Asymptotic Equivalence of Evolution Equations Governed by Cocoercive Operators and Their Forward Discretizations

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The purpose of this work is to study discrete approximations of evolution equations governed by cocoercive operators by means of Euler iterations, both in a finite and in an infinite time horizon. On the one hand, we give precise estimations for the distance between iterates of independently generated Euler sequences and use them to obtain bounds for the distance between the state, given by the continuous-time trajectory, and the discrete approximation obtained by the Euler iterations. On the other hand, we establish the asymptotic equivalence between the continuous- and discrete-time systems, under a sharp hypothesis on the step sizes, which can be removed for operators deriving from a potential. As a consequence, we are able to construct a family of smooth functions for which the trajectories/sequences generated by basic first-order methods converge weakly but not strongly, extending the counterexample of Baillon.