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MDTA: MARKOVIAN DYNAMIC TRAFFIC ASSIGNMENT, A NEW APPROACH FOR STOCHASTIC DTA

TESIS PARA OPTAR AL GRADO DE
DOCTOR EN SISTEMAS DE INGENIERÍA

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POR: RICARDO FELIPE DE LA PAZ GUALA
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Esta tesis doctoral se centra en un nuevo enfoque de modelamiento para la asignación dinámica del tráfico. Considerando los avances actuales en aspectos que en trabajos de asignación de tráfico eran bastante restrictivos, tales como la tecnología o la disponibilidad de datos, el problema puede afrontarse desde un punto de vista más realista abordando el proceso de asignación desde una perspectiva dinámica. De hecho, una característica importante del concepto de asignación dinámica de tráfico (DTA por sus siglas en inglés) es reconocer explícitamente la evolución del estado de la red de transporte a lo largo de un período de tiempo, a diferencia de los típicos modelos estáticos en los que esa información es agregada. En una dimensión diferente, previamente a concentrarse en la versión dinámica del problema, los investigadores han dedicado tiempo y esfuerzo a representar otro aspecto importante del comportamiento de los automovilistas que viajan por una red de transporte, que es la incertidumbre de sus decisiones a la hora de elegir cómo proceder para llegar a sus destinos. Así, muchos trabajos sobre asignación y equilibrio de tráfico han integrado la estocasticidad como parte de sus formulaciones. La forma de modelar ese comportamiento ha sido a nivel de elección de ruta, considerando que el criterio de la elección se basa en los costos de las rutas desde el origen hasta el destino percibidos por los automovilistas. En esta tesis doctoral se propone un nuevo enfoque que aborda la estocasticidad en el contexto de los problemas de DTA. La principal contribución de este trabajo es el modelo Markovian Dynamic Traffic Assignment (MDTA), desarrollado primero para el caso general de múltiples orígenes y un destino, que luego se extiende a redes generales de transporte. La base de los resultados presentados es la integración del concepto de Markovian Traffic Equilibrium propuesto por Baillon y Cominetti, para el caso de una asignación estática realizada por un modelo logit, y los desarrollos en DTA propuestos por Addison y Heydecker. El enfoque propuesto se basa en arcos, a diferencia de los modelos típicos en la literatura basados en rutas. La estructura anidada de los costos en la formulación resultante es un aspecto relevante de este enfoque, en que los automovilistas toman sus decisiones de elección de ruta de manera dinámica, según los costos percibidos de lo que resta de su viaje, es decir, desde su nodo actual hasta su destino. El modelo MDTA tiene como característica fundamental que, dado su modelo de elección basado en arcos, permite trabajar con rutas superpuestas sin asumir la independencia de sus costos y, por lo tanto, no requiere la enumeración de rutas de una red de transporte.

ABSTRACT OF THE THESIS TO APPLY
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This doctoral thesis focuses on a new modeling approach for dynamic traffic assignment. Considering the advances at present in aspects that in earlier stages of the study of traffic assignment were more restrictive, such as technology or data availability, the problem can be faced from a more realistic point of view by approaching the process of assignment from a dynamic perspective. In fact, a major feature of the Dynamic Traffic Assignment (DTA) concept is to recognize explicitly the evolution of the transport network status over a time period unlike the typical static models where such information is aggregated. On a different dimension, before getting interested in the dynamic version of the problem, researchers have devoted time and effort in representing another important aspect of the behavior of motorists traveling over a transport network, which is the uncertainty of their decisions when choosing how to proceed to their destinations. Thus, many works on traffic assignment and equilibrium have integrated the stochasticity as part of their formulations. The way to model that behavior has been at a route-choice level, considering that the route-choice criterion is based on the perceived costs by motorists of the routes from the origin to the destination. In this doctoral thesis, a novel approach that tackles the stochasticity in the context of DTA problems is proposed. The core contribution of this work is the Markovian Dynamic Traffic Assignment (MDTA) model, developed first for the multiple origins and a single destination general case, which is later extended to general transport networks. The basis of the presented results is the integration of the concept of the Markovian Traffic Equilibrium proposed by Baillon and Cominetti, for the case of an assignment performed by a logit model for the static case, and the developments on DTA modelling proposed by Addison and Heydecker. The proposed approach is arc-based unlike the typical route-based formulations generally found in the specialized literature. The nested structure of the costs in the resulting formulation is a relevant feature of this approach, in which motorists make their route choice decisions dynamically, according to the perceived costs of the remaining portion of the trip, namely, from their current node to their final destination. The MDTA model has as a key feature that, given its arc-based choice model, it allows working with overlapping routes with no assumptions of independence of their costs and, thus, it does not require the enumeration of the many transport network routes.

To my friends, my sister, my mother and, especially, to my father, who supported me from the beginning of this journey and now guides me as my brightest star.

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Introduction

For traffic analysis, the study of how the the motorists are assigned over a transportation network is relevant for the evaluation of strategic and tactical transportation projects in the context urban policy studies. In the last few decades, due to new methodological and technological advances, researchers have put attention on the dynamics behind the way in which the assignment of vehicles is performed over a transportation network, which is more realistic than traditional static assignments developed and implemented in the past. This way, the interest in the called *Dynamic Traffic Assignment* (*DTA*) problem, that considers the time dependence of the demand in the process of assignment, has grown considerably, to develop more realistic modelling approaches in the context of transport networks planning and policy studies.

Conceptually, *DTA* approaches consider the relationship between the characteristic of a transport network and the dependency of demand as a function of time, incorporating the dynamism in traffic behaviour or the route choice of the motorists. A DTA model is a natural extension of previous static assignment/equilibrium models, in which routing decisions of motorists are assumed static over a determined period of time. Conversely, DTA models assume that traffic conditions evolve in time and change as motorists move through the network. Research on DTA has been based widely on motorists' behaviour assumptions, model formulations and solution methods in order to represent the time dependency consistently, according to the observed congestion dynamics. For traffic analysis, the study of how the traffic assignment is performed is fundamental. Particularly, given the access to data of mobility patterns on networks, it would be desirable to build a model that represents real life situations properly. In that sense, the dynamic component of the assignment along with stochasticity in travel decisions are two aspects to be incorporated in the construction of a proper modelling approach for traffic assignment. The correct development of a model that adequately represents both aspects combined, dynamics and stochasticity, can generate approaches that cover a variety of phenomena that occur in real urban deployements.

The *DTA* problem was first introduced by Merchant and Nemhauser [41]. Their work is considered to be groundbreaking, as it opened the subject of how to approach the traffic assignment problem, typically tackled as static, from a dynamic point of view. Since its formal introduction, DTA has been addressed through several approaches. In fact, the stochastic version of DTA has been studied and analyzed, considering different ways to integrate the concept of uncertainty in the routing decisions with the dynamic evolution of traffic during the modelling period, as it is approached by DTA models.

Among the variety of literature on DTA, a work of Addison and Heydecker stands out by establishing the necessary requirements for a proper formulation of a suitable DTA model [2]. The same authors offer a comparison among different traffic models in order to show how they could contribute to a DTA formulation, based on the pursued objectives [3].

Stochastic route-choice models have been studied in literature, but while they may vary on how to tackle uncertainty on the motorists routing decisions, they generally consider as the choice criterion the individual perceived costs of traveling from their origin to their destination. About a decade ago, the concept of *Markovian traffic equilibrium* (*MTE*) was introduced for the static case of traffic assignment [7]. It differs from most of the approaches as its traffic assignment model considers that motorists choose according to the expected minimum cost from their current node to their destination.

In this doctoral thesis, the route-choice model proposed by Baillon and Cominetti [7] in their *MTE* model is adapted under a novel approach that considers the dynamic features associated with a DTA formulation, by following the modelling considerations established by Addison and Heydecker [2]. This generates what is denoted as *Markovian Dynamic Traffic Assignment model* (*MDTA*). The formulations and developments behind the basics of *MDTA* will be further addressed in Section 1.3 of Chapter 1.

Objectives and Motivation

In general terms, the main objective of this doctoral thesis is to develop a new dynamic traffic assignment model that considers uncertainty in motorists decisions while they are moving through the network. The motivation behind this goal comes from the idea of integrating the traffic assignment associated with the MTE model by Baillon and Cominetti[7] with the DTA formulation framework proposed by Addison and Heydecker [2]. A complementary and significant specific objective of this work is to provide a solution algorithm for the *MDTA* formulations. The idea is to contribute not only a new DTA model, but also a tool with the potential of applying the proposed approach in the analysis of different scenarios of interest when dealing with dynamic traffic assignment problems.

Methodology

The stages followed in this research, considering the motivation and objectives stated above, can be summarized as follows:

- **Literature Review:** An exhaustive bibliographic analysis was performed in order to understand the state of art regarding DTA and stochasticity in traffic assignment.

Next, as approached in many works in the literature regarding DTA, the plan was to develop a first version of the model for simple transport networks, and then to scale the results until accomplish a version for general transport networks.

- **Analysis of the "one-to-one" case:** The first notions resulting from the new MDTA approach were conceived in this stage. As noticed later in Section 2.1 of Chapter 2, the basics introduced here are extended with subtle changes to the next stage for the more general "many-to-one" case.

- **Analysis of the "many-to-one" case:** The first formal version of the MDTA model was developed at this stage for the multiple origins and a single destination general case. A solution algorithm for this model was constructed and applied to different scenarios.
- **Analysis of the "many-to-many" case:** The final version of the MDTA model was developed for the multiple origins and multiple destinations general case, as this case covers general transport networks. A solution algorithm for this model was constructed and tested for different cases.

Beyond the goals of this doctoral thesis, an alternative version of the MDTA is being currently developed by varying one of its main structures. This will be later discussed in Chapter 4.

Thesis Structure

This doctoral thesis starts with Chapter 1, by presenting some of the results of an exhaustive review of DTA literature. The chapter ends by highlighting the two basic models that were the foundations for the MDTA model. In Chapter 2, the first MDTA model is introduced, one that covers the multiple origins and a single destination general case, along with a solution algorithm and how the method works over an illustrative example and an analysis regarding its computational implementation. This model serves as the foundation for the one presented in Chapter 3, the MDTA model for general transport networks, which is the main contribution of this doctoral thesis. Its solution algorithm, its performance and the results regarding its computational implementation are also presented. In Chapter 4,one specific line of ongoing research in further developments of the MDTA is presented. Finally, this thesis closes with some important conclusions, final comments and some potentialities associated with further research lines.

Chapter 1

A Review on DTA

In this chapter, a comprehensive revision of the literature performed in the development of this doctoral research is summarized, providing an insight on different aspects regarding the DTA problem. In the first section, general topics are addressed, such as why and when to apply a DTA-based modelling approach when performing traffic assignment. The second section presents some model classifications and illustrates the potentialities of DTA by presenting some models from the existing literature. The third section closes this chapter by highlighting two different lines of work that ended up becoming the basis and motivation of the work developed on this doctoral thesis.

1.1. The interest on the DTA approach

This chapter begins by briefly contextualizing on some general topics regarding the DTA analysis. This section addresses the origins of DTA, and why and when DTA is convenient for performing traffic assignment. In summary, it is discussed what aspects are relevant to decide about whether or not to move from the static to the dynamic analysis.

1.1.1. Origin of the DTA concept

In literature, traffic assignment with static characteristics has been widely studied by establishing the first models and solution methods associated with what is currently known as *Static Traffic Assignment* (*STA*). Merchant and Nemhauser [41] present a new point of view to approach the problem of the traffic assignment by proposing a model, and its underlying solution algorithm, that addresses the dynamic version of STA, defined as the *Dynamic Traffic Assignment* (*DTA*). Their proposed formulation represents a model that considers a transport network of a single origin-destination pair and a temporal dependent demand over a period of time. The model manages to explicitly express congestion by representing the traffic behaviour through a *whole link model*. The algorithm works over a discretization of the time period. The analytical features of the model generate a non-linear and non-convex problem. This groundbreaking contribution is referenced, directly or indirectly, by the DTA literature, as it is the first work that formally introduces the DTA concept. It establishes not only the first model and solution method associated with the DTA definition, but also the first questions and challenges regarding the theoretical, computational and empirical analysis associated with the subject.

1.1.2. Why DTA?

Depending on the objective of the analysis, there are different ways to answer the question of why is DTA a suitable approach to take when studying traffic assignment. In what follows, a compilation work [18] is presented, which helps get an intuition on what to consider as common aspects when checking DTA suitability.

From a transport planning point of view, DTA models describe the dynamics of a transport network, understood as an evolution on time of some of the aspects that interact with its demand, by analyzing the time-dependent behaviour of the motorists. The results that come from applying the DTA modelling approach can be used to evaluate decisions regarding time and cost of travel, from an individual point of view, and also to assess decisions regarding the whole transport network system, considering general planning goals.

Traffic engineering is an area of study that currently bases most of its analysis on deterministic or simulation-based approaches to address the performance of the transport network systems, from micro to macroscopic scales. From this point of view, DTA is a practical approach to take, considering that the solution methods that look for a DTA based on equilibrium are generally obtained through recursive algorithmic procedures. The general objective is to describe the evolution of different choice dimensions and its influence, from the route and departure time choice to the interpretation of how these decisions relate to the behaviour of the entire transport network. DTA models apply iterative procedures that involve the load of traffic vehicles on the routes and algorithms that assign the flow on those routes. This is performed in order to compute departure times that allow motorists to experience the minimum possible total travel time.

There are a variety of simulation-based DTA models that, from a computational point of view, adopt efficiently the traffic requirements and considerations to properly describe the changes of traffic flows. The analysis can be performed at a regional level, which represents a larger geographic scale compared to what is considered in microscopic simulation-based models. This, without compromising a realistic representation of the aggregated individual behaviour of the motorists in the network while, simultaneously, integrating car-following and traffic flow theories. Depending on how the model is built, the DTA approach is capable of avoiding the unnecessary details of the interactions between motorists, for example, line changing for gap acceptance, according to the goals of each particular analysis.

1.1.3. When is DTA a good approach to take?

A Norway Institute of Transport Economics report [32] establishes a benchmark for the appropriate use of DTA models, in the sense of how suitable they are, by considering a classification given by its level of analysis scale. In this context, the scale is understood as the microscopic, mesoscopic or macroscopic aggregation of the parameters of interest. Different methodological approaches for dynamic traffic assignment models arose. These approaches can be classified based on the context of their respective applications, the properties of the demand model and the features and practical characteristics of the model. Such a classification is summarized in Table 1.1.

	Micro	**Meso**	**Macro**
Congestion Mitigation	Suitable	Suitable	Suitable
Intelligent Transport Systems	Suitable	Acceptable	Not Suitable
Demad Management	Suitable	Suitable	Acceptable
Equity Analysis	Suitable	Suitable	Not Suitable
Standar Cost-Utility Analysis	Acceptable	Acceptable	Suitable

Table 1.1: Classification propposed by the Norway Institute of Transport Economics [32].

In this compilation, it is worth mentioning that other models from the literature, that have not been directly referenced, could also be suitable or acceptable, according to their particular features depending on the context of their use.

1.1.4. STA vs DTA

According to the reviewed literature, a comparison of some aspects of interest between STA and DTA can be established. First, general comments on some of this aspects are discussed and then summarized in a table. The contents in the table can be used as benchmark to decide whether to apply STA or DTA to address a traffic assignment analysis.

Generally, most of the STA models found in the literature are defined over a relatively long period of the day (an interval of time long enough to include the peak hours, for example). In these models, the congestion of the transport network is usually described through a *Volume Delay Function* (*VDF*). A VDF is a performance/time function that represents the average total travel time experienced by motorists on each arc according to its traffic load and the behaviour of the motorists. One of the limitations of using VDF functions is that they allow arcs with volume/capacity ratio greater than 1, which leads to a measure that does not hold an intuitive meaning in terms of traffic technical features. Another limitation is that it is necessary to assume that the motorists entry satisfies the *FIFO* rule, which discards some phenomenon that does happen in reality, like overtaking. Also, there is no differentiation of types of lanes on a highway and it is assumed that at each arc the traffic that enters is equal to one that exits. Because of this, it is implied that there is no representation of the effects of a congested arc on the interaction with its immediately preceding/following arc and, thus, some phenomena can not be analyzed, such as the *spillback effect*.

On the other hand, in DTA models, each arc can be defined by its own *Traffic Fundamental Diagram* (*TFD*), if this condition is required to accomplish the particular goals of the analysis. Even though it could be perceived as the dynamic analogue version of a VDF associated with a static model, actually, it is not. The reason is that, mathematically, they represent different aspects of their respective models. In a dynamic model, given a certain arc in the transport network, the TFD describes how congestion in the ending node of the arc propagates until it generates an effect in its consecutive arcs. This phenomenon can not be represented in a STA model through a VDF function. DTA models that use TFD allow recognizing motorists in different lanes, and their interactions, which permits overtaking in the modelling, for example. Consequently, it is no longer needed to impose the FIFO rule. Being capable of modelling different lanes also allows representing phenomena that occurs in reality. For example, considering that the most external lane of a highway have motorists moving at a slower speed.

A guidebook contributed by Sloboden et al. [49] presents a comparison between the two approaches, the STA and the DTA. The analysis performed on these approaches has been summarized in a very general way by showing their advantages and disadvantages in Table 1.2, considering a PRO/CON comparison on four different aspects.

	STA		DTA	
On the time horizon analysis	CON:	Limited to aggregated data of a time interval	PRO:	Allows continuity between consecutive traffic stages
On the variables that uses	CON:	Functions depend only on the traffic load	PRO:	Functions depend on traffic load and on time
On the analytical formulations	PRO:	Approachable given the static time	CON:	Difficult given the time dynamics
On the computational implementation	PRO:	Requieres affordable calibration/validation	CON:	Requieres big volume of data for calibration/validation

Table 1.2: Summarized STA VS DTA in Sloboden et al. Guidebook [49].

1.2. A general bibliographic review on DTA modelling

Literature regarding DTA covers a wide spectrum of aspects, like classifications of models under different criteria or applications of new modelling approaches or definitions. In this section, some of these aspects are addressed and, particularly, some models that can help to illustrate the potentialities of the DTA analysis are commented.

1.2.1. First expectations regarding DTA earlier developments

Thanks to the analytical developments and the technological advances, that allow to computationally implement what has been theoretically proposed, DTA models have transformed in an increasingly viable modelling option for traffic analysis. This has been accomplished by becoming a key tool in studies of prediction of motorists and their travel behaviour on a transport network and by being applied to micro, meso and macroscopic simulation-based traffic models. Microscopic models allow the analysis of the behaviour of the motorists traveling at a lane level on highways. Mesoscopic models move gradually from these characteristics to the level of aggregation of data associated with the macroscopic ones. Macroscopic models allow the analysis of journeys at a regional level over a transport network. Chiu [18] highlights that the DTA modelling approach has the potential to work, simultaneously, at microscopic and macroscopic levels. From this feature, DTA is capable to work at different mesoscopic levels, by allowing the representation of the dynamics of the traffic behaviour at an aggregated global transport network level without missing the smaller scales of data that can be obtained at a lane level.

Peeta and Ziliaskopoulos [44] present a comprehensive study of the past, present and future of the DTA modelling and its applications. This work provides a reference point of view of what were the basis and research context that they faced back then and, most importantly, it gives a perception of how much of what was expected to explore and develop has been actually accomplished at our current times. Always considering the technological limitations at the time of this research, the authors point out that in the 80s and 90s the analytical development of DTA models experienced a quick evolution. This was mainly motivated by the need of managing the analysis on different aspects, from the real-time traffic behaviour to the long-term planning. Such evolution of the DTA modelling led to a growing diversification of the literature regarding this subject and the emergence of different assumptions and different analysis goals. The authors establish that, at their time, research was still focusing on the general characteristics of the problem, with few approaches with the potential of motivating further research. On the other hand, they state that the debate would be centred around the applicability of DTA modelling in short as well as in long term planning, while still highlighting that the advantages of DTA over STA were still difficult subjects to discuss properly. This issue has been partially answered, considering the work that addressed this topic [49], summarized in Table 1.2. They close their analysis by establishing that, if the technological, analytical and simulation developments are the necessary, dynamic traffic assignment modelling will be the recommended approach to be use by default.

1.2.2. Models Classifications Criteria

Recalling again the work presented by Sloboden et al. [49], in the second chapter they present a classification of different DTA modelling approaches. Models are analyzed according to two criteria: whether they are equilibrium-based or not and whether if they consider one or multiple types of motorists travelling through the network.

- **Equilibrium-based DTA approaches:** First, let us recall the notion of *Wardrop Equilibrium* for the STA case, understood as:

  In a network model with multiple possible routes for each O-D pair, all used routes experience equal, and minimum, travel time (generalized cost) and no user can improve his/her experienced travel time by unilaterally change to another route.

  According to this intuitive definition, a Wardrop Equilibrium for the DTA case can be understood as:

  In a network model with multiple possible routes for each O-D route and in a specific period of time, for each O-D pair and for each time increment in the departure time of the travel, all used routes experience equal, and minimum, travel time (generalized cost) and no user can improve his/her experienced travel time by unilaterally change to other route or change his/her departure time.

  Considering this, the algorithmic procedures associated with equilibrium-based DTA models can, generally, be compared to the ones that are applied in the analysis of the STA case. These are:

  1. Detecting cheapest routes;
  2. Travel assigning over the routes;
  3. Analysis of the traffic load assigned to the routes and the resulting traffic conditions.

Regarding these algorithmic aspects, one of the points that generate multiple diversifications is the stop criterion of the solution methods. Some of the conditions that may need to be satisfied are, for example, respecting a limit amount of motorists waiting in queues or reaching a certain *relative GAP*.

- **Non equilibrium-based DTA approaches:** In traffic analysis, it is sometimes desirable to study cases in which unexpected events may occur when the system is not necessarily in an equilibrium state. This, in order to analyze the decisions that the motorist would make with the limited information that he/she can perceive at the moment. Such incidents can be structural works, accidents, evacuations, or even information updates while travelling, for example.

  Under this type of modelling approach, each motorist has already chosen a route once his/her trip has started. This chosen route is not assumed to be a result of an optimization process, as that could have been chosen without considering any established criteria, like a usual route, for example. This does not mean that the assignment cannot be the one that results from an equilibrium assignment of the network. Also, it is worth noticing that, under this approach, exogenous agents and different types of structural events that are not generally part of other models become a significant influence in the decisions made by the motorists in the network.

  As for the solution method aspects, there are important considerations when comparing to the usual procedures that are applied to the case of the equilibrium-based DTA modelling. This is because, when this type of network state is not assumed, the routes start as a set configuration that will be reassigned according to the events that may occur later and, thus, dependent on the response of the environment. This modelling approach still preserves that the assignment aims to minimize the experienced travel time of the rest of the trip.

- **DTA approaches that consider different types of motorists:** These DTA models consider that the behaviour of the motorists in the transport network can be classified under different types of criterion. As an example, a model can consider that part of the motorists chooses strictly the routes that an equilibrium-based DTA model would assign while other motorists chooses routes that have been set for them, without any established criterion. The remaining motorists are willing to change their initial routes reacting to different stimulation that they can perceive once their trip has started.

  This type of model, assuming a proper estimation of the proportions of the classification of the behaviour of the motorists, offers an advantage considering that they represent a more realistic situation.

Szeto and Wong [50] present a new classification for DTA modelling approaches. This contribution is summarized in Table 1.3, where some of the multiple aspects of the DTA modelling potentialities that the authors considered are shown. Some of these aspects include the expected level of realistic representation of the situation to be achieved, the efficiency of the solution or the approximation method and the feasibility of the problem. Moreover, they point out the way how the route choice is addressed in the DTA models opposed to their equivalent version under a STA approach. This, considering that for previously commented reasons, when facing some DTA modelling approaches the complexity increases and the difficulty of finding solutions also increases considerably. Under this analysis, a specially illustrative DTA modelling approach is found in Lo and Szeto [39].

<table>
<tr><th>Criterion</th><th>Subcriterion</th><th>Category</th></tr>
<tr><td rowspan="2">Modelling on the choices dimension</td><td>Route and/or departure time choice</td><td>· Only departure time choice
· Only route choice
– With on-route reaction
– Without on-route reaction
· Route and departure time choice</td></tr>
<tr><td>If motorists travel or not</td><td>· Fixed demand
· Static demand</td></tr>
<tr><td rowspan="2">Modelling on the temporal dimension</td><td>Time horizon length</td><td>· Day-to-day
· Within a day</td></tr>
<tr><td>Time horizon modelling</td><td>· Continuous
· Discrete</td></tr>
<tr><td rowspan="4">Other formulation approaches</td><td>Decision variable (or formulation)</td><td>· Arc flow
· Route flow</td></tr>
<tr><td>Queues representation</td><td>· Physical representation
· Non-physical representation</td></tr>
<tr><td>Number of motorists types</td><td>· Single type
· Multiple types</td></tr>
<tr><td>Methodological approaches</td><td>· Simulation-based
· Analytical
– NCP
– VIP
– MPP
– FPP
– OCP
– CMP</td></tr>
</table>

Table 1.3: Classification presented by Szeto and Wong [50].

More recently, Abdulhafedh [1] highlights some particular types of DTA modelling approaches that, under the author's perception, are especially practical when addressing the most recurring DTA problems in the literature. These approaches are:

1. DTA modelling approaches for highly congested urban transport networks: These models tackle the problem of transport networks that, in order to be considered as urban and highly contested, must satisfy:
   - Thousands of directed arcs;
   - A big amount of short arcs;
   - Frequent ramps that have to be close to each other;
   - Long queues and presence of spillback effect in the network;
   - High interference of non-motorized vehicles in traffic.

   A model contributed by Ben-Akiva et al. [12] is pointed out to be a suitable example for this case.
2. DTA modelling approaches for Optimal System with and without spillback effect: Their formulation and solution method use approximations of the marginal costs of the routes. Some of this models offer a method to find the marginal cost of the path through the

path marginal cost, which represents the changes in the costs of flows in the network caused by an additional unit of flow in a path that has been used since a certain instant. As an example, the author refers to a work contributed by Qian et al. [45].

3. DTA modelling approaches to evaluate moving bottlenecks: These models are developed to evaluate the effects of bottlenecks that can change, over time, the transport network performance, in terms of travel times. These approaches require a strong microscopic simulation basis. As an example, the author refers to a contribution by Juran et al. [32].

1.2.3. Modelling and Solving approaches of DTA problems in the literature

Some favorable and desirable conditions to hold for DTA Problems

Zhang and Nie [55] establish some conditions in order to reduce the difficulty of approaching DTA modelling, considering that a basic and necessary requirement is that FIFO rule needs to be satisfied at both arc and route levels in the transport network. The fundamental property of the FIFO rule applied in DTA modelling is that directly relates inflow, outflow and travel time of a given arc. Recalling the concepts used in this work, a *state set* is referred to as the set of time-dependent variables that are used to fully characterize the dynamics of the whole transport network system (such as flows over all the arcs). Then, a *minimum state set* is the set of variables such that if one of them is taken out from the set, then, the evolution of the dynamics of the system cannot be determined. They prove that if the system satisfies the FIFO rule for every arc of the network, then the dimension of the minimum state set is two. Given this, they proposed two ways of modelling the behaviour of the traffic over the arcs, in order to be used in the implementation of the DTA model. In this work, the authors do point out that, even though satisfying the FIFO rule is something desirable in a transport environment, that is not observed in all cases to be analyzed, as commented earlier [49]. Thus, there are situations that are of particular interest when studying traffic, such as overtaking, where the FIFO rule is not satisfied.

Analytical models

There are several analytical DTA modelling approaches in the literature that could be useful to illustrate how the approach has diversificated. In this subsection, three particular DTA models are presented, as they are suitable examples to show how the evolution of the DTA extensions has led to deeply specialized results.

Before commenting on these examples, it is worth noticing that even though analytical models base their developments on theoretical formulations for solving DTA problems, in literature, they generally end up being validated by computational implementations, to check how close to reality the models are.

- **A model of interrelated traffic flows:** Jayakrishnan et al. [31] present a DTA model that interrelates the traffic flows at different levels of the network. By establishing a convex monotonously increasing behaviour of travel times on the density, they used arc-cost functions generated from a modified form of the *Greenshield's* speed-density relationship, on their dynamic assignment part of the model. It differs from previous models by incorporating explicitly the relationships between the traffic flows in the model and by using considerably short intervals of time to represent in a better way the dynamics of traffic. This leads to a remarkable communication between their assignment algorithm and their applied traffic theory. They propose, for later works, to relax the notion of strict equilibrium in DTA, intuitively getting close to those classified by Sloboden et al. [49], earlier commented and summarized in Table 1.2.

- **A model that improves the integration of the generated demand data:** Ramachandra et al. [6] present a model that considers mixed traffic conditions, but what is worth noticing is that the generation of demand is based on drivers instead of vehicles. They based their approach on the *Activity-Based Demand Generation* (*ABDG*) in order to generate the demand chains within a day, according to people's activities. They propose a method that allows using the advantages of overlapping the potentials of the ABDG methodology in the context of DTA modelling. It was accomplished by incorporating to ABDG changes that allow recognizing dynamic evolutions in the system within a day and also day-to-day, besides differentiating types of motorists behaviours when making choices. Their methodology was later applied to the city of Patna, Bihar's capital, in India, given the particularities of its transport system, where motorists are known for not respecting traffic laws.
- **A model that seeks to minimize the amount of data needed to do DTA analysis:** Laval et al. [34] present formulations to study the dynamic user equilibrium through the use of urban macroscopic models (fundamental diagram) in order to require the least possible number of parameters. Their work considers an O-D pair with two alternative routes on a highway, of fixed capacity and generic streets, and generic urban traffic levels. They managed to prove that, under appropriate transformations, the capacity of the highway and the fundamental diagram of the capacity ratio are enough to characterize the solution of the dynamic user equilibrium. This solution is determined by two critical accumulation values that define if the stable state is at free-flow or bottleneck levels, depending on the initial accumulation.
- **A Cell-Based model:** Islam et al. [30] present an analysis of the impact of signal control on the system optimal by evaluating the accuracy of the *Cell-Based* methodology applied to DTA modelling. This uses the concept of *Cell Transmission model* (*CTM*) [20, 21], widely applied as a *Dynamic Loading* model (time-dependent flow assignment). The authors present the *Signal Control with Realistic Cycle-length* (*SCRC*), that has as one of its main features how it manages the trade-off between the size of the geographic zone and the length of the signal control cycle. Some advantages of this model are the following: its linear and continuous formulation and that the model behaviour through traffic congestion variations allows the use of fewer decision variables, which reduces the complexity of the problem. The analysis considers the system optimal instead of the user equilibrium, as it refers to signal control, which is an activity managed by a central decision-maker according to the global benefit of the network. The notion of DTA modelling according to a system optimal will be discussed later in Subsection 1.2.4.

Simulation-based DTA models

Simulation-based DTA models vary depending on their objective. They can use analytical models to generate, for example, O-D matrices, or use real data and calibrate parameters of other models to adapt them to a particular situation to be analyzed. Some cases of DTA analysis based on simulation are commented below:

- **A model that considers multimodal transport:** Meng et al. [40] present a mesoscopic simulator comprising a supply and a time-dependent demand simulator. It is characterized by the fact that it considers different types of vehicles intervening in the traffic (private cars, the subway system, buses and bicycles). Each of these types of vehicles absorbs part of the demand of people moving through the network. They emphasize its usefulness in the application of *Intelligent Transport Systems* (*ITS*). Five scenarios were successfully implemented to test the efficiency of the model.
- **The CONTRAM model:** Taylor [53] presents CONTRAM, a computational model of time-dependent traffic on a transport network. The transport network parameters, the time-varying demand between a given set of O-D zones are te inputs of the model. It delivers as outputs the resulting flows, selected routes and travel times. The approach combines traffic models for short time intervals with disaggregated traffic assignment, thus moving between macroscopic equilibrium and microscopic models. The author emphasizes that it can be efficiently complemented by various methods for the choice model, either theoretical or empirical. This research caused interest in the British Government, which injected funds for the continuation and extension of the study of its potential functionalities.
- **A mesoscopic simulator based on queues:** Zhou and Taylor [57] present *DTALite*, a light, open-source DTA analysis package developed to enable rapid and advanced use of dynamic traffic analysis capabilities. Three modelling components that work behind the DTALite procedures are described:
  - A light simulator of network dynamic loading that integrates *Newell*'s wave model;
  - A mesoscopic agent-based procedure to generate heterogeneous motorists;
  - A calibration system for the demand of the O-D pairs and for the traffic assignment.

  DTALite was tested in real cases and was found to be effective for various networks and available data. The analysis of the queuing model is similar to the one developed by Mirchandani Zou [42], in which the methodology for the adaptive control study of signals is applied.
- **Simulation in DTA and emissions studies:** Zhou et al. [56] present a DTA model characterized by its methodology and its solution for studying fuel consumption and emissions generation. Although the operational background of their implemented DTA model is made in DTALite [57], the interesting aspect is how the results are cross-checked with those of an emission estimation software package, *MOVES Lite*. This allows studying the impact of different strategies that can be taken according to the traffic behaviour, both in terms of dynamic traffic, and emissions and fuel consumption.
- **A congestion simulator:** A work by Mirchandani and Zou [42] developed tools associated with DTA suitable for ITS applications. Two models of dynamic user equilibrium are proposed in which the evolution of flows is simulated for long paths networks in order to obtain real-time responses from the system. The first model considers very short time intervals and process all the nodes for each temporal layer in chronological order. The second model considers long time intervals and, for each sub-model associated with each interval, processes everything that would happen with its variables. Both models are regarded as successful in their implementation to simulate congestion in urban networks.
- **The LADTA model:** Gentile [26] presents *LADTA*, a mesoscopic DTA model designed and implemented as a continuation of a previous static version of the model [4]. The

objective is to work with networks whose set of variables exceeds the amount of memory available in a single computer. To this end, the model architecture and its implementation are based on algorithms capable of distributing tasks over more than one processor and executing the most expensive ones in reasonable periods of time. Some of its defining features are as follows:

  - Route flows propagation over time;
  - route choice and/or departure time choice;
  - Dynamic operation of the exit capacity on arcs and its reference travel time;
  - Punctual queues located at the end of the arcs.

- **A model to study vulnerability under structural events:** Alam et al. [5] present a study of the vulnerability of transport networks to infrastructure renewal, using as a tool a DTA microsimulation model that assesses the impact on the transport network during the sudden closure of a *critical infrastructure* (*CI*). This is defined by *Public Safety Canada* [14] as processes, systems, facilities, technologies, networks, goods and services essential to the health, safety or economic well-being of Canadians and the proper functioning of government institutions. The methodology of the model is based on the integration of a network model, the necessary data and the calibration of the DTA model (driving behaviour and route choice parameters). For the calibration and validation, field traffic data was used to predict traffic flows in the network. They then evaluated the scenario at a system performance level and local impact on traffic. Their conclusion is that, for the analyzed case, a 6 minutes increase in the average traffic delay is observed along with a resulting 24.5 per cent reduction in the number of vehicles arriving at the destination during the simulated incident.
- **DTA combined with Traffic Control:** Mitsakis et al. [43] present a compilation work of results and criteria about the combination of methodologies after DTA modelling and urban traffic control models. They consider that nowadays urban traffic control systems have the ability to forecast traffic conditions in a way that is increasingly closer to reality by means of artificial intelligence tools. Because of this, the use of fluid-type information generates opportunities through DTA approaches that are missed in case of approaching the traffic assignment through STA models. They implemented an integration of DTA and control models to the large scale urban transport network of Thessaloniki, Greece.

Solution Methods

The models that are presented in the literature, regardless of their different variations depending on their particular context, do follow solution methods that are based on methods and techniques first developed for STA models [49]. Thus, some equivalent methods are found in DTA with respect to the following typical STA procedures:

1. Detecting cheapest routes;
2. Assignment of motorists over the available routes;
3. Analysis of traffic loads assigned to the routes and the resulting traffic conditions.

For example, Jayakrishnan et al. [31] present a heuristic of solution for the model expanded network model proposed in their work. The method could be considered similar to the solution search of the *Stackelberg's followers* problem. The fomrulation is solved, first, by fixing the routes-arcs incidence matrix using equilibrium estimates in the arrival time intervals to the nodes. Then, it proceeds by assigning the traffic demand in the extended time-space transport network through some conventional static traffic assignment algorithm adapted to work with the features of the dynamic case. The computational implementation of this heuristic was successfully tested with different network sizes.

Another interesting and more recent solution method case is the one presented by Aguilera et al. [24], in which a solution algorithm for their DTA model, based on paths instead of arcs, is proposed. The traffic flow in the roads is combined with the *one-point queueing model* (*PQ*) [42] and the queueing generation model (*M/G/c/c*) [35]. This algorithm uses the *generalized method of expansion* [35], which keeps the record of vehicles trying to enter a node once it has reached its capacity. The key component of this methodology is the correct estimation of marginal costs per road of the whole transport network. In the same work, the real case of the Sydney transport network is studied by comparing an arc-based approach with their proposed road-based approach, with satisfactory computational results to solve traffic assignment problems.

1.2.4. Other definitions derived from DTA

The extensive literature on DTA modelling analysis allows to find different approaches to define particular problems or cases of DTA, varying from the general use of the strict definition of user equilibrium in its dynamic version. Some of these different approaches are presented below:

- **SO-DTA:** Most of the literature related to DTA modelling bases its analyses on the definition of Wardrop's Equilibrium. One of the works developed outside that line is presented by Samaranayake et al. [47], in which the study is conducted for computing the *System Optimal*. In this case, a central authority chooses the route to follow for each motorist while seeking to minimize the total travel time aggregated by all motorists from a global point of view. A model and optimization criteria are presented to solve the problem of assigning flow according to the *System Optimal Dynamic Traffic Assignment with Partial Control* (*SO-DTA-PC*) for transport networks. It considers an horizontal dynamic queuing generation that requires the complete information of the O-D pairs of the subset of agents to which the route and the division radius of the non-controllable agents can be assigned. They apply this methodology to generate optimal routing strategies in response to network capacity loss and show the reduction in congestion that can be achieved. As a real case analysis, they tested their results for the Southern California transport network.
- **Q-DTA:** Tajtehranifard et al. [52] present a combination of methodologies developed in other papers focusing on roads instead of arcs [24]. This work considers the User Optimal System Equilibrium, as proposed in a previously commented work [47], but applied to a model under the concept of *Quasi Dynamic Traffic Assignment* (*QDTA*). In the literature, this concept refers to static models of limited capacity with residual queues. It is based on the *Quasi-Dynamic Loading Network* models [13], which are used to compute changes in flows and residual queues in nodes and at bottleneck points. This leads to what is considered as an improvement on the accuracy of the travel time estimation. The implementation of this work and its subsequent testing on the classic

Sioux Falls transport network validates their proposed approach. They show how that optimal system traffic flow patterns improves the total travel time compared to the results obtained from a user equilibrium-based DTA modelling approach.

1.2.5. DTA and Public Transport

Although there is an extensive literature on DTA modelling approaches that covers different types of cases, when looking for results associated exclusively with public transport the options are considerably reduced, and there is no established definition of DTA in this type of context. However, some approaches regarding public transport assignment and tools that have been used in the development of DTA models are combined in selected key works. These works are commented on below:

- **Heterogeneity in Vehicles:** For this case, the contribution by Meng et al. [40] is recalled again. This mesoscopic simulation-based model considers the existence of private cars, a subway system, buses and bicycles as different transport options in the system. The generation of private transport supply is conducted according to an O-D matrix of time-dependent demand. For the case of buses and subways (exclusively public transport vehicles), the demand is generated by obeying predetermined routes and schedules. The generation of private cars and bicycles is perfomed according to independent stochastic processes. The demand generation involves a *nested C-logit model* that takes into account the interaction between the route-choice model and the traffic assignment model. Five experiments were successfully conducted and showed good response to, for example, increasing demand on the O-D pairs.
- **Dynamic Network Loading with Multiple types of Agents:** Cats [15] presents a Multi-Agent Transit Operation and Assignment model. It provides a framework that captures supply uncertainties and adaptive decisions by the motorists in the system. It develops a day-to-day learning process that consists of a dynamic load within the day [52, 13], which simulates the interaction between supply and demand generation. The model integrates a traffic simulator, an operation and transit control simulator, a dynamic route-choice model and a real-time information generator. The most interesting points in this model can be described as follows:
  - A population generator: Public transport vehicles are generated with pairs of depot zones that will act as O-D pairs and a predefined list of trips, with data on length, number of doors and number of seats, among others. Public transport users and private transport drivers are generated with time-dependent O-D matrices according to a probability function.
  - A traffic and a transit simulator: The flow of private cars results from a traffic assignment model according to the fundamental theory of traffic flow. The progress of public transport vehicles between stops is determined by the interaction with other vehicles and the waiting time in stops by the interaction with public transport users.
  - Dynamic route choice: A dynamic route-choice model from a previous work by the same author is used [16]. In this model, users make successive decisions after comparing different alternatives of actions they could take during their trip (getting on a bus vs. waiting for the next one, for example).

  - Adaptive Operation of Transit: For public transport, the service is adjusted by applying real-time management strategies or operations planning decisions from a central control choice maker, without physical-spatial representation. This decision maker uses as inputs, in real-time, the traffic predictions and conditions and can apply various control strategies (*holding*, decide services *express*, etc).

  The author highlights as potential applications: network design analysis, operational strategies, reliability and resilience measures of networks. He also notes that, in the future, the model can adopt adaptive strategies of the users, such as change of transport mode or adjustments of travel departure time.

1.2.6. Stochasticity and DTA

As in the case of public transport and DTA analysis, it is not common to find literature that directly relates stochasticity and DTA. However, there are considerably more research efforts than those developed for public transport. Some of the most important works are commented below:

- **A heuristic solution for stochasticy in route choice.** In general, analytical DTA models do not represent the spillback effect by themselves. This can be modelled if the approach is complemented by simulation, generating two components: an analytical model that determines route-dependent flow rates and a simulation-based model that performs a dynamic loading to determine the traffic volume on the arcs. Barceló et al. [9] present a DTA heuristic using two alternative analytical components to generate flow rates in arcs, combining stochasticity and dynamic traffic assignment (according to the user equilibrium principle). Its dynamic loading is performed through a microscopic simulation-based model, which is even able to recognize more than one type of motorist in the network. The implementation is based on **AIMSUN**, an earlier work that involved the authors [8, 11, 10]. The methodology of the model obeys the following points:
  - Determine path-dependent flow rates: Assign a candidate path to each vehicle entering the network considering:
    * Paths defined by the user, that can be given by default or they can be outputs of a simulator;
    * Shortest path trees, that can be obtained according to default costs or they can defined by the user. There are two types: initial shortest paths or stochastic shortest paths.

    Then, for each vehicle moving between an O-D pair, there is a probability of choosing one of the roads defined above.
  - Dynamic network load: It will determine how flow rates assigned to each route increase traffic volumes in such a route according to time, travel times of each arc and travel times of each route.

  Next, the simulation process based on the time-dependent flow rates on routes works as follows:
  1. Initial computing of shortest routes for each O-D pair using defined costs;
  2. Simulate a predefined period of time by assigning to the available routes a fraction of trips between each O-D pair for that period according to the selected routes and obtain a new average travel time per arc;

3. Compute again the shortest routes based on current average travel times;
4. If there are guided vehicles or the simulator suggests rerouting, give the information from step 3 to the drivers who are allowed to reroute;
5. If the termination criteria are not met, return to step 2.

This simulation model includes stochasticity in the choice of the first route to be decided by each motorist while performing the assignment iteratively in a dynamic way.

- **DTA with stochastic demand:** Waller and Ziliaskopoulos [54] address the problem of System Optimal DTA when demands are stochastic and time-dependent. They present a model that is a stochastic extension of a deterministic linear programming formulation for System Optimal DTA, an earlier work by one of the authors [58]. The methodology behind this approach consists of three parts:
  - Identifying characteristics of the deterministic reference model: They extended the CTM model [20, 21] to one with stochasticity on the System Optimal definition previously commented [58].
  - Development of the stochastic model: In order to include stochasticity into the model they use the *Chance Constraint Programming* formulation (*CCP*)[17, 23]. This has two major advantages: the ability to explicitly include reliability constraints and to derive robust solutions with exogenously specified functions that are, generally, linear.
  - Computational testing: They validate and verify efficiency and performance by testing two transport networks with 50 randomly generated scenarios. The implementation, described in *RouteSim*, showed consistency in the results over the total travel time of the whole system.

  The proposed formulation proves to be able to generate a robust solution to the DTA problem according to the System Optimal with a specific level of user reliability. It can serve as a reference for similar analyzes and results in a useful tool to produce robust control and management strategies when dealing with uncertainty, where DTA on System Optimal could be an advantageous approach.

- **DTA and uncertain traffic:** Szeto et al. [51] present a cell-based DTA model with more than one type of motorist. The model considers the random evolution of the traffic states with a strong computational basis.

  In order to deal with the effect of travel time variability associated with the route choice, the random evolution of traffic states and the effects of spatial queues (as opposed to models that generally consider punctual queues), a DTA problem called *Multi-Class Double Stochastic Dynamic User Equilibrium Problem* (*MDS-DUE-P*) is proposed. This problem can be understood as determining the temporal pattern of flows in a stochastic traffic network given a fixed demand in each period of departure time. In this scenario, different classes of users have imperfect information about the network conditions and have different attitudes towards risk. The model has two basic components:
  - The route-choice model: A dynamic extension of the concept of *effective travel time* [38, 48], which considers the standard deviation of travel times rather than their variance.
  - Traffic model: For this, they propose a representation through a stochastic version of the CTM, based on the *Monte Carlo method*, generating the *Monte-Carlo-Based Stochastic Cell Transmission model* (*MC-SCTM*).

Then, the MDS-DUE-P model is formulated as a fixed-point problem and is solved through the *Self-Regulated Averaging Method* (*SAM*) [37]. They list the following important conclusions regarding the computational implementation context of their work:

- Reducing error perception in traffic conditions may not result in a reduction of the uncertainty when estimating system performance;
- SAM can converge quickly, but its rate of convergence can be much worse if the combination of parameters for step size is not well done;
- Three things can lead to higher computing times: higher demand, better quality information or high-risk aversion of drivers;
- More types of drivers does not always mean more running time;
- The running time can be significantly reduced if small samples are used in the early stages of the solution search process.

1.3. Two research lines regarding Stochasticity: The motivation of this Doctoral Thesis

After performing a deeper search regarding the DTA modelling approaches and stochasticity applied to traffic assignment, two lines of work ended up becoming the basis and motivation of this doctoral thesis. The first one proposes a strong formulation basis on DTA modelling while the second, even though it is conceived for static traffic analysis, proposes an interesting definition for stochastic traffic assignment that is used in the present work.

1.3.1. Addison and Heydeckers's research

The first line of work to highlight, and the one that motivates the formulational basis, specifically, how to approach the development of the modelling process of this thesis, is started by Addison and Heydecker. In their first work [2], they show the general conditions that are required to reach a *dynamic user equilibrium*. They also establish that a DTA model has three fundamental parts: a generation demand model, a dynamic traffic behaviour model and a route-choice model. The same authors analyze some dynamic traffic models [3], where they concluded that, in general, the deterministic punctual queueing model is one of the most suitable traffic behaviour approaches for studying DTA. Later on, they add the departure time choice dimension to their formulations [29]. Then, they add uncertainty to their modelling framework [28] by assuming that route costs are perceived differently by the motorists. Here, the dynamic traffic assignment obeys a logit model for the discrete choice of routes, considering the generalized cost as the dominant criterion. These costs include an error that distributes iid Gumbel, which generates the stochastic version of their original DTA model.

Han [27] generalizes the modelling approach proposed by Addison and Heydecker in a model that considers general transport networks and discrete time, incorporating the stochasticity through a similar logit model for the route choice split. This framework considers the deterministic punctual queueing model on the arcs of an underlying digraph (N, A) defined to represent a transport network, where N and A denote the sets of nodes and arcs, respectively. In this work, as in Addison and Heydecker's approach, given a transport network represented by the digraph (N, A), for each arc $a \in A$, ϕ_a is its free flow travel time and Q_a is its queue unloading capacity. Also, at a time $t > 0$, $E_a(t)$ is the inflow rate entering arc a at t, $G_a(t)$ is the outflow rate leaving arc a at t, $L_a(t)$ is the length of the queue in arc a at t, $C_a(t)$ is the total travel time of arc a having entered it at t, and $d_a(t)$ is the delay for waiting in the queue of arc a having joined the queue at t. Then, the following equations apply:

$$\frac{dL_a}{dt} = \begin{cases} 0, & \text{if } L_a(t) = 0 \text{ and } E_a(t - \phi_a) < Q_a, \\ E_a(t - \phi_a) - Q_a, & \text{otherwise,} \end{cases} \tag{1.1}$$

$$G_a(t) = \begin{cases} E_a(t - \phi_a), & \text{if } L_a(t) = 0 \text{ and } E_a(t - \phi_a) < Q_a, \\ Q_a, & \text{otherwise,} \end{cases} \tag{1.2}$$

$$C_a(t) = \phi_a + d_a(t), \tag{1.3}$$

and

$$d_a(t) = \frac{L_a(t + \phi_a)}{Q_a}. \tag{1.4}$$

Among the features of this model, it is worth mentioning that it satisfies the FIFO rule, flow propagation and causality. The latter means that the route choice of a motorist will not be affected by the ones entering the network after him/her. Note that in a stochastic dynamic route choice context not all the used routes have effective minimum cost. Considering this, the *stochastic dynamic user equilibrium* (*SDUE*) traffic assignment model is presented. In this model, according to the chosen logit model of parameter θ over a time discretization of timestep size Δt, the probability $P_p^{od}(t)$ of choosing route p at time t to go from the origin to the destination of the O-D pair od, among the set R_{od} of all routes that go from o to d, is given by:

$$P_p^{od}(t) = \frac{\exp(-\theta C_p^{od}(t + \Delta t))}{\sum_{q \in R_{od}} \exp(-\theta C_q^{od}(t + \Delta t))}, \tag{1.5}$$

where $C_p^{od}(t)$ is the travel time of route p to go from the origin to the destination of O-D pair od, starting at time t. They provide an accurate and intuitive idea of this approach, given by the following definition for SDUE: *At every instant, no driver believes that he/she can improve his/her perceived travel cost by changing routes unilaterally* [28]. Analytically, for continuous time, considering that $q^{od}(t)$ is the demand for the O-D pair od at time t, $f_p^{od}(t)$ is the flow assigned to route $p \in R_{od}$ at time t and $P_p^{od}(t) = Pr(\hat{C}_p(t) \leq \hat{C}_{p'}(t), \forall p' \in R_{od} | \mathbf{C}(t))$, where $\hat{C}_p(t)$ is the least perceived cost among the routes in R_{od} at time t (that depends on the costs pattern of all routes at time t, $\mathbf{C}(t)$), then, the definition is expressed as follows:

$$P_p^{od}(t) = \frac{f_p^{od}(t)}{q^{od}(t)}, \forall p \in R_{od}, \forall od, \tag{1.6}$$

$$\sum_{p \in R_{od}} f_p^{od}(t) = q^{od}(t), \forall od \tag{1.7}$$

and

$$f_p^{od}(t) \geq 0, \forall od. \tag{1.8}$$

Considering the already named notations, the formulation for the SDUE as a variational inequality [46] can be expressed as:

$$\begin{aligned} f_p(t) &\geq 0, \forall p \in R_{od}, \forall t, \\ \sum_{p \in R_{od}} f_p(t) &= q^{od}(t), \forall t, \end{aligned} \tag{1.9}$$

then, a flow assignment at time t for all routes $p \in R_{od}$ and for all O-D pairs od, in the form of a route flows configuration $f^*(t)$, corresponds to an equilibrium if and only if:

$$\sum_{od} \sum_{p} K_p^{od}(t) \left\{ f_p^{od}(t) - K_p^{*od}(t) \right\} \geq 0, \forall f \in F, \tag{1.10}$$

where F is the set of all feasible route flows configurations of all O-D pairs of the whole network, $K_p^{od}(t) = \left\{ f_p^{od}(t) - q^{od}(t) P_p^{od}(t) \right\} \frac{\partial C_p^{od}(t)}{\partial f_p^{od}(t)}$ and $\frac{\partial C_p^{od}(t)}{\partial f_p^{od}(t)} \geq 0$.

Later [36], the same authors provide an extension of their previous model considering the dynamic departure time choice in addition to the route choice, defining the *dynamic departure time/stochastic user equilibrium* (*DDSUE*) condition, as follows: *No traveller can improve his/her perceived travel cost by unilaterally changing his/her departure time and route combination.*

1.3.2. Baillon and Cominetti's research

The second research line to highlight is the one started by Baillon and Cominetti. They propose a stochastic, although static, user equilibrium model, which is built by applying notions related to Markovian chains [7]. They generate what is introduced as the *Markovian traffic equilibrium* (*MTE*), in which the flow on the routes is obtained by assigning flow on the arcs according to the expected minimum costs to the destinations. Given the construction of the model under its *arc-choice approach*, rather than in a *route-choice* one, no enumeration of the routes is required and no independence of the route costs is assumed. In order to get an intuitive idea behind this approach, let us consider Fig. 1.1 [7], where the number on each arc represents its cost and there is a demand going from node 1 to node 3 that will be assigned under a logit rule. From a route-choice approach, as independence assumptions are not well suited for overlapping paths and the three paths from node 1 to node 3 have cost 1, the logit model operates by assigning 1/3 of the demand to each route. On the other hand, as the two lower routes differ from one another just at their ends, under the arc-choice model presented in this work, the solution is an assignment of 1/2 of the demand to the upper route and 1/4 of the demand to each of the lower routes.

On a digraph (N, A), with set of nodes N and set of arcs A, given a destination node $d \in D$ and the sets of routes from each node $i \in N$ to destination d, R_i^d, the cost $\tilde{c}_r$ to proceed through route r to go to d is a random variable. Then, the demand proportion g_i^d from a node i to destination node d through route r, is given by:

$$h_r = g_i^d Pr(\tilde{c}_r \leq \tilde{c}_p, \forall p \in R_i^d). \tag{1.11}$$

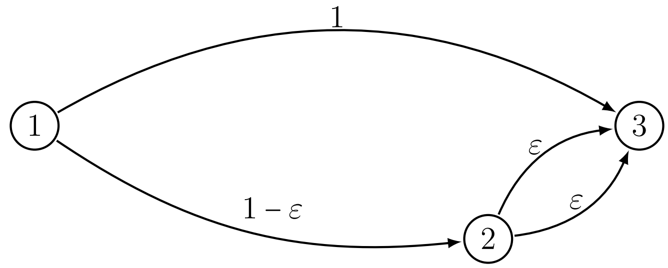

Figure 1.1: Simple network to compare path choice versus arc-choice apporaches.

Given a destination d, the uncertainty is given by the motorists' perception of the travel costs, towards d, on the arcs. Thus, in the case of arc $a \in A$, the perceived cost is computed as $\tilde{t}_a = t_a + \varepsilon_a$, with $\mathbb{E}(\varepsilon_a) = 0$. Analyzing the equilibrium at a level of route $r \in R_i^d$, from node $i \in N$, the perceived cost of choosing that route is $\tilde{c}_r = \sum_{a \in r} \tilde{t}_a$. Then, the *MTE* model relies on the estimation of the expected optimal cost of travelling from node i to destination d, which is $\tilde{\tau}_i^d = \text{mín}_{r \in R_i^d} \tilde{c}_r$. Thus, the cost of taking a route that starts on arc a to next proceed to destination d is computed as $\tilde{z}_a^d = \tilde{t}_a + \tilde{\tau}_{j_a}^d$, with j_a representing the ending node of a. For all random variables of the form $\tilde{\tau}_i^d = \tau_i^d + \theta_i^d$ and $\tilde{z}_a^d = z_a^d + \varepsilon_i^d$, they are split into a systematic component and a random component, where it is assumed that the latter satisfy $\mathbb{E}(\theta_i^d) = \mathbb{E}(\varepsilon_a^d) = 0$. Then, for every destination node d, there is a Markov Chain of finite states on graph (N, A), with transition probabilities computed as follows: for all $i, j \in N$,

$$P_{ij}^d = \begin{cases} Pr\left(\tilde{z}_{(i,j)}^d \leq \tilde{z}_b^d, \forall b \in A_i^+\right), & \text{if } (i,j) \in A, i \neq d, \\ 1, & \text{if } i = d, j = d, \\ 0. & \text{otherwise,} \end{cases} \tag{1.12}$$

where A_i^+ is the set of outgoing arcs from node i.

Thus, given a destination $d \in N$ and an arc $a = (i,j) \in A, i \neq d$, the expected flow in arc a towards d, v_a^d, and the expected flow from node i to d, x_i^d, satisfy:

$$v_a^d = x_i^d Pr(\tilde{z}_a^d \leq \tilde{z}_b^d, \forall b \in A_i^+). \tag{1.13}$$

Expression (1.13) can be related to the demand that goes from i to d considering that $x_i^d = g_i^d + \sum_{a \in A_i^-} v_a^d$, where A_i^- is the set of incoming arcs to node i. Defining the functions $\varphi_i^d(z^d) = \mathbb{E}(\text{mín}_{a \in A_i^+}(z_a^d + \varepsilon_a^d))$, the flow at both levels, arcs and nodes to destination d, for all nodes $\forall i \neq d$, can be expressed by:

$$v_a^d = x_i^d \frac{\partial \varphi_i^d}{\partial z_a^d}(z^d), \qquad \forall a \in A_i^+ \tag{1.14}$$

and

$$x_i^d = g_i^d + \sum_{a \in A_i^-} v_a^d. \tag{1.15}$$

In addition, the systematic component of the expected minimum cost to destination d through arc a, z_a^d, is computed as

$$z_a^d = t_a + \varphi_{j_a}^d(z^d) \tag{1.16}$$

Now, considering the previous expressions, given a set D of destination nodes, a vector w is defined as a *Markovian traffic equilibrium* if and only if, for each $a \in A$:

$$w_a = \sum_{d \in D} v_a^d, \tag{1.17}$$

where the v_a^d satisfy (1.14) and (1.16) and z^d satisfyies (1.16) with t_a being the underlying cost of carrying flow w_a on arc a.

Before closing this subsection of the current chapter, it is important to mention two dynamic models for traffic assignment that, because they are deeply associated with the *MTE* approach, which is one of the motivations of this work. First, Koch and Skutella [33] study a Nash equilibrium formulation modelling a sequence of static flows with some particular properties. At every instant θ, the *current shortest paths graph* G_θ is calculated. Then, a chain of static flows is used to form a *Nash flow*, defined as a flow over time that satisfies a Nash equilibrium. Later, Cominetti et al. [19] further develop Koch and Skutella's approach by adding new concepts and showing some fundamental properties of the dynamic features of traffic. The authors prove existence and uniqueness of the solution of the problem for the single origin and single destination general case, considering as a traffic model the deterministic punctual queuing model, same one highlighted by Addison and Heydecker [3] as a suitable option to work with when studying DTA.

Chapter 2

The first step: The Markovian Dynamic Traffic Assignment Model for the Multiple Origins and a Single Destination General Case

Motivated by the *MTE* concept proposed by Baillon and Cominetti [7], the mathematical basis of their related works [33, 19] and the advances in formulations and approaches started by Addison and Heydecker [2, 3, 29, 28, 27, 36], this doctoral thesis pursues, as its core goal, to develop a new dynamic traffic assignment model.

Conceptually, the proposed model apply notions associated with the MTE concept that incorporates the uncertainty in the route choice dimension in dynamic traffic assignment context. The model is called *Markovian dynamic traffic assignment* model (*MDTA* model), and it is an integration of the *MTE* model but considering that demand evolves over time, in the way it is approached in the formulation proposed by Addison and Heydecker. In this new scenario, costs will be time-dependent, because travel times associated with delays due to the formation of queues are directly influenced by how the flows are being assigned through the network. In the approach taken in this work, the costs are computed considering the concept of deterministic punctual queues.

In this chapter the focus is on the problem that considers a transport network where time-dependent demand is originated at different origins but in which all motorists joining the system are going to the same destination. The result is the *MDTA* model for the multiple origins and a single destination general case. This model serves as a first approach to accomplish the model that covers general transport networks. As it will be shown later on this thesis, a more generalized case from multiple origins to multiple destinations is also developed. The latter represents a second stage of this work, which requires some determinant considerations that makes the problem more difficult to approach in both the formulation of the model and the solution algorithm, as it is highlighted further in this work. The "many-to-one" version discussed in the current chapter creates the foundations for the general "many-to-many" case, the core result of this doctoral thesis.

The chapter starts by highlighting how the case was first approached. Then, the concept of *reasonable arc* is introduced, a notion that becomes a defining one on this thesis. Then, the building process of the MDTA model for the multiple origins and a single destination general case is presented. Next, its algorithm solution, an illustrative example of how it works and results regarding its computational implementation are provided.

Before showing the model construction, the relevance of analyzing the multiple origins and a single destination case at this stage of work is highlighted. Then, according to this, a new concept is defined with the goal of reflecting an aspect of the motorists behaviour when they enter the transport network.

2.1. From the "one-to-one" to the "many-to-one" case: Focusing the travel on the destination

Before describing the development of the model to be presented on this chapter, it is worth commenting on how it was first conceived. Even though the following sections are dedicated to the analysis of the case of multiple origins and a single destination case, as it has clearly been established, it would be natural, first, to wonder how to tackle the general case of a single O-D pair. This case is is usually the first one to be studied in literature when the idea is to escalate in the complexity of the transport network to be analyzed. This work started, in fact, by addressing the single origin and single destination general case.

The approach taken in this thesis aims to have as one of its features a realistic representation of how motorists consider which arcs are actually a convenient option, even before choosing an arc to move forward to. To do so, it is assumed that motorists have a correct perception of how the transport network behaves, when the level of congestion is low, in the sense of computation of travel times. This idea has been applied before in the literature for the case of routes instead of arcs, as it is going to be further referenced in the next section. It is usually assumed that, given an O-D pair, a motorist considers getting farther from his/her origin and getting closer to his/her destination as simultaneous conditions to be satisfied by a route to be considered as an option. Under this idea, the aggregated flow rate of motorists that enters the network at a given time through an origin would need to be labeled by both, its origin and its destination.

In this work it will be assumed that a motorist is only considering what is left of the trip towards his/her destination. With this, the aggregated flow rate of motorists only needs a labeling regarding its destination. Thus, whenever a demand rate going to given destination d enters the network through an origin o at an instant t and meets an already existing flow rate of incoming arcs to o at t that is going to the same destination d, all the flow rate going to d is aggregated. From this, the origin is not needed when the flow rate has to be assigned to the outgoing arcs from o to move forward to d. This allows to scale the one origin and one destination general case to the multiple origins and a single destination general case with few considerations.

The following section offers a formal definition of the reasonable arc concept in the context of traffic assignment and its dynamism according to what has been introduced in the previous literature. That definition allows moving naturally from the "one-to-one" case to the "many-to-one" case analysis.

2.2. Reasonable Arcs

As established in the previous section, as one of the main features of the approach of this work is to consider a stochastic behaviour of motorists, it can be assumed that every route has a certain positive probability of being chosen, which means that every route will have a positive inflow assigned. In reality, it is observed that not all routes are actually considered as options by the motorists. Then, in a maneagable as well as realistic model, it is desirable to reduce the options of routes for users.

The definition of *reasonable route*, introduced by Dial [22], is recalled here and adapted to the dynamic context. The original definition states that given an O-D pair (o, d) then, a route that starts at node i and ends in node j, is a *reasonable route* for the pair (o, d) if the minimum cost from o to i is smaller than the minimum cost from o to j and, simultaneously, the minimum cost from node i to d is greater than the minimum cost from j to d. Intuitively, this means that the route has to lead the vehicle farther from the origin and closer to the destination, if minimum cost routes are meant to be used. This concept is adapted defining a less restrictive version of "reasonability" defined now on the arcs. Thus, it results in a manageable dynamic traffic assignment approach since enumeration of routes is not required.

In the context of the problem faced in this chapter, let us consider d as the destination of every O-D pair. Then, an arc (i, j) is a *reasonable arc* if the minimum cost of going from j to d is less or equal to the minimum cost of going from i to d. Intuitively, this means that a motorist will not use arcs that will get him/her farther from his/her destination, if minimum cost routes are taken. This definition not only reduces the set of arcs that a motorist may have as possible options to move forward to, but also, it helps to properly construct the solution algorithm. In the proposed model, the set of reasonable arcs towards destination d is defined over the empty network, as it is assumed that motorists have a proper perception of travel times when the arcs are not congested. Even though this reduction in the option of arcs has no direct influence on the pieces of the model explained here, this definition does become fundamental later in the development of the solution algorithm, where its preponderance applies.

2.3. Building the MDTA model for the multiple origins and a single destination case

In order to build the MDTA model for the case of the multiple origins and a single destination general case, the model structure established by Addison and Heydecker [2] is replicated. Here, three parts have to be properly defined: the demand profile, the traffic model and the route-choice model. In this approach the last one is in particular, an arc-choice model.

Let us consider a transport network represented by the digraph (N, A), where N and A are the sets of nodes and arcs, respectively. For each node $i \in N$, the set of outgoing arcs from node i and the set of incoming arcs to node i are denoted as A_i^+ and A_i^-, respectively.

For each arc $a \in A$ are known two paremeters, its free flow travel time, which is the constant time that takes to travel the arc when uncongested, and its queue unloading capacity, which is the constant maximum rate at which vehicles can leave the arc, denoted by ϕ_a and Q_a, respectively. A set of origin nodes $O \subseteq N$ and a single destination node $d \in N$ are also assumed known. From those, we consider exogenous temporal dependent demand rate functions $\mathcal{D}_o(\cdot)$ from each origin $o \in O$ towards destination node d and a temporal horizon that starts at time 0 and ends at time T.

2.3.1. The Demand Profile

The temporal dependent demand rate functions $\mathcal{D}_o(\cdot)$, from each origin $o \in O$ to the destination d are exogenous. Therefore, to start with, it is relevant to clearly establish *the demand profile*, which is the first of the three parts of the model.

2.3.2. The Traffic Model

For the construction of the second of the three parts, *the traffic model*, the *Deterministic Punctual Queueing model* is considered, which has major features properly explained by Addison and Heydecker[3]. Here, the relationships between inflow rates, outflow rates and queues of the arcs are established and the costs functions on the arcs, in the representation of total travel times, are defined. In order to compute these expressions, it is necessary to consider the free flow travel time ϕ_a and the queue unloading capacity Q_a of each arc $a \in A$.

Let us consider an arc $a \in A$ and a time $t \in [0, T]$. The *inflow rate* of a at t, which is the flow rate that enters the arc a through its initial node at t, and the *outflow rate* of a at t, which is the flow rate that leaves the arc a through its ending node at t, are denoted as $E_a(t)$ and $G_a(t)$, respectively. Also in this work, as queues generate when arcs get congested, the *queue length* of a at t is referred to as the number of motorists in the queue of the arc a at the instant t, denoted as $L_a(t)$ and, thus, if there is no congestion in a at t, $L_a(t) = 0$.

Next, the relationships between the inflow rate, outflow rate and queue length of each arc $a \in A$, at each time $t \in [\phi_a, T + \phi_a]$, are analytically expressed as:

$$G_a(t) = \begin{cases} E_a(t - \phi_a), & \text{if } E_a(t - \phi_a) \leq Q_a \wedge L_a(t) = 0 \\ Q_a, & \text{otherwise} \end{cases} \tag{2.1}$$

$$\frac{dL_a}{dt} = \begin{cases} 0, & \text{if } E_a(t - \phi_a) \leq Q_a \wedge L_a(t) = 0 \\ E_a(t - \phi_a) - Q_a, & \text{otherwise.} \end{cases} \tag{2.2}$$

To explain these expressions, let us consider an arc $a \in A$ with free flow travel time ϕ_a and queue unloading capacity Q_a at an instant $t \in [0, T]$. Then, (2.1) represents that if the inflow rate at t is less or equal to Q_a and, simultaneously, there is no queue, then, the outflow rate at $t + \phi_a$ equals the inflow rate at t, as all motorists are able to travel through the arc under no congestion and, thus, the total cost equals ϕ_a. Otherwise, the outflow rate at $t + \phi_a$ is equal to Q_a, and this can happen for two reasons. First, if the inflow rate at t is greater than Q_a, then, some motorists won't be able to leave the arc and will be added to the end of a queue, if there is one. Second, if there is already a queue, then all motorists associated with the inflow rate at t will be added to the end of that queue, as the latter has to be unloaded at capacity and FIFO rule has to be satisfied. (2.2) represents that if the inflow rate at t is not greater than Q_a and, simultaneously, there is no queue, then the queue length does not change, as all motorists are able to travel the uncongested arc. Otherwise, two things can happen. First, if the inflow rate at t is greater than Q_a, then the queue length will increase at a rate given by the inflow rate minus Q_a, as motorists will join or form a queue at a rate given by the inflow rate and, simultaneously, others will leave the arc at a rate given by Q_a. Second, if there is a queue and the inflow rate at t is less or equal to Q_a, then the queue length will decrease at a rate given by Q_a minus the inflow rate, as motorists will be added to the queue at a rate given by inflow rate and, simultaneously, others will leave the arc at a rate given by Q_a.

Considering what has been explained, the total travel cost (or, indistinctly, total cost or cost) faced by a motorists that enters an arc at a certain time is given by the addition of two components: the free flow travel time of the arc and the delay because of waiting in its queue, having joined said queue once traveled the arc. Analytically, for each arc $a \in A$ at a time $t \in [0, T]$, the total cost of a at t, denoted by $C_a(t)$, is expressed as:

$$C_a(t) = \phi_a + \frac{L_a(t + \phi_a)}{Q_a}. \tag{2.3}$$

A remarkable feature of the model, given the arc-based construction of the cost functions, is that it allows working with overlapping routes, as it does not assume or require independence on the route costs and, in fact, no conditions regarding routes interaction need to be established.

At this point, two out of the three model structures required to define a proper DTA model have been provided: the demand profile, as a given function, and the traffic model, the one just presented. Next, we develope the third part.

2.3.3. The Arc-Choice Model

The third model structure that has to be built is the route-choice model. In this work, the MTE approach proposed by Baillon and Cominetti [7] is adapted. As mentioned before, this model was originally developed to be applied to a static traffic assignment.

The original arc-choice model associated with the MTE concept [7] states that given a node $i \in N$, the flow towards destination d arriving to that node from incoming arcs will split by selecting arcs instead of complete routes. In the formulation here proposed, this choice is made by a logit rule of known dispersion parameter θ, considering the cost of using each arc $a = (i, j) \in A$ plus the expected minimum cost from j to d. In other words, in a static context, being C_a the cost of the arc $a = (i, j)$, the expected minimum cost of going from i to d by choosing the arc a, namely Z_a, is computed as follows:

$$Z_a = C_a - \frac{1}{\theta} \ln \left(\sum_{b \in A_j^+} \exp\left(-\theta Z_b\right) \right). \tag{2.4}$$

Now, in this thesis, this expression is extended to a dynamic version, considering the general case of multiple origins and a single destination, d, and based on the expected cost of going to the destination by choosing an arc given in equation (2.4). Then, given a known dispersion parameter θ, for each $a = (i, j) \in A$ at time $t \in [0, T]$, and with $C_a(t)$ the cost of the arc a at t, defined in 2.3, the expected minimum cost of going from node i to destination d by choosing arc a at time $t \in [0, T]$, namely $Z_a(t)$, can be written as follows:

$$Z_a(t) = C_a(t) - \frac{1}{\theta} \ln \left(\sum_{b \in A_j^+} \exp\left(-\theta\left(Z_b\left(t + C_b\left(t\right)\right)\right)\right) \right). \tag{2.5}$$

On the other hand, for each node $i \in N$ at each time $t \in [0, T]$, the expected minimum cost of going from node i to destination d starting at t, namely $W_i(t)$, can be expressed by:

$$W_i(t) = -\frac{1}{\theta} \ln \left(\sum_{a=(i,j) \in A_i^+} \exp\left(-\theta\left(C_a(t) + W_j\left(t + C_a\left(t\right)\right)\right)\right) \right). \tag{2.6}$$

From expressions (2.5) and (2.6), given arc $a = (i, j) \in A$ and at a time $t \in [0, T]$, the following equations must be satisfied:

$$Z_a(t) = C_a(t) + W_j\left(t + C_a(t)\right) \tag{2.7}$$

and

$$W_i(t) = -\frac{1}{\theta} \ln \left(\sum_{a \in A_i^+} \exp\left(-\theta Z_a(t)\right) \right). \tag{2.8}$$

Therefore, given a node that is not at origin, namely $i \in N/O$, the inflow rate assignment over the outoging arcs a from i at time t, denoted as $E_a(t)$, can be computed. Then, for each arc $a = (i, j) \in A_i^+$ at each time $t \in [0, T]$, the inflow rate of a at t is given by:

$$E_a(t) = \frac{\exp\left(-\theta\left(Z_a\left(t + C_a\left(t\right)\right)\right)\right)}{\sum_{b \in A_i^+} \exp\left(-\theta\left(Z_b\left(t + C_b\left(t\right)\right)\right)\right)} \sum_{b \in A_i^-} G_b(t), \tag{2.9}$$

while, for nodes that are origins, namely $o \in O$, the inflow rate assgined to each arc $a = (o, j) \in A_o^+$ at a time $t \in [0, T]$, is calculated as follows:

$$E_a(t) = \frac{\exp\left(-\theta\left(Z_a\left(t + C_a\left(t\right)\right)\right)\right)}{\sum_{b \in A_o^+} \exp\left(-\theta\left(Z_b\left(t + C_b\left(t\right)\right)\right)\right)} \left(\sum_{b \in A_o^-} G_b(t) + \mathcal{D}_o(t) \right), \tag{2.10}$$

where, as established earlier, $G_b(t)$ is the outflow rate of arc b at instant t and $\mathcal{D}_o(t)$ is the given demand rate starting at an origin node o at instant t, $\forall o \in O$. In other words, (2.9) and (2.10) represent how flow rates arriving to a node, which are entirely outflow rates if it the node is not an origin and are outflow rates and a demand rate if it is, are assigned as inflow rates among the outgoing arcs from that node at a given instant. Now that the choice model has been built, the *MDTA* model for the multiple origins and a single destination general case has been defined.

Given its arc-based construction, the arc-choice model preserves the property of not assuming any conditions regarding routes interactions, allowing to work with overlapping routes without assumptions of independence on their costs.

In this section, the three structures that define this proposed dynamic traffic assignment stochastic approach, denoted as *Markovian dynamic traffic assignment*, *MDTA*, have been described, as well as the relationships required for a proper definition of this novel MDTA approach. In the following section, an algorithm to solve this problem is built, again, established for a multiple origins and a single destination general case.

2.4. The MOSD-MDTA Algorithm

In this section, a solution method for the MDTA model for the case of multiple origins and a single destination, the *MOSD-MDTA algorithm*, is summarized. The method works over a discretization of the time period, namely $[0, T]$. The procedure is mainly, but not exclusively, based on *Dial's algorithm* [22]. In this dynamic approach, the process is repeated at every time increment. At each time interval, the algorithm starts with a backward step, in order to compute the expected minimum costs, and then it proceeds with a forward step, in order to assign the inflow rates to the different arcs of the network.

The inputs of the MOSD-MDTA algorithm are: the digraph (N, A), the set of origins $O \subseteq N$, the destination d, the free flow travel time and the unloading queue capacitiy of the arcs, Q_a and ϕ_a, respectively, parameters that are aggregated as the vectors ϕ and Q. It is also considered the length of the time of analysis, T, the size of the time steps, Δt, and the number of time intervals of the discretization, defined as K ($K = T/\Delta t$). For each $k \in \{1, ..., K\}$, the k-th *time increment* refers to the interval given by $[(k-1)\Delta t, k\Delta t]$. Also given, are the time-dependent demand rate function from every origin o to the destination, $\mathcal{D}_o(\cdot)$. This demand is aggregated as the vectorial function $\mathcal{D}(\cdot)$. Finally, a dispersion parameter θ associated with the logit specification for the route choice is assumed to be known.

The outputs are a matrix of inflow rates $E = (E_a^k)_{a \in A, k=1,...,K}$, a matrix of outflow rates $G = (G_a^k)_{a \in A, k=1,...,K}$ and a matrix of queue lengths $L = (L_a^k)_{a \in A, k=1,...,K}$, where, given an arc $a \in A$ and a time increment $k \in \{1, ..., K\}$, E_a^k, G_a^k and L_a^k represent the inflow rate, the outflow rate and the queue length of a at k.

The details of how the algorithm proceeds over time can be described as follows:

- **Initial Settings:** Before starting any computation, it is necessary to set parameters and provide initial values to the structures that will change over every time increment:
  - **STEP 0: INITIALIZATION:** The sets of incoming(outgoing) arcs from(to) every node $i \in N$, $A_i^+(A_i^-)$, as well as the number of time increments, K, are set. As the network starts empty, for each arc $a \in A$ and at each time increment $k = 1, ..., K$,

the inflow rate of a at k, the outflow rate of a at k and the queue length of a at k are set to zero, that is $E_a^k = 0$, $G_a^k = 0$ and $L_a^k = 0$, respectively. A default time increment 0 is set to define $L_a^0 = 0$. For each $a \in A$ and at each $k = 0, ..., K$, as a is uncongested, the total cost of using arc a is initialized equal to its the free flow travel time ϕ_a, this is $C_a^k = \phi_a$. For each $i \in N$, the initial minimum cost S_i from node i to d is computed. In increasing order of these initial costs, an order π of all nodes is defined starting from d itself. Also, the set of reasonable arcs $R = \{(i,j) \in A : S_i \geq S_j\}$ is set. Finally, for each origin node $o \in O$ and at each time increment $k = 1, ..., K$, the average of the demand rate generated during time increment k from origin o to destination d is set as:

$$\overline{\mathcal{D}}_o^k = \frac{\int_{(k-1)\Delta t}^{k\Delta t} \mathcal{D}_o(t)\, dt}{\Delta t}. \tag{2.11}$$

- **Proceedings on every Time Increment:** Starting with the previous settings, a default time increment $k = 0$ is established. Then, the following steps are executed repeatedly until the last time increment, $k = K$ or until stop contidition is satisfied. At every time increment k, the algorithm proceeds as follows:
  - **STEP 1: BACKWARD:** In this step, the expected minimum costs from nodes and from arcs to the destination are updated. First, in order for the algorithm to work, at each $t = 1, ..., K$, for each $a \in A$ and for each $i \in N$, the expected minimum cost of using arc a at t going from its starting node to d and the expected minimum cost of going from i to d at t, are set as $Z_a^t = \infty$ and $W_i^t = \infty$, respectively. Then, following the order π, for each node $j \in N$ and each incoming arc a to j that is reasonable, this is $a = (i,j) \in A_j^- \cap R$, and for each time increment $t = 1, ..., K$, W_i^t and Z_a^t are computed. If j is the destination, this is $j = d$, then the expected minimum cost of going from j to d at t is 0 and the expected minimum of going from i to j through $a = (i,j)$ at t is its total cost, this is $W_i^t = 0$ and $Z_a^t = C_a^t$, respectively. Otherwise, if j is not a destination and will not be reached after the period of analysis, this is $j \neq d$ and $t + \lfloor C_a^t \rfloor \leq K$, then

    $$W_j^t = -\frac{1}{\theta} \log \left(\sum_{b \in A_j^+} \exp\left(-\theta Z_b^{t+\lfloor C_a^t \rfloor}\right) \right) \tag{2.12}$$

    and

    $$Z_a^t = C_a^t + W_j^{t+\lfloor C_a^t \rfloor}. \tag{2.13}$$

  - **STEP 2: COMPUTING OF ASSIGNMENT FACTORS:** For each $a \in A$, the *Assignment Factor* of a at k, denoted as F_a^k, is computed. If a is reasonable, this is $a \in R$, then

    $$F_a^k = \exp\left(-\theta Z_a^k\right) \tag{2.14}$$

    and, otherwise, $F_a^k = 0$.

    This terms are part of the structure that ends up computing the assignment under the logit model over the expected minimum costs.

– **STEP 3: FORWARD:** In this step, the assignment of inflow rates is performed. Outflow rates and queue lengths are computed from these assignments. For a node i, there is flow to be assigned if:

$$\sum_{b \in A_i^-} G_b^k + \overline{\mathcal{D}}_i^k + \frac{\sum_{a \in A_i^+} L_a^{k+\phi_a-1}}{\Delta t} > 0, \tag{2.15}$$

where $\overline{\mathcal{D}}_i^k = 0$ if i is not an origin, and $k + \max_{a \in A_i^+}\{\phi_a\} \leq K$. That is, when there is non-zero outflow rates of incoming arcs to i at k, non-zero demand at i at k or a residual queue that needs to be unloaded and, simultaneously, the end of the outgoing arcs from i will be reached not later than the end of the time period. In this case, the algorithm proceeds as follows. If i is not an origin, then all the outflow rates of incoming arcs to i at k are aggregated to be assigned as inflow rates among the outgoing arcs from i and the inflow rate of $a \in A_i^+$ at k is given by:

$$E_a^k = \frac{F_a^k}{\sum_{a' \in A_i^+} F_{a'}^k} \sum_{b \in A_i^-} G_b^k. \tag{2.16}$$

Otherwise, if i is an origin, then the average demand rate generated at i during k is added to the aggregation of outflow rates and the inflow rate of $a \in A_i^+$ at k is given by:

$$E_a^k = \frac{F_a^k}{\sum_{a' \in A_i^+} F_{a'}^k} \left(\sum_{b \in A_i^-} G_b^k + \overline{\mathcal{D}}_i^k \right). \tag{2.17}$$

Next, once the inflow goes through a, it reaches its ending node at $k + \phi_a$. At this node, if there is residual queue from its previous time increment $k + \phi_a - 1$, the inflow rate of a at k queues behind those motorists that are waiting. Then, if the arc has not exceeded its queue unloading capacity Q_a, namely $\frac{L_a^{k+\phi_a-1}}{\Delta t} + E_a^k \leq Q_a$, then all motorists at the end of the arc a will be able to leave, from where the outflow rate of a at $k + \phi_a$ is given by:

$$G_a^{k+\phi_a} = \frac{L_a^{k+\phi_a-1}}{\Delta t} + E_a^k \tag{2.18}$$

and, as there is no queue, the queue length of a at $k + \phi_a$ is given by:

$$L_a^{k+\phi_a} = 0. \tag{2.19}$$

Otherwise, if the arc exceeds its queue unloading capacity Q_a, then some motorists will leave the arc at capacity and the outflow rate of a at $k + \phi_a$ is given by:

$$G_a^{k+\phi_a} = Q_a \tag{2.20}$$

and, as there will be a queue formed by the motorists that were not able to leave, the queue length of a at $k + \phi_a$ is given by:

$$L_a^{k+\phi_a} = L_a^{k+\phi_a-1} + \left(E_a^k - Q_a\right)\Delta t. \tag{2.21}$$

- **STEP 4: COSTS UPDATE:** For each $a \in A$, the total cost of using arc a entering it at k is updated. Queues of positive length that may be joined by the motorists once they have traveled the arc result in a delay because of the waiting time $\frac{L_a^{k+\phi_a}}{Q_a}$. The total cost of the arc a is then updated to

$$C_a^k = \phi_a + \frac{L_a^{k+\phi_a}}{Q_a}. \tag{2.22}$$

- **STEP 5: STOP CONDITION:** The algorithm stops when the last time increment $k = K$ is reached or there is no more flow rates to be assigned in later time increments:

$$\sum_{l=k+1}^{l=K}\left(\sum_{i\in N}\left(\sum_{b\in A_i^-} G_b^l + \overline{\mathcal{D}}_i^l\right) + \sum_{a\in A}\frac{L_a^{l+\phi_a-1}}{\Delta t}\right) = 0, \tag{2.23}$$

  (where $\overline{\mathcal{D}}_i^l = 0$ if i is not an origin). If none of these conditions is satisfied, the algorithm processes the next time increment $k+1$ starting again at **STEP 1**.

It is worth remarking that the MOSD-MDTA algorithm can be intialized with a non-empty transport network. Even though this particular feature is not further developed in this work, it opens interesting research opportunities.

Next, the algorithm is summarized as a pseudocode:

Algorithm 1 $(E, G, L) = \text{MOSD-MDTA}((N, A), O, d, \phi, Q, T, \Delta t, \mathcal{D}(\cdot), \theta)$

```
STEP 0: INITIALIZATION Technical settings
for k=1,...,K do
    STEP 1: BACKWARD
    for all i ∈ N, in the order given by π, do
        for all a ∈ A_i^- incoming arcs to i, do
            Compute expected minimum costs from i, through a, to d
        end for
        Compute expected minimum costs from i to d
    end for
    STEP 2: ASSIGNMENT FACTORS COMPUTING
    STEP 3: FORWARD
    for all i ∈ N do
        for all a ∈ A_i^+ outgoing arcs from i, do
```

```
            Compute inflow rate, outflow rate and queue length of arc a going to d
        end for
    end for
    STEP 4: COSTS UPDATES
    for all a ∈ A do
        Update cost of a according to the delays given by the current queue lengths
    end for
    STEP 5: STOP CONDITION
    if there are no more flow rates to assign then
        End
    end if
end for
```

In Appendix A, the complete pseudocodes of the *MOSD-MDTA algorithm* are presented.

2.5. How the *MOSD-MDTA algorithm* works: an illustrative example

To show in a simple way how the *MOSD-MDTA algorithm* works, in this section the development of its process for the case of a simple transport network is presented. Once introduced and explained all the inputs, it is graphically shown how the inflow rates and queue lengths evolve over all time increments. This process starts at the first moment a positive demand enters the network at an origin and ends at the last moment a vehicle reaches the destination.

Let us consider the transport network represented by the digraph (N, A) in Figure 2.1. For this analysis, the demand has nodes 1 and 2 as its origins and node 6 as its destination. On every arc a, the pair (ϕ_a, Q_a) shows its free flow travel time $[sec]$ and its queue unloading capacity $[veh/sec]$, respectively.

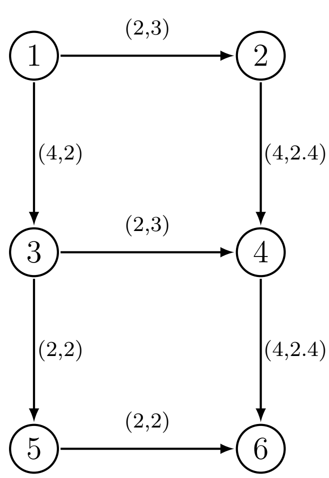

Figure 2.1: Network (N, A), with $(\phi_a,\ Q_a)$ on each arc a.

In Figure 2.2, the demand rate functions from each origin to the destination, $\mathcal{D}_1(t)$ and $\mathcal{D}_2(t)$ $[veh/sec]$, are shown, over continuous time t $[sec]$. The time period to analyze is $T = 18$ sec with a timestep size of $\Delta t = 1$ sec, from where there are $K = 18$ time increments. From origin nodes 1 and 2, at each time increment $k = 1, \ldots, 18$, the demand rate to be assigned, denoted as $\overline{\mathcal{D}}_1^k$ and $\overline{\mathcal{D}}_2^k$, respectively, is given by the average of the demand rate functions over the corresponding time interval $[(k-1)\Delta t, k\Delta t]$. Finally, as for the specifications for the logit model, the dispersion parameter used is θ =0.2 sec^{-1}.

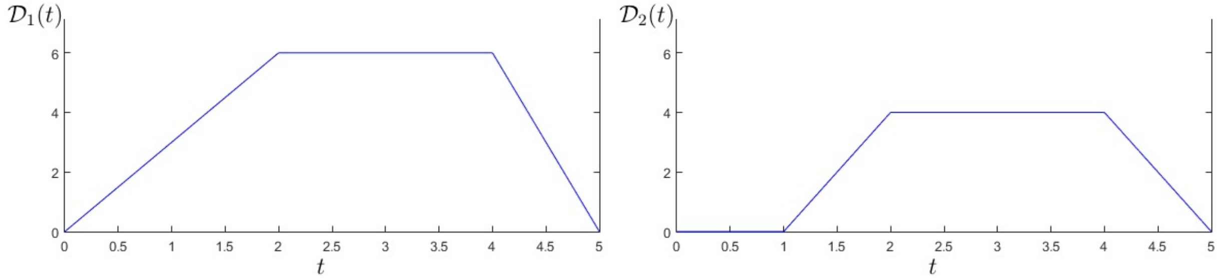

Figure 2.2: Demand rate functions from origin nodes 1 and 2, respectively.

Figure 2.4 and Figure 2.5 show, for all time increments, how the *MOSD-MDTA algorithm* assigns the inflow rates and, when they overpass the queue unloading capacity, how queues start and empty later on time. In Figure 2.3 the notations used in Figures 2.4 and 2.5 are presented. At each time increment k, given an origin node o: $a)$ represents a positive average demand rate $\overline{\mathcal{D}}_o^k$ generated at o; next, given an arc $a = (i, j)$, $b)$ and $c)$ represent how a positive inflow rate E_a^k (blue on the right or above the arc) starts travelling a and how it can possibly get behind a positive inflow rate that may have entered earlier, as well as the positive queue length L_a^k that the arc may have (red on the left or under the arc); Finally, at destination node 6, $d)$ represents a flow rate that has arrived destination, which can be an inflow rate arriving directly to it as an outflow rate or an unload of an existing queue.

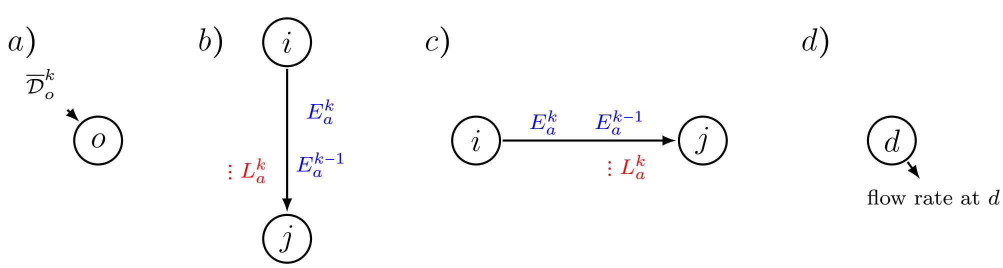

Figure 2.3: Notations used in Figures 2.4 and 2.5

Before showing the evolution of inflow rates and queue lengths of this simple example, it is important to note that, for simplicity, outflow rates are not shown, although, they can be implied. Given an arc $a = (i, j)$, the set A_j^+ of outgoing arcs of j and a time increment k, then, if j is not an origin, $G_a^k = \sum_{b \in A_j^+} E_b^k$ or, if j is an origin, $G_a^k = \sum_{b \in A_j^+} E_b^k - \overline{\mathcal{D}}_j^k$.

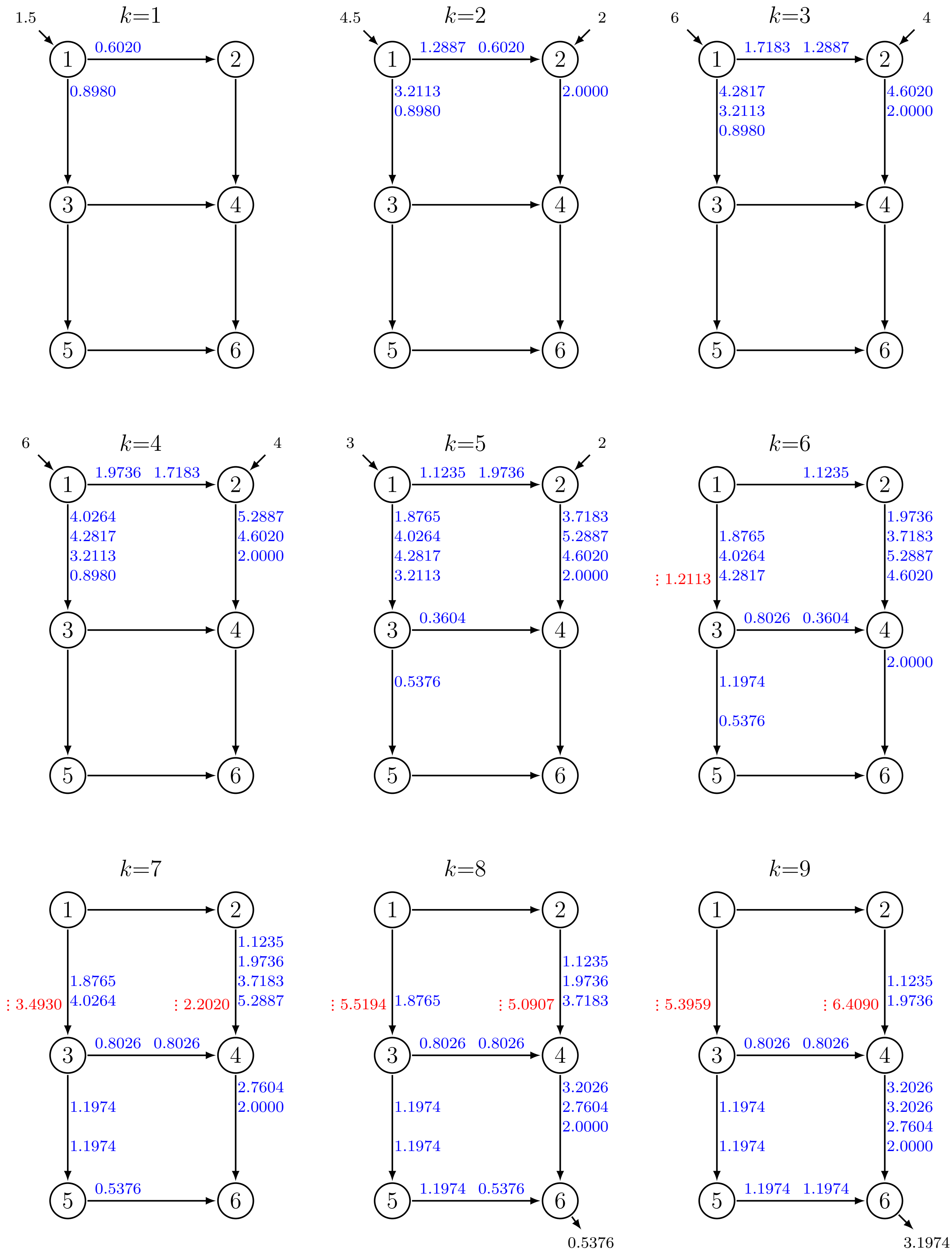

Figure 2.4: Evolution of *MOSD-MDTA algorithm* from $k = 1$ to $k = 9$.

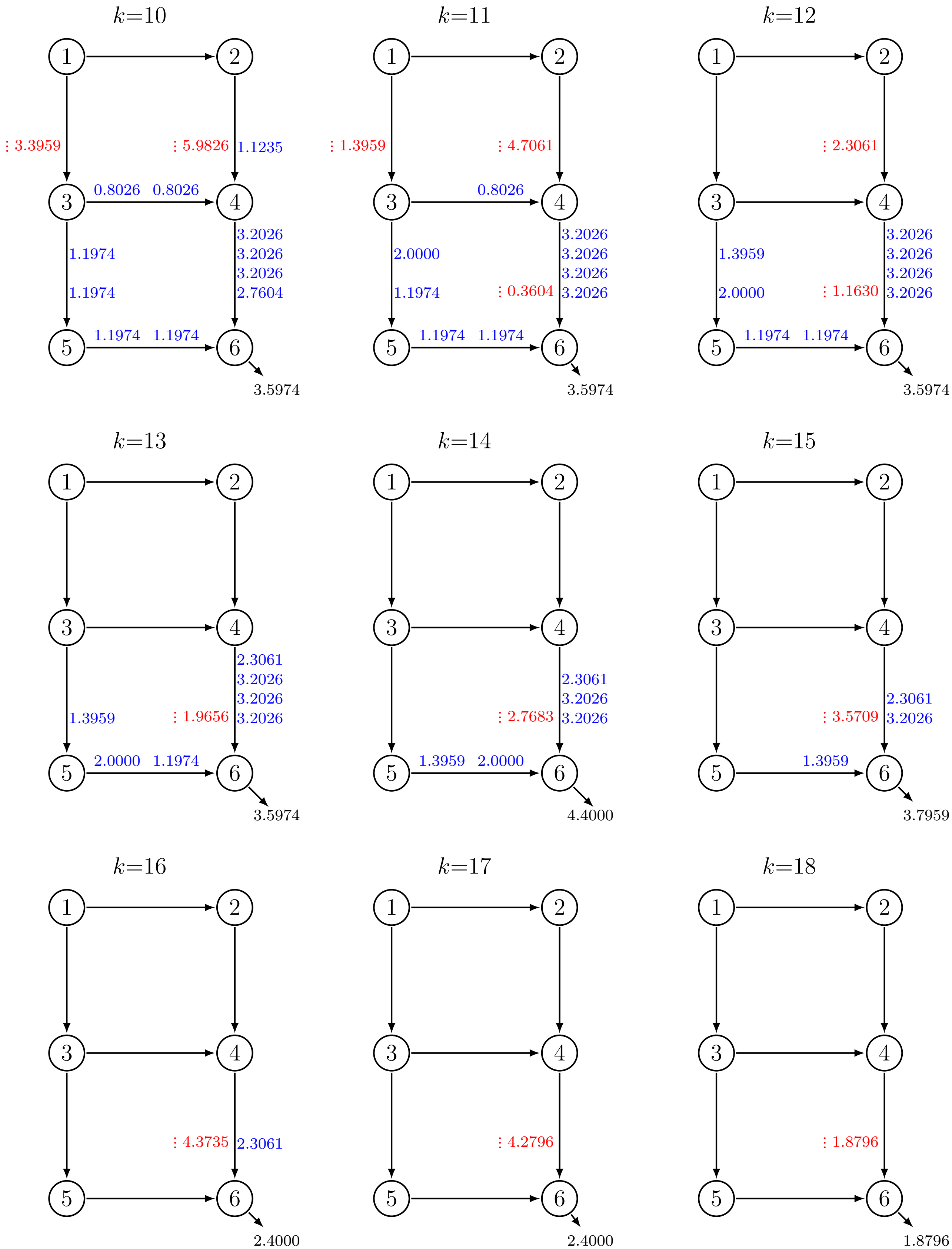

Figure 2.5: Evolution of *MOSD-MDTA algorithm* from $k = 10$ to $k = 18$.

2.6. Computational Implementation of the *MOSD-MDTA algorithm*

In order to be able to analyze different instances, the *MOSD-MDTA algorithm* has been implemented in MATLAB. One of the tested cases is given by the transport network represented by the digraph (N, A) in Figure 2.6, where nodes 1 and 2 are the origins, node 9 is the destination and, on each arc a, the pair (ϕ_a, Q_a) shows the free flow travel time $[sec]$ and the queue unloading capacity of the arc, respectively. The outputs of this case and a technical aspect regarding the computational implementation are later commented.

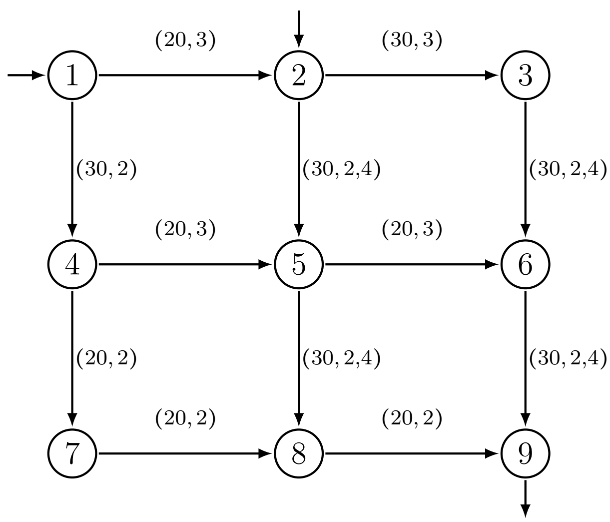

Figure 2.6: Network (N, A), with $(\phi_a,\, Q_a)$ on each arc a.

The demand rate functions from origin nodes 1 and 2 to the destination, $\mathcal{D}_1(t)$ and $\mathcal{D}_2(t)$, respectively, are shown in Figure 2.7.

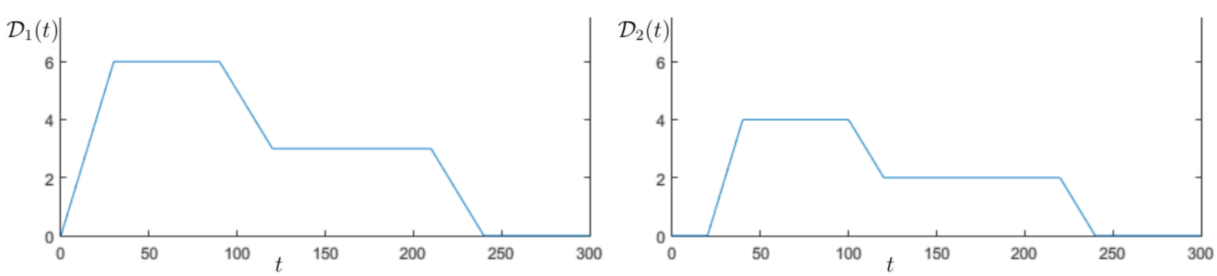

Figure 2.7: Demand rates from nodes 1 and 2, $\mathcal{D}_1(t)$ and $\mathcal{D}_2(t)$, respectively.

The computational implementation has been run over a period of $T = 600$ sec with timestep size of $\Delta t = 1$ sec and a dispersion parameter of θ =0.2 sec^{-1}. From T and Δt, there are $K = 600$ time increments. The algorithm takes the demand rate functions, $\mathcal{D}_1(t)$ and $\mathcal{D}_2(t)$, and considers their average, $\overline{\mathcal{D}}_1^k$ and $\overline{\mathcal{D}}_2^k$, respectively, over each time increment $k = 1, ..., K$. It took an average of 26.56 seconds to run the entire experiment.

2.6.1. The outputs: The behaviour of inflow rates, outflow rates and queue lengths

Next, the evolution over the 600 time increments of the three main outputs of the algorithm, inflow rates, outflow rates and queue lengths, are shown. Some insights and comments on the observed behaviour are later provided.

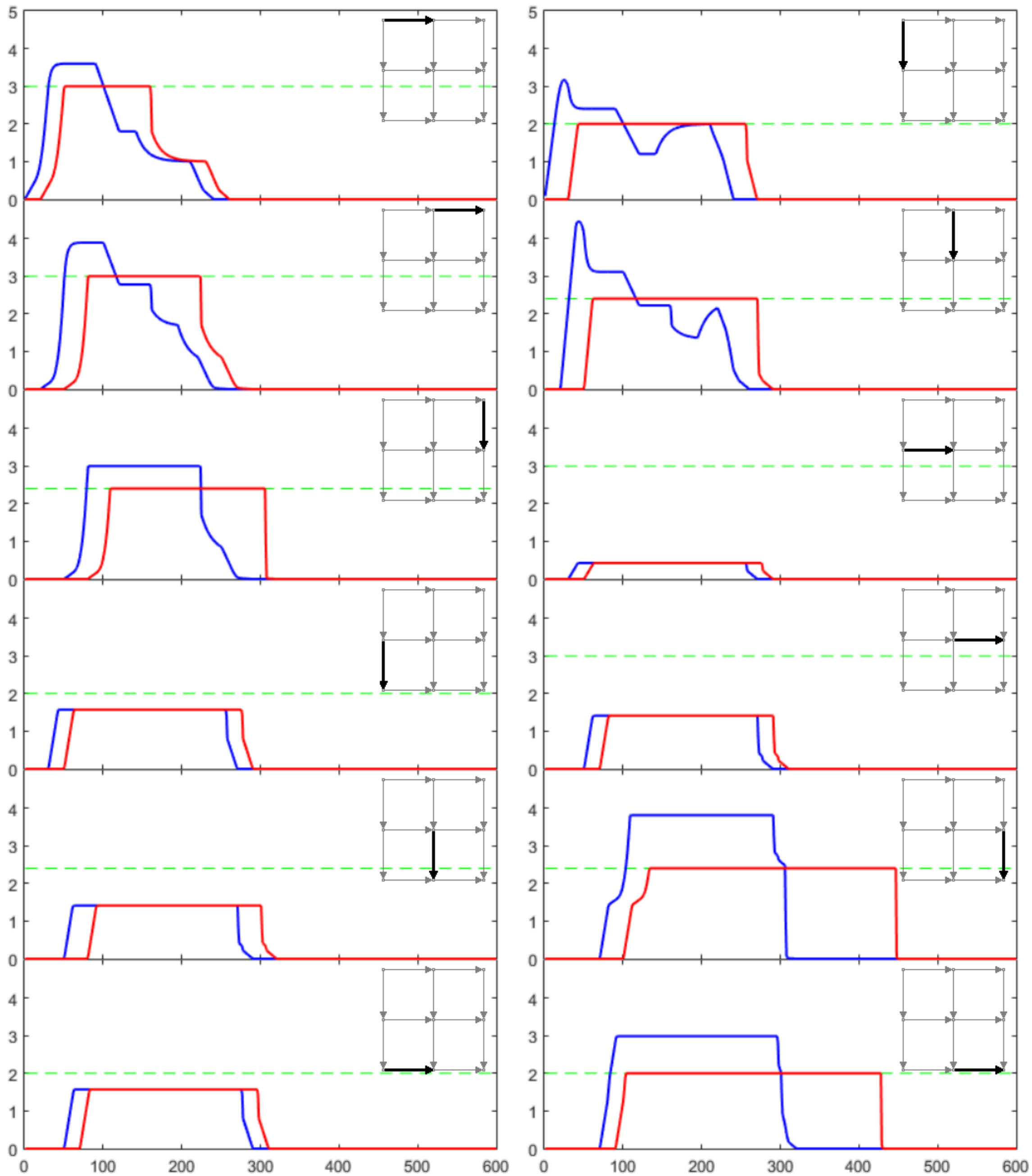

Figure 2.8: Evolution of inflow and outlfow rate of each arc of the network.

Figure 2.8 shows the evolution of the inflow rate (blue) and the outflow rate (red) of each arc of the network (indicated on the upper right corner of each plot), as well as its queue unloading capacity (dashed green). Given $a \in A$, and regarding the relationship between its inflow and outflow rates, there are two cases. First, if the inflow rate of a never overpasses Q_a, there is no congestion and no delay, thus, the cost of a is its free flow travel time ϕ_a, and inflow and outflow rate curves are identical except for a lag given by ϕ_a. Second, if the inflow rate of a overpasses Q_a, then a queue is formed due to congestion, thus, a delay is experienced and the cost of a is given by ϕ_a plus the delay because of the queue, and the outflow rate equals Q_a until the queue dissipates. Also, for each period of M consecutive time increments in which the inflow rate overpasses Q_a, after ϕ_a, outflow rate is equal to Q_a for longer than those M time increments, as the arc takes longer to empty. How this relates to the queue lengths evolution is further developed in the Figure 2.9 comments.

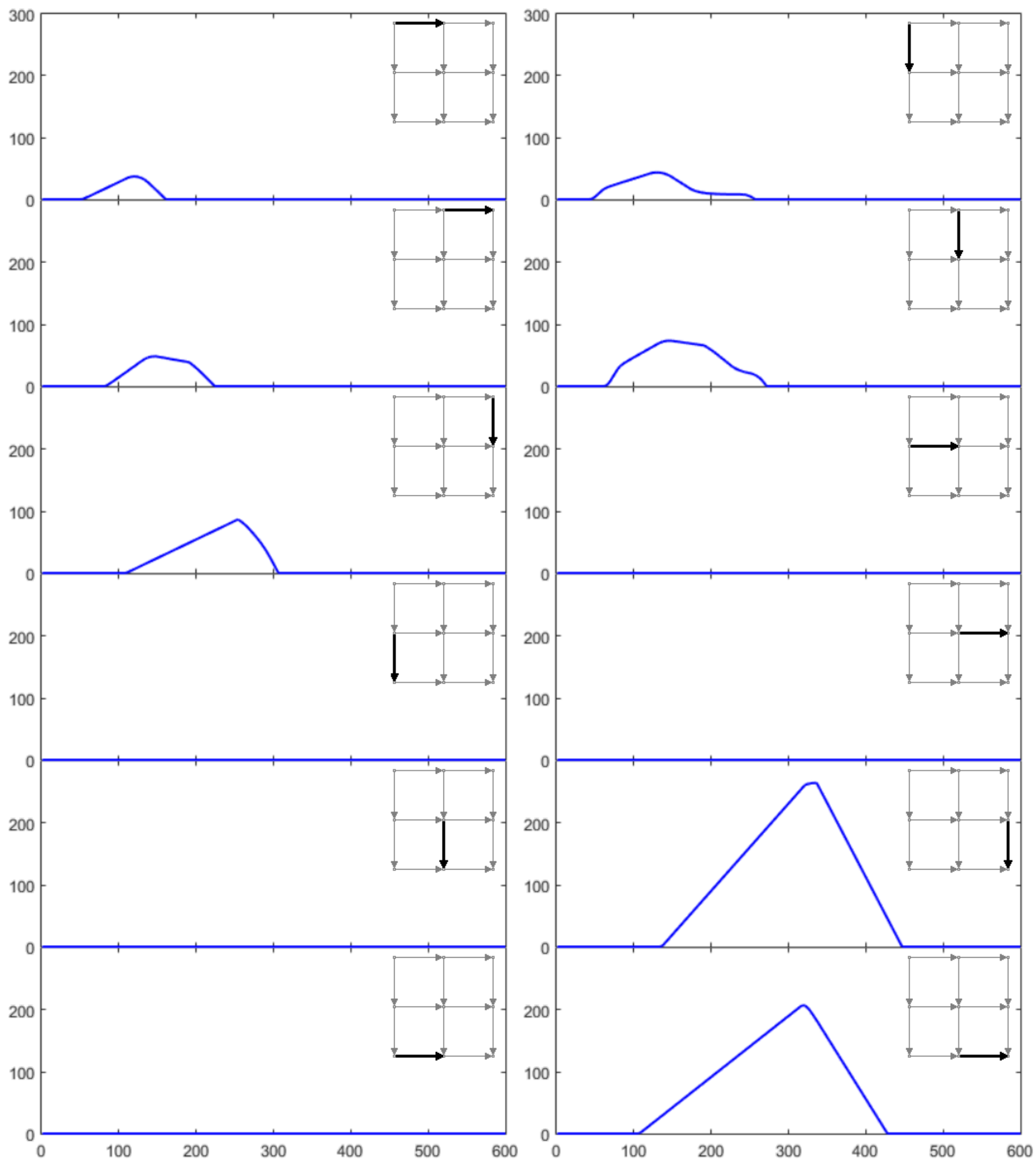

Figure 2.9: Evolution of the queue lengths for each arc of the network.

In order to explain the plots in Figure 2.9, let us consider the two cases of inflow and outflow behaviour described earlier for Figure 2.8. Given an arc a, when the first case happens in Figure 2.8, the queue length in Figure 2.9 is always 0, as there is no congestion and, thus, no queues. Otherwise, when the second case happens in Figure 2.8, in Figure 2.9 there is a non-negative queue length in the same time increments in wich the outflow rate is equal to Q_a, until the queue dissipates, as arcs always unload their queues at capacity.

It is worth noticing that, as the only destination is node 6, the total flow arriving to it is the aggregation of the demand that enters origin nodes 1 and 2. That is why the inflow rates assigned to arcs arriving to 6 are larger than in other arcs (Figure 2.8). This causes congestions, explaining why their queue lengths are considerably larger than the queue lengths of the other arcs (Figure 2.9).

2.6.2. A technical aspect: The timestep size Δt

In addition, it is worth to acknowledge of the importance of choosing a proper timestep value Δt to perform a stable and sufficiently accurate dynamic traffic assignment. This, in the sense of obtaining reliable outputs (inflow rates, outflow rates and queue lengths) and, simultaneously, not to be very expensive from the computational implementation standpoint. In fact, in the previous case study an already chosen timestep of one second has been used, and, in what follows, it is briefly discussed and shown why that value is appropriate to support the conclusions here presented.

Before the analysis, it is necessary to set a couple of conditions over Δt:

- The timestep size can not be greater than the minimum among the free flow travel times of the arcs of the network:

  That is to say, $\Delta t \leq \text{mín}\{\phi_a : a \in A\}$, as the method needs flow rates to arrive to the ending node of the arc on a later, and thus different, time increment. Without this condition, the flow rates departing from the starting node of those arcs a where $\phi_a < \Delta t$ will arrive to their ending node within the same time increment, something that, under the approach used in this work, has the same effect as having $\phi_a = 0$. A consequence of this may result in the existance of demand that enters and leaves the network within the same time increment. Both of this cases are non realistic phenomena and certainly, not desirable for the development of the method here presented;

- The free flow travel times of the arcs must be a multiple of the the timestep size:

  That is to say, for each arc $a \in A$, $\phi_a = m\Delta t$, for some $m \in \mathbb{N}$. Satisfying this condition allows all flow rates moving through the arcs to fit their travel times on an integer number of time increments.

Now, regarding the computational aspect, let us consider, again, the transport network represented by the digraph (N, A) shown in Figure 2.6, with the same time period of length $T = 600$ *sec* and the demand rate functions shown in Figure 2.7. This time, the implemented algorithm has been executed for five different timestep sizes, $\Delta t_1 = 10$ *sec*, $\Delta t_2 = 5$ *sec*, $\Delta t_3 = 1$ *sec*, $\Delta t_4 = 0.5$ *sec* and $\Delta t_5 = 0.1$ *sec*, which leads to five different number of time increments, $K_1 = 60$, $K_2 = 120$, $K_3 = 600$, $K_4 = 1200$, and $K_5 = 6000$, respectively.

There are important differences between the outputs of the cases Δt_1 and Δt_2. However, smaller differences between the outputs of the cases Δt_2 and Δt_3 were detected. For the cases Δt_3, Δt_4 and Δt_5, differences are practically undetectable. As the most important variations were detected on the outputs associated with the arcs that start at origin nodes (1 and 2) and, among this outputs, the inflow rates suffered the most notorious variations, these are used as illustrative examples to compare their behaviour under the different timestep sizes. Figure 2.10 and Figure 2.11 show the differences resulting from this different timestep sizes of the inflows associated with arcs starting at node 1 and node 2, respectively.

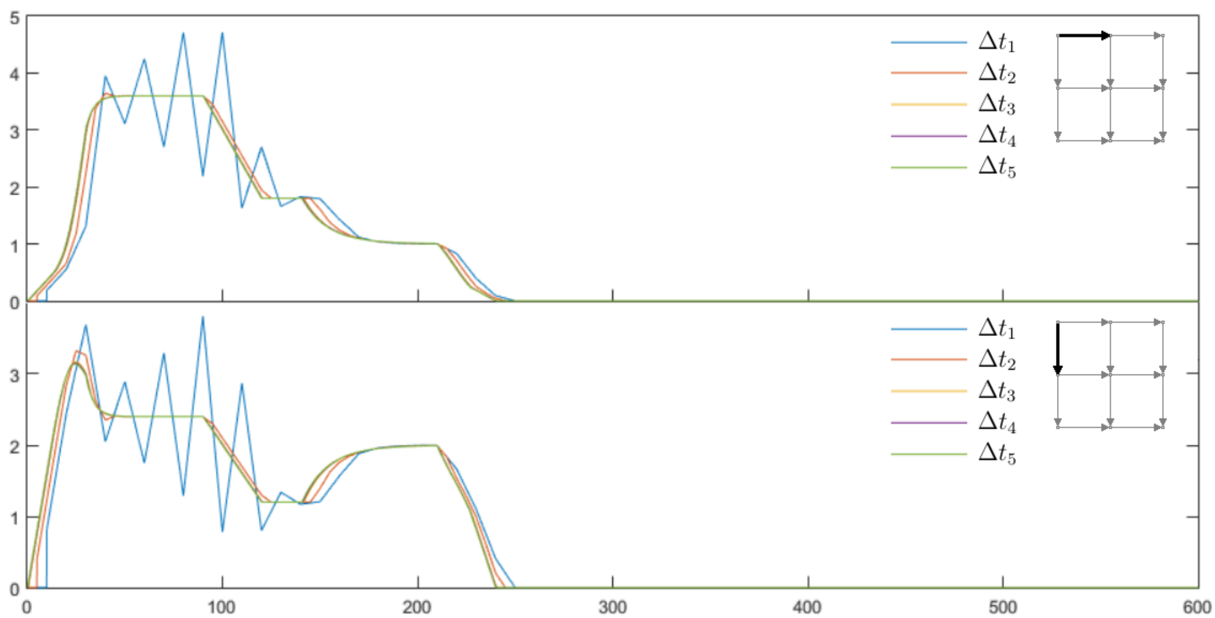

Figure 2.10: Inflow rates of arcs starting at origin node 1, for every $\Delta t_i, i = 1, ..., 5$,

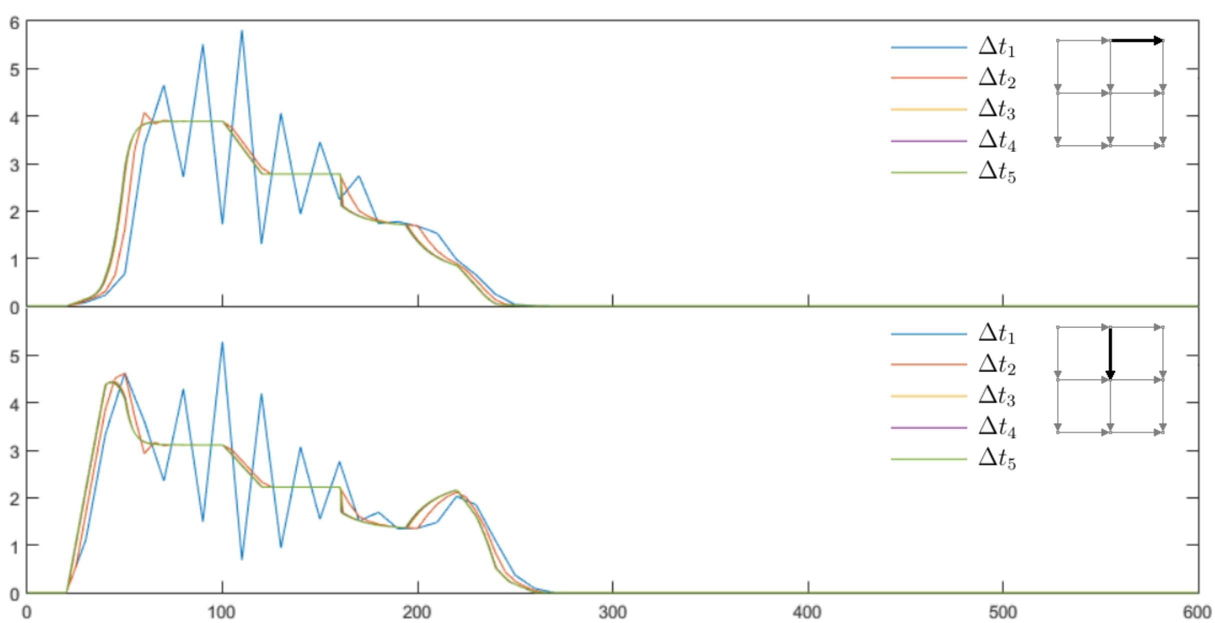

Figure 2.11: Inflow rates of arcs starting at origin node 2, for every $\Delta t_i, i = 1, ..., 5$.

In the case of the other outputs (outflows and queue lengths) for these and the rest of the arcs, even though the differences may not be as noticeable as the ones previously shown, the same conclusions can be made, as the Δt_1 case keeps having the biggest differences with Δt_2, then, the Δt_2 still has some differences with the Δt_3 case, but the variations between cases Δt_3, Δt_4 and Δt_5 are imperceptible, and, in fact, their curves overlap one another, as can be seen in Figure 2.10 and Figure 2.11, where the curve of the case Δt_5 overlaps the curves of the Δt_3 and Δt_4 cases.

In order to add another aspect for this analysis of the timestep size, the average execution time associated with each case $\Delta t_i, i = 1, ..., 5$, in $[sec]$, has been registered. Now, in Table 2.1, the average execution time for each case is shown.

Case	Number of time increments	Execution time
$\Delta t_1 = 10\ sec$	$K_1 = 60$	3.14
$\Delta t_2 = 5\ sec$	$K_2 = 120$	5.27
$\Delta t_3 = 1\ sec$	$K_3 = 600$	26.56
$\Delta t_4 = 0.5\ sec$	$K_4 = 1200$	101.21
$\Delta t_5 = 0.1\ sec$	$K_5 = 6000$	2428.98

Table 2.1: Execution time of every case of $\Delta t_i, i = 1, ..., 5$, in seconds.

From this results, it can be concluded that a timestep size of 1 second is small enough to ensure that smaller timestep sizes will result in practically identical outputs and, simultaneously, its running time is acceptably expensive. This, in the sense that it is considerable cheaper compared with smaller timestep sizes that will have equally significant results.

In this chapter, a novel approach to obtain a Dynamic Traffic Assignment for the general case of multiple origins and a single destination has been introduced, generating the MOSD MDTA model. This model integrates notions of stochasticity from a Markovian point of view with a logit route choice model and a deterministic punctual queueing traffic model. A solution method for this case is also highlighted, the MOSD-MDTA algorithm, that solves the assignment problem associated with the MOSD MDTA model by providing an approximation of the solution over a discretization of the time period. Its computational implementation, together with some testing and a sensibility analysis have also been provided. An important feature of the model presented in this chapter, given its embedded arc-based approach, is the fact that it is not needed to impose conditions for route interactions allowing to work with overlapping routes, and consequently the assumption of independence between routes is not necessary. In addition, enumeration of routes is not required for the functioning of the algorithms, which is usually a drawback of the route-based approaches.

The results presented in this chapter of the doctoral thesis will be submitted to Transportation Research Part C, Special Issue in Dynamic Transportation Network modelling: Emerging Technologies, Data Analytics and Methodology Innovations. It is important to mention that this special issue is associated with the DTA 2020 Conference, rescheduled for July 2021, because of the pandemia, in which a paper regarding this results has already been accepted and scheduled for that conference, which turns out to be one of the most prestigious events in topics related to DTA.

In the following chapter, the approach taken in this work is further developed and integrated with new ways to tackle a more general problem, the MDTA for the case of general transport networks.

Chapter 3

The Markovian Dynamic Traffic Assignment Model for the Multiple Origins and Multiple Destinations General Case

The MDTA model for the multiple origins and a single destination general case presented in Chapter 2 comes as a first, but fundamental, result of the integration of the markovian notions and the formulation approaches further referenced in Section 1.3 of Chapter 1. Even though it is already an important contribution by itself, it is the foundation for a model that covers the case for more general transport networks: the Markovian Dynamic Traffic Assignment model for the multiple origins and multiple destinations general case. This model is the core result of this doctoral thesis.

In this chapter, the results obtained and presented in Chapter 2, developed for the case when motorists enter the system through different origins but are heading to the same single destination, will be extended and adapted. This is relevant in order to to generate results that cover the general case of motorists with different destinations entering the transport network from different origins. The motivation behind the approach considered in the developments of this work stage remains the same as in Chapter 2. Again, the markovian aspects are integrated through the definition of an arc-based choice model rather than a route-based choice model, with the adaptation of the deterministic punctual queueing model to represent the behaviour of the interactions within each arc of the transport network. The main difference is how each part of the whole model is built, now considering the existence of multiple types of motorists within an arc, which requires a detailed treatment at each level of the modelling process.

Chapter 3 starts by explaining the reasoning that allows using the results for the "many-to-one" general case to accomplish the results for the "many-to-many" general case. Then, a new definition of the reasonability of arcs is introduced, considering for this case the fact that there are multiple destinations. Later, the building process of the MDTA model for the multiple origins and multiple destinations general case is presented and explained. Then, a solution method for the model, an example of a simple transport network to show how it works and, finally, results regarding the computational implementation are all provided.

3.1. From the “many-to-one” to the “many-to-many” case: Different types of motorists within an arc

At this stage, the challenge is to be able to properly represent interactions within an arc to address the “many-to-many” general case. Such interactions are much more difficult to handle than those of the “many-to-one” general case, where there was an inflow rate, an outflow rate and a queue length that needed to be correctly related. In the “many-to-many” case there are multiple inflow rates, multiple outlfow rates and a queue length comprising motorists going to different destinations. These, in an aggregated way, need to satisfy the same conditions that were fulfilled in the modelling process presented in Chapter 2.

The fundamental aspect now is to correctly represent the interactions of the heterogeneous flow rate traveling each arc of the transport network in the model. The technical modelling aspects will be properly developed later on this chapter but, the following idea helps understanding the intuition of how the within-arc behaviours are approached in the “many-to-many” case. Given a transport network, let us consider a set of destinations D and an arc $a = (i, j)$ that contains motorists going to each destination $d \in D$. Now, at a given time, the inflow rate going to each destination d that has been assigned to arc a, after traversing a, those vehicles will reach the end of the arc. Once there, if there is no congestion, namely, if the queue unloading capacity of the arc is enough to unload the aggregation of motorists going to all destinations, the outflow rate going to each destination d will become equal to the inflow rate to d that entered earlier. In the congested case, on the other hand, the inflow rates will generate a queue and the outflow rate to each destination d will be computed by splitting the queue unloading capacity proportionally according to the number of motorists going to each destination waiting to leave the arc.

Now, to illustrate in a simple way the intuition of the idea here presented, let us consider Figure 3.1. Given an arc (i, j), with queue unloading capacity Q, where two inflow rates with different destinations, A and B, have entered, there are two possible cases once the motorists have reached the end of the arc. If there is no congestion, namely, if $A + B \leq Q$, then the outflow rates going to each destination will be equal to the inflow rates that entered earlier. Otherwise, if there is congestion, namely, $A + B > Q$, due to the fact that not all motorists will be able to leave the arc, the queue unloading capacity will split proportionally as outflow rates while the motorists that did not leave the arc will join a single queue.

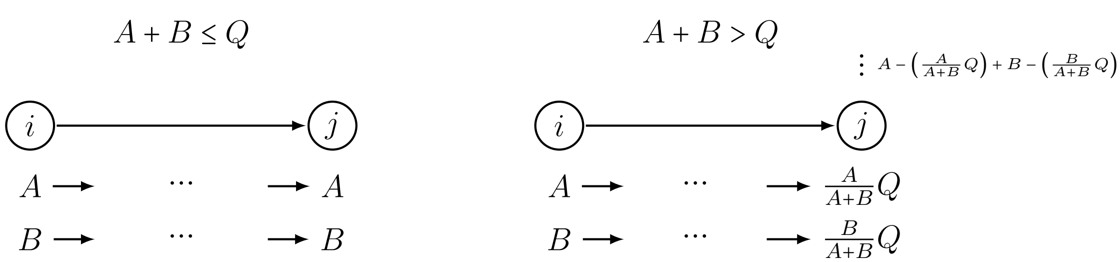

Figure 3.1: Cases when the arc (i, j) is uncongested and when congested.

3.2. Reasonable Arcs towards a particular destination

Before covering the details of the general model, it is first recalled the definition of reasonable arc introduced in Section 2.2 of Chapter 2. It can be noticed that, now that it is allowed the existence of multiple destinations in the transport network, a new take on this definition is required in order to be able to proceed to the formulation of the model for this case.

Let us consider the context of the problem of a general transport network with multiple origins and multiple destinations, the latter represented by the set D. Given a destination $d \in D$, for an O-D pair (o, d), the arc $a = (i, j)$ is a *reasonable arc towards destination* d if the minimum cost of going from node j to destination node d is less or equal to the minimum cost of going from node i to d. Again, as in Section 2.2 of Chapter 2, this concept represents the idea that a motorist will not use arcs that take him/her farther from his/her destination, if routes of minimum cost are meant to be used to reach d.

An important difference that is worth pointing out with respect to the previous version of the definition is that in this general "many-to-many" case an arc can be, simultaneously, reasonable towards some destinations but not reasonable towards others. As before, even though this definition is not reflected directly in the formulations, it is fundamental for the construction of the solution algorithm, as in the previous "many-to-one" case.

3.3. Building the MDTA model for the multiple origins and multiple destinations case

The development of the MDTA model for the multiple origins and multiple destinations general case is performed according to the structure for dynamic traffic assignment models established by Addison and Heydecker [2]. This, consistently with what was exposed in Chapter 2, where the analysis was made according to a single destination case. In what follows in the current section, the demand profile, the traffic model and the arc-choice model, that serves as the route-choice model, will all be explained and developed under these new considerations.

Let us consider a transport network with an underlying digraph (N, A), where N is the set of nodes, A is the set of arcs and, for each $i \in N$, A_i^+ and A_i^- are the sets of outgoing arcs from i and incoming arcs to i, respectively. For each arc $a \in A$, its free flow travel time ϕ_a and its queue unloading capacity Q_a are parameters assumed known. Next, regarding the characteristics of the demand, there are a set of origin nodes $O \subseteq N$, a set of destination nodes $D \subseteq N$, a set of O-D pairs $OD \subseteq O \times D$ and temporal dependent demand rate functions from the origin to the destination of every O-D pair $(o, d) \in OD$, $\mathcal{D}_{(o,d)}(\cdot)$. The temporal horizon, represented by the time interval $[0, T]$, is also known. Next, the three main structures of the DTA model here presented will be developed according to these definitions and notations.

3.3.1. The Demand Profile

For each O-D pair $(o, d) \in OD$, the time-dependent demand rate function from the origin node o to the destination node d, $\mathcal{D}_{od}(\cdot)$, is given as it is considered to be exogenous. In an aggregated way, these functions determine *the demand profile*, which is the first of the three parts of the model.

3.3.2. The Traffic Model

For the second structure, *the traffic model*, the Deterministic Punctual Queueing model is, again, adapted to represent the behaviour of the traffic within each arc. Given that in this case there are multiple destinations, it is necessary to identify independently each inflow rate, each outflow rate and the queue composition. Simultaneously, it is important to represent their aggregated interactions, which requires a detailed interpretation of the considerations associated with the way to handle multiple types of motorists within a single arc. This, as the queue unloading capacity of an arc applies over the total inflow rate, regardless of the destinations. Thus, whenever the aggregation of the inflow rates corresponding to all destinations overpasses the capacity, a queue composed of motorists heading to all destinations with positive inflow rate will be generated, as not all of them will be able to leave the arc.

For each destination $d \in D$, for each arc $a \in A$ and at each time $t \in [0, T]$, the inflow rate and outflow rate of arc a going to destination d at time t are denoted as $E_{ad}(t)$ and $G_{ad}(t)$, respectively. The number of motorists with destination d in a queue on arc a at time t is denoted as $L_{ad}(t)$, that for simplicity will be referred at as the *queue length going to d of a at t*. Considering this, now the expression for the queue length of arc a at time t is given by $\sum_{d' \in D} L_{ad'}(t)$. Thus, the way in which the outflow rate of an arc a behaves, as briefly commented earlier in Section 3.1 is as follows. In the uncongested case, which means that no queue is observed and the total inflow rate does not overpasses Q_a, the inflow rate going to each destination that enters the arc at t exits an uncongested arc after ϕ_a. This means that the outflow rate at $t + \phi_a$ going to each destination is equal to its respective inflow rate at t. In the other case, where traffic congestion occurs, the unload of flow rate that exits a at $t + \phi_a$ is given by all the possible unload that the arc can perform, which is Q_a. This value will be split proportionally as outflow rates going to each destination, according to the composition of the different types of motorists going to each destination at that moment in the queue.

The described behaviours, for each destination node $d \in D$, for each arc $a \in A$, at each time $t \in [\phi_a, T + \phi_a]$ and considering the introduced notation, can be analytically expressed as:

$$G_{ad}(t) = \begin{cases} E_{ad}(t - \phi_a), & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ \frac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise}, \end{cases} \tag{3.1}$$

$$\frac{dL_{ad}}{dt} = \begin{cases} 0, & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ E_{ad}(t - \phi_a) - \frac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise}. \end{cases} \tag{3.2}$$

Regarding the construction of the total travel costs, for a given arc at a given time, any motorist in a queue, regardless of his/her destination, experiences the same delay as the other motorists in the queue. Therefore, a definition of delay associated with a particular destination is not necessary. Having said that, for each arc $a \in A$ and at each time $t \in [0, T]$, the total travel cost of using the arc a if the entering time is t, denoted as $C_a(t)$, is given by the free flow travel time of a plus the delay due to the waiting time in the queue. Analytically, this can be expressed as:

$$C_a(t) = \phi_a + \frac{\sum\limits_{d' \in D} L_{ad'}(t + \phi_a)}{Q_a}. \tag{3.3}$$

Now the approach addresses multiple destinations rather than a single one, however it still applies the same arc-based construction of the formulations that was used earlier in Chapter 2. This allows preserving the defining property of not needing any kind of conditions regarding the routes interactions, from where overlapping routes are allowed and the independence of their costs is not needed.

3.3.3. The Arc-Choice Model

Even though, as it is usually approached in the literature, the choice of routes to move through the network decided by motorists is built as a route-choice model, here it is actually constructed as an arc-choice model, given the arc-based approach developed in this doctoral thesis. From this, it is the recursive choices of arcs that end up forming the route that a motorist travels through to go to his/her destination. This arc-choice model is the last of the three structures developed as part of the MDTA model for the multiple origins and multiple destinations general case. This model is again adapted from the static flow assignment embedded in the *MTE* concept [7], which is here extended to a dynamic traffic assignment context, as it was first approached in Chapter 2. The difference in this case, however, is that different destinations have to be considered, as the expected minimum cost perceived by a motorist is computed according to his/her destination.

For each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, the expected minimum cost of going from i to d by choosing arc a, entering it at t, denoted $Z_{ad}(t)$, is computed as:

$$Z_{ad}(t) = C_a(t) - \frac{1}{\theta} \ln \left(\sum_{b \in A_j^+} \exp\left(-\theta \left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right) \right). \tag{3.4}$$

Next, for each destination node $d \in D$, for each node $i \in N$ and at each time $t \in [0, T]$, the expected minimum cost of going from node i to destination d, starting at t, is given by:

$$W_{id}(t) = -\frac{1}{\theta} \ln \left(\sum_{a=(i,j) \in A_i^+} \exp\left(-\theta \left(C_a(t) + W_{jd}\left(t + C_a\left(t\right)\right)\right)\right) \right). \tag{3.5}$$

Therefore, from expressions (3.4) and (3.5), for each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, it holds that

$$Z_{ad}(t) = C_a(t) + W_{jd}\left(t + C_a(t)\right) \tag{3.6}$$

and

$$W_{id}(t) = -\frac{1}{\theta} \ln\left(\sum_{a \in A_i^+} \exp\left(-\theta Z_{ad}(t)\right) \right). \tag{3.7}$$

The logit model used in this approach assigns the inflow rate going to each destination from a given node according to the expected costs of using its outgoing arcs to go to the destination. Now, extending what was addressed in Subsection 2.3.3 of Chapter 2 for the previously studied case, the inflow rate going to a destination d assigned from a given node can have two sources: the aggregated outflow rates going to d from incoming arcs to that node or that and, simultaneously, the demand rate going to d originated at that node.

Analytically, for each destination node $d \in D$, there are two cases. First, for each node $i \in N$ such that $(i, d) \notin OD$, that is, for nodes that are not origins of destination d, for each arc $a = (i, j) \in A_i^+$ and at each time $t \in [0, T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \frac{\exp\left(-\theta\left(Z_{ad}\left(t + C_a\left(t\right)\right)\right)\right)}{\sum\limits_{b \in A_i^+} \exp\left(-\theta\left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right)} \sum_{b \in A_i^-} G_{bd}(t). \tag{3.8}$$

Second, for each $o \in O$ such that $(o, d) \in OD$, that is, for nodes that act as origins for destination d, for each arc $a = (o, j) \in A_o^+$ and at each time $t \in [0, T]$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \frac{\exp\left(-\theta\left(Z_{ad}\left(t + C_a\left(t\right)\right)\right)\right)}{\sum\limits_{b \in A_o^+} \exp\left(-\theta\left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right)} \left(\sum_{b \in A_o^-} G_{bd}(t) + \mathcal{D}_{(o,d)}(t) \right). \tag{3.9}$$

Just like the arc-choice model developed for the previous case in Subsection 2.3.3 of Chapter 2, this one preserves the property that results from the arc-based approach of not assuming any kind of conditions regarding routes interaction. From this, the model allows working with overlapping routes and does not requires independence on their costs.

Thus, the three fundamental structures of a DTA model have been properly constructed and established under the proposed approach developed in this doctoral thesis. This framework defines the Markovian Dynamic Traffic Assignment model for the multiple origins and multiple destinations general case. Next, in Section 3.4 a solution method for this model is presented.

3.4. The MOMD-MDTA Algorithm

In Section 2.4 of Chapter 2, the MOSD-MDTA algorithm was presented as a solution method for the MDTA problem regarding the multiple origins and a single destination general case. In this chapter it is developed the *MOMD-MDTA algorithm* for the multiple origins and multiple destinations general case. As the previous one, this algorithm works over a discretization of the analyzed time period. As the MOSD-MDTA algorithm, this version also integrates the idea of the backward and forward steps of *Dial's algorithm* [22], but now considering multiple types of flow rates, one for each destination. As in the previous method, these steps are repeated in every time increment of the resulting time discretization.

The inputs of the MOMD-MDTA algorithm are: the digraph (N, A) associated with the transport network, the set of origins $O \subseteq N$, the set of destinations $D \subseteq N$, the set of O-D pairs $OD \subseteq O \times D$, the free flow travel time ϕ_a and the queue unloading capacity Q_a of every arc $a \in A$, aggregated as vectors ϕ and Q, respectively. The number of time intervals of the discretization, K, can be obtained, as $K = T/\Delta t$, where T is the length of the period, and Δt is the size of the timestep of the discretization, both known. Then, given $k \in \{1, ..., K\}$, the interval $[(k-1)\Delta t, k\Delta t]$ corresponds to the *time increment* k. The time-dependent rate demand functions from the origin to the destination of each O-D pair $(o, d) \in OD$, $\mathcal{D}_{(o,d)}(\cdot)$ are also known, which can be written in a vectorial form as the function $\mathcal{D}(\cdot)$. As for the logit model specifications, the dispersion parameter θ is also provided.

The outputs are three sets of matrices of size $K \times |A|$, $\mathcal{E} = \left\{E_d = \left(E^k_{ad}\right)_{a \in A, k=1,...,K} : d \in D\right\}$, $\mathcal{G} = \left\{G_d = \left(G^k_{ad}\right)_{a \in A, k=1,...,K} : d \in D\right\}$ and $\mathcal{L} = \left\{L_d = \left(L^k_{ad}\right)_{a \in A, k=1,...,K} : d \in D\right\}$. Here, given $a \in A$, $d \in D$ and $k \in \{1, ..., K\}$, E^k_{ad} and G^k_{ad} are the inflow rate and outflow rate of arc a of motorists going to destination d at time increment k, respectively, and L^k_{ad} is the queue length going to destination d of arc a at time increment k.

The algorithm proceeds as follows:

- **Initial Settings:** Parameters, sets, and initial values to the structures that change over every time increment, are all set.
  - **STEP 0: INITIALIZATION:** For each node $i \in N$, the set of incoming arcs to i and the set of outgoing arcs from i are set as $A^-_i = \{(j, i) \in A : j \in N\}$ and $A^+_i = \{(i, j) \in A : j \in N\}$, respectively, as well as the number of time increments, $K = T/\Delta t$. As the network starts empty, for each destination node $d \in D$, for each arc $a \in A$ and at each time increment $k = 1, ..., K$, the inflow rate of a going to d at k, the outflow rate of a going to d at k and the queue length going to d of a at k are set as $E^k_{ad} = 0$, $G^k_{ad} = 0$ and $L^k_{ad} = 0$, respectively. A default time increment 0 is set to define $L^0_{ad} = 0$. For each $a \in A$ and at each $k = 1, ..., K$, as a is uncongested, the total cost of using arc a is initialized equal to its free flow travel time ϕ_a, this is $C^k_a = \phi_a$. For each destination node $d \in D$ and for each $i \in N$, the initial minimum cost S_{id} from node i to d is computed and set. According to an increasing order of these values, an order π_d of all nodes, starting from d itself, is set, as well as the set of reasonable arcs towards d, given by $R_d = \{(i, j) \in A : S_{id} \geq S_{jd}\}$. Finally, for each O-D $(o, d) \in OD$ and at each time increment $k = 1, ..., K$, the average demand rate generated during time increment k from the origin node o to the destination

node d is set as:

$$\overline{\mathcal{D}}^k_{(o,d)} = \frac{\int_{(k-1)\Delta t}^{k\Delta t} \mathcal{D}_{(o,d)}(t)\, dt}{\Delta t}. \tag{3.10}$$

- **Time Increment update:** Considering the initial settings, the following steps are executed from $k = 1$ to $k = K$ or until the stop condition is satisfied. At every time increment k, the algorithm proceeds as follows:
  - **STEP 1: BACKWARD:** In this step, the expected minimum costs from nodes and from arcs to each destination are updated. By default, at each time increment $t = 1, ..., K$, for each destination node $d \in D$, for each arc $a \in A$ and for each node $i \in N$, the expected minimum cost of using a at t, going from its starting node, to d, and the expected minimum cost of going from i to d at t, are set as $Z^t_{ad} = \infty$ and $W^t_{id} = \infty$, respectively. Next, for each destination node $d \in D$ and for each node $j \in N$, in the order given by π_d, for each incoming arc a to j that is reasonable towards d, namely $a = (i,j) \in A^-_j \bigcap R_d$, and for each time increment $t = 1, ..., K$, Z^t_{ad} and W^t_{id} are computed. If j is the destination, namely $j = d$, then the expected minimum cost of going from j to d at t is 0 and the expected minimum cost of going from i to j through $a = (i,j)$ at t is its total cost, this is $W^t_{jd} = 0$ and $Z^t_{ad} = C^t_a$, respectively. Otherwise, if j is not a destination and will not be reached after the period of analysis, that is to say $j \neq d$ and $t + \lfloor C^t_a \rfloor \leq K$, then the expected minimum cost of going from j to destination d at t is updated by:

    $$W^t_{jd} = -\frac{1}{\theta} \log\left(\sum_{b \in A^+_j} \exp\left(-\theta Z^{t+\lfloor C^t_a \rfloor}_{bd}\right)\right). \tag{3.11}$$

    Then, the expected minimum cost of going from i to d by using a at t is updated as follows:

    $$Z^t_{ad} = C^t_a + W^{t+\lfloor C^t_a \rfloor}_{jd}. \tag{3.12}$$

  - **STEP 2: COMPUTING OF ASSIGNMENT FACTORS:** For each destination node $d \in D$ and for each arc $a \in A$, the *Assignment Factor of arc a to destination d at k*, denoted as F^k_{ad}, is computed. If a is reasonable towards d, that is to say $a \in R_d$, then

    $$F^k_{ad} = \exp\left(-\theta Z^k_{ad}\right) \tag{3.13}$$

    and, otherwise, $F^k_{ad} = 0$.

    As in the previous version of the algorithm, this terms are used to compute the assignment under the logit model over the expected minimum costs.
  - **STEP 3: FORWARD:** In this step, the assignment of inflow rates going to each destination is performed and, with this, the computation of outflow rates to each

destination and queue lengths to each destination is also conducted. For each node i, it is first explored if a flow rate needs to be assigned. This flow rate can come from outflow rates to any destination of incoming arcs to i at k, from demand rate going to any destination generated at i at k or from residual queues that need to be unloaded. This will happen if

$$\sum_{d \in D} \left(\sum_{b \in A_i^-} G_{bd}^k + \overline{\mathcal{D}}_{(i,d)}^k + \frac{\sum_{a \in A_i^+} L_{ad}^{k+\phi_a-1}}{\Delta t} \right) > 0, \tag{3.14}$$

where $\overline{\mathcal{D}}_{(i,d)}^k = 0$ if $(i,d) \notin OD$. Simultaneously, the end of the outgoing arcs from i needs to be reached not later than the end of the time period under analysis, that is to say $k + \max_{a \in A_i^+} \{\phi_a\} \leq K$. If the previous conditions are fulfilled, the algorithm proceeds as follows. If i is not an origin, that is $i \notin O$, then all the outflow rates going to d from incoming arcs to i at k are aggregated to be assigned as inflow rates going to d among the outgoing arcs from i. Thus, the inflow rate of a going to d at k is given by:

$$E_{ad}^k = \frac{F_{ad}^k}{\sum_{a' \in A_i^+} F_{a'd}^k} \sum_{b \in A_i^-} G_{bd}^k. \tag{3.15}$$

Otherwise, if i is an origin, namely $i \in O$, then the average demand rate going to d generated at i during k is added to the aggregation of outflow rates going to d in order to perform the assignment. Then, the inflow rate of a going to d at k is given by:

$$E_{ad}^k = \frac{F_{ad}^k}{\sum_{a' \in A_i^+} F_{a'd}^k} \left(\sum_{bd \in A_i^-} G_{bd}^k + \overline{\mathcal{D}}_{(i,d)}^k \right). \tag{3.16}$$

Next, once the inflow rate going to d has traversed arc a, it will reach its ending node at $k + \phi_a$. There, if a residual queue is observed from its previous time increment $k + \phi_a - 1$, the inflow rate from a going to d at k will first get behind those motorists that are waiting to leave the arc. This, aggregately with the other inflow rates of a going to other destinations that also entered at k. Then, if the arc has not exceeded its queue unloading capacity Q_a, which happens if

$$\sum_{d \in D} \left(\frac{L_{ad}^{k+\phi_a-1}}{\Delta t} + E_{ad}^k \right) \leq Q_a, \tag{3.17}$$

then all motorists at the end of the arc a will be able to leave. This means that the outflow rate of a going to d at $k + \phi_a$ is given by:

$$G_{ad}^{k+\phi_a} = \frac{L_{ad}^{k+\phi_a-1}}{\Delta t} + E_{ad}^{k}. \tag{3.18}$$

Then, as all motorists are able to leave the arc, there is no congestion, thus, there is no queue and the queue length going to d of a at $k+\phi_a$ will be equal to zero, that is to say

$$L_{ad}^{k+\phi_a} = 0. \tag{3.19}$$

Otherwise, if the arc has exceeded its queue unloading capacity Q_a, then some motorists will leave the arc at capacity by splitting Q_a proportionally according to the total of motorists going to each destination waiting to leave the arc a. Then, the outflow rate of a going to d at $k+\phi_a$ is given by:

$$G_{ad}^{k+\phi_a} = \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum\limits_{d'\in D}\left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t\right)} Q_a. \tag{3.20}$$

As there will be a queue formed by the motorists going to all destinations that had positive inflow rate at k and were not able to leave the arc, the queue length going to d of a at $k+\phi_a$ is given by:

$$L_{ad}^{k+\phi_a} = L_{ad}^{k+\phi_a-1} + \left(E_{ad}^{k} - \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum\limits_{d'\in D}\left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t\right)} Q_a\right)\Delta t. \tag{3.21}$$

- **STEP 4: COSTS UPDATE:** For each $a \in A$, the total cost of using arc a entering the arc at k is updated. This, considering that a queue of positive length that may be joined by the motorists once they have traversed the arc, regardless of their destination, results in a delay because of the waiting time in the queue, which is given by $\frac{\sum_{d\in D} L_{ad}^{k+\phi_a}}{Q_a}$. Therefore, the total cost of arc a is updated to

$$C_a^k = \phi_a + \frac{\sum\limits_{d\in D} L_{ad}^{k+\phi_a}}{Q_a}. \tag{3.22}$$

- **STEP 5: STOP CONDITION:** The algorithm stops for two reasons. First, if the current time increment is the last one of the discretization, namely $k = K$. Second, if there are no more flow rates going to any destination to be assigned in later time increments. These flow rates can come from outflow rates going to any destination from incoming arcs to any node, from demand rates going to any destination generated at any node or from residual queues on any arc. This will happen if

$$\sum_{l=k+1}^{l=K}\left(\sum_{d\in D}\left(\sum_{i\in N}\sum_{b\in A_i^-}G_{bd}^l+\sum_{a\in A}\frac{L_{ad}^{l+\phi_a-1}}{\Delta t}\right)+\sum_{(o,d)\in OD}\overline{\mathcal{D}}_{(o,d)}^l\right)=0. \tag{3.23}$$

Otherwise, the algorithm starts over again from **STEP 1** at time increment $k+1$.

As the MOSD-MDTA algorithm, presented in Section 2.4 of Chapter 2 as a solution method for the multiple origins and a single destination general case, the MOMD-MDTA algorithm has the property of allowing initialization with non-empty transport networks. Again, this feature is not further developed in this doctoral thesis, but, given its potentialities, it is intended to do analysis in later stages of this research on some interesting cases that could be studied because of this property.

Expressed as a summarized pseudocode, the *MOMD-MDTA algorithm* can be written as follows:

Algorithm 2 $(\mathcal{E},\mathcal{G},\mathcal{L})$ = MOMD-MDTA$((N,A),O,D,OD,\phi,Q,T,\Delta t,\mathcal{D}(\cdot),\theta)$

```
STEP 0: INITIALIZATION Technical settings
for k=1,...,K do
    STEP 1: BACKWARD
    for all d ∈ D do
        for all i ∈ N, in the order given π_d do
            for all a ∈ A_i^- incoming arcs to i, do
                Compute expected minimum costs starting from i, through a, to d
            end for
            Compute expected minimum costs starting from i to d
        end for
    end for
    STEP 2: ASSIGNMENT FACTORS COMPUTING
    STEP 3: FORWARD
    for all i ∈ N do
        for all d ∈ D do
            for all a ∈ A_i^+ outgoing arcs from i, do
                Compute inflow rate, outflow rate and queue length going to d through a
            end for
        end for
    end for
    STEP 4: COSTS UPDATES
    for all a ∈ A do
        Update cost of a because the delays given by the current queue lengths
    end for
    STEP 5: STOP CONDITION
    if there are no more flow rates to assign then
        End
    end if
end for
```

A detailed version of the *MOMD-MDTA algorithm* pseudocode, along with the ones of its subalgorithms, is presented in Appendix B.

3.5. How the *MOMD-MDTA algorithm* works: an illustrative example

To illustrate how the *MOMD-MDTA algorithm* works, an example over a simple transport network is presented. The goal is to get an intuition of how the inflow rates, outflow rates and queues lengths going to their respective destinations behave from the first instant of positive demand entering the system through the origins until the last instant in which a motorist reaches his/her destination.

Let us consider the transport network with an underlying digraph (N, A) represented in Figure 3.2. Here, nodes 1, 3 and 5 are origins and nodes 6 and 8 are destinations, while the O-D pairs are (1,6), (3,8) and (5,8). On each arc a, the pair (ϕ_a, Q_a) represents its free flow travel time $[sec]$ and its queue unloading capacity $[veh/sec]$, respectively. In order to keep the example simple, the (ϕ_a, Q_a) pair is $(2, 3)$ for every arc a. Also, a period of time of $T = 14$ *sec*, a timestep size of $\Delta t = 1$ *sec* and a dispersion parameter of $\theta = 0.2$ sec^{-1} are considered. With this, $K = 14$ time increments are set, starting when the first demand becomes positive.

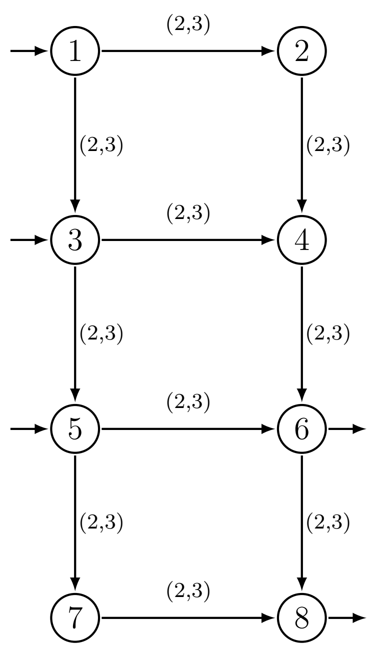

Figure 3.2: Network (N, A), with $(\phi_a,\ Q_a)$ on each arc a.

In Figure 3.3, are presented the time-dependent demand rate functions from the origin to the destination of each O-D pair of the transport network, $\mathcal{D}_{(1,6)}(t), \mathcal{D}_{(3,8)}(t)$ and $\mathcal{D}_{(5,8)}(t)$ (veh/sec), over continuous time t $[sec]$. Next to each plot, it is indicated the origin and the destination of its correspondant O-D pair.

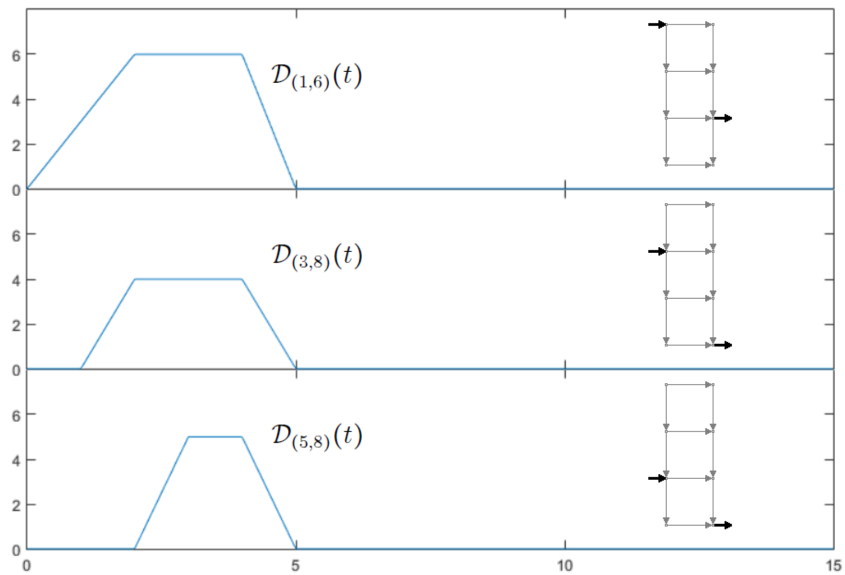

Figure 3.3: Demand rate of each O-D pair.

Figures 3.5, 3.6 and 3.7 show, for all time increments, how the *MOMD-MDTA algorithm* assigns the inflow rates going to each destination. Given an arc, when the aggregated inflow rate overpasses the queue unloading capacity, the figures depict how queues of motorists with different destinations are first formed and then emptied later on time. Figure 3.4 explains the notation used in Figures 3.5, 3.6 and 3.7, where motorists going to 6 and motorists going to 8 are shown blue and red, respectively. Given a time increment k, item a) represents a positive average demand rate $\overline{\mathcal{D}}^k_{(o,d)}$ entering origin o with destination d; for a given arc $a = (i, j)$, items b) and c) represent how positive inflow rates with destinations 6 and 8, $E^k_{a,6}$ and $E^k_{a,8}$, respectively, traverse arc a and, possibly, get behind inflow rates that may have entered earlier, as well as the queue lengths going to 6 and 8, $L^k_{a,6}$ and $L^k_{a,8}$, respectively; finally item d) represents flow rates that have arrived to destination d, which can be either inflow rates arriving directly to it or unloads of an existing queue.

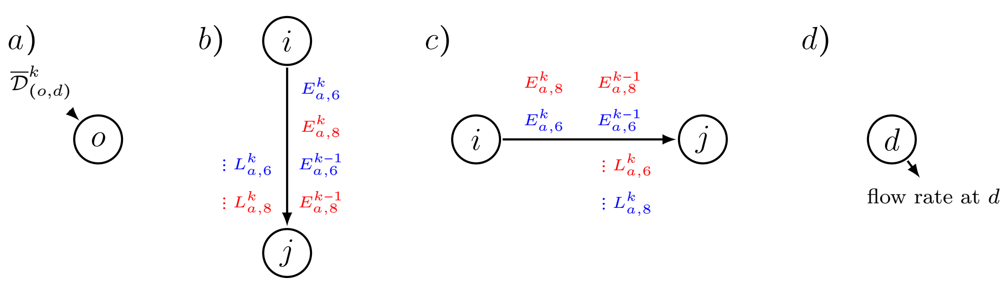

Figure 3.4: Notation for Figures 3.5, 3.6 and 3.7

Before presenting the figures of the example, it is worth pointing out that, for simplicity, outflow rates are not shown, although, they can, in fact, be computed. Given a destination d, an arc $a = (i, j)$, the set A^+_j of outgoing arcs of j and a time increment k, if j is not an origin, $G^k_{ad} = \sum_{b \in A^+_j} E^k_{bd}$ or, if j is an origin, $G^k_{ad} = \sum_{b \in A^+_j} E^k_{ad} - \overline{\mathcal{D}}^k_{(j,d)}$.

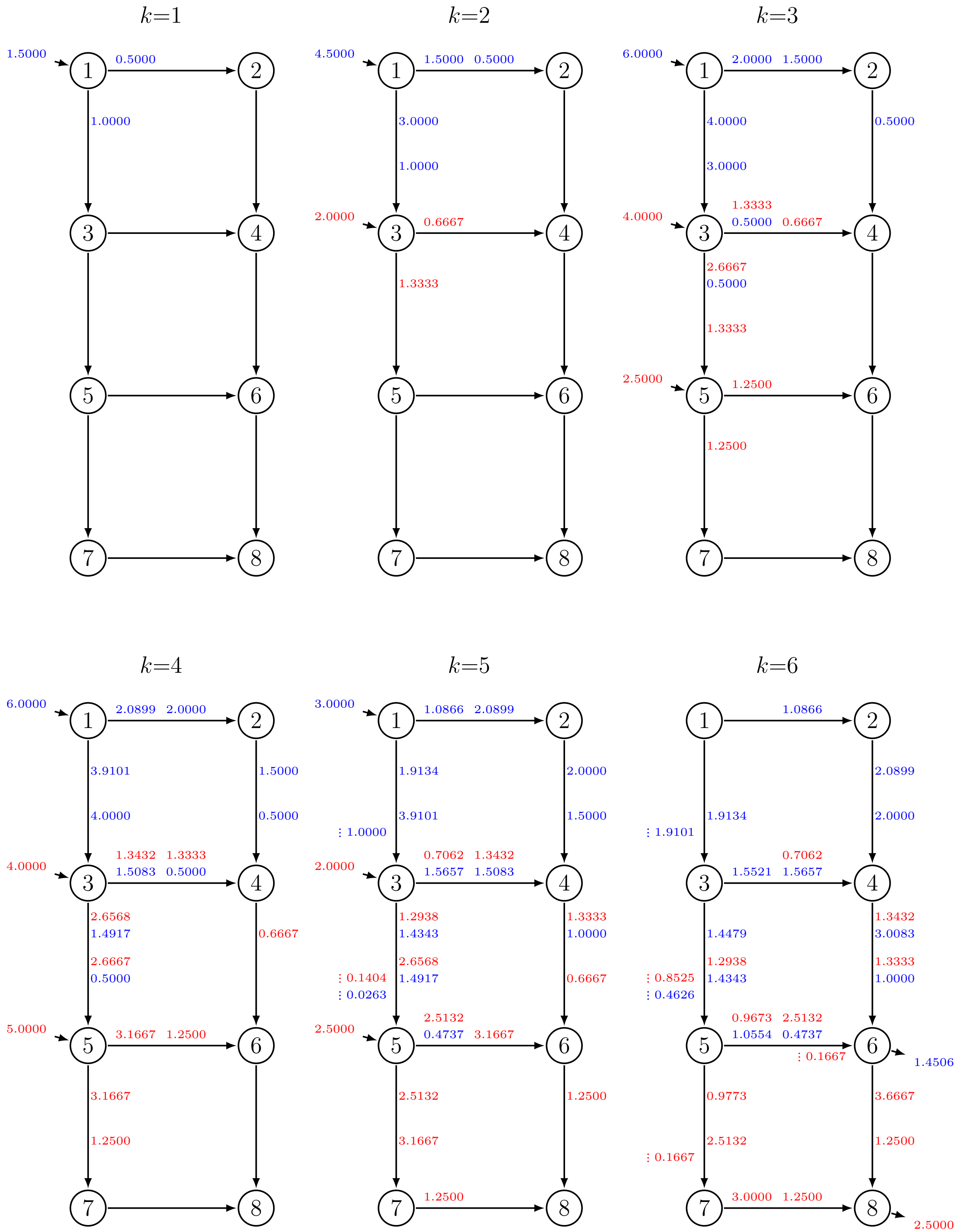

Figure 3.5: Evolution of *MOMD-MDTA algorithm* from $k = 1$ to $k = 6$.

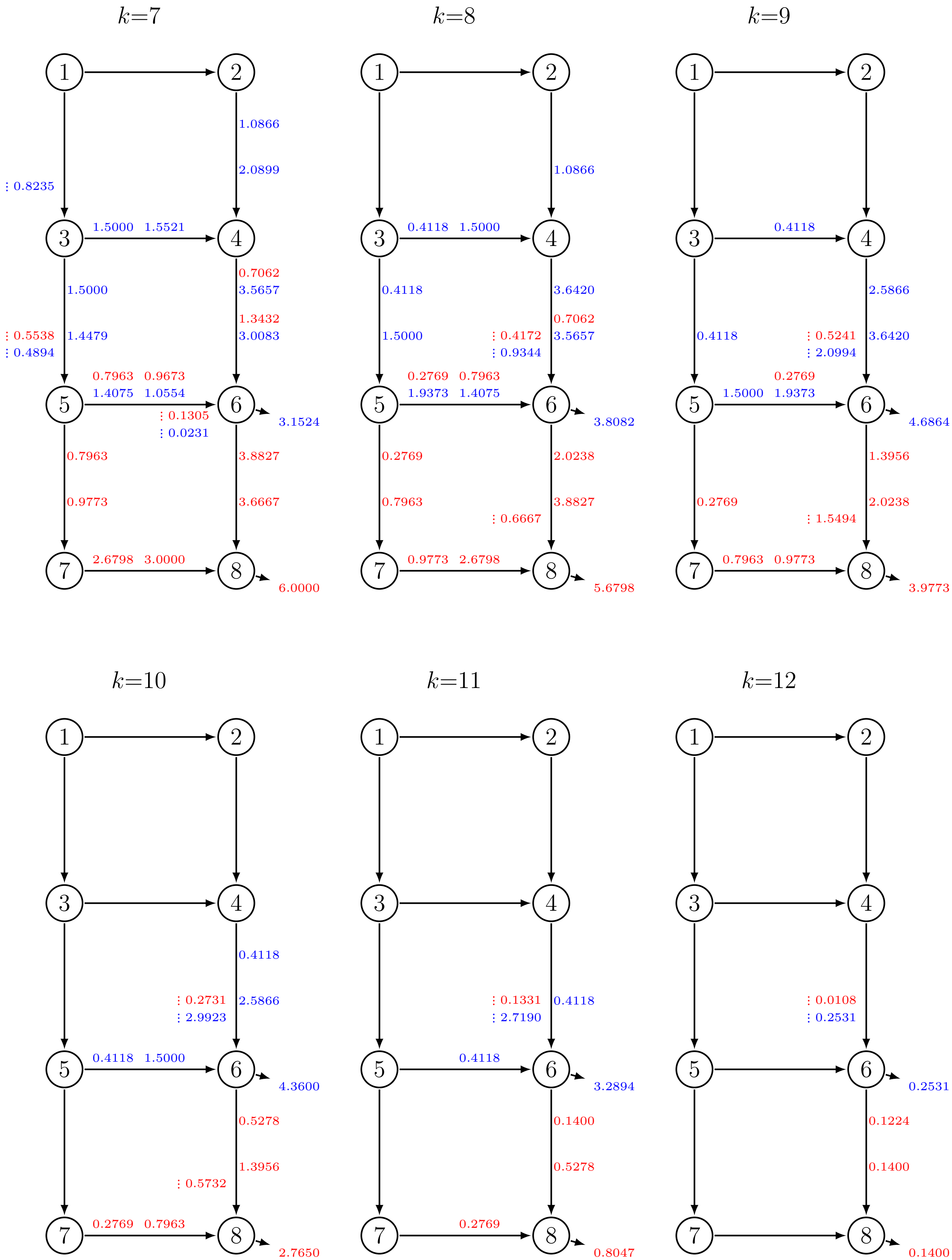

Figure 3.6: Evolution of *MOMD-MDTA algorithm* from $k = 7$ to $k = 12$.

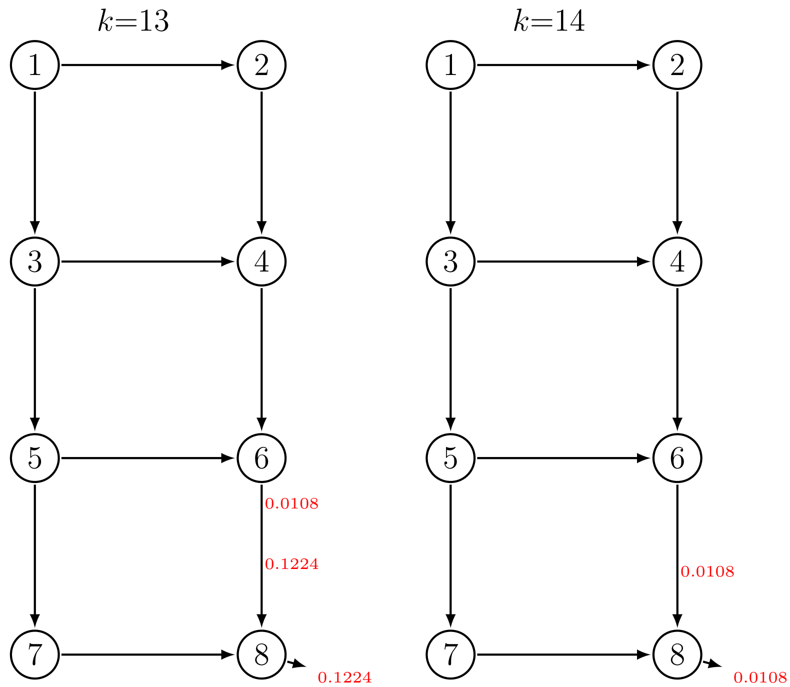

Figure 3.7: Evolution of *MOMD-MDTA algorithm* from $k = 13$ to $k = 14$.

In Figures 3.5, 3.6 and 3.7, it can be noticed that in time increments where an arc whose queue unloading capacity has been overpassed by the aggregation of inflow rates going to 6 and 8, the queue formed is composed by motorists going to both destinations. Then, such a queue dissipates in a later time increment. This happens because the process splits the queue unloading proportionally as outflow rates going to each destination depending on the total of motorists going to that destination waiting to leave the arc. This is the case of arc $(3,5)$ in $k = 4$, where the inflow rates going to 6 and 8 arriving to 5, are equal to 2.6667 veh/sec and 0.5 veh/sec, respectively. These rates are, in combination, larger than the queue unloading capacity of the arc, namely 3 veh/sec. Thus, in $k = 5$, as not all motorists are able to leave the arc, there is a queue with 0.1404 veh waiting to go to 6 and 0.0263 veh waiting to go to 8. This queue varies on later time increments, until it is dissipated in $k = 8$.

3.6. Computational Implementation of the *MOMD-MDTA algorithm*

As one of the goals of this work is to develop a tool that allows the use of the presented *MDTA* models, and its associated solution methods, in order to apply the methodology to different instances, the *MOMD-MDTA algorithm* has been implemented on MATLAB.

Among the tested cases, the transport network with underlying digraph (N, A), represented in Figure 3.8, where $(1, 13)$, $(2, 14)$ and $(4, 14)$ are the O-D pairs, has been chosen to be presented. For each arc a the pair (ϕ_a, Q_a) is shown, where ϕ_a is the free flow travel time of a $[sec]$ and Q_a is the queue unloading capacity of arc a $[veh/sec]$. The time-dependent demand rate functions of each O-D pair, $\mathcal{D}_{(1,13)}(t)$, $\mathcal{D}_{(2,14)}(t)$ and $\mathcal{D}_{(4,14)}(t)$, are shown in Figure 3.9.

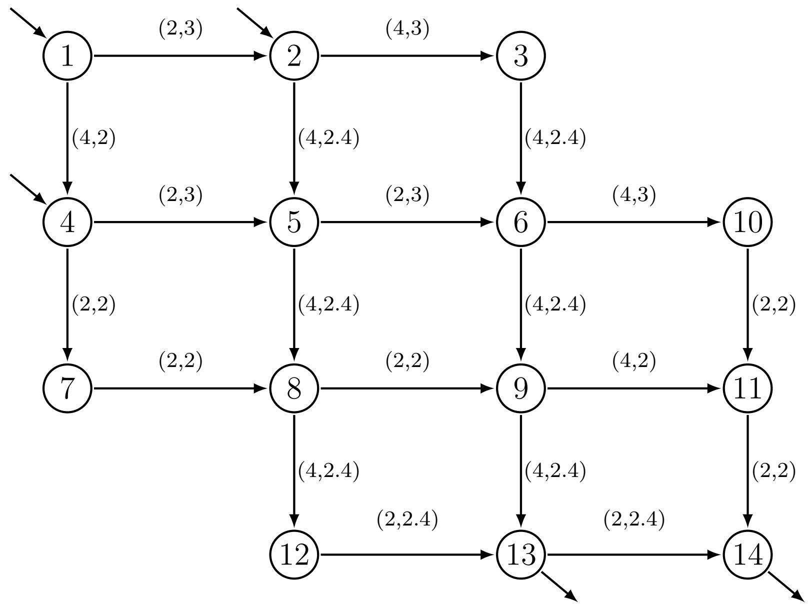

Figure 3.8: Network (N, A), with (ϕ_a, Q_a) on each arc a.

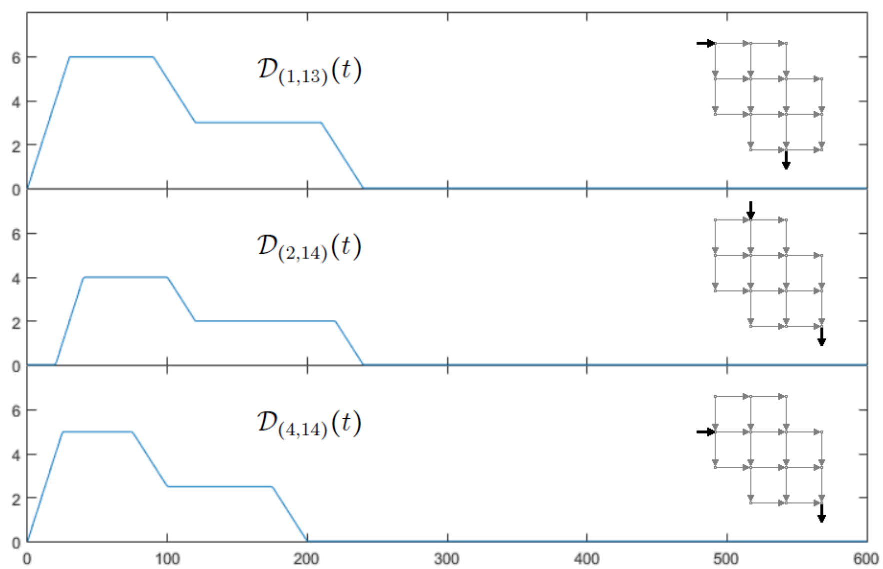

Figure 3.9: Demand rate functions $\mathcal{D}_{(1,13)}(t)$, $\mathcal{D}_{(2,14)}(t)$ and $\mathcal{D}_{(4,14)}(t)$.

The implementation has been executed over a period of $T = 600$ *sec* with a timestep size of $\Delta t = 1$ *sec* and a dispersion parameter of θ =0.2 sec^{-1}. The execution time was 32.79 *sec*.

3.6.1. The outputs: the behaviour of inflow rates, outflow rates and queue lengths going to each destination

Next, Figures 3.10, 3.11 and 3.12 show the evolution of all inflow rates, outflow rates and queue lengths going to each destination, respectively. They will be later commented.

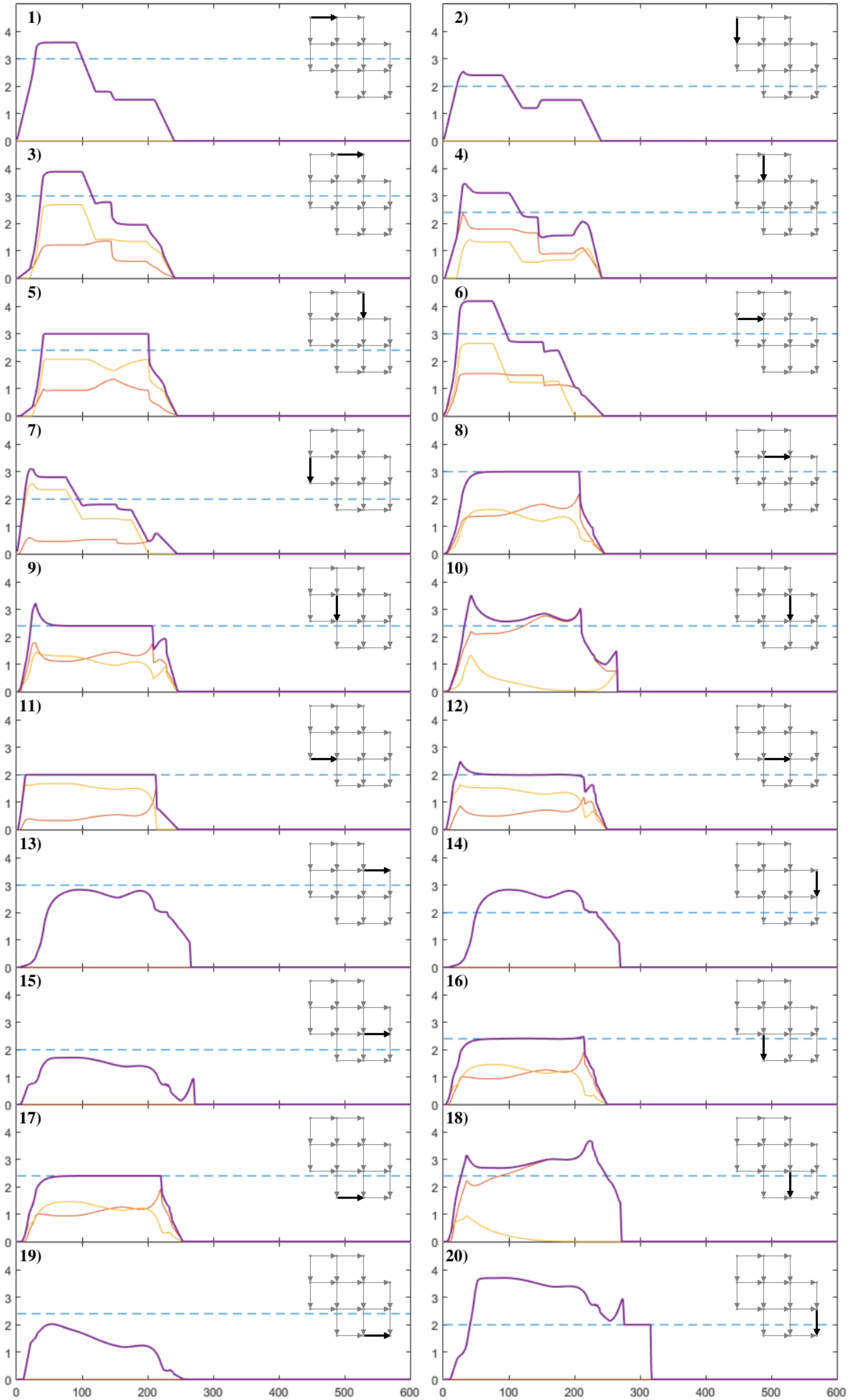

Figure 3.10: Evolution of inflow rates going to each destination [veh/sec].

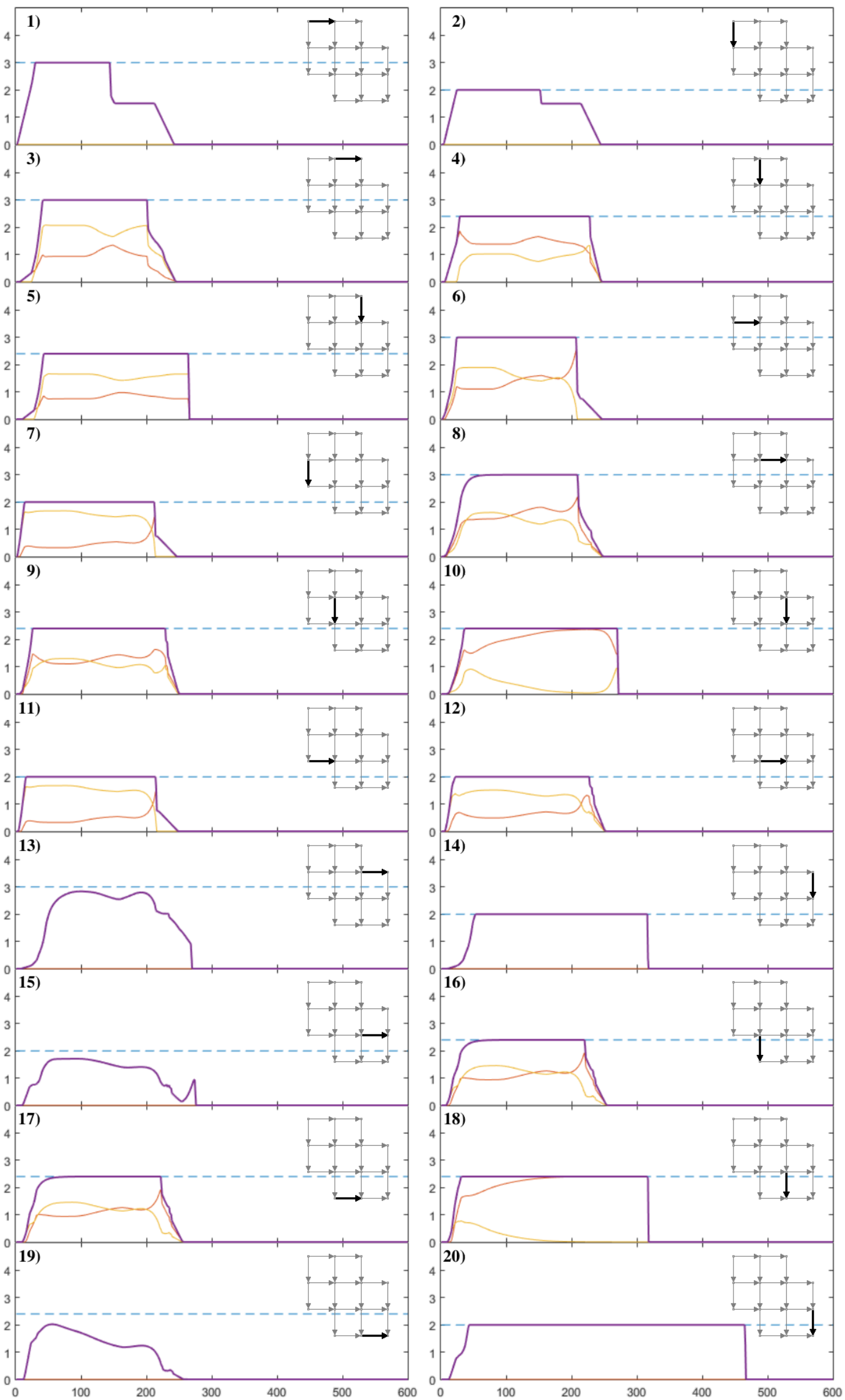

Figure 3.11: Evolution of outflow rates going to each destination [veh/sec].

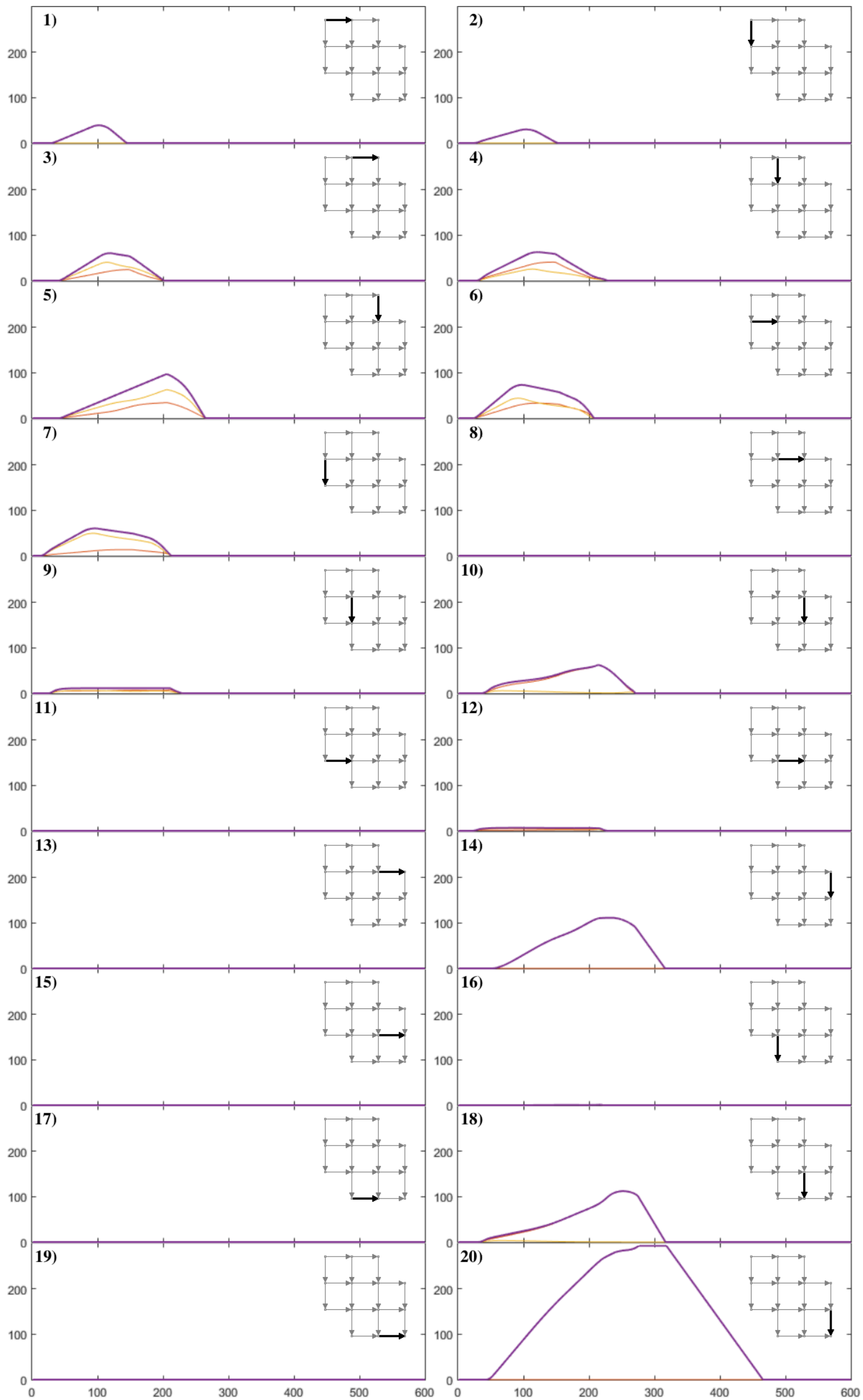

Figure 3.12: Evolution of queue lenghts going to each destination [veh].

Specifically, for all the arcs of the transport network, Figure 3.10 shows the evolution of inflow rates going to each destination and Figure 3.11 shows the evolution of outflow rates going to each destination. Figure 3.12 shows the evolution of the queue lengths going to each destination as well as the total queue length given by all the motorists waiting to leave the arc. In the three figures, curves in orange are associated with motorists going to destination node 13 and curves in yellow are associated with motorists going to destination node 14. The totals (sums of curves going to both destinations) are shown in thick purple curves and, thus, whenever the orange or the yellow curve is zero, the other one equals the total. In Figures 3.10 and 3.11, dashed blue lines indicate the queue unloading capacities.

There are three types of analysis that are worth highlighting regarding the behaviour of the outputs. To do so, in each one, different examples will be addressed in order to illustrate situations that can happen in cases of general transport networks.

How to relate the three types of plots

In order to explain how the plots on the three figures relate, four different cases of arcs, and how each one of their outputs interacts with the other ones, are commented. The classification of the arcs for these cases is conducted according to the presence or not of congestion and if there are multiple types of motorists or not. In this analysis, the arc as well as the number of its underlying plot are indicated, which are always the same for the three figures (3.10, 3.11 and 3.12).

- **One type of motorist on an uncongested arc** (**Arc**$(6,10)$, Plot **13**)): In this case, in Figure 3.10, just the inflow rate of motorists going to destination 13 is positive; thus, it equals the total inflow rate of the arc. As the latter does not overpass the queue unloading capacity, the outflow rate curve in Figure 3.11 is the same as the inflow rate curve except for a displacement in time given by the free flow travel time of the arc. This, because there is no queue forming on it, as it is observed in Figure 3.12, where the queue length is always 0, meaning that there are no delays.
- **Two types of motorists on an uncongested arc** (**Arc** $(7,8)$, Plot **11**)): In Figure 3.10, inflow rates of motorists going to both destinations are simultaneously positive for a consecutive number of time increments. It can be noticed that the total inflow rate does not overpass the queue unloading capacity of the arc (in fact, the total inflow rate equals that most of the time). Thus, the curves of the outflow rates going to each destination in Figure 3.11 are the same as their respective inflow rate curves in Figure 3.10, except for a displacement in time given by the free flow travel time of the arc. This, because there is no queue forming on it, as shown in Figure 3.12, meaning that there is no delay.
- **One type of motorist on a congested arc** (**Arc** $(1,2)$, Plot **1**)): In this case, in Figure 3.10, the inflow rate going to destination 13 is the only positive one and equals the total inflow rate. This time, the queue unloading capacity of the arc is overpassed; thus, the outflow rate curve in Figure 3.11 has maximum value (capacity) and, as the excess can not leave the arc, such excess gets stuck in a queue, as shown in Figure 3.12. Therefore, a delay is observed and the motorists end up leaving the arc after a period of time given by the free flow travel time plus the delay due to the waiting time in queue.
- **Two types of motorists on a congested arc** (**Arc** $(2,5)$, Plot **4**)): In Figure 3.10,

inflow rates going to both destinations are simultaneously positive for several consecutive time increments. In this case, the total inflow rate overpasses the queue unloading capacity of the arc. This results in the formation of a queue of positive length, as shown in Figure 3.12, composed in this case by two types of motorists waiting in it, one type with destination node 13 and the other one with destination node 14. The way this reflects in how vehicles leave the arc, is by assigning the outflow rate towards each destination proportionally, considering the number of motorists waiting to leave the arc going to each destination. Simultaneously, the sum of both outflow rates has to be equal to the queue unloading capacity. This, because whenever there is a queue the outflow rate has to be, necessarily, equal to said capacity, except for the last time increment in which the queue length is positive. In that case, it unloads just what is left, which is less or equal to the queue unloading capacity, as shown in Figure 3.11.

What happens in cases of general transport networks, regarding the aspect of how the plots of the three figures relate for a given arc, can be extended from these comments. This, because the behaviour of the chosen arcs are simple and more understandable versions of what could happen on the arcs belonging to larger networks with multiple destinations and, consequently, multiple types of motorists moving through the arcs.

Behaviour near origins and destinations

Besides the analysis that can be performed regarding the behaviour of the motorists according to the outputs of a particular arc, it is also worth to analyze their behaviour around origins and destinations. This, given that origins concentrate the demand rates and destinations concentrate the flow rates at the end of their trips. The focus on the comments is the behaviour of the inflow rates, going to both destinations, of outgoing arcs from origins and inflow rates, going to both destinations, of incoming arcs to destinations. It will be mainly referred Figure 3.10, as conclusions about the outflow rates and queue lengths related to the inflow rate of an arc going to each destination have already been covered in the previous comments. For each case, the plots considered in the context of the comment are indicated.

- **Origin node** 1 (Plots **1)** and **2)**): As origin node 1 has no incoming arcs and is an origin for the demand rate going to node 13, it only has one type of motorists that get split as inflow rates between the outgoing arcs of node 1, namely arcs $(1, 2)$ and $(1, 4)$, as shown in Figure 3.10. As these assignments are traversing the first arc on their trip to destination node 13 and have not yet been split into more inflow rates, they are large enough to cause congestion on the arcs and, thus, they cause the formation of queues. The time increments in which the demand rate at node 1 varies, as shown in Figure 3.9, can be observed in Figure 3.10 noting that the demand rate first changes and then becomes constant. Thus, there are some time increments in which the inflow rate of each arc adjusts before becoming constant for a period of time increments. This can be explained by the fact that when the expected cheapest arc becomes more expensive than the second cheapest one, the assignment will result in more inflow rate to the latter, until both equal their expected minimum costs. This, given an increment of the waiting time of the cheapest one because of its queue. Said increment happens gradually according to the dispersion parameter of the logit model.
- **Origin node** 2 (Plots **3)** and **4)**): Origin node 2 receives outflow rate from its only inco-

ming arc, namely $(1,2)$, with motorists going to destination node 13 and, simultaneously, is an origin of demand rate going to destination node 14. Thus, two types of inflow rates, one for each destination, have to be assigned among its outgoing arcs, namely $(2,3)$ and $(2,5)$. Let us first recall that each arc is associated not only with its travel cost (deterministic and equal for all motorists) but also with two expected minimum costs, one for each destination. Now, the way in which the inflow rates are assigned from node 2 among its outgoing arcs is not as clear as in the previously commented case. At each arc, the aggregation of inflow rates is large enough to cause congestion and, thus, to form a queue composed by motorists waiting to go to both destinations. Such a queue affects the cost of the arc and, thus, affects both of the expected minimum costs of using that arc to reach each destination. Subsequently, the inflow rate assingment among both arcs is affected by these expected minimum costs, as it is computed according to these values. Because of these embedded interactions, a pattern of behaviour of the inflow rates cannot be established.

- **Origin node** 4 (Plots **6)** and **7)**): For the case of origin node 4, the analysis is the same as the one commented for origin node 2, since the evolution is similar, although in this case the outflow rate going to destination 13 comes from arc $(1,4)$.
- **Destination node** 13 (Plots **17)** and **18)**): In this case, even though node 13 is the destination only for one type of inflow rate, its incoming arcs also carry inflow rate going to destination node 14. Thus, as all motorists going to node 13 are simultaneously using these arcs along with a portion of the motorists going to node 14, in a way that the total inflow rate overpasses their queue unloading capacities, it causes congestion and queues. In this case, again, a clear description of a behaviour pattern is not actually feasible, as the inflow rates of these arcs are the result of consecutive combinations of outflow rates going to each destination arriving from incoming arcs of each visited node.
- **Destination node** 14 (Plots **19)** and **20)**): As the incoming arcs to node 14 are not reasonable towards destination 13, only motorists that have such destination travel through the arc. Thus, the inflow rate for this case just refers to inflow rate going to node 14. The inflow rate assigned to arc $(13,14)$ is considerably less than the one assigned to arc $(11,14)$, and this happens because the first results from the aggregation of the outflow rates of incoming arcs to node 13. It means that, before leaving that node, the motorists going to node 14 were sharing arcs with the motorists that exited the network on node 13. This means that the total inflow rates of these arcs where composed by both types of inflow rates. The inflow rate of arc $(11,14)$ is larger because it comes from the aggregation of outflow rates of the incoming arcs to node 11. Those arcs only have motorists going to destination 14, meaning that the assignment has not been shared with motorists going to other destinations.

Interaction between arcs

As a last analysis regarding the evolution of the outputs, some comments about the interaction between arcs are provided. Figure 3.10 is referenced to make comments regarding the inflow rates on three cases. Once again, its relation with the behaviour of outflow rates and queue lengths going to each destination can be concluded by following an analysis similar to the previously done.

- **Arcs** (6,10) and (10,11) (Plots **13)** and **14)**): In this case, the inflow rate assigned to arc $(6, 10)$ is composed only of motorists going to destination node 14. As there is no congestion, the inflow rate traverses and exits the arc as outflow rate after the free flow travel time of the arc. Because of this, as $(6, 10)$ is the only incoming arc to node 10 and $(10, 11)$ is the only outgoing arc from node 10, all motorists leave the first arc and enter the second one inmediatly. Then the outflow rate of arc $(6, 10)$ is equal to the inflow rate of arc $(10, 11)$ at all time increments. For the same reasons, the inflow rates of arcs $(6, 10)$ and $(10, 11)$ are the same except for a time displacement given by the free flow travel time of arc $(6, 10)$.

- **Arcs** (4,7) and (7,8) (Plots **7)** and **11)**): In this case, the aggregation of inflow rates of arc $(4, 7)$ going to each destination overpasses the queue unloading capacity of the arc. The inflow rates traverse and later exit the arc as outflow rates going to each destination, after covering the free flow travel time plus the delay because of waiting in the queue. Like the previous case, as this is the only incoming arc to node 7, the outflow rate of arc $(4, 7)$ is exactly equal to the inflow rate of arc $(7, 8)$, altough the curves of the inflow rates of both arcs are not similar due to the observed congestion.
- **Arcs** (2,5), (4,5), (5,6) and (5,8) (Plots **4)**, **6)**, **8)** and **9)**): This type of interaction represents what will happen in most of the cases. This, because for a certain node in a general transport network, the aggregated outflow rates going to all destinations of its incoming arcs will be assigned as inflow rates going to all destinations among its outgoing arcs. Trying to describe a pattern of behaviour in a general case is not feasible, as the assignment is affected by several interactions that, simultaneously, depend on different aspects arising in a general and realistic network.

3.6.2. A technical aspect: The dispersion parameter θ

An important technical aspect regarding computational implementation is the decision of a proper time-step to obtain stable results while controlling the execution time. Subsection 2.6.2 discuss this issue concluding that values for that parameter smaller than one second resulted in practically identical outputs. Therefore, time-steps one second long seem small enough to be convenient for the implementation.

In this section, another technical aspect regarding the parameters of the computational implementation is discussed: the value of the dispersion parameters of the logit model needed to obtain a realistic implementation of the MDTA model.

As the dispersion parameter affects how the arc-choices are decided by the motorists, this modelling issue has an effect in the way in which the inflow rates that go to different destinations interact within the arcs as a result of these choices. To understand this effect, the analysis is focused on how the demand rate going to a particular destination from an origin node behaves under different values for this parameter. Specifically, it has been graphically analyzed the behaviour of the inflow rates of the outgoing arcs from origin node 1 going to destination node 13. These are the flows chosen to be presented, as they reflect how the demand rate entering that node is assigned and, since they have larger values compared to the inflow rates of other arcs in the network, the variations between different dispersion parameters will be more perceivable on them.

The evolution of the inflow rates of arcs $(1, 2)$ and $(1, 4)$ with the dispersion parameter used in the previous example, θ =0.2 sec^{-1}, is used as reference to be compared with the inflow rates resulting from the proposed sensitivity using 5 different values for the dispersion parameter. These are given by: 0.01 sec^{-1}, 0.1 sec^{-1}, 1.5 sec^{-1}, 1.75 sec^{-1} and 5 sec^{-1}.

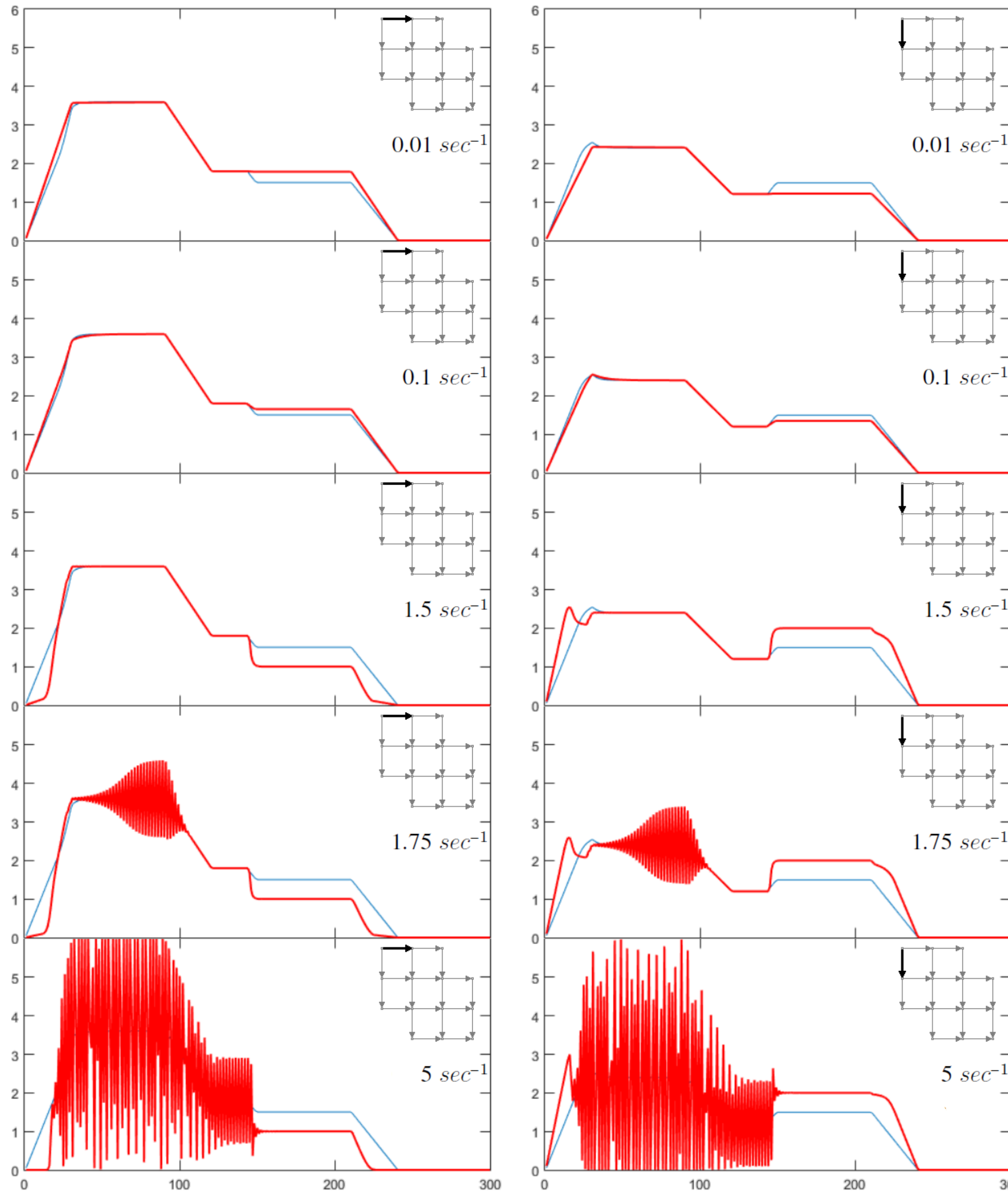

Figure 3.13: Comparison of inflow rates under different θ values.

All curves associated with the inflow rates of arcs $(1, 2)$ and $(1, 4)$ under the different dispersion parameters that were previously set and numbered are presented in Figure 3.13. In each plot, the θ value used is indicated and its associated curve of inflow rates is shown in thick red. The curve of the inflow rates associated with the reference dispersion parameter, 0.2 sec^{-1}, is shown in blue for comparison. Plots on the left side are associated with arc $(1, 2)$ and plots on the right side are associated with arc $(1, 4)$, as it is shown in the upper right corner of each plot.

Regarding the plots shown in Figure 3.13, and according to the established reference value, an analysis on two cases is then conducted:

- θ **smaller than the reference value**: For the dispersion parameters smaller than 0.2 sec^{-1}, the behaviour of the curves, when compared to the ones associated with the reference case, can be understood as less sensitive to changes on the cost of the arcs. This can be seen in both cases, but more notoriously on the curves where 0.01 sec^{-1} is used. In this case, the behaviour is closer to the assignment using a dispersion parameter equal to 0. Tested values that were smaller than 0.01 sec^{-1} resulted in curves that were practically identical.

  Extremely small values may cause numerical errors, as they might be too small to be processed correctly, considering the recursive use of these values over a single computation of a given step of the algorithm.
- θ **greater than the reference value**: For values greater than 0.2 sec^{-1}, it can be noticed how the assignment becomes more sensitive to cost variations. This happens gradually, according to the increment on the tested dispersion parameter values. This phenomenon is particularly observed in periods of consecutive time increments in which the queue lengths increase, from where the waiting times increase and, thus, the total costs, before starting to unload until the queues dissipate.

  The high value of 1.5 sec^{-1} for this parameter accentuates the changes in the curves of inflow rates with respect to the reference case, showing how the choices are more affected by the variation of the costs. The larger value 1.75 sec^{-1} leads to a behaviour of the inflow rates that is not totally explainable in real-life situations. In the peak of the assignment of the inflow rates, as the use of one arc becomes more expensive from one time increment to the next, the logit model makes a greater variation on the assignment, as it assigns more inflow to the other one. This generates a more expensive total cost, causing that, in the next time increment, the model assigns more inflow rate back to the first arc, generating an oscillatory behaviour, assigning more to one arc in one time increment and more to the other arc in the next one. For the even larger value of 5 sec^{-1}, this phenomenon becomes more drastic, as the oscillatory behaviour starts earlier on time and with larger differences on the assignments between a time increment and the next one. Larger values of the dispersion parameter were tested and the conclusions were the same as in the last case.

  Extremely large values caused numerical errors, as in the case of the extremely small ones.

The observed behaviour was also replicated in the outputs of the other arcs of the network. Therefore, the conclusions about the influence of the different dispersion parameter values on the assignment of the arcs were the same in all cases.

From this analysis, the dispersion parameter can be interpreted as a measure of adjustment of how sensitive to changes of the arc costs in the network the model is going to be. For values closer to 0, it generates a model more insensitive to changes, even if they are big, such as great accumulations of motorists in queues. For values greater than 1.5 sec^{-1} values, the increasing sensitivity to changes in the costs of the arcs results in an oscillatory behaviour, in the sense of how the model prioritizes which arc to assign more inflow rate to, and this could hardly represent a real-life scenario. From these sensitivity analyzes, a range of values for the dispersion parameter between 0 and 1.5 sec^{-1} leads to a model that has feasible real interpretations, representing a reasonable set of values to cover different behaviours of real motorists responding to variations of the costs of the arcs.

Regarding the execution time of the experiments, there were no significant differences, being all approximately 33 seconds for any of the values tested.

Finally, the analysis of which values of the dispersion parameters are better suited for each particular case will depend on a calibration process. This process should replicate in a proper way the real behaviour observed in different transport networks. Given their technical properties it would be difficult to define a benchmark and decide a priori which value is the best to use.

This concludes the presentation of the results of the last completed stage of this research, that serves as the core of this doctoral thesis and its main contribution. The final comments regarding this chapter will be presented later in the Conclusions section. A paper with the contents of this chapter will be submitted to Transportation Research Part B.

Chapter 4

A glimpse of a work in development: Extending the Reasonability concept

The motivations regarding both of the presented MDTA model versions, for the multiple origins and a single destination general case in Chapter 2 and for general transport networks in Chapter 3, have already been explained earlier. One of the main aspects of the reasoning behind the approach and, thus, its formulations, is that motorists travel through the network making successive decisions according to their perceived costs towards their respective destinations. In both cases, it has been assumed that motorists have a correct perception of the travel times when the arcs of the transport network are not congested. According to this perception, and depending on their destinations, they have a fixed set of arcs that they consider reasonable to move forward to, as these reasonable arcs will take them not farther from their destination, if only minimum cost routes are considered. The concepts of *reasonable arc* and *reasonable arc towards a particular destination* are introduced in Section 2.2 of Chapter 2 and Section 3.2 of Chapter 3, respectively, to properly define this concept under the considerations of each version of the MDTA model.

Now, a new scenario will be faced. As for the characteristics of the transport network, as in Chapter 3, it will be considered the general case. As for the motorists behaviour, it will be considered that they use their current time-dependent cost perception of what is left of their trip to their respective destinations to determine if an arc is or not reasonable at that instant. Also, they will only consider as reasonable options those arcs that they expect not to get them farther from their destination, if routes with minimum expected cost are meant to be taken.

The main characteristic of this reasoning, and that states the most significant difference with the one used for the previous versions of the MDTA model, is that it tackles a different aspect of reality. This, by addressing the question of what if motorists classify dynamically which arcs are, or not, reasonable according to their current perceived minimum costs. The idea comes, actually, from the construction of the algorithms for both versions of the MDTA model. As at each time increment they allow computing the expected minimum costs towards destinations, motorists could use that perception to decide wether an arc is either reasonable or not, under an adapted definition for this new considerations.

4.1. The expected set of reasonable arcs towards a destination

The intuition behind the extension of the reasonability concept that will be established for this new case is, basically, the same as in its previously defined versions. The difference is that now it will be defined according to a given time and a new criterion. To decide if an arc, at a certain time, is reasonable or not, the criterion will be based on the time-dependent perceived minimum costs rather than being based on the constant deterministic minimum costs of an uncongested network.

Let us consider a transport network with underlying digraph (N, A), with set of nodes N and set of arcs A, a set of destinations $D \subseteq N$ and a time interval $[0, T]$. Let us also recall the notation introduced in Subsection 3.3.3 of Chapter 3. Given a destination $d \in D$ and a time $t \in [0, T]$, for each node $i \in N$, $W_{id}(t)$ is the expected minimum cost of going from node i to destination d at time t, and, for each arc $a \in A$, $C_a(t)$ is the cost of a at t. Then, the *set of expected reasonable arcs towards destination d at time t* is defined as the set $\widehat{R}_d(t) = \{a = (i, j) \in A : W_{jd}(t + C_a(t)) \leq W_{id}(t)\}$.

Now, under this new scenario where motorists have a different way of defining reasonability, the MDTA model works slightly different, as explained next.

4.2. The MDTA model for general transport networks and the expected reasonable arcs sets

As the reasoning behind the construction of the formulations of the MDTA model for general transport networks has been already explained in detail in Section 3.3 of Chapter 3, that aspect will not be analyzed again in what follows. However, the explicit formulations according to this new intuition of reasonability will presented.

As in the previous cases, let us consider a transport network with an underlying digraph (N, A), with set of nodes N and set of arcs A. For each $i \in N$, A_i^+ and A_i^- are the sets of outgoing arcs from i and incoming arcs to i, respectively. For each arc $a \in A$, ϕ_a is its free flow travel time and Q_a is its queue unloading capacity, both known parameters. The time interval $[0, T]$ is also known. The set origin nodes is $O \subseteq N$, the set of destination nodes is $D \subseteq N$, the set of O-D pairs is $OD \subseteq O \times D$. The temporal dependent demand rate functions from the origin to the destination of every O-D pair $(o, d) \in OD$ are denoted as $\mathcal{D}_{(o,d)}(\cdot)$.

The three parts of the MDTA model, considering the sets of expected reasonable arcs towards each destination, are formulated as follows.

4.2.1. The Demand Profile

As, for each O-D pair $(o, d) \in OD$, its demand rate function $\mathcal{D}_{(o,d)}(\cdot)$ is exogenous information, the *the demand profile* is considered given, as in the previously studied cases.

4.2.2. The Traffic Model

Let us recall that for a given destination node $d \in D$, a given arc $a \in A$ and at given time $t \in [0, T]$, $E_{ad}(t)$ is the inflow rate of a going to d, $G_{ad}(t)$ is the outflow rate of a going to d and $L_{ad}(t)$ is the queue length of a going to d. Then, for each $d \in D$, for each $a \in A$ and at each time $t \in [\phi_a, T + \phi_a]$, the *traffic model* is given by the following expressions:

$$G_{ad}(t) = \begin{cases} E_{ad}(t - \phi_a), & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ \frac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise,} \end{cases} \tag{4.1}$$

$$\frac{dL_{ad}}{dt} = \begin{cases} 0, & \text{if } \sum\limits_{d' \in D} E_{ad'}(t - \phi_a) \leq Q_a \wedge \sum\limits_{d' \in D} L_{ad'}(t) = 0, \\ E_{ad}(t - \phi_a) - \frac{L_{ad}(t)}{\sum\limits_{d' \in D} L_{ad'}(t)} Q_a, & \text{otherwise,} \end{cases} \tag{4.2}$$

from where the cost of arc $a \in A$ at $t \in [\phi_a, T + \phi_a]$ is given by:

$$C_a(t) = \phi_a + \frac{\sum\limits_{d \in D} L_{ad}(t + \phi_a)}{Q_a}. \tag{4.3}$$

4.2.3. The Arc-Choice Model

For each destination node $d \in D$, for each arc $a = (i, j) \in A$ and at each time $t \in [0, T]$, the expected minimum cost of going from i to d choosing arc a at t, denoted $Z_{ad}(t)$, is given by:

$$Z_{ad}(t) = C_a(t) - \frac{1}{\theta} \ln\left(\sum_{b \in A_j^+} \exp\left(-\theta\left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right) \right). \tag{4.4}$$

For each destination node $d \in D$, for each node $i \in N$ and at each time $t \in [0, T]$, the expected minimum cost of going from node i to destination d, starting at t, denoted $W_{id}(t)$, is given by:

$$W_{id}(t) = -\frac{1}{\theta} \ln\left(\sum_{a = (i,j) \in A_i^+} \exp\left(-\theta\left(C_a(t) + W_{jd}\left(t + C_a\left(t\right)\right)\right)\right) \right). \tag{4.5}$$

As for the inflow rate assignments, for each destination node $d \in D$, there are two cases. For each node $i \in N$ such that $(i, d) \notin OD$, this is, for nodes that are not origins for the destination d, at each time $t \in [0, T]$ and for each outgoing arc from i that is in the set of expected reasonable arcs towards d at t, this is $a = (i,j) \in A_i^+ \cap \widehat{R}_d(t)$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \frac{\exp\left(-\theta\left(Z_{ad}\left(t + C_a\left(t\right)\right)\right)\right)}{\sum_{b \in A_i^+} \exp\left(-\theta\left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right)} \sum_{b \in A_i^-} G_{bd}(t). \tag{4.6}$$

Then, for each $o \in O$ such that $(o,d) \in OD$, this is, for nodes that are origins for destination d, at each time $t \in [0,T]$ and for each outgoing arc from o that is in the set of expected reasonable arcs towards d at t, this is $a = (o,j) \in A_o^+ \cap \widehat{R}_d(t)$, the inflow rate of a going to d at t is given by:

$$E_{ad}(t) = \frac{\exp\left(-\theta\left(Z_{ad}\left(t + C_a\left(t\right)\right)\right)\right)}{\sum_{b \in A_o^+} \exp\left(-\theta\left(Z_{bd}\left(t + C_b\left(t\right)\right)\right)\right)} \left(\sum_{b \in A_o^-} G_{bd}(t) + \mathcal{D}_{(o,d)}(t)\right). \tag{4.7}$$

Note that the formulation for the MDTA model remains the same as in the case for general networks where the concept of reasonable arc was applied. In the technical aspect of the formulation, the difference is in the stage where the assignment is performed, as in this case only arcs that are in the current set of expected reasonable arcs towards a destination will be considered to be assigned with positive inflow rate.

4.3. The MOMD-ERS-MDTA Algorithm

Now, a solution method for this new scenario is presented, the *MOMD-ERS-MDTA algorithm*. It comes directly from the MOMD-MDTA algorithm presented in Section 3.4 of Chapter 3 but with changes in its initalization, at each time increment it has a new step and its original steps suffer minor modifications. Again, as it is an adaptation with no fundamental changes in its more important procedures, the reasoning behind every step is not fully developed, as this has already been done in the presentation of the MOMD-MDTA algorithm.

Specifically, the first change comes in the **INITIALIZATION**. In this step, for each destination d, for all nodes it is computed their minimum cost to d to set a default initial set of expected reasonable arcs towards each destination. A first order of the nodes, increasingly according to their minimum costs to d is set as well, as these structures are required to execute the **BACKWARD** step. The next change comes in the **BACKWARD** step. For each time increment k in which the MOMD-ERS-MDTA algorithm is being exectued, the arcs that were required to be reasonable arcs towards a destination d in the MOMD-MDTA algorithm now should be in the set of expected reasonable arcs towards d. Then the **COMPUTING OF SETS OF EXPECTED REASONABLE ARCS** step is added. In this new step, for each node d, the expected minimum costs of going from each node i to d are used to update the set of expected reasonable arcs towards d and to update the order of the nodes that considers these values increasingly. In the **COMPUTING OF ASSIGNMENT FACTORS** step, when it was required an arc to be reasonable towards a destination in MOMD-MDTA algorithm, now it should to be in the set of expected reasonable arcs towards that destination. The **FORWARD**, **COSTS UPDATE** and **STOP CONDITION** steps are executed as in the MOMD-MDTA algorithm.

The inputs of the MOMD-ERS-MDTA algorithm are: the digraph (N, A) associated with the transport network, the set of origins $O \subseteq N$, the set of destinations $D \subseteq N$, the set of O-D pairs $OD \subseteq O \times D$, the free flow travel time ϕ_a and the queue unloading capacity Q_a of every arc $a \in A$, aggregated as the vectors ϕ and Q, respectively. The length of the period of time, T, and the size of the timestep, Δt, are known, from where the number of time intervals, K, is computed as $K = T/\Delta t$. k. The demand rate functions of each O-D pair $(o, d) \in OD$, $\mathcal{D}_{(o,d)}(\cdot)$, aggregated as the vector function $\mathcal{D}(\cdot)$, are given. The dispersion parameter of the logit model, θ, is also given.

The outputs of the algorithm are trhee sets of matrices: $\mathcal{E} = \left\{E_d = \left(E_{ad}^k\right)_{a\in A, k=1,\dots,K} : d \in D\right\}$, $\mathcal{G} = \left\{G_d = \left(G_{ad}^k\right)_{a\in A, k=1,\dots,K} : d \in D\right\}$ and $\mathcal{L} = \left\{L_d = \left(L_{ad}^k\right)_{a\in A, k=1,\dots,K} : d \in D\right\}$. In those sets, given $a \in A$, $d \in D$ and $k \in \{1, \dots, K\}$, E_{ad}^k and G_{ad}^k are the inflow and outflow rates of arc a going to destination d at time increment k, respectively, and L_{ad}^k is the queue length of arc a going to destination d at time increment k.

The algorithm proceeds as follows:

- **STEP 0: INITIALIZATION:** Sets $K = T/\Delta t$ and, for each $i \in N$, sets $A_i^- = \{(j, i) \in A : j \in N\}$ and $A_i^+ = \{(i, j) \in A : j \in N\}$. For each $d \in D$, for each $a \in A$ and at each $k = 1, \dots, K$, sets $E_{ad}^k = 0$, $G_{ad}^k = 0$ and $L_{ad}^k = 0$, and $k = 0$ is set to define $L_{ad}^0 = 0$. For each $a \in A$ and at each $k = 1, \dots, K$, it sets $C_a^k = \phi_a$. For each $d \in D$, for each $i \in N$, the initial minimum cost from node i to d is computed and set, then, according to this values, increasingly, an order π_d^0 of all nodes, starting from d itself, is set, as well as the first set of expected reasonable arcs towards d, $\widehat{R}_d^0$. For each $(o, d) \in OD$ and at $k = 1, \dots, K$, it is computed and set

$$\overline{\mathcal{D}}_{(o,d)}^k = \frac{\int_{(k-1)\Delta t}^{k\Delta t} \mathcal{D}_{(o,d)}(t)\, dt}{\Delta t}. \tag{4.8}$$

Then, the following steps are executed from $k = 1$ to $k = K$ or until the stop condition is satisfied.

- **STEP 1: BACKWARD:** In this step, the expected minimum costs are computed. At each $t = 1, \dots, K$, for each $d \in D$, for each $a \in A$ and for each $i \in N$, sets $Z_{ad}^t = \infty$ and $W_{id}^t = \infty$. Then, for each $d \in D$ and for each node $j \in N$, in the order given by π_d^{k-1}, for each $a = (i, j) \in A_j^- \cap \widehat{R}_d^{k-1}$ and for each $t = 1, \dots, K$, if $j = d$, then $W_{id}^t = 0$ and $Z_{ad}^t = C_a^t$. Otherwise, if $j \neq d$ and $t + \lfloor C_a^t \rfloor \leq K$, then

$$W_{jd}^t = -\frac{1}{\theta} \log\left(\sum_{b\in A_j^+} \exp\left(-\theta Z_{bd}^{t+\lfloor C_a^t \rfloor}\right)\right) \tag{4.9}$$

and

$$Z_{ad}^{t} = C_{a}^{t} + W_{jd}^{t+\lfloor C_{a}^{t} \rfloor}. \tag{4.10}$$

- **STEP 2: COMPUTING OF SETS OF EXPECTED REASONABLE ARCS:** For each $d \in D$, an order π_{d}^{k} of all nodes $i \in N$, according to the increasing values of W_{jd}^{k} and starting from d itself, is set, and the set of expected reasonable arcs towards d at k is computed as:

$$\widehat{R}_{d}^{k} = \left\{ a = (i,j) \in A : W_{jd}^{k+\lfloor C_{a}^{k} \rfloor} \leq W_{id}^{k} \right\}. \tag{4.11}$$

- **STEP 3: COMPUTING OF ASSIGNMENT FACTORS:** For each $d \in D$ and for each $a \in A$, the assignment factor of a to d at k is computed. If a is in the set of expected reasonable arcs towards d at k, this is $a \in \widehat{R}_{d}^{k}$, then

$$F_{ad}^{k} = \exp\left(-\theta Z_{ad}^{k}\right) \tag{4.12}$$

and, otherwise, $F_{ad}^{k} = 0$.

- **STEP 4: FORWARD:** In this step, the assignment is performed. It runs the *MOMD-Forward algorithm*, the same used in the MOMD-MDTA algorithm. For each i, if

$$\sum_{d \in D} \left(\sum_{b \in A_{i}^{-}} G_{bd}^{k} + \overline{\mathcal{D}}_{(i,d)}^{k} + \frac{\sum\limits_{a \in A_{i}^{+}} L_{ad}^{k+\phi_{a}-1}}{\Delta t} \right) > 0, \tag{4.13}$$

where $\overline{\mathcal{D}}_{(i,d)}^{k} = 0$ if $(i,d) \notin OD$, and, simultaneously, $k + \max_{a \in A_{i}^{+}} \{\phi_{a}\} \leq K$, then the algorithm proceeds as follows. If $i \notin O$, then the inflow rate of a going to d at k is given by:

$$E_{ad}^{k} = \frac{F_{ad}^{k}}{\sum\limits_{a' \in A_{i}^{+}} F_{a'd}^{k}} \sum_{b \in A_{i}^{-}} G_{bd}^{k}. \tag{4.14}$$

Otherwise, if $i \in O$, then the inflow rate of a going to d at k is given by:

$$E_{ad}^{k} = \frac{F_{ad}^{k}}{\sum\limits_{a' \in A_{i}^{+}} F_{a'd}^{k}} \left(\sum_{b \in A_{i}^{-}} G_{bd}^{k} + \overline{\mathcal{D}}_{(i,d)}^{k} \right). \tag{4.15}$$

Next, if $\sum_{d \in D} \left(\frac{L_{ad}^{k+\phi_{a}-1}}{\Delta t} + E_{ad}^{k} \right) \leq Q_{a}$, the outflow rate of a going to d at $k + \phi_{a}$ is given by

$$G_{ad}^{k+\phi_a} = \frac{L_{ad}^{k+\phi_a-1}}{\Delta t} + E_{ad}^{k} \tag{4.16}$$

and the queue length going to d of a at $k+\phi_a$ is given by

$$L_{ad}^{k+\phi_a} = 0. \tag{4.17}$$

Otherwise, the outflow rate of a going to d at $k+\phi_a$ is given by

$$G_{ad}^{k+\phi_a} = \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum\limits_{d'\in D}\left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t\right)} Q_a \tag{4.18}$$

and the queue length going to d of arc a at $k+\phi_a$ is given by

$$L_{ad}^{k+\phi_a} = L_{ad}^{k+\phi_a-1} + \left(E_{ad}^{k} - \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum\limits_{d'\in D}\left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t\right)} Q_a\right)\Delta t. \tag{4.19}$$

- **STEP 5: COSTS UPDATE:** For each $a \in A$ its total cost is uptaded to

$$C_a^k = \phi_a + \frac{\sum\limits_{d\in D} L_{ad}^{k+\phi_a}}{Q_a}. \tag{4.20}$$

- **STEP 6: STOP CONDITION:** If $k = K$ or

$$\sum_{l=k+1}^{l=K}\left(\sum_{d\in D}\left(\sum_{i\in N}\sum_{b\in A_i^-} G_{bd}^{l} + \sum_{a\in A}\frac{L_{ad}^{l-1}}{\Delta t}\right) + \sum_{(o,d)\in OD}\overline{\mathcal{D}}_{(o,d)}^{l}\right) = 0, \tag{4.21}$$

  the algorithm ends. Otherwise, the algorithm starts again from **STEP 1** for time increment $k+1$.

This work in development is currently in its computational implementation stage. The idea is to take advantage of the already computed expected minimum costs and use them to compute the set of expected reasonable arcs for each destination at each time increment.

As has been presented, this approach uses the formulation of the MDTA model and applies it to a new context regarding the motorists behaviour. Now, it is addressed and represented a scenario where the perception of the travelers not only determines how the inflow assignments will be performed over the arcs and over time, but also it now determines dynamically if said arcs are perceived as reasonable or not. This, contrary to what happened in the previous cases, where it was assumed that motorists have a correct perception of the travel times of an uncongested network and, according to this, determine if an arc is or not reasonable, where that classification remains constant in time.

Conclusion

The main contribution presented in this doctoral thesis is the development of a novel model that allows facing a DTA problem under stochasticity in the motorists decisions, the *Markovian Dynamic Traffic Assignment model* (*MDTA model*). The approach that generated this model comes from the integration of the contributions of two lines of work. First, the traffic assignment that results from the Markovian Traffic Equilibrium (MTE) concept by Baillon and Cominetti [7], originally applied for static cases. Second, the basis of the formulation presented by Addison and Heydecker [2], where it is established that the DTA modelling process requires the development of a demand profile, a traffic model and a route-choice model. According to the approach here proposed, the demand profile is considered to be exogenous, the traffic model adapts the deterministic punctual queueing model and the route-choice model is built as an arc-choice model. The latter results from a dynamic extension of a traffic assignment conducted according to the MTE.

The first fundamental result that has been presented is the MDTA model for the multiple origins and a single destination general case. The approach applies the notion that a motorist decides how to move forward considering what is left of the trip, and does not decide according to his/her origin once he/she enters the transport network. To represent that, the *reasonable arc* concept is introduced, which is an arc that, once traveled through, takes the motorist not farther from the destination if minimum cost routes are meant to be taken. Then, it is assumed that motorists only travel through reasonable arcs. As origins do not influence the formulations in the modelling process and given that there is a single destination, the flow rates that move through the network only have one type of motorist. Even though this version of the model still does not cover all cases of transport networks, it is an important contribution and a fundamental step for what has been accomplished in this research and, particularly, in this doctoral thesis.

Next, using as foundation the previously commented result, and with the goal of developing a model that is able to cover the multiple origins and multiple destination general case, the MDTA model for general transport networks, the main contribution of this work, is presented. As the previous model is not able to work with multiple destinations and the general transport network case is understood as the multiple origins and multiple destinations general case, it was necessary to integrate the first version with a reasoning that allows identifying different destinations on the network. In order to do this, the formulations were adapted for representing the interactions of multiple types of motorists in the arcs, one type defined for each destination of the network. While the idea may seem straightforward to apply at first, it is not. This, given that the model needs to represent the within-arc interactions of all types of motorists and, simultaneously, represent the aggregated aspects that affect these interactions. For example, in the case of arcs that get congested because the sum of flow rates going to all destinations overpasses the queue unloading capacity, where this phenomenon affects independently the way in which the motorists going to each destination leave the arc. The notion of reasonable arc is still applied but extended to a *reasonable arc towards a particular destination* concept, because in this general model, an arc can be reasonable for some destinations but not for others. Then, it is assumed that a motorist going to a particular destination is only using reasonable arcs towards that destination. The arc-choice modelling process is an extension from its previous version, as every formulation that was conducted for the single destination is now repeated for all the destinations. The traffic model, on the other hand, required a different type of treatment, as the aggregated behaviour of all motorists of an arc affects the congestion and, subsequently, affects the queue lengths and the outflow rates going to each particular destination. With this, the MDTA model has been accomplished as a complete model that is able to work with general transport networks.

Apart from presenting a novel approach to tackle the dynamic traffic assignment problem with stochasticity, the MDTA model in both versions, has properties that are not usually found in DTA models from the literature, particularly in those approaches that consider uncertainty. Given the arc-based approach rather than the usually assumed route-based approach, and along with the within-arc interactions defined and formulated for the traffic model, the MDTA framework allows working with overlapping routes. This, given that the route selection is actually a recursive decision process over the arcs. From applying this reasoning, it happens that independence on the route costs is not assumed, as the formulations are constructed according to the arcs. Thus, the only aspect regarding routing behaviour, which is the computation of the expected minimum costs from a current node to the destination experienced by the motorist, is constructed through nested arc costs operators. Also, route enumeration, usually applied to analyze and compare the motorists options, is not required. In other aspect, even though the arc-choice model assigns the inflow rates according to the expected minimum costs through a logit rule, it is not limited only to this as, given its construction, it has the potential of using different models to perform the assignment. The same can be concluded for the costs functions, where other models, apart from the deterministic punctual queueing model, can be potentially used.

Other important results that are worth to be highlighted, which are considered strong features of this doctoral thesis, are the solution methods developed for both versions of the MDTA model: the *MOSD-MDTA algorithm*, for the multiple origins and a single destination general case, and the *MOMD-MDTA algorithm*, for the case of general transport networks. Both methods allow getting solutions for a discretized version of their respective problems. They work efficiently considering the fact that it is expected to have a notorious computational effort in the execution of the algorithms, since a dynamic and repeated computing of the flow assignment has to be executed. Along with presenting an analysis regarding the results of their computational implementations testing, two technical aspects were studied. First, it is concluded that one second is an appropriate timestep size for the studied cases, which sets a minimum on this parameter for the general case. However, it can be tested if larger values could be equally significant if real case networks are meant to be analyzed, depending on the scale of its studied time period and its free flow travel times. Then, it is studied the range of dispersion parameters for the logit model that would lead to feasible interpretations. This analysis, even though is conducted according to the particular studied case, can be replicated for other transport networks, in order to obtain the range for this parameter that would have realistic interpretations. Also, the construction of the algorithms allows initializations with transport networks that are not necessary empty. From this feature, it can be studied how an already loaded network empties on time if an MDTA is applied. Even though these results are complementary to the core of this doctoral thesis, they are important contributions, as they enforce the applicability of the model, and remarkable accomplishments, as traffic assignment solution methods are already complex to deal with. In this work, efficient methods that solves the proposed DTA arc-based approach through elaborated dynamic programming algorithms were developed, which is also a major achievement of this research.

As for the ongoing research regarding the use of the *set of expected reasonable arcs towards a destination*, it offers a new point of view in how to represent the motorists behaviour when traveling through the network. This, by assuming that they use their current perceived cost to decide whether an arc is reasonable or not. This approach, rather than being an extension of the previously proposed model, presents an alternative in how to model the reasoning that motorists apply behind their decisions.

Before closing this document, it is worth pointing out that this doctoral thesis achieves what was established as the long term goal proposed in its initial project stage: the development of a new dynamic traffic assignment model to tackle stochasticity. Under the novel approach here proposed, this research has been able to accomplish the contribution of the MDTA model, along with solution methods for two versions, the MOSD-MDTA algorithm and the MOMD-algorithm. Among the potential research opportunities that this work and its results open, there is special interest from the author in the following aspects:

- Study situations in which the network is not empty and not necessary in an equilibrium state and apply a MDTA model to analyze how it empties;
- Study the response to incidents and capacity reductions, not all known in advance;
- Use the algorithms outputs as an initial solution and apply an improvement method, such as *MSA*;
- Use of different traffic models, such as Friesz' *Divided Link model* [25];
- Use of different route-choice models, such as the *Probit model*.

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Appendix

A. MOSD-MDTA algorithm pseudocodes

Notation

(N, A)	Diagraph with set of nodes N and set of arcs A
$O \subseteq N$	Set of origins
$d \in N$	Destination
ϕ_a	Free flow travel time of arc a
ϕ	Vector aggregation of ϕ_a, for all $a \in A$
Q_a	Queue unloading capacity of arc a
Q	Vector aggregation of Q_a, for all $a \in A$
T	Length of the period of time to analize
Δt	Timestep size
K	Amount of time increments
$\mathcal{D}_o(\cdot)$	Demand rate from origin o to the destination
$\mathcal{D}(\cdot)$	Vectorial function aggregation of $\mathcal{D}_o(\cdot)$, for all $o \in O$
θ	Dispersion parameter for the logit model
E_a^k	Inflow rate of arc a at time increment k
E	Matrix aggregation of E_a^k, for all $a \in A$ at every $k = 1, ..., K$
G_a^k	Outlow rate of arc a at time increment k
G	Matrix aggregation of G_a^k, for all $a \in A$ at every $k = 1, ..., K$
L_a^k	Queue length of arc a at time increment k
L	Matrix aggregation of L_a^k, for all $a \in A$ at every $k = 1, ..., K$
A_i^+	Set of outgoing arcs from node i
$\mathcal{A}^+$	Family aggregation of sets A_i^+, for all $i \in N$
A_i^-	Set of incoming arcs to node i
$\mathcal{A}^-$	Family aggregation of sets A_i^-, for all $i \in N$
C_a^k	Total cost of arc a at time increment k
C	Matrix aggregation of C_a^k, for all $a \in A$ at every $k = 1, ..., K$
S_i	Minimum cost of going from i to the destination
S	Vector aggregation of S_i, for all $i \in N$
π	Increasing order of all nodes $i \in N$ according to S_i
R	Set of reasonable arcs towards the destination

Z_a^k	Expected minimal cost from initial node of a to the destination, throuh a, at k
Z	Matrix aggregation of Z_a^k, for all $a \in A$ at every $k = 1, ..., K$
W_i^k	Expected minimal cost from node i to the destination at k
$\overline{\mathcal{D}}_o^k$	Average Demand rate from origin o at time increment k
$\overline{\mathcal{D}}$	Matrix aggregation of $\overline{\mathcal{D}}_o^k$, for all $o \in O$ at every $k = 1, ..., K$
F_a^k	Assignment Factor of arc a at time increment k

MOSD-MDTA main algorithm

The complete pseudocode of the *MOSD-MDTA algorithm* is given by:

Algorithm 3 MOSD-MDTA $((N, A), O, d, \phi, Q, T, \Delta t, \mathcal{D}(\cdot), \theta) = (E, G, L)$

STEP 0: INITIALIZATION:
$K \leftarrow \dfrac{T}{\Delta t}$
for all $i \in N$ **do**
    $A_i^+ \leftarrow \{(i, j) \in A : j \in N\}$
    $A_i^- \leftarrow \{(j, i) \in A : j \in N\}$
    $S_i \leftarrow \text{MinCost}(i, d, \phi)$
end for
$\pi \leftarrow \text{IncreasingOrder}(N, d, S)$
$R \leftarrow \varnothing$
for all $a = (i, j) \in A$ **do**
    if $S_j \leq S_i$ **then**
        $R \leftarrow R \cup \{a\}$
    end if
end for
for all $k = 1, ..., K$ **do**
    for all $a \in A$ **do**
        $E_a^k \leftarrow 0$
        $G_a^k \leftarrow 0$
        $L_a^k \leftarrow 0$
        $C_a^k \leftarrow \phi_a$
    end for
    for all $i \in N$ **do**
        if $i \in O$ **then**
            $\overline{\mathcal{D}}_i^k \leftarrow \dfrac{\int_{(k-1)\Delta t}^{k\Delta t} \mathcal{D}_i(t)\, dt}{\Delta t}$
        else
            $\overline{\mathcal{D}}_i^k \leftarrow 0$
        end if
    end for
end for
for all $k = 1, ..., K$ **do**
    STEP 1: BACKWARD:
    $Z \leftarrow \text{MOSD-Backward}((N, A), d, C, \mathcal{A}^+, \mathcal{A}^-, K, R, \pi, \theta)$

STEP 2: ASSIGNMENT FACTORS COMPUTING:
for all $a \in A$ **do**
  if $a \in R$ **then**
    $F_a^k = \exp\left(-\theta Z_a^k\right)$
  else
    $F_a^k = 0$
  end if
end for
STEP 3: FORWARD:
$(E, G, L) \leftarrow \text{MOSD-Forward}\big((N, A), O, \mathcal{A}^-, \mathcal{A}^+, E, G, L, k, \phi, Q, \overline{\mathcal{D}}\big)$
STEP 4: COSTS UPDATE:
for all $a \in A$ **do**
  if $k < K$ **then**
    $C_a^{k+1} \leftarrow \phi_a + \dfrac{L_a^{k+\phi_a}}{Q_a}$
  end if
end for
STEP 5: STOP CONDITION:
if $\displaystyle\sum_{l=k+1}^{l=K}\left(\sum_{i \in N}\left(\sum_{b \in A_i^-} G_b^l + \overline{\mathcal{D}}_i^l\right) + \frac{\sum_{a \in A} L_a^{l+\phi_a-1}}{\Delta t}\right) = 0$ **then**
  STOP
end if
end for

Generic MOSD-MDTA subalgorithms

While *MinCost* can be any algorithm that computes the minimum cost between two nodes and *IncreasingOrder* can be any algorithm that orders increasingly the elements of a set according to their given associated values, starting by the destination, *MOSD-Backward algorithm* and *MOSD-Forward algorithm* were specially developed for this work.

MOSD-Backward algorithm

Algorithm 4 MOSD-Backward$\big((N, A), d, C, \mathcal{A}^+, \mathcal{A}^-, K, R, \pi, \theta\big) = Z$

for all $k = 1, \ldots, K$ **do**
  for all $i \in N$ **do**
    $W_i^k \leftarrow \infty$
  end for
  for all $a \in A$ **do**
    $Z_a^k \leftarrow \infty$
  end for
end for
for all $j \in N$ in the order given by π **do**
  for all $a \in A_j^-$ **do**
    for all $k = 1, \ldots, K$ **do**

if $j = d$ **then**
if $a \in R$ **then**
$W_j^k \leftarrow 0$
$Z_a^k \leftarrow C_a^k$
end if
else
if $a \in R$ and $k + \lfloor C_a^k \rfloor \leq K$ **then**
$W_j^k \leftarrow -\frac{1}{\theta} \log \left(\sum_{b \in A_j^+} \exp \left(-\theta Z_b^{k+\lfloor C_a^k \rfloor} \right) \right)$
$Z_a^k \leftarrow C_a^k + W_j^{k+\lfloor C_a^k \rfloor}$
end if
end if
end for
end for
end for

MOSD-Forward algorithm

Algorithm 5 MOSD-Forward$\left((N, A), O, \mathcal{A}^-, \mathcal{A}^+, E, G, L, k, \phi, Q, \overline{\mathcal{D}}\right) = (E, G, L)$

for all $i \in N$ **do**
if $\sum_{b \in A_i^-} G_b^k + \overline{\mathcal{D}}_i^k + \frac{\sum_{a \in A_i^+} L_a^{k+\phi_a - 1}}{\Delta t} > 0$ **then**
$F_{sum} \leftarrow \sum_{b \in A_i^+} F_b^k$
for all $a \in A_i^+$ **do**
if $i \in O$ **then**
$E_a^k \leftarrow \frac{F_a^k}{F_{sum}} \left(\sum_{b \in A_i^-} G_b^k + \overline{\mathcal{D}}_i^k \right)$
else
$E_a^k \leftarrow \frac{F_a^k}{F_{sum}} \sum_{b \in A_i^-} G_b^k$
end if
if $\frac{L_a^{k+\phi_a-1}}{\Delta t} + E_a^k \leq Q_a$ **then**
$G_a^{k+\phi_a} \leftarrow \frac{L_a^{k+\phi_a-1}}{\Delta t} + E_a^k$
$L_a^{k+\phi_a} \leftarrow 0$
else
$G_a^{k+\phi_a} \leftarrow Q_a$
$L_a^{k+\phi_a} \leftarrow L_a^{k+\phi_a-1} + (E_a^k - Q_a)$
end if
end for
end if
end for

B. MOMD-MDTA algorithm pseudocodes

Notation

MOMD-MDTA algorithm and it subalgorithms use the following notations:

(N, A)	Diagraph with set of nodes N and set of arcs A
$O \subseteq N$	Set of origins
$D \in N$	Set of destinations
$OD \in N \times N$	Set of O-D pairs
ϕ_a	Free flow travel time of arc a
ϕ	Vector aggregation of ϕ_a, for all $a \in A$
Q_a	Queue unloading capacity of arc a
Q	Vector aggregation of Q_a, for all $a \in A$
T	Length of the period of time to analize
Δt	Timestep size
K	Amount of time increments
$\mathcal{D}_{(o,d)}(\cdot)$	Demand rate of O-D pair (o, d)
$\mathcal{D}(\cdot)$	Vectorial function aggregation of $\mathcal{D}_{(o,d)}(\cdot)$, for all $(o, d) \in OD$
θ	Dispersion parameter for the logit model
E^k_{ad}	Inflow rate of arc a going to d at time increment k
E_d	Matrix aggregation of E^k_a, for all $a \in A$, at every $k = 1, ..., K$
$\mathcal{E}$	Set aggregation of E_d, for all $d \in D$
G^k_{ad}	Outlow rate of arc a going to d at time increment k
G_d	Matrix aggregation of G^k_a, for all $a \in A$, at every $k = 1, ..., K$
$\mathcal{G}$	Set aggregation of G_d, for all $d \in D$
L^k_{ad}	Queue length of arc a going to d at time increment k
L_d	Matrix aggregation of L^k_a, for all $a \in A$, at every $k = 1, ..., K$
$\mathcal{L}$	Set aggregation of L_d, for all $d \in D$
A^+_i	Set of outgoing arcs from node i
$\mathcal{A}^+$	Family aggregation of sets A^+_i, for all $i \in N$
A^-_i	Set of incoming arcs to node i
$\mathcal{A}^-$	Family aggregation of sets A^-_i, for all $i \in N$
C^k_a	Total cost of arc a at time increment k
C	Matrix aggregation of C^k_a, for all $a \in A$ at every $k = 1, ..., K$
S_{id}	Minimum cost of going from i to d
S_d	Vector aggregation of S_{id}, for all $i \in N$
π_d	Increasing order of all nodes $i \in N$ according to S_i
Π	Set family of orders π_d, for all $d \in D$
R_d	Set of reasonable arcs towards destination d
$\mathcal{R}$	Family of sets R_d, for all $d \in D$
Z^k_{ad}	Expected minimal cost from initial node of a to d, throuh a, at k
Z_d	Matrix aggregation of Z^k_{ad}, for all $a \in A$, at every $k = 1, ..., K$
$\mathcal{Z}$	Set aggregation of Z_d, for all $d \in D$
W^k_{id}	Expected minimal cost from node i to d at k

$\overline{\mathcal{D}}^k_{(o,d)}$	Average Demand rate of O-D pair (o,d) at time increment k
$\overline{\mathcal{D}}$	Vector aggregation of $\overline{\mathcal{D}}^k_{(o,d)}$, for all $(o,d) \in OD$ at every $k = 1,...,K$
F^k_{ad}	Assignment Factor of arc a going to d at time increment k

MOMD-MDTA main algorithm

The complete pseudocode of *MOMD-MDTA algorithm* is given by:

Algorithm 6 MOMD-MDTA $((N,A),O,D,\phi,Q,T,\Delta t,\mathcal{D}(\cdot),\theta) = (\mathcal{E},\mathcal{G},\mathcal{L})$

STEP 0: INITIALIZATION:
$K \leftarrow \dfrac{T}{\Delta t}$
for all $i \in N$ **do**
  $A_i^+ \leftarrow \{(i,j) \in A : j \in N\}$
  $A_i^- \leftarrow \{(j,i) \in A : j \in N\}$
  for all $d \in D$ **do**
    $S_{id} \leftarrow \text{MinCost}(i,d,\phi)$
  end for
end for
for all $d \in D$ **do**
  $\pi_d \leftarrow \text{IncreasingOrder}(N,d,S_d)$
  $R_d \leftarrow \varnothing$
  for all $a = (i,j) \in A$ **do**
    if $S_{jd} \le S_{id}$ **then**
      $R_d \leftarrow R_d \cup \{a\}$
    end if
  end for
end for
for all $k = 1,...,K$ **do**
  for all $a \in A$ **do**
    for all $d \in D$ **do**
      $E^k_{ad} \leftarrow 0$
      $G^k_{ad} \leftarrow 0$
      $L^k_{ad} \leftarrow 0$
    end for
    $C^k_a \leftarrow \phi_a$
  end for
  for all $(i,d) \in N \times D$ **do**
    if $(i,d) \in OD$ **then**
      $\overline{\mathcal{D}}^k_{(i,d)} \leftarrow \dfrac{\int_{(k-1)\Delta t}^{k\Delta t} \mathcal{D}_{(i,d)}(t)\,dt}{\Delta t}$
    else
      $\overline{\mathcal{D}}^k_{(i,d)} \leftarrow 0$
    end if
  end for

end for
for all $k = 1, ..., K$ **do**
  STEP 1: BACKWARD:
  $\mathcal{Z} \leftarrow$ MOMD-Backward$\big((N, A), D, C, \mathcal{A}^{+}, \mathcal{A}^{-}, K, \mathcal{R}, \Pi, \theta\big)$
  STEP 2: ASSIGNMENT FACTORS COMPUTING:
  for all $d \in D$ **do**
    for all $a \in A$ **do**
      if $a \in R$ **then**
        $F_a^k = \exp\left(-\theta Z_a^k\right)$
      else
        $F_a^k = 0$
      end if
    end for
  end for
  STEP 3: FORWARD:
  $(\mathcal{E}, \mathcal{G}, \mathcal{L}) \leftarrow$ MOMD-Forward$\Big((N, A), O, D, \mathcal{A}^{-}, \mathcal{A}^{+}, \mathcal{E}, \mathcal{G}, \mathcal{L}, k, \phi, Q, \overline{\mathcal{D}}\Big)$
  STEP 4: COSTS UPDATE:
  for all $a \in A$ **do**
    if $k < K$ **then**
      $$C_a^{k+1} \leftarrow \phi_a + \frac{\sum_{d \in D} L_{ad}^{k+\phi_a}}{Q_a}$$
    end if
  end for
  STEP 5: STOP CONDITION:
  if $\sum_{l=k+1}^{l=K} \left(\sum_{d \in D} \left(\sum_{i \in N} \sum_{b \in A_i^-} G_{bd}^l + \frac{\sum_{a \in A} L_{ad}^{l+\phi_a-1}}{\Delta t} \right) + \sum_{(o,d) \in OD} \overline{\mathcal{D}}_{(o,d)}^l \right) = 0$ **then**
    STOP
  end if
end for

Generic MOMD-MDTA subalgorithms

Again, as in its previous version, the *MOSD-MDTA algorithm*, *MinCost* and *IncreasingOrder* can be any algorithm for minimum cost routes and for ordering, respectivelly. *MOMD-Backward algorithm* and *MOMD-Forward algorithm* were addapted from their previous versions for the general case.

MOMD-Backward algorithm

Algorithm 7 MOMD-Backward$\big((N, A), D, C, \mathcal{A}^{+}, \mathcal{A}^{-}, K, \mathcal{R}, \Pi, \theta\big) = \mathcal{Z}$

for all $k = 1, ..., K$ **do**
  for all $d \in D$ **do**
    for all $i \in N$ **do**
      $W_{id}^k \leftarrow \infty$

end for
for all $a \in A$ **do**
$Z^k_{ad} \leftarrow \infty$
end for
end for
end for
for all $d \in D$ **do**
for all $j \in N$ in the order given by π_d **do**
for all $a \in A^-_j$ **do**
for all $t = 1, \ldots, K$ **do**
if $j = d$ **then**
if $a \in R_d$ **then**
$W^t_{jd} \leftarrow 0$
$Z^t_{ad} \leftarrow C^t_a$
end if
else
if $a \in R_d$ and $t + \lfloor C^t_a \rfloor \le K$ **then**
$$W^t_{jd} \leftarrow -\frac{1}{\theta} \log \left(\sum_{b \in A^+_j} \exp\left(-\theta Z^{t+\lfloor C^t_a \rfloor}_{bd} \right) \right)$$
$Z^t_{ad} \leftarrow C^t_a + W^{t+\lfloor C^t_a \rfloor}_{jd}$
end if
end if
end for
end for
end for
end for

MOMD-Forward algorithm

Algorithm 8 MOMD-Forward$\left((N, A), D, OD, \mathcal{A}^-, \mathcal{A}^+, \mathcal{E}, \mathcal{G}, \mathcal{L}, k, \phi, Q, \overline{\mathcal{D}}\right) = (\mathcal{E}, \mathcal{G}, \mathcal{L})$

for all $i \in N$ **do**
if $\sum_{d \in D} \left(\sum_{b \in A^-_i} G^k_{bd} + \overline{\mathcal{D}}^k_{(i,d)} + \frac{\sum_{a \in A^+_i} L^{k+\phi_a-1}_{ad}}{\Delta t} \right) > 0$ **then**
for all $d \in D$ **do**
$F_{sum,d} \leftarrow \sum_{b \in A^+_i} F^k_{bd}$
end for
for all $a \in A^+_i$ **do**
for all $d \in D$ **do**
if $(i, d) \in OD$ **then**
$$E^k_{ad} \leftarrow \frac{F^k_{ad}}{F_{sum,d}} \left(\sum_{b \in A^-_i} G^k_{bd} + \overline{\mathcal{D}}^k_{(i,d)} \right)$$
else

$$E_{ad}^{k} \leftarrow \frac{F_{ad}^{k}}{F_{sum,d}} \sum_{b \in A_i^-} G_{bd}^{k}$$

end if

end for

if $\dfrac{\sum_{d \in D} L_{ad}^{k+\phi_a-1}}{\Delta t} + \sum_{d \in D} E_{ad}^{k} \leq Q_a$ **then**

for all $d \in D$ **do**

$$G_{ad}^{k+\phi_a} \leftarrow \frac{L_{ad}^{k+\phi_a-1}}{\Delta t} + E_{ad}^{k}$$

$$L_{ad}^{k+\phi_a} \leftarrow 0$$

end for

else

for all $d \in D$ **do**

$$G_{ad}^{k+\phi_a} \leftarrow \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum_{d' \in D} \left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t \right)} Q_a$$

$$L_{ad}^{k+\phi_a} \leftarrow L_{ad}^{k+\phi_a-1} + \left(E_{ad}^{k} - \frac{L_{ad}^{k+\phi_a-1} + E_{ad}^{k}\Delta t}{\sum_{d' \in D} \left(L_{ad'}^{k+\phi_a-1} + E_{ad'}^{k}\Delta t \right)} Q_a \right) \Delta t$$

end for

end if

end for

end if

end for