

# On the existence of weak solutions of semilinear elliptic equations and systems with Hardy potentials

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© 2018 Elsevier Inc. Let  $\Omega \subset \mathbb{R}^N$  ( $N \geq 3$ ) be a bounded  $C^2$  domain and  $\varphi(x) = \text{dist}(x, \partial\Omega)$ . Put

$L^q = L^q(\Omega, \varphi^{-\alpha} dx)$  with  $\alpha > 0$ . In this paper, we provide various necessary and sufficient conditions for the existence of weak solutions to  $\Delta u = \lambda u + \mu$  in  $\Omega$ ,  $u = \psi$  on  $\partial\Omega$  where  $\lambda > 0$ ,  $p > 0$ ,  $\mu$  and  $\psi$  are measures on  $\Omega$  and  $\partial\Omega$  respectively. We then establish existence results for the system  $\begin{cases} \Delta u = \lambda u + \mu \\ \Delta v = \lambda v + \nu \end{cases}$  in  $\Omega$ ,  $u = \psi$ ,  $v = \chi$  on  $\partial\Omega$  where  $\lambda = \pm 1$ ,  $p > 0$ ,  $p' > 0$ ,  $\mu$  and  $\nu$  are measures on  $\Omega$  and  $\psi$  and  $\chi$  are measures on  $\partial\Omega$ . We also deal with elliptic systems where the nonlinearities are more general.