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RELIABLE AND RESILIENT NETWORK DESIGN WITH DISTRIBUTIONALLY ROBUST OPTIMIZATION

TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS DE LA INGENIERÍA, MENCIÓN ELÉCTRICA

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL ELÉCTRICO

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RESUMEN DE LA MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL ELÉCTRICO Y DE LA TESIS PARA OPTAR AL GRADO DE MAGÍSTER EN CIENCIAS DE LA INGENIERÍA, MENCIÓN ELÉCTRICA
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Transmission expansion models, so far, have not recognized properly the limited data and knowledge associated with the underlying process behind the realization of system contingencies. Therefore, investment in new transmission assets has traditionally been decided by models that either overlook the likelihood of different system outages, or assume perfect knowledge on their probability distribution, which can lead to non-optimal decisions.

In this context, this work contributes with the development of two models. The first one proposes a distributionally robust approach to network security in order to acknowledge the ambiguity on reliability information, and analyzes the contribution that distributed energy resources (DER) can make to network security, potentially releasing latent capacity of existing transmission assets. To do so, a two-stage optimization model is developed, where the first stage determines the transmission expansion plan and the scheduling of post-contingency services, while the second one minimizes the expected cost of corrective actions.

The second model is a two-stage mathematical program that determines the optimal portfolio of resilience enhancing strategies to harden the grid against earthquakes, considering the costs of investment, operation, and the costs of different contingency scenarios the system can undergo. To deal with the limited information regarding outage likelihoods during earthquakes, it minimizes against the worst-case probability distribution within an ambiguity set. However, since it is of great importance to assess the benefits of substation hardening, this ambiguity set depends on the decision taken.

Through a number of quantitative assessments obtained by running the first model, this work demonstrates both the benefits of security services provided by DER, and the advantages of the proposed distributionally robust approach against alternative n-1 security and fixed probabilities (stochastic) solutions. Showing that, while the n -1 approach significantly undermines the value of DER in displacing network capacity, the fixed probabilities counterpart is too optimistic. Through the second model, we show that it is critical to consider the possibility of investing on substation hardening in order to determine the optimal array of measures to hedge the system against earthquakes, and that overlooking them may yield to unnecessary investments on new network infrastructure.

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Los modelos de expansión de la transmisión, hasta ahora, no han reconocido apropiadamente la cantidad limitada de información y conocimiento asociados al proceso subyacente por el cual se producen las contingencias en el sistema. Por lo tanto, la inversión en nuevos activos de transmisión ha sido tradicionalmente decidida mediante modelos que, o no consideran la probabilidad de las distintas contingencias en el sistema, o asumen un conocimiento total de su distribución de probabilidad, lo que puede llevar a decisiones subóptimas.

Es en este conexto que el presente trabajo contribuye con el desarrollo de dos modelos. El primero trata la seguridad del sistema con un enfoque distribucionalmente robusto, de manera de reconocer la ambigüedad en la información de confiabilidad, y analiza la contribución que los recursos energéticos distribuidos (DER) pueden hacer a la seguridad del sistema, potencialmente liberando capacidad de activos de transmisión existentes. Para esto, se desarrolla un modelo de optimización de dos etapas, donde la primera etapa determina el plan de expansión de la transmisión y la disponibilidad de servicios post-contingencia, mientras que la segunda etapa minimiza el costo esperado de las acciones correctivas.

El segundo es un modelo de programación matemática que determina el portafolio óptimo de estrategias para mejorar la resiliencia del sistema ante terremotos, considerando el costo de inversión, operación, y el costo de distintos escenarios a los que puede verse sometido el sistema. Para lidiar con la limitada información acerca de las probabilidades de falla durante los terremotos, el costo es minizado contra la peor distribución de probabilidad en un conjunto de ambigüedad. Sin embargo, como es de gran importancia evaluar los beneficios de reforzar subestaciones, este conjunto de ambigüedad depende de la decisión que se tome.

A través de numerosos análisis cuantitativos obtenidos resolviendo el primer modelo, este trabajo demustra tanto los beneficios de los servicios a la seguridad provistos por los DER, como las ventajas del enfoque distribucionalmente robusto comparado con la utilización del criterio n-1 y la utilización de probabilidades fijas (estocástico). Mostrando que, mientras el criterio n-1 socava el valor de los DER desplazando capacidad de la red, el enfoque de probabilidades fijas es demasiada optimista. A través del segundo modelo, mostramos que es crítico considerar la opción de invertir en robustecer subestaciones para determinar el conjunto óptimo de medidas para proteger el sistema ante terremotos, ya que ignorarla puede llevar a inversiones innecesarias en infraestructura.

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Chapter 1

Introduction

1.1 Motivation

The need for transmission network infrastructure in reality is mainly driven by both increasing the economic efficiency and the reliability of electricity systems [1]. In fact, new network investments can present significant benefits in terms of both reduced operational costs and/or unsupplied energy cost. In terms of the latter, networks are designed with a certain level of redundancy in order to securely deal with network (or other system) outages without curtailing (high volumes of) demand, which may be costly. According to empirical evidence [1], the application of reliability criteria has been the most important and predominant reason to undertake network investments within system operators' jurisdictions.

A transmission expansion model that aims to determine the optimal portfolio of measures to be taken to increase grid security has to take into account system outages and their consequences. It is critical to consider the likelihood of each contingency, otherwise, balancing the costs of investment, operation and corrective actions would be an impossible task [2]. However, these probabilities are rarely known, and usually can only be estimated through historical information or by utilizing fragility models developed for that purpose. This difficulty to obtain a reliable probability distribution for different contingency scenarios is the reason why it is convenient to utilize Distributionally Robust Optimization models, in which the probability distribution is not assumed to be known, and the decision is taken considering the worst-case distribution within a predefined ambiguity set, that can be adjusted according to the quality of the information available.

Furthermore, it is envisaged that network security, which has been historically delivered through asset redundancy, should evolve and hence be provided by emerging and innovative non-network technologies, especially those at the demand side in order to release latent network capacity and thus make more efficient use of the existing assets. In this vein, advanced technologies for post-contingency control, which can take advantage of a range of distributed energy resources (DER), can effectively provide security services and thus displace the need for redundant network capacity. The set of DER includes distributed generation (DG), backup generation, an array of storage technologies and demand itself (by utilizing the inherent demand-side flexibility, particularly from non-essential loads and demand associated with the heat and transport sectors). It is, therefore, paramount to determine the real cost of demand response as an alternative to further network investments.

To enhance the ability of the system to resist natural disasters, it is important to assess the benefits of a set of different measures, like investing in new transmission routes, reinforcing existing corridors, or investing in making existing network assets more resistant to external hazards. Earthquakes are natural disaster that affect a large amount of countries, and that can produce devastating effects on the power network. As analyzed in [3], the most common outages at transmission level during the earthquake that struck Chile in 2010 were on substations. It is, therefore, of great importance to assess strategies to protect substations against seismic hazards, if the objective is to enhance system resilience against earthquakes. The aforementioned strategy is to harden substations, which makes them less prone to fail, however, assessing this alternative poses an important modelling problem, because hardening substations changes the probability distribution of outages, so the model must be able to deal with decision dependent uncertainty. Actually, as the intention is not to depend on a single probability distribution, but rather on an ambiguity set, the decision to harden substations changes the ambiguity set, making necessary the development of a Decision Dependent Ambiguity model.

1.2 Proposed Hypothesis

Distributionally Robust Optimization can be utilized to address the problem of incomplete or inaccurate reliability and fragility data in the context of transmission expansion planning, recognizing actual levels of information available, and allowing the participation of demand response, distributed generation, and other forms of DER in the provision of security of supply, substituting the role of network redundancy that is commonly needed when historical information regarding reliability and fragility data is ignored, as in the current operating and planning practices.

1.2.1 General Objective

- To propose and solve two transmission expansion models. First, a transmission network investment model with distributionally robust security, that can assess the contribution of distributed energy resources to security. And second, a decision dependent ambiguity model that is able to determine the optimal investment to enhance resilience against earthquakes, considering the alternative of hardening substations.

1.2.2 Specific Objectives

- Through study cases on the 24 busbar RTS, analyze the differences in network investment and security operational measures when utilizing the proposed distributionally robust security framework, compared to other deterministic and probabilistic approaches.

- Demonstrate scalability of the first model by applying it on different configurations to the IEEE 118 busbar system.
- Show the benefits of utilizing a decision dependent ambiguity model to determine the optimal resilient network design, by applying the second model to a 3-bus test system.
- Apply the second model to a simplified version of the Chilean system to demonstrate scalability, and to assess the benefits of hardening substations on a real system.

1.3 Contributions

- Develop a distributionally robust, 2-stage optimization model for the treatment of network security in TEP problems that appropriately captures the participation of DER in the provision of network security.
- Demonstrate that $n-1$ security approaches significantly undermine the value of DER in displacing network investments and generation reserves.
- Demonstrate that alternative stochastic approaches (fixed probabilities approaches) are optimistic regarding the value of DER in displacing network investments and generation reserves.
- Formulate a resilient network design model with endogenous ambiguity and outages on substations.
- Demonstrate the benefits of substation hardening in the context of transmission investment to enhance system resilience.

1.4 Structure of the Document

The document presents a five chapter structure, beginning by stating the motivations and objectives in Chapter 1. Chapter 2 presents a literature review on the fundamental concepts that will be used throughout this work. The transmission expansion model with distributionally robust security is formulated in Chapter 3, alongside with different study cases. Resilient transmission design with decision dependent ambiguity is formulated and studied in Chapter 4. In Chapter 5 main conclusions are drawn, and future work is proposed.

Chapter 2

Literature Review

2.1 Network Investment with Distributionally Robust Security

Smart control of distributed energy resources, can render important benefits to the grid, like enhancing energy efficiency, peak shaving, generation following, regulation ancillary services, and fast frequency response [4, 5]. Moreover, new technologies can be used to make them participate in the provision of security to the main system. It is important to assess the contribution that these resources spread throughout the distribution network can make at the transmission design stage, because they can potentially be used to decrease the levels of transmission redundancy, or generation reserves needed for the future network. References [6, 7, 8, 4] report on the possible contributions from DER to the security of supply of the main system.

Within the context of transmission expansion planning, outages have been considered either in a deterministic manner [9, 10, 11, 12] or in a probabilistic/stochastic fashion [13, 14, 15, 16]. When the deterministic approach is chosen, likelihood of generation and network outages is ignored. Such likelihood is key to balance pre- and post-contingency costs and therefore determine the right portfolio of network assets, generation reserves, and DER participation in the provision of security. On the other hand, probabilistic/stochastic approaches have been proposed to properly balance pre- and post-contingency generation, network and demand-side measures (by minimizing cost of network investment and expected cost of operation, including demand shedding/curtailment). However stochastic approaches assume perfect information of reliability data, which may be impractical. A more comprehensive review and discussion on deterministic and/versus probabilistic treatment of system security in network planning problems can be found in [2].

Within the context of robust optimization (RO), conservativeness has always been a matter of concern. The first approach presented by [17] proposed to deal with uncertainty by means of box constraints, which led to over-conservative solutions. In [18], ellipsoidal uncertainty is considered to alleviate conservativeness at the expense of sacrificing the benefits of linearity.

In order to propose a less conservative approach with linear robust counterparts, the work in [19] has developed a methodology that guarantees feasibility for a predefined number of coefficient changes. In addition, adjustable robust optimization (ARO) [20] has been proposed as a way to consider recourse decisions within the RO framework. Over the years, multiple problems have been addressed via robust optimization and several important theoretical results in RO have been derived [21]. Within power systems literature, some of the many relevant contributions are [22, 23, 24, 25] for operations and [9, 26, 27, 28] for expansion planning.

More recently, distributionally robust optimization (DRO) has been attracting a great deal of attention. Unlike stochastic optimization (SO), DRO does not assume full knowledge of the underlying process behind the realization of uncertainty. Nevertheless, DRO can properly take advantage of available moment information. From a theoretical point of view, some of the main contributions are [29, 30, 31]. While the use of distributionally robust approaches to uncertainty has been already proposed to analyze various problems in power system operation (see [32, 33, 34, 35]) and investment planning (see [36] and [37]), it has never been proposed a distributionally robust approach to network security for determining the right portfolio of DER services in transmission planning.

2.2 Resilient Network Design with Decision Dependent Ambiguity

Over the last decade several climatic catastrophes have occurred all over the globe, and they have produced loss of supply on hundreds of thousands, or even millions of people, and it is important to distinguish these disasters from traditional blackouts. A reliable power grid should be able to minimize the impacts of a blackout, however, the aforementioned high-impact low-probability catastrophes, are severe events the system has most likely never experienced before, and can lead to prolonged incapacitations of big areas of the system. A well-designed power grid must be able to withstand properly traditional blackouts, but also be resilient against less probable but severe disasters [38].

As stated in [39], the process that the power system undergoes when facing a high impact event is divided in three main phases. Immediately after the event hits resiliency level drops (phase I), until it reaches a degraded post-contingency state (phase II), in which the system stays until restoration begins to take place (phase III). How fast and severe is the degradation, how much time it takes to begin the recovery, and how promptly it is carried out, are used to assess the resilience of the power system. In [40] an end-to-end framework was developed to assess the effects of HILP events, and thus support decision making regarding the maintenance of a resilient power grid.

There are several strategies to enhance the resilience of the system [41], that can be classified as hardening if they entail changes to the grids infrastructure, or operational if they do not. Among hardening strategies, we can find adding redundant transmission paths, undergrounding lines, upgrading structures with more robust materials and elevating substations.

While the utilization of distributed energy resources (including demand), preventive control, taking advantage of microgrids, and switching grid configuration fall in the category of operational strategies [42]. In the literature, some of these operational strategies have been studied, like the utilization of demand side response (DSR) [43], microgrids formations at distribution level [44], generators re-dispatch [45] and topology switching [46]. Hardening strategies to increase resilience levels have also gained a lot of attention in the literature in recent years. An investment model based on relaxed AC Optimal Power Flow (OPF), is proposed in [47], which determines the optimal portfolio of network assets to improve the resilience of the system. In [48] the design of the power grid and the gas transportation system is cooptimized in such a way that it enhances the power grid resilience by replacing the function of transporting energy of some of its segments by underground gas pipelines.

Earthquakes are natural hazards that affect many countries around the world, including China, Chile, New Zealand, USA and Mexico, and they can inflict severe damage to critical infrastructure, and specifically to the power system, leading to losses of supply that can span several days [49]. Chile is a country that is constantly affected by these events, and it is always looking for ways to improve the response of its power system to seismic hazards. This has been clearly stated in the Chilean energy policy to 2050 [50], when in its first pillar, called "Security and Quality of Supply", it says that by that year the national energy system should be robust, resilient, and able to anticipate and face hazards like natural disasters. To reach this goal one of the lineaments proposed is to promote cost-effective infrastructure to face these critical situations, so it is of great importance to determine properly the infrastructure to be invested in.

The last big earthquake that struck Chile occurred in February 2010, reaching a magnitude of 8.8 on the momentum magnitude scale, and producing a massive blackout that affected more than 90% of the population of the country. According to [3], by far the biggest impact on transmission infrastructure was on substations, more than one of each four of the substations of the main transmission company were damaged by the earthquake. Seismic response of substations and their components has not been satisfactory, prompting research on this topic [51, 52, 53, 54]. Comparatively, only 1.6 out of 7280 km of transmission lines suffered damages, representing a 0.02%. This is why a model that aims to determine an optimal portfolio of design (hardening) measures to enhance resilience against earthquakes must take into account measures to make substations less prone to failure, alongside with investment in new transmission assets.

Substations with anchored components are less likely to fail in case of an earthquake [55], so hardening the substation by anchoring its components is a hardening strategy worth considering. However, adding the possibility to invest on hardening substations pose an important modelling difficulty when trying to balance the costs of the investment with the savings they will yield in case an earthquake strikes. Since hardening substations alters the probability distribution of outages, these probabilities are no longer exogenous, but parameters endogenously modified by the decision maker, meaning that is needed a model with Decision-Dependent Uncertainty (DDU).

Optimization models with DDU have been extensively studied lately [56, 57, 58], and they have been applied to global climate policy making [59] and to plan oil and gas infrastructure [60]. In the context of power systems they have been applied to model endogenous uncertainty on demand side response participation [61], and on technology innovation [62]. Regarding the decision of hardening network components, in order to enhance grids resilience, in [63] the authors propose a model capable of assessing the benefits of hardening substations, circumventing the inherent nonlinearity using Optimization via Simulation. In [64] and [65] hardening of network assets (transmission and distribution lines respectively) is decided utilizing an attacker-defender scheme, however, decisions were made to be hedged against the worst-case contingency rather than considering the likelihoods of each one.

To optimally balance the cost of resilience-enhancing strategies with their benefits, the model has to take into account the probability distribution of outages, either in normal operation and when the earthquake strikes. Historical data can be used to estimate the failure rates in the case of normal operation, and earthquakes models and fragility curves when the hazard occurs. However, the decision maker never truly knows the probability distribution, therefore, distributionally robust optimization can be proposed to address this problem. As stated above, DRO has been used to tackle the uncertainty on failure rates [32, 66], but always within the context of reliability and not on network resiliency. In [64] the authors apply DRO to a resiliency problem, but they consider ambiguity on wind availability rather than in failure rates.

Chapter 3

Network Investment with Distributionally Robust Security

3.1 Mathematical Formulation

3.1.1 Overview

The model proposed in this chapter aims to determine the optimal set of transmission network investments by balancing the costs of network investments against the corresponding costs of network operation pre- and post-fault, including the costs of network congestions, generation reserves, demand and generation curtailments through special protection schemes (SPS) and, importantly, the costs of an array of DER post-contingency services. Note that our model has been designed from the transmission network planner's perspective and thus it selects the right portfolio of DER services among those being offered by aggregators. As transmission network operators and planners have no jurisdiction over distribution networks, DER sizing is out of the scope of this work.

In this context, Fig. 3.1 illustrates that there are several alternatives to network investment in order to increase secured power transfers during a pre-fault condition, comprising utilization of DER (to increase flexible demand in the exporting area and reduce flexible demand in the importing area in case, for instance, a line fault occurs), SPS (to curtail generation in the exporting area and demand in the importing area in case a line fault occurs), and even generation reserves (to reschedule generation post-fault and accommodate power transfers in case a line fault occurs). Furthermore, doing nothing is also an option if the cost of congestion (i.e. cost of network operation without the increase in power transfers pre-fault, that can include the cost of wind spillage) is proved very small.

Note that although the expansion of the transmission network has to be undertaken, in general terms, due to the increasing connections of renewable generation in exporting areas and demand resources in importing areas (for example, from the transport and heat sectors), the investment levels needed to deal with such transmission expansion can be alleviated if

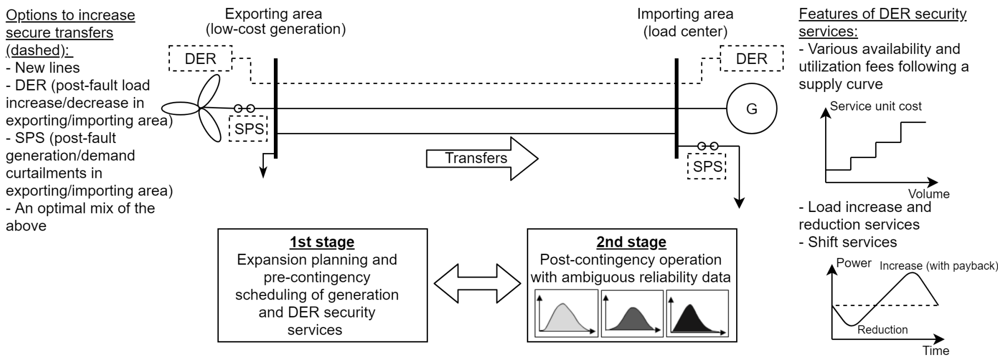


Figure 3.1: Diagram illustrating the TEP problem and the general methodology

the right portfolio of corrective, post-contingency measures is deployed to control the outputs/levels of such renewables and demand resources. As explained in [16], post-contingency, corrective control actions can successfully reduce network congestions and thus release latent network capacity of the existing assets, displacing the need for further network investments. In the case of DER, we assume that aggregators can provide three instrumental services to system operators from DER under a contingency state, namely net load reduction, net load increase, and net load shifting. Although these services may be provided by a very large set of DER within a distribution network (including DG, energy storage systems and flexible load itself), they are presented/offered to transmission system operators in an aggregated fashion, using supply curves as shown in Fig. 3.1, where various response/volume levels (in MW) present different availability and utilization fees. In the case of the load shift service from DER, we also consider a payback effect [67], where the total energy consumption (in MWh) results, overall, higher when load is shifted.

To determine the optimal portfolio of DER security services within the TEP problem, we propose a two stage model as depicted in Fig. 3.1. The first stage of the model determines the transmission expansion plan and the scheduling of generation, up- and down-spinning reserves as well as the availability of DER post-contingency services. The second stage minimizes the expected cost of corrective actions under various contingencies. Since it is typically challenging to calculate precise probabilities of contingent scenarios (that is critical to determine DER support to post-contingency network congestion), in this work, we assume limited knowledge of the underlying process behind the realization of such contingencies. Hence, we propose a formulation capable to solve the TEP problem while assuming "ambiguous" reliability data, simultaneously comprising several probability distributions of failure rates as shown in Fig. 3.1.

Note that as our focus is on the role of network redundancy for providing network security and how DER can efficiently compete as an alternative measure against such network redundancy, we focus on system outages rather than other sources of uncertainty in the short term. Note also that we use a static (rather than dynamic) model in the sense that it considers only one year (not many) and comprises two stages, pre and post-fault across various operating conditions. These assumptions are commonly used and well accepted in network reliability studies [11, 13, 68, 14, 16, 2].

3.1.2 Model

The two-stage TEP model with post-contingency DER services and ambiguity in the underlying process behind the realization of outages is mathematically written as follows:

$$\underset{\substack{\Delta D^+_{bte}, \Delta D^-_{bte}, \Delta^{FD}_{bt}, \theta_{bt},\\ f_{lt}, p_{it}, r^{d}_{it}, r^{u}_{it}, u_{it}, v_l}}{\text{Minimize}} \sum_{t \in T} h_t \Big[\sum_{i \in I} \big(C^p_i p_{it} + C^{d}_i r^{d}_{it} + C^u_i r^u_{it} \big)$$

$$+ \sum_{b \in N} \big(C^{FD}_b \Delta^{FD}_{bt} + \sum_{e \in \mathcal{E} \setminus \{NE\}} \big(C^{I+}_{be} \Delta D^+_{bte} + C^{I-}_{be} \Delta D^-_{bte} \big) \big)$$

$$+ \sup_{\mathcal{Q} \in \mathcal{P}_t} \mathbb{E}_{\mathcal{Q}} \{ H_t \big(p_{it}, r^{d}_{it}, r^u_{it}, \Delta^{FD}_{bt}, \Delta D^+_{bte}, \Delta D^-_{bte}, v_l, \boldsymbol{a}_t \big) \} \Big]$$

$$+ \sum_{l \in \mathcal{L}^C} C^L_l v_l \tag{3.1}$$

subject to:

$$\sum_{i \in I_b} p_{it} + \sum_{l \in \mathcal{L} | to(l) = b} f_{lt} - \sum_{l \in \mathcal{L} | fr(l) = b} f_{lt} = D_{bt};$$

$$\forall b \in N, t \in \mathcal{T} \tag{3.2}$$

$$-\overline{F}_l \leq f_{lt} \leq \overline{F}_l; \forall l \in \mathcal{L}^E, t \in \mathcal{T} \tag{3.3}$$

$$-v_l \overline{F}_l \leq f_{lt} \leq v_l \overline{F}_l; \forall l \in \mathcal{L}^C, t \in \mathcal{T} \tag{3.4}$$

$$f_{lt} = \frac{1}{x_l} (\theta_{fr(l),t} - \theta_{to(l),t}); \forall l \in \mathcal{L}^E, t \in \mathcal{T} \tag{3.5}$$

$$-M(1 - v_l) + \frac{1}{x_l} (\theta_{fr(l),t} - \theta_{to(l),t}) \leq f_{lt}$$

$$\leq \frac{1}{x_l} (\theta_{fr(l),t} - \theta_{to(l),t}) + M(1 - v_l); \forall l \in \mathcal{L}^C, t \in \mathcal{T} \tag{3.6}$$

$$p_{it} - r^{d}_{it} \geq \underline{P}_i u_{it}; \forall i \in I, t \in \mathcal{T} \tag{3.7}$$

$$p_{it} + r^u_{it} \leq \overline{P}_i u_{it}; \forall i \in I, t \in \mathcal{T} \tag{3.8}$$

$$0 \leq r^{d}_{it} \leq \overline{R}^D_i; \forall i \in I, t \in \mathcal{T} \tag{3.9}$$

$$0 \leq r^u_{it} \leq \overline{R}^U_i; \forall i \in I, t \in \mathcal{T} \tag{3.10}$$

$$p_{wt} \leq \zeta_{wt} \overline{P}_i; \forall w \in \mathcal{W}, t \in \mathcal{T} \tag{3.11}$$

$$0 \leq \Delta^{FD}_{bt} \leq \overline{\Delta}^{FD}_{bt}; \forall b \in N, t \in \mathcal{T} \tag{3.12}$$

$$0 \leq \Delta D^+_{bte} \leq \overline{\Delta D^+}_{bte}; \forall b \in N, t \in \mathcal{T}, e \in \mathcal{E} \setminus \{NE\} \tag{3.13}$$

$$0 \leq \Delta D^-_{bte} \leq \overline{\Delta D^-}_{bte}; \forall b \in N, t \in \mathcal{T}, e \in \mathcal{E} \setminus \{NE\} \tag{3.14}$$

$$v_l \in \{0, 1\}; \forall l \in \mathcal{L}^C \tag{3.15}$$

$$u_{it} \in \{0, 1\}; \forall i \in I, t \in \mathcal{T}, \tag{3.16}$$

where sets $\mathcal{E}$, I, I_b, $\mathcal{L}$, $\mathcal{L}^C$, $\mathcal{L}^E$, N, $\mathcal{P}_t$, $\mathcal{T}$, $\mathcal{W}$ include (in this order) indexes of steps for power imbalance costs (in this work, steps refer to the segments −or piecewise constant values− in the supply curve shown in Fig. 3.1), indexes of all generators, indexes of generators connected to bus b, indexes of all transmission lines, indexes of candidate transmission lines, indexes of existing transmission lines, indexes of buses, probability distributions, indexes of time blocks (used to discretize yearly operation into a few operating points), and indexes of renewable generators such as wind and solar. The decision variables are scheduled load increase (or disconnection of DG), $\Delta D^{+}_{bt\mathrm{e}}$, scheduled DG output increase (or load disconnection), $\Delta D^{-}_{bt\mathrm{e}}$, scheduled load shifting, Δ^{FD}_{bt}, voltage angles, θ_{bt}, power flows, f_{lt}, power outputs, $p_{\mathrm{i}t}$, scheduled down- and up-spinning reserves, $r^{\mathrm{d}}_{\mathrm{i}t}$ and $r^{u}_{\mathrm{i}t}$, as well as construction of candidate lines, v_l, and commitment of dispatchable units, $u_{\mathrm{i}t}$. Coefficients C^{p}_{i}, $C^{\mathrm{d}}_{\mathrm{i}}$, C^{u}_{i}, C^{FD}_{b}, $C^{I+}_{b\mathrm{e}}$, $C^{I-}_{b\mathrm{e}}$, and C^{L}_{l} represent cost of generation, scheduling of down- and up-spinning reserves, scheduling of load shifting, scheduling of load increase (or disconnection of DG), scheduling of DG output increase (or load disconnection), and investment in transmission lines. Parameters $\overline{\Delta D^{+}}_{bt\mathrm{e}}$, $\overline{\Delta D^{-}}_{bt\mathrm{e}}$, $\overline{\Delta}^{FD}_{bt}$, ζ_{wt}, D_{bt}, $\overline{F}_l$, M, NE, $\underline{P}_{\mathrm{i}}$, $\overline{P}_{\mathrm{i}}$, $\overline{R}^{D}_{\mathrm{i}}$, $\overline{R}^{U}_{\mathrm{i}}$, x_l, and h_t correspond to maximum load increase (or disconnection of DG), maximum DG output increase (or load disconnection), maximum load shifting, available fraction of renewable generation at a given period, nominal demands, power flow capacities, sufficiently large constant, number of steps for power imbalance cost, minimum stable generations, maximum stable generations, maximum capacities of down-spinning reserve, maximum capacities of up-spinning reserve, reactances of lines, and number of hours in each time block, respectively. Finally, $H_t(\boldsymbol{q}_t, \boldsymbol{a}_t)$ represents generation cost for a given first-stage decision, $\boldsymbol{q}_t$, and $\boldsymbol{a}_t$ is a random vector associated with the availability of system elements (generators and transmission lines), i.e, $\boldsymbol{a}_t = (\boldsymbol{a}^{G^T}_t, \boldsymbol{a}^{L^T}_t)^T$.

The objective function (3.1) to be minimized includes costs of generation and scheduling of post-contingency services, namely, down- and up-spinning reserves, load shifting, load increase (or disconnection of DG), and DG output increase (or load disconnection) as well as expected value of second-stage operation cost, and cost of construction of candidate lines. Nodal power balance is modeled by constraints (3.2). Constraints (3.3) and (3.4) enforce power flow limits for existing and candidate lines, respectively. In a DC load flow fashion, constraints (3.5) and (3.6) represent power transfers through existing and candidate lines, respectively. Constraints (3.7)–(3.10) impose limits to pre-contingency power generation as well as down- and up-spinning reserves scheduling for each dispatchable unit. Similarly, constraints (3.11) limit the generation of renewable generation units. Constraints (3.12)–(3.14) model maximum capacities of post-contingency DER services. It should be noted that the last step e of variables $\Delta D^{+}{}_{bt\mathrm{e}}$ and $\Delta D^{-}{}_{bt\mathrm{e}}$ is not scheduled in the first stage since such step corresponds to generation and demand curtailments (i.e. SPS), which are highly penalized in the second stage. Finally, constraints (3.15) and (3.16) enforce the binary nature of investment and commitment variables.

The operation under contingency can be modeled as:

$$H_t(\boldsymbol{q}_t, \boldsymbol{a}_t) = \underset{\substack{\Delta^{FD+c}_{bty}, \Delta^{FD-c}_{bty},\\ \Delta D^{+c}_{bty\mathrm{e}}, \Delta D^{-c}_{bty\mathrm{e}},\\ \theta^{c}_{bty}, f^{c}_{lty}, p^{c}_{\mathrm{i}ty},\\ r^{\mathrm{dc}}_{\mathrm{i}ty}, r^{uc}_{\mathrm{i}ty}}}{\text{Minimize}} \sum_{y \in \mathcal{Y}} \Big[\sum_{\mathrm{i} \in I} \left(C^{\mathrm{dc}}_{\mathrm{i}} r^{\mathrm{dc}}_{\mathrm{i}ty} + C^{uc}_{\mathrm{i}} r^{uc}_{\mathrm{i}ty} \right) \Delta t$$

$$+\sum_{b\in N,\mathrm{e}\in\mathcal{E}}\Big(C^{I+^c}_{\mathrm{be}}\Delta D^{+c}_{bty\mathrm{e}}+C^{I-^c}_{\mathrm{be}}\Delta D^{-c}_{bty\mathrm{e}}\Big)\Delta t\Big]$$

$$+\sum_{b\in N}C^{FD^c}_{b}\Delta^{FD-c}_{bt1} \tag{3.17}$$

subject to:

$$\sum_{\mathrm{i}\in I_b}p^c_{\mathrm{i}ty}+\sum_{l\in\mathcal{L}|to(l)=b}f^c_{lty}-\sum_{l\in\mathcal{L}|fr(l)=b}f^c_{lty}+$$
$$\sum_{\mathrm{e}\in\mathcal{E}}\left(\Delta D^{-c}_{bty\mathrm{e}}-\Delta D^{+c}_{bty\mathrm{e}}\right)+\Delta^{FD-c}_{bty}-\Delta^{FD+c}_{bty}=D_{bt}:$$
$$(\lambda_{bty});\forall b\in N,y\in\mathcal{Y} \tag{3.18}$$

$$-a^L_{lt}\overline{F}_l\le f^c_{lty}\le a^L_{lt}\overline{F}_l:(\phi^+_{lty},\phi^-_{lty});\forall l\in\mathcal{L}^E,$$
$$y\in\mathcal{Y} \tag{3.19}$$

$$-a^L_{lt}v_l\overline{F}_l\le f^c_{lty}\le a^L_{lt}v_l\overline{F}_l:(\phi^{n+}_{lty},\phi^{n-}_{lty});\forall l\in\mathcal{L}^C,$$
$$y\in\mathcal{Y} \tag{3.20}$$

$$-M(1-a^L_{lt})+\frac{1}{x_l}(\theta^c_{fr(l),ty}-\theta^c_{to(l),ty})\le f^c_{lty}$$
$$\le\frac{1}{x_l}(\theta^c_{fr(l),ty}-\theta^c_{to(l),ty})+M(1-a^L_{lt}):(\mu^+_{lty},\mu^-_{lty});$$
$$\forall l\in\mathcal{L}^E,y\in\mathcal{Y} \tag{3.21}$$

$$-M(1-v_la^L_{lt})+\frac{1}{x_l}(\theta^c_{fr(l),ty}-\theta^c_{to(l),ty})\le f^c_{lty}$$
$$\le\frac{1}{x_l}(\theta^c_{fr(l),ty}-\theta^c_{to(l),ty})+M(1-v_la^L_{lt}):(\mu^{n+}_{lty},\mu^{n-}_{lty});$$
$$\forall l\in\mathcal{L}^C,y\in\mathcal{Y} \tag{3.22}$$

$$p^c_{\mathrm{i}ty}=p_{\mathrm{i}t}a^G_{\mathrm{i}t}+r^{uc}_{\mathrm{i}ty}-r^{\mathrm{dc}}_{\mathrm{i}ty}:(\eta_{\mathrm{i}ty});\forall\mathrm{i}\in I,y\in\mathcal{Y} \tag{3.23}$$

$$0\le r^{\mathrm{dc}}_{\mathrm{i}ty}\le r^{\mathrm{d}}_{\mathrm{i}t}a^G_{\mathrm{i}t}:(\kappa^-_{\mathrm{i}ty});\forall\mathrm{i}\in I,y\in\mathcal{Y} \tag{3.24}$$

$$0\le r^{uc}_{\mathrm{i}ty}\le r^{u}_{\mathrm{i}t}a^G_{\mathrm{i}t}:(\kappa^+_{\mathrm{i}ty});\forall\mathrm{i}\in I,y\in\mathcal{Y} \tag{3.25}$$

$$p^c_{\mathrm{i}t1}\le\left(p_{\mathrm{i}t}+RU_{\mathrm{i}}\frac{\Delta t}{2}\right)a^G_{\mathrm{i}t}:(\chi^+_{\mathrm{i}t});\forall\mathrm{i}\in I \tag{3.26}$$

$$p^c_{\mathrm{i}t1}\ge\left(p_{\mathrm{i}t}-RD_{\mathrm{i}}\frac{\Delta t}{2}\right)a^G_{\mathrm{i}t}:(\chi^-_{\mathrm{i}t});\forall\mathrm{i}\in I \tag{3.27}$$

$$p^c_{\mathrm{i}ty}-p^c_{\mathrm{i}ty-1}\le RU_{\mathrm{i}}\Delta t:(o^+_{\mathrm{i}ty});\forall\mathrm{i}\in I,y\in\mathcal{Y}\setminus\{1\} \tag{3.28}$$

$$p^c_{\mathrm{i}ty-1}-p^c_{\mathrm{i}ty}\le RD_{\mathrm{i}}\Delta t:(o^-_{\mathrm{i}ty});\forall\mathrm{i}\in I,y\in\mathcal{Y}\setminus\{1\} \tag{3.29}$$

$$0\le\Delta D^{+c}_{bty\mathrm{e}}\le\Delta D^+_{bt\mathrm{e}}:(\psi^+_{bty\mathrm{e}});\forall b\in N,y\in\mathcal{Y},$$
$$\mathrm{e}\in\mathcal{E}\setminus\{\mathrm{NE}\} \tag{3.30}$$

$$0\le\Delta D^{-c}_{bty\mathrm{e}}\le\Delta D^-_{bt\mathrm{e}}:(\psi^-_{bty\mathrm{e}});\forall b\in N,y\in\mathcal{Y},$$
$$\mathrm{e}\in\mathcal{E}\setminus\{\mathrm{NE}\} \tag{3.31}$$

$$0\le\Delta^{FD-c}_{bty}\le\Delta^{FD}_{bt}:(\rho_{bty});\forall b\in N,y\in\mathcal{Y} \tag{3.32}$$

$$\delta \sum_{y\in\mathcal{Y}} \Delta^{FD-c}_{bty} = \sum_{y\in\mathcal{Y}} \Delta^{FD+c}_{bty} : (\iota_{bt}); \forall b \in N \tag{3.33}$$

$$\Delta^{FD-c}_{bt1} \geq \Delta^{FD-c}_{bty} : (\beta_{bty}); \forall b \in N, y \in \mathcal{Y}\setminus\{1\} \tag{3.34}$$

$$\Delta D^{+c}_{bt1\mathrm{e}} \geq \Delta D^{+c}_{bty\mathrm{e}} : (\gamma^{+}_{bty\mathrm{e}}); \forall b \in N, y \in \mathcal{Y}\setminus\{1\}, \quad \mathrm{e} \in \mathcal{E}\setminus\{\mathrm{NE}\} \tag{3.35}$$

$$\Delta D^{-c}_{bt1\mathrm{e}} \geq \Delta D^{-c}_{bty\mathrm{e}} : (\gamma^{-}_{bty\mathrm{e}}); \forall b \in N, y \in \mathcal{Y}\setminus\{1\}, \quad \mathrm{e} \in \mathcal{E}\setminus\{\mathrm{NE}\} \tag{3.36}$$

$$\Delta D^{+c}_{b,t,y,NE} \leq D_{bt} : (\sigma^{+}_{bty}); \forall b \in N, y \in \mathcal{Y} \tag{3.37}$$

$$\Delta D^{-c}_{b,t,y,NE} \leq \sum_{\mathrm{i}\in I_b} p_{\mathrm{i}t} a^{G}_{\mathrm{i}t} : (\sigma^{-}_{bty}); \forall b \in N, y \in \mathcal{Y}, \tag{3.38}$$

where $\mathcal{Y}$ is the set of indexes of snapshots under contingency within each time block $t \in \mathcal{T}$. These snapshots are used to discretize post-contingency operation and capture evolution of relevant variables within 1 hour. For instance, if set $\mathcal{Y}$ comprises two snapshots, the first one is related to the first 30 minutes after the occurrence of an outage and the second one corresponds to the remaining 30 minutes. Parameters Δt, δ, RD_{i}, and RU_{i} represent time length of each snapshot (1 hour divided by the number of snapshots), payback for the load shifting service, ramp-down and ramp-up limit of each generator, respectively. Coefficients $C^{\mathrm{dc}}_{\mathrm{i}}$, C^{uc}_{i}, $C^{I+^c}_{b\mathrm{e}}$, $C^{I-^c}_{b\mathrm{e}}$, and $C^{FD^c}_{b}$ correspond to costs of utilizing scheduled down-spinning reserve, up-spinning reserve, DG disconnection (or load increase), load decrease (or DG output increase), and load shifting, respectively. The decision variables correspond to the positive and negative deviation of flexible demand from its nominal value, Δ^{FD+c}_{bty} and Δ^{FD-c}_{bty}, actual DG disconnection (or load increase), $\Delta D^{+c}_{bty\mathrm{e}}$, actual load decrease (or DG output increase) $\Delta D^{-c}_{bty\mathrm{e}}$, voltage angles under contingency, θ^{c}_{bty}, power transfers under contingency, f^{c}_{lty}, generation output under contingency, $p^{c}_{\mathrm{i}ty}$, utilized down- and up-spinning reserves under contingency, $r^{\mathrm{dc}}_{\mathrm{i}ty}$ and $r^{uc}_{\mathrm{i}ty}$.

The objective function (3.17) to be minimized in the system operation problem includes costs of utilizing scheduled post-contingency services of down- and up-spinning reserves, DG disconnection (or load increase), load decrease (or DG output increase), and load shifting. Analogously to (3.2)–(3.6), constraints (3.18)–(3.22) model nodal balance and power transfers under contingency. Constraints (3.23) relate post-contingency generation outputs to pre-contingency generation and scheduled reserves. Constraints (3.24) and (3.25) limit the utilization of down- and up-spinning reserves to the amounts scheduled in the first-stage. Constraints (3.26)–(3.29) impose ramp rate limits to the post-contingency generation. Note that constraints (3.26) and (3.27) (which are imposed only on the first snapshot) present a term equal to $\Delta t/2$ since this is the middle or reference point in time to which the variables of the first snapshot are referred. In (3.28) and (3.29), we do not divide Δt by 2 as the time length between the middle/reference points of two consecutive snapshots is equal to Δt. Constraints (3.30) and (3.31) enforce limits for actual DG disconnection (or load increase) and actual load decrease (or DG output increase), respectively, for all steps of power imbalance costs, except the last one which corresponds to involuntary generation curtailment $\left(\Delta D^{+c}_{b,t,y,NE}\right)$ or demand curtailment $\left(\Delta D^{-c}_{b,t,y,NE}\right)$. Constraints (3.32) and (3.33) model actual load shifting limits according to first-stage decision and load shifting payback, respectively. Constraints (3.34)–(3.36) impose as a rule that the first snapshot ($y = 1$) should present the highest values of load shifting as well as DG disconnection (or load increase) and load decrease (or DG output increase). We use this rule because the first snapshot represents the first minutes right after an outage occurs and therefore when the volume of corrective control measures is the highest. Finally, (3.37) and (3.38) impose limits on involuntary generation curtailment $\left(\Delta D^{+c}_{b,t,y,NE}\right)$ and demand curtailment $\left(\Delta D^{-c}_{b,t,y,NE}\right)$, respectively.

Note that, due to our focus on network security, ramp rate limits (and other operational details) have been ignored in the first stage but considered in the second stage in order to properly compare DER against utilization of reserve services right after an outage occurs. Interestingly, considering the lack of generation flexibility in the first stage (ignored in this work) can enhance the scope of the benefits associated with DER (which are associated with further ancillary services, apart from network security), but this is beyond the scope of this work.

3.1.3 Ambiguity Sets

Similar to [32], for each time block $t \in \mathcal{T}$, we consider the ambiguity set $\mathcal{P}_t$ which is constituted by the probability distributions associated with the $n-K$ criterion for a given knowledge level of failure probabilities. The ambiguity set $\mathcal{P}_t$ can be mathematically described as:

$$\mathcal{P}_t = \left\{ \mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \mathbb{E}_{\mathcal{Q}}[S\hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t \right\}, \tag{3.39}$$

where $\hat{\boldsymbol{a}}_t = \mathbb{1} - \boldsymbol{a}_t$. Vector $\overline{\boldsymbol{\mu}}_t$ and matrix S correspond to estimated values of means and an auxiliary matrix of coefficients.

In addition, we define:

$$\mathcal{A} = \Big\{ (\boldsymbol{a^G}, \boldsymbol{a^L}) \in \{0,1\}^{|I|} \times \{0,1\}^{|\mathcal{L}|} : \sum_{\mathrm{i} \in I} a^G_{\mathrm{i}} + \sum_{l \in \mathcal{L}} a^L_l \geq n - K \Big\}, \tag{3.40}$$

where n is the number of system elements ($n = |I| + |\mathcal{L}|$) and K is the security parameter, which is a predefined number of simultaneous outages.

In the presented methodology, $\boldsymbol{a}_t$ is a random vector that represents the availability of system elements (in our case, generators and transmission lines). In addition, set $\mathcal{A}$ is the support of $\boldsymbol{a}_t$. Within this context, $\mathcal{M}_+(\mathcal{A})$ contains all probability distributions on $\mathcal{A}$. Thus, $\mathcal{Q}$ is a probability distribution that belongs to $\mathcal{M}_+(\mathcal{A})$ such that the condition $\mathbb{E}_{\mathcal{Q}}[S\hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t$ is satisfied. Hence, for each time block t, $\mathcal{P}_t$ contains all probability distributions in $\mathcal{M}_+(\mathcal{A})$ that comply with $\mathbb{E}_{\mathcal{Q}}[S\hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t$.

In this work, within the context of transmission expansion planning, we compare three types of ambiguity sets, namely fixed probabilities, $n-K$ security, and interval probabilities, which are described next.

Fixed probabilities approach

In this case, we consider that failure probabilities are well-known, therefore, we have:

$$\begin{aligned}\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \mathbb{E}_{\mathcal{Q}}[1 - a^G_{\text{it}}] = p^G_{\text{it}}, \forall \text{i} \in I;\\ \mathbb{E}_{\mathcal{Q}}[1 - a^L_{lt}] = p^L_{lt}, \forall l \in \mathcal{L}\big\}; \forall t \in \mathcal{T}.\end{aligned} \tag{3.41}$$

For this approach $\overline{\boldsymbol{\mu}}_t$ should be chosen as $[p^T, -p^T]^T$, where p is the column vector of estimated failure rates. Matrix S would be $[\mathbb{I}, -\mathbb{I}]^T \in \mathbb{R}^{2(|I|+|L|)\times(|I|+|L|)}$.

$n - K$ security approach

Opposite to the previous approach, in this case, we assume that failure probabilities are completely unknown. Hence, the ambiguity set is defined as:

$$\begin{aligned}\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : 0 \le \mathbb{E}_{\mathcal{Q}}[1 - a^G_{\text{it}}] \le 1, \forall \text{i} \in I;\\ 0 \le \mathbb{E}_{\mathcal{Q}}[1 - a^L_{lt}] \le 1, \forall l \in \mathcal{L}\big\}; \forall t \in \mathcal{T}.\end{aligned} \tag{3.42}$$

In this case, $\overline{\boldsymbol{\mu}}_t$ should be chosen as $[\vec{1}^T, \vec{0}^T]^T$, where $\vec{1}$ and $\vec{0}$ are column vectors of only ones and zeros in $\mathbb{R}^{|I|+|L|}$, respectively. Whereas matrix S is the same used for the previous case.

Interval probabilities approach

In this approach, we assume limited knowledge of the underlying process behind the realization of outages. This knowledge is characterized by a range of failure probabilities (i.e. ambiguity intervals), whose length depends on the quality of the historical data regarding outages of the element (more details on the definition of distributional sets under moment uncertainty can be found in [29]), and a bound of the overall system's failure rate. In this manner, the overall ambiguity set is written as:

$$\begin{aligned}\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \underline{p}^G_{\text{it}} \le \mathbb{E}_{\mathcal{Q}}[1 - a^G_{\text{it}}] \le \overline{p}^G_{\text{it}}, \forall \text{i} \in I;\\ \underline{p}^L_{lt} \le \mathbb{E}_{\mathcal{Q}}[1 - a^L_{lt}] \le \overline{p}^L_{lt}, \forall l \in \mathcal{L};\\ \sum_{\text{i}\in I} \mathbb{E}_{\mathcal{Q}}[1 - a^G_{\text{it}}] + \sum_{l\in\mathcal{L}} \mathbb{E}_{\mathcal{Q}}[1 - a^L_{lt}] \le \overline{p}_t\big\}; \forall t \in \mathcal{T}.\end{aligned} \tag{3.43}$$

The vector of estimated means, $\overline{\boldsymbol{\mu}}_t$, should be selected as $[\overline{p}^T, -\underline{p}^T, p]^T$ for this case. Where $\overline{p}$ is the column vector of upper bounds for the failure rates, $\underline{p}$ is the column vector for lower bounds, and p is the system wide failure rate. This time matrix S should be chosen as $[\mathbb{I}, -\mathbb{I}, \vec{1}]^T \in \mathbb{R}^{(2(|I|+|L|)+1)\times(|I|+|L|)}$.

3.2 Solution Methodology

The two-stage model (3.1)–(3.16) mathematically describes the TEP problem with an optimal portfolio of DER security services under ambiguity in failure probabilities of system equipments. Due to convenient convexity properties, this model is suitable for the use of Benders decomposition.

First, for simplicity purposes, the two-stage model (3.1)–(3.16) can be written in the following compact manner.

$$\begin{aligned}
&\underset{\boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in T} h_t\Big[\boldsymbol{c}^T\boldsymbol{q}_t^C + \boldsymbol{c}^{B^T}\boldsymbol{q}_t^B\Big] \\
&\qquad + \boldsymbol{c}^{L^T}\boldsymbol{q}^L + \sum_{t\in T} h_t \sup_{\mathcal{Q}\in\mathcal{P}_t} \mathbb{E}_{\mathcal{Q}}(H_t(\boldsymbol{q}_t,\boldsymbol{a}_t)) && (3.44)\\
&\text{subject to:}\\
&A\boldsymbol{q}_t \geq \boldsymbol{b}_t; \forall t\in\mathcal{T} && (3.45)\\
&\boldsymbol{q}^L \in \{0,1\}^{|\mathcal{L}^C|} && (3.46)\\
&\boldsymbol{q}_t^B \in \{0,1\}^{|I||T|}, && (3.47)
\end{aligned}$$

where $\boldsymbol{q}_t^B$, $\boldsymbol{q}_t^C$, and $\boldsymbol{q}^L$ represent vectors of binary operational variables, continuous operational variables, and binary investment variables, respectively. Note that $\boldsymbol{q}_t = [\boldsymbol{q}_t^{B^T}, \boldsymbol{q}_t^{C^T}, \boldsymbol{q}^{L^T}]^T$. In addition, the objective function (3.44) corresponds to (3.1), whereas constraints (3.45) group (3.2)–(3.14). Moreover (3.46) and (3.47) are related to (3.15) and (3.16), respectively.

Also, the operation under uncertainty (3.17)–(3.38) can be written as:

$$\begin{aligned}
H_t(\boldsymbol{q}_t,\boldsymbol{a}_t) = \quad & \underset{\boldsymbol{y}_t}{\text{Minimize}}\ \mathrm{d}_t^T\boldsymbol{y}_t && (3.48)\\
& \text{subject to:}\\
& B_t\boldsymbol{y}_t \geq \mathrm{e}_t : (\Theta_t) && (3.49)\\
& C_t\boldsymbol{y}_t \geq D_t\boldsymbol{q}_t + g_t : (\Phi_t) && (3.50)\\
& E_t\boldsymbol{y}_t \geq F_t(\boldsymbol{a}_t)\boldsymbol{q}_t + h_t(\boldsymbol{a}_t) : (\Omega_t) && (3.51)\\
& G_t\boldsymbol{y}_t \geq J_t(\boldsymbol{a}_t)\boldsymbol{q}_t + j_t : (\Lambda_t) && (3.52)\\
& K_t\boldsymbol{y}_t \geq s_t(\boldsymbol{a}_t) : (\Gamma_t), && (3.53)
\end{aligned}$$

where (3.48) represents (3.17). Expression (3.49) groups constraints (3.18), (3.28), (3.29), (3.33)–(3.37). Constraint (3.50) is associated with (3.30)–(3.32), whereas (3.51) is related to (3.26) and (3.27). Expression (3.52) represents (3.20), (3.22)–(3.25), and (3.38). Finally, constraint (3.53) corresponds to (3.19) and (3.21).

The steps related to the proposed solution methodology are described next.

3.2.1 Problem reformulation

Formulation (3.44)–(3.47) can be equivalently written as the following bilevel program:

$$\underset{\alpha_t, \boldsymbol{q}_t}{\text{Minimize}} \sum_{t \in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B \Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \sum_{t \in T} h_t \alpha_t \tag{3.54}$$

subject to:

$$\text{Constraints (3.45)–(3.47)} \tag{3.55}$$

$$\alpha_t = \Big\{ \underset{\mathcal{Q} \in \mathcal{P}_t}{\text{Maximize}} \sum_{a_t \in \mathcal{A}} H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big) \mathcal{Q}(\boldsymbol{a}_t) \tag{3.56}$$

subject to:

$$\sum_{a_t \in \mathcal{A}} S \hat{\boldsymbol{a}}_t \mathcal{Q}(\boldsymbol{a}_t) \leq \overline{\boldsymbol{\mu}}_t : (\boldsymbol{\pi}_t) \tag{3.57}$$

$$\sum_{a_t \in \mathcal{A}} \mathcal{Q}(\boldsymbol{a}_t) = 1 : (\varphi_t) \Big\}, \forall t \in \mathcal{T}, \tag{3.58}$$

where the adequate choice of matrix S and vector $\overline{\boldsymbol{\mu}}_t$ in expression (3.57) indicate which of the ambiguity sets defined in (3.41), (3.42), and (3.43) is considered.

In light of duality theory, model (3.54)–(3.58) can be rewritten as:

$$\underset{\boldsymbol{\pi}_t \geq 0, \varphi_t, \boldsymbol{q}_t}{\text{Minimize}} \sum_{t \in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B \Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \sum_{t \in T} h_t \big(\boldsymbol{\pi}_t^T \overline{\boldsymbol{\mu}}_t + \varphi_t \big) \tag{3.59}$$

subject to:

$$\text{Constraints (3.45)–(3.47)} \tag{3.60}$$

$$\boldsymbol{\pi}_t^T S \hat{\boldsymbol{a}}_t + \varphi_t \geq H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big), \forall t \in \mathcal{T}, \boldsymbol{a}_t \in \mathcal{A}. \tag{3.61}$$

It should be noted that constraints (3.61) can render formulation (3.59)–(3.61) computationally intractable due to their combinatorial nature. In order to circumvent this dimensionality curse, we rewrite (3.61) in the following manner.

$$\varphi_t \geq \max_{\boldsymbol{a}_t \in \mathcal{A}} \Big\{ H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big) - \boldsymbol{\pi}_t^T S \hat{\boldsymbol{a}}_t \Big\}, \forall t \in \mathcal{T}. \tag{3.62}$$

Based on the presented reformulation, we describe next subproblem, master problem, and solution algorithm.

3.2.2 Subproblem

At each iteration j, for a given first-stage solution $\boldsymbol{q}_t^{(j)}$, the subproblem identifies its associated worst-case contingency, $\boldsymbol{a}_t$, for each time block $t \in \mathcal{T}$. This identification is done by means of the problem formulated in the right hand side of (3.62). This problem is bilevel program since $H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big)$ corresponds to a minimization problem and it is therefore not aligned with the outer maximization in $\boldsymbol{a}_t \in \mathcal{A}$. However, since $H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big)$ is a linear program, we replace it by its dual (which is a maximization problem) in (3.62), include constraints (3.66)–(3.68), and linearize products of binary and continuous decision variables to develop a mixed integer linear programming (MILP) formulation hereinafter called subproblem. Hence, the subproblem can be represented by the compact formulation (3.63)–(3.68). The complete formulation of the subproblem can be found in [69].

$$\begin{aligned}
&\underset{\Theta_t,\Phi,\Omega_t,\Lambda_t,\Gamma_t,\boldsymbol{a}_t}{\text{Maximize}} \quad \sum_{t\in T} \big[\, \mathrm{e}_t^T \Theta_t + (g_t + D_t \boldsymbol{q}_t)^T \Phi_t \\
&+ (F_t(\boldsymbol{a}_t)\boldsymbol{q}_t + h_t(\boldsymbol{a}_t))^T \Omega_t + (J_t(\boldsymbol{a}_t)\boldsymbol{q}_t + j_t)^T \Lambda_t \\
&\qquad\qquad\qquad + s_t(\boldsymbol{a}_t)^T \Gamma_t - \boldsymbol{\pi}_t^T S \hat{\boldsymbol{a}}_t \big] \qquad (3.63)
\end{aligned}$$

subject to:

$$B_t^T \Theta_t + C_t^T \Phi_t + E_t^T \Omega_t + G_t^T \Lambda_t + K_t^T \Gamma_t = \mathrm{d}_t; \forall t \in T \qquad (3.64)$$

$$\Theta_t, \Phi_t, \Omega_t, \Lambda_t, \Gamma_t \geq 0; \forall t \in T \qquad (3.65)$$

$$\sum_{l\in\mathcal{L}} a_{lt}^L + \sum_{\mathrm{i}\in\mathrm{i}} a_{\mathrm{i}t}^G \geq n - K; \forall t \in T \qquad (3.66)$$

$$a_{lt}^L \in \{0,1\}; \forall l \in \mathcal{L}, t \in T \qquad (3.67)$$

$$a_{\mathrm{i}t}^G \in \{0,1\}; \forall \mathrm{i} \in I, t \in T \qquad (3.68)$$

3.2.3 Master problem

The master problem (3.69)–(3.71) is a relaxation of the original problem (3.1)–(3.16). This relaxation is achieved by replacing (3.62) by cutting planes in the equivalent reformulation of the original problem (3.59)–(3.60) and (3.62). The complete formulation of the master problem can be found in [69].

$$\begin{aligned}
&\underset{\boldsymbol{\pi}_t\geq 0,\varphi_t,\boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L \\
&\qquad\qquad\qquad + \sum_{t\in T} h_t \big(\boldsymbol{\pi}_t^T \overline{\boldsymbol{\mu}}_t + \varphi_t\big) \qquad (3.69)
\end{aligned}$$

subject to:

Constraints (3.45)–(3.47) (3.70)

$$\varphi_t \geq \mathrm{e}_t^T \Theta_t^{(j)} + (g_t + D_t \boldsymbol{q}_t)^T \Phi_t^{(j)}$$

$$+ (F_t(\boldsymbol{a}_t^{(j)})\boldsymbol{q}_t + h_t(\boldsymbol{a}_t^{(j)}))^T \Omega_t^{(j)} + (J_t(\boldsymbol{a}_t^{(j)})\boldsymbol{q}_t + j_t)^T \Lambda_t^{(j)}$$
$$+ s_t(\boldsymbol{a}_t^{(j)})^T \Gamma_t^{(j)} - \boldsymbol{\pi}_t^T S \hat{\boldsymbol{a}}_t^{(j)}; \forall j \in \mathcal{J}, t \in T \tag{3.71}$$

3.2.4 Solution algorithm

The procedure proposed in this section is an outer algorithm based on Benders decomposition. This outer algorithm is an iterative process that is carried out until the included cutting planes rend the solution of the relaxed problem (master problem) sufficiently close to optimality. The solution algorithm can be summarized as follows.

1. Initialization: set $j \leftarrow 0$.
2. Solve the optimization model (3.69)–(3.71), store $\boldsymbol{q}_t^{(j)}$, $\boldsymbol{\pi}_t^{(j)}$ and $\varphi_t^{(j)}$, and calculate
$$LB^{(j)} = \sum_{t \in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^{C(j)} + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^{B(j)} \Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^{L(j)} + \sum_{t \in T} h_t \big(\boldsymbol{\pi}_t^{(j)T} \overline{\boldsymbol{\mu}}_t + \varphi_t^{(j)} \big)$$
3. Identify the worst case contingency for stored $\boldsymbol{q}_t^{(j)}$ and $\boldsymbol{\pi}_t^{(j)}$ by running the subproblem, store values of its decision variables and calculate
$$UB^{(j)} = \sum_{t \in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^{C(j)} + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^{B(j)} \Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^{L(j)} + \sum_{t \in T} h_t \big(\boldsymbol{\pi}_t^{(j)T} \overline{\boldsymbol{\mu}}_t + \Psi_t^{(j)} \big),$$
where $\Psi_t^{(j)}$ is the value of the objective function of the subproblem for time block t.
4. If $\big(UB^{(j)} - LB^{(j)}\big)/UB^{(j)} \leq \varepsilon$, then
   STOP;
   else,
   CONTINUE.
5. Include in (3.69)–(3.71) a cutting plane of the format (3.71) with decision variables stored in step 3, set $j \leftarrow j + 1$, and go to step 2.

3.3 IEEE RTS Case Study

This section studies the economic and reliability performance of various security services provided by DER when co-optimized with network investments and other alternative operational measures to release network capacity. To do so, we introduce three approaches for the treatment of DER security services, namely improved $n-1$ security, fixed probabilities and interval probabilities. These three approaches will be compared against the traditional $n-1$ security approach that prevents the use of DER services to provide network security.

3.3.1 Input data

We modified the IEEE RTS described in [70] by adding:

1. 500 MW of thermal generation in buses 13,14,16,23.
2. 700 MW of peak load in buses 9,10, and 400 MW of peak load in bus 3.
3. 300 MW of wind generation in bus 12.

Also, as shown in 3.2, we consider the following 10 candidate network assets (in addition to the existing infrastructure informed in [70]) in the network planning problem (indicating end buses): 3-24, 9-11, 9-12, 10-11, 10-12, 11-13, 11-14, 12-13, 12-23, 15-24. Network investment costs for lines and transformers are 60 \$/MW.km.year and 20 k\$/MW.year, respectively. The annuity of the investment cost is balanced against the cost of one year of system operation, representing the state of the system when the transmission assets are already built, that is, years after the investment decisions have been originally made. Other relevant cost data includes a VoLL equal to 12 k\$/MWh, SPS utilization cost for generation curtailment equal to 1 k\$/MWh, and reserve utilization costs equal to the fuel costs of the corresponding generation technologies (fuel costs can be found in [71]). Cost of holding/scheduling generation reserves is considered to be costlier for fast generation technologies and equal to 20 \$/MW/h. For slow generation technologies, we consider a lower cost equal to 7 \$/MW/h. Regarding the pre-contingency operating conditions, they were clustered in 40 blocks that represent combinations of different demand and wind outputs across a year. Regarding post-fault operation, all $n-1$ contingencies are modeled throughout 1 hour, divided into 2 30-min snapshots. Outage rates of network and generation equipment are those presented in [70].

We consider DER post-contingency services available in 10 nodes with the following features:

1. Downwards DER service: Disconnections of flexible, non-essential loads and DG outputs increases that can contribute up to 13% of the nodal demand and respond right after a contingency occurs. Scheduling costs follow a 2-step supply curve similar to that illustrated in Fig.3.1 whose values are equal to 5 and 10 \$/MW/h for the first 8% and the following 5%, respectively. Likewise, utilization costs are equal to 50 and 80 \$/MWh.
2. Upwards DER service: Disconnections of DG and demand increases that can contribute up to an equivalent of 6% of nodal demand and respond right after a contingency occurs. Scheduling costs follow a 2-step supply curve similar to that illustrated in Fig.3.1 whose values are equal to 1 and 2 \$/MW/h for the first 4% and the following 2%, respectively. Likewise, utilization costs are equal to 20 and 30 \$/MWh.
3. Shift DER service: Shifts of flexible, non-essential loads and storage plants that can contribute up to 5% of nodal demand, responding right after a contingency occurs and recovering 30 min later. Scheduling and utilization costs are equal to 2 \$/MW/h and 5 \$/MW, respectively. The payback considers a 10% penalization in terms of energy consumption (parameter $\delta = 1.1$ in (3.33)).

3.3.2 Case studies

We analyze 4 approaches to consider DER security services in network investment planning:

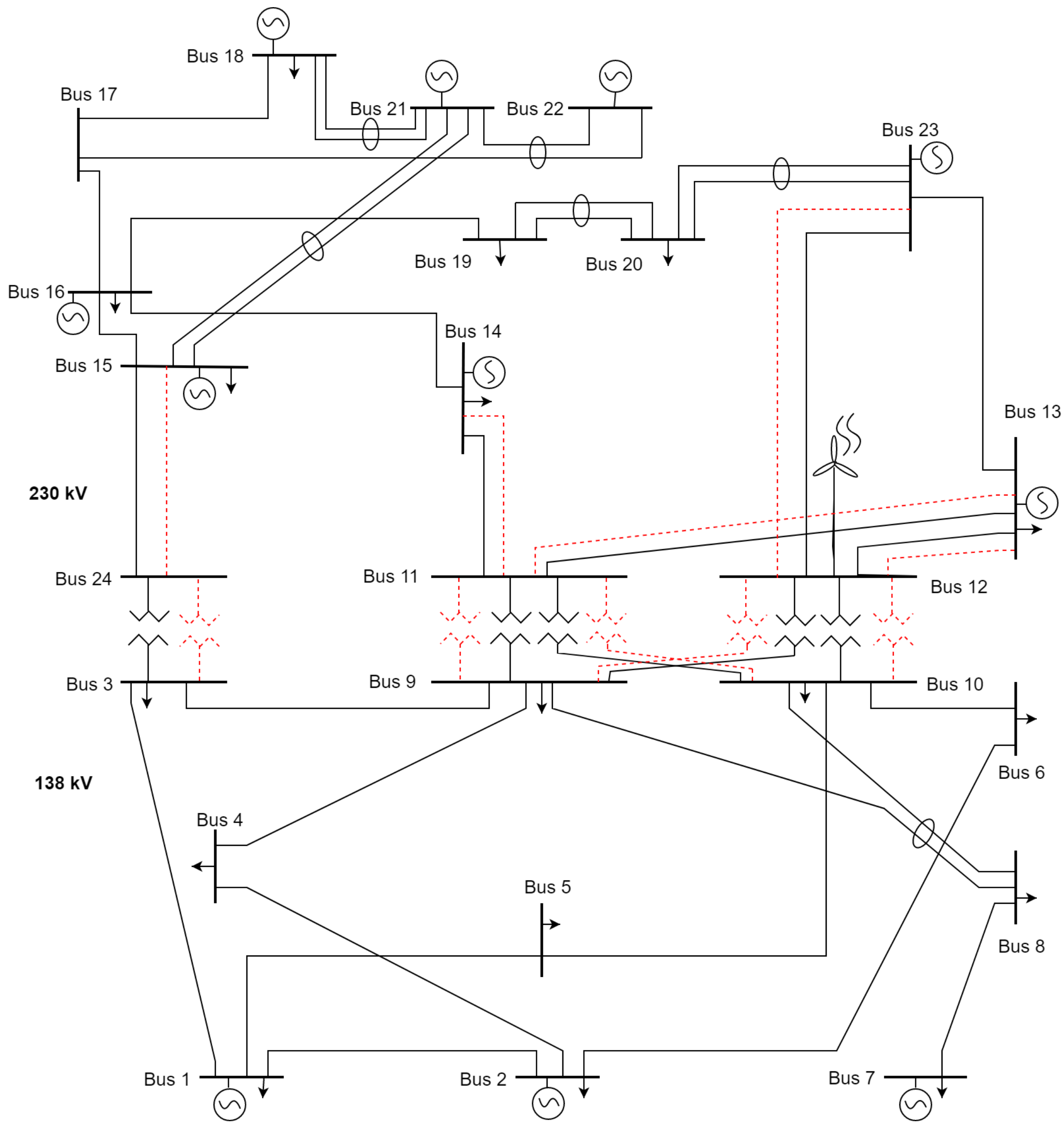


Figure 3.2: Modified 24-bus IEEE reliability test system.

1. Traditional $n-1$ security approach (baseline), where no DER is permitted.
2. Improved $n-1$ security approach, where DER services are used to provide security as long as involuntary demand curtailments through SPS are avoided. Post-contingency costs are, evidently, neglected since probabilities are ignored.
3. Fixed probabilities, where DER services are used to provide security in coordination with further post-contingency control measures (i.e. SPS).
4. Interval probabilities, where DER services are used to provide security in coordination with further post-contingency control measures, recognizing ambiguity in reliability data.

We use Julia version 0.6 and Gurobi [72] on a server with two 10-core processors (Intel Xeon E5-2660) and 48 GB of RAM.

3.3.3 Results and discussion

Table 3.1 presents a general overview of the results for each approach, where economic, reliability and physical output data are shown and compared. Discussion is provided next.

Table 3.1: Overall results of 4 DER approaches

Item	Traditional n-1 (baseline, no DER)	Improved n-1	Fixed probabilities	Interval probabilities
Investment cost [million $]	38.3	27.1	17.7	19.1
Operating cost (planned, pre-fault) [million $]	295.7	289.3	283.7	283.5
Reserve holding cost (all services) [million $]	60.6	19.1	12.5	12.7
DER holding cost (all services) [million $]	0	15.7	15.3	15.3
Expected costs of generation reserves DER SPS [million $]*	0.46 0 0.75	0.31 0.59 0.61	0.22 0.66 4.00	0.22 0.64 2.75
Total cost [million $]	395.8	352.71	334.1	334.2
$CVaR_{99\%}$ of SPS cost [million $]*	75.5	60.7	398.4	273.9
LOLE [h]*	0.48	0.40	7.67	4.06
No. of new network assets installed	7	5	3	4
Averaged upwards/ downwards generation reserve available [MW]	398/99	155/92	150/0	152/0
Averaged downwards/ upwards DER services available [MW]	0/0	250/29	250/9	250/17
Averaged shift DER service available [MW]	0	10	7	8

*Results obtained from an out-of-sample analysis of 30 million scenarios (3,000 random vectors where each vector element contains the failure rate of a system component uniformly distributed in the [-30%, 30%] ambiguity interval with respect to its reference value, times 10,000 contingencies (beyond $n-1$) for each of these vectors, considering exponentially distributed outages as indicated in [73]).

Case 1: Traditional $n-1$ solution (baseline, no DER)

Table 3.1 shows that, in the baseline case, the model invests significantly, including 7 network assets (3 lines and 4 transformers) at a total investment cost of \$38.3 million. Also, the amount of reserve scheduled is the highest and equal to 398 MW (on average across the year) in order to deal with all $n-1$ generation (and network) outages without demand participation, totalizing an annual scheduling cost equal to \$60.6 million. Given that DER is prevented to provide security services, high levels of redundancy in both transmission and generation are needed to secure the system.

Case 2: Improved $n-1$ solution with DER participation

In this case, Table 3.1 shows that the model invests in 5 network assets (2 lines and 3 transformers), totalizing an investment cost of \$27.1 million, which is considerably lower than that of the traditional $n-1$ approach. Furthermore, the model also schedules considerably less generation reserves, with a total reserve scheduling cost of \$19.1 million per year and an average of 155 MW of generation reserve margin across the year. Expectedly, this is possible due to the scheduling of DER services, which totalizes a cost equal to \$15.7 million. All of the above demonstrate, at least from a robust/deterministic point of view, the significant benefits of DER.

Case 3: Fixed probabilities approach solution with DER participation

In this case, the optimal portfolio of DER and further post-contingency control actions can significantly displace redundancy (network capacity and generation reserves), reducing network investment and generation reserve availability costs up to \$17.7 and 12.5 million, respectively.

Regarding the more efficient use of generation reserves, we observe an interesting interaction with DER services. In particular, we notice that optimally shifting non-essential loads allows operators to delay (rather than reduce) the need for generation reserve utilization. This enables the use of slower, less flexible thermal units (but more cost-effectively) to provide reserves services. Fig. 3.3 shows exercised volumes of reserves and DER services, demonstrating that faster (and more costly) generation reserves can be reduced and interchanged by slower (and lower-cost) generation reserves due to the use of shift DER services, improving the overall economic performance of post-contingency control actions to secure the power network.

Case 4: Interval probabilities approach solution with DER participation

We run the proposed interval probabilities approach to DER, where each failure rate is assumed to be within the $\pm 30\%$ ambiguity interval with respect to its reference value. Also, we assume (following [32]) that the aggregated system failure rate, i.e. overall number of

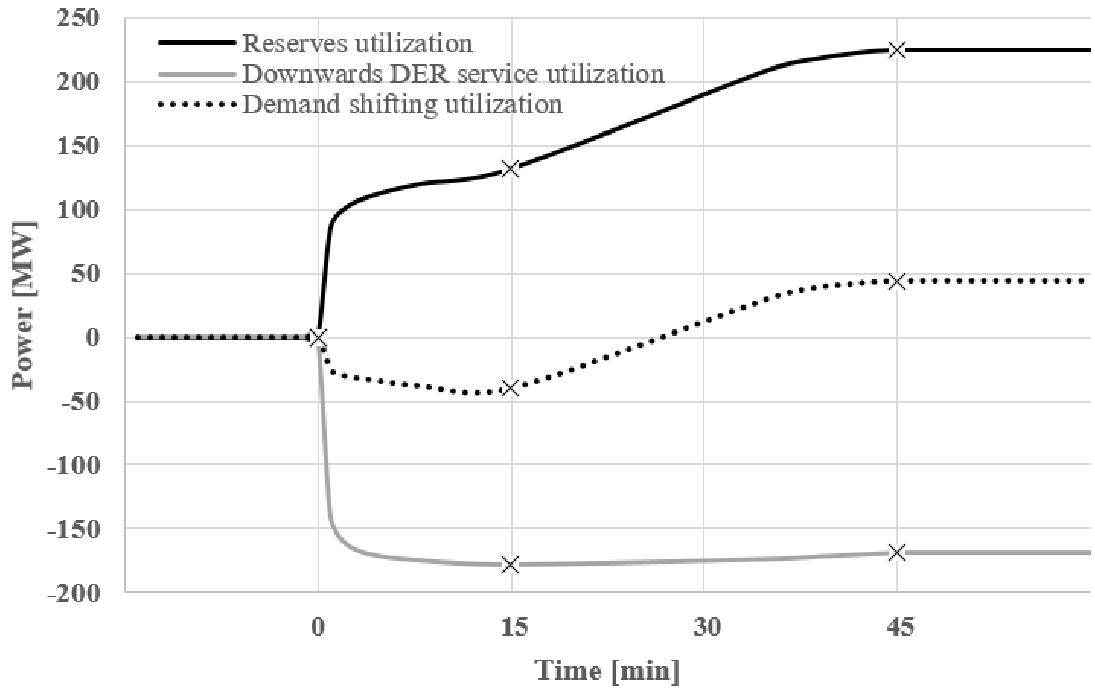


Figure 3.3: Aggregated utilization of reserves and DER services when facing an outage of a 350 MW generator on bus 23. Model results are indicated by crosses; for the sake of visualization, referential lines are provided.

outages at a system level, presents an upper bound and equal to that of the previous fixed probabilities approach (see Eq. (3.43)). This assumption properly captures that, at an aggregated level, uncertainty is more limited.

Because of the ambiguity in reliability data, the model hedges the system operation and investment solutions through higher volumes of network investment and generation reserves as shown in Table 3.1 in comparison with the fixed probabilities model. Furthermore, we found that the interval probabilities approach solution seeks to protect the system against the outages of highly loaded generating units and lines that, due to the ambiguity framework, become more prone to fail due to their potentially large impact. Similarly to the $n-1$ criterion that protects the system against the worst $n-1$ outage, the interval probabilities approach protects the system against the worst expected impact, that with the largest risk (i.e. worst expected cost; note that worst cases are decision dependent). In this context, this model decides to reinforce transmission corridors from/to busbar 11, which, after examining the resulting network operation, is proved to be the main hub of the system (the one with the highest volumes of energy transferred across a year).

3.3.4 Overall costs and risks: out-of-sample analysis

Table 3.1 shows material benefits of DER on total costs and risks, although the ultimate value of DER depends on the approach used towards probabilities. In fact, Table 3.1 shows that ignoring reliability data (like in the improved $n-1$ approach) significantly undermines the value of DER services. Furthermore, Table 3.1 demonstrates that the fixed probabilities approach can minimize total costs mainly through less network investment. Similarly, the interval probabilities approach also attempts to minimize total costs but when assuming that reliability data is not fully known, which drives a slightly higher network investment cost (in comparison with the fixed probabilities approach) in order to efficiently hedge the solution and thus decrease risks, as demonstrated in Table 3.1 through the $\text{CVaR}_{99\%}$ and LOLE ($\text{CVaR}_{99\%}$ corresponds to SPS curtailment cost on average in the worst 1% events and LOLE is calculated across the year following the general method described in [73]).

We have undertaken a sensitivity analysis in order to study the differences among various solutions determined by using different intervals applied on outage rates (e.g. 0%, ±10%, ±30%, ±50%, ±70%, ±90%, ±100%). These results are shown in Table 3.2. As expected, the larger the interval, the more conservative the solution. In fact, the network investment solution associated with an interval of ±10% is equal to that obtained by running the model with fixed probabilities (i.e. 0% interval), which leads to the larger cost of network investment. Also, for intervals higher than ±30%, we observed more network investment (than that of the case with ±10% interval), but no differences in network investments within the entire range between ±30% and ±100%. Significant changes in the investment propositions can be observed, however, in both n-1 solutions (improved and traditional), which are equivalent to use the complete 0-1 ambiguity interval (with and without DER, respectively). This demonstrates that, from a network investment perspective, the results (in this case) obtained by the proposed probabilities interval approach are reasonably robust against changes in the length of the intervals used, but fundamentally different from those classical solutions obtained through the application of the n-1 security approach and the fixed probabilities approach.

3.4 118-busbar System Case Study

3.4.1 Input data

This section demonstrates the scalability of our distributionally robust approach to DER. To do so, we apply it on the IEEE 118-busbar test system presented in [74]. We add 1300 MW of extra demand so as to increase congestion levels and consider 10 candidate lines. Regarding reliability data, reference outage rates are 0.001 occ/h and 0.00011 occ/h for generators and lines (every 100km), respectively considering a ±30% ambiguity interval. A total of 10 buses provide DER security services in a similar way as in the previous IEEE RTS case study.

3.4.2 Results and discussion

Table 3.3 demonstrates the scalability of our modelling approach against different volumes of data. In effect, 10 and 20 demand levels (or blocks) are considered across a year. Also,

Table 3.2: Sensitivity analysis

Item	**0% interval**	**±10% interval**	**±30% interval**	**±100% interval**
Investment cost [million $]	17.7	17.7	19.1	19.1
Operating cost (planned, pre-fault) [million $]	283.7	283.8	283.5	283.4
Reserve holding cost (all services) [million $]	12.5	12.8	12.7	13.5
DER holding cost (all services) [million $]	15.3	15.4	15.3	15.5
Expected costs of				
generation reserves	0.22	0.22	0.22	0.22
DER	0.66	0.66	0.64	0.63
SPS	4.00	3.65	2.75	2.16
[million $]*				
Total cost [million $]	334.1	334.2	334.2	334.5

*Results obtained from an out-of-sample analysis of 30 million scenarios (3,000 random vectors where each vector element contains the failure rate of a system component uniformly distributed in the [-30%, 30%] ambiguity interval with respect to its reference value, times 10,000 contingencies (beyond $n-1$) for each of these vectors, considering exponentially distributed outages as indicated in [73]).

we demonstrate that the time resolution in the post-fault conditions can be improved in order to more accurately represent the post-contingency evolution of demand services. These results also demonstrate the advantages of parallel computing, reducing computational time by almost 5 times, although RAM memory resources are increased by more than 3 times. In this particular case, Table 3.3 shows that investment decisions are more sensitive to post-fault rather than pre-fault time resolution, which demonstrates the importance of modelling appropriately the evolution of different DER services during post-contingency conditions in transmission network investment, as proposed in this work.

Table 3.3: Results of the 118-busbar System Case Study

Item	Case #1	Case #2	Case #3	Case #4
Number of time blocks	10	20	10	10
Number of snapshots	2	2	3	2
Serial or parallel	Parallel	Parallel	Parallel	Serial
Execution time [min]	177	289	353	861
Maximum RAM used [GB]	14	16	15	4
Investment cost [million $]	0.84	0.84	0.72	0.84
DER holding cost (all services) [million $]	10.19	10.13	10.38	10.19

Chapter 4

Resilient Network Design with Decision Dependent Ambiguity

Nomenclature

Constants

Δt Duration of each snapshot [h].

δ Energy payback for load shifting.

$\overline{\Delta D^{+}}_{be}$ Maximum load increase/generation reduction to be scheduled [MW].

$\overline{\Delta D^{-}}_{be}$ Maximum load reduction/generation increase to be scheduled [MW].

$\overline{\Delta}_{b}^{FD}$ Maximum flexible demand to be scheduled [MW].

$\overline{\mu}_{ts}$ Auxiliary vector used to construct the ambiguity set.

$\overline{F}_{l}$ Maximum power flow through lines [MW].

$\overline{P}_{\mathrm{i}}$ Maximum power output [MW].

$\overline{R}_{\mathrm{i}}^{D}$ Maximum capacity of downward reserves [MW].

$\overline{R}_{\mathrm{i}}^{U}$ Maximum capacity of upward reserves [MW].

$\underline{P}_{\mathrm{i}}$ Minimum stable generation [MW]

ζ_{w} Fraction of the total capacity available for a renewable generator.

$C_{\mathrm{i}}^{\mathrm{d}}$ Scheduling cost of downwards reserves [$/MW].

C_b^h Cost of hardening substations [$].

C_l^L Investment cost of candidate lines [$].

C_{i}^p Generation cost [$/MWh].

C_{i}^u Scheduling cost of upward reserves [$/MW].

$C_{\mathrm{i}}^{\mathrm{dc}}$ Downward reserves utilization cost [$/MWh].

C_b^D Load shedding cost [$/MWh].

$C_b^{FD^c}$ Demand shifting utilization cost [$/MW].

C_b^{FD} Scheduling cost of demand shifting [$/MW].

C_{i}^G Generation shedding cost [$/MWh].

$C_{b\mathrm{e}}^{I+^c}$ Load increase/generation reduction utilization cost [$/MWh].

$C_{b\mathrm{e}}^{I+}$ Scheduling cost of load increase/generation reduction [$/MW].

$C_{b\mathrm{e}}^{I-^c}$ Load reduction/generation increase utilization cost [$/MWh].

$C_{b\mathrm{e}}^{I-}$ Scheduling cost of load reduction/generation increase [$/MW].

C_{i}^{uc} Upward reserves utilization cost [$/MWh].

D_b Nominal demand [MW].

h_t Number of hours represented by the time block.

K_{ts} Auxiliary matrix used to modify the ambiguity set when a substation is hardened.

M Large constant.

p_{ts} Probability of a given scenario of each time block.

S_{ts} Auxiliary matrix used to construct the ambiguity sets.

x_l Reactance [Ω].

Sets

$\mathcal{D}_D$ Indexes for damage states of demand.

$\mathcal{D}_N$ Indexes for damage states of substations.

$\mathcal{E}$ Indexes for power imbalance steps.

$\mathcal{L}$ Indexes of all lines.

$\mathcal{L}^C$ Indexes of candidate lines.

$\mathcal{L}^E$ Indexes of existing lines.

$\mathcal{P}_{ts}$ Probability distributions.

$\mathcal{S}_t$ Indexes of scenarios for every time block.

$\mathcal{W}$ Indexes of renewable generators.

$\mathcal{Y}$ Indexes of snapshot to discretize under-contingency operation.

I Indexes of generators.

I_b Indexes of generators connected to bus b.

N_h Indexes of buses that can be hardened.

N Indexes of buses.

Variables

$\Delta D^+_{\mathrm{bet}}$ Scheduled load increase/generation reduction [MW].

$\Delta D^-_{\mathrm{bet}}$ Scheduled load reduction/generation increase [MW].

Δ^{FD}_{bt} Scheduled load to be shifted [MW].

θ^c_{byt} Voltage angles under contingency [Rad]

θ_{bt} Voltage angles [Rad].

f^c_{lyt} Power flow through lines under contingency [MW].

f_{lt} Power flow through lines [MW].

$GS_{\mathrm{i}yt}$ Generation shedding [MW].

LS_{byt} Load shedding [MW].

$p^c_{\mathrm{i}yt}$ Power outputs under contingency [MW].

p_{it} Power output of generators [MW].

$r^{\mathrm{d}}_{\mathrm{i}yt}$ Scheduled downward reserves [MW].

$r^u_{\mathrm{i}yt}$ Scheduled upward reserves [MW].

$r^{\mathrm{dc}}_{\mathrm{i}yt}$ Downward reserve utilization [MW].

$r^{uc}_{\mathrm{i}yt}$ Upward reserve utilization [MW].

u_{it} Commitment of generators (binary).

v_l Investment in candidate assets (binary).

z_b Substation hardening decision (binary).

Δ_{byt}^{FD+c} Positive deviation of flexible demand from its nominal value [MW].

Δ_{byt}^{FD-c} Negative deviation of flexible demand from its nominal value [MW].

ΔD_{byet}^{+c} Actual load increase/generation reduction [MW].

ΔD_{byet}^{-c} Actual load reduction/generation increase [MW].

4.1 Resilient grid planning

In the present chapter we propose a model that can determine the optimal portfolio of operational and hardening strategies to enhance power systems resilience against seismic hazards. As outages and their probabilities drive the decision of enhancing systems resilience, it is critical to take into consideration that these probabilities are not completely known by the decision maker, which is recognized by the model by applying a DRO approach, in which the probability distribution lies within an ambiguity set specific to the system condition (normal operation or facing an earthquake). On this work, at operation level resilience can be enhanced by utilizing generation reserves or taking advantage of distributed energy resources via a coordinated demand response. Hardening strategies considered are investment in new network infrastructure, and anchoring (hardening) existing substations. Since hardening substations modifies the likelihood of an outage on that element, the ambiguity set is modified by the decision taken, making this a model with Decision-Dependent Ambiguity (DDA).

4.1.1 Earthquake effects on the grid

We firstly need a mathematical model that can propagate the effects of an earthquake of given characteristics, to assess how it would affect the elements of the power system. In [75] the authors used regression methods on a data set of seismic events in Chile to develop a formula to compute the peak ground acceleration (PGA) an earthquake produces on a certain location. This PGA is computed based on the moment magnitude, focal depth, closest distance to the rupture surface, and whether the the ground is soil or rock.

Once the PGA is known, we need to compute how likely would it be the outage of an element given that acceleration. In [55] the Federal Emergency Management Agency developed methods for assessing the damage produced by an earthquake to different facilities, including power system components. They present fragility curves for generators and substations, which represent the likelihood of exceeding certain level of damage for a given ground motion (represented by its PGA). In [76], the authors developed fragility curves for transmission line towers, and we will assume a line gets out of service when at least one of its towers collapses. In the case of substations, [55] present fragility curves for different percentages of damage (namely 5, 40, 70 and 100%) on them. These partial outages in substations capture that when an earthquake strikes, a given number of elements of the substation (circuit breakers and switches for instance) may get out of service, which hinders its capacity to deliver power to its bays, but the substation is still able to work partially.

A substation with a given percentage of damage will be modelled as a busbar that is not able to deliver to each one of the elements connected to it the amount of power it was designed to deliver, instead that capacity is reduced by the damage percentage. This means that if a 100 MW generator is directly connected to a substation that gets 50% damaged by an earthquake, the generator will only be able to inject 50 MW to the bus. If the generator had a power output lower than 50 MW prior to the event, the outage will not affect it (as long as it does not intend to increase its power output beyond 50 MW). If the output was higher than 50 MW, the generator will need to ramp down its power, or spill its energy otherwise. This same principle will apply for other elements that may be connected to the bus, considering that the bay by which a load is connected, was designed to transport power equal to its peak consumption, and bays for transformers and lines were designed to transport up to the maximum capacity of the respective element. Finally, the earthquake can damage directly the demand, for example, by different failures at distribution level. If this happens, demand curtailment associated with the level of damage suffered is inescapable.

4.1.2 Resilience enhancing strategies

As previously stated, an operational strategy that could enhance grids resilience, is to take advantage of resources at distribution level, making them contribute in case of a high impact event. The main power system can benefit from these distributed energy resources, provided that they can be controlled in a coordinated fashion, for instance, some loads can readily modify their consumption levels or change their consumption pattern without major problems, relieving the system from stress, that may contribute to decrease the need for involuntary demand shedding. Other controllable resources that can be used to alleviate post-hazard stress are batteries and distributed generation, that can contribute in the same sense of changing the net load perceived by the main grid.

Just like on the previous chapter, throughout this chapter we will be modelling services supplied by DER, assuming the presence of aggregators, that will group different resources that will physically provide the service, and present to the TSO the supply curves, so he can decide whether it is beneficial to schedule these services or not, and how much, without needing to interact directly with the agents that actually provide them. The first service modelled is a demand reduction one, that would be issued through disconnection of loads, increase in DG, backup generation, or by managing adequately storage systems (such as electric vehicles). The second service is a demand increase one, and can be seen as the counterpart of the first. It would be issued by disconnection of DG, connection of loads, or by charging storage systems, and would be used in case of decreased network capacity due to an outage, preventing the tripping of generators by consuming the power they are injecting to the grid. The third and last service would be provided by non-essential loads, whose energy demand can be delayed in time, like loads with thermal inertia, and would be a shift in time of the energy consumed, considering a payback effect, meaning that they will need more energy overall if their consumption is shifted [67].

The first hardening strategy is investment in new transmission assets. Adding new routes for power to flow will make outages from other lines less critical. It is important to notice that as shown in the Chilean experience, substations are much more prone to suffer an outage than transmission lines, however, investment can be made in new corridors that do not exist yet, so power can be transferred between two substations without the need of other substations that can suffer outages. Finally, as previously stated, we will assess the benefits of hardening substations by anchoring their components, so they can withstand better the impacts of ground acceleration. To capture this effect, if a substation is hardened, the model will change the ambiguity set considering that outages on that substation will most likely follow the fragility curves from an anchored substation.

4.2 Mathematical model

4.2.1 Overview

The optimization model aims to minimize the total cost of the system, which comprises the transmission expansion and substation hardening investment cost, the operating cost on different conditions, and the worst-case expected cost of corrective actions taken in case of contingency. These operating conditions are divided in a set of scenarios that the system can undergo after the operation is decided. It is important to notice that the system operation is not decided for one specific scenario, but for the whole set of scenarios weighted by their respective probabilities. Each one of these scenarios can represent normal operation, or a specific earthquake within a predefined set, and they differ in the ambiguity set in which the probability distribution of outages lies. For each scenario, the model considers outages on generators, transmission lines, transformers, and a set of different outages on substations and directly on demand.

As stated above, the operation of the system is divided into different conditions, that are decoupled time blocks with different demand levels and availability of renewable energy resources. The operation under contingency is also modelled, and it is divided in time-coupled snapshots to capture the evolution of the utilization of post-contingency demand services, generation reserves and further corrective actions, such as demand shedding and generation spilling.

4.2.2 Ambiguity set

To properly acknowledge our ignorance regarding the probability of each state, an ambiguity set is constructed utilizing first-moment information, specifically letting the outage probability of each element to lie in an interval adjusted utilizing the quality of the information available. The model minimizes the cost against the worst-case probability distribution within that ambiguity set. It is important to notice that as only information of the first moment is considered, so in case of simultaneous outages (that are particularly common when a HILP event strikes) the solution will consider the worst correlation of outages (that also depends on the solution). In no case assuming statistical independence would be a good choice, considering outages are caused by the same source, increasing the probability of having multiple outages simultaneously.

For each time block (representing an operating condition) and scenario, there will be a random vector $\boldsymbol{a}_{ts}$, containing one binary variable for each generator, line, and for every damage state of each substation and each load, that takes the value 0 if the corresponding line or generator got out of service, or if the substation or load got at least to that damage state. The ambiguity set will be constructed utilizing linear inequalities on the expected value of the elements of $\boldsymbol{a}_{ts}$. For ease of notation we will call $\hat{\boldsymbol{a}}_t$ to $\mathbf{1}$ minus $\boldsymbol{a}_t$, so the ambiguity set is:

$$\mathcal{P}_{ts}(\boldsymbol{z}) = \left\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \mathbb{E}_{\mathcal{Q}}[S_{ts}\hat{\boldsymbol{a}}_{ts}] \leq \overline{\boldsymbol{\mu}}_{ts} + K_{ts}\boldsymbol{z}\right\}, \tag{4.1}$$

Where $\boldsymbol{z}$ is the vector of decision variables on hardening the substations, K_{ts} is the matrix that contains the information about changes on the ambiguity set made by hardening decisions, while S_{ts} and $\overline{\boldsymbol{\mu}}_{ts}$ are a auxiliary matrix and vector respectively, utilized to construct the ambiguity set. The set $\mathcal{A}$ corresponds to the set of all contingency conditions to be considered. This work considers failures on generation, network elements, and different damage levels on substations and demand, therefore, if we want to consider up to k simultaneous outages, $\mathcal{A}$ would be the set presented on (4.2).

$$\begin{aligned}\mathcal{A} = \Big\{(\boldsymbol{a}^{\boldsymbol{G}}, \boldsymbol{a}^{\boldsymbol{L}}, \boldsymbol{a}^{\boldsymbol{N}}, \boldsymbol{a}^{\boldsymbol{D}}) \in \{0,1\}^{|I|} \times \{0,1\}^{|\mathcal{L}|} \times \{0,1\}^{|\mathcal{D}_N|\cdot|N|} \times \{0,1\}^{|\mathcal{D}_D|\cdot|N|} :\\ \sum_{\mathrm{i}\in I} a_{\mathrm{i}}^G + \sum_{l\in\mathcal{L}} a_l^L + \sum_{b\in N} a_b^{N_1} + \sum_{b\in N} a_b^{D_1} \geq |I| + |\mathcal{L}| + |N| + |N| - k,\\ a_b^{N_m} \geq a_b^{N_{m-1}} \quad \forall b \in N, m \in \mathcal{D}_N \setminus \{1\},\\ a_b^{D_m} \geq a_b^{D_{m-1}} \quad \forall b \in N, m \in \mathcal{D}_D \setminus \{1\}\Big\},\end{aligned} \tag{4.2}$$

Note that the binary variables used to define a given damage state must be greater or equal than the one representing less severe states (both for substations and demand). In this way we can compute the level of damage as the weighted sum of these binary variables, as shown in (4.3) and (4.4), provided that the weights add up to one.

$$\mathrm{d}_b^N = \sum_{m\in\mathcal{D}_N} \sigma_m^N \cdot a_b^{N_m} \quad \forall b \in N \tag{4.3}$$

$$\mathrm{d}_b^D = \sum_{m\in\mathcal{D}_D} \sigma_m^D \cdot a_b^{N_m} \quad \forall b \in N \tag{4.4}$$

4.2.3 Complete formulation

The objective function to be minimized in the model is presented in (4.5). It contains the operating costs, consisting of the costs of energy and the costs of scheduling post-contingency services provided by generation and demand, that is, reserves and DER services. It also considers the investment cost of new assets and hardened substations, as well as the worst-case (in terms of probability distribution) expected value of corrective action costs of every time block with their respective scenarios weighted by their probability.

It is important to notice that the worst-case probability distribution for each scenario depends on the specific time block, since different demand levels and renewable resources availability change the generation dispatch, modifying the criticality of different failures. Although we know there is a unique (and unknown) probability distribution for each scenario, which does not depend on the time block, the model minimizes against the worst-case distribution for each scenario and each time block, because it considers the presence of a risk-averse system operator, which hedges the operation against the worst case on every time block.

Constraint (4.6) enforces power balance at each bus of the system, noting that there is no demand-side participation, since DER services modelled are only intended to be utilized in case of contingency. Constraints (4.7) and (4.8) model the DC power flow, while constraints (4.9) and (4.10) ensure that thermal limits of existing and candidate lines are met. Constraints (4.11) and (4.12) make sure that the power of each generator is within its technical limits, even after delivering its reserves. The limit on upward and downward reserves are enforced by constraints (4.13) and (4.14), while maximum available DER to be scheduled on post-contingency services are represented by constraints (4.15), (4.16) and (4.17). Finally, availability of resources on renewable generator is modelled by constraint (4.18), while constraints (4.19), (4.20), and (4.21) state that commitment, network investment, and substation hardening are binary decisions.

$$\begin{aligned}
\underset{\substack{\Delta D^{+}_{bet},\Delta D^{-}_{bet},\Delta^{FD}_{bt},\theta_{bt},\\ f_{lt},p_{it},r^{d}_{iyt},r^{u}_{iyt},u_{it},v_{l},z_{b}}}{\text{Minimize}} \; & \sum_{t\in\mathcal{T}} h_t \Bigg[\sum_{i\in I} \Big(C^{p}_{i} p_{it} + \sum_{y\in\mathcal{Y}} \big(C^{d}_{iy} r^{d}_{iyt} + C^{u}_{iy} r^{u}_{iyt} \big) \Big) \\
& + \sum_{b\in N} \Big(C^{FD}_{b} \Delta^{FD}_{bt} + \sum_{e\in\mathcal{E}} \big(C^{I+}_{be} \Delta D^{+}_{bet} + C^{I-}_{be} \Delta D^{-}_{bet} \big) \Big) \\
& + \sum_{s\in\mathcal{S}_t} p_{ts} \Big(\sup_{\mathcal{Q}\in\mathcal{P}_{ts}(\mathbf{z})} \mathbb{E}_{\mathcal{Q}} \{ H_t(\boldsymbol{p}, \boldsymbol{r}^{\mathbf{d}}, \boldsymbol{r}^{\boldsymbol{u}}, \boldsymbol{\Delta}^{\boldsymbol{FD}}, \boldsymbol{\Delta D}^{+}, \boldsymbol{\Delta D}^{-}, \boldsymbol{v}, \boldsymbol{a}_{ts}) \} \Big) \Bigg] \\
& + \sum_{l\in\mathcal{L}^C} C^{L}_{l} v_l + \sum_{b\in N_h} C^{h}_{b} z_b
\end{aligned} \tag{4.5}$$

subject to:

$$\sum_{i\in I_b} p_{it} + \sum_{l\in\mathcal{L}|to(l)=b} f_{lt} - \sum_{l\in\mathcal{L}|fr(l)=b} f_{lt} = D_{bt}; \forall b\in N, t\in\mathcal{T} \tag{4.6}$$

$$f_{lt} = \frac{1}{x_l}(\theta_{fr(l),t} - \theta_{to(l),t}); \forall l\in\mathcal{L}^E, t\in\mathcal{T} \tag{4.7}$$

$$-M(1-v_l)+\frac{1}{x_l}(\theta_{fr(l),t}-\theta_{to(l),t})\leq f_{lt}$$
$$\leq\frac{1}{x_l}(\theta_{fr(l),t}-\theta_{to(l)},t)+M(1-v_l);\forall l\in\mathcal{L}^C,t\in\mathcal{T} \tag{4.8}$$
$$-\overline{F}_l\leq f_{lt}\leq\overline{F}_l;\forall l\in\mathcal{L}^E,t\in\mathcal{T} \tag{4.9}$$
$$-v_l\overline{F}_l\leq f_{lt}\leq v_l\overline{F}_l;\forall l\in\mathcal{L}^C,t\in\mathcal{T} \tag{4.10}$$
$$p_{\text{it}}-\sum_{y\in\mathcal{Y}}r^{\text{d}}_{\text{iy}t}\geq\underline{P}_{\text{i}}u_{\text{it}};\forall\text{i}\in I,t\in\mathcal{T} \tag{4.11}$$
$$p_{\text{it}}+\sum_{y\in\mathcal{Y}}r^{u}_{\text{iy}t}\leq\overline{P}_{\text{i}}u_{\text{it}};\forall\text{i}\in I,t\in\mathcal{T} \tag{4.12}$$
$$0\leq r^{\text{d}}_{\text{iy}t}\leq\overline{R}^{D}_{\text{iy}};\forall\text{i}\in I,y\in\mathcal{Y},t\in\mathcal{T} \tag{4.13}$$
$$0\leq r^{u}_{\text{iy}t}\leq\overline{R}^{U}_{\text{iy}};\forall\text{i}\in I,y\in\mathcal{Y},t\in\mathcal{T} \tag{4.14}$$
$$0\leq\Delta^{FD}_{bt}\leq\overline{\Delta}^{FD}_{b};\forall b\in N,t\in\mathcal{T} \tag{4.15}$$
$$0\leq\Delta D^{+}_{\text{be}t}\leq\overline{\Delta D^{+}}_{\text{be}};\forall b\in N,\text{e}\in\mathcal{E},t\in\mathcal{T} \tag{4.16}$$
$$0\leq\Delta D^{-}_{\text{be}t}\leq\overline{\Delta D^{-}}_{\text{be}};\forall b\in N,\text{e}\in\mathcal{E},t\in\mathcal{T} \tag{4.17}$$
$$p_{wt}\leq\zeta_w\overline{P}_{\text{i}};\forall w\in\mathcal{W},t\in\mathcal{T} \tag{4.18}$$
$$u_{\text{it}}\in\{0,1\};\forall\text{i}\in I,t\in\mathcal{T} \tag{4.19}$$
$$v_l\in\{0,1\};\forall l\in\mathcal{L}^C \tag{4.20}$$
$$z_b\in\{0,1\};\forall b\in N_h, \tag{4.21}$$

4.2.4 Operation under contingency

The operation under contingency model is an optimization model that decides the corrective actions to be taken in case of a given outage, by minimizing the system cost. As mentioned before, outages can affect either generation, transmission lines, transformers, substations, or demand. When a generator faces a contingency, it is modelled the traditional way, it cannot deliver its power, and it is therefore disconnected. When the outage affects a line or transformer, power can no longer flow across them, so they have to be disconnected. In the case of substations, different damage levels are represented by percentages of damage. When the damage is complete, energy cannot go through the substation, meaning that every line connected has to be disconnected, generators cannot inject and demands withdraw power either. To model partial damage of the substation we will assume that the equipment to drive power from an element into the substation is sized considering the maximum capacity of the element. For instance, if a substation faces a 50% damage, any generator would be able to inject up to half of its maximum power to the substation, loads would be able to withdraw half of their peak demand, and the capacity of lines connected to the substation would be reduced in half. When an outage occurs directly at demand level, a fraction of demand is unavoidably lost. Next, we will detail the operation under contingency model, noting that expression (4.3) will be utilized for the sake of simplicity.

$$H_t(\boldsymbol{q}_t, \boldsymbol{a}_{ts}) = \underset{\substack{\Delta_{by}^{FD+c}, \Delta_{by}^{FD-c}, \\ \Delta D_{bye}^{+c}, \Delta D_{bye}^{-c}, \\ \theta_{by}^{c}, f_{ly}^{c}, p_{iy}^{c}, \\ r_{iy}^{dc}, r_{iy}^{uc}}}{\text{Minimize}} \sum_{y\in\mathcal{Y}} \Big[\sum_{i\in I} \left(C_{iy}^{dc} r_{iy}^{dc} + C_{iy}^{uc} r_{iy}^{uc} \right) + \sum_{b\in N, e\in\mathcal{E}} \left(C_{be}^{I+^c} \Delta D_{bye}^{+c} + C_{be}^{I-^c} \Delta D_{bye}^{-c} \right) + \sum_{b\in N} C_b^D LS_{by} + \sum_{i\in I} C_i^G GS_{iy} \Big] \Delta t + \sum_{b\in N} C_b^{FD^c} \Delta_{b1}^{FD-c} \tag{4.22}$$

subject to:

$$\sum_{i\in I_b} p_{iy}^c + \sum_{l\in\mathcal{L}|to(l)=b} f_{ly}^c - \sum_{l\in\mathcal{L}|fr(l)=b} f_{ly}^c + \Delta_{by}^{FD-c} - \Delta_{by}^{FD+c} + \sum_{e\in\mathcal{E}} \left(\Delta D_{bye}^{-c} - \Delta D_{bye}^{+c} \right) = D_{bt} - LS_{by} : (\lambda_{by}); \forall b \in N, y \in \mathcal{Y} \tag{4.23}$$

$$-a_l^L \overline{F}_l \le f_{ly}^c \le a_l^L \overline{F}_l : (\phi_{ly}^+, \phi_{ly}^-); \forall l \in \mathcal{L}^E, y \in \mathcal{Y} \tag{4.24}$$

$$-a_l^L v_l \overline{F}_l \le f_{ly}^c \le a_l^L v_l \overline{F}_l : (\phi_{ly}^{n+}, \phi_{ly}^{n-}); \forall l \in \mathcal{L}^C, y \in \mathcal{Y} \tag{4.25}$$

$$-\mathrm{d}_{to(l)}^N \overline{F}_l \le f_{ly}^c \le \mathrm{d}_{to(l)}^N \overline{F}_l : (\phi_{ly}^{1+}, \phi_{ly}^{1-}); \forall l \in \mathcal{L}^E, y \in \mathcal{Y} \tag{4.26}$$

$$-\mathrm{d}_{fr(l)}^N \overline{F}_l \le f_{ly}^c \le \mathrm{d}_{fr(l)}^N \overline{F}_l : (\phi_{ly}^{2+}, \phi_{ly}^{2-}); \forall l \in \mathcal{L}^E, y \in \mathcal{Y} \tag{4.27}$$

$$-\mathrm{d}_{to(l)}^N v_l \overline{F}_l \le f_{ly}^c \le \mathrm{d}_{to(l)}^N v_l \overline{F}_l : (\phi_{ly}^{n1+}, \phi_{ly}^{n1-}); \forall l \in \mathcal{L}^C, y \in \mathcal{Y} \tag{4.28}$$

$$-\mathrm{d}_{fr(l)}^N v_l \overline{F}_l \le f_{ly}^c \le \mathrm{d}_{fr(l)}^N v_l \overline{F}_l : (\phi_{ly}^{n2+}, \phi_{ly}^{n2-}); \forall l \in \mathcal{L}^C, y \in \mathcal{Y} \tag{4.29}$$

$$-M \left(3 - (a_l^L + a_{fr(l)}^{N_{|\mathcal{D}|}} + a_{to(l)}^{N_{|\mathcal{D}|}}) \right) + \frac{1}{x_l} (\theta_{fr(l),y}^c - \theta_{to(l),y}^c) \le f_{ly}^c \le \frac{1}{x_l} (\theta_{fr(l),y}^c - \theta_{to(l),y}^c) + M \left(3 - (a_l^L + a_{fr(l)}^{N_{|\mathcal{D}|}} + a_{to(l)}^{N_{|\mathcal{D}|}}) \right) : (\mu_{ly}^+, \mu_{ly}^-); \forall l \in \mathcal{L}^E, y \in \mathcal{Y} \tag{4.30}$$

$$-M \left(3 - v_l(a_l^L + a_{fr(l)}^{N_{|\mathcal{D}|}} + a_{to(l)}^{N_{|\mathcal{D}|}}) \right) + \frac{1}{x_l} (\theta_{fr(l),y}^c - \theta_{to(l),y}^c) \le f_{ly}^c \le \frac{1}{x_l} (\theta_{fr(l),y}^c - \theta_{to(l),y}^c) + M \left(3 - v_l(a_l^L + a_{fr(l)}^{N_{|\mathcal{D}|}} + a_{to(l)}^{N_{|\mathcal{D}|}}) \right) : (\mu_{ly}^{n+}, \mu_{ly}^{n-}); \forall l \in \mathcal{L}^C, y \in \mathcal{Y} \tag{4.31}$$

$$p_{iy}^c = p_i a_i^G + r_{iy}^{uc} - r_{iy}^{dc} - GS_{iy} : (\eta_{iy}); \forall i \in I, y \in \mathcal{Y} \tag{4.32}$$

$$p_{iy}^c \le \overline{P}_i \mathrm{d}_{bus(i)}^N : (\eta_{iy}); \forall i \in I, y \in \mathcal{Y} \tag{4.33}$$

$$0 \le r_{iy}^{dc} \le r_i^d a_i^G : (\kappa_{iy}^-); \forall i \in I, y \in \mathcal{Y} \tag{4.34}$$

$$0 \le r_{iy}^{uc} \le r_i^u a_i^G : (\kappa_{iy}^+); \forall i \in I, y \in \mathcal{Y} \tag{4.35}$$

$$0 \le \Delta D_{bye}^{+c} \le \Delta D_{be}^+ : (\psi_{bye}^+); \forall b \in N, y \in \mathcal{Y}, e \in \mathcal{E} \tag{4.36}$$

$$0 \le \Delta D_{bye}^{-c} \le \Delta D_{be}^- : (\psi_{bye}^-); \forall b \in N, y \in \mathcal{Y}, e \in \mathcal{E} \tag{4.37}$$

$$0 \le \Delta_{by}^{FD-c} \le \Delta_b^{FD} : (\rho_{by}); \forall b \in N, y \in \mathcal{Y} \tag{4.38}$$

$$\delta \sum_{y\in\mathcal{Y}} \Delta_{by}^{FD-c} = \sum_{y\in\mathcal{Y}} \Delta_{by}^{FD+c} : (\iota_b); \forall b \in N \tag{4.39}$$

$$\Delta_{b1}^{FD-c} \geq \Delta_{by}^{FD-c} : (\beta_{by}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\} \tag{4.40}$$

$$\Delta D_{b1\mathrm{e}}^{+c} \geq \Delta D_{by\mathrm{e}}^{+c} : (\gamma_{by\mathrm{e}}^{+}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\}, \mathrm{e} \in \mathcal{E} \tag{4.41}$$

$$\Delta D_{b1\mathrm{e}}^{-c} \geq \Delta D_{by\mathrm{e}}^{-c} : (\gamma_{by\mathrm{e}}^{-}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\}, \mathrm{e} \in \mathcal{E} \tag{4.42}$$

$$D_{bt} - LS_{by} - \sum_{\mathrm{e} \in \mathcal{E}} \left(\Delta D_{by\mathrm{e}}^{-c} - \Delta D_{by\mathrm{e}}^{+c} \right) - \Delta_{by}^{FD-c} + \Delta_{by}^{FD+c} \leq \overline{D}_b \mathrm{d}_b^N : (\sigma_{by}^{+}); \forall b \in N, y \in \mathcal{Y} \tag{4.43}$$

$$LS_{by} \geq D_{bt}(1 - \mathrm{d}_b^D) : (\sigma_{by}^{+}); \forall b \in N, y \in \mathcal{Y}, \tag{4.44}$$

As stated in (4.22) the objective function considers the costs of load shedding, generation curtailment, and utilization costs of all services, namely, delivery of reserves and DER utilization. It can be seen that the load shifting service is paid for power of the first (and also largest) disconnection, while the other two DER services are paid for the energy they inject or withdraw. Constraint (4.23) enforces power balance when considering the utilization of voluntary and involuntary corrective actions. Constraints (4.24), (4.25) limit the maximum flow through lines, considering they may be out of service or not constructed, cases in which the maximum flow must be zero. Maximum capacity of lines can also be reduced by an outage in the substations they connect, which is expressed on constraints (4.26), (4.27), (4.28) and (4.29). Once again, DC power flow is utilized, as stated by constraints (4.30) and (4.31), that also ensure no Kirchhoff's law is applied when the line is disconnected, either because it is out of service, or because one of the substations of its extremes suffered complete damage. Constraint (4.32) models the total output of a given generator when the utilization of reserves and spillage is considered, while (4.33) limits the maximum injected power according to the damage state of the substation. Constraints (4.34) and (4.35) limit the maximum amount of reserves to be delivered, while constraints (4.36), (4.37) and (4.38) limit the utilization of DER services by the amount scheduled beforehand. Constraint (4.39) ensures power balance on loads providing the demand shift service, while constraint (4.40) guarantees that the first disconnection is the biggest, and only reconnections can be made afterwards. For the other DER services, constraints (4.41) and (4.42) enforces that the first demand reduction (or increase) is the biggest one. The maximum amount of power withdrawn by a load, given the damage state of the substation, is modelled by constraints (4.43). Finally, (4.44) states that damages that occur directly at demand level make unavoidable to shed certain levels of demand.

4.3 Solution methodology

4.3.1 Compact formulation

The first step to explain how the previously presented problem is solved, is to reformulate the multilevel problem (4.5)–(4.21) in a compact fashion.

$$\underset{\boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in\mathcal{T}} h_t \left[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{\boldsymbol{B}^{\boldsymbol{T}}} \boldsymbol{q}_t^B + \sum_{s\in\mathcal{S}_t} p_{ts} \left(\sup_{\mathcal{Q}\in\mathcal{P}_{ts}(\boldsymbol{q}^H)} H_t(\boldsymbol{q}_t, \boldsymbol{a}_{ts}) \right) \right] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \boldsymbol{c}^{H^T} \boldsymbol{q}^H \quad (4.45)$$

subject to:

$$A\boldsymbol{q}_t \leq \boldsymbol{b}_t \quad (4.46)$$

$$\boldsymbol{q}_t^B \in \{0,1\}^{|I||\mathcal{T}|} \quad (4.47)$$

$$\boldsymbol{q}_t^L \in \{0,1\}^{|\mathcal{L}^C|} \quad (4.48)$$

$$\boldsymbol{q}_t^H \in \{0,1\}^{|N_h|}, \quad (4.49)$$

where $\boldsymbol{q}_t^C$ and $\boldsymbol{q}_t^B$ are vectors that contain all the continuous and binary operating variables, respectively. Binary investment variables in new transmission assets are represented by $\boldsymbol{q}^L$, while $\boldsymbol{q}^H$ contains all the substation hardening binary variables. Vector $\boldsymbol{q}_t$ contains all operating variables of a given time block, and investment variables, both in new network elements and in substation anchoring, that is $\boldsymbol{q}_t = [\boldsymbol{q}_t^C, \boldsymbol{q}_t^B, \boldsymbol{q}^L, \boldsymbol{q}^H]$. Objective function (4.45) corresponds to (4.5), whereas (4.46) represents constraints (4.6) to (4.18), and constraints (4.47)-(4.49) are the reformulation of (4.19)-(4.21), respectively.

Similarly, now we present a compact formulation of problem (4.22)–(4.44).

$$H_t(\boldsymbol{q}_t, \boldsymbol{a}_{ts}) = \underset{\boldsymbol{y}}{\text{Minimize}}\ \mathrm{d}_t^T \boldsymbol{y}_t \quad (4.50)$$

subject to:

$$B_t \boldsymbol{y}_t \geq \mathrm{e}_t : (\Theta_t) \quad (4.51)$$

$$C_t \boldsymbol{y}_t \geq D_t \boldsymbol{q}_t + g_t : (\Phi_t) \quad (4.52)$$

$$G_t \boldsymbol{y}_t \geq J_t(\boldsymbol{a}_{ts}) \boldsymbol{q}_t + j_t : (\Lambda_t) \quad (4.53)$$

$$K_t \boldsymbol{y}_t \geq u_t(\boldsymbol{a}_{ts}) : (\Gamma_t), \quad (4.54)$$

where the objective function (4.50) corresponds to (4.22). Constraint (4.51) groups constraints (4.23) and (4.39)–(4.42). In the same way, (4.52) is associated to (4.36) – (4.38). Constraint (4.53) corresponds to (4.25),(4.28),(4.29), (4.31),(4.32),(4.34) and (4.35). Finally, (4.54) is related to constraints (4.24), (4.26), (4.27), (4.30), (4.33), (4.43) and (4.44).

4.3.2 Problem reformulation

Next, we reformulate the compact model (4.45)– (4.49) by expanding the worst-case expected corrective actions cost.

$$\underset{\alpha_{ts}, \boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B \Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \boldsymbol{c}^{H^T} \boldsymbol{q}^H$$

$$+\sum_{t\in T} h_t \sum_{s\in\mathcal{S}_t} p_{ts}\alpha_{ts} \tag{4.55}$$

subject to:

Constraints (4.46)–(4.49) (4.56)

$$\alpha_{ts} = \Big\{ \underset{\mathcal{Q}}{\text{Maximize}} \sum_{a_{ts}\in\mathcal{A}} H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_{ts}\big)\mathcal{Q}(\boldsymbol{a}_{ts}) \tag{4.57}$$

subject to:

$$\sum_{a_{ts}\in\mathcal{A}} S_{ts}\hat{\boldsymbol{a}}_{ts}\mathcal{Q}(\boldsymbol{a}_{ts}) \le \overline{\boldsymbol{\mu}}_{ts} + K_{ts}\boldsymbol{q}^H : (\boldsymbol{\pi}_{ts}) \tag{4.58}$$

$$\sum_{a_{ts}\in\mathcal{A}} \mathcal{Q}(\boldsymbol{a}_{ts}) = 1 : (\varphi_{ts})\Big\}, \forall t \in \mathcal{T}, s \in \mathcal{S}_t. \tag{4.59}$$

Replacing the inner maximization problem by its dual, we obtain the following model.

$$\underset{\boldsymbol{\pi}_{ts}\ge 0,\varphi_{ts},\boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in T} h_t\Big[\boldsymbol{c}^T\boldsymbol{q}_t^C + \boldsymbol{c}^{B^T}\boldsymbol{q}_t^B\Big] + \boldsymbol{c}^{L^T}\boldsymbol{q}^L + \boldsymbol{c}^{H^T}\boldsymbol{q}^H$$

$$+\sum_{t\in T} h_t \sum_{s\in\mathcal{S}_t} p_{ts}\left(\boldsymbol{\pi}_{ts}^T\left(\overline{\boldsymbol{\mu}}_{ts} + K_{ts}\boldsymbol{q}^H\right) + \varphi_{ts}\right) \tag{4.60}$$

subject to:

Constraints (4.46)–(4.49) (4.61)

$$\boldsymbol{\pi}_{ts}^T S_{ts}\hat{\boldsymbol{a}}_{ts} + \varphi_{ts} \ge H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_{ts}\big), \forall t \in \mathcal{T}, s \in \mathcal{S}_t, \boldsymbol{a}_{ts} \in \mathcal{A}, \tag{4.62}$$

Noting that we can rewrite (4.62) as (4.63) to solve the problem more efficiently.

$$\varphi_{ts} \ge \max_{\boldsymbol{a}_{ts}\in\mathcal{A}} \left\{H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_{ts}\big) - \boldsymbol{\pi}_{ts}^T S_{ts}\hat{\boldsymbol{a}}_{ts}\right\}, \forall t \in \mathcal{T}, s \in \mathcal{S}_t. \tag{4.63}$$

4.3.3 Subproblem

The subproblem will be one of the two problems that will be used to iteratively get the resilient planning solution. It corresponds to the optimization problem on the right side of (4.63), noting that, as H_t is a minimization problem, it has to be replaced by its dual, so only one maximization problem is obtained.

$$\underset{\Theta_{ts},\Phi_{ts},\Omega_{ts},\Lambda_{ts},\boldsymbol{a}_{ts}}{\text{Maximize}} \sum_{t\in T}\sum_{s\in\mathcal{S}_t}\Big[\,\mathrm{e}_t^T\Theta_{ts} + (g_t + D_t\boldsymbol{q}_t)^T\Phi_{ts}$$

$$+ (J_t(\boldsymbol{a}_{ts})\boldsymbol{q}_t + j_t)^T\Lambda_{ts} + u_t(\boldsymbol{a}_{ts})^T\Gamma_{ts} - \boldsymbol{\pi}_{ts}^T S_{ts}\hat{\boldsymbol{a}}_{ts}\Big] \tag{4.64}$$

subject to:

$$B_t^T\Theta_{ts} + C_t^T\Phi_{ts} + G_t^T\Lambda_{ts} + K_t^T\Gamma_{ts} = \mathrm{d}_t; \forall t \in T, s \in \mathcal{S}_t \tag{4.65}$$
$$\Theta_{ts}, \Phi_{ts}, \Lambda_{ts}, \Gamma_{ts} \geq 0; \forall t \in T, s \in \mathcal{S}_t \tag{4.66}$$
$$\boldsymbol{a}_{ts} \in \mathcal{A}, \tag{4.67}$$

It is important to notice that (4.67) determines which outages will be considered for every scenario of each time block, Also, the multiplication of continuous and binary decision variables are linearized to produce a mixed integer linear program.

4.3.4 Master problem

The master problem is a relaxation of (4.55) – (4.57), in which constraint (4.57) is replaced by a set of cutting planes that are added iteratively.

$$\underset{\boldsymbol{\pi}_{ts}\geq 0, \varphi_{ts}, \boldsymbol{q}_t}{\text{Minimize}} \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \boldsymbol{c}^{H^T} \boldsymbol{q}^H$$
$$+ \sum_{t\in T} h_t \sum_{s\in\mathcal{S}_t} p_{ts} \left(\boldsymbol{\pi}_{ts}^T \overline{\boldsymbol{\mu}}_{ts} + \boldsymbol{\pi}_{ts}^T K_{ts} \boldsymbol{q}^H + \varphi_{ts}\right) \tag{4.68}$$

subject to:

Constraints (4.46)–(4.49) (4.69)

$$\varphi_{ts} \geq \mathrm{e}_t^T \Theta_{ts}^{(j)} + (g_t + D_t \boldsymbol{q}_t)^T \Phi_{ts}^{(j)} + (J_t(\boldsymbol{a}_t^{(j)})\boldsymbol{q}_t + j_t)^T \Lambda_{ts}^{(j)}$$
$$+ s_t(\boldsymbol{a}_t^{(j)})^T \Gamma_{ts}^{(j)} - \boldsymbol{\pi}_{ts}^T S_{ts} \hat{\boldsymbol{a}}_{ts}^{(j)}; \forall j \in \mathcal{J}, t \in T, s \in \mathcal{S}_t, \tag{4.70}$$

noting that $\boldsymbol{\pi}_{ts}^T K_{ts} \boldsymbol{q}^H$ is multiplication between the vector $\boldsymbol{\pi}_{ts}$ and a linear transformation of $\boldsymbol{q}^H$. In order to get a mixed integer linear program we can take advantage of the binary nature of $\boldsymbol{q}^H$, identifying the products between continuous and binary variables, and linearizing them through additional constraints.

4.3.5 Solution methodology

In this subsection we present the solution algorithm, based on the Benders decomposition. Basically, on every iteration the subproblem takes the optimal decision variables of the master problem and constructs a cutting hyperplane to be added to be master. This procedure renders the solution of the master problem closer to the optimal solution each iteration, until it eventually converges.

1. Initialization: set $j \leftarrow 0$.
2. Solve the optimization model (4.68)–(4.70), store $\boldsymbol{q}_t^{(j)}$, $\boldsymbol{\pi}_{ts}^{(j)}$ and $\varphi_{ts}^{(j)}$, and calculate

$$LB^{(j)} = \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^{C(j)} + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^{B(j)}\Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^{L(j)} + \boldsymbol{c}^{H^T} \boldsymbol{q}^{H(j)}$$

$$+\sum_{t\in T} h_t \sum_{s\in\mathcal{S}_t} p_{ts}\left(\boldsymbol{\pi}_{ts}^{(j)T}\left(\overline{\boldsymbol{\mu}}_{ts}+K_{ts}\boldsymbol{q}^{H(j)}\right)+\varphi_{ts}^{(j)}\right)$$

3. Identify the worst case contingency for stored $\boldsymbol{q}_t^{(j)}$ and $\boldsymbol{\pi}_t^{(j)}$ by running the subproblem, store values of its decision variables and calculate

$$UB^{(j)}=\sum_{t\in T} h_t\Big[\boldsymbol{c}^T\boldsymbol{q}_t^{C(j)}+\boldsymbol{c}^{B^T}\boldsymbol{q}_t^{B(j)}\Big]+\boldsymbol{c}^{L^T}\boldsymbol{q}^{L(j)}+\boldsymbol{c}^{H^T}\boldsymbol{q}^{H(j)}$$
$$+\sum_{t\in T} h_t \sum_{s\in\mathcal{S}_t} p_{ts}\left(\boldsymbol{\pi}_{ts}^{(j)T}\left(\overline{\boldsymbol{\mu}}_{ts}+K_{ts}\boldsymbol{q}^{H(j)}\right)+\Psi_{ts}^{(j)}\right),$$

where $\Psi_{ts}^{(j)}$ is the value of the objective function of the subproblem for time block t and scenario s.

4. If $\left(UB^{(j)}-LB^{(j)}\right)/UB^{(j)}\leq\varepsilon$, then
   STOP;
   else,
   CONTINUE.
5. Include in (4.68)–(4.70) a cutting plane of the format (4.70) with decision variables stored in step 3, set $j\leftarrow j+1$, and go to step 2.

4.4 Illustrative 3-bus Study Case

4.4.1 Description

To illustrate the capabilities of the model presented on previous sections, we are going to apply it to the small-scale system presented on Fig. 4.1. For the sake of simplicity, yearly operation will be represented by one time block of peak demand. This time block will be divided in two scenarios, the first one being normal operation, that comprises 8710 hours, whereas the remaining 50 hours will represent the time the system is affected by an earthquake near bus 2.

The total demand of the system is 250 MW, distributed in a 50 MW load on bus 2, and a 200 MW load on bus 3. At every load and generator, demand and generation can be curtailed at a cost of 3000 \$/MWh. DER post-contingency services can be provided by resources at bus 3 by the following amounts and prices:

- The first service is divided in two steps of 10 MW, with availability costs of 5 and 7 \$/MW, and utilization costs of 10 and 20 \$/MWh.
- The second service is divided in two steps 10 MW, with availability costs of 5 and 7 \$/MW, and utilization costs of 10 and 20 \$/MWh.
- There are 10 MW available for the demand shifting service, with availability cost of 1 \$/MW, and utilization cost of 3 \$/MWh.

There are two generators on the system, the first one located at bus 1, with a maximum capacity of 150 MW and a 10 \$/MWh energy production cost, whereas the second one is located on bus 2, with the same capacity as the first, but a considerably higher energy production cost of 50 \$/MWh. Availability costs for upward and downward reserves are 5 \$/MW and 3 \$/MW for both generators, while utilization costs of each generator is equal to its variable cost.

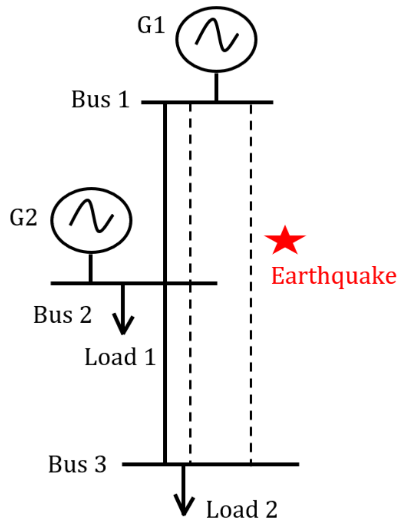


Figure 4.1: 3-bus system

There are two existing transmission lines on the system, that are represented by continuous lines, whereas the three investment candidates are depicted by dashed lines. Two of the three candidates are circuits exactly equal to the existing ones, while the third one is a long line connecting directly cheap generation with the main load. The most important characteristics of existing and candidate transmission lines are presented on Tables 4.1 and 4.2 respectively. Note that we identify the lower bound for failure rates as λ_{lb}, and the upper bound as λ_{ub}, both in occ/hr.

Table 4.1: Characteristics of existing lines

From	To	Capacity [MW]	Scenario 1 λ_{lb}	Scenario 1 λ_{ub}	Scenario 2 λ_{lb}	Scenario 2 λ_{ub}
Bus 1	Bus 2	200	$0.91 \cdot 10^{-4}$	$0.137 \cdot 10^{-3}$	0.019	0.039
Bus 2	Bus 3	200	$0.91 \cdot 10^{-4}$	$0.137 \cdot 10^{-3}$	0.018	0.036

Table 4.2: Characteristics of candidate lines

From	To	Investment Cost [MM\$]	Capacity [MW]	Scenario 1 λ_{lb}	Scenario 1 λ_{ub}	Scenario 2 λ_{lb}	Scenario 2 λ_{lb}
Bus 1	Bus 2	3	200	$0.91 \cdot 10^{-4}$	$0.137 \cdot 10^{-3}$	0.019	0.039
Bus 2	Bus 3	3	200	$0.91 \cdot 10^{-4}$	$0.137 \cdot 10^{-3}$	0.018	0.036
Bus 1	Bus 3	6	200	$0.18 \cdot 10^{-3}$	$0.27 \cdot 10^{-3}$	0.037	0.075

Three different damage levels will be considered for substations, namely 30%, 70%, and 100%, whereas direct damage on demand will be neglected. As we are trying to protect the system against an earthquake near bus 2, we will also consider the alternative of investing on hardening that substation, which will make it less prone to failure. On Table 4.3 lower and upper bounds for the probabilities of undergoing different damage levels are presented, whereas the probabilities for substation 2 if it is hardened, are presented on Table 4.4. It is important to notice that the probability of any damage state must be interpreted as the likelihood of undergoing a damage at least equal to the one of that state.

Table 4.3: Failure rates for different damage states of substations

Bus Number	λ_{lb}^1	λ_{lb}^2	λ_{lb}^3	λ_{ub}^1	λ_{ub}^2	λ_{ub}^3
1	0.0076	0.0002	0.0000	0.0272	0.0015	0.0000
2	0.5577	0.2740	0.0018	0.7411	0.4891	0.0100
3	0.0067	0.0002	0.0000	0.0243	0.0013	0.0000

Table 4.4: Failure rates of substation 2 if it is hardened

Bus Number	λ_{lb}^1	λ_{lb}^2	λ_{lb}^3	λ_{ub}^1	λ_{ub}^2	λ_{ub}^3
2	0.4251	0.1435	0.0011	0.6014	0.3115	0.0051

4.4.2 Results and analysis

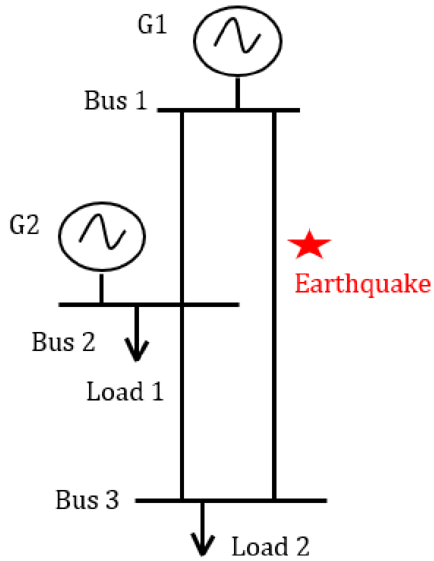
Baseline

In this first case, we will determine the optimal investment considering that hardening substation 2 is not an alternative. Notice that this case emulates the capabilities of a traditional analytical model that is not decision dependent, so it is the baseline to which we will later compare.

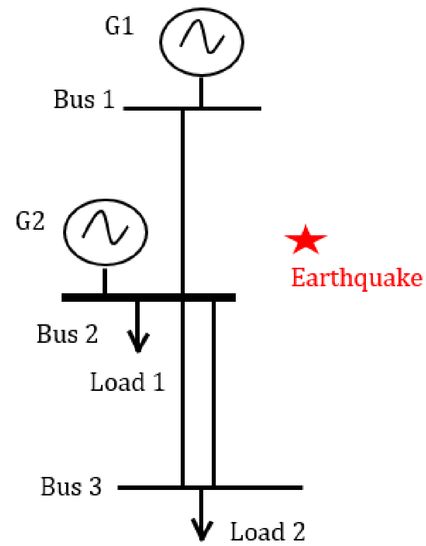
In this case, optimal dispatch of generators consists of 150 MW on generator 1, and 100 MW on generator 2. Optimal investment considers the construction of one of the three transmission assets, the long line that connects bus 1 and bus 3. Notice that as cheap generation resources are located at bus 1, without investment, any outage on a transmission line would trigger a major problem to supply demand on bus 3.

Also, outages on substation 2 have a high probability, and that asset is essential to deliver power to demand connected to bus 3, any outage on that bus, even if it is partial, entails significant demand curtailment. This is why investing in an alternative route to deliver power that doesn't involve substation 2 is an intelligent measure, hence the construction of line 1-3.

No upward reserves were scheduled, but 30 MW of downward primary and secondary reserves were held on generator 1, and 45 MW on generator 2. Downward reserves are very important in case of decreased transmission capacity, do to an outage on a line, or on one of the substations it connects. Note that outages on substations don't only decrease lines capacity, but also the ability of generator to inject power into the system, therefore it is very likely that downward reserves are needed.



(a) Baseline case



(b) Substation hardening allowed

Figure 4.2: Optimal investment plans

Interestingly, the first DER service (demand disconnection/DG increase) was proven to be more effective than upward reserves, since the maximum amount of that service (20 MW) was scheduled. This can be attributed that the service acts physically over resources on the distribution network, and it is not affected by a decrease on transmission capacity, or substation bays capacity, it is not even affected by a reduction on the capacity of the bay that connects that load to the substation. The second and third DER services were not scheduled. Demand increase/DG reduction is particularly useful when it is provided on a bus were generation can be spilled, and it may replace downward reserves. However, in this case downward reserves are better, because when the outage is on the substation, increasing the demand on the bus does not necessarily mean that there are saving on spillage, since the generator may not be able to inject its power by restrictions on the bay it is connected. Demand shifting is particularly useful to enable the utilization of slow but cost-effective upwards reserves, but in this case generator are often not capable of injecting more power, so shifting demand in time, and paying an overall higher consumption is not useful.

Optimal portfolio of strategies

In this case, we analyze the optimal portfolio of resilience enhancing strategies when the whole array of alternatives is considered, even substation hardening. We will compare this solution to the one of the previous case.

The optimal dispatch did not change, it is still 150 MW on generator 1 and 100 MW on generator 2. The investment decision changed, now it is not optimal to invest in line 1-3, instead, it is better to invest in anchoring the components of the substation on bus 2, and in a cheaper line connecting bus 2 and 3. The main reason that motivated the investment in line 1-3 was that the likelihood of outages on substation 2 were too high, therefore there was need for another path for the energy. However, investment on hardening bus 2 decreased failure rates of different damage levels on it, rendering the investment on other route non optimal because of its price. In this case is better to harden substation 2 and construct a shorter line to be hedged against outages on transmission lines.

The analysis made on the previous case about the need for upward reserves, and different DER services still holds. Upward reserves were not scheduled, and neither were the second and third DER services, but the demand disconnection/DG increase service again proved to be be useful, as all its capacity was scheduled. Similarly, in this case downward reserves are needed in order to prevent excessive involuntary generation spillage. However, only 15 MW of primary and secondary downward reserves were scheduled on generator 1, as opposed to the 75 MW that were needed in total on the first case. This happened because hardening the substation made limitations on power injection of generators less likely, therefore the levels of downwards reserves needed decreased.

As observed in this illustrative study case, failing to make a decision considering all hardening strategies simultaneously can lead to an incorrect decision about the optimal network expansion plan that enhances resilience of the grid against earthquakes. Particularly, for this study case investment on new transmission assets would have been overestimated if substation hardening had not been a strategy evaluated, for lack of a suitable model or any other reason.

4.5 Real-Scale Study Case

To show scalability and applicability of the formulation, we will also apply the resilient network design model to the Chilean system as projected to 2024 (by the "E" scenario of the Chilean long term energy planning [77]), in order to assess the potential benefits in resilience of the candidate assets of the country's expansion plan. This plan consists of 10 circuits, grouped in 5 double circuits, one of which is an HVDC link connecting the north of the country with the main demand area, as shown in the Table 4.5.

Additionally, we will consider the possibility of investing on hardening 5 substations of the system, that are presented on Tables 4.6.

4.5.1 Description

The simplified 42-bus version of the Chilean system is shown in Fig. 4.3, which considers 114 transmission assets (most of the lines drawn are double or triple circuits), and 226 generating units, adding up to a 30 GW installed capacity, that includes thermal generators, big and small hydroelectric power plants, and wind and solar generators. During the operation without earthquakes, failure rates used will lie in a 5% confidence interval around the value estimated by the Chilean ISO utilizing 5 years of data [78].

Planning was made considering one time block of peak demand, where three equally likely

Table 4.5: Candidate lines

Line	Type	Capacity [MW]	From	To	Length [km]	Cost [MM$/yr]
1	500kV DC	2x750	Crucero - Encuentro	Cerro Navia - Lo Aguirre	1239	212
2	500kV AC	2x660	Laberinto-Domeyko	Cumbre	316	20.8
3	500kV AC	2x375	Ciruelos	Pichirropulli	67	2.5
4	500kV AC	2x375	Cautín	Charrúa	182	6.8
5	500kV AC	3x375	Ciruelos	Cautín	97	3.6

Table 4.6: Candidate substations to be anchored

Candidate number	Substation name	Cost [MM$/yr]
1	Charrúa	3.94
2	Crucero - Encuento	3.94
3	Laberinto - Domeyko	1.35
4	Los Vilos	1.35
5	Temuco - Cautín	1.35

earthquakes can impact on different locations of the country, meaning that 4 total scenarios will be considered. One earthquake is located at the north, near the city of Antofagasta, another near Santiago (the capital), and the third on the south of the country, near the second biggest city, Concepcion. To model the impact on the power system, there will be considered outages on generators, transmission lines, and two different damage states for substations, one in which their capacity is reduced to 30%, and another one in which there is no capacity left at all. Direct damage at demand level (which enforces demand shedding) will not be considered on this study case.

All the events considered are 7.5-magnitude earthquakes with a 50 km depth of focus. Utilizing the model developed in [75], the effects of the earthquakes were propagated, and the peak ground acceleration was calculated on every point of the system. Once the PGA is obtained, an upper and lower bound will be computed for the outage probability of every element (and for every damage state on substations), utilizing the fragility curves presented on [55, 76]. The lower and the upper bounds will be obtained by varying the computed PGA over an interval centered on its nominal value. Transmission lines were considered to be straight lines connecting two substations, with towers each 300 m. As stated previously, if any of the towers collapses, the transmission line will be considered out of service.

During normal operation only simple outages will be taken into account, whereas on the earthquake scenarios double outages are allowed on the five nearest substations to the event (meaning all the elements connected to them and the substations themselves). Furthermore, outages on other substations, as they will be far away from the earthquake's focus, will be neglected.

Post-contingency operation will consist of two snapshots, in which generators can deliver 2 different reserves services, namely, primary and secondary reserves. DER services will be available on the the substations with the highest residential load, which are the nearest to Santiago and Concepcion. Demand shifting will present a 10% energy payback, while demand reduction/increase services will be divided into 2 steps each with different costs.

The number of hours of the year affected by earthquakes was estimated utilizing historical information from USGS. According to their historical data, 73 earthquakes of magnitude 7 or higher have occurred in Chile throughout the last 113 years. We will assume that a total of 50 hours of operation are affected each time one of these events occurs, meaning that on average 32 hours of operation are affected by earthquakes of those magnitudes each year.

4.5.2 Results and analysis

We are going to use 32 hours as the baseline for the number of hours affected by earthquakes, that determines the number of hours that the different earthquakes scenarios comprise. Two other cases are going to be tested, one in which the amount of hours is increased to 100, and another one in which the number of hours is further increased to 200 hours. Moreover, three different levels of ambiguity were utilized. In the first, lower and upper bounds for failure rates were obtained by using an interval of ±10%, whereas in the second and third the intervals were ±30% and ±50%, respectively.

The model, implemented in Julia 0.6, was solved on a server with 2 Intel Xeon E5-2660 processors, with 10 cores each. Main results regarding investment, and the detail of different first-stage costs are presented on Table 4.7.

The execution time was reasonable for a real-scale problem, and only in one case it exceeded 8 hours. As expected, for a given ambiguity percentage, increasing the amount of hours the earthquakes affect the system is going to impact both operation and investment. Total cost of reserves increases, as well as the total cost of energy production, due to the need to hold more reserves and have a more secure distribution of generation across the country. Substation hardening proved to be a good alternative to harden the system against earthquakes, as it is a measure taken in every case presented on Table 4.7.

It is interesting to notice that the amount of ambiguity being considered has a deep impact on the optimal solution obtained. This ambiguity tries to represent how much do we *believe* on the fragility models utilized to determine failure rates, and as it has been shown, the optimal operation and investment depends on that.

On Table 4.8 the optimal investments for the different cases are presented. It is clear that the most cost-efficient hardening strategies are to anchor the components of substations Charrúa and Laberinto-Domeyko. The correct operation of those two substations is critical

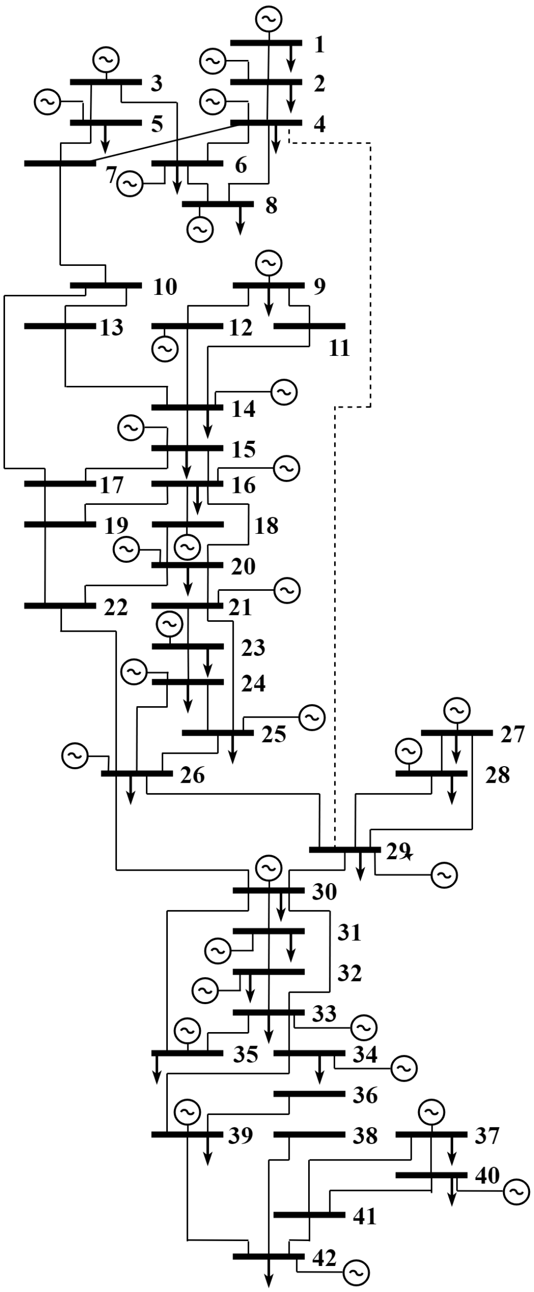


Figure 4.3: Simplified Chilean electric system.

for the performance of the system. The former connects the southern part of the system with the rest, and most of hydro-power generation is connected there. On the other hand, the latter is a fundamental substation in the core of the northern part of the system, were many industrial loads are located (associated with mining activity). In two cases, both with a large ambiguity interval, there are additional substations that need to be hardened in the optimal solution.

It is interesting to notice that lines can contribute with benefits in both resilience and alleviating congestion during normal operation, whereas substation anchoring can only contribute to resilience during earthquakes, notwithstanding, the latter proved to be the preferred

Table 4.7: Costs and investments for different study cases

Case	**32 hr ±10%**	**100 hr ±10%**	**200 hr ±10%**	**32 hr ±30%**	**100 hr ±30%**	**200 hr ±30%**	**32 hr ±50%**	**100 hr ±50%**	**200 hr ±50%**
Execution time [hours]	3.00	5.03	4.13	2.95	5.73	8.71	5.66	5.37	4.03
Number of substations hardened	2	2	2	2	2	2	2	3	3
Number of new HVDC circuits	0	0	0	0	0	2	0	0	2
Number of new AC circuits	0	0	0	0	0	0	0	0	2
Total first stage cost [MM$]	914	957	1028	922	1009	1186	923	1026	1238
Cost of hardening [MM$]	5.29	5.29	5.29	5.29	5.29	5.29	5.29	6.64	9.23
Cost of new HVDC lines [MM$]	0.00	0.00	0.00	0.00	0.00	212.00	0.00	0.00	212.00
Cost of new AC lines [MM$]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.83
Energy cost [MM$]	895.4	917.1	958.6	900.2	944.9	936.5	899.8	953.6	969.0
Total reserves cost [MM$]	5.53	26.66	55.97	8.40	51.05	24.25	9.83	57.10	32.89
Total DER services cost [MM$]	8.71	8.71	8.71	8.71	8.72	8.71	8.71	8.72	8.71

hardening strategy. It is also important to notice that, albeit being by far the most expensive asset, the HVDC line was the transmission asset constructed in the most amount of cases (one other line was part of the optimal portfolio on a single case). This line has the peculiarity that connects 2 very distant substations, in such a way that power can be transferred from the north of the country, where solar resources are located, to the main load, on the center of the country, without other substations in the middle that can be affected by the earthquakes.

Table 4.8: Investment solution for every case

Case	**32 hr** ±10%	**100 hr** ±10%	**200 hr** ±10%	**32 hr** **±30%**	**100 hr** **±30%**	**200 hr** **±30%**	**32 hr** **±50%**	**100 hr** **±50%**	**200 hr** **±50%**
Line 1 Crucero-Encuentro Cerro Navia-Lo Aguirre						x			x
Line 2 Laberinto-Domeyko Cumbre									
Line 3 Ciruelos Pichirropulli									
Line 4 Cautín Charrúa									x
Line 5 Ciruelos Cautín									
Substation 1 Charrúa	x	x	x	x	x	x	x	x	x
Substation 2 Crucero-Encuentro									x
Substation 3 Laberinto-Domeyko	x	x	x	x	x	x	x	x	x
Substation 4 Los Vilos								x	
Substation 5 Temuco-Cautín									

Chapter 5

Conclusions and Further Work

5.1 Conclusions

This work proposed a distributionally robust approach to network security for planning future network infrastructure that can properly recognize both the participation of DER security services and limited knowledge of the underlying process behind the realization of system contingencies. To do so, we proposed a two-stage optimization model, where the first stage determines the transmission expansion plan and the scheduling of generation, up- and down-spinning reserves, and availability of DER post-contingency services, and the second stage minimizes the expected cost of corrective actions under various contingencies. Overall, the proposed model is capable to solve the TEP problem while simultaneously comprising several probability distributions of failure rates, necessary to properly determine the right portfolio of demand-based security services.

Through a number of quantitative assessments, we demonstrated the benefits of security services provided by DER and the advantages of our proposed interval probabilities approach against alternative $n-1$ security and fixed probabilities solutions. In particular, we demonstrated that while the $n-1$ approach significantly undermines the value of DER in displacing network capacity, the fixed probabilities counterpart is optimistic. In this vein, the interval probabilities approach properly utilizes DER services to displace network investments (and other security services from generation reserves), while providing hedged and secured solutions against the partially (un)known reliability data available in reality.

This work also proposed a model capable of determining the optimal network design that enhances its resilience against earthquakes. The aforementioned model does not only consider outages on generation and transmission lines, it also takes into account partial and complete outages on demand and on transmission substations, since they have proven to be of big importance in reality. It also does not assume perfect information on failure rates, that are usually estimated utilizing historical data or fragility models, but treat them in a distributionally robust fashion. Furthermore, the model has decision dependent ambiguity, since it is capable of determining when it is optimal to harden substations by anchoring its components, decision that alters the ambiguity set.

Through an illustrative case study, we showed that considering substation hardening decisions when designing a resilient grid is of paramount importance, since overlooking them may yield an inefficient solution with higher transmission investment than needed in reality. By applying the resilient transmission design model to a simplified version of the Chilean system, we didn't only prove that the formulation is indeed scalable, but also that substation hardening is a decision worth taking into account in the system.

5.2 Further Work

In both models the ambiguity set is restricted to a certain structure, that makes the problem suitable to be solved through Bender's Decomposition. Specifically, the ambiguity set must be constructed utilizing linear inequalities on failure rates of the system. It would be very interesting to propose a model in which other moments of the probability distribution can be adjusted, which is particularly relevant when multiple outages are considered. Another type of ambiguity sets that would be worth studying are sets that consider every distribution sufficiently close to a nominal one, utilizing a certain a metric on the probability distribution space.

It would be also interesting to analyze the effects of considering investment on adding new distributed energy resources that can provide security/resilience to the system, proposing a model that is not constructed from the system operator's perspective, but rather from the point of view of a centralized decision maker. Additionally, uncertainty regarding DER and how this may affect network investments was not considered on this work, and is proposed as further research.

Chapter 6

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