Natural Resources Research (© 2020)
https://doi.org/10.1007/s11053-020-09692-6

Original Paper

Simulation of Synthetic Exploration and Geometallurgical Database of Porphyry Copper Deposits for Educational Purposes

Mauricio Garrido,[1,5] **Exequiel Sepúlveda**,[2,3] **Julián Ortiz**,[4] **and Brian Townley**[1]

Received 23 December 2019; accepted 29 April 2020

The access to real geometallurgical data is very limited in practice, making it difficult for practitioners, researchers and students to test methods, models and reproduce results in the field of geometallurgy. The aim of this work is to propose a methodology to simulate geometallurgical data with geostatistical tools preserving the coherent relationship among primary attributes, such as grades and geological attributes, with mineralogy and some response attributes, for example, grindability, throughput, kinetic flotation performance and recovery. The methodology is based in three main components: (1) definition of spatial relationship between geometallurgical units, (2) cosimulation of regionalized variables with geometallurgical coherence and (3) simulation of georeferenced drill holes based on geometrical and operational constraints. The simulated geometallurgical drill holes generated look very realistic, and they are consistent with the input statistics, coherent in terms of geology and mineralogy and produce realistic processing metallurgical performance responses. These simulations can be used for several purposes, for example, benchmarking geometallurgical modeling methods and mine planning optimization solvers, or performing risk assessment under different blending schemes. Generated datasets are available in a public repository.

KEY WORDS: Geometallurgy, Geostatistics, Synthetic database, Uncertainty.

INTRODUCTION

At present, access to large mining exploration and/or geometallurgical databases from industry, for academic and/or educational purposes, is difficult, and this due to confidentiality restrictions and/or budget limitations. Development of realistic synthetic geometallurgical databases as proposed in this paper may allow an alternative to such problem and may also offer a robust tool for the purposes of benchmarking exploration and/or geometallurgical modeling, mine planning methods or reserves estimations (Garrido et al. 2019).

Geometallurgy has become an important field in mining engineering because of its benefits on the ore quality on mine planning, plant performance, lower costs and product quality. To incorporate these benefits into the mining value chain, key metallurgical responses and proxy variables need to be incorporated into the block model, which is the main input to solve many optimization problems in

[1]Department of Geology, Universidad de Chile, Santiago, Chile.
[2]School of Civil, Environmental and Mining Engineering, The University of Adelaide, Adelaide, Australia.
[3]School of Mining Engineering, Universidad de Talca, Talca, Chile.
[4]The Robert M. Buchan Department of Mining, Queen's University, Kingston, ON, Canada.
[5]To whom correspondence should be addressed; e-mail: maumap@ug.uchile.cl

Published online: 14 May 2020

mine planning (Ortiz et al. 2015; Dominy et al. 2018). This enriched block model with geometallurgical variables is commonly termed a geometallurgical block model (GMBM), and the methodology of this research is based on the transfer of the simulated attributes of the GMBM to a geometallurgical database (GMDB), referred to a drill holes of a geological exploration campaign. From the point of view of practitioners, researchers, teachers and students, there is another issue with GMDB; that is, the important lack of available GMDB that can be used because the data needed are usually subject to confidentiality agreements. This fact is the motivation to offer a methodology for the simulation of GMDB, exemplified here with a porphyry copper type deposit, but it can be applied to any other type of mineral deposit.

There are several methodologies for building such GMBM (Garrido et al. 2018b). The primary-response framework for building geometallurgical models is a very solid methodology for geometallurgical modeling (Coward et al. 2009). Primary attributes, such as grades, lithology and alteration, can be proxies to response attributes such as grindability indices, recovery, metallurgical rock properties (Deutsch 2016), among others. As many of those response attributes are not additive, traditional linear estimation methods are not valid and should not be used to build the block model (Carrasco et al. 2008). Typically, there are three complementary approaches to populate the GMBM with response variables. The first approach is the use of predictive regression models, from simple linear regressions (Montoya et al. 2011; Boisvert et al. 2013), nonlinear regressions (Carmona and Ortiz 2010; Keeney and Walters 2011; Sepúlveda et al. 2017) and clustering (Hunt and Jorgensen 2011). The second approach is simulating the processing stage (Suazo et al. 2010). The third approach is the use of mineralogy as the main proxy. Mineralogy is of enormous importance for geometallurgy as it plays a fundamental role in the characterization of metallurgical responses (Lamberg 2011; Hunt et al. 2013; Yildirim et al. 2014; Lund et al. 2015). This approach, nevertheless, requires having the mineralogy characterization of the deposit, which is expensive, often resulting in limited data available.

The only related research on methodologies for the simulation of geometallurgical block models, so far according to the literature review done in this paper, is Lishchuk (2016) thesis. In this thesis, a methodology, termed geometallurgical testing framework, was proposed for building a synthetic ore deposit model with focus on geometallurgy. This framework has three main modules: (1) a geological module, (2) a mineral processing module and (3) an economic module. The first two modules are the most relevant modules for the simulation of synthetic geometallurgical ore bodies. Imposing multivariate spatial correlations, which is missing in Lishchuk's methodology, is critical to ensure that the desired spatial characteristics are reproduced with geological sense and coherence (Maksaev et al. 2007).

In the mineral processing stage, there are very limited simulation models available. A few commercial simulators exist, but these do not disclose the methods and parameters used to create the models, and in most cases, are simple nonlinear predictors that do not consider the uncertainty associated with the response variable. Commercial simulators are not designed to estimate the uncertainty associated with geological variability, since mineral characterization is a ''constant'' input and does not vary over time processing.

The contribution of this paper is a robust methodology to simulate a GMDB using openly available geostatistical tools, which preserves the coherent relationship among primary attributes, mineralogy and geometallurgical response attributes.

METHODOLOGY

To simulate a GMDB, the following steps are needed:

1. Identification of variable types
2. Generation of a consolidated database
3. Simulation of geological primary variables
    a. Definition of geometallurgical domains
    b. Simulation of domains
    c. Compositional geostatistical simulation of minerals
    d. Geochemical simulation
4. Simulation of geometallurgical responses
    a. Simulation of variables for comminution process
    b. Simulation of variables for flotation process

5. Simulation of spatial drill holes
   a. Topographic simulation
   b. Simulation of density of drill holes
   c. Survey and length simulation

These steps and some tools recommended for this stage are provided in Figure 1. This methodology allows simulating a GMDB for different purposes. In this research, we show an application to geometallurgical uncertainty in mine planning (long term).

SIMULATING A GEOMETALLURGICAL DATABASE

Through a case study, we illustrate the application of the proposed methodology in a synthetic typical porphyry copper deposit. The methods presented are not new, and their details are available in the published references cited herein (Table 1); however, the proposed workflow is novel in the sense that it provides the logical steps for the construction of the exploration and geometallurgical database, including the design of a realistic drilling campaign, according to the typical exploration process.

Identification of Variable Types

Different types of variables must be treated differently. Conventionally, variables are classified as categorical or continuous; however, some considerations must be kept in mind before modeling:

Categorical variables take a unique discrete value within a pool of exhaustive and mutually exclusive outcomes, in other words, at every location one and only one of the K categories prevail. However, categorical variables may be nominal or ordinal:

- Nominal categories have no order relation between them. Typical examples are the lithological codes assigned to samples, which can also be represented with numerical codes, or the mineralization zone assigned to each sample or location. In general, estimation and simulation domains can be seen as nominal categorical variables.
- Ordinal variables are ranked categories usually with unknown distance between the categories. An example of an ordinal variable is the alteration intensity, labeled with a scale of the type absent, low, moderate, high, or the corresponding numerical values 0, 1, 2 and 3.

Continuous variables take values with an arbitrary precision, defined by the number of digits and decimal places, within a continuous range. They may be unbounded, but are most often bounded, e.g., positive. Furthermore, some continuous variables are labeled as compositional, when they are part of a multivariate observation and each represents a relative part of a whole. Typical examples of compositional variables are mineral proportions, relative weight in a particle size distribution or geochemistry. The main complication associated with compositional variables is that their pairwise correlations depend on the other variables considered in the whole.

Although not formally a variable type, it is important to distinguish between continuous variables sampled abundantly, typically grades of valuable or detrimental elements, some geotechnical parameters such as rock quality designation, fracture frequency or uniaxial compressive strength, and those sampled scarcely, typically the case for geometallurgical variables such as grindability, acid consumption, flotation kinetics, to name a few.

From a geometallurgical perspective, there are also two types of variables. Variables that are intrinsic rock properties, termed primary variables, and variables that reflect a response to a specific process, termed response variables (Coward et al. 2009). Primary variables are among others, grade, alteration, mineralization styles and density, whereas examples of response variables are grindability indices, e.g., Bond work index (BWI), semi-autogenous grinding (SAG) power index and recoveries, e.g., flotation recovery and consumption of acid in leaching. In general, response variables are not additive, which complicates the way in that these variables can be propagated in the GMBM (Carrasco et al. 2008).

We carried out a case study to show the application of the proposed methodology. The database consists of (1) geological information (logging of mineral zones and alterations, categorical variables) to define and simulate the geometallurgical units or domains (GMU), (2) geochemistry (percentage of total copper by analysis X ray fluorescence or induced plasm coupled), (3) mineral characterization (percentage of most important minerals by infrared

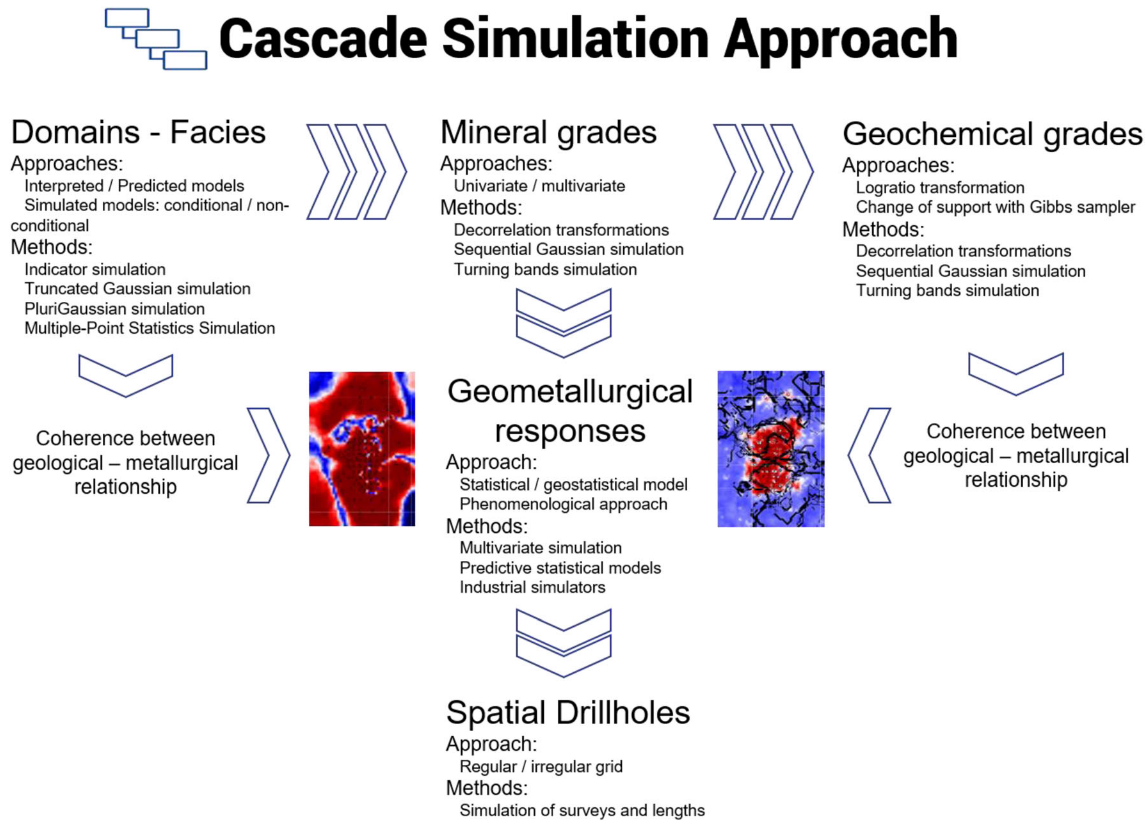

Figure 1. Global methodology and tools for each stage.

spectroscopy, continuous variables) to simulate with compositional geostatistics, and (4) geometallurgical responses (BWI test and rougher recovery test) to cosimulate with conventional geostatistics, e.g., sequential Gaussian simulation.

Generation of a Consolidated Database

The available information must be formatted for processing by the different modeling methods. This apparently trivial task may consume a significant amount of time, so it should not be minimized. The main objective is to prepare the database for the application of conventional geostatistical tools. This requires that every piece of information must be attached to spatial coordinates. This allows the calculation of spatial correlations, and also the cross correlations between variables, which are necessary for the application of estimation and simulation techniques (Isaaks and Srivastava 1989; Goovaerts 1997; Deutsch and Journel 1998);

For most multivariate statistical techniques, it is also required that the data be homotopic, that is, all variables must be available at the same location for cokriging or cosimulation, and they must be measured at a consistent volumetric support (Carrasco et al. 2008; Chiles and Delfiner 2012; Garrido et al. 2016). Imputation is necessary to replace missing data by values that are statistically consistent with the non-missing data, both in a statistical and spatial sense (Munoz et al. 2010; Barnett et al. 2013). These values should reproduce the variability expected at their location and honor the spatial relationship with neighboring samples. There are several imputation methods, among the most used are (a) impute missing values by Gibbs sampling methods, (b) multiple imputation from predictive distribution, (c) impute with regressions or (d) optimization approach.

Regarding the issue of the volumetric support, the idea is to bring all the available data to the same support. For example, geochemistry analyses, geological logging, structural information, geometallur-

Table 1. Summary of references by each stage of the methodology

Stage of the methodology	References	Comments
Consolidate database	Tran et al. (1999), Pardo-Iguzguiza et al. (2006), Carrasco et al. (2008), Munoz et al. (2010), Chiles and Delfiner (2012), Barnett et al. (2013), Garrido et al. (2016), Deutsch et al. (2016), Garrido et al. (2018a)	Discussion on the problem of multivariate simulation of heterotopic attributes, imputation of missing data, upscaling and down-scaling problems
Simulation of geological domains	Isaaks and Srivastava (1989), Goovaerts (1997), Deutsch and Journel (1998), Armstrong et al. (2003), Deutsch (2006), Maksaev et al. (2007), Carmona and Ortiz (2010), Sillitoe (2010), Mariethoz and Caers (2015), Beucher and Renard (2016), Jackson and Young (2016), Sepúlveda et al. (2017)	Discussion on ore type concept, clustering and the importance of understanding geological setting to simulate geological domains. References to conventional geostatistics methods to simulate ore body deposits, such as sequential indicator simulation, truncated Gaussian, pluri-Gaussian, multi-point simulation algorithms
Simulation of geological continuous attributes	Davis (1986), Webster and Oliver (1990), Goovaerts (1997), Desbarats and Dimitrakopoulos (2000), Pawlowsky-Glahn and Olea (2004), Babak and Deutsch (2009), Manchuk and Deutsch (2012), Mueller and Ferreira (2012), Barnett et al. (2013), Boluwade and Madramootoo (2014), Bolgkoranou and Ortiz 2019)	Tools for dimensionality reduction in the modeling process, such as principal components analysis, minimum/maximum autocorrelation factors, independent component analysis, uniformly—weighted exhaustive diagonalization with gauss iterations and projection-pursuit multivariate transform. Log ratio transformation to simulate mineralogical attributes and use of geostatistical simulation in continuous attributes
Simulation of geometallurgical industrial responses	King 2001), Suthers et al. (2004), Coleman et al. 2007), Vann et al. 2011)	To support industrial simulation of geometallurgical variables. Use of industrial processing or prediction of plant process performance, upscaling of laboratory to industrial scale, use of JKSim and mathematical models
Mine planning	Gholamnejad and Osanloo (2007), Suazo et al. (2010), Lamghari and Dimitrakopoulos (2012), Kumral (2013), Silva et al. (2015), Garrido et al. (2017)	Incorporation of geometallurgical models in mine planning and quantification of the uncertainty of the inputs to mine planning optimization problems

gical samples should be considered to enrich the model, but their volumetric supports may be different by orders of magnitude. Geochemical samples may be taken over diamond drill holes samples at 1 m support, while geometallurgical samples may be taken over bulk volumes representing 15 or 30 m of a reverse circulation hole. Upscaling by compositing is common practice (Chiles and Delfiner 2012). Down-scaling techniques are sometimes required to bring the data to the smallest support where more abundant information exists. This can be achieved by using geostatistical cosimulation and applying constraints to the simulated values to impose reproduction of the sample value at the larger support (Tran et al. 1999; Pardo-Iguzguiza et al. 2006; Deutsch et al. 2016; Garrido et al. 2018a).

Variables can be dropped if deemed irrelevant for the model, by using statistical techniques for variable selection or machine learning, and accounting for domain knowledge, that is understanding of the geological setting (Carmona and Ortiz 2010). They can also be merged, to reduce the dimensionality in the modeling process, using data integration such as cokriging (Babak and Deutsch 2009), or dimensionality reduction techniques, such as principal components analysis (Davis 1986; Webster and Oliver 1990; Goovaerts 1997). In this case study application, the consolidated database is used to learn the geometallurgical relationships required to generate realistic simulations.

Simulation of Primary Geological Variables

Definition of Geometallurgical Domains

Hydrothermal ore deposits, in general, present zoning of different mineral associations (Sillitoe 2010), which correspond to GMU and, within these domains, there is also variability in the composition of rock. The concept of ore type provides a framework to form a common perspective around the performance of material, to make decisions (Jackson and Young 2016). This implies that, depending on the perspective, the definition of GMU is given through the orebody knowledge, rock characteristics and performance engineering. For example, from a blasting perspective, the performance type is the fragmentation distribution (target to optimize process), and this depends on the geological domains (joint characteristics, rock strength, rock density and rock mass description rating) and the blast design (operational factor). However, from a mill perspective, the performance target is the throughput that depends on the fragmentation distribution, impact resistance and grinding hardness (material type and geological domains) and the milling circuit (operational factor).

To support the definitions of GMU, supervised and unsupervised algorithms can be used. For example, Sepúlveda et al. (2018a) give a methodology to clustering with spatial corrections to define GMU. It is important that the definition has geological foundation that validates the behavior of an ore type. Another common option is to combine different criteria to define the GMU through several iterations: geological knowledge, statistical analysis, multivariate iso-grades and spatial modeling. Finally, the GMU can be validated through geostatistical tools, such as spatial data analysis, cumulative probability plots and boxplot by category, among others, to discriminate the different statistical population. This is a subjective process requiring many iterations, and, in this context, there may be many valid interpretations of GMU for the same deposit.

In the case study, five GMUs have been modeled. These are associated with the copper mineral zones of the deposit:

- GMU1: Oxidized copper ores with evidence of leaching on the groundwater level of the deposit;
- GMU2: Sulfides such chalcocite and digenite (enrichment sulfides layer);
- GMU3: Primary hypogene sulfides with high chalcopyrite—pyrite ratio;
- GMU4: Primary hypogene sulfides with low chalcopyrite—pyrite ratio; and
- GMU5: Waste and gravel without economic content associated with copper.

Simulation of Domains

Simulation of categorical variables can be carried out with many different algorithms. In the geostatistical toolbox, the following methods are widely known and could be used for this stage: pluri-Gaussian simulation (Armstrong et al. 2003), sequential indicator simulation (Deutsch 2006), multiple-point simulation (Mariethoz and Caers 2015) and truncated Gaussian simulation (Beucher and Renard 2016). Most of these methods aim at reproducing the indicator variograms between the different categories. This entails reproducing the number of transitions from one category to different categories. Control over the transition's changes with different methods; therefore, some methods work well under mostly unstructured (mosaic type) categorical models (indicator simulation), while others aim at preserving specific features such as hierarchies (truncated Gaussian and pluri-Gaussian simulation) or even curvilinear features and trends (multiple-point statistical simulation).

In this research, we simulate categorical GMU and calculate the probability of occurrence. Figure 2 shows a plan view with (left) the expected GMU and the iso-curves with probability of GMU contacts, and (right) the confidence level of model (40% to 100% of confidence).

GMU are simulated in the deposit by indicator simulation. The actual implementation used here is the block sequential indicators simulations algorithm (Deutsch 2006), which implements the mapping pixel smoothing algorithm—maximum a posteriori selection (Deutsch 1998) to improve the contact among categories and preserve their imposed proportions.

Compositional Geostatistical Simulation of Minerals

Compositional variables are modeled after a so-called log ratio transformation (Pawlowsky-Glahn and Olea 2004). A full review of this approach is given by Tolosana-Delgado et al. (2019). There are several ways to approach this transformation, but the simplest is presented here. Assume $(p-1)$ variables are available and form a composition, for example, a set of mineralogical proportions. Since

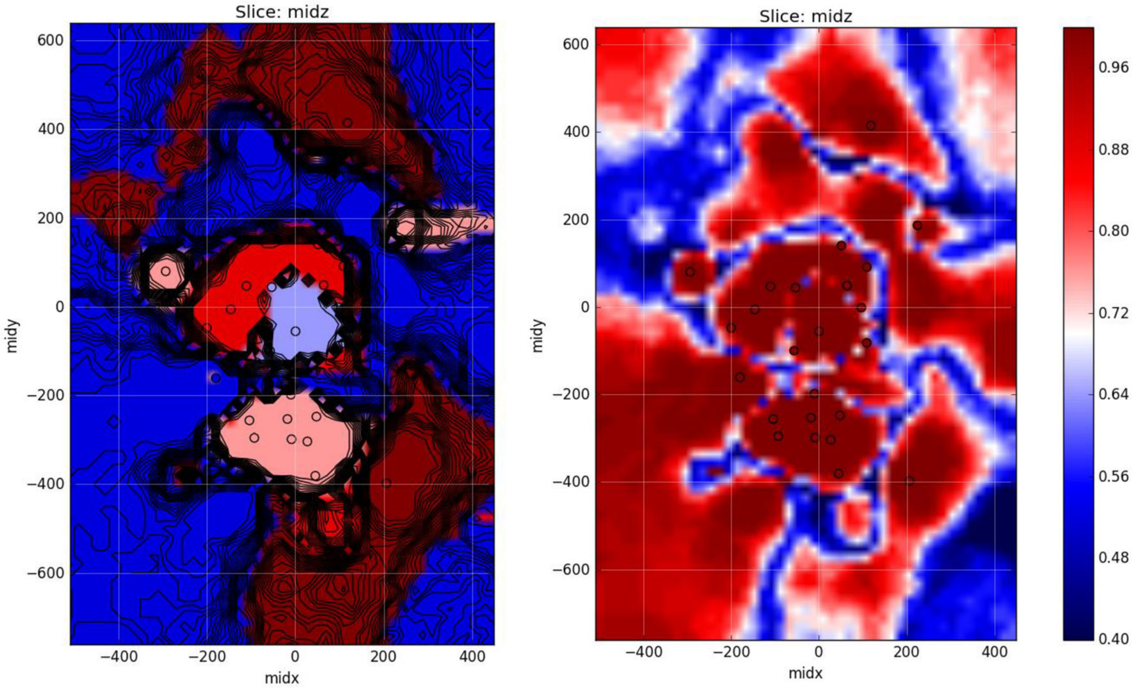

Figure 2. (Left) Plan view with the expected GMU and (right) the confidence level of model.

these variables form part of a whole, a filler variable is calculated to complete the set. For example, if proportions are reported in percent, this filler variable can be:

$$R(u_\alpha) = 100\% \; - \sum_{i=1}^{p} X_i(u_\alpha)$$

The additive log ratio transforms (Aitchison 1982) can be computed:

$$Z_i(u) = \log\left(\frac{X_i(u)}{R(u)}\right)$$

These new variables are unbounded, that is, they can take values between $-\infty$ and $+\infty$, but are also spatially correlated. Therefore, simulation can be done by applying a decorrelation transformation and simulating independently each component, or jointly simulating all the log ratio transformed variables using conventional geostatistical methods.

Decorrelation can be done by using a collocated factorization such as principal component analysis, which does not impose decorrelation of the variables in space, but most of the time significantly reduces the spatial cross-correlation of the principal components (Bolgkoranou and Ortiz 2019). Other methods for decorrelation are maximum autocorrelation factors (Desbarats and Dimitrakopoulos 2000), uniformly—weighted exhaustive diagonalization with gauss iterations (Mueller and Ferreira 2012) and independent component analysis (Boluwade and Madramootoo 2014). Projection-pursuit multivariate transform (Barnett et al. 2013) finds successively directions where the projection has the maximum univariate non-Gaussian index and performs the normal score transformation to that specific direction.

In this step, we relate mineralogy with geology. Mineralogy is often determined by mineralogical test work, such as quantitative evaluation of materials by scanning electron microscopy (Fennel et al. 2015), which provides mineralogical proportions. For each geological domain, a multivariate spatial lineal model of coregionalization (LMC) is imposed, if a correlation between variables exists. This LMC is determined according to the relationships between minerals in each geological domain, for example, cuprite and chalcocite should be found in the mixed or secondary enriched zone. The relationship can be determined by correlation matrices. The simulation

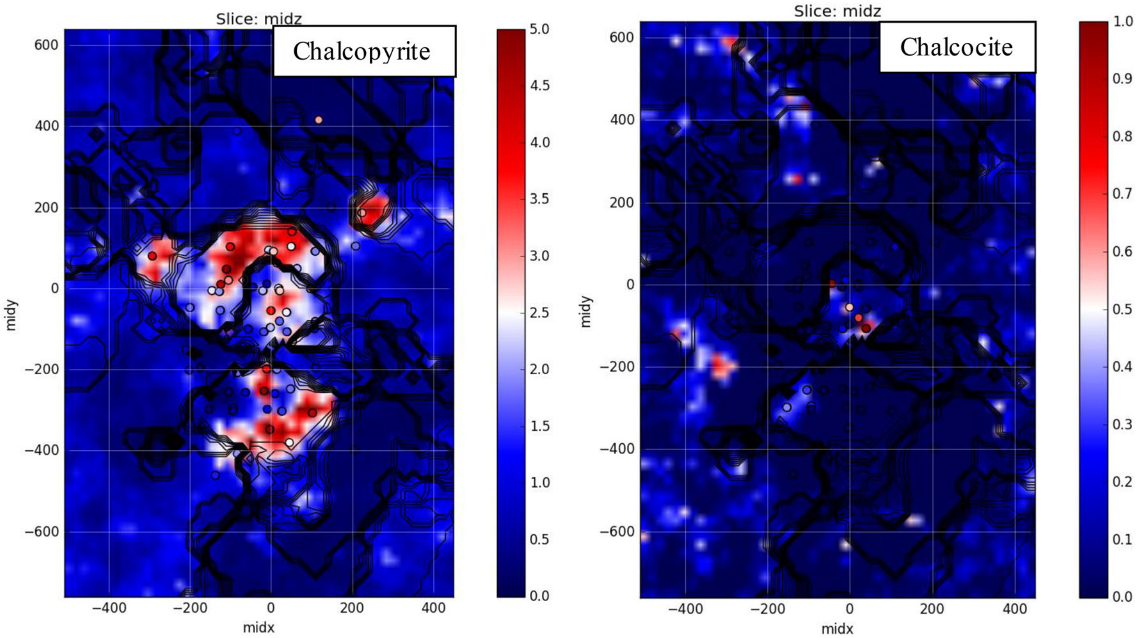

Figure 3. (Left) simulated proportions of chalcopyrite. (Right) simulated proportions of chalcocite in percent.

within each geologic domain is performed by the ultimate sequential Gaussian simulation algorithm (Manchuk and Deutsch 2012).

In this case study, compositional mineralogical simulation was performed over each realization of geological simulation (cascade approach), in order for the geological uncertainty be propagated to the simulation of mineralogical proportions. Figure 3 shows the E-Type (average of 50 realizations) of chalcopyrite and chalcocite simulated over the same plan view shown previously.

Correlations found in the exploratory data analysis were replicated in the compositional simulation. The relative proportions of minerals are preserved, which are different for each GMU. Figure 4 shows that GMU2 has a higher proportion of chalcocite's sulfides and GMU3 has a higher proportion of primary hypogene sulfides with high chalcopyrite-pyrite ratio, which is congruent with simulated GMUs.

Geochemistry

As minerals contain the chemical elements of interest, simulating the geochemistry could significantly improve the simulation of responses at the plant, which will be dependent on the mineral occurrence of these elements. In geometallurgy, the elements of interest should not only be those of economic interest, such as copper, gold, molybdenum, silver and iron, but also deleterious elements, such as sulfur, fluorine or arsenic. From the geometallurgical perspective, deleterious elements could be crucial in the beneficiation process and in minimizing contaminants that affect the quality and economic value of the final product (Lane 1988). A geometallurgical block model should include both kinds of elements.

There are two approaches to have elements and minerals in the GMBM: (1) predicting mineralogy from grades and (2) predicting grades from mineralogy. Some researchers have linked chemistry composition to mineralogy to predict the mineral proportions from element concentrations (Lamberg 2011; Townley et al. 2018; Abildin et al. 2019).

The other approach is deducing element concentrations from mineral proportions. The grade of each element is a function of the minerals present:

$$g_e = f(m_1, m_2, \ldots, m_M)$$

The g_e function is derived from the chemical composition of the M minerals. For example, if there are three minerals hosting copper: bornite, chalcopyrite and chalcocite; we have:

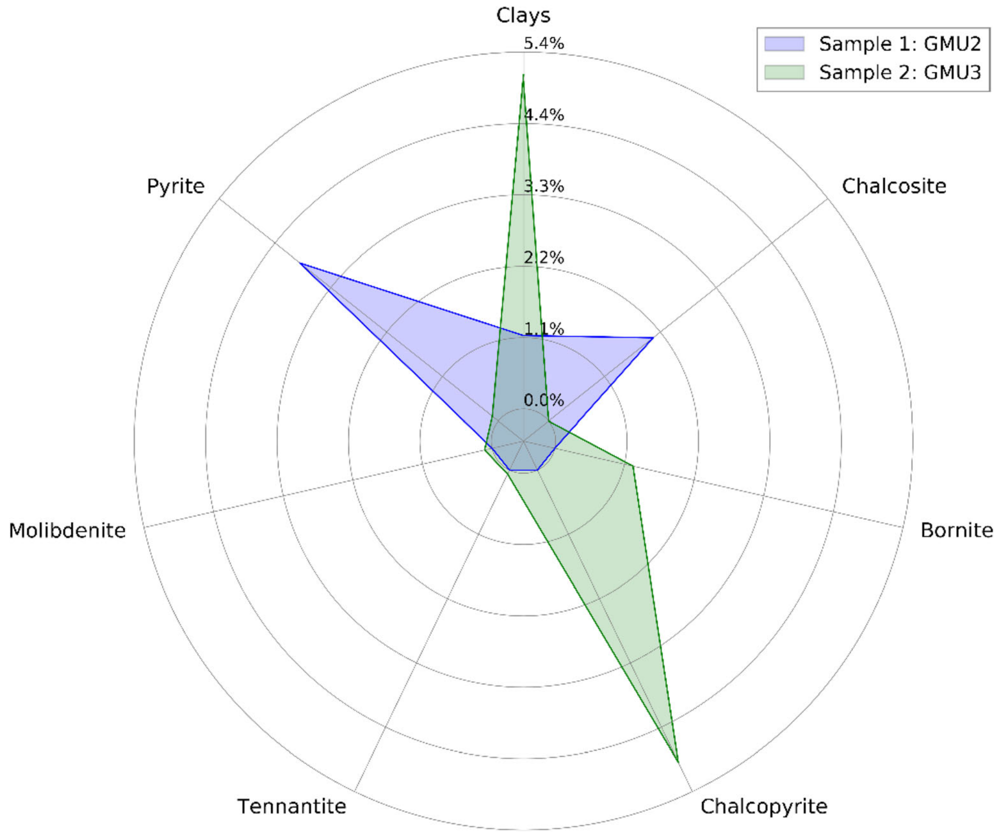

Figure 4. Radial map of mineral percentage for two samples of different GMU, validating the definition based on mineralogical approach.

$$\begin{aligned}
&\text{Bornite} \equiv \mathrm{Cu_5FeS_4} \rightarrow 63.31\%\mathrm{Cu} \\
&\text{Chalcopyrite} \equiv \mathrm{CuFeS_2} \rightarrow 34.63\%\mathrm{Cu} \\
&\text{Chalcocite} \equiv \mathrm{Cu_2S} \rightarrow 79.85\%\mathrm{Cu} \\
&g_{\mathrm{Cu}}(m_1, m_2, m_3) = m_1 63.31\% + m_2 34.63\% + m_3 79.85\%
\end{aligned}$$

where m_1, m_2 and m_3 are the percentage of bornite, chalcopyrite and chalcocite, respectively. The limitation of this methodology is that mineral proportions are most commonly derived from qualitative or semi-qualitative estimates, usually with high degrees of uncertainty. In addition, mineral proportions estimates would only account for the theoretical copper present but exclude trace elements such as gold or silver that may be present.

Mineralogy also helps establishing the relationship of grade and mineralization zones. For illustration, in a porphyry copper deposit, we could find the following relationships of copper grade in different mineralization zones: the total copper content in secondary enrichment, which is characterized by minerals with high copper content such as chalcocite and covellite, is in general higher than the total copper in primary rocks characterized by sulfur with high content of chalcopyrite.

In the case study, mineral proportions were used to calculate geochemical composition of total copper, molybdenum (commodities) and arsenic (pollutant). For example, the sum of copper in chalcopyrite, bornite, covellite and oxides minerals represents the total copper in minerals. The original database contains total copper (in samples), and it was compared through quantile–quantile plot with the Total copper in minerals (Fig. 5). In addition, spatial continuity was validated for total copper in minerals, at a range of 70 m approx.

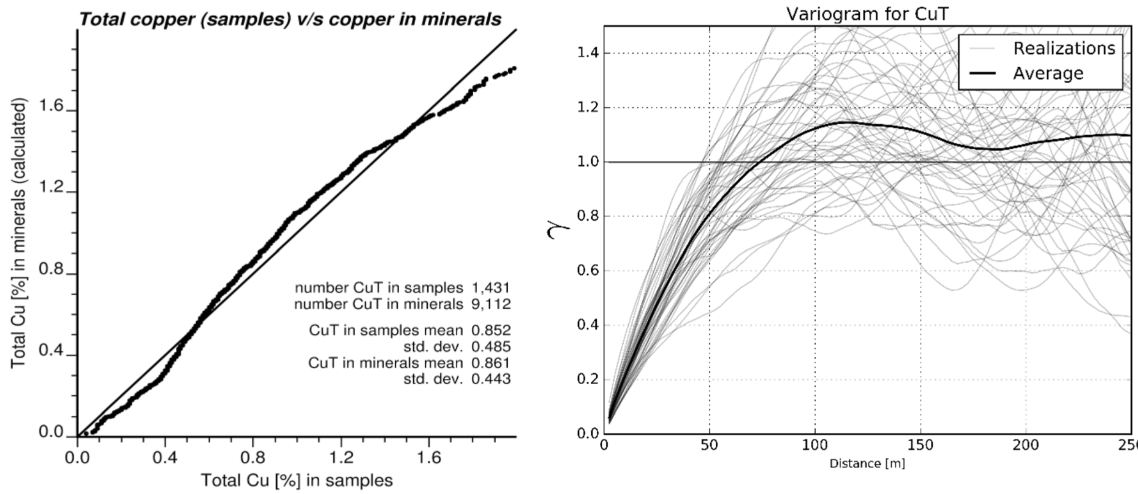

Figure 5. Validation of statistical distribution and spatial variability for total copper grade in minerals.

Simulation of Geometallurgical Response from Drill Core Samples

Metallurgical batch tests are performed on drill core samples to generate mineral processing predictive models. However, such tests are not enough to predict industrial performance as these do not necessarily account for ore rock blending of feed through the process and scale-up factors from batch to industrial scale are not always known, especially in exploration projects. Batch laboratory tests are a useful tool to identify, through geometallurgical modeling, trends and optimal conditions that are proposed to be implemented later in the plant. In this work, two instances of geometallurgical modeling are presented: comminution and flotation processes.

Simulation of Comminution Process

There are roughly two kinds of models: power-based models, which are based on grinding parameters that allow estimating the energy consumption associated with a given size reduction, and population mass balance models that can also be used to predict the behavior of the rocks, from a particulate system perspective, and how the particle size distribution evolves during grinding. In each case, the product particle size, characterized by P80, e.g., the 80% passing size is an important variable since it is directly related to the liberation degree.

The following is a list of common comminution tests: Bond work index for ball mill: the grindability test determines the hardness of the ore rock. The work index is used when determining the size of the mill and grinding power required in producing the required ore throughput in a ball mill. SAG power index or Starkey test for SAG mill: provides the time (minutes) required to perform a specific milling work, from a feed size to an output size. SAG mill comminution (Morrell 2006): it is a function between the specific energy applied and the percentage of product generated in the impact fracture of a specific particle size.

From these comminution tests, the specific energy consumption can be calculated, and later used to optimize the process at industrial scale. When simulating these variables, the use of multivariate tools is recommended as it allows improving the models' robustness.

Simulation of Flotation Process Performance

Flotation is a selective separation process that is based on the difference in hydrophobicity of minerals at given physical–chemical conditions. In this case, unlike comminution tests, the flotation tests are not standardized, and, in general, the flotation results correspond to a combination of ore characteristics and the way the flotation tests were performed, expressed in operational variables such as pH, P80, solid weight and aeration conditions. In the case of

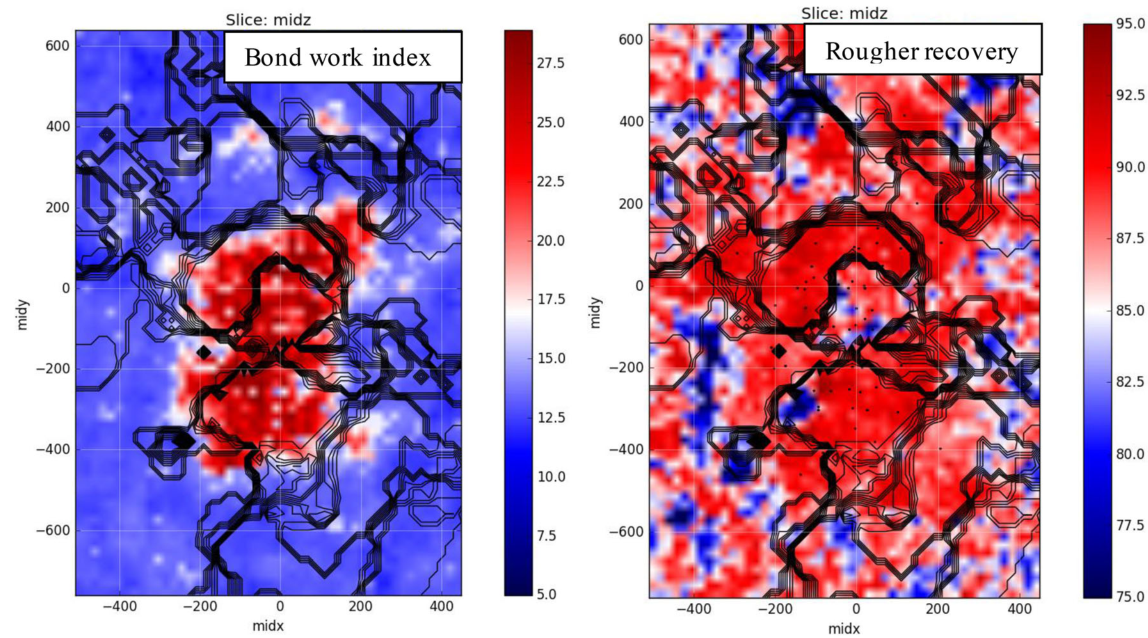

Figure 6. (Left) Bond Work Index BWI simulated (kwh/tc). (Right) Rougher recovery simulated (%).

flotation modeling, the performance depends on the head grade, rate constant, mineralogy and liberation degree, which determines the maximum recovery. All these variables can be simulated using multivariate tools to improve models' robustness without increasing unnecessarily the number of tests.

The flotation test can have many variants, for example, the most common is the open cycle test: (1) rougher primary flotation, (2) secondary flotation optional cleaner, (3) optional scavenger tertiary flotation, (4) optional re-grinding before the flotation cleaner. Combinations of these tests can be performed to replicate the industrial flotation cell to maximize the recovered ore and its concentration. The following information is usually obtained from these tests: kinetics of flotation k Klimpel, maximum recovery with prolonged flotation time or ''infinity'', mineral characterization and head geochemistry (feed), mineral and geochemical characterization of concentrate, at 1.5 min, 3 min, 6 min, 12 min and 15 min of flotation, mineral and geochemical characterization of tailings, at 1.5 min, 3 min, 6 min, 12 min and 15 min of flotation, among others.

The mineral characterization consists of briquette preparation, quantitative mineralogy, mineral association identifications, granulometric distribution, among others. From these tests, a database is obtained with many variables that are used to optimize the flotation process performance at an industrial scale (Jackson et al. 2011).

The metallurgical process is related with the industrial processing or prediction of plant process performance (Suthers et al. 2004) of the ore that has been removed from the in situ ore deposit. The ore is processed continuously at industrial scale, through a process of crushing, conveyor belt, grinding, flotation, thickening and filtration, among others, and this can be modeled and simulated (King 2001). Many industrial simulators are used for this purpose, for example the JKTech simulators: JKSimMet for comminution and classification circuits, or JKSimFloat (Vann et al. 2011) for simulation for steady state performance in flotation plants. The software can simulate operational parameters (e.g., flowsheet of the processing plant) and tests its performance (metal recovery and concentrate grade, water recovery, residence time, gas holdup, froth recovery, mass balance on a size by assay basis) to achieve the best consistent data set and simulate the effect of changes in the flowsheet to predict flows, size distributions and element distribution, among others.

Predictive models can be implemented by different mathematical adjustment models, such as Australian minerals industry research association to

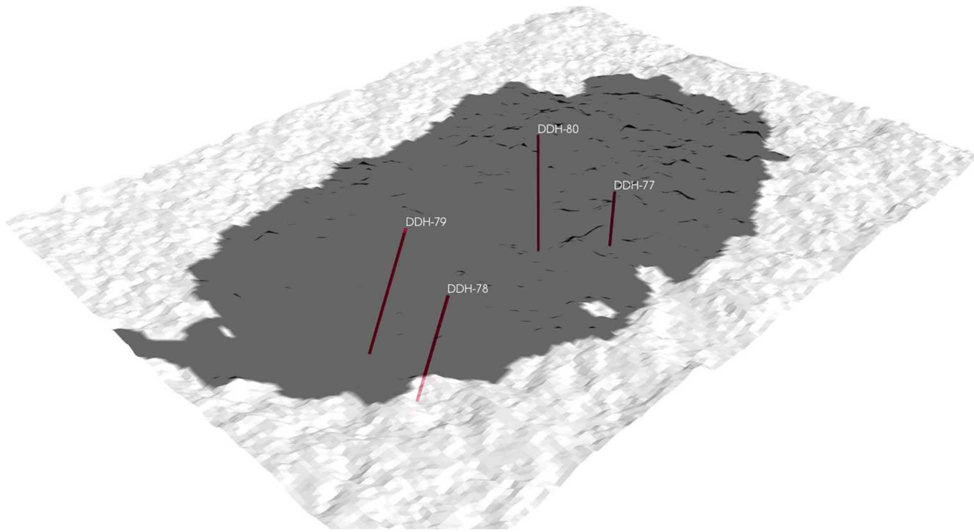

Figure 7. Topography area that is an exploration target with four drill holes.

floatability component (Coleman et al. 2007). The most important limitation of these simulators is that they allow varying the configuration of industrial machines (and other operational factors) considering a constant mineral feed, neglecting the geological variability associated with the deposit and blending factors. Another limitation of these simulators is that the simulation of geometallurgical attributes generates many possible processing scenarios, which are not directly used in these simulators (industrial simulators receive a deterministic input, not a stochastic geological input). Metallurgical response can be estimated by regression models calibrated from test work or reconciliation data, which is the approach used in this paper.

In this case study, cosimulation for BWI and rougher recovery was performed for each GMU (Fig. 6). Rougher recovery was calculated as a sum of individual mineral recoveries, assuming there is no cross-interference that affects the flotation process. Rougher recovery has a negative correlation with chalcocite, consistent with laboratory performance of assays to flotation.

Finally, the block model has been simulated with geological variables, mineralogical variables, geochemical variables and geometallurgical response. Each simulation was simplified for research purposes, but the methodology is flexible and can be implemented to other more sophistically types of simulations (see ''Simulation of Spatial Drill Holes'' section).

Simulation of Spatial Drill Holes

To generate a database that looks realistic, different conditions must be simulated for exploration campaigns. In this context, simulations of topographical area, density of information and survey of drill holes are simulated.

Topographic Simulation

To simulate elevation, non-conditional simulation was performed. Smooth simulations are appropriate for realistic surface modeling. Z-elevation collar position of drill holes is a known function of x-east and y-north coordinates. Figure 7 shows an exploration target area (estimated with geochemistry, petrological, geophysics and geochronology knowledge). This area can be simulated with categorical simulation of boundary and synthetic pseudo-drill holes. This is the first campaign whose objective is to find deep mineralized bodies.

Simulation of Density of Drill Holes

If the ore body is found in the first campaign, a second campaign is performed with different objectives: define the prospect dimensions, develop the first estimation of ore grades and improve the geological knowledge for interpretation.

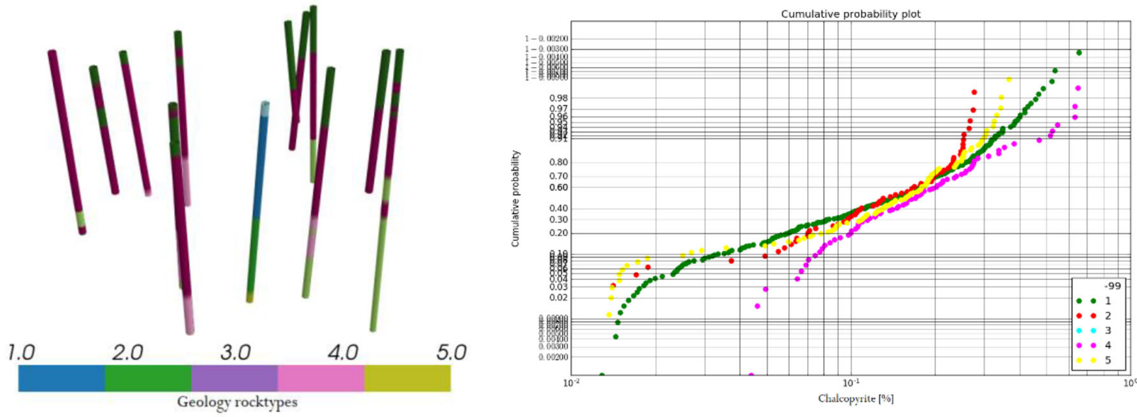

Figure 8. (Left) Plot of five geological codes in drill holes logging and (right) categorical cumulative probability plot of chalcopyrite percent.

The geological interpretation of metallogenic controls of mineralization is important at this stage to design the next exploration plan. Geostatistical tools can improve the geological knowledge to define domains, for example exploratory data analysis may help showing distributions of different geological properties (Figure 8 shows of the logging of drill holes with 5 geological codes and the cumulative probability plot of chalcopyrite in each unit), for the purpose of identifying relevant economic domains and quantifying the possible mining resources.

Depending on the exploration stage, a given drilling spacing is targeted, in consideration of the associated risks of finding the resources and budgetary constraints. The expected orebody geometry determines the orientation and depth of the drill holes. Figure 9 shows a regular mesh (collars of drill holes, in surface) that depends of the geological continuity of ore body.

Survey and Length Simulation

In depth, drill holes may be oriented in particular directions with the objective of intersecting perpendicularly a tabular body, structural vein, etc. At this stage, structural information is important to define the orientation of the drill holes (azimuth and dip). Structural zones must be identified through the oriented drill holes. Figure 10 shows oriented drill holes with an azimuth and dip calculated based on structural information.

The lengths of the drill holes depend on the depth of ore body and long-term scheduling (based on feasibility studies). In regular deposits near to surface, the length drill holes can range from 50 m to 500 m in depth. We use a normal distribution to simulate the length of the drill holes, for example a normal distribution with mean of 300 m and standard deviation of 100 m. Finally, a random subset of the available drill holes samples is informed with geometallurgical attributes, to represent the typical scarcity of geometallurgical information. In our example, 10% of all simulated samples contains geometallurgical test values.

Application to Uncertainty in Mine Planning

Uncertainty in mine planning optimization plays a critical role not only in finding the optimal economic valuation through the maximization of net present value, but also in risk assessment. Most of the research focuses on incorporating grade uncertainty in strategic mine planning, and medium- and short-term scheduling optimization problems. Because scheduling transforms a three-dimensional resource model into a temporal model, one cannot assign a profit value at block scale (or selective mining unit scale) before the decision on where, when and what to mine is made by the optimizer. Traditionally, this simplification is often done, but a realistic schedule of the profit of a set of blocks in a temporal interval should depend on geological

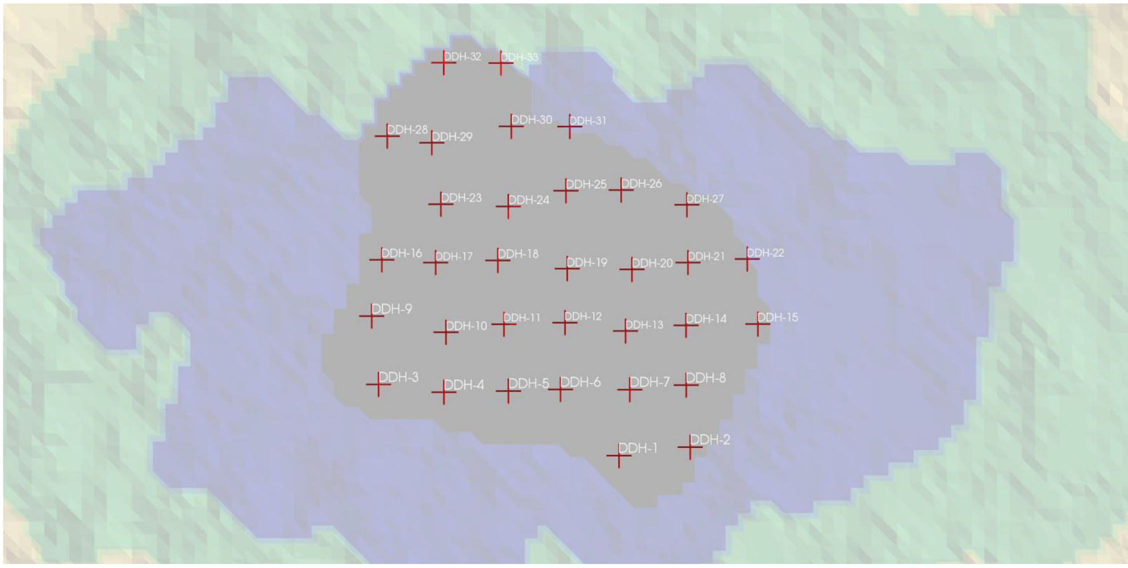

Figure 9. Collars of drill holes in regular mesh in surface.

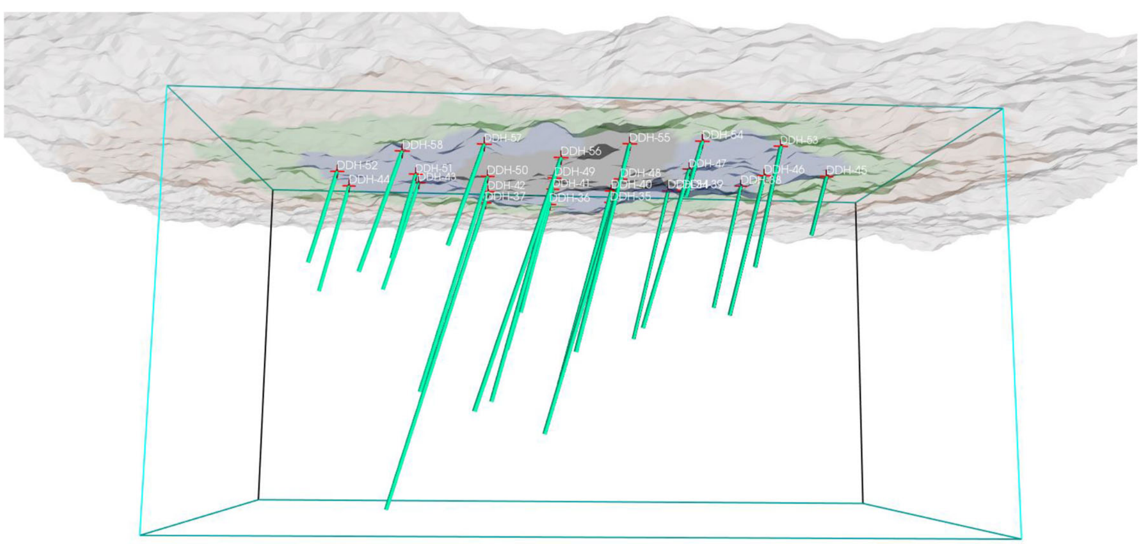

Figure 10. Oriented drill holes campaign with an azimuth and dip calculated based on structural information.

properties and response properties of the complete set of blocks, and to the specific plant conditions. In case of early stages, design plant condition needs to be used, whereas in productive stages, real plant conditions need to be considered.

Any response attribute can be modeled as a transfer function f with inputs: set of blocks B and their attributes, a set of plant parameters P, and a timeframe Δt.

$$\rho = f(B, P, \Delta t)$$

Therefore, accounting for the uncertainty of processes requires not only carrying the uncertainty of inputs to the model, but also, the uncertainty of the processes themselves. Incorporating the uncertainty of the inputs to mine planning optimization problems is a very active research topic. Grade is the main geological attribute that was incorporated to

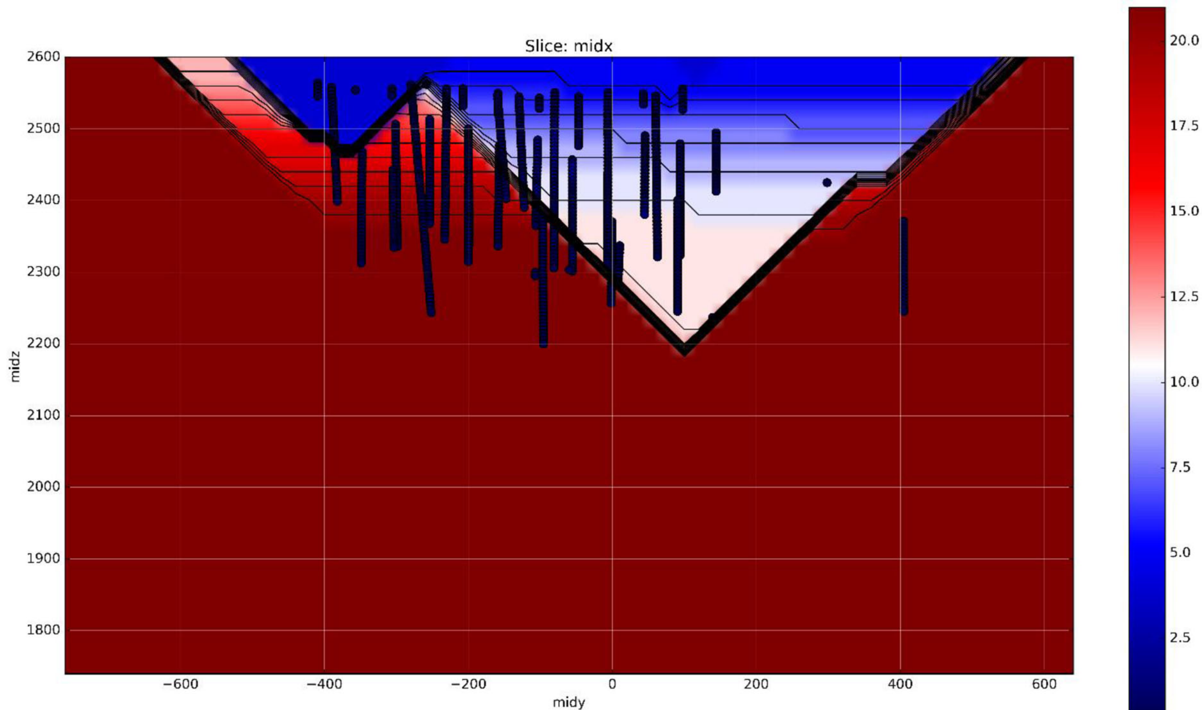

Figure 11. Graphical scheduling of 20 phases of project, LOM.

many production planning optimization problems (Gholamnejad and Osanloo 2007; Lamghari and Dimitrakopoulos 2012; Kumral 2013; Silva et al. 2015; Goodfellow and Dimitrakopoulos 2016). Nevertheless, accounting for geometallurgical uncertainty is very limited. Kumral (2011) incorporated uncertainty of revenue and cost by many scenarios based on simulations. Sepúlveda et al. (2018b) used several geometallurgical variables under uncertainty to optimize production scheduling in a block caving operation by a multi-objective approach. The uncertainty of geometallurgical attributes was quantified by geostatistical simulations of primary variables and nonlinear regression models for response variables.

One approach, which is the most used and the simplest, is defining transfer functions from standardized response variables to specific plant conditions. The approach that reflects the responses to processes of a set of blocks in a timeframe is by simulating the processes, while the optimization is being performed. Obviously, this approach is very challenging because simulating the processes requires large computing power. Populating any geometallurgical resource model with response attributes should be avoided because it implicitly assumes (1) a block responds independently to the other extracted blocks, which is not the case, and (2) the throughput is constant. However, there is limited research in this direction. Garrido et al. (2017) defined the concept of geometallurgical dilution to account for the impact of feeding blocks of different geometallurgical domains to the plant, if different geometallurgical domains have different responses. They showed that geometallurgical attributes can be effectively included as part of the optimization process. More research needs to be done to incorporate the response simulation as part of the optimization process.

In our case study, to transfer spatial variability of geometallurgical variables to temporal variability, a mining scheduling (life of mine) was calculated with the Lerchs and Grossman algorithm (Lerchs and Grossman 1965) for the E-Type of the generated simulations. Figure 11 shows the 20 phases of the project, calculated with real economic and design parameters of a porphyry copper ore body open pit.

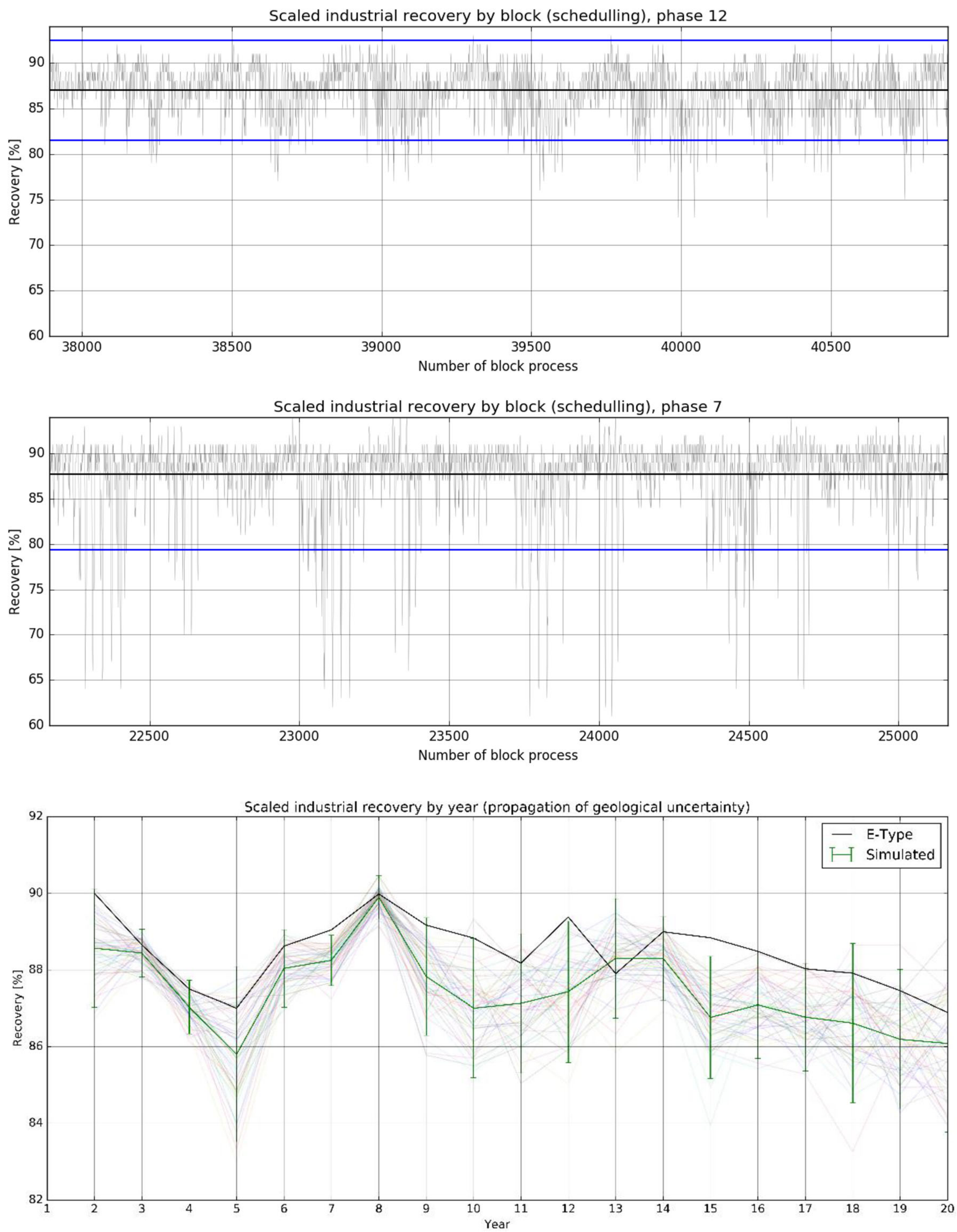

Figure 12. Re-dimensioned scaled recovery variability by blocks (for one realization) and by year (for all realizations). Mean recovery by block with 95% confidence interval is shown.

Geometallurgical variability can be measured by period (in this case, by phase or year). Known temporal behavior of mineral processing may allow industrial metallurgical simulations. In this case, to simplify the research application, a correction factor of rougher recovery was applied to calculate industrial recovery. This correction factor was calculated for each GMU, as in the Collahuasi case study (Suazo et al. 2010).

Figure 12 shows the rougher recovery variability by year (E-Type vs. uncertainty in simulations). It shows how geological variability is propagated to metallurgical variability. The results show years with low metallurgical variability (for example, phase 12) and years with high metallurgical variability (phase 7) for one realization. This variability is fully attributed to geological variability, and it does not consider the operational variability. Case phase 7 shows low recovery because sulfides mineral zones include secondary enrichment (GMU2), and chalcocite affects recovery negatively, increasing uncertainty.

This application shows how geological uncertainty is propagated to metallurgical responses. Usually, conciliations show differences that can be attributed to geological variability and operational interferences.

CONCLUSIONS

We have presented a reproducible methodology for the simulation of a synthetic geometallurgical drill holes dataset, with special interest in preserving the coherence between geology, mineralogy and grades. Response attributes were included in the drill hole database, comminution process and flotation performance. Simulations can be self-explained, any algorithm aligned with the generating method will appear to work well—other algorithms will appear to have poor performance, and this condition is a limitation to any method. One of the main contributions of this article is the summary of geostatistics tools that can be used.

Starting with real or synthetic drill holes and following the six steps in the proposed methodology, a GMDB can be successfully simulated. All programs used in the methodology can be found in open source software, free software or commercial software.

The article discusses how we understand a geometallurgical unit, which may depend on the geological setting, the metallurgical process and the implementation in the operation, unlike the conventional geological domains that only depend on geological characteristics associated with the rock.

The geometallurgical variable (associated with a rock process in situ) is differentiated from the metallurgical variable (associated with a continuous process in time). The geometallurgical variables (such as BWI, rougher recovery, specific acid consumption and soluble copper) can be simulated by geostatistical tools in spatial block model, subject to the correct definition of the GMU. The loss of predictive processing capacity generates problems in mining reconciliation, increased uncertainty and increased costs. With a correct and careful application of this methodology, the geometallurgical uncertainty can be evaluated by implementing preventive protocols to reduce processing costs.

We have also included in the complementary material the simulated inputs and GMDB of the case study for academic and teaching purposes which are also available for downloading in the public repository https://github.com/exepulveda/geomet_datasets.

ACKNOWLEDGMENTS

The authors would like to thank the financial support from project FONDEF 'Caracterización y Modelamientos Geo-Minero-Metalúrgicos Predictivos: Camino a la Minería del Futuro' (IT16M10021). The author acknowledges the support of the CONICYT PAI Concurso Nacional Tesis de Doctorado en el Sector Productivo, Convocatoria 2018 PAI7818D20001 and industrial support of IMDEX Limited—REFLEX INSTRUMENT South America SpA. We acknowledge the support of the Natural Sciences and Engineering Council of Canada (NSERC), funding reference number RGPIN-2017-04200 and RGPAS-2017-507956.

REFERENCES

Abildin, Y., Madani, N., & Topal, E. (2019). A hybrid approach for joint simulation of geometallurgical variables with inequality constraint. *Minerals, 9,* 24. https://doi.org/10.3390/min9010024.

Aitchison, J. (1982). The statistical analysis of compositional data (with discussion). *Journal of the Royal Statistical Society Series B (Statistical Methodology), 44*(2), 139–177.

Armstrong, M., Galli, A., Le Loc'h, G., Geffroy, F., & Eschard, F. (2003). *Plurigaussian simulations in geosciences*. Berlin: Springer.

Babak, O., & Deutsch, C. (2009). Improved spatial modeling by merging multiple secondary data for intrinsic collocated cokriging. *Journal of Petroleum Science and Engineering, 69,* 93–99.

Barnett, R., Manchuk, J., & Deutsch, C. (2013). Projection pursuit multivariate transform. *Mathematical Geosciences*. https://doi.org/10.1007/s11004-013-9497-7.

Beucher, H., & Renard, D. (2016). Truncated Gaussian and derived methods. *Comptes Rendus Geoscience, 348*(7), 510–519.

Boisvert, J., Rossi, M., Ehrig, K., & Deutsch, C. (2013). Geometallurgical modelling at Olympic dam mine, South Australia. *Mathematical Geosciences, 10,* 10. https://doi.org/10.1007/s11004-013-9462-5.

Bolgkoranou, M., & Ortiz, J. (2019). Multivariate geostatistical simulation of compositional data using Principal Component Analysis: Application to a Nickel laterite deposit. In *39th APCOM conference: Application of computers and operations research in the mineral industry*, 4–6 June 2019.

Boluwade, A., & Madramootoo, C. (2014). Geostatistical independent simulation of spatially correlated soil variables. *Computers and Geosciences*. https://doi.org/10.1016/j.cageo.2015.09.002.

Carmona, S., & Ortiz, J. (2010). Geological modelling and metallurgical prediction supported by linear and non-linear statistics. In *Proceedings of the 4th international conference on mining innovation, MININ 2010, Santiago, Chile* (pp. 459–470).

Carrasco, P., Chiles, J., & Seguret, S. (2008). Additivity, metallurgical recovery and grade. Paper presented at the Geostats 2008, Santiago, Chile.

Chiles, J., & Delfiner, P. (2012). *Geostatistics: Modelling spatial uncertainty* (2nd ed., p. 734). New York: Wiley. ISBN 978-0-470-18315-1.

Coleman, R., Franzidis, J., & Manlapig, E. (2007). Validation of the AMIRA P9 flotation model using the floatability characterization test rig (FCTR). In *Ninth mill operators conference 2007, Fremantle, WA, Australia, 19–21 March 2007.* Carlton, VIC: AusIMM.

Coward, S., Vann, J., Dunham, S., & Stewart, M. (2009). The primary-response framework for geometallurgical variables. Paper presented at the seventh international mining geology conference 2009, Perth, Western Australia.

Davis, J. (1986). *Statistics and data analysis. Geology* (2nd ed., p. 646). New York: Wiley.

Desbarats, A., & Dimitrakopoulos, R. (2000). Geostatistical simulation of regionalized pore-size distributions using min/max autocorrelation factors. *Mathematical Geology*. https://doi.org/10.1023/A:1007570402430.

Deutsch, C. (1998). Cleaning categorical variable (lithofacies) realizations with maximum a posteriori selection. *Computers and Geosciences, 24,* 551–562. https://doi.org/10.1016/S0098-3004(98)00016-8.

Deutsch, C. (2006). A sequential indicator simulation program for categorical variable with point and block data: BlockSIS. *Computers and Geosciences, 32,* 1669–1681.

Deutsch, J. (2016). Multivariate spatial modeling of metallurgical rock properties. Thesis for the degree of doctor of philosophy, mining engineering. Department of Civil and Environmental Engineering, University of Alberta.

Deutsch, C., & Journel, A. (1998). *GSLIB: Geostatistical software library and user's guide* (p. 340). New York, NY: Oxford University Press.

Deutsch, J., Palmer, K., Deutsch, C., Szymanski, J., & Etsell, T. (2016). Spatial modeling of geometallurgical properties: Techniques and a case study. *Natural Resources Research*. https://doi.org/10.1007/s11053-015-9276-x.

Dominy, S., O'Connor, L., Parbhakar-Fox, A., Glass, H., & Purevgerel, S. (2018). Geometallurgy: A route to more resilient mine operations. *Minerals, 8,* 560.

Fennel, M., Guevara, J., & Canchaya, S. (2015). QEMSCAN mineral analysis for ore characterization and plant support at Cerro Verde. In *XXVII Convención Minera, Arequipa, Perú*.

Garrido, M., Ortiz, J., Villaseca, F., Kracht, W., Townley, B., & Miranda, R. (2018a). Change of support using non-additive variables with Gibbs Sampler: Application to metallurgical recovery of sulfide ores. *Computers and Geosciences*. https://doi.org/10.1016/j.cageo.2018.10.002.

Garrido, M., Ortiz J., Sepúlveda, E., Farfan, L., & Townley, B. (2019) An overview of good practices in the use of geometallurgy to support mining reserves in copper sulfides deposits. In *Proceeding procemin GEOMET 2019, November Santiago, Chile*.

Garrido, M., Sepúlveda, E., & Navarro, F. (2016). A case study of geometallurgical modelling of metal recovery with unequal sampling. In *Proceeding GEOMET 2016, December Santiago Chile.*.

Garrido, M., Sepúlveda, E., & Navarro, F. (2017). Optimization of planning and scheduling of ore body with open pit extraction considering homogeneity in clays as geometallurgical variables. In *Proceeding Geomin Mineplanning, August 2017*.

Garrido, M., Sepúlveda, E., Ortiz, J., Navarro, F., & Townley B. (2018b). A methodology for the simulation of synthetic geometallurgical block models of porphyry ore bodies. In *Proceeding procemin GEOMET 2018, Santiago, Chile.*.

Gholamnejad, J., & Osanloo, M. (2007). Incorporation of ore grade uncertainty into the push back design process. *Journal of the Southern African Institute of Mining and Metallurgy, 107,* 177–185.

Goodfellow, R., & Dimitrakopoulos, R. (2016). Global optimization of open pit mining complexes with uncertainty. *Applied Soft Computing*. https://doi.org/10.1016/j.asoc.2015.11.038.

Goovaerts, P. (1997). *Geostatistics for natural resources evaluation* (p. 483). Oxford: Oxford University Press.

Hunt, L., & Jorgensen, M. (2011). Clustering mixed data. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*. https://doi.org/10.1002/widm.33.

Hunt, J., Kojovic, T., & Berry, R. (2013) Estimating comminution indices from ore mineralogy, chemistry and drill core logging. In *The second AusIMM international geometallurgy conference, Brisbane, QLD* (pp. 173–176).

Isaaks, E., & Srivastava, M. (1989). *An introduction to applied geostatistics* (p. 561). New York: Oxford University Press.

Jackson, J., McFarlane, A., & Olson, K. (2011) Geometallurgy—Back to the future: Scoping and communicating geomet programs. In *The first AusIMM international geometallurgy conference, Brisbane, Queensland, 5–7 June 2011* (pp. 115–123).

Jackson, J., & Young, M. (2016). Ore type: Everything to someone but nothing to anyone. Paper presented to 3rd international geometallurgy conference 2016, Perth, Western Australia.

Keeney, L., & Walters, S. (2011). A methodology for geometallurgical mapping and orebody modelling. Paper presented at the the first AusIMM international geometallurgy conference, Brisbane, Queensland.

King, R. (2001). Modeling and simulation of mineral processing systems. *Modeling and Simulation of Mineral Processing Systems*. https://doi.org/10.1016/B978-0-08-051184-9.50004-3.

Kumral, M. (2011). Incorporating geo-metallurgical information into mine production scheduling. *Journal of the Operational Research Society, 62,* 60–68.

Kumral, M. (2013). Optimizing ore–waste discrimination and block sequencing through simulated annealing. *Applied Soft Computing*. https://doi.org/10.1016/j.asoc.2013.03.005.

Lamberg, P. (2011). Particles the bridge between geology and metallurgy. In *Conference in mineral engineering*. Luleå, Sweden.

Lamghari, A., & Dimitrakopoulos, R. (2012). A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2012.05.029.

Lane, K. (1988). The economic definition of ore: cut-off grades in theory and practice. *Mining Journal*, Books London.

Lerchs, H., & Grossman, F. (1965). Optimum design of open-pit mines. *Transaction CIM, 58,* 47–54.

Lishchuk, V. (2016). Geometallurgical programs—Critical evaluation of applied methods and techniques. Doctoral thesis, Lulea University of Technology.

Lund, C., Lamberg, P., & Lindberg, T. (2015). Development of a geometallurgical framework to quantify mineral textures for process prediction. *Minerals Engineering*. https://doi.org/10.1016/j.mineng.2015.04.004.

Maksaev, V., Townley, B., Palacios, C., & Camus, F. (2007). Metallic ore deposits. In *The geology of Chile* (pp. 179–199). London: The Geological Society.

Manchuk, J., & Deutsch, C. (2012). A flexible sequential Gaussian simulation program: USGSIM. *Computers and Geosciences*. https://doi.org/10.1016/j.cageo.2011.08.013.

Mariethoz, G., & Caers, J. (2015). *Multiple-point geostatistics: Stochastic modeling with training images, 374*. Hoboken, NJ: Wiley.

Montoya, P., Keeney, L., Jahoda, R., Hunt, J., Berry, R., Drews, U., et al. (2011). Geometallurgical modelling techniques applicable to pre-feasibility projects—La Colosa case study. In *Proceedings of the international geometallurgy conference, Brisbane, Australia, 5–7 September 2011; Australasian Institute of Mining and Metallurgy: Melbourne, Australia, 2011* (pp. 103–111).

Morrell, S. (2006). *Determination of the DWi using the SMC test and its use in orebody profiling, comminution circuit design and optimisation. Monograph series*. Indooroopilly: SMCC Pty Ltd.

Mueller, U., & Ferreira, J. (2012). The U-WEDGE transformation method for multivariate geostatistical simulation. *Mathematical Geosciences*. https://doi.org/10.1007/s11004-012-9384-7.

Munoz, B., Lesser, V., & Smith, R. (2010). Applying multiple imputation with geostatistical models to account for item nonresponse in environmental data. *Journal of Modern Applied Statistical Methods, 9,* 274–286.

Ortiz, J., Kracht, W., Townley, B., Lois, P., Cárdenas, E., Miranda, R., et al. (2015) Workflows in geometallurgical prediction: Challenges and outlook. In *Proceedings of the 17th annual conference of the international association for mathematical geosciences IAMG 2015*.

Pardo-Iguzguiza, E., Chica-Olmo, M., & Atkinson, P. (2006). Downscaling cokriging for image sharpening. *Remote Sensing of Environment*. https://doi.org/10.1016/j.rse.2006.02.014.

Pawlowsky-Glahn, V., & Olea, R. (2004). *Geostatistical analysis of compositional data*. Oxford: Oxford University Press.

Sepúlveda, E., Dowd, P. A., & Xu, C. (2018a). The optimisation of block caving production scheduling with geometallurgical uncertainty: A multi-objective approach. *Transactions of the Institution of Mining and Metallurgy, Section A: Mining Technology*. https://doi.org/10.1080/25726668.2018.1442648.

Sepúlveda, E., Dowd, P. A., & Xu, C. (2018b). Fuzzy clustering with spatial correction and its application to geometallurgical domaining. *Mathematical Geosciences*. https://doi.org/10.1007/s11004-018-9751-0.

Sepúlveda, E., Dowd, P. A., Xu, C., & Addo, E. (2017). Multivariate modelling of geometallurgical variables by projection pursuit. *Mathematical Geosciences*. https://doi.org/10.1007/s11004-016-9660-z.

Sillitoe, R. (2010). Porphyry copper systems. *Economic Geology, 105,* 3–41.

Silva, M., Dimitrakopoulos, R., & Lamghari, A. (2015). Solving a large SIP model for production scheduling at a gold mine with multiple processing streams and uncertain geology. *Mining Technology*. https://doi.org/10.1179/1743286314Y.0000000075.

Suazo, C., Kracht, W., & Alruiz, O. (2010). Geometallurgical modelling of the Collahuasi flotation circuit. *Minerals Engineering*. https://doi.org/10.1016/j.mineng.2009.11.005.

Suthers, S., Clout, J., & Donskoi, E. (2004). Prediction of plant process performance using feed characterisation: An emerging tool for plant design and optimisation. In *MetPlant, Perth, WA, 6–7 September 2004* (pp. 203–217).

Tolosana-Delgado, R., Mueller, U., & Van den Boogaart, K. (2019). Geostatistics for compositional data: An overview. *Mathematical Geosciences*. https://doi.org/10.1007/s11004-018-9769-3.

Townley, B., Luca, R., Lopez, L., Muñoz, M., & Castillo, P. (2018) Geochemistry of hydrothermal alteration associations in porphyry copper deposits: Applications to geometallurgical modeling. In *Proceeding RFG resources for future generations, June 16–21, 2018, Vancouver Convention Center, BC, Canada*.

Tran, T., Wen, X., & Behrens, R. (1999) Efficient conditioning of 3D fine-scale reservoir model to multiphase production data using streamline-based coarse-scale inversion and geostatistical downscaling. In *Annual technical conference and exhibition held in Houston, Texas, 3–6 October*. Society of Petroleum Engineers Inc.

Vann, J., Jackson, J., Coward, S., & Dunham, S. (2011) The geomet curve: A model for implementation of geometallurgy. In *The first AusIMM international geometallurgy conference, Brisbane, Queensland, 5–7 June 2011* (pp. 115–123). Melbourne: The Australasian Institute of Mining and Metallurgy.

Webster, R., & Oliver, M. (1990). *Statistical methods in soil and land resource survey*. New York: Oxford University Press.

Yildirim, B., Bradshaw, D., & Powell, M. (2014). Development of an effective and practical Process Alteration Index (PAI) for predicting metallurgical responses of Cu porphyries. *Minerals Engineering*. https://doi.org/10.1016/j.mineng.2014.07.009.