Contents

1 Preliminar Concepts **1**
1.1 Liquid Crystals . . . 1
1.2 The Frank-Oseen model . . . 2

2 Ginzburg-Landau equations **4**
2.1 Pitchfork Bifurcation . . . 4
2.1.1 Degenerated Pitchfork Bifurcation . . . 5
2.2 Vortex solutions of Ginzburg-Landau Equation . . . 6
2.2.1 Symmetries of the Ginzburg-Landau equation . . . 8
2.3 Energy . . . 9
2.4 Linearized operator of Ginzburg-Landau equation around vortex solution . . . 11
2.4.1 Kernel of Linearized operator around vortex solution in the plane . . . 11
2.4.2 Linearized operator on a bounded domain . . . 13

3 Anisotropic Ginzburg Landau Equation **15**
3.1 Experimental Setup . . . 15
3.2 Amplitude Equation Derivation . . . 16
3.2.1 Linear Analysis . . . 17
3.2.2 Weakly Nonlinear Analysis . . . 18
3.3 Anisotropic Ginzburg Landau Equation . . . 20
3.3.1 Fourfold symmetry . . . 21
3.3.2 Positive Vortex Solution in Anisotropic Ginzburg-Landau . . . 22

4 Anisotropic Vortices: Existence, Stability and Energy **24**
4.1 Construction of the solution . . . 24
4.2 Anisotropic energy . . . 31
4.2.1 Stability . . . 31
4.2.2 Anisotropic energy expansion of anisotropic vortex solution . . . 32
4.2.3 Linear approximation of the perturbation . . . 34
4.2.4 Fourier Series decomposition . . . 36

5 Numerical Results **38**
5.1 Finite element method . . . 38
5.1.1 Introduction . . . 38
5.1.2 Finite element method for the linear approximation of anisotropic vortex solution . . . 39
5.1.3 Quadratic coefficient of negative anisotropic vortex solution . . . 42

5.1.4 Numerical calculation of anisotropic Energy 43

6 Analysis for an O.D.E. system from negative anisotropic vortex 46
6.1 Introduction 47
6.2 Asymptotic behavior of solutions of the homogeneous system 48
6.2.1 The possible behaviours at zero 48
6.2.2 The possible behaviours at infinity 52
6.2.3 The homogeneous system does not admit globally bounded solutions 59
6.2.4 The fastest vanishing at zero is related with the fastest blow-up at infinity 60
6.3 Bounded solutions of the inhomogeneous system 63

Conclusion 67

A Invariant function subspace of the anisotropic Ginzburg-Landau equation 68

B Computation details of the quadratic coefficient in energy expansion 70

Bibliography 73