

# Contents

<b>1</b>	<b>General Introduction</b>	<b>1</b>
1.1	Inverse problems for hyperbolic equations . . . . .	1
1.1.1	On the well-posedness of the wave equation with potential . . . . .	3
1.1.2	Potential reconstruction for the wave equation . . . . .	4
1.1.3	Inverse problem for the wave speed of the wave equation . . . . .	8
1.2	Controllability issues in PDE's . . . . .	10
1.2.1	Classical results on controllability of parabolic equations . . . . .	11
1.2.2	Null controllability of the heat equation for parabolic equations with dynamic boundary conditions . . . . .	13
1.3	Main results of the thesis . . . . .	15
<b>2</b>	<b>Potential reconstruction for a class of hyperbolic systems in cascade</b>	<b>19</b>
2.1	General Setting . . . . .	19
2.1.1	Literature review . . . . .	21
2.1.2	Main result . . . . .	22
2.2	Carleman estimates . . . . .	23
2.2.1	Technical results . . . . .	23
2.2.2	A new Carleman estimate for a hyperbolic system . . . . .	28
2.3	Proof of Theorem 2.1 . . . . .	34
<b>3</b>	<b>Controllability properties of a class of heat equations with dynamic boundary conditions</b>	<b>39</b>
3.1	Introduction and main results . . . . .	39
3.2	Well-posedness of the heat equation with dynamic boundary conditions . . . . .	43
3.2.1	Variational approach . . . . .	43
3.2.2	Semigroup approach . . . . .	45
3.3	Controllability properties of the original problem . . . . .	47
3.4	Convergence of the approximate system . . . . .	56
3.5	Plugging the limit control in the approximate system . . . . .	58
<b>4</b>	<b>Controllability of 1-D heat equation with discontinuous diffusion coeffi- cients</b>	<b>60</b>
4.1	Introduction, setting and main result . . . . .	60
4.2	Proof of the Theorem 4.1 . . . . .	65
4.2.1	Setting . . . . .	65
4.2.2	End of the proof of Theorem 4.1 . . . . .	68
4.3	Proof of Lemma 4.3 . . . . .	77
4.3.1	First estimates . . . . .	79

4.3.2	Global estimates in the Fourier domain . . . . .	83
4.3.3	Estimates of the spatial derivatives . . . . .	88
4.4	Proof of the Corollary 4.2 . . . . .	91

**Conclusions** **93**

**A Carleman estimate for heat equation with dynamic boundary conditions  
by using Classical weights** **99**

A.1	Introduction and main result . . . . .	99
A.2	Proof of the Carleman estimate . . . . .	100

**Bibliography** **106**