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Bonferroni means with induced ordered weighted average operators

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Abstract
The induced ordered weighted average is an averaging aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. This paper presents some new generalizations by using Bonferroni means (BM) forming induced BM. The main advantage of this approach is the possibility of reordering the results according to complex ranking processes based on order-inducing variables. The work also presents some additional extensions by using the weighted ordered weighted average, immediate weights, and hybrid averages. Some further generalizations with generalized and quasi-arithmetic means are also developed to consider a wide range of particular cases including quadratic and geometric aggregations. The article also considers the applicability of the new approach in-group decision-making developing an application in sales forecasting.

KEYWORDS
Bonferroni means, group decision-making, induced ordered weighted average operator, sales forecasting

1 | INTRODUCTION

Sales forecasting has become one of the most important issues in strategic management and planning among the organizations. This occurs because a poor forecast leads to bad decisions in inventory, profitability, and risk in losing the competitive position of the company.[1,2] The main difficulty with sales forecasting is that the problem is complex, ill-structured and the environment where business works presents a lot of uncertainty.[3]

A very common technique that is used to forecast sales is the common average. But this technique main limitation is that it only considers the historical data. In the decision-making process, the knowledge, expertise and expectations of the decision-maker[4] is essential, which can improve the sales forecasting to establish future scenarios where the company will be. One of the most common aggregation operators is the ordered weighted average (OWA) operator developed by Yager.[5] Aggregation operator allows aggregating different types information and attitudinal character of decision-maker that represents the degree of subjectivity and the degree of uncertainty.[6] Since then many applications have been made.[7,8]

One extension that will be taken into account in the paper to improve the sales forecasting is the induced OWA (IOWA) operator.[9] The main attribute of this operator is that the reordering step is not based on the value of the arguments, instead they are based on induced values, that are related to the appreciation of the decision-maker or special characteristics of the problem.[10,11]

Finally, it is important to not only include the traditional average in the formulation because there is sometimes information that can be included if different functions are used. Among them, there is the Bonferroni means (BM)[12] that are useful because they take into the formulation the interrelationship between the arguments, in this sense new scenarios can be seen.

The main objective of this paper is to introduce a new operator that takes into the same formulation the IOWA operator and the BM. This is important because it will be possible to generate better results that will not consider only the expectations, knowledge, and attitude of the decision-maker but also the interrelationship of the data. This new operator is called Bonferroni IOWA (BON-IOWA) operator. Also, some of the particular cases using quasi-arithmetic means are presented.

The BON-IOWA operator is used in a case of sales forecasting for a Mexican enterprise based on the historical data from 2010 to 2016 and considering the experience of the decision-maker. The results obtained are compared with other operators to visualize the different scenarios when additional information is added to the formulation or not.

The paper is structured as follows: section 2 shows some of the preliminaries and main definitions that will be used in the rest of the paper. Sections 3, 4, and 5 present the BON-IOWA operator, the BON-IOWA operator with hybrid averages, and the generalized BON-IOWA operator respectively. Section 6 shows an application of the BON-IOWA operator in sales forecasting, and finally, in Section 7, the conclusions of the paper are presented.

2 | PRELIMINARIES

In this section, we briefly review BM, OWA and IOWA operators and BON-OWA to develop new tools based on BM in combination with IOWA operator.

2.1 | Bonferroni means

The BM[12] are an averaging aggregation function that allows capturing the interrelationship between arguments. Recently several authors have used it with OWA operators,[13,14] uncertain data,[15] linguistic variables,[16,17] intuitionistic information,[18,19] hesitant representation,[20,21] and distance measures.[22-26] By rearranging the terms,[13] it can be formulated in the following way:

$$B(a_1, a_2, \ldots, a_n) = \left(\sum_{k=1}^{n} a_i^r \left(\frac{1}{1-n} \sum_{\substack{j=1 \\ j \neq i}}^{n} a_j^q \right) \right)^{\frac{1}{r+q}} \tag{1}$$

2.2 | OWA and IOWA operators

The OWA operator[5] is a method that allows aggregating information with the possibility to obtain the maximum operator and the minimum operator and providing a parameterized class of mean-type of aggregation operators. It can be defined as follows.

Definition 1 An OWA operator of dimension n is a mapping $OWA: R^n \rightarrow R$ that has an associated weighing vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_i is the jth largest of the a_i.

Based on the OWA operator, great deals of extensions have been developed. One of these extensions is the IOWA operator, which is proposed by Yager and Filev.[9] In this operator, we shall reorder the arguments by a inducing variable, such that:

Definition 2 An IOWA operator of dimension n is an application $IOWA: R^n \times R^n \rightarrow R$ that has a weighting vector associated, W of dimension n where the sum of the weights is 1 and $w_j \in [0, 1]$, where an induced set of ordering variables are included (u_i) such that the formula is

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the a_i value of the OWA pair $< u_i, a_i >$ having the jth largest u_i . u_i is the order-inducing variable and a_i is the argument variable.

2.3 | Bonferroni OWA

BON-OWA is an operator proposed by Yager[13] which allows aggregating information and making multiple comparison between input arguments and capturing its interrelationship to present information. It can be defined as follows:

$$BON - OWA(a_1, \ldots, a_n) = \left(\frac{1}{n} \sum_{i} a_i^r OWA_W(V^i) \right)^{\frac{1}{r+q}}, \tag{4}$$

where $OWA_W(V^i) = \left(\frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} a_j^q\right)$ with (V^i) being the vector of all a_j except a_i and w being an

$n-1$ vector W_i associated with α_i whose components w_{ij} are the OWA weights. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$. Then, we can define this aggregation as $OWA_W(V^i) = (\sum_{j=1}^{n-1} w_i a_{\pi_k(j)})$, where $a_{\pi_k(j)}$ is the largest element in the tuple V^i and $w_i = \frac{1}{n-1}$ for all i.

3 | BM WITH IOWA OPERATORS

As previously noted, BON-OWA aggregates, makes multiple comparison, and captures interrelationship of the present information. This allows obtaining the maximum and minimum operators in a comparative and continuous interrelationship of each one of the arguments. Now, we propose a new operator that also allows us to reorder the information by using induced variables. This proposition is defined as follows:

Proposition 1 *The BON-IOWA is a mean-type continuous aggregation operator that can be defined as follows:*

$$BON-IOWA(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_i b_i^r IOWA_W(V^i)\right)^{\frac{1}{r+q}}, \tag{5}$$

where b_i is the a_i value of the BON-IOWA pair $<u_i, a_i>$ having the jth largest u_i and $IOWA_W(V^i) = (\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} b_j^q)$ with (V^i) being the vector of all b_j except b_i and w being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the OWA weights. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0,1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i and u_i is the order-inducing variable.

Furthermore, BON-IOWA has the following properties. Note that the proofs are trivial and thus omitted. Commutativity-OWA aggregation: assume f is the BON-IOWA operator, the $f(\langle u_i, a_i\rangle, \dots, \langle u_n, a_n\rangle) = f(\langle u_i, b_i\rangle, \dots, \langle u_n, b_n\rangle)$. Monotonicity: assume f is the BON-IOWA operator; if $|u_i, a_i| \geq |u_i, b_i|$ for all i_i, then $f(\langle u_i, a_i\rangle, \dots, \langle u_n, a_n\rangle) \geq f(\langle u_i, b_i\rangle, \dots, \langle u_n, b_n\rangle)$. Bounded: assume f is the BON-IOWA operator, then $\min\{a_i\} \leq f(\langle u_i, a_i\rangle, \dots, \langle u_n, a_n\rangle) \leq \max\{a_i\}$. Idempotency: assume f is the BON-IOWA operator; if $|u_i, a_i| = a$ for all i, then $f(\langle u_i, a_i\rangle, \dots, \langle u_n, a_n\rangle) = a$.

In addition, if $q = 0$, then by (5), then it follows that:

$$BON-IOWA^{r,0}(\langle u_1, a_1\rangle, \dots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_i a_i^r\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q\right)\right)^{\frac{1}{r+0}} = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{1/r}. \tag{6}$$

If $r = 2$ and $q = 0$, then (13) reduces to square mean:

$$BON-IOWA^{2,0}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_{k=1}^{n} a_i^2\right)^{1/2}. \tag{7}$$

If $r = 1$ and $q = 0$, then (13) reduces to average:

$$BON-IOWA^{1,0}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = \frac{1}{n}\sum_{k=1}^{n} a_i. \tag{8}$$

If $r \to +\infty$ and $q = 0$, then (13) reduces to the max operator:

$$\lim_{r\to+\infty} BON-IOWA^{r,0}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = \max\{a_i\}. \tag{9}$$

If $r \to 0$ and $q = 0$, then (13) reduces to geometric mean:

$$\lim_{r\to 0} BON-IOWA^{r,0}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\prod_{i=1}^{n} a_i\right)^{1/n}. \tag{10}$$

If $r = q = 1$, then BON-IOWA reduces to the following expression:

$$BON-IOWA^{1,1}(\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\left(\frac{1}{n(n-1)}\right)\sum_{\substack{i,j=1\\ i\neq j}}^{n} b_i b_j\right)^{\frac{1}{2}}. \tag{11}$$

Likewise, it is considered that the different measures have been used in the OWA literature to characterize the weighting vector.[27] In Blanco-Mesa et al,[23] it is mentioned that the weighting vector can be fixed by a numbers of manners. Thus, the entropy of dispersion, the balance operator, the divergence of W, and the degree of orness[5,27] are defined as follows:

The entropy of dispersion is defined as follows

$$H(W) = -\left(\frac{1}{n}\sum_{i} \ln(w_i)\left(\sum_{\substack{j=1\\ j\neq i}}^{n} w_i \ln(w_i)\right)\right)^{\frac{1}{r+q}}. \tag{12}$$

For the balance operator, we obtain

$$Bal(W) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1-2_i}{n-1}\right)\left(\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{n+1-2_j}{n-1}\right)w_i\right)\right)^{\frac{1}{r+q}}. \tag{13}$$

For the divergence of W, we obtain

$$Div(W) = \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{n-i}{n-1} - \alpha(W) \right)^2 \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} w_i \left(\frac{n-j}{n-1} - \alpha(W) \right)^2 \right) \right)^{\frac{1}{r+q}}. \tag{14}$$

For the degree of orness, we obtain

$$\alpha(W) = \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{n-i}{n-1} \right) \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} w_i \left(\frac{n-j}{n-1} \right) \right) \right)^{\frac{1}{r+q}}. \tag{15}$$

Now, the following simple example illustrates the proposition:

Example 1 We have assumed that IOWA pair $\langle \mu_i, a_i \rangle$ is given by $\langle 3, 0.2 \rangle$, $\langle 6, 0.5 \rangle$, $\langle 1, 0.9 \rangle$, $\langle 5, 0.8 \rangle$. w_i is the weighting vector of the $\langle \mu_i, a_i \rangle$ associated with α_i whose components v_{ij}. Here we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.7$, and $\alpha_4 = 0.5$. The ordered OWA pair is $\langle 6, 0.5 \rangle$, $\langle 5, 0.8 \rangle$, $\langle 3, 0.2 \rangle$, $\langle 1, 0.9 \rangle$, that is, the ordered list a_i is 0.5, 0.8, 0.2, 0.9. We take $r = q = 0.5$. In addition: $V^1 = (0.8 + 0.2 + 0.9)$, $V^2 = (0.5 + 0.2 + 0.9)$, $V^3 = (0.5 + 0.8 + 0.9)$, and $V^4 = (0.5 + 0.8 + 0.2)$. Then,

$$IOWA_{v_1}(V^1) = 0.4 \times (0.8 + 0.2 + 0.9) = 0.76,$$

$$IOWA_{v_2}(V^2) = 0.3 \times (0.5 + 0.2 + 0.9) = 0.48,$$

$$IOWA_{v_3}(V^3) = 0.7 \times (0.5 + 0.8 + 0.9) = 1.54,$$

$$IOWA_{v_4}(V^4) = 0.5 \times (0.5 + 0.8 + 0.2) = 0.75,$$

$$BON - IOWA = \left(\frac{1}{4} \times ((0.5 \times 0.76) + (0.8 \times 0.48) + (0.2 \times 1.54) + (0.9 \times 0.75)) \right)^{0.5}$$
$$= 0.6608.$$

4 | BM WITH INDUCED WEIGHTED OWA OPERATORS AND HYBRID AVERAGES

Further extension to the BON-IOWA could be developed following the current developments on the aggregation operators.[28,29] In this sense, in this study, we introduce new approaches that unify framework between BM, OWAWA operator,[30] and immediate weighted (IW) average.[31] Hence, we would obtain the BON-IOWA weighted average (BON-IOWAWA) operator, the Bonferroni induced IW OWA (BON-IIWOWA) operator, and Bonferroni induced hybrid weighted average (BON-IHWA) operator. The main advantage of this approach is that it can combine classical Bonferroni aggregation, OWAWA, IW, and HWA operators at the same formulation, which allows considering the attitudinal character characteristic of the decision-maker. These new formulations are presented as follows: it is important to observe that the formulas proposed by Merigó[30] are followed.

The OWAWA operator is an aggregation operator proposed by Merigó[30] in which the WA and OWA operators are unified in the same formulation. In this operator, the degree is considered that each concept has in the analysis. It can be defined as follows:

Definition 3 An OWAWA operator of dimension n is a mapping $OWAWA{:}\ R^n \times R^n \rightarrow R$ that has an associated weighting vector W, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$ such that:

$$OWAWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{16}$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (WA) v_i with $\sum_{j=1}^{n} v_j = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

By using this approach, let us extend the BON-IOWA operator with the OWAWA operators, forming the BON-IOWAWA operator.

Proposition 2 *A BON-IOWAWA is a mean-type continuous aggregation operator that can be defined as follows:*

$$\begin{aligned} &BON-IOWAWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) \\ &= \beta \times \left(\frac{1}{n}\sum_{i} a_i^r IOWA_W(V^i)\right)^{\frac{1}{r+q}} + (1-\beta) \\ &\times \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r IWA_{V_i}(V^i)\right)^{\frac{1}{r+q}}, \end{aligned} \tag{17}$$

where b_i is the a_i value of the OWA pair $< u_i, a_i >$ having the jth largest u_i and $\beta \in [0,1]$. Observe that $\beta = 1$ forms the BON-IOWA operator and $\beta = 0$ the induced weighted Bonferroni (BON-IWA) that is expressed as:

$$BON-IWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_{i=1}^{n} b_i^r IWA_{v_i}(V^i)\right)^{\frac{1}{r+q}}, \tag{18}$$

where b_i is the a_i value of the WA pair $< u_i, a_i >$ having the jth largest u_i and $\mathrm{WA}_{v_i}(v^i) = (\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} b_j^q)$ with (V^i) being the vector of all b_j except b_i and v_i being an $n-1$ vector V_i associated with λ_i whose components v_{ij} are the IWA weights.

Now, the following simple example illustrates the proposition:

Example 2 We have assumed that the IOWA pair $\langle \mu_i, a_i \rangle$ is given by $\langle 3, 0.2\rangle$, $\langle 6, 0.5\rangle$, $\langle 1, 0.9\rangle$, $\langle 5, 0.8\rangle$. w_i is the weighting vector of the $\langle \mu_i, a_i \rangle$ associated with α_i whose components v_{ij}. Here we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.7$ and $\alpha_4 = 0.5$. The ordered OWA pair is $\langle 6, 0.5\rangle$, $\langle 5, 0.8\rangle$, $\langle 3, 0.2\rangle$, $\langle 1, 0.9\rangle$, that is, the ordered list a_i is 0.5, 0.8, 0.2, 0.9. We take $r = q = 0.5$. In addition: $V^1 = (0.8 + 0.2 + 0.9)$, $V^2 = (0.5 + 0.2 + 0.9)$, $V^3 = (0.5 + 0.8 + 0.9)$ and $V^4 = (0.5 + 0.8 + 0.2)$. Then

$$IOWA_{v_1}(V^1) = 0.4 \times (0.8 + 0.2 + 0.9) = 0.76,$$

$$IOWA_{v_2}(V^2) = 0.3 \times (0.5 + 0.2 + 0.9) = 0.48,$$

$$IOWA_{v_3}(V^3) = 0.7 \times (0.5 + 0.8 + 0.9) = 1.54,$$

$$IOWA_{v_4}(V^4) = 0.5 \times (0.5 + 0.8 + 0.2) = 0.75,$$

$$\begin{aligned} BON - IOWA &= \left(\frac{1}{4} \times ((0.5 \times 0.76) + (0.8 \times 0.48) \right. \\ &\qquad \left. + (0.2 \times 1.54) + (0.9 \times 0.75))\right)^1 \\ &= 0.6608. \end{aligned}$$

Since BON-IOWA is part of BON-IOWAWA corresponding to $\beta \times \left(\frac{1}{n}\sum_i b_i^r IOWA_{w_i}(V^i)\right)^{\frac{1}{r+q}}$, now we develop the other part $(1-\beta) \times \left(\frac{1}{n}\sum_{i=1}^n b_i^r IWA_{V_i}(V^i)\right)^{\frac{1}{r+q}}$. $v_i = (0.1, 0.2, 0.1, 0.2)$ is the weighting vector associated with IWA and $\beta = 0.3$. Then,

$$IWA_{v_1}(V^1) = 0.1 \times (0.8 + 0.2 + 0.9) = 0.19,$$

$$IWA_{v_2}(V^2) = 0.2 \times (0.5 + 0.2 + 0.9) = 0.32,$$

$$IWA_{v_3}(V^3) = 0.1 \times (0.5 + 0.8 + 0.9) = 0.22,$$

$$IWA_{v_4}(V^4) = 0.2 \times (0.5 + 0.8 + 0.2) = 0.30,$$

$$BON - IWA = \left(\frac{1}{4} \times ((0.5 \times 0.19) + (0.8 \times 0.32) + (0.2 \times 0.22) + (0.9 \times 0.30))\right)^{0.5} = 0.4076,$$

$$BON - IOWAWA = 0.3 \times 0.\ 6608 + (1 - 0.3) \times 0.4076 = 0.48352.$$

The immediate weighting (IW)[32] is an operator that the extended concept of immediate probabilities,[33-35] which considers the information used in the weighted average. It can be defined as follows:

Definition 4 An IW operator is a mapping $IW: R^n \to R$ of dimension n, which has an associated weighting vector W with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such as:

$$IW(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{19}$$

where b_j is the jth largest of the a_i, each a_i has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

As we can see, if $w_j = 1/n$ for all j, then we get the weighted average and if $v_j = 1/n$ for all j, then the OWA operator. Thus, Merigó and Gil-Lafuente[32] extended this measure using OWA for getting the IWOWA operator, which is defined as follows:

Definition 5 An IWOWA operator of dimension n is a mapping $IWOWA: R^n \mathrm{x} R^n \to R$ that has an associated weighted vector W of dimension n $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$IWOWA(\langle x_1, y_1 \rangle, \ldots, \langle x_n, y_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{20}$$

where b_j is the jth largest of the a_i, each a_i has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

Recently, Blanco and Merigó[26] proposed Bonferroni IW ordered weighted ordered distance (BON-IWOWAD)

Proposition 3 *Bonferroni induced IW (BIIW) is a mean-type continuous aggregation operator that can be defined as follows:*

$$BIIW(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = \left(\frac{1}{n} \sum_{i=1}^{n} b_i^r IW_{w_i}(V^i)\right)^{\frac{1}{r+q}}, \tag{21}$$

where b_i is the a_i value of the BIIW pair $< u_i, a_i >$ having the jth largest u_i and $IW_{w_i}(V^i) = (\frac{1}{1-n}\sum_{\substack{j=1 \\ j\neq i}}^{n}(v_j/\sum_{j=1}^{n} v_j)b_j^{\,q})$ *with (V^i) being the vector of all b_j except b_i, w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the weighting vector and a weighting vector v_i associated with the WA.*

Proposition 4 *A BON-IIWOWA is a mean-type continuous aggregation operator that can be defined as follows:*

$$BON - IIWOWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\frac{1}{n}\sum_{k=1}^{n} b_i^r \; IIWOWA_{w_i}(V^i)\right)^{\frac{1}{r+q}}, \tag{22}$$

where b_i is the a_i value of the BON-IIWOWA pair $< u_i, a_i >$ having the jth largest u_i and $\mathrm{II}WOWA_{w_i}(V^i) = (\frac{1}{1-n}\sum_{\substack{j=1 \\ j\neq k}}^{n}(v_j/\sum_{j=1}^{n} v_j)b_j^{\,q})$ *with (V^i) being the vector of all b_j except b_i and w_i being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the BON-OWA weights and a weighting vector v_i associated with the WA. In this case, if $w_j = 1/n$ for all j, we get the BON-IW and if $v_j = 1/n$ for all j, the BON-IOWA operator. If one of the sets is empty, we get the BIIW operator.*

Now, the following simple example illustrates the proposition:

Example 3 We have assumed that the OWA pair $\langle \mu_i, a_i\rangle$ is given by $\langle 3, 0.2\rangle$, $\langle 6, 0.5\rangle$, $\langle 1, 0.9\rangle$, $\langle 5, 0.8\rangle$. $v_j = (0.12,\ 0.09,\ 0.1,\ 0.2)$ is the weighting vector associated with WA and w_i is the weighting vector of the argument b_i associated with α_i whose component is v_{ij}. Here we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.7$ and $\alpha_4 = 0.5$. The ordered OWA pair is $\langle 6, 0.5\rangle$, $\langle 5, 0.8\rangle$, $\langle 3, 0.2\rangle$, $\langle 1, 0.9\rangle$, that is the ordered list a_i is 0.5, 0.8, 0.2, 0.9. We take $r = q = 0.5$. In addition: $V^1 = (0.8 + 0.2 + 0.9)$, $V^2 = (0.5 + 0.2 + 0.9)$, $V^3 = (0.5 + 0.8 + 0.9)$, and $V^4 = (0.5 + 0.8 + 0.2)$. Also: $\sum_{j=1}^{n} w_i v_j = (0.4 \times 0.12) + (0.3 \times 0.09) + (0.7 \times 0.1) + (0.2 \times 0.5) = 0.245$. Using this, we get:

$$IIWOWA_{v_1}(V^1) = \frac{0.09 \times 0.4}{0,\ 245} \times 0.8 + \frac{0.10 \times 0.4}{0,\ 245} \times 0.2 + \frac{0.20 \times 0.4}{0, 245} \times 0.9 = 0.444,$$

$$IIWOWA_{v_2}(V^2) = \frac{0.12 \times 0.3}{0,\ 245} \times 0.5 + \frac{0.10 \times 0.3}{0,\ 245} \times 0.2 + \frac{0.20 \times 0.3}{0,\ 245} \times 0.9 = 0.318,$$

$$IIWOWA_{v_3}(V^3) = \frac{0.12 \times 0.7}{0,\ 245} \times 0.5 + \frac{0.09 \times 0.7}{0,\ 245} \times 0.8 + \frac{0.20 \times 0.7}{0,\ 245} \times 0.9 = 0.891,$$

$$IIWOWA_{v_4}(V^4) = \frac{0.12 \times 0.5}{0,\ 245} \times 0.5 + \frac{0.09 \times 0.5}{0,\ 245} \times 0.8 + \frac{0.10 \times 0.5}{0,\ 245} \times 0.2 = 0.310,$$

$$BON - IIWOWA = \left(\frac{1}{4} \times ((0.5 \times 0,\ 444) + (0.8 \times 0.318) + (0.2 \times 0.891) + (0.9 \times 0.310))\right)^{0.5} = 0.\ 4832.$$

Another approach to unify the OWA operator with the weighted average is by using the HWA.[36] Recently, Blanco and Merigó[26] has proposed Bonferroni hybrid weighted distance (BON-HWD). With induced aggregation operators, the HWA operator becomes the IHWA operator. Hence, the BON-IOWA operator can be extended with this approach forming the BON-IHA operator. Note that the main advantage of this operator is the possibility of using hybrid averages with BM in a complex environment where the data are reordered with order inducing variables.

Definition 6 A HWA is a mapping $HWA: R^n \rightarrow R$ of dimension n, it has an associated weighting vector W of the dimension n, with $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0, 1]$, such as

$$HWA(a_1, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j (a_j)^{\lambda}\right)^{1/\lambda}, \quad \lambda > 0, \tag{23}$$

where a_j is the the jth largest of the weighted arguments $mw_i a_i$ and m is a balancing coefficient which plays a balancing role.

Proposition 5 *Bonferroni HWA (BON-HWA) operator is a mean-type continuous aggregation operator that can be defined as follows:*

$$BON - HWA(a_1, \ldots, a_n) = \left(\frac{1}{n} \sum_{i=1}^{n} a_i^r HWA_{w_i}(V^i)^{\frac{1}{\lambda}}\right)^{\frac{1}{r+q}}, \tag{24}$$

where $HWA_{w_i}(V^i) = (\frac{1}{1-n} \sum_{\substack{j=1 \\ j \neq i}}^{n} v_i m (a_j^q)^{\lambda})$ *with* (V^i) *being the vector of all* a_j *except* a_i, w_i *being an* $n-1$ *vector* V_i *associated with* α_i *whose components of the argument* a_i *are weights, a weighting vector* v_i *associated with the HWA and* m *is a balancing coefficient which plays a balancing role.*

Proposition 6 *BON-IHWA operator is a mean-type continuous aggregation operator that can be defined as follows:*

$$BON-IHWA(a_1, \ldots, a_n) = \left(\frac{1}{n}\sum_{i=1}^{n} b_i^r HWA_{w_i}(V^i)^{\frac{1}{\lambda}}\right)^{\frac{1}{r+q}}, \tag{25}$$

where b_i is the a_i value of the BON-IHWA pair $<u_i, a_i>$ having the jth largest u_i and $\mathrm{IHWA}_{w_i}(V^i) = \left(\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} v_i m\left(b_j^q\right)^{\lambda}\right)$ *with (V^i) being the vector of all b_j except b_i, w_i being an $n-1$ vector V_i associated with α_i whose components of the argument a_i are weights, a weighting vector v_i associated with the HWA and m is a balancing coefficient which plays a balancing role.*

Note that if the reordering of the order inducing variables is the same than the ordering of the OWA operator based on a decreasing or increasing perspective, then, the BON-IHWA operator becomes the BON-HWA operator.

To understand the BON-IHWA numerically, let us present a simple example.

Example 4 We have assumed that OWA pair $\langle \mu_i, a_i \rangle$ is given by $\langle 3, 0.2\rangle$, $\langle 6, 0.5\rangle$, $\langle 1, 0.9\rangle$, $\langle 5, 0.8\rangle$. $v_i = (0.12,\ 0.09,\ 0.1,\ 0.2)$ is the weighting vector associated with HWA and w_i is the weighting vector of the argument b_i associated with α_i whose components v_{ij}, these values are specified by a value α_i. Here we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\alpha_3 = 0.7$ and $\alpha_4 = 0.5$.

The ordered OWA pair is $\langle 6, 0.5\rangle$, $\langle 5, 0.8\rangle$, $\langle 3, 0.2\rangle$, $\langle 1, 0.9\rangle$, that is, the ordered list a_i is 0.5, 0.8, 0.2, 0.9. We take $r = q = 0.5$ and $\lambda = 1$. In addition: $V^1 = (0.8; 0.2; 0.9)$, $V^2 = (0.5; 0.2; 0.9)$, $V^3 = (0.5; 0.8; 0.9)$, and $V^4 = (0.5; 0.8; 0.2)$. Using this, we get

$$\mathrm{IHWA}_{v_1}(V^1) = 4 \times 0.09 \times 0.4 \times 0.8 + 4 \times 0.10 \times 0.4 \times 0.2 + 4 \times 0.20 \times 0.4 \times 0.9 = 0.7808,$$

$$\mathrm{IHWA}_{v_2}(V^2) = 4 \times 0.12 \times 0.3 \times 0.5 + 4 \times 0.10 \times 0.3 \times 0.2 + 4 \times 0.20 \times 0.3 \times 0.9 = 0.3120,$$

$$\mathrm{IHWA}_{v_3}(V^3) = 4 \times 0.12 \times 0.7 \times 0.5 + 4 \times 0.09 \times 0.7 \times 0.8 + 4 \times 0.20 \times 0.7 \times 0.9 = 0.8736,$$

$$\mathrm{IHWA}_{v_4}(V^4) = 4 \times 0.12 \times 0.5 \times 0.5 + 4 \times 0.09 \times 0.5 \times 0.8 + 4 \times 0.10 \times 0.5 \times 0.2 = 0.3040,$$

$$\begin{aligned} BON-\mathrm{IHWA} &= \left(\frac{1}{4} \times ((0.5 \times 0.\ 7808) + (0.8 \times 0.\ 3120) + (0.2 \times 0.\ 8736) + (0.9 \times 0.\ 3040))\right)^{0.5} \\ &= 0.5216. \end{aligned}$$

TABLE 1 Particular cases of the BON-IGOWA and QBON-IOWA operators

Particular cases	BON-IGOWA	QBON-IOWA
BON-IOWA	$\lambda = 1, \delta = 1$	$g = (b_i^r), h = (b_j^q)$
Harmonic BON-IOWA	$\lambda = -1, \delta = -1$	$g = (b_i^r)^{-1}, h = (b_j^q)^{-1}$
Quadratic BON-IOWA	$\lambda = 2, \delta = 2$	$g = (b_i^r)^2, h = (b_j^q)^2$
Cubic BON-IOWA	$\lambda = 3, \delta = 3$	$g=(b_i^r)^3, h=(b_j^q)^3$
Geometric BON-IOWA	$\lambda \to 0, \delta \to 0$	$g = (b_i^r)^0, h = (b_j^q)^0$
BON-Max	$\lambda = \infty, \delta = \infty$	$g = (b_i^r)^{\infty}, h = (b_j^q)^{\infty}$
BON-Min	$\lambda = -\infty, \delta = -\infty$	$g = (b_i^r)^{-\infty}, h = (b_j^q)^{-\infty}$

Abbreviations: BON-IGOWA, Bonferroni induced generalized ordered weighted averaging; QBON-IOWA, Quasi-arithmetic Bonferroni induced ordered weighted averaging.

5 | BM WITH INDUCED GENERALIZED AGGREGATION OPERATORS

Using generalized and quasi-arithmetic means can also further extend the BON-IOWA operator.[37,38] Thus, the Bonferroni induced generalized ordered weighted averaging (BON-IGOWA) operator allows presenting particular cases from its general formulation. It is defined as follows:

The BON-IGOWA operator is a generalized mean-type continuous aggregation operator that can be defined as follows:

$$BON-IGOWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\left(\frac{1}{n}\sum_{i}(b_i^r)^{\lambda}\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}(b_j^q)^{\delta}\right)\right)^{\frac{1}{r+q}}\right)^{\frac{1}{\lambda}}, \tag{26}$$

where b_i is the a_i value of the BON-IGOWA pair $< u_i, a_i >$ having the jth largest u_i, λ and δ are the parameters such that $\lambda \in (-\infty, \infty)$ and $(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}(b_j^q)^{\delta})$ can be expressed as $IGOWA_W(V^i)$ where (V^i) is the vector of all b_j except b_i and w being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the OWA weights. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i, and u_i is the order-inducing variable.

Also, by using quasi-arithmetic means, we can further extend the BON-IOWA operator. So, we obtain the quasi-arithmetic BON-IOWA. It is defined as follows:

Definition 7 The quasi-arithmetic Bonferroni induced ordered weighted averaging (QBON-IOWA) operator is a quasi-arithmetic mean-type continuous aggregation operator that can be defined as follows:

$$QBON-IOWA(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle) = g^{-1}(h^{-1})\left(\frac{1}{n}\sum_i g(b_i^r)\left(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} h(b_j^q)\right)\right)^{\frac{1}{r+q}} \quad (27)$$

where b_i is the a_i value of the QBON-IOWA pair $< u_i, a_i >$ having the jth largest u_i, g and h are strictly continuous monotonic functions and $(\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} \mathrm{hb}_j^q)$ can be expressed as $QIOWA_W(V^i)$ where (V^i) is the vector of all b_j except b_i and w being an $n-1$ vector W_i associated with α_i whose components w_{ij} are the OWA weights. Let W be an OWA weighing vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$, where the weights are associated according to the largest value of u_i, and u_i is the order-inducing variable. Furthermore, it includes the BON-IGOWA operator as a particular case when $g = (b_i^r)^\lambda$ and $h = (b_j^q)^\lambda$. Moreover, in Table 1 shows particular cases that it is important to mentioning.

6 | APPLICATION IN GDM SALES FORECASTING

The environment in business has change drastically in the recent years, within the factors it is possible to identify globalization, technology, e-business, competition, product proliferation, and so on.[39-41] With this situation, one of the main effects within the organizations it is in sales, that is why sales manager has been focusing in forecasting research to develop better objectives and strategies to achieve them.[42,43] Among the problems to select adequate sales forecasting

TABLE 2 Historical sales of the company

Year	January	February	March	April	May	June
2010	145	123	135	138	162	178
2011	160	136	149	152	179	197
2012	147	125	137	140	165	181
2013	187	159	174	178	209	230
2014	154	131	143	146	172	189
2015	169	144	157	161	189	208
2016	174	148	162	165	195	214

Year	July	August	September	October	November	December
2010	168	148	144	128	132	139
2011	186	163	158	141	146	154
2012	171	150	146	129	134	141
2013	217	191	185	165	170	180
2014	179	157	152	136	140	148
2015	196	172	167	149	154	162
2016	202	177	172	153	158	167

method is the vast amount of methods that exist, but more important is the need of the model to be accurate and to measure the complexity of the market conditions.[44,45] In this sense, some new methodologies have been developed for sales forecasting treatment that allow measuring the complexity and uncertainty of the environment.[8,46]

6.1 | Group decision-making approach

In this paper, the objective is to use the BON-IOWA operator as a new sales forecasting method, that help the decision-maker to include his experience, knowledge, and expectations of the market conditions for the following year in combination with the historical data. The steps to use this new method are as follows:

Step 1. Determine the number of years that will be considered in the analysis based on the impact that will have in future results (eg, 12 months, 5 years, and 10 years).
Step 2. Determine the weights that will apply to each month or year according to the importance that they will have in the forecasting.
Step 3. An order-inducing vector has to be done according to the expectation of the decision-maker.
Step 4. In this step is necessary to include the information that is needed to do the BM, that is, the α and r.
Step 5. Once all the information is obtained, it is possible to forecast the sales with different aggregation operators such as moving average (MA), BM, OWA, IOWA, BON-OWA, and BON-IOWA.
Step 6. With the information provided by the different operators, it is possible to analyze different scenarios that will help the decision-maker in doing objectives and strategies for the following years.

TABLE 3 Sales forecasting for 2017 according to expert 1

Operator	January	February	March	April	May	June
MA	162	138	151	154	182	200
BM	130	110	121	139	164	180
IOWA	181	154	168	172	203	223
OWA	183	156	170	174	205	225
BON-OWA	147	125	136	157	185	203
BON-IOWA	145	123	135	155	182	200
Operator	**July**	**August**	**September**	**October**	**November**	**December**
MA	188	166	161	143	148	156
BM	207	182	177	114	118	125
IOWA	210	185	179	159	165	174
OWA	213	187	181	161	167	176
BON-OWA	234	206	200	129	133	141
BON-IOWA	231	203	197	143	148	156

Abbreviations: BM, Bonferroni means; BON, Bonferroni; IOWA, induced ordered weighted average; MA, moving average; OWA, ordered weighted average.

TABLE 4 Sales forecasting for 2017 according to expert 2

Operator	January	February	March	April	May	June
MA	162	138	151	154	182	200
BM	130	110	121	139	164	180
IOWA	188	159	174	178	210	231
OWA	183	156	170	174	205	225
BON-OWA	147	125	136	157	185	203
BON-IOWA	150	128	140	160	189	208
Operator	**July**	**August**	**September**	**October**	**November**	**December**
MA	188	166	161	143	148	156
BM	207	182	177	114	118	125
IOWA	218	191	186	165	171	180
OWA	213	187	181	161	167	176
BON-OWA	234	206	200	129	133	141
BON-IOWA	239	210	204	149	154	162

Abbreviations: BM, Bonferroni means; BON, Bonferroni; IOWA, induced ordered weighted average; MA, moving average; OWA, ordered weighted average.

TABLE 5 Sales forecasting for 2017 according to expert 3

Operator	January	February	March	April	May	June
MA	162	138	151	154	182	200
BM	130	110	121	139	164	180
IOWA	181	154	168	172	202	222
OWA	183	156	170	174	205	225
BON-OWA	147	125	136	157	185	203
BON-IOWA	145	123	134	154	182	200
Operator	**July**	**August**	**September**	**October**	**November**	**December**
MA	188	166	161	143	148	156
BM	207	182	177	114	118	125
IOWA	210	184	179	159	164	173
OWA	213	187	181	161	167	176
BON-OWA	234	206	200	129	133	141
BON-IOWA	231	203	197	143	148	156

Abbreviations: BM, Bonferroni means; BON, Bonferroni; IOWA, induced ordered weighted average; MA, moving average; OWA, ordered weighted average.

6.2 | Numerical example

In this section, a real situation for a Mexican enterprise is developed. The sales manager wants to forecast the sales of the enterprise to make a plan that can be achieved and that takes into account different information like the expectations and market conditions. To do this, the steps defined in section 6 are used.

TABLE 6 Sales forecasting for 2017 according to unification 1

Operator	January	February	March	April	May	June
MA	162	138	151	154	182	200
BM	130	110	121	139	164	180
IOWA	183	156	170	174	205	225
OWA	183	156	170	174	205	225
BON-OWA	147	125	136	157	185	203
BON-IOWA	147	125	136	156	184	203
Operator	**July**	**August**	**September**	**October**	**November**	**December**
MA	188	166	161	143	148	156
BM	207	182	177	114	118	125
IOWA	213	187	181	161	167	176
OWA	213	187	181	161	167	176
BON-OWA	234	206	200	129	133	141
BON-IOWA	234	205	199	145	150	158

Abbreviations: BM, Bonferroni means; BON, Bonferroni; IOWA, induced ordered weighted average; MA, moving average; OWA, ordered weighted average.

TABLE 7 Sales forecasting for 2017 according to unification 2

Operator	January	February	March	April	May	June
MA	162	138	151	154	182	200
BM	130	110	121	139	164	180
IOWA	184	156	170	174	206	226
OWA	183	156	170	174	205	225
BON-OWA	147	125	136	157	185	203
BON-IOWA	147	125	137	157	185	203
Operator	**July**	**August**	**September**	**October**	**November**	**December**
MA	188	166	161	143	148	156
BM	207	182	177	114	118	125
IOWA	213	187	182	161	167	176
OWA	213	187	181	161	167	176
BON-OWA	234	206	200	129	133	141
BON-IOWA	234	206	200	145	150	158

Abbreviations: BM, Bonferroni means; BON, Bonferroni; IOWA, induced ordered weighted average; MA, moving average; OWA, ordered weighted average.

Step 1. The company provides us with the monthly sales from 2010 to 2016; the information is presented in thousands of pesos (see Table 2)

Step 2. In this case we have three different decision-makers, each one proposed a different weighting vector, and these are as follows:

The expert1 $W_1 = 0.10, 0.10, 0.15, 0.15, 0.15, 0.20, 0.25$

The expert2 $W_2 = 0.15, 0.15, 0.15, 0.15, 0.15, 0.20, 0.20$

The expert3 $W_3 = 0.10, 0.10, 0.10, 0.10, 0.15, 0.25, 0.30$

as it can be seen the latest year is higher because it is the nearest scenario to the future market conditions.

Step 3. The proposed inducing vector is $U = 5{,}10, 20{,}15, 30, 35, 25$
Step 4. The α data will be divided quarterly as follows $\alpha_1 = 0.8$, $\alpha_2 = 0.9$, $\alpha_3 = 1.1$, $\alpha_4 = 0.8$, respectively, and $r = q = 0.5$.
Step 5. With the information provided by the decision-maker, the results using MA, BM, OWA, IOWA, BON-OWA, and BON-IOWA operators (see Tables 3-5) are as follows:

With the information from Tables 3-5, we can observe that different sales scenarios that will help the decision-maker to understand better the market and the future of the company. It is noteworthy that the BON-IOWA is the operator that includes more information and the MA the one that includes the less. We can use different operators according to the information that is available or if the problem is an easy one or a very important one that has to include all the information available. Analyzing the results, it is possible to observe that in most of the cases the BM forecast the lowest sales and the BON-OWA the one with the highest sales, and this will help the decision-maker to make better strategies and policies according to the market information.

Also, it is possible to unify the results taking into account the information provided individually by the decision-makers. In this sense, two different unifications will be presented: (a) taking into account that all the information provided by each decision-maker is equally important and (b) considering that the importance is 0.3, 0.4, and 0.3, respectively. The results are presented in Tables 6 and 7.

Using these techniques, it is possible to not only generate different scenarios based in the individual knowledge of the decision-makers but also is possible to unify them and prioritize which information is more relevant based on the expertise, knowledge, and impact that the decision-maker has in the final decision.

7 | CONCLUSIONS

In this paper, we have studied the operators related to aggregation theory, such as, the OWA and IOWA operators and BM. We have presented a new aggregation operator combining IOWA operators and BM, which is called BON-IOWA operator. This new operator allows reordering the information by using order-inducing variables to obtain the maximum and minimum operators in a comparison and continuous interrelationship of each one of the arguments. Likewise, we have introduced some new operators, which are called BON-IOWAWA, BON-IIWOWA, and BON-IHWA operators. The main advantage of these approaches is that they can combine classical Bonferroni aggregation, OWAWA, IW, and HWA operators in the same formulation, which allows considering the attitudinal character characteristic of the decision-maker. Thus, we have proposed a set of operators that form a new family of aggregation operators that allows combining classical operators in the same formulation to analyze multiple comparison, interrelationship, and reorder of the present information. Furthermore, the work also presents some generalizations by using generalized and quasi-arithmetic means.

We have developed a mathematical application, which is focused on the sales forecasting problem. An important aspect in sales is to select the adequate sales forecasting method since it is necessary that the model to be accurate and measure the complexity of the market. In this sense, we have used the BON-IOWA operator as a new sales forecasting method. This method allows us to aggregate experience, knowledge, and expectations of the market conditions for the following year of the decision-maker in combination with the historical data. To develop this application, we have considered data set of the monthly sales from 2010 to 2016 for a Mexican enterprise. Also, to observe the feasibility and versatility of the operator, we have compared with other aggregation operators to generate more scenarios. Results have shown two main features. First, the BON-IOWA operator includes more information and attitudes of the decision-maker. Second, this information can help the decision-maker to understand the environment and market information to make a better strategic planning and information policies.[47]

In future research, we expect to develop further the operator by using new aggregations functions, such as heavy weights,[48] heavy MAs,[49,50] and group decision-making problems.[51] Note also that this operator can be extended to uncertain environments with intervals, fuzzy numbers, linguistic information, and extension with distance measures and norms.

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