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On dealing with strategic and tactical decision levels in forestry planning under uncertainty☆

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ABSTRACT

A new scheme for dealing with the uncertainty in the scenario trees is considered in the presence of strategic and tactical stochastic parameters for a dynamic mixed 0–1 optimization model in a forest harvesting network along a time horizon under uncertainty. The strategic level of the model presented in this work is included by a several years time horizon, where the uncertainty lies in the timber production. It is represented in a multistage stochastic scenario tree, such that each stage comprises one or several years. Each node in the strategic tree has associated a multi-period scenario graph, where each period in the stages is related to a summer /winter season. The nodes in the graph represent the tactical uncertainty, whose stochastic parameters are the timber price and demand. The strategic decisions aim to the optimal design of the logistic timber harvesting and distribution network at each first period in the stages. The tactical decisions aim to timber harvesting, stocking and distribution from the stands until the markets at the periods in the stages. The model has been validated by using data from a real-life problem.

1. Introduction

Forest companies must plan the sustainable harvest of their resources over a given time horizon. Cut timber is then sold in specific local and international markets. They have to meet demand, primarily from pulp plants and sawmills. The main aim of the companies is to maximize profit while complying with environmental regulations. In previous studies we formulated and solved a specific problem addressing various issues that arise in forestry planning, namely, planning the harvest of forest land designated for timber production and the construction of access roads needed to transport the timber. Good surveys of forest-based supply chain planning cover such aspects as planting, cutting, construction of access roads for transportation. See Bredstrom et al. (2004), Marques et al. (2011), Pinho et al. (2015) and Rönnqvist (2003), among others. Starting in the 70s, environmental and wildlife issues were increasingly considered in forest management models at different planning levels.

In the last 30 years the twin problems of planning harvesting and access road construction have been addressed jointly using mathematical optimization models and computational tools. The advantage of integrating the two processes in a single mixed 0–1 model was demonstrated in Jones et al. (1986), whose solutions are from 15% to 45% better than with models that optimized the processes separately.

There exist relevant studies on the different phases of forestry planning, especially regarding access road construction and harvesting. The problem to be dealt with in this work can be formulated in terms of a partition of the forest into harvesting units, called stands. For a chosen time horizon one must determine which stands will be cut in each period, which roads need to be constructed to access those stands and when, and what quantity of wood will be transported from one point to another. These decisions are made in relation and references therein, among others. Our approach did benefit from these earlier reports. Some ion to an optimization criterion, typically profit maximization. A model for solving the harvesting problem considering road building and adjacency is provided in Candia (2010), which constrains the pos-

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sibility of harvesting adjacent stands for observing the maximum clearfell areas regulations.

Selling prices of forest products are a key element in forestry planning. Price fluctuations have a direct impact on profits from sales and figure prominently in the planners' decision-making. The role played by randomness in forestry planning is closely related to the length of the chosen time horizon. Planners who must make tactical decisions are therefore concerned about price variations at a time horizon of two to five years. Although the most relevant source of uncertainty is prices, uncertainty in tree growth, timber demand and losses due to fires is also significant. The approach developed in this paper analyzes decision-making under uncertainty in wood selling prices and demand. We assume that they can be modeled over time by means of a set of scenarios with different associated probabilities.

In mathematical terms, the deterministic version of the problem, which assumes that all parameters are known, may be formulated as a mixed 0–1 linear optimization model. Even this case is difficult to solve, due to its size and the presence of thousands of binary variables. Approaches for solving this problem have been described in Andalaft et al. (2003), Constantino and Martins (2017), Guignard et al. (1998), Henningsson et al. (2007) and Weintraub and Navon (1976) of them use strengthening of the formulation and decomposition techniques such as Lagrangean relaxation to obtain very good solutions in reasonable computation times with low residual gaps. We should point out that the forest planning problems studied in Andalaft et al. (2003), Guignard et al. (1998) and Weintraub and Navon (1976) consider either the expected scenario or a single scenario.

A stochastic optimization model enables the planner to make more robust decisions by taking into account the stochastic behavior of the selling price and demand of timber. It considers a representative range of timber price scenarios over time, maximizing the expected value instead of merely analyzing a single (e.g., average) scenario as performed in the deterministic version of the problem. It is assumed that the realization of the scenarios at a given period is probabilistically conditioned by the realization of these scenarios in the earlier periods. So, the values of the decision variables at a given node in a multi-period scenario tree also depend on the realization of the uncertain parameters in the ancestors of the node. That is, the values of the variables depend on the values of the parameters and the value of the variables in the scenario groups with one-to-one correspondence with the nodes up to the period that the node belongs to, being a unique solution for those scenarios. So, the non-anticipativity principle is satisfied. See e.g. Birge and Louveaux (2011) and Pflug and Pichler (2014) for the main concepts on stochastic optimization via scenario tree analysis.

There is a variety of papers incorporating risk and uncertainty into forest models, comparing it with a deterministic approach. A good survey and analysis is presented in Pasalodos-Tato et al. (2013), where different sources of risk and uncertainty are considered, namely, forest inventory, timber growth prediction, material hazards as fires, markets (timber prices), climate change, etc. The development of the approach for the forest harvesting planning under uncertainty to be dealt in this work is based on our work Alonso-Ayuso et al. (2018), where a review of the literature on the subject is presented. In any case there is a sizable amount of overlapping between both works, since for completeness reasons we will take from the previous work all the required elements introduced in that work. Anyway the main differences between both works, and the motivation of the new one, are stressed. A real forest harvesting and road building / upgrading planning problem is considered for Forestal Millalemu, see Andalaft et al. (2003), where a much simplified versions was used. However, the original problem is considered in the current work, not the simplified one.

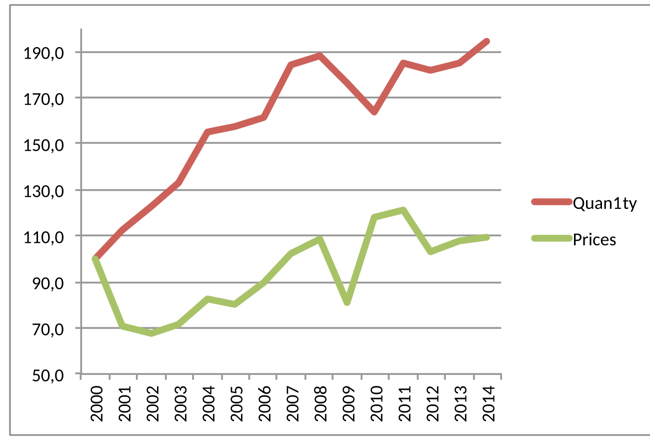


Fig. 1. Chilean forest exports index of wood quantity and prices. (base: Avr. year 2000=100).

Many forest stands are considered in the instances to experiment with.

The case study under consideration is representative of the forest industry and it presents a realistic planning problem of timber harvesting and road building under uncertainty in Chile. Forestry is Chile's second largest source of exports, surpassed only by copper mining. According to data from INFOR (Instituto de Investigación Forestal de Chile - the Chilean Institute of Forest Research), the forest industry exports in 2014 exceeded for the first time the barrier of US$6 billions, registering a sum of US$ 6,094.3 millions, which represents an increase of 6.7% over 2013. Such a figure confirms the magnitude of the industry and underlines the importance of providing its planners with efficient decision-making tools. See in Fig. 1 the evolution of the wood demand and price from 2000 to 2014.

The main difference contribution of this work lies in the multistage multi-period scenario tree setting, see Section 2, that strongly impact the type of model to be dealt with, see Section 4, for solving the problem presented in Section 3. Basically, the idea consists of splitting the scenario set in two subsets, namely, the strategic and the tactical ones. In fact, it helps the scenario reduction without a high impact in the solution goodness and, then, allowing to increase the set of representative scenarios of the uncertain parameters. The instances considered in Sections 5 and 5.3 for validating the approach introduced in this work, have three stages, several period in each stage, and a stage-dependent strategic scenario tree included by 12 successor nodes from the first stage node and 6 successors nodes for each second stage node, in total 85 nodes and 72 strategic scenarios. There are also 8 tactical scenarios as sons of each strategic node. Each of these scenarios are related to a multi-period environment for each stage, such that some variables in the related model are inter-period linking (i.e., state) ones. So, the full strategic-tactical scenario tree has $72 \times 8^3 = 36864$ scenarios. The purpose of this work is to present a scheme and the related model for dealing with the, in this case, $1 + 12 + 72 = 85$ strategic nodes in the tree and 8 tactical scenarios each, such that although the solution of the approximated model is not pretended to be an optimal one of the original model, a (hopefully, small) optimality gap is guaranteed with a reasonable computing effort.

2. Multistage strategic multi-period tactical stochastic tree

Strategic and tactical scenario trees

Let some definitions for a multistage scenario tree setting: A *stage* of a given horizon is a set of consecutive time periods where the realizations of the uncertain parameters take place; A *scenario*

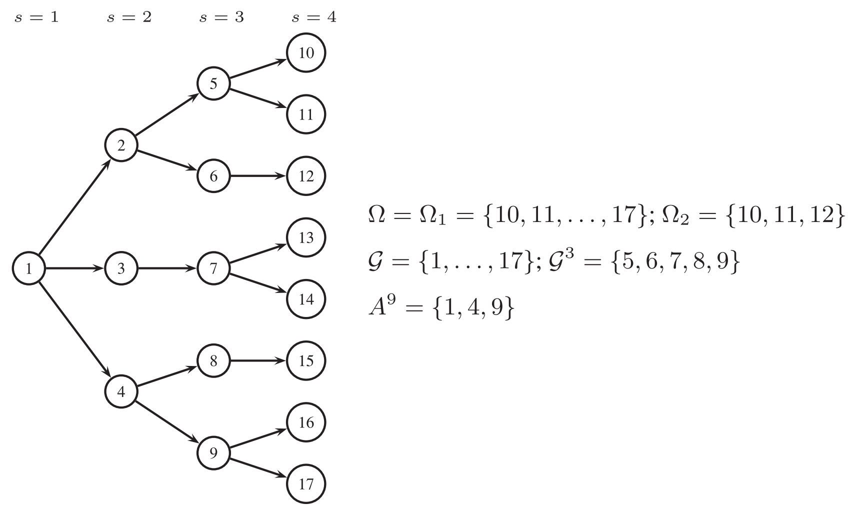


Fig. 2. Multistage nonsymmetric scenario tree.

is a realization of the uncertain parameters along the stages of a given horizon; A *node* for a given stage in the scenario tree has a one-to-one correspondence with the group of scenarios that have the same realization of the uncertain parameters up to the stage; and *Nonanticipativity principle*: Given a node of the scenario tree, the scenarios of the corresponding group have a unique solution up to the stage where the node belongs to.

Let Fig. 2 be a bad approach for long-term Forestry Planning Problem, FPP, unless it is restricted to the strategic elements of the problem.

In this work we consider a *multistage multi-period scenario tree* setting, where the multistage strategic tree is included by the nodes in the first period of each stage, and the multi-period setting is related to tactical scenario multi-period graphs rooted. Each graph is rooted with a strategic node, and it is included by a set of (so-named tactical) nodes, each one for a period of the stage. The goal consists of maximizing the NPV of the expected investment cost (i.e., logistic network building) over the set of strategic nodes plus the related NPV of the expected tactical cost (i.e., timber socking and transportation) plus the unmet demand penalization over the set of tactical nodes. So, the logistic network investment is performed at the strategic nodes and the timber's operations are performed at the tactical nodes.

Multistage strategic scenario tree. Notation

$\mathcal{T}$, set of consecutive time periods (usually, semesters, years) in the time horizon, where $\mathcal{T} = \{1, 2, \dots, T\}$ and $T = |\mathcal{T}|$.

$\mathcal{E}$, set of consecutive stages in the strategic tree, where the set $\mathcal{T}$ is partitioned, $E = |\mathcal{E}|$.

$\mathcal{T}^e$, set of consecutive time periods (usually, semesters, years) in stage e, for $e \in \mathcal{E}$, such that $\mathcal{T} = \cup_{e \in \mathcal{E}} \mathcal{T}^e$ and $\mathcal{T}^e \cap \mathcal{T}^{e'} = \emptyset$, for $e, e' \in \mathcal{E} : e \neq e'$.

$\underline{t}^e$, and $\bar{t}^e$, first and last period in set $\mathcal{T}^e$, resp., for $e \in \mathcal{E}$. Note: $\underline{t}^q = 1$, $\bar{t}^q = |\mathcal{T}^1|$ and $\underline{t}^{e+1} = \bar{t}^e + 1$, for $e \in \mathcal{E} \setminus \{E\}$.

$\mathcal{G}$, set of strategic nodes, where the investment on the logistic network is made

$\mathcal{G}^e \subset \mathcal{G}$, set of strategic nodes that belong to stage e, for $e \in \mathcal{E}$, such that $\mathcal{G} = \cup_{e \in \mathcal{E}} G^e$ and $\mathcal{G}^e \cap \mathcal{G}^{e'} = \emptyset$, for $e, e' \in \mathcal{E} : e \neq e'$. Let us assume that $|\mathcal{G}^1| = 1$ and $0 \in \mathcal{G}^1$.

Ω, strategic scenario set.

$e(g)$, stage which strategic scenario node g belongs to, for $g \in \mathcal{G}$.

$t(g)$, period which strategic node g belongs to, for $g \in \mathcal{G}$. Note: Let us assume that it is the first period $\underline{t}^e$ of set $\mathcal{T}^{e(g)}$.

$\Omega_g \subseteq \Omega$, set of strategic scenarios in the (unique) group with one-to-one correspondence with node g, for $g \in \mathcal{G}$. By construction, the scenarios that belong to set Ω_g share the same realizations of the uncertain parameters up to stage $t(g)$. Note: Ω_g is singleton for any leaf node g of the strategic scenario tree (i.e., $g \in \mathcal{G}^E$). Let us assume that $\omega = g$, for $\omega \in \Omega$.

w^ω, weight or probability assigned to strategic scenario ω, for $\omega \in \Omega$, such that $\sum_{\omega \in \Omega_g} w^\omega = 1$ and $w^g = \sum_{\omega \in \Omega_g} w^\omega$, for $g \in \mathcal{G}$.

$\mathcal{A}_g$, set included by strategic node g and its (strategic) ancestors in the scenario tree, for $g \in \mathcal{G}$. Note: $\mathcal{A}_0 = \{0\}$.

$\mathcal{S}_g$, set included by the immediate strategic nodes g of node g in the scenario tree, for $g \in \mathcal{G}$. Note: $\mathcal{S}_g = \emptyset$, for $g \in \mathcal{G}^E$.

$\sigma(g)$, immediate ancestor strategic node to node g, for $g \in \mathcal{G}$. So, $\sigma(g) \in \mathcal{A}_g$ and $\sigma(0)$ is null.

Tactical multi-period scenario graphs and two-stage trees. Notation

The uncertainty of the tactical parameters, related to the periods in set $\mathcal{T}^e$, $e \in \mathcal{E}$, is represented for the problem subject of this work in a scenario multi-period graph. Here, the tactical (state) variables linking one stage with the next one are some of the variables related to the last period of the stage and the first period of the next one. See the right part of Fig. 3. There are other type of problems where the inter-stage tactical relationship is assumed to be non-existent and, then, the periods from different stages only share the availability of the segments of the logistic network and, so, the tactical graph becomes a multi-period two-stage tree. See the left part of Fig. 3.

Finally, there are other type of problems where the period linking in any stage only lies only on the availability of the segments of the logistic network, see e.g., Fig. 4 for pure operational problems as the rapid transport network design problem (Cadarso et al., 2018) and energy planning (Kaut et al., 2014; Werner et al., 2013).

Fig. 5 shows how the strategic and tactical scenario structures representations are integrated in the forestry problem subject of this work. The scheme to be presented can also be considered in such problems as supply chain planning (Escudero et al., 2017b), electricity transmission and generation capacity planning (Alonso-Ayuso et al., 2016), and others.

For any strategic node $g \in \mathcal{G}$, the elements of the tactical scenario graph to be used throughout this work are as follows:

$\mathcal{Q}_g$, (tactical) node set in the tactical scenario graph rooted with node g.

$\mathcal{Q}_g^t$, (tactical) node set that belongs to period t, for $t \in \mathcal{T}^e$ such that $\mathcal{Q}_g = \cup_{t \in \mathcal{T}^e} \mathcal{Q}_g^t$, for $e \equiv e(g)$.

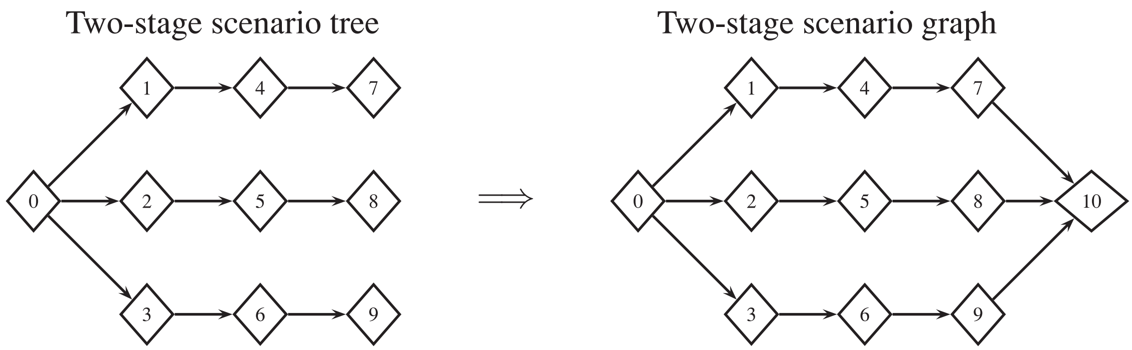


Fig. 3. A multi-period scenario two-stage tree and a multi-period scenario graph for strategic nodes.

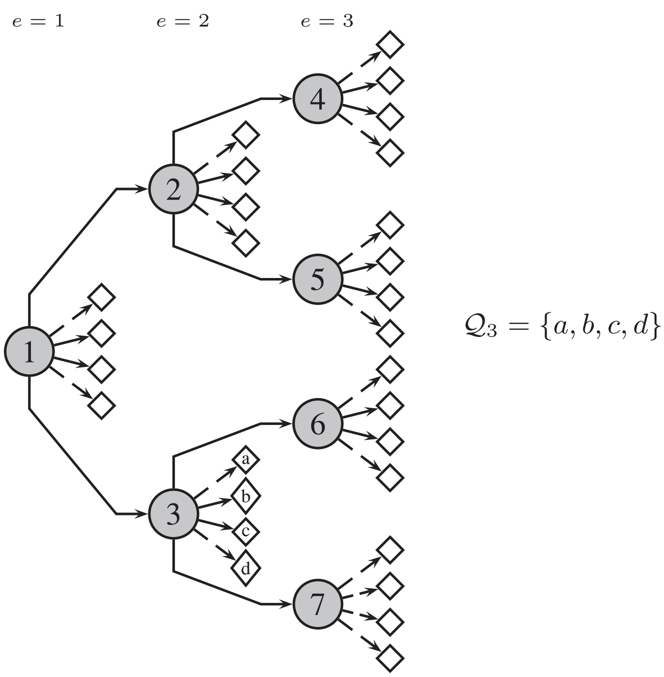


Fig. 4. Strategic multistage scenario tree with operational two-stage scenario trees.

By construction, $\mathcal{Q}_g^{t(g)}$ gives the realizations of the tactical uncertainty in period $t(g)$, being obviously the first period in set $\mathcal{T}^e$, for $e \equiv e(g)$.

$\mathcal{Q}$, node set in the whole scenario tree, where $\mathcal{Q} = \cup_{g \in \mathcal{G}} \mathcal{Q}_g$.

$t(q)$, period which tactical node q belongs to, for $q \in \mathcal{Q}_g$.

$\mathcal{L}_g$, leaf node set in the tactical graph for strategic node g. Note: $\mathcal{L}_g \equiv \mathcal{Q}_g^t$ where $t \equiv \bar{t}^e$, $e \equiv e(g)$.

$\tilde{\mathcal{A}}_g^q$, set included by tactical node q and its (tactical) ancestor nodes in the graph, for $q \in \mathcal{Q}_g$.

Notice that strategic node g *is not* in $\tilde{\mathcal{A}}_g^q$, $q \in \mathcal{Q}_g$, but its replicas q, and $t(q) = t(g)$ for $q \in \mathcal{Q}_g^{t(g)}$.

$\sigma(q)$, immediate (tactical) ancestor to tactical node q, for $q \in \mathcal{Q}_g$ in the graph.

w^q, weight of tactical scenario associated to leaf tactical node q, for $q \in \mathcal{L}_g$, such that $\sum_{q \in \mathcal{L}_g} w^q = 1$.

3. The case study

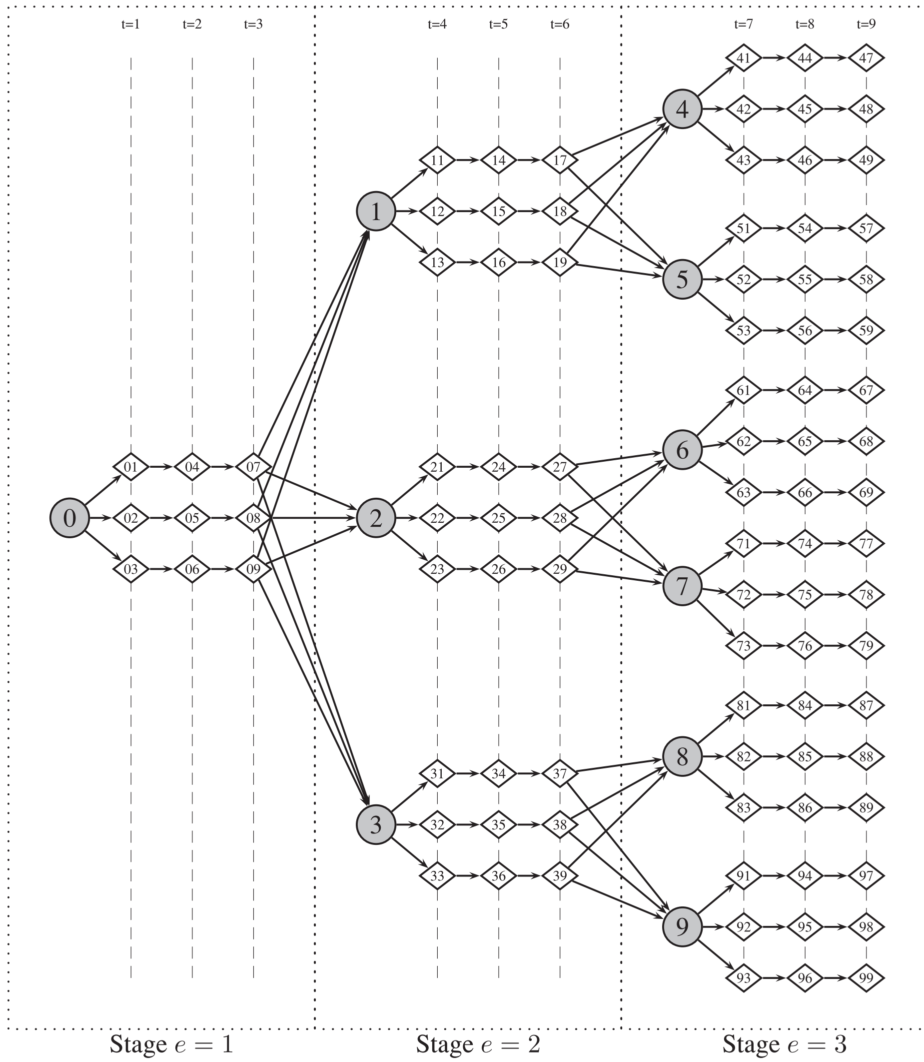
A deterministic mixed 0–1 model as introduced in Andalaft et al. (2003) for forest harvesting and related road building. A scheme for generating the multistage scenario tree for representing the uncertainty due to the variability on the timber price and demand along the time horizon has been presented in Alonso-Ayuso et al. (2018) as well as the tightening of the related stochastic model. The known data used in the experiment are taken from Andalaft et al. (2003). The multistage strategic scenario tree for representing the uncertainty on the volume of (the different qualities of) the available timber per ha in the forest stands under consideration is generated in this work by an ad-hoc scheme. The multi-period tactical scenario tree for representing the uncertainty in the timber price and demand is generated according to the scheme presented in Alonso-Ayuso et al. (2018). A detailed description of the forestry planning problem is presented in Section 3.1 and the main characteristics of the six instances considered for validating the proposal made in this work are presented in Section 5.1.

3.1. The forestry problem

Consider the following management planning problem in the timber industry. The firm under consideration owns plantation lands that are divided into areas. Within each area there are different stands, considered homogeneous as defined by age of trees, soil quality (site index), and volume available per hectare, see Fig. 6). All areas are planted with pine trees, which mature at age 22 to 28. The stands that can be harvested along the time horizon are therefore known. Growth-simulator models developed by the forest firms are used to estimate timber yields in future periods. In this kind of problems, the time horizon considered is usually two to five years (in the computational experience three years are considered).

On the demand side, timber production goes to export, to sawmills, and to pulp plants, as logs. While in reality there are many different products, defined mainly by log length and diameter, at this level of planning only a few basic aggregate products is defined, referred to as export, sawmill, and pulp. Usually a higher-level quality can be used for lower-level purposes, at a loss in sale revenue. For example, the pulp mill takes any type of timber, while only export quality can be exported. The main goal of the planning process is to match the supply of standing timber with demand for timber products of specific grades, lengths and diameters, and, thus, reducing losses in revenues due to down-grading and non-profitable additional cutting.

The problem also considers the logistics of producing and delivering those timber products. Most timber areas are near paved public roads, but in order to get access to the different stands in each area, inside the areas private roads are needed. At the beginning of the time horizon there are potential roads, i.e., roads that can be built as well as existing roads. In any other period there are roads already built and projected ones. In addition to taking into account the existence or nonexistence of roads, one also has to consider their surface quality. First, private roads can be built of



$$\mathcal{E} = \{1, 2, 3\}, \mathcal{T} = \{1, \ldots, 9\}, \mathcal{T}^2 = \{4, 5, 6\}, \Omega = \{4, \ldots, 9\}, \Omega_3 = \{8, 9\}$$

$$\mathcal{G} = \{0, \ldots, 9\}, \mathcal{Q} = \{01, \ldots, 99\}, \mathcal{Q}_2 = \{21, \ldots, 29\}, \mathcal{Q}_2^5 = \{24, 25, 26\}, \mathcal{L}_2 = \{27, 28, 29\}$$

$$e(0) = 1, e(3) = 2, \sigma(2) = 0, \mathcal{A}_2 = \{0, 2\}, \tilde{\mathcal{A}}_2^{27} = \{21, 24, 27\}$$

(i) Strategic node $\quad$ $\diamond_{ij}$ Tactical node in $\mathcal{Q}_i$

Fig. 5. A multistage strategic tree with multi-period tactical graphs rooted with strategic nodes.

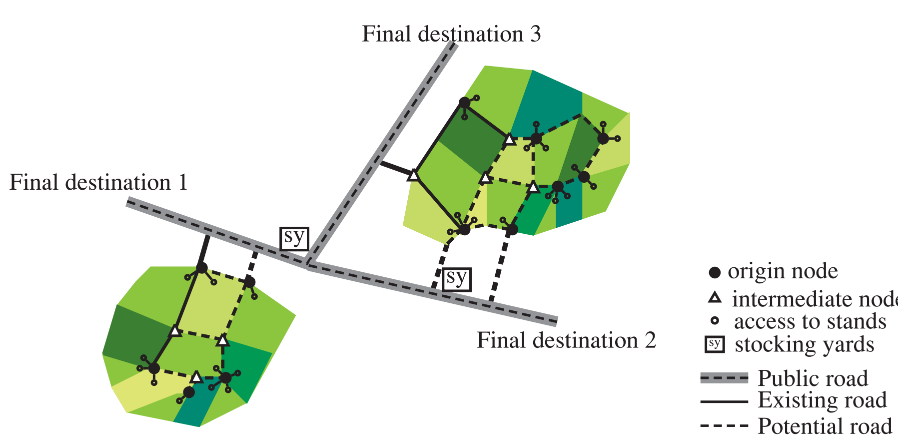


Fig. 6. Areas and (potential and existing) logistic structure.

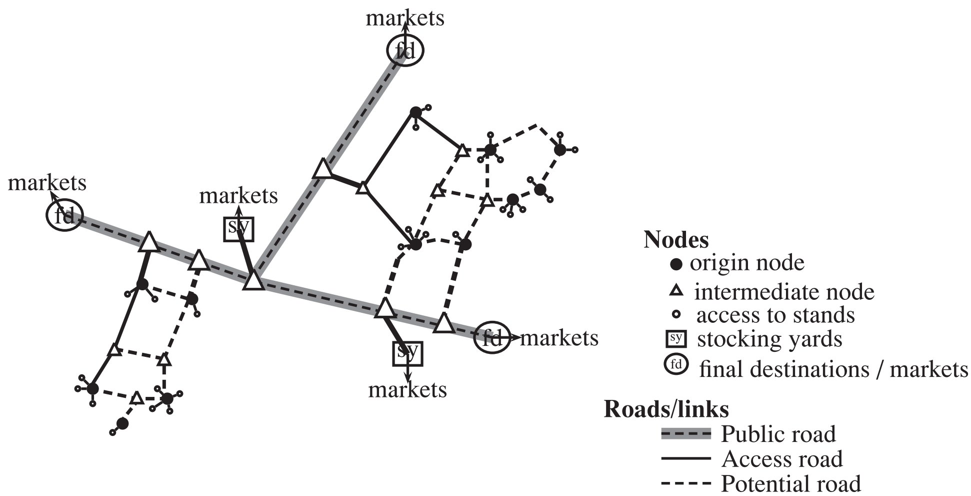


Fig. 7. Logistic network.

either dirt or gravel, and this has an impact on operations. Gravel roads are more expensive to build, but lead to lower transportation costs and can be used year-round, while dirt roads are only useful in the dry summer. Next, road building and upgrading should be carried out in proper sequence so as to be consistent, timed with stand harvesting, as well as to avoid excessive road building. In addition, road building can only be carried out in summer.

Harvested timber can be stocked from summer to winter in stocking yards; it makes sense to keep in the stocking yards from summer to winter some of the timber harvested in stands accessed via dirt roads, which can only be harvested in summer. The stocking yards are located where there exist gravel road connections to the area exit, so that timber harvested in summer can also be sent to destinations in winter.

Finally, consider the production and delivery of timber demand. Aggregate demands are projected to future periods, often as lower and upper bounds and so are the expected prices. Cable logging (or towers) carry out harvesting for steep areas, while skidders harvest flat terrain. Timber hauling is carried out by truck to such destinations as ports, pulp plants, sawmills, or stocking yards. Harvesting machinery and crews are usually subcontracted with yearly contracts. There is no clear way to evaluate the fixed costs needed to install the harvesting machinery process, so firms replace this cost by a policy of harvesting at least 10 or 15 hectares for larger stands, and harvest the whole stand for smaller areas.

To summarize, the basic decisions to be considered in each period are as follows:

- stands to be harvested;
- roads to be built (in gravel or dirt) and roads to be upgraded from dirt to gravel;
- amount of timber production, by aggregate product for harvesting to satisfy demand;
- amount of timber transported to destinations or stocked from summer to winter, if applicable.

As an example, a logistic infrastructure for the problem is depicted in Fig. 6. Observe that there are public roads for transporting the material from the two harvesting areas and two stocking yards. Each harvesting area is accessible through an existing gravel road, but there are other possible access roads. Not all stands are accessible through the existing roads and additional roads are needed to be able to access them.

The logistic structure can be modeled as a network where the nodes can be defined as follows:

- Stands: They can be represented by nodes in the network associated to an access point.
- Origins: Each access point to a stand is linked to an origin node, such that one or more stands are accessible from each origin point, but the stands are only accessible from one origin.
- Intermediate points: road junctions (linking different pieces of roads, public or private).
- Stocking yards.
- Final destinations.

Notice that products are sent to the markets from the final destination nodes or directly from the stocking yards.

The set of links in the network includes all roads in the model (public and private, existing or potential for the latter) and the connections between origins and stands. Fig. 7 shows the network associated with the logistic structure depicted in Fig. 6.

In the stochastic model, all parameters are assumed to be known at the beginning of the time horizon except (strategic) productivity, and (tactical) timber prices and demand along the time horizon.

- Uncertainty in strategic tree: amount of timber (m^3) of different qualities available per ha in each stand if harvested. The amount of timber available at the beginning of the time horizon is known, but the availability for future periods (years) is considered uncertain. In the literature, see Alonso-Ayuso et al. (2018) and Ríos et al. (2016), this parameter is determined through a growth simulator. However, the proposal in this work is to generate different scenarios for the growth/degrowth rates.
- Uncertainty in tactical graphs: Timber demand and prices can vary along the time horizon, see Fig. 1. Notice the volatility of the uncertain parameters which is, therefore, very difficult to predict. The proposal is to represent this uncertainty via a representative set of tactical scenarios in the graph structure type depicted in Fig. 5 for each strategic stage.

4. Multistage stochastic multi-period forestry planning model

The additional notation to be used throughout the work is as follows, where the parameters and variables are denoted with capital and small letters, resp.

Sets

$\mathcal{T}_S$ and $\mathcal{T}_W$, consecutive summer and winter time periods in stage e, resp., for $e \in \mathcal{E}$, such that $\mathcal{T} = \mathcal{T}_S \cup \mathcal{T}_W$ and $\underline{t}^e \in \mathcal{T}_S$

(i.e, the stages start with the summer periods in any stage). Note: $\mathcal{T}_S \cap \mathcal{T}_W = \emptyset$, for $e \in \mathcal{E}$.

$\mathcal{P}$, harvest products. It is an ordered set, such that $\mathcal{P} = \{p_1, p_2, \ldots, p_{|\mathcal{P}|}\}$, where the timber indexed with p has higher quality than the p'th timber, provided that $p < p'$. A higher-level quality can be used for lower level purposes, at a loss in sale price, see below.

$\mathcal{N}$, such that $\mathcal{N} = (\mathcal{I}, \mathcal{R})$, where $\mathcal{I}$ is the set of nodes in the network and $\mathcal{R}$ is the set of links.

$\mathcal{I}$, nodes in the road network, such that

$$\mathcal{I} = \mathcal{I}^O \cup \mathcal{I}^I \cup \mathcal{I}^S \cup \mathcal{I}^F,$$

where $\mathcal{I}^O$ is the set of origin nodes (it has some stands associated), $\mathcal{I}^I$ is the set of intermediate nodes (a junction in the roads network), $\mathcal{I}^S$ is the set of stocking yards, and $\mathcal{I}^F$ is the set of final destination nodes (it is directly connected to the markets).

$\mathcal{C}$, stands.

$\mathcal{C}_i$, stands associated with origin node i, such that $\mathcal{C}_i \subset \mathcal{C}$, for $i \in \mathcal{I}^O$.

$\mathcal{R}$, links (potential or existing) in the road network. A link is an edge linking two consecutive nodes in the road network.

$\mathcal{K}$, set of types of link (i.e., road) standards, being $k = 1$ for dirt and $k = 2$ for gravel.

$\mathcal{R}^E$, and $\mathcal{R}^P$, existing and potential links, resp., such that $\mathcal{R} = \mathcal{R}^E \cup \mathcal{R}^P$.

$\mathcal{R}^E_k$, existing links in standard k, for $k \in \mathcal{K}$.
Note 1: All public roads exist from the beginning of the time horizon and they are in gravel.
Note 2: Existing links in dirt (i.e., private roads) can be upgraded to gravel.
Note 3: Existing or potential links in dirt cannot be used in winter, so, they should be upgraded to gravel in case that they are to be used.

$\mathcal{M}$, markets.

$\mathcal{M}_i$, markets served from node i (being a final destination or stocking yard), such that $\mathcal{M}_i \subseteq \mathcal{M}$, for $i \in \mathcal{I}^F \cup \mathcal{I}^S$.

Γ_i, adjacency set of node i, for $i \in \mathcal{I}$, where $j \in \Gamma_i \iff \{i, j\} \in \mathcal{I} \iff i \in \Gamma_j$.

$\mathcal{R}^{(i,j)}$, adjacent to link (i, j), for $(i, j) \in \mathcal{R}$.

Deterministic parameters

$\overline{A}_c$, upper bound in the area (ha) of stand c that can be harvested, for $c \in \mathcal{C}$.

$\underline{A}_c$, lower bound in the area (ha) of stand c to be harvested in any time period, if any, for $c \in \mathcal{C}$.

N_c, maximum number of periods that stand c can be harvested, for $c \in \mathcal{C}$.
Note 1: It depends on $\overline{A}_c$, such that combined with $\underline{A}_c$ it tries to concentrate the harvesting of a stand in a reasonable number of time periods with a minimum area to be harvested, at least.
Note 2: A possible value for N_c could be $\left\lceil \frac{\overline{A}_c}{\underline{A}_c} \right\rceil$, for $c \in \mathcal{C}$.

N^*_c, maximum number of stages that stand c can be harvested. This is a strategic parameter.

U^e_{ijk}, flow capacity (m^3) on link (i, j), built in standard k, available at any period of stage e, for $(i, j) \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}$. Note: The flow in a link can be in both directions.

$\overline{S}_i$, capacity (m^3) of stocking yard i, for $i \in \mathcal{I}^S$.

P^t_c, unit harvesting cost per ha in stand c at period t, for $c \in \mathcal{C}, t \in \mathcal{T}$.

$\overline{P}^{pt}_i$, unit production cost per m^3 of timber of quality p in any stand associated with node i at period t, for $i \in \mathcal{I}^O, p \in \mathcal{P}, t \in \mathcal{T}$.

D^{pt}_{ijk}, unit transportation cost of timber of quality p through link (i, j) in standard k at period t, for $(i, j) \in \mathcal{R}, k \in \mathcal{K}, p \in \mathcal{P}, t \in \mathcal{T}$.

$\overline{D}^{pt}_{im}$, unit transportation cost of timber of quality p from node i to market m at period t, for $m \in \mathcal{M}_i, i \in \mathcal{I}^F \cup \mathcal{I}^S, p \in \mathcal{P}, t \in \mathcal{T}$.

H^t_{ijk}, cost of building link (i, j) in standard k, for $(i, j) \in \mathcal{R}^P, k \in \mathcal{K}$.

$\overline{H}^t_{ij}$, cost of upgrading link (i, j) from standard dirt (i.e., $k = 1$) to gravel (i.e., $k = 2$) at period t, for $(i, j) \in \mathcal{R}^P \cup \mathcal{R}^E_1, t \in \mathcal{T}$.

$\hat{H}^t_i$, unit stocking cost in yard i at period t, for $i \in \mathcal{I}^S, t \in \mathcal{T}$.

Latency and ι-strategic node for road standard availability

τ_1, and τ_2, latency (i.e., number of periods) required for making available a potential link in dirt $k = 1$ and in gravel $k = 2$, resp., from the period that is decided to build it.
Note 1: The latency refers to periods and the building is made at the first period of a stage.
Note 2: If a link is available in the first stage, then it is assumed that the decision to build it was is made before the beginning of the time horizon.
Note 3: It is assumed that the latency is the same for all potential links along the time horizon.
Observe that one distinguishes between the stage when the decision is made to build a link and the stage when the link becomes available.

ι^g_k, strategic node whose period $t(\iota^g_k)$ is the *latest* one *by* which the link (i.e., road) built in standard k, for $k \in \mathcal{K}$, can start its construction, so that it is available for (tactical) use at any period that belongs to stage $e(g)$, for $g \in \mathcal{G}$. It can be computed as follows

$$\iota^g_k = argmax_{g' \in \mathcal{A}_g}\{t(g') \in \mathcal{T} : t(g') \leq t(g) - \tau_k\}$$

Stochastic parameters

- Strategic tree:

B^{pg}_c, amount of timber (m^3) of quality p produced per ha in stand c if harvested at stage $e(g)$ in strategic node g, for $c \in \mathcal{C}, p \in \mathcal{P}, g \in \mathcal{G}$. It is determined through a growth simulator, see Alonso-Ayuso et al. (2018) and Ríos et al. (2016).

- Tactical graph:

$\underline{Z}^{pq}_m$, and $\overline{Z}^{pq}_m$, lower and upper bound, resp., on demand (m^3) of timber of quality p from market m in tactical node q of strategic node g, for $p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}_g, g \in \mathcal{G}$. Note: $t(q) \in \mathcal{T}^{e(g)}$, for $g \in \mathcal{G}$.

R^{pq}_m and S^{pq}_m, unit selling price and unit penalization cost for unmet demand of timber of quality p, resp., from market m in tactical node q of strategic node g, for $p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}_g, g \in \mathcal{G}$. Note: $S^{pq}_m >> R^{pq}_m$.

Strategic (binary) variables

w^g_{ijk}, its value 1 means that potential link (i, j) is built in standard k *by* stage $e(g)$ in strategic node g and otherwise, 0, for $(i, j) \in \mathcal{R}^P, k \in \mathcal{K}, g \in \mathcal{G}$. Notice that w^g_{ijk} is a so-named *step variable*, it makes the model stronger than when using its counterpart so-called impulse variable, see e.g., Guignard et al. (1998) for forest harvesting.

ν^g_{ij}, its value 1 means that link (i, j) is upgraded from dirt (i.e., $k = 1$) to gravel (i.e., $k = 2$) *by* stage $e(g)$ in strategic node g and otherwise, 0, for $(i, j) \in \mathcal{R}^P \cup \mathcal{R}^E_1, g \in \mathcal{G}$. Notice that, by construction, the link cannot be upgraded in the same stage it is built. It is a *step variable*.

u_c^g, its value 1 means that stand c is opened for harvesting from the first period $\underline{t}^e$ of stage $e(g)$ in strategic node g and otherwise, 0, for $c \in \mathcal{C}, g \in \mathcal{G}$.
u_i^{*g}, its value 1 means that one stand in set $\mathcal{C}_i$ associated with origin node i, at least, has been opened for harvesting *by* stage $e(g)$ in strategic node g and otherwise, 0, for $i \in \mathcal{I}^O, g \in \mathcal{G}$. It is a *step variable*.

Tactical variables

e_c^q, binary variable whose value 1 means that stand c is harvested at period $t(q)$ in tactical node q, and otherwise, 0, for $c \in \mathcal{C}, q \in \mathcal{Q}$.
e_c^{*g}, maximum number of (non-necessarily consecutive) time periods that stand c has been harvested *by* stage $e(g)$ in the tactical scenarios, for $c \in \mathcal{C}, g \in \mathcal{G}$. It is a *step variable*.
x_c^q, area (ha) of stand c that is harvested at period $t(q)$ in tactical node q, for $c \in \mathcal{C}, q \in \mathcal{Q}$.
x_c^{*g}, maximum area (ha) of stand c that has been harvested *by* stage $e(g)$ in the tactical scenarios, for $c \in \mathcal{C}, g \in \mathcal{G}$. It is a *step variable*.
y_i^{pq}, volume (m^3) of timber of quality p harvested in all stands associated with origin i at period $t(q)$ in tactical node q, for $i \in \mathcal{I}^O, p \in \mathcal{P}, q \in \mathcal{Q}$.
f_{ijk}^{pq}, flow (m^3) of timber of quality p transported on link (i, j) (built in standard k) at period $t(q)$ in tactical node q, for $(i, j) \in \mathcal{R}, k \in \mathcal{K}, p \in \mathcal{P}, q \in \mathcal{Q}$.
Note: $f_{ij1}^{pq} = 0$ for timber of quality p, for $p \in \mathcal{P}$, in tactical node q on existing link in dirt ($(i, j) \in \mathcal{R}_1^E$) or potential link in dirt (i.e, $(i, j) \in \mathcal{R}^P$), provided that node q belongs to a winter period (i.e., $q \in \mathcal{Q} : t(q) \in \mathcal{T}_W$).
f_{im}^{pq}, flow (m^3) of timber of quality p transported from node i (i.e., a final destination or a stocking yard) to market m at period $t(q)$ in tactical node q, for $m \in \mathcal{M}_i, i \in \mathcal{I}^F \cup \mathcal{I}^S, p \in \mathcal{P}, q \in \mathcal{Q}$.
b_i^{pq}, stock (m^3) of timber of quality p in stocking yard i at the end of period $t(q)$ in tactical node q, for $i \in \mathcal{I}^S, p \in \mathcal{P}, q \in \mathcal{Q}$.
z_m^{pq}, amount (m^3) of timber delivered as quality p to market m at period $t(q)$ in tactical node q, for $p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}$.
Note: The timber delivered to the market at the price of quality p can actually be (in part or totally) of a higher quality.
z_m^{-pq}, unmet demand (m^3) of timber of quality p requested from market m at period $t(q)$ in tactical node q, for $p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}$.

Strategic constraints

s1 *Road network design*:

$$w_{ijk}^{\sigma(g)} \le w_{ijk}^g \qquad \forall k \in \mathcal{K}, (i, j) \in \mathcal{R}^P, g \in \mathcal{G} \tag{1a}$$

$$v_{ij}^{\sigma(g)} \le v_{ij}^g \qquad \forall (i, j) \in \mathcal{R}^P \cup \mathcal{R}_1^E, g \in \mathcal{G} \tag{1b}$$

A link cannot be upgraded from dirt to gravel if it has not been built earlier enough:

$$v_{ij}^g \le w_{ij1}^{\iota_1^g} \qquad \forall (i, j) \in \mathcal{R}^P, g \in \mathcal{G} \tag{1c}$$

Road incompatibility: A link cannot simultaneously be built in dirt and in gravel:

$$\sum_{k \in \mathcal{K}} w_{ijk}^g \le 1 \qquad \forall (i, j) \in \mathcal{R}^P, g \in \mathcal{G} \tag{1d}$$

Note: A link that already is built in dirt can be *upgraded* to gravel and, in that case, it cannot not be built in gravel.

s2 *Stand selection*:
A stand can only be harvested in a limited number of stages along the time horizon:

$$\sum_{g' \in \mathcal{A}_g} u_c^{g'} \le N_c^* \qquad \forall c \in \mathcal{C}, g \in \mathcal{G}^E \tag{1e}$$

s3 *Network connectivity*:
Origins-to-roads triggers: If a origin is not connected to an existing link at the beginning of the time horizon and one of its stands is to be harvested, then one potential link has to be built, at least. By noticing that $w_{ijk}^{\iota_k^g}$ is penalized in the objective function (9), observe that $w_{ijk}^g = 0$ for $u_i^{*g} = 0$. Then:

$$u_i^{*\sigma(g)} \le u_i^{*g} \qquad \forall i \in \mathcal{I}^O, g \in \mathcal{G} \tag{1f}$$

$$u_c^g \le u_i^{*g} \qquad \forall c \in \mathcal{C}_i, i \in \mathcal{I}^O, g \in \mathcal{G} \tag{1g}$$

$$u_i^{*g} \le \sum_{\{i,j\} \in \mathcal{R}^P} \sum_{k \in \mathcal{K}} w_{ijk}^{\iota_k^g} \qquad \forall i \in \mathcal{I}^O : \left\{(i, j) \in \mathcal{R}^E : j \in \Gamma_i\right\} = \emptyset, g \in \mathcal{G} \tag{1h}$$

Road-to-road triggers (Andalaft et al., 2003; Guignard et al., 1998): If a potential link (i, j) that is not yet connected to an existing one is to be built, then one of the links, say $(i', j') \in \mathcal{R}^{(i,j)}$, must be built, at least:

$$\sum_{k \in \mathcal{K}} w_{ijk}^g \le \sum_{(i', j') \in \mathcal{R}^{(i,j)}} \sum_{k \in \mathcal{K}} w_{i'j'k}^g \qquad \forall (i, j) \in \mathcal{R}^P : \mathcal{R}^{(i,j)} \cap \mathcal{R}^E = \emptyset, g \in \mathcal{G} \tag{1i}$$

Strategic-tactical link constraints

ts1 *Harvesting decisions*: A stand cannot be harvested if it has not been opened yet:

$$e_c^q \le u_c^g \qquad \forall c \in \mathcal{C}, q \in \mathcal{Q}_g, g \in \mathcal{G} \tag{2a}$$

ts2 *Capacity constraints*:
Product flow through a potential link in dirt (i.e., $k = 1$) is only allowed at any period of a stage, provided that the link has been built in dirt and not upgraded to gravel *by* that stage:

$$\sum_{q' \in \tilde{\mathcal{A}}_g^q} \sum_{p \in \mathcal{P}} \left(f_{ij1}^{pq'} + f_{ji1}^{pq'}\right) \le U_{ij1}^{e(g)} \left(w_{ij1}^{\iota_1^g} - v_{ij}^{\iota_2^g}\right) \quad \forall (i, j) \in \mathcal{R}^P, q \in \mathcal{L}_g, g \in \mathcal{G} \tag{3a}$$

Product flow through a potential link in gravel is only allowed at any time period of a stage, provided that the link has been built in gravel or upgraded from dirt to gravel *by* that stage:

$$\sum_{q' \in \tilde{\mathcal{A}}_g^q} \sum_{p \in \mathcal{P}} \left(f_{ij2}^{pq'} + f_{ji2}^{pq'}\right) \le U_{ij2}^{e(g)} \left(w_{ij2}^{\iota_2^g} + v_{ij}^{\iota_2^g}\right) \quad \forall (i, j) \in \mathcal{R}^P, q \in \mathcal{L}_g, g \in \mathcal{G} \tag{3b}$$

Product flow through an existing link in dirt is only allowed, provided that it has not been upgraded to gravel:

$$\sum_{q' \in \tilde{\mathcal{A}}_g^q} \sum_{p \in \mathcal{P}} \left(f_{ij1}^{pq'} + f_{ji1}^{pq'}\right) \le U_{ij1}^{e(g)} (1 - v_{ij})^{\iota_2^g} \qquad \forall (i, j) \in \mathcal{R}_1^E, q \in \mathcal{L}_g : t(q) \in \mathcal{T}_S, g \in \mathcal{G} \tag{3c}$$

Since a link in dirt can only be used (for timber flow) in summer periods, then:

$$f_{ij1}^{pq} = 0 \qquad \forall (i,j) \in \mathcal{R}_1^E \cup \mathcal{R}^P,\, p \in \mathcal{P},\, q \in \mathcal{Q}_g : t(q) \in \mathcal{T}_W,\, g \in \mathcal{G} \tag{3d}$$

Product flow through an existing link, that was built in dirt, is only allowed in gravel, provided that previously it has been upgraded:

$$\sum_{q' \in \tilde{A}_g^q} \sum_{p \in \mathcal{P}} \left(f_{ij2}^{pq'} + f_{ji2}^{pq'} \right) \le U_{ij2}^{e(g)} v_{ij}^{t_2^g} \quad \forall (i,j) \in \mathcal{R}_1^E,\, q \in \mathcal{L}_g : t(q) \in \mathcal{T}_S,\, g \in \mathcal{G} \tag{3e}$$

Product flow through an existing link in gravel is upper bounded:

$$\sum_{q' \in \tilde{A}_g^q} \sum_{p \in \mathcal{P}} \left(f_{ij2}^{pq'} + f_{ji2}^{pq'} \right) \le U_{ij2}^{e(g)} \qquad \forall (i,j) \in \mathcal{R}_2^E,\, q \in \mathcal{L}_g,\, g \in \mathcal{G} \tag{3f}$$

Tactical constraints

t1 *Harvesting bounding in tactical nodes related to the strategic ones*:

$$\underline{A}_c e_c^q \le x_c^q \le \overline{A}_c e_c^q \qquad \forall c \in \mathcal{C},\, q \in \mathcal{Q}_g,\, g \in \mathcal{G} \tag{4a}$$

$$x_c^{*\sigma(g)} + \sum_{q' \in \tilde{A}_g^q} x_c^{q'} \le x_c^{*g} \qquad \forall c \in \mathcal{C},\, q \in \mathcal{L}_g,\, g \in \mathcal{G} \tag{4b}$$

$$x_c^{*g} \le \overline{A}_c \qquad \forall c \in \mathcal{C},\, g \in \mathcal{G}^E \tag{4c}$$

$$e_c^{*\sigma(g)} + \sum_{q' \in \tilde{A}_g^q} e_c^{q'} \le e_c^{*g} \qquad \forall c \in \mathcal{C},\, q \in \mathcal{L}_g,\, g \in \mathcal{G} \tag{4d}$$

$$e_c^{*g} \le N_c \qquad \forall c \in \mathcal{C},\, g \in \mathcal{G}^E \tag{4e}$$

Constraints (4a) bound the harvested area per stand.
Constraints (4b) compute the maximum harvested area per stand in the tactical scenarios for each strategic node. Notice that a tactical scenario is included by the nodes in the path from a tactical replica of the strategic node, say g, (and, then, it belongs to period $t(g)$) to its tactical leaf node (and, then, it belongs to period $\overline{t}^{e(g)}$). Constraints (4c) bound the maximum harvested area per stand in the tactical scenarios up to the end of the time horizon.
Constraints (4d) compute the maximum number of periods that each stand has been harvested in the tactical scenarios for each strategic node. Constraints (4e) bound the maximum number of periods that a stand can be harvested in the tactical scenarios up to the end of the time horizon.
Notice that the robust constraint systems (4b)–(4c) and (4d)–(4e) are based on the 'worst case' approach.

t2 *Production of timber products by origin nodes in tactical nodes*:

$$\sum_{c \in \mathcal{C}_i} B_c^{pq} x_c^q = y_i^{pq} \qquad \forall i \in \mathcal{I}^O,\, p \in \mathcal{P},\, q \in \mathcal{Q} \tag{5}$$

Remember that $\mathcal{Q} = \cup_{g \in \mathcal{G}} \mathcal{Q}_g$.

t3 *Flow constraints of timber products for origin nodes (6a), intermediate nodes (6b), and final destination nodes (6c) in the tactical nodes of the scenario graphs*:

$$y_i^{pq} + \sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{jik}^{pq} = \sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{ijk}^{pq} \qquad \forall i \in \mathcal{I}^O,\, p \in \mathcal{P},\, q \in \mathcal{Q} \tag{6a}$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{jik}^{pq} = \sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{ijk}^{pq} \qquad \forall i \in \mathcal{I}^I,\, p \in \mathcal{P},\, q \in \mathcal{Q} \tag{6b}$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{jik}^{pq} = \sum_{k \in \mathcal{K}} \sum_{j \in \Gamma_i} f_{ijk}^{pq} + \sum_{m \in \mathcal{M}_i} f_{im}^{pq} \qquad \forall i \in \mathcal{I}^F,\, p \in \mathcal{P},\, q \in \mathcal{Q} \tag{6c}$$

t4 *Flow constraints of timber products for stocking yards in the tactical nodes of the scenario graphs*:
Constraint system (6d)–(6e) forces that the stock at the end of the previous node plus the arrivals (in summer periods) minus the dispatches (in winter periods) in a given tactical node must be equal to the final stock in that node). It is worth to point out that a winter season (and the same for a summer one) can be composed by several (consecutive) periods in the same yearly stage, since the seasons could be split in different periods, say months or quarters. So, without of loss of generality it has been assumed that the stages start with summer periods and end with winter ones.

$$b_i^{pq} = \sum_{q' \in \mathcal{L}_{\sigma(g)}} \omega^q b_i^{pq'} + \sum_{k \in \mathcal{K}, j \in \Gamma_i} f_{jik}^{pq} - \sum_{m \in \mathcal{M}_i} f_{im}^{pq}$$
$$\forall i \in \mathcal{I}^S,\, p \in \mathcal{P},\, q \in \mathcal{Q}_g : \left\{ t(q) = t(g) \right\},\, g \in \mathcal{G} \tag{6d}$$

Let us consider any scenario graph structure where the tactical uncertainty is represented as well as the tactical variables, see Fig 5. Observe the approximation that is made in eq. (6d) by considering the expected stock at the end of the immediate previous stage to be used as input in the tactical replicas of the strategic nodes in the stages. Notice that those replicas belong to set $\mathcal{Q}_g^{t(g)}$ for $g \in \mathcal{G}$. Note: Remember that σ^g is null for $g = 0$.

$$b_i^{pq} = b_i^{p\sigma(q)} + \sum_{k \in \mathcal{K}, j \in \Gamma_i} f_{jik}^{pq} - \sum_{m \in \mathcal{M}_i} f_{im}^{pq}$$
$$\forall i \in \mathcal{I}^S,\, p \in \mathcal{P},\, q \in \mathcal{Q}_g : \left\{ t(q) > t(g) \right\},\, g \in \mathcal{G} \tag{6e}$$

Constraints (6f) forces that the stocking yards be emptied at the end of the last winter period. On the other hand, constraints (6g) and (6h) force that there is not any product arrival in winter periods as well as there is not any product dispatch in summer ones, resp.

$$b_i^{pq} = 0 \qquad \forall i \in \mathcal{I}^S,\, p \in \mathcal{P},\, q \in \mathcal{Q}_g : \left\{ t(q) \in \mathcal{T}_W \wedge t(q) = \overline{t}^{e(g)} \right\},\, g \in \mathcal{G} \tag{6f}$$

$$f_{jik}^{pq} = 0 \qquad \forall k \in \mathcal{K},\, i \in \mathcal{I}^S,\, j \in \Gamma_i,\, p \in \mathcal{P},\, q \in \mathcal{Q}_g : \left\{ t(q) \in \mathcal{T}_W \right\},\, g \in \mathcal{G} \tag{6g}$$

$$f_{im}^{pq} = 0 \quad \forall p \in \mathcal{P},\, i \in \mathcal{I}^S,\, m \in \mathcal{M}_i,\, q \in \mathcal{Q}_g : \left\{ t(q) \in \mathcal{T}_S \right\},\, g \in \mathcal{G} \tag{6h}$$

t5 *Demand constraints*:

$$\sum_{p' \in \mathcal{P}: p' \le p} z_m^{p'q} \le \sum_{p' \in \mathcal{P}: p' \le p} \sum_{i \in \mathcal{I}^F \cup \mathcal{I}^S} f_{im}^{p'q} \qquad \forall p \in \mathcal{P},\, m \in \mathcal{M},\, q \in \mathcal{Q} \tag{7a}$$

$$\underline{Z}_m^{pq} \le z_m^{-pq} + z_m^{pq} \le \overline{Z}_m^{pq} \qquad \forall p \in \mathcal{P},\, m \in \mathcal{M},\, q \in \mathcal{Q}_g \tag{7b}$$

Observe that the amount of timber delivered to a market as quality p is bounded by the flow arriving to the nodes that can serve that market. On the other hand, notice that the timber's quality can be higher than requested at a lost, as state above.

t6 *Capacity of stocking yards (in summer periods)*:

$$\sum_{p\in\mathcal{P}} b_i^{pq} \le \bar{S}_i \qquad \forall i \in \mathcal{I}^S, q \in \mathcal{Q} \tag{8}$$

Variables' domain definition

$$w_{ijk}^g \in \{0,1\} \quad \forall (i,j) \in \mathcal{R}^P, k \in \mathcal{K}, g \in \mathcal{G}$$
$$v_{ij}^g \in \{0,1\} \quad \forall (i,j) \in \mathcal{R}^P \cup \mathcal{R}_1^E, g \in \mathcal{G}$$
$$u_c^g \in \{0,1\} \quad \forall c \in \mathcal{C}, g \in \mathcal{G}$$
$$u_i^{*g} \in \{0,1\} \quad \forall i \in \mathcal{I}^O, g \in \mathcal{G}$$
$$e_c^{*g} \in N \quad \forall c \in \mathcal{C}, g \in \mathcal{G}$$
$$x_c^{*g} \ge 0 \quad \forall c \in \mathcal{C}, g \in \mathcal{G}$$
$$e_c^q \in \{0,1\} \quad \forall c \in \mathcal{C}, q \in \mathcal{Q}$$
$$x_c^q \ge 0 \quad \forall c \in \mathcal{C}, q \in \mathcal{Q}$$
$$y_i^{pq} \ge 0 \quad \forall i \in \mathcal{I}^O, p \in \mathcal{P}, q \in \mathcal{Q}$$
$$b_i^{pq} \ge 0 \quad \forall i \in \mathcal{I}^S, p \in \mathcal{P}, q \in \mathcal{Q}$$
$$f_{ijk}^{pq} \ge 0 \quad \forall (i,j) \in \mathcal{R}, k \in \mathcal{K}, p \in \mathcal{P}, q \in \mathcal{Q}$$
$$f_{im}^{pq} \ge 0 \quad \forall p \in \mathcal{P}, i \in \mathcal{I}^F \cup \mathcal{I}^S, m \in \mathcal{M}_i, q \in \mathcal{Q}$$
$$z_m^{pq} \ge 0 \quad \forall p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}$$
$$z_m^{-pq} \ge 0 \quad \forall p \in \mathcal{P}, m \in \mathcal{M}, q \in \mathcal{Q}$$

Objective function

The objective function under consideration consists of maximizing the NPV expected profit. It can be expressed as follows:

$$\max \sum_{g\in\mathcal{G}} \omega^g \Bigg\{ \sum_{(i,j)\in\mathcal{R}^P} \sum_{k\in\mathcal{K}} (-H_{ijk}^t) w_{ijk}^g + \sum_{(i,j)\in\mathcal{R}^P\cup\mathcal{R}_1^E} (-\overline{H}_{ij}^t) v_{ij}^q + \sum_{q'\in\mathcal{L}_g} \omega^{q'} \sum_{q\in\tilde{\mathcal{A}}_g^{q'}} \Bigg(\sum_{m\in\mathcal{M}} \sum_{p\in\mathcal{P}} R_m^{pq} z_m^{pq} - \Bigg[\sum_{m\in\mathcal{M}} \sum_{p\in\mathcal{P}} S_m^{pq} z_m^{-pq} + \sum_{c\in\mathcal{C}} P_s^q x_c^q - \sum_{i\in\mathcal{I}^O} \sum_{p\in\mathcal{P}} \overline{P}_i^{pq} y_i^{pq} + \sum_{(i,j)\in\mathcal{R}} \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}} D_{ijk}^{pq} f_{ijk}^{pq} + \sum_{i\in\mathcal{I}^S} \hat{H}_i^q \sum_{j\in\Gamma_i} \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}} f_{jik}^{pq} + \sum_{i\in\mathcal{I}^F\cup\mathcal{I}^S} \sum_{m\in\mathcal{M}_i} \sum_{p\in\mathcal{P}} \overline{D}_{im}^{pq} f_{im}^{pq} \Bigg] \Bigg) \Bigg\} \tag{9}$$

5. Computational experiments

The instances in the experiments carried out for testing RN model (1)–(9) are based on Andalaft et al. (2003), in which a deterministic version of the problem was solved using real data from the forest company *Forestal Millalemu*. It consisted of 17 forests, geographically separated, each connected through public roads to demand nodes. It produced three wood qualities (for export, sawmills and pulp plants) that were sent to different destinations, either final markets or processing plants. The instances in the testbed have been created by selecting subsets of the areas in order to obtain small but yet realistic examples where the stochastic version could be solved in a reasonable computing time. The time horizon considered is three years and each year is divided into two seasons (summer and winter). Each year defines a strategic stage and a season defines a period in the stage. So, $E=3, T=6, \mathcal{T}_S=\{1,3,5\}, \mathcal{T}_W=\{2,4,6\}, \mathcal{T}_1=\{1,2\}, \mathcal{T}_2=\{3,4\}, \mathcal{T}_3=\{5,6\}, \underline{t}_1=1, \overline{t}_1=2, \underline{t}_2=3, \overline{t}_2=4$ and $\underline{t}_3=5, \overline{t}_3=6$.

Table 1
Instance dimensions.

Ins.	na	$\mathcal{C}$	$\mathcal{I}^S$	$\mathcal{I}$	$\mathcal{R}^P$	$\mathcal{R}_1^E$	$\mathcal{R}_2^E$	Ha	$\mathcal{P}$	$\mathcal{M}$
i1	2	32	0	25	7	7	10	579.9	3	7
i2	4	15	0	38	7	10	21	745.4	3	7
i3	2	21	1	44	8	16	20	216.1	3	7
i4	3	19	0	43	5	14	22	388.3	3	7
i5	1	32	1	53	4	22	25	404.1	3	7
i6	7	29	0	43	11	23	33	989.2	3	7

Table 2
Model dimensions.

	Deterministic model			Stochastic model		
Ins.	m	nc	$n01$	m	nc	$n01$
i1	1543	2667	242	313,790	559,289	20,138
i2	1957	3678	326	400,927	774,764	28,468
i3	2397	4563	427	491,372	958,109	37,307
i4	2210	4230	367	459,369	893,934	33,145
i5	3166	5529	578	660,633	1,154,459	54,566
i6	3146	6603	592	641,606	1,391,949	51,584

5.1. Instances' description

The main characteristics of the instances considered in this work are shown in Table 1. The headings are as follows: *na*, number of areas; $\mathcal{C}$, number of stands; $\mathcal{I}^S$, number of stocking yards; $\mathcal{I}$, number of forest network nodes; $\mathcal{R}^P$, number of potential roads; $\mathcal{R}_1^E$ and $\mathcal{R}_2^E$, number of existing roads in dirt and in gravel, resp.; *Ha*, total forest surface; $\mathcal{P}$, number of harvest products; and $\mathcal{M}$, number of markets.

The strategic scenario tree structure corresponds to a $1 \times 12 \times 6$ model, where the second stage has 12 nodes and each of the second-stage nodes has 6 sons, resulting in $|\Omega| = 12 \times 6 = 72$ scenarios and $|\mathcal{G}| = 85$ strategic nodes, see Fig. 8.

Each strategic node has associated a two-stage scenario graph with 8 different scenarios and 2 periods. Then, the model has $|\mathcal{Q}| = 85 \times 8 = 680$ two period tactical nodes, being the tactical uncertain situations, where an inter-period dependent deterministic submodel is optimized for each one.

To build the data for the sons of a node, a base increment of 5%, 6% and 7% of the timber has been considered from one year to the next one for export, sawmills and pulp plants quality, resp. That increment has been increased and decreased by using a step of 3% in order to obtain a different value for each son. The scenarios of each tactical two-period graph have been built by a combination of 4 product price scenarios and 2 demand ones by following the scheme described in Alonso-Ayuso et al. (2018).

The dimensions of the deterministic model (only one scenario) and the compact formulation of the strategic-tactical risk neutral strategic model RN (1)–(9)are shown in Table 2. The headings are as follows: *m*, number of constraints, *nc*, number of continuous variables, and *n*01, number of binary variables. Observe the high differences between both formulations.

The computational experiments were conducted in the HW/SW platform given by a WS under the Linux operating system (version Ubuntu GNU/linus 14.04.1) with 64 bits, 2 processors Intel(R) Xeon(R) CPU E5-2630 @ 2.3 GHz, 64 Gb of RAM DDR3 1600 MHz ECC and 24 virtual cores. The model has been implemented with GAMS 24.3.2. The optimization has been carried out by considering a state-of-the-art commercial optimization engine, CPLEX 12.6.1. The optimality gap has been set to 1%, while the elapsed time limit has been set to 15 h.

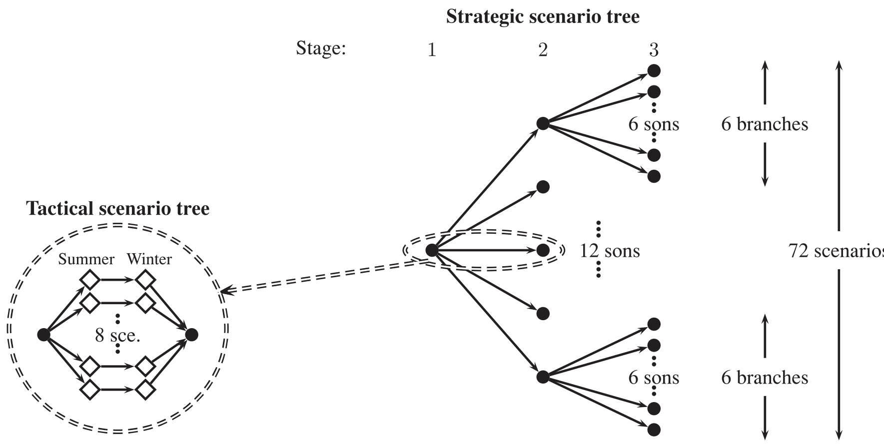


Fig. 8. Forestry multistage strategic multi-period tactical scenario tree.

Table 3
Main results for RN model (1)–(9).

Ins.	Z_{LP}	Z_{IP}	GAP_{LP}	GAP_{OPT}	Elapsed time	
i1	2098715.07	2044393.36	2.66%	0.97%	5 h	30 min
i2	2391157.05	2339693.40	2.20%	1.41%	15 h	0 min
i3	3176521.19	3140017.68	1.16%	0.70%	6 h	6 min
i4	2168936.59	2137675.68	1.46%	1.00%	5 h	57 min
i5	4557525.13	4501659.78	1.24%	0.71%	8 h	56 min
i6	3374674.50	3286739.55	2.68%	1.95%	15 h	0 min

5.2. Some results

Table 3 presents the main results obtaining in the optimization process. The headings are as follows: Z_{LP} is the optimal solution for the continuous relaxation fo the problem; Z_{IP} is the best feasible obtained by the solver, GAP_{LP} is the integrality gap, defined as the relative gap between Z_{IP} and Z_{LP}; GAP_{OPT} is the optimality gap, i.e., the relative difference between the objective function for the incumbent solution and the best known upper bound (ie.e., the highest solution value among the active branch-and-cut nodes at the optimization stopping time instant. Finally, the elapsed time is provided (a limit of 15 h has been imposed). Notice that in despite of the model's dimensions (some with more than 1.3 million variables), the approach can provide a solution in a reasonable time, guaranteeing an optimality gap of less than 1 percent in 4 out of the 6 instances in the testbed.

An interesting scheme for evaluating the usefulness of the strategic-tactical proposal is its comparison with some traditional alternatives, where only a strategic tree is considered. That is, the tactical graph associated to each strategic node is replaced with just one scenario that is built by considering the expected value of each uncertain parameter at tactical level. Let the two following approaches:

Model EV, where the (tactical and strategic) uncertain parameters have been replaced with their expected values in the related stage. Let the well-known Expected profit of the Expected Value (EEV) be obtained by applying the EV solution to the scenarios. The methodology for obtaining the EEV is very well established for the two-stage setting, see Birge and Louveaux (2011), but it is not for the multistage one, see Escudero et al. (2007). Alternatively, we propose the following methodology for obtaining EEV in a rolling horizon type of calculation (see Agustín et al., 2012 for more details): (1) The solution for the first stage is taken from the EV solution. (2) Once the solution up to stage $e-1$ is fixed, $|\mathcal{G}^e|$ independent strategic scenario subtrees remain for stage e, where the strategic uncertainty is taken back for each of their root nodes, say g, for $g \in \mathcal{G}^e$, instead of the expect value in the nodes of the stage, and each strategic uncertain parameter is replaced with is its expected value in the successor nodes for each stage. So, each uncertain strategic parameter in node g', for $g' \in \mathcal{G}^{e(g')}$, where g' belongs to the successor set (i.e., set $\mathcal{S}_g$) of node g is replaced with the expected one in set $\mathcal{S}_g \cap \mathcal{G}^{e(g')}$. On the other hand, each uncertain tactical parameter for the nodes g' in set $\{g\} \cup \mathcal{S}_g$ is replaced with the expect value in set $\Pi_{g'}$. (3) So, the EV solution is independently obtained for each scenario subtree, such that each one is rooted with a node g in $\mathcal{G}^e$, then, the solution for each root node is fixed to its EV solution, (4) The models where only the stages $E-1$ and E are involved are mixed 0–1 strategic two-stage problems, where the first stage nodes belongs to set $\mathcal{G}^{E-1}$, each uncertain tactical parameter is replaced with their expected value for each strategic node and, finally, they are solved. (5) At the end of the process there is a solution for each strategic scenario, such that EEV is the weighting of the solution values of the scenarios as calculated by the procedure.

Strategic model RNst, where each uncertain parameter in the tactical scenario graph in model RN model (1)-(9) is replaced with the corresponding expected value. So, the model is the traditional strategic stochastic one.

It is worthy to point out the main difference between the solution of the approaches EEV and RNst for the submodels, being each one rooted with a strategic node g in set $\mathcal{G}^{e(g)}$. Notice that the approach EV considers the expected value of the strategic parameters in the submodels supported by the subtrees rooted with the successor nodes of node g, i.e., nodes in set $\mathcal{S}^g$. On the other hand, the solution of RNst considers the strategic tree that supports the model. In both approaches the uncertainty in the tactical parameters is represented by their expected values in the related subtree at the corresponding stage. They both also consider the strategic solution attached to the previous stages and the tactical solution attached to the last period of the immediate previous stage.

Both EEV and RTst solutions are evaluated in the original strategic-tactical model RN (1)–(9), by fixing their strategic variables as well as the production and transportation ones. Additionally, the WS (Wait-and-See) profit is provided as a reference. It gives the expected profit for the whole strategic-tactical scenario set, included by all the combinations of the tactical scenarios along the time horizon (each one joined by the related strategic node). In total, $72 \times 8^3 = 36864$ different scenarios are considered. Notice that the non-anticipativity constraints (NAC) in the scenario tree

Table 4
Comparison of the different approaches (1).

Ins.	EEV			RNst			RN			WS
	Z_{IP}	Elapsed time		Z_{IP}	Elapsed time		Z_{IP}	Elapsed time		Z_{IP}
i1	1812.57	0 h	3 min	1947.79	2 h	18 min	2044.53	5 h	30 min	2112.29
i2	2084.79	0 h	8 min	2237.13	15 h	0 min	2339.89	15 h	0 min	2469.11
i3	–			3003.07	3h	10 min	3140.22	6 h	6 min	3193.58
i4	1858.55	0 h	7 min	1978.96	7 h	48 min	2137.88	5 h	57 min	2232.21
i5	4070.99	0 h	5 min	4380.91	1 h	23 min	4501.94	8 h	56 min	4627.56
i6	2879.51	0 h	17 min	3173.39	5 h	30 min	3287.06	15 h	0 min	3497.03

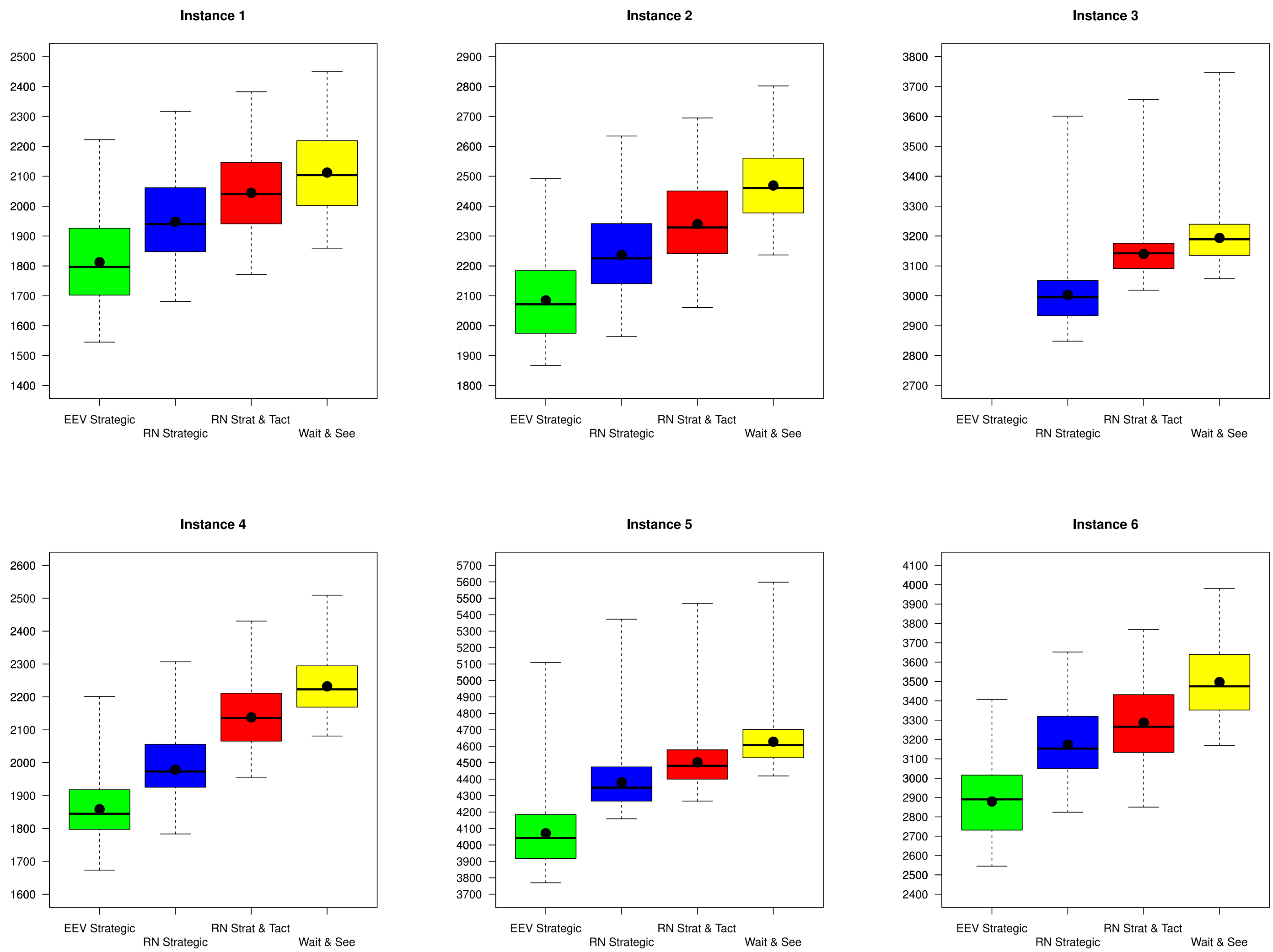


Fig. 9. Profit distribution.

are relaxed and, therefore, it is not an implementable policy. However, it is an upper bound of the expected profit of any feasible solution for model RN and, then, it can be used for computing the solutions's goodness provided by the three approaches under consideration.

Fig. 9 depicts the profit distribution over the set of scenarios for both approaches. The results of the experiment are reported by taken benefit of the use of boxplots. It is a standardized way that, in this case, allows to display the summary of the distribution of the scenario profit as quantified in the following five statistical measures: minimum, first quartile, median, third quartile, and maximum. The central box spans from the first quartile to the third one, the segment inside the rectangle shows the median, and the *whiskers* above and below the box show the locations of the minimum and maximum. The cumulative expected profit up to the last stage is also presented. All measures are depicted for the four alternatives in each instance. Notice that the proposed RN (1)–(9) is always dominant (all measures represented in the boxplot are always better) In fact, it can be observed the increasing profit in all measures from the worst alternative, EEV, to the best one, RN, apart the non-implementable measure, WS, for all of the 6 instances in the testbed.

5.3. Discussion

Table 5 shows the comparison of the incumbent solution value provided by the approaches EEV, RNst and RN as well as the upper bound WS of the optimal solution value for the original prob-

Table 5
Comparison of the different approaches (2).

Ins.	EEV		RNst		RN	
	#best	*dev.best*	*#best*	*dev.best*	*#best*	*dev.best*
i1	0	11.3	0	4.7	72	0.0
i2	0	10.9	0	4.4	72	0.0
i3	–	–	0	4.4	72	0.0
i4	0	13.1	0	7.4	72	0.0
i5	0	9.6	2	2.8	70	1.3
i6	0	12.4	0	3.5	72	0.0

lem with the 36,864 combinations of the strategic-tactical scenarios, see below. Note: Due to space limitations, the columns headed Z_{IP} (expected profit) should be read as $Z_{IP} \times 10^3$ what is a rounding of the true values. It can be observed in the table that approach EV gives an infeasible solution for RN model (1)–(9). Approach RNst is stopped while reaching the allowed time (as it is RN for instances i2 and i6). And, surprisingly, RNst requires much more time for obtaining the RN solution that it provides for instance i4 than the time required by approach RN (its expected profit being smaller). On the other hand, notice that Z_{IP} for RN is between 2.76% and 8.03% higher than for RNst, and it is for the latter between 6.45% and 10.17% higher than for EEV (whenever EEV gives a feasible solution).

Table 5 shows some other illustrative results. Let the following notation related to approach $a \in \{EEV, RNstRN\}$, for $\omega \in \Omega$: $best^{\omega} = max_{\{a\}}\{profit_a^{\omega}\}$, highest profit among the three approaches in scenario ω, for $\omega \in \Omega$; and $\Omega_a \subseteq \Omega$, subset of scenarios where the profit $profit_a^{\omega}$ obtained by approach a was not the highest one, $best^{\omega}$. The headings are as follows for each approach a: Z_{IP}, expected profit provided by approach a (the values have been normalized, being 100 the profit of RN) (i.e., cardinality of set $\{\omega \in \Omega_a : profit_a^{\omega} < best^{\omega}\}$; and *#best*, number of scenarios where approach a has the highest profit out of the $|\Omega| = 72$ strategic scenarios in the experiment; and *dev.best*, expected difference (in percentage) between the highest profit $best^{\omega}$ and $profit_a^{\omega}$ obtained by approach a, among the subset of scenarios Ω_a (i.e., the set of scenarios where approach a does not provide the best profit). It can be expressed as $dev.best = 100\frac{\sum_{\omega\in\Omega_a} w^{\omega}(best^{\omega}-profit_a^{\omega})}{\sum_{\omega\in\Omega_a} w^{\omega} best^{\omega}}$. Notice that the higher the number *#best* and the smaller the difference *dev.best*, the higher the quality of approach a. Finally, the WS expected profit is given as a reference. It can be shown that RN provides the best result in almost all scenarios for the six instances. Observe also that the upper bound Z_{IP} for WS is only between 1.71% and 6.38% higher than for RN.

Finally, notice that the strategic-tactical original scenario tree has $1 + 8 + 8 \times 128 + 8^2 \times 12 + 8^2 \times 72 + 36,864 = 42345$ nodes. So, it is unrealistic to seek for the optimal solution of the original, by-large. Additionally, since the constraint system (6d)–(6e) in model RN performs only an approximation of the product stored volume at the end of the stages, the RN solution is not guaranteed to be the optimal one in the original model. However, a hint on its quality can be given by the WS solution value.

6. Conclusions and outline of future research plans

In this work, we have presented a model for a multistage multi-period stochastic mixed 0–1 model for forestry planning. Its framework can be extended to problem solving for capacity expansion planning (CEP) problems in a broad sense along a long-term time horizon. Two types of decisions are considered, namely strategic ones (i.e., decisions on the selection, capacity and timing of forest stands and road building / upgrading) and tactical decisions (related to product production, transportation, stoking and market distribution) based on the available infrastructure elements in the periods of the time horizon. Those periods are partitioned in stages and, without loss of generality, the first period of each stage is chosen for the strategic decision making. Given the dynamic nature of the problem, the realization of the main parameters is uncertain. Two types of uncertainties are considered, namely the strategic and tactical ones. A finite set of discrete scenarios, represented in a multistage scenario tree, is taken into account for considering the two types of uncertainties, contrary to the traditional approach in practice that considers expected values for the uncertain parameters.

Additionally, contrary to the traditional approach in stochastic optimization literature for dynamic forestry planning and others, the uncertain parameters in the scenario tree are not independently considered of their strategic or tactical character and, additionally, they are stagewise dependent. A huge reduction on the dimensions of the full strategic-tactical scenario tree has been performed since the latter has 36,864 scenarios in the tree with 42,345 nodes in the testbed that we have experimented with. The set of tactical multiperiod scenarios has been represented as a graph-based structure. The approximation that is proposed in this work has 72 strategic scenarios with 85 nodes in the tree, plus 8 two-period tactical scenarios in each strategic node, such that the value of the inter-stage linking variables is replaced with the expected one. However, in spite of the strategic-tactical partition in the scenario tree, the model's dimensions are still very high. The dimensions of the instances in the testbed we have experimented with are up to 641,606 constraints, 1,391,949 continuous variables and 51,584 0–1 variables. The optimality gap of the proposal is only between 1.71% and 6.38% with respect to the WS upper bound in the original model.

Notice that for solving real-life multistage strategic problems with tactical graph structures, the RN-based model could provide solutions with high cost variability in the scenarios. Thus, risk averse functionals should be dealt with, so that the negative impact in the profit function should be prevented for low-probability small profit scenarios. The type of risk management to address in our future research plan considers a mixture of the following strategic node-based time-consistent and time-inconsistent multi-function functionals for risk averse:

1. The time-inconsistent stochastic dominance functional on the (strategic and tactical-based) values of the chosen functions up to the nodes in modeler-driven intermediate stages along the time horizon, see Escudero et al. (2017a). Notice that it destroys the nice structure of the RN model, due to the many related cross scenario node constraints that are involved.
2. The expected conditional stochastic dominance functional on the (strategic and tactical-based) values of the chosen functions for selected scenario groups, see Escudero et al. (2017b). Notice that the functional does not destroy too-much the structure of the RN model. Ir is time-consistent, see the definition in Escudero and Monge (2018), Escudero et al. (2017b), Homem-de Mello and Pagnoncelli (2016), Rudloff et al. (2014) and Shapiro et al. (2009), among others.

A matheuristic version of the Nested Stochastic Decomposition (NSD) methodology, in particular, the Stochastic Dynamic Programming (SDP) algorithm for stagewise dependent uncertainty is one of the most suitable methodologies for solving dynamic problems, see Escudero et al. (2017b). Given the character of the state step variables, our future NSD research would benefit from the splitting variable scheme considered in Zou et al. (2018) for state 0–1 variables.

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