014), (Solozhenkin et al., 2016), (Haga et al., 2012). Clay minerals associated with alteration negatively affect the process to recover copper (Bulatovic et al., 1999). Faced with these materials, different operational responses can increase recovery by improving the effectiveness of the process. Other sulphide minerals such as pyrite also negatively affect the copper recovery (Mular and Barratt, 2002). Usual recovery values range from 90% to 95%, but in the face of these geological variations the recovery may decrease to 80% or less (Metso, 2006).

Linear regression models are used to estimate the recovery of an ore body (Weisberg, 2005). This methodology requires to find statistical correlations between recovery and other variables such as total copper, solubility ratio, analytical acid consumption, etc. Gaussian simulations have been used to model metallurgical parameters since they do not assume additivity of the study variable (Deutsch et al., 2015). The methodology proposed in this research allows estimating/simulating the recovery using samples of variable length/support, maintaining basic statistics, the spatial variability and the physical conditions or restrictions associated to the problem of additivity. The methodology can be easily applied or adapted to other cases related to geometallurgical performance.

Geostatistical modelling needs defining the estimation units (domains) on which the study is being performed (Hunt et al., 2014). These domains assume a constant statistical behavior of the variable within the entire volume (second order stationarity) (Matheron, 1973).

In the case of recovery, domains are established on geological and metallurgical information. Within these domains, variables are estimated or simulated using classical statistical and geostatistical algorithms such as multivariate regression and Gaussian simulation (Deutsch, 2016). If the geological characteristics are constant in two flotation tests, then the metallurgical response must have the same behavior in both tests without making operational changes. This hypothesis is debatable given the high variability of geological conditions in the ore body, and many of these geometallurgical relationships have not been exhaustively described yet.

Another complication associated with modelling the recovery is the scaling problem going from laboratory small scale test to production volumes (Truter, 2010). Many models have been created based on these tests (Boisvert et al., 2013), (Coward and Dowd, 2015). The models generated based on laboratory tests are used to determine the expected behavior in the processing plant, and must be over-dimensioned on an industrial scale (Suazo et al., 2009). Laboratory tests are performed as a batch process. On the other hand, in industrial applications flotation is performed through a continuous flow (serial and parallel Rougher, Scavenger and Cleaner cells) (Mular and Barratt, 2002). These and other problems complicate the correct modelling for the prediction of the geometallurgical variables.

This research deals with the issue of downscaling the support of geometallurgy tests done at the laboratory to match the support of geochemical and mineralogical composites. Downscaling methodologies have been developed by different authors, where change of support accounts for the statistical consequences (Pardo-Iguzguiza et al., 2006) (Tran et al., 1999) (Deutsch, 2016). This article covers the problem of downscaling integrating different sources of information through auxiliary variables (in the case of the flotation of sulphur minerals, the metal in feed W_f and metal in concentrate W_c explained in section 2 Methodology. It is based on the Gibbs Sampler (Geman and Geman, 1984) and allows to estimate/simulate samples on a smaller support consistent with the original data reproducing their basic statistics, spatial variability and constraints associated to the nature of the variable. The article is explained through a simulated synthetic case and applied to a case study (exploratory drillholes).

The objective of applying this methodology is to reduce the support of the metallurgical recovery variable in order to facilitate the application of conventional geostatistics tools. The upscaling procedure is not considered in this article because it generates a decrease in the variance of data and, consequently, predictive models with low resolution. The objective of this research is the assimilation of small support samples (eg, geochemical variables) with large support samples (eg, geometallurgical variables) to generate high resolution models. To find multivariate correlations usual statistical tools are correlation coefficients, principal component analysis, cross variograms, scatter plots, etc. (Wackernagel, 2003). These tools require that the data be collocated (all variables measured in the same sample (Chiles and Delfiner, 2012),) and at the same support, a condition that can be achieved through this methodology without losing resolution of the local variability. The proposed methodology is built on the hypothesis that specific geometallurgical parameters and hence mineral behavior in processing are a function of geological/mineralogical properties. These properties may be identified on a much smaller scale, hence improving resolution of geometallurgical properties and models.

2. Methodology

Samples with geological information are usually measured on supports different from the metallurgical tests. Metallurgical tests require much larger sample volumes, in order to analyze the different attributes and their operational characteristic ranges. To integrate geological and metallurgical information, it is convenient to change the variable different supports to a standardized support for all measurements. For non-additive categorical variables (e.g. lithology or mineralogy code), the majority code can be assigned to the composite or the sample code located in its center. In the case of continuous variables, there are different tools to increase or decrease the sampling support:

- Composite: method of up-scaling, is based on averaging an attribute based on the sampling lengths. This is not recommended for non-additive variables because it biases the result by using a linear average.
- Gibbs sampling: down-scaling method, allows to simulate samples with higher sampling density by scaling basic statistics and spatial continuity. It also allows to consider mathematical restrictions in the simulation.

A schematic example of down-scaling with 3 samples (Z_1, Z_2 and Z_3) is shown in Fig. 1.

The value of Z_1 with support of 3 m is downscaled to the values of z_{11}, z_{12} and z_{13} with supports of 1 m respectively. These values are related through $f(\cdot)$ which represents the physical constraints associated with metallurgical processes, for example mass balance. The change of support implementation considers the following aspects:

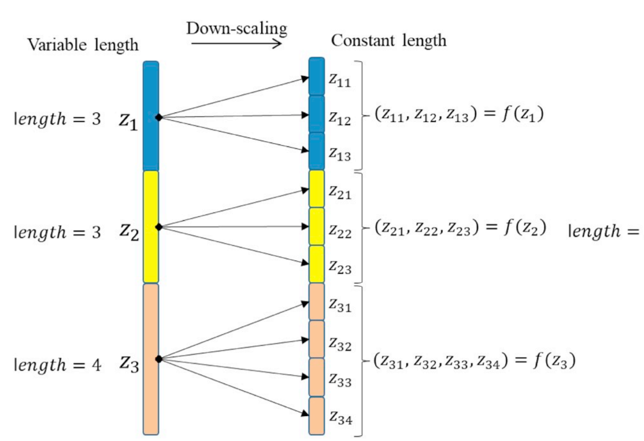

Fig. 1. Diagram of down-scaling for 3 samples Z_1, Z_2 and Z_3.

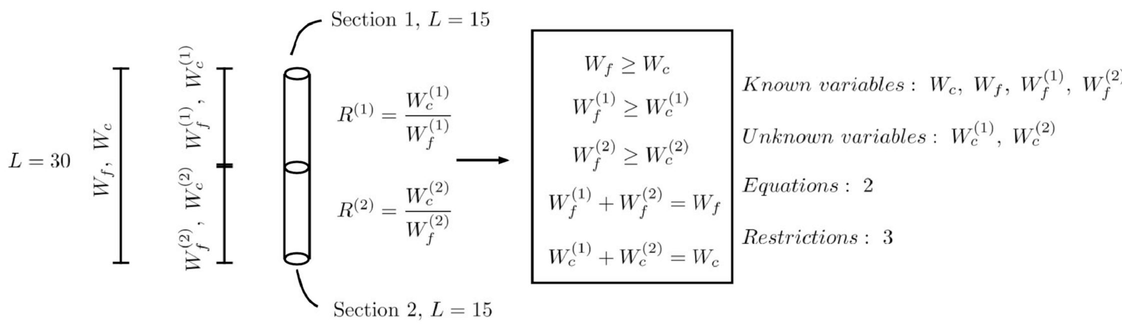

Fig. 2. Diagram of variables, equations and physical restrictions of the change of support problem for recovery.

1. Scaling of basic statistics.
2. Scaling of spatial variability (smaller support implies increased variability).
3. Integration of measured geochemical variables to small support.
4. Physical constraints of the problem (inequalities and relationships).

The methodology is now presented for the particular case of separating a composite of length L into 2 composites of length $L/2$ subject to the conditions described above. For the composite of length L, let W_f be the mass of metal in the feed and W_c the mass of metal in the concentrate. Fig. 2 shows a diagram depicting the known variables, equations and constraints of the problem.

$W_f^{(1)}$ and $W_f^{(2)}$ are the mass of metal in the feed from the top and bottom halves of the original composite. Similarly, $W_c^{(1)}$ and $W_c^{(2)}$ are the mass of metal in the concentrate. $R^{(1)}$ and $R^{(2)}$ are the corresponding recoveries, which are not additive variables:

$$R \neq \frac{R^{(1)} + R^{(2)}}{2} \tag{1}$$

$$\frac{W_c}{W_f} \neq \frac{\left(\frac{W_c}{W_f}\right)^{(1)} + \left(\frac{W_c}{W_f}\right)^{(2)}}{2} \tag{2}$$

Equation (2) is the expanded form of equation (1). Under unusual conditions, the equality of equation (1) can be met, for example when the sample has homogeneous behavior, i.e., $R^{(1)} = R^{(2)} = R$. The variables W_f and W_c are additive variables (mass). The methodology uses these variables to calculate R in each sub-composite. The methodology consists of the following steps:

1. **Transformation.** Given a set of data at support L, transform W_f and W_c independently to standard Gaussian distributions y_f and y_c, respectively. The anamorphosis functions are given by the equations:

$$y_f(x) = \phi_f(W_f(x)) \tag{3}$$

$$y_c(x) = \phi_c(W_c(x)) \tag{4}$$

where $W_f(x)$ and $W_c(x)$ are the mass of the metal in the feed and in the concentrate, respectively, from composites at support L. ϕ_f and ϕ_c are the transformation functions (anamorphosis) and $y_f(x)$ and $y_c(x)$ are the transformed variables (also at support L). These variables are distributed as Gaussian distributions with mean of 0.0 and variance of 1.0.

2. **Variogram analysis.** The variograms of the transformed variables $y_f(x)$ and $y_c(x)$ are calculated and modelled to capture their spatial continuity and anisotropy. The variogram models obtained for the transformed variables at support L are used to simulate the same variables at support $L/2$. This is a reasonable assumption when the nugget effect is small, considering that the variance is normalized to 1.0, since we are considering the normal scores and we can assume the shape of the variogram does not change significantly. The variance reduction is corrected after back transformation, as explained later, to account for the change of support. In cases of larger nugget effect, the variable at support $L/2$ can be simulated with a variogram that includes the increase on the relative nugget effect, as computed using the variogram scaling approach (see, for example (Chiles and Delfiner, 2012)).

3. **Simulation at support $L/2$ using a Gibbs sampler.** The Gibbs sampler to obtain the downscaled values of $W_f(x)$ and $W_c(x)$, specifically the values $W_f^{(1)}(x)$, $W_f^{(2)}(x)$, $W_c^{(1)}(x)$ and $W_c^{(2)}(x)$, which represent the mass at support $L/2$, is implemented as follows (we illustrate the process for $W_f^{(1)}$, but the four variables are simulated at every location in order to check compliance with the constraints):

(a) Every location where a sample at support L exists is divided into two simulation locations, representing the two downscaled values at support $L/2$ (Fig. 3).

4. Backtransformation to calculate $W_c^{(1)sim}$, $W_f^{(1)sim}$, $W_c^{(2)sim}$ and. $W_f^{(2)sim}$

5. Calculation of the recovery at support $L/2$ through the empirical formula of metallurgical recovery, for each sub-composite:

$$R^{(1)sim} = \frac{W_c^{(1)sim}}{W_f^{(1)sim}} \tag{9}$$

$$R^{(2)sim} = \frac{W_c^{(2)sim}}{W_f^{(2)sim}} \tag{10}$$

The condition $0 \leq R \leq 1$ is verified by the rejection conditions imposed earlier.

Notice that the back transformations $\phi_{c(sub)}^{-1}$ and $\phi_{f(sub)}^{-1}$ account for the variance increase due to the smaller support of sub-composites. An affine correction is applied over the distribution of the original composites, to account for the new support. In the case of small variance reductions, an affine correction will do well. If larger variance corrections are needed, a different model such as the Discrete Gaussian model could be used, although this does not change the suggested approach. The variance increase (used in the affine correction for the back transformation, step 4) is calculated using classic variance-support relationships (for more information see (Chiles and Delfiner, 2012)). In particular, the scaling of the variance is given by the following relation:

$$C(0) = C(V, V) + \bar{\gamma}(V, V) \tag{11}$$

Where $C(V, V) = \gamma_V(\infty)$ is the sill (or modelled variance) of the variogram at support L before transformation to Gaussian units, $C(0) = \gamma(\infty)$ is the sill of the variogram at point support and $\bar{\gamma}(V, V)$ is the average variogram value of vectors defined within the volume V. It is given by the following relation:

$$\bar{\gamma}(V, V) = \frac{1}{|V|^2} \int_V \int_V \gamma(x - x')dxdx' \tag{12}$$

In our case, it is easy to show that:

$$\sigma_L^2 = \sigma_{L/2}^2 - (\bar{\gamma}(L, L) - \bar{\gamma}(L/2, L/2)) \tag{13}$$

The relationship can be graphically observed in Fig. 4.Where σ^2 is

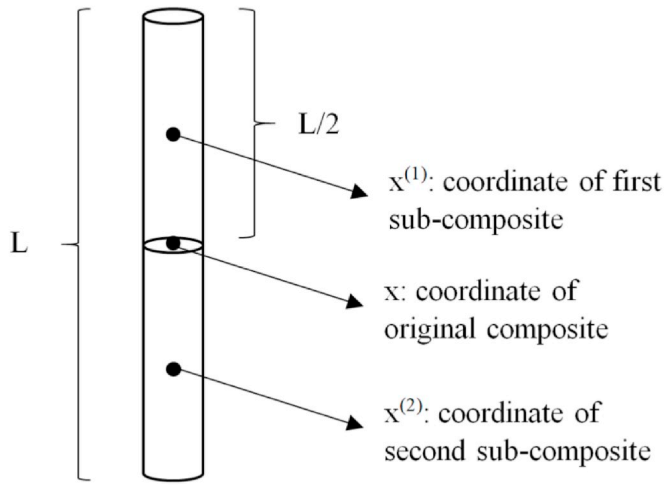

Fig. 3. Support L division to two sub-composites $L/2$.

(b) Downscaled simulation locations are visited in a random order.

(c) At every simulation location, perform simple kriging of the sub-composites previously simulated to determine the mean and variance of the conditional distribution at a new sub-composite location.

$$y_f^{(1)*}(x^{(1)}) = \sum_{i=1}^{n} \lambda_i \cdot y_f^{(1)}(x_i) \tag{5}$$

$$\sigma_{SK}^2(x^{(1)}) = 1.0 - \sum_{i=1}^{n} \lambda_i \cdot C(x_i - x^{(1)}) \tag{6}$$

$$y_c^{(1)*}(x^{(1)}) = \sum_{i=1}^{n} \lambda_i \cdot y_c^{(1)}(x_i) \tag{7}$$

$$\sigma_{SK}^2(x^{(1)}) = 1.0 - \sum_{i=1}^{n} \lambda_i \cdot C(x_i - x^{(1)}) \tag{8}$$

(d) Simulate $y_f^{(1)sim}(x^{(1)})$ and $y_c^{(1)sim}(x^{(1)})$ by Monte Carlo simulation, from the Gaussian conditional distribution with mean $y_f^{(1)*}(x^{(1)})$, and variance $\sigma_{SK}^2(x^{(1)})$ for $y_f^{(1)sim}(x^{(1)})$ and with mean $y_c^{(1)*}(x^{(1)})$, and variance $\sigma_{SK}^2(x^{(1)})$ for $y_c^{(1)sim}(x^{(1)})$.

(e) $y_f^{(1)sim}$ is back-transformed to get $W_f^{(1)sim}$ and $y_c^{(1)sim}$ is back-transformed to get $W_c^{(1)sim}$.

(f) Test 1: rejection condition $W_c^{(1)sim}(x^{(1)}) \geq W_f^{(1)sim}(x^{(1)})$, where $W_c^{(1)sim}(x^{(1)}) = \phi_{c(sub)}^{-1}(y_c(x^{(1)}))$ and $W_f^{(1)sim}(x^{(1)}) = \phi_{f(sub)}^{-1}(y_f(x^{(1)}))$. If rejected, return to (*d*) and resimulate $y_f^{(1)sim}(x^{(1)})$ and $y_c^{(1)sim}(x^{(1)})$.

(g) Calculate $W_f^{(2)sim}(x^{(2)}) = W_f(x) - W_f^{(1)sim}(x^{(1)})$ and $W_c^{(2)sim}(x^{(2)}) = W_c(x) - W_c^{(1)sim}(x^{(1)})$.

(h) Test 2: rejection condition $W_c^{(2)sim}(x^{(2)}) \geq W_f^{(2)sim}(x^{(2)})$, where $W_c^{(2)sim}(x^{(2)}) = \phi_{c(sub)}^{-1}(y_c(x^{(2)}))$ and $W_f^{(2)sim}(x^{(2)}) = \phi_{f(sub)}^{-1}(y_f(x^{(2)}))$. If rejected, return to (*d*) and resimulate $y_f^{(1)sim}(x^{(1)})$ and $y_c^{(1)sim}(x^{(1)})$.

(i) Once simulated values are accepted, add to conditioning information and go back to (*c*) until all nodes have been simulated.

the variance of the data W_c at point support (unknown), σ_L^2 variance at support L and $\sigma_{L/2}^2$ variance at support $L/2$. More details in (Chiles and Delfiner, 2012).

Rejection tests (conditions of inequality) are based on the physical constraints in the recovery calculation. It can be observed that with more rejection conditions, the variance of the data simulated at support $L/2$ increases. Simulation using Gibbs sampler has been used in other conditional simulation methodologies based on random fields with Gaussian distribution (Geman and Geman, 1984). The simulation is done sequentially (Gomez-Hernandez et al., 1993).

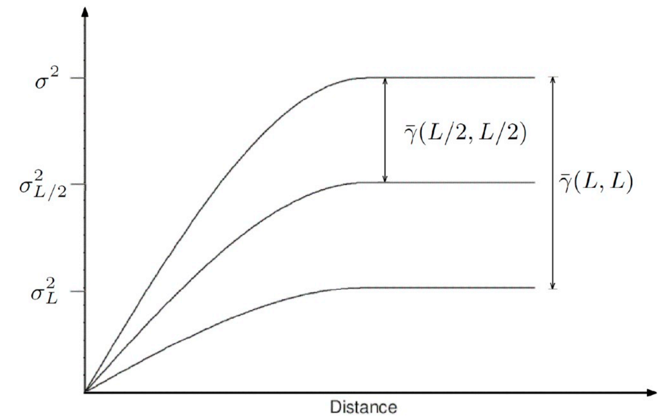

Fig. 4. Schematic relationship between variances of the same variable at different supports.

3. Synthetic case analysis

The change of support methodology was applied to a basic case study in 2D. Statistical validations are presented.

3.1. Synthetic case study

Copper grades and metallurgical recovery values were simulated in 10 drillholes. The support used is $L = 30\ m$. Fig. 5 shows the simulated values, the grade distribution of W_c and the variogram of this variable transformed to Gaussian scores.

The 30 m composites are uniformly spaced every 150 m in the horizontal and every 30 m in the vertical direction. The distribution of mass of metal concentrate is log-normal with a coefficient of variation of 50%. The experimental variogram is calculated on the Gaussian values of the variable W_c at support of 30 m. The distribution of W_f is known at support 30 m. From metallurgical tests, the recoveries R are known over 30 m samples. Thus, W_c can be inferred.

We apply the methodology previously described to obtain W_c and W_f in sub-composites (support 15 m) at composite locations (Fig. 6).

3.2. Statistical validations

Using the copper grade (W_f in mass) and metallurgical recovery, the mass of recovered ore (W_c in mass) was calculated. Fig. 7 (A) shows the statistical reproduction of W_c at a support of 15 m and (B and C) shows the multivariate reproduction of the relation $W_c \leq W_f$.

The quantile-quantile comparison of the distributions of W_c using supports of L and $L/2$ is shown in Fig. 7, (A). The number of data points was doubled, the mean of the data remained constant (W_c additive variable) and the standard deviation increased (the decrease in support implies an increase in the variance of the data). The relation $W_c \leq W_f$ with support at $L/2$ remained similar in comparison to this relation with support L. A slight decrease in the correlation coefficient is observed due to the increased variance at smaller support.

Estimated W_c was plotted at support $L = 30\ m$ and $L = 15\ m$ for two of the drillholes (Fig. 8). In Fig. 9 the increase in the variance that entails the decrease of the support can be observed. From 100 simulations the variogram reproduction at support of $L = 15$ m was checked in Fig. 9.

4. Case study: application to drillhole samples

The following case study corresponds to a campaign with 50 real drillholes where some samples have been selected for flotation analysis calculating the metallurgical recovery of copper in sulphide minerals. The samples for the flotation rougher test have different lengths with a

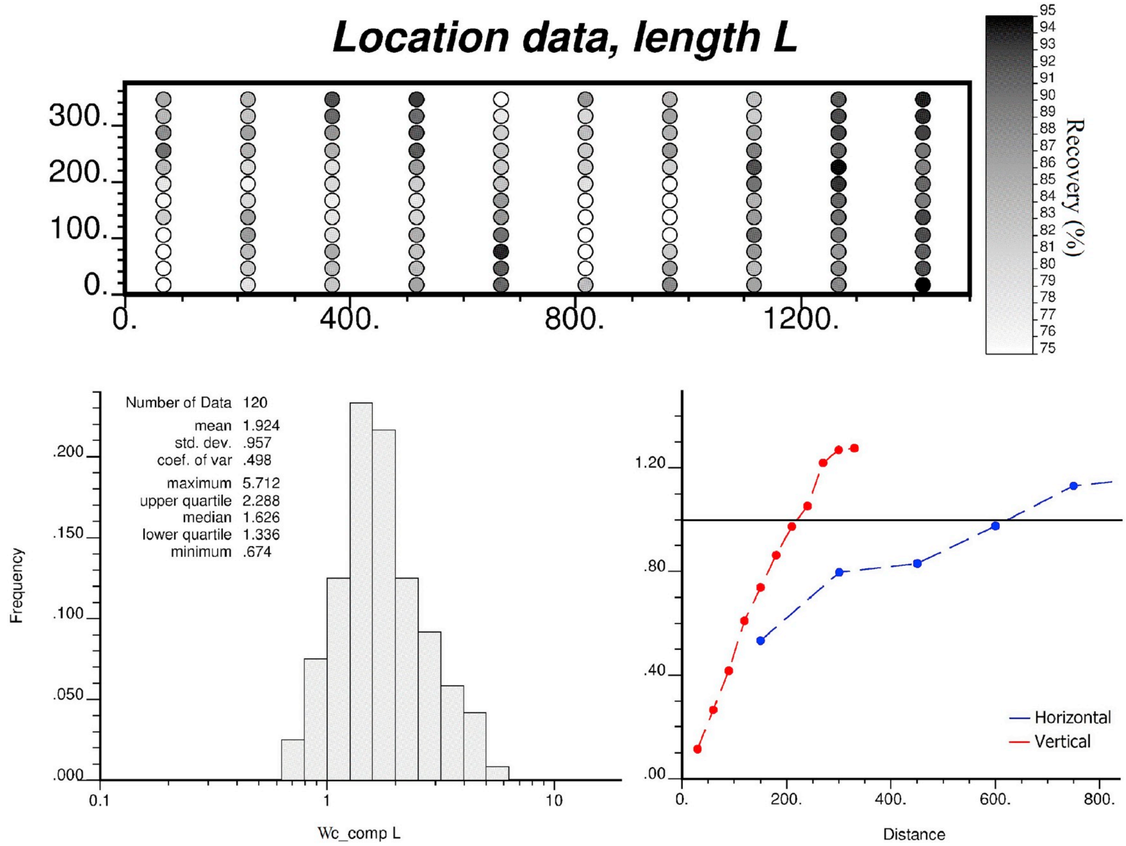

Fig. 5. Simulated case study, 10 drillholes with their statistical distribution and normal score variogram for mass of metal in concentrate.

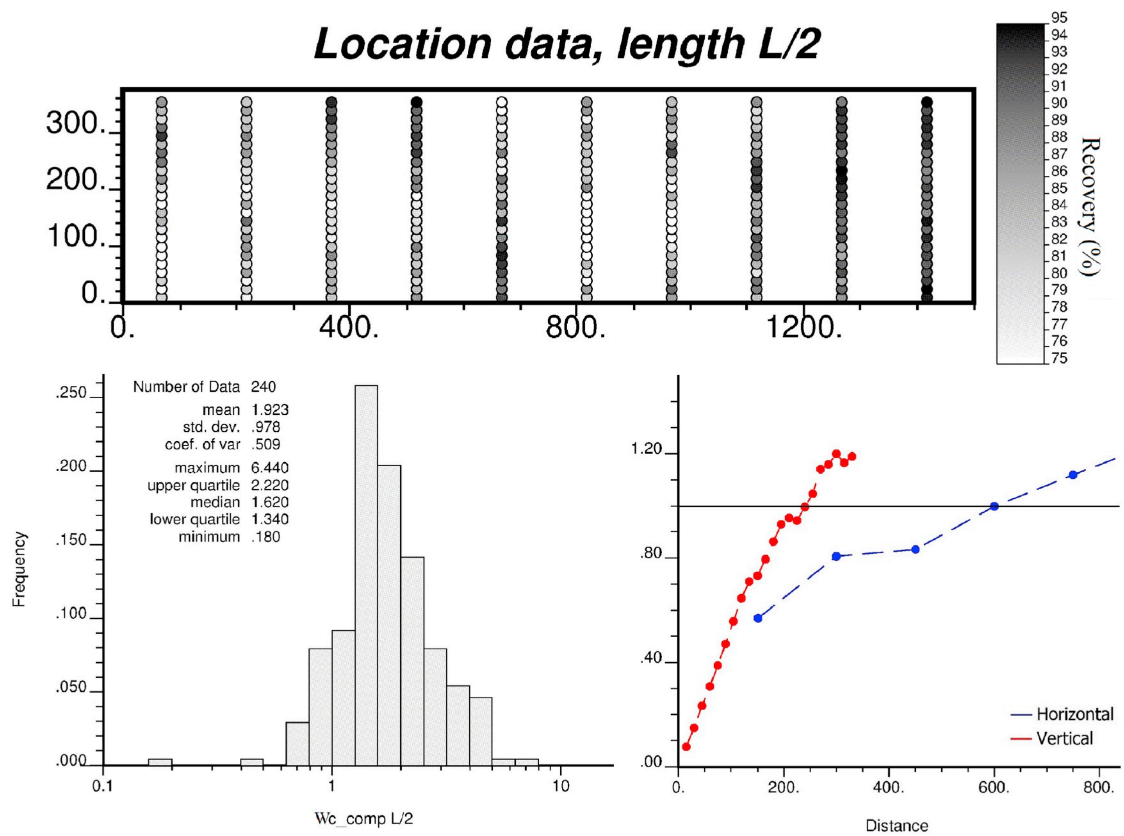

Fig. 6. Resulting simulated 15 m composites, their histogram and normal score variogram (only 1 simulation is shown).

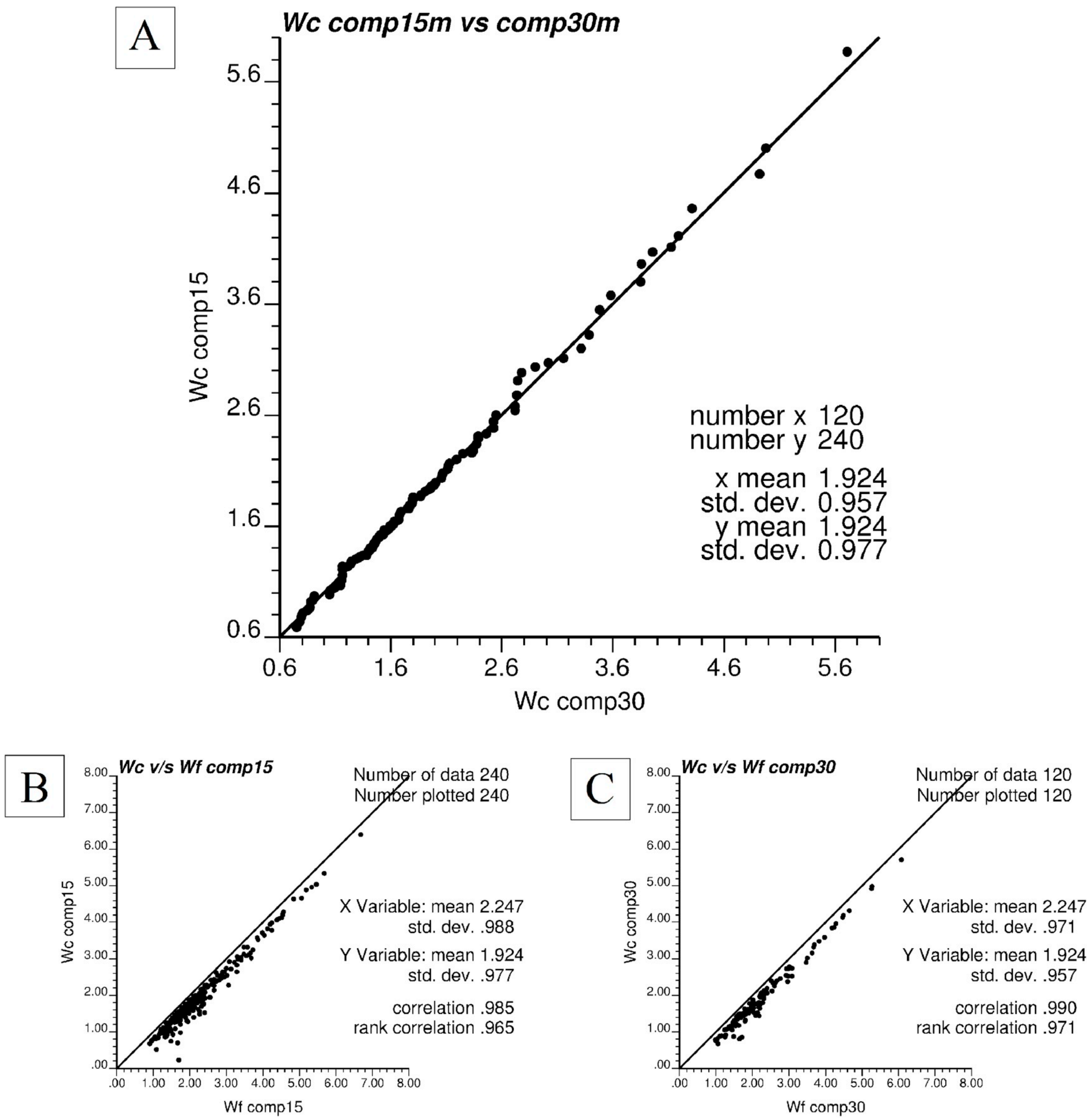

Fig. 7. Statistical comparison of W_c and bivariate relations $W_c \leq W_f$ using support of L and $L/2$.

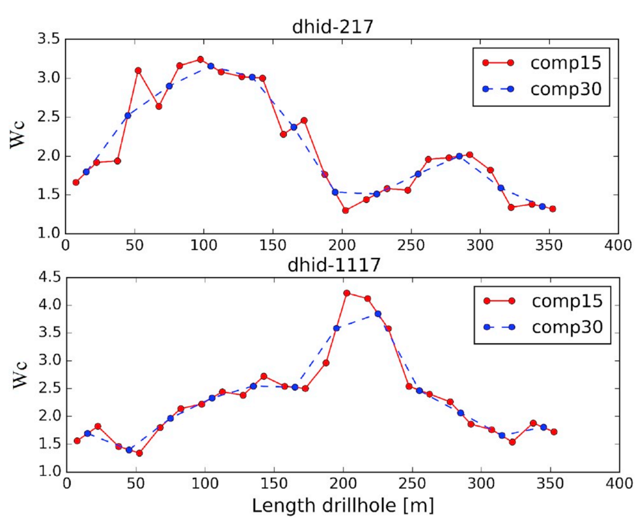

Fig. 8. Reproduction of grades from two drillholes, length $L = 15\ m$ and $L = 30\ m$.

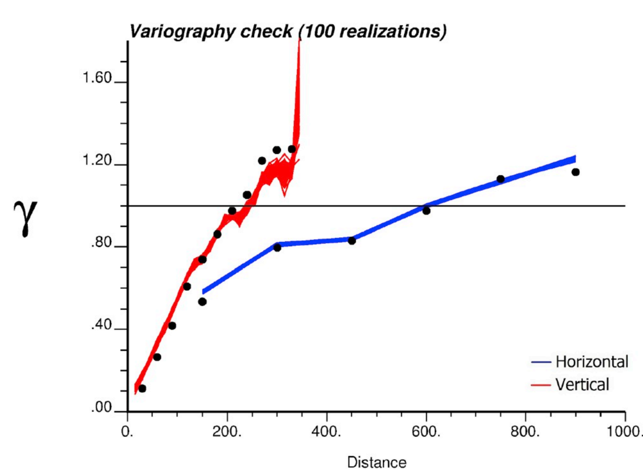

Fig. 9. Reproduction of spatial variability.

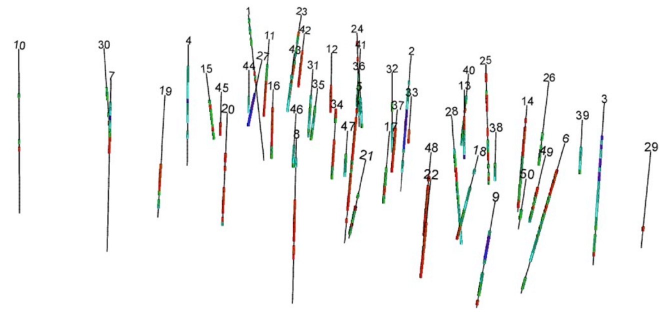

Fig. 10. Graphical display using metallurgical recovery samples.

mean of 40 m. As a simplification (given the low variability in sample length) these have been regularized to a nominal length of 40 m. The samples have similar geological conditions, and are part of the same geometallurgical domain. Fig. 10 shows a graphical display of the samples selected for the flotation analysis at laboratory scale.

The copper grades have been composited at a nominal length of 20 m (block size of the estimation model) using the proposed methodology. 99 simulations using the Gibbs sampler were performed at the 20 m support. For each realization, the cooper recovery in each block of the model was calculated as ratio between the sum of W_c divided by the sum of W_f and the average/uncertainty expected were calculated. The respective validations were done obtaining satisfactory results from a statistical point of view.

Fig. 11 shows the quantile-quantile comparison for W_c using 20 and 40 m composite length. The number of composites was doubled, the mean remained constant and the variance increased as expected. The graphs (B) and (C) show the bivariate relations between W_f and W_c to the supports of 20 and 40 m, evidencing notorious similarities and conservation of the behavior $W_c \leq W_f$.

The next step is to estimate metallurgical recovery in the block model with 40 m composites (conventional methodology) and using the 20 m composites obtained with the proposed methodology.

The estimates from 20 m composites show greater variability. Smoothing can be observed in Fig. 12. Fig. 13 shows a histogram of the block-to-block estimation difference between estimated recovery with 20 m and 40 m composites.

The mean difference is close to 0.0 (-0.073) with a standard deviation of 0.9. Approximately 1% of blocks is estimated with a bias of 3% (red box in histogram). This is explained by the difference of the change of support using a non-additive variable. This bias is not statistically significant with respect to the total of the blocks but locally it can generate important differences on the ore which is recovered in short-term planning.

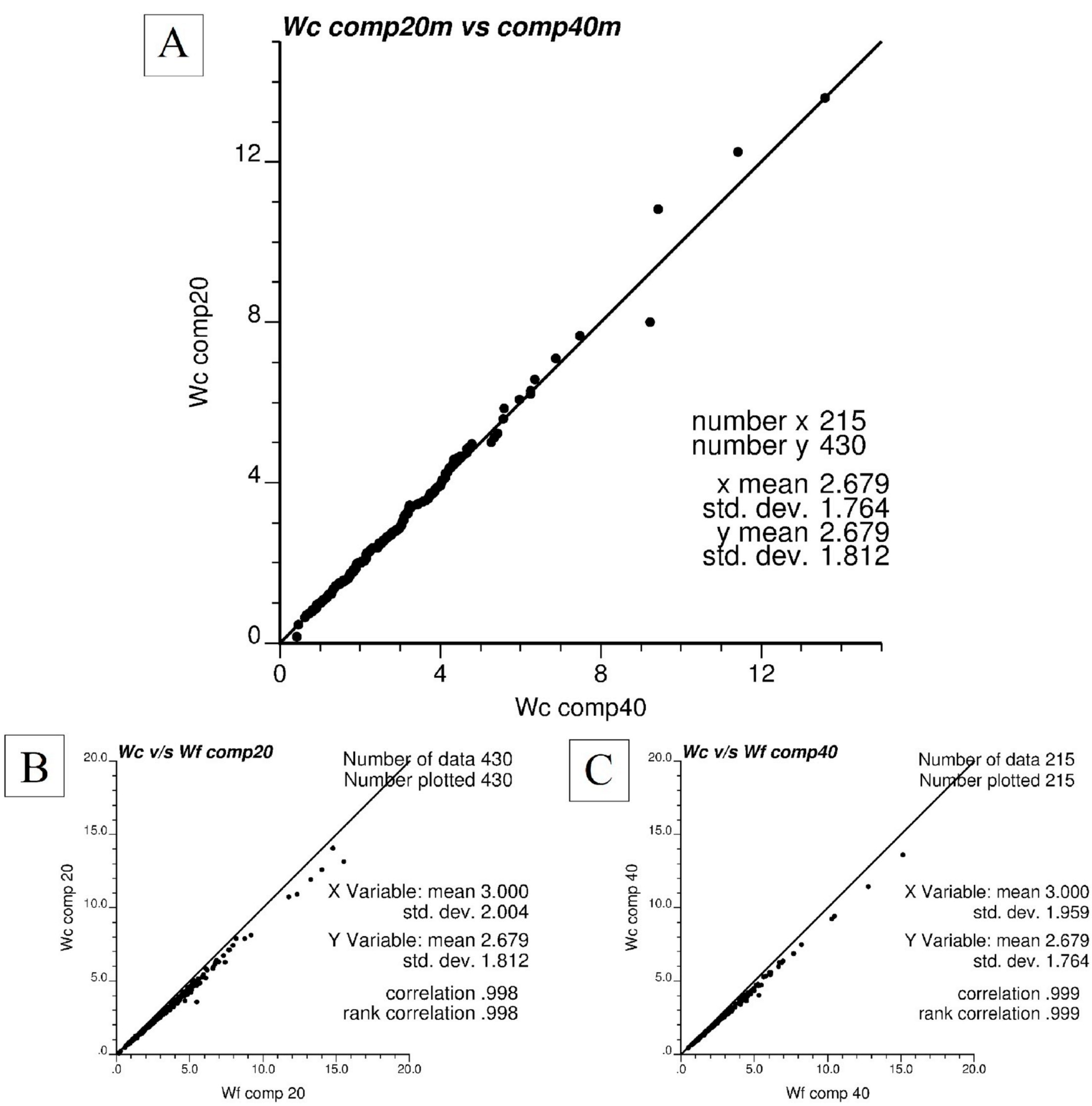

Fig. 11. Statistical validation of composites at 40 m–20 m.

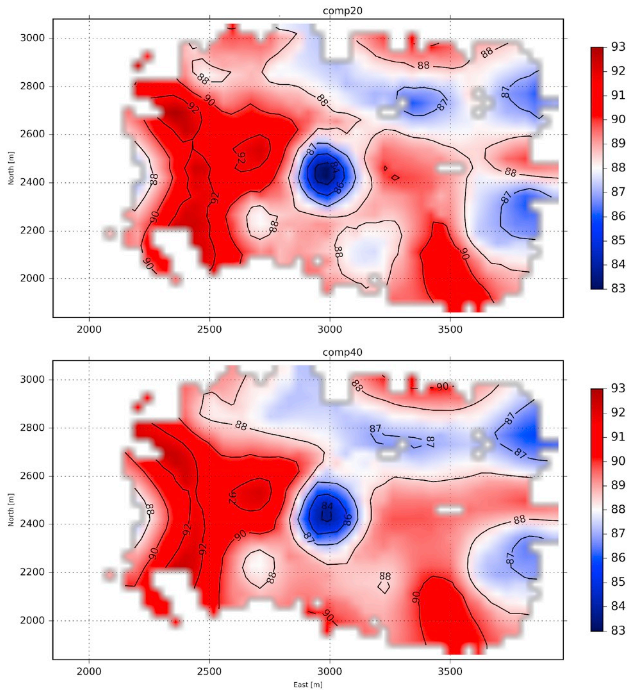

Fig. 12. (Left) Estimation of metallurgical recovery using composites of 20 m. (Right) Estimation using composites of 40 m.

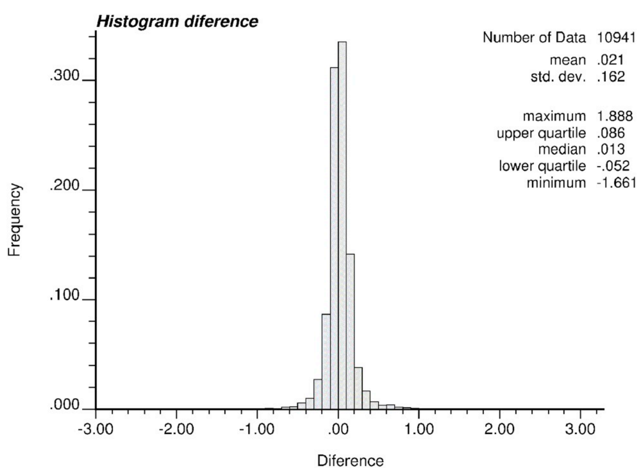

Fig. 13. Histogram of diference between estimated recovery using composites 40 m and E-Type of simulation.

Fig. 14 shows a scatter plot between estimated values with 20 m and 40 m composites. Fig. 14 highlights the area with metallurgical recovery lower than 80% These values may generate operational problems at the metallurgical process plant. The estimation using a support of 40 m does not capture their range due to smoothing. This could be used as an alarm from a predictive point of view to apply operational modifications to the treatment of this material.

This result shows the difference in the estimate considering the size of the sampling support. This affects the resolution of a predictive model. The estimate with a larger support will not capture extreme data (high or low metallurgical recovery) that are usually important results from an operational point of view. Therefore reducing the sampling support to perform an estimation or simulation allows generating models of better resolution to capture small-scale variability, which may help improving the performance of the mining project.

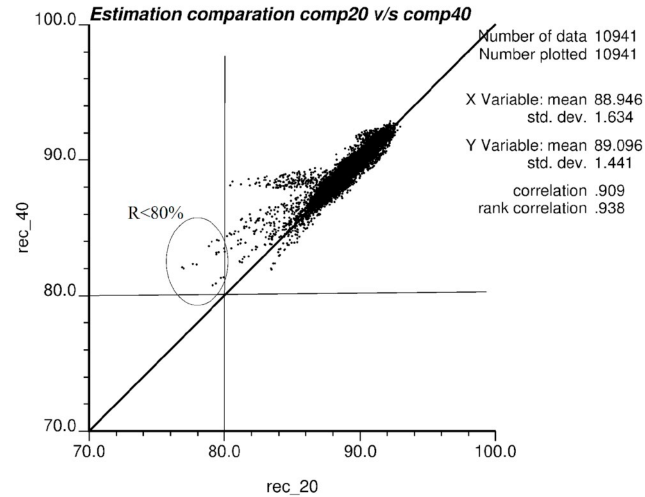

Fig. 14. Cross-validation between estimated recovery using composites of 20 and 40 m.

5. Conclusions

Modelling geometallurgical variables often causes problems when conventional geostatistics tools are applied. The main causes are the condition of heterotopic sampling and differences in the measurement supports. This article provides a new compositing approach based on simulations through a Gibbs Sampler. Statistical and geostatistical characteristics are preserved with this methodology: global mean, spatial variability and bivariate relations. The simulated values at smaller support can be used as input for simulations, in order to account for the uncertainty stemming from the variability of small support samples, or they can be averaged for estimation purposes, as shown in the case studies presented in this paper.

In this article, two advantages of the method were highlighted:

– Reduction of support allows data assimilation (geological samples and metallurgical tests) for the application of conventional geostatistics tools; for example search of multivariable correlations for generation of geometallurgical predictive models.
– Reduction of support allows generating estimation models or simulation models with higher resolution and less smoothing. These models allow the description of local variability on a smaller scale, identifying extreme value zones that are important from a metallurgical point of view.
– The expected metal of the simulation is 28,263 ± 870 tons of Cu. If the metal is calculated based on an estimate with 40 m support, the result is 28,484 tons of Cu.

The advantages of estimating metallurgical recovery using this compositing methodology was illustrated through a case study. The results were compared with the traditional methodology to estimate recovery, with important local biases that can generate operational problems in the mineral processing plant.

Reducing the size of the composite is associated with an increase in local variability that was captured in the estimation. This information is captured in the methodology through scaling the variance of the distribution. The methodology was applied to metallurgical recovery data that fulfil the physical conditions associated to the non-additivity of this variable.

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