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RISK ANALYSIS OF THE COPPER CONCENTRATE SALE MODEL

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RISK ANALYSIS OF THE COPPER CONCENTRATE SALE MODEL

This thesis focuses on presenting the relevance of the risk analysis of the copper concentrate sale model, through the analysis of a real case study based of the historical sale data one of the copper mining company in Chile.

The methodology in this thesis contains a real case study of the copper concentrate sales contracts with smelters and traders, concluding with this relevant information that provide the mining company with information that supports future decision making process.

In general, copper smelting business consists of the gross profit elements, among the other, copper treatment charges and refining charges, gold and silver refining charges, free metals, by-product credits, metal premiums and penalties. Therefore, both smelters and traders, put the biggest pressure and are focused only on those elements, especially the copper treatment charges and refining charges.

The uncertainty of the value of the copper treatment charges and refining charges was a motivation to develop the copper concentrate sale's analysis model. The analysis model considers historical copper treatment charges and refining charges, gold and silver refining charges, as well as paid penalties in order to optimize annual future sales distribution. To perform this analysis model considering the uncertainty and verifiability to predict the future outcome, the Monte Carlo simulation as high-tech quantitative analysis technique was used.

The results obtained from the analysis model suggests that the annual optimal distribution is Trader A and Trader AAA of 15 lots each, Smelter B of 12 lots and Other clients of 41 lots. Having this optimal distribution, total copper treatment charges and refining charges, gold and silver refining charges, as well as paid penalties are amounted to US$ 87,524,294. However, the company uses this distribution Trader A, AA, AAA of six lots each, Smelter B and BB of five lost each, Smelter BBB of 25 lots and Other clients of 30 lots. This company distribution is amounted to US$ 109,651,518. Therefore, the difference gives the profit of US$ 22,002,650, which means lower cash outflows. Applied methodology allows generating savings by 20.1% in the expenses of the company.

Based on the above mentioned result, recommended optimal copper concentrate sale distribution under uncertainty will incorporate greater agility and methodological flexibility to the execution of copper concentrate sale with more integrated vision to limit the risk of loss of cash flow and improve the economic value of the company.

RESUMEN DE LA MEMORIA PARA OPTAR AL GRADO DE: Magíster en Gestión y Dirección de Empresas
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ANÁLISIS DE RIESGOS DEL MODELO DE VENTA DE CONCENTRADO DE COBRE

Esta tesis se enfoca en presentar la relevancia del análisis de riesgo del modelo de venta de concentrado de cobre, a través del análisis de un estudio de caso real basado en los datos históricos de venta de cobre de la empresa minera en Chile.

La metodología de esta tesis contiene un estudio de caso real de los contratos de venta de concentrado de cobre con fundiciones y distributores, concluyendo con esta información relevante que brinda a la empresa minera información que respalda la futura toma de decisiones.

En general, el negocio de fundición de cobre se compone de los elementos de utilidad bruta, entre otros, cargos por tratamiento y refinación de cobre, oro y plata, metales libres, créditos por subproductos, primas y multas por metales. Por lo tanto, fundiciones y distributores ejercen la mayor presión y se centran solo en esos elementos, especialmente los cargos por tratamiento y refinación de cobre.

La incertidumbre del valor de los cargos por tratamiento y por refinación de cobre fue una motivación para desarrollar el modelo de análisis de venta de concentrado de cobre. El modelo de análisis considera los cargos históricos de tratamiento y refinación de cobre, de oro y plata, así como las multas pagadas con el fin de optimizar la distribución anual de las ventas futuras. Para realizar este modelo de análisis considerando la incertidumbre y verificabilidad para predecir el resultado futuro, se utilizó la simulación de Monte Carlo como técnica de análisis cuantitativo de alta tecnología.

Los resultados obtenidos del modelo de análisis sugieren que la distribución óptima anual es Trader A y Trader AAA de 15 lotes cada uno, Fundición B de 12 lotes y Otros clientes de 41 lotes. Con esta distribución óptima, los cargos totales por tratamiento y refinación de cobre, de oro y plata, así como las multas pagadas ascienden a US $ 87.524.294. Sin embargo, la empresa utiliza esta distribución Trader A, AA, AAA de seis lotes cada uno, Fundición B y BB de cinco lotes cada uno, Fundición BBB de 25 lotes y Otros clientes de 30 lotes. Esta distribución de la empresa asciende a US $ 109.651.518. Por lo tanto, la diferencia arroja una ganancia de US $ 22,002,650, lo que significa disminución de flujo de caja. La metodología aplicada permite generar un ahorro del 20,1% en los gastos de la empresa.

Con base en el resultado, la distribución óptima recomendada de venta de concentrado de cobre bajo incertidumbre incorporará mayor agilidad y flexibilidad metodológica para la ejecución de la venta de concentrado de cobre con una visión más integrada para limitar el riesgo de pérdida de flujo de caja y mejorar el valor económico de la compañía.

DEDICATION

Pracę dedykuje Mojej Żonie w podziękowaniu za Jej bezwarunkową miłośc.
Dziękuje, że dzięki Tobie mogłem rozwinąć skrzydła.

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1. INTRODUCTION

1.1. General introduction

Risk analysis is an emerging science, and it is a decision-making paradigm. Terje Aven (2018) makes a powerful argument for risk analysis as a new emerging science. Although it is rapidly developing, it is not yet widely regarded as a science unto itself. As a paradigm, it is capable of producing knowledge about risks and risky activities in the real world. As a science, it also produces knowledge about concepts, theories, frameworks, methods, and the like to understand, assess, communicate, and manage risks. This latter knowledge set makes risk analysis as much a science as statistics is, for example. Especially those who practice enterprise risk management frequently refer to the risk analysis paradigm presented in this text as risk management [19].

A risk and an opportunity can be considered the opposite sides of the same coin, i.e. a positive and negative. It is usually easiest to consider a potential event to be a risk, if it would have a negative impact and its probability is less than 50%. If the risk has a probability in excess of 50 %, can be included it in a base plan and then consider the opportunity of it not occurring [18].

The main objective of risk analysis is to establish a rational foundation for an objective decision-making. The risk analysis aims at quantifying the undesirable effects that a given activity may impose on economical values. The objective of the decision process is then to identify the solution that in some sense minimizes the risk of the considered activity [3].

The copper concentrate sales process is a complicated process and contains many variables that depend on movements in world markets. When creating a sales model and selecting clients, the most common is to consider the price offered for a ton of copper concentrate, mainly limited to determining the value of copper Treatment Charges-TC and Refining Charges-RC, together TCRCs.

TCRCs are commonly used in the terms of purchase for copper concentrate or nickel ore for refining. They are amounts designed to cover refining costs, i.e. the remuneration received by a smelter for processing smelting material and extracting metals. Copper concentrate contracts may define a purchase price based on the London Metal Exchange (LME) price at a certain date, minus the TCRCs being used at the time.

Copper TCRCs are a keystone for pricing copper concentrates, which are the actual feedstock for copper smelters. The potential evolution of TCRCs is a question of both economic and technical significance for miners, as their value decreases the potential final selling price of concentrates. Additionally, copper miners’ revenues are more narrowly related to the market price of copper, as well as to other technical factors such as ore dilution or the grade of the concentrates produced. Smelters, on the contrary, are hugely affected by the discount, which they succeed in getting when purchasing the concentrates, since that makes up the largest part of their gross revenue, besides other secondary sources. In addition, eventual differences between TCRCs may give commodity traders ludicrous arbitrage opportunities. In addition, differences between short- and long-term TCRCs agreements offer arbitrage opportunities for traders, hence comprising a part of their revenue in the copper concentrate trading business, as well copper price fluctuations and the capacity to make economically optimum copper blends [7].

The Net Smelter Revenue (NSR) method is commonly used to analyze the economic impact of the concentration grade of enriched minerals on mine revenue in the light of processing costs and metal market prices. The method involves calculating profits achievable from the sale of the main product of the mine, i.e. the Cu concentrate, after deduction of the TCRCs charges. This is an important piece of information for the mine, which may be a criterion for optimizing extraction and beneficiation of ore according to the quality of the concentrates [10].

In general, smelting business consists of the following gross profit elements, among the other, copper TCRCs, gold and silver refine charges (Au & Ag RCs), free metals, By-product credits, metal premiums and penalties. Therefore, mining clients, both smelters and traders, put the biggest pressure and are focused only on those elements, in particular copper TCRCs.

Unfortunately, other key conditions such as Au & Ag RCs, penalties for purity of concentrate, quotation period (QP), transportation, customer financial position, payment terms and environmental protection etc. are ignored in this process. Each of the conditions carries different risks, which can ultimately cause difficulties in completing the sales contract and cause losses to the mining company.

Therefore, the mining company should also consider the other key elements and conditions while performing the risk analysis of sale model and choosing a client.

The human striving of predicting a future outcome has been a wanted skill for many decades. Uncertainty and variability satisfies our inability to be able to predict the future meaning that if we are able to determine these two components we would be able to predict the future outcome [3].

To perform the copper concentrate sale model considering the uncertainty and verifiability to predict the future outcome, the Monte Carlo simulation (MCS) as high-tech quantitative analysis technique is recommended.

The Monte Carlo method is now one of the most powerful and commonly used techniques for analyzing complex problems. The different types of applications can be found in many fields from radiation transport to river basin modeling. Furthermore, it is not only on stochastic processes the MCS is applicable, but also at deterministic problems, this method is usable. There are three major points suggesting a Monte Carlo method instead of traditional simulation methods:

1. In the Monte Carlo method, time does not play as substantial role as it does in stochastic simulation in general.
2. The observations in the Monte Carlo method, as a rule, are independent. In simulation, however, the experiment with the observations is over time so, as a rule, these are serially correlated and hence dependent of each other.
3. In the Monte Carlo method, it is possible to express the responses in a rather straightforward manor by simple functions of the stochastic input variables. In

simulation, the response is often a very complicated one and can only be expressed explicitly by computer programs [3].

This technique not only considers cost and schedule risk for individual activities, but also for the entire project. In many cases, there is the temptation to assume that all project risks must be accounted for in the worst case. The Monte Carlo analysis technique, however, takes a more holistic approach. As such, the total project cost risk and the total project schedule risk are usually expressed as a cumulative probability distribution of total project cost and total project schedule, respectively. Such distribution of information can be used to reflect project risk by computing the probability that the project will be accomplished within particular cost or schedule targets. It can also be used to assess what level of funding or schedule would be required to guarantee success. A computer is necessary to use this technique because the analysis requires repetitive computations [14].

Considering the above, Oracle Crystal Ball tool was selected to realize copper concentrate sale model simulation. Oracle Crystal Ball is a spreadsheet-based application for predictive modeling, forecasting, simulation, and optimization. It gives unparalleled insight into the critical factors affecting risk, which support the right tactical decisions to reach objectives and gain a competitive edge-even in an uncertain market.

This thesis contains an approach to risk analysis (RA) using Monte Carlo simulation (MCS) related to sale of copper concentrate. The thesis is not a full review on how to perform a RA and MCS, nor intended to be a complete and rigorous treatment of probability and statistics. Thus, it has been avoided to give a formal examination of the theory concerning statistics and probability in which case the thesis instead tries to present a more functional description of RA and MCS. However, some statistical terms considered when applying probability distribution functions are discussed. A special emphasis has been put on the separation between inherent randomness in the modeling system and the lack of knowledge. These two concepts have been defined in terms of variability (ontological uncertainty) and uncertainty (epistemic uncertainty).

Based on an example of copper concentrate sale agreements with smelters and traders, the risk of the copper concentrate sale model will be analyzed to find the optimal distribution of the sale limiting the risk of loss of cash flow and prepare the most efficient composition of sales.

2. OBJECTIVES AND METHODOLOGY

2.1. General objetive

The world's copper production is achieved through alternative processes, which depend on the chemical and physical characteristics of the copper ores extracted. According to the USGS 2020 Mineral Commodity Summary on Copper, global identified copper resources contained 2.1 billion tonnes of copper as of 2014, of which about 80% are mainly copper sulphides, whose copper content has to be extracted through pyrometallurgical processes. In 2010, the average grades of ores being mined ranged from 0.5% to 2% Cu, which makes direct smelting unfeasible for economic and technical reasons. Therefore, copper sulphide ores undergo a process known as froth-flotation to obtain concentrates containing ≈ 30% Cu, which makes concentrates the main products offered by copper mines. Concentrates are later smelted and, in most cases, electrochemically refined to produce high-purity copper cathodes. Copper cathodes are generally regarded as pure copper, with a minimum copper content of 99.9935% Cu. Cathodes are normally produced by integrated smelters that purchase concentrates at a discounted price of the copper market price and sell copper cathode at the market price, adding a premium when possible [7].

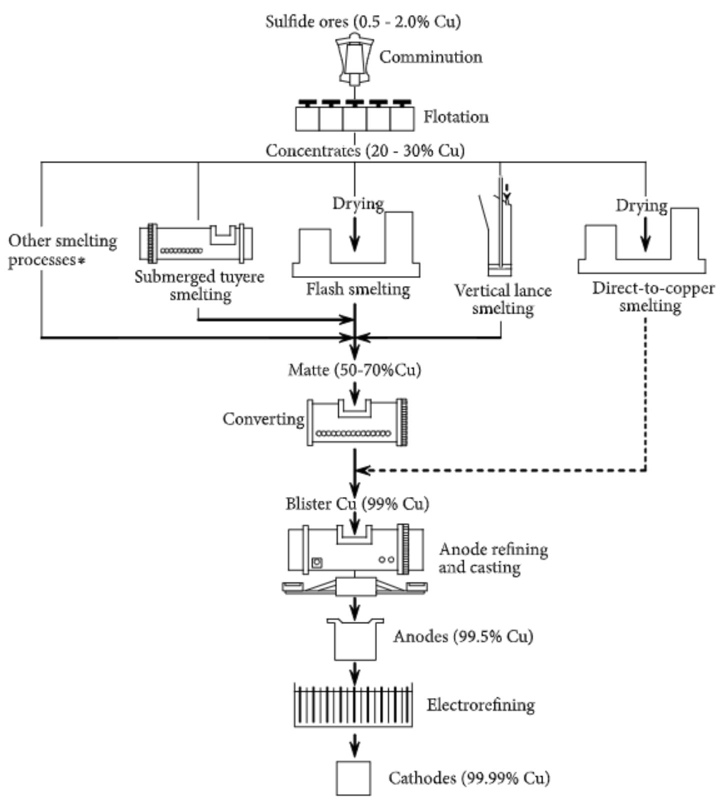


Figure 1 Industrial processing of copper sulphides ore to obtain copper cathodes. (Díaz-Borrego et al., 2019).

The current information available for market participants may be regarded as sufficient to draw an accurate assumption of market sentiment about current TCRCs levels, but not enough to foresee potential market trends regarding these crucial discounts, far less as a reliable tool, which may be ultimately applicable by market participants to their decision-making framework or their risk-management strategies. Hence, from an organizational standpoint, providing accurate forecasts of copper TCRCs benchmark levels, as well as an accurate mathematical model to render these forecasts, is a research topic that is yet to be explored in depth. This is a question with undeniable economic implications for traders, miners, and smelters alike, due to the role of TCRCs in the copper trading revenue stream [7].

The essence of the traditional risk analysis approach is to give the decision-maker a mean by which he can look ahead to the totality of any future outcome. The advantage of using any risk analysis approach is the possibility of differentiating the feature of risk information in terms of outcome criteria such as Net Present Value (NPV), the Internal Rate of Return (IRR) or the Benefit/Cost rate (B/C-Rate) by probability distributions [3].

The thesis contributes to improving the decision-making process related to the sales of copper concentrate through the assessment and analysis of the risks associated based on example of copper concentrate sale agreements with smelters and traders. The objective is to incorporate greater agility and methodological flexibility to the execution of copper concentrate sale with more integrated vision to limit the risk of loss of cash flow and prepare the most efficient composition of sales.

2.2. Particular objetives

Four particular objectives addressed in this document can be summarized in:

- Present the risk management plan of the copper concentrate sale (refer section 3.1).
- Present qualitative and quantitative analysis techniques to analyze risks (refer section 3.2).
- Perform an analysis a copper concentrate sales model using selected Monte Carlo analysis technique (refer chapter 4).
- Recommend the copper concentrate sale model to limit the risk of loss of cash inflows and prepare the most efficient clients' composition (refer chapter 5).

2.3. Methodology

The methodology in this thesis contains a real case study of the copper concentrate sales contracts with smelters and traders, concluding with this relevant information that provide the mining company with information that supports future decision making process.

The thesis is divided into the following parts:

- A first part relates to a presentation of risk assessment process including the guidelines and practice how to perform it.
- A second part relates to the description and selection of the risk analysis techniques that supports the performance of a copper concentrate sale model analysis presented in the third part.
- A third part relates to the performance of a copper concentrate sale model's analysis.
- A fourth part relates to the recommendation of the copper concentrate sale model to limit the risk of loss of cash flow and prepare the most efficient sales' composition.

The risk analysis methodology is a process for identifying and analyzing undesirable events or results of a process, and determining whether the risks are acceptable. If risks are unacceptable, the process may include recommendations and assessments of risk control measures.

MCS has been used in much earlier research as a way of modelling prices that are believed not to follow any specific rule or pattern and hence seen as random. Hence, concerning whether TCRCs might vary randomly, there should not exist a main driving factor that would determine TCRCs future benchmark levels and therefore MCS could to a certain extent to be a feasible model for them.

However, although MCS may be well suited to modelling immediate or short-term price paths for TCRCs, it lacks the ability to include the underlying long-term price trend should be assumed that there is one. Thus, on benchmark TCRCs behavior, levels would move in line with copper concentrate supply and demand as well as the smelters and refineries' available capacity to transform concentrates into metal copper. Hence, a relationship between TCRCs levels and copper supply and demand is known to exist and, therefore, is linked to its market price, so to some extent they move together coherently.

3. RISK ASSESSMENT PROCESS

Risk assessment is a process to determine the probability of losses by analyzing potential hazards and evaluating existing conditions of vulnerability that could pose a threat or harm to property, people, livelihoods and the environment etc. ISO 31000 (2009) defines risk assessment as a process made up of three processes: risk identification, risk analysis, and risk evaluation.

Risk identification is the process that is used to find, recognize, and describe the risks that could affect the achievement of objectives. Risk analysis is the process that is used to understand the nature, sources, and causes of the risks that have been identified and to estimate the level of risk. It is also used to study impacts and consequences and to examine the controls that currently exist. Risk evaluation is the process that is used to compare risk analysis results with risk criteria in order to determine whether or not a specified level of risk is acceptable or tolerable [17].

The result of risk assessment is as shown in Figure 2 using the ‘dendogram’ method where risk assessment comprises of four main elements: (1) approach, (2) definition, (3) process and (4) technique. The ‘qualitative’ and ‘quantitative’ clusters form the ‘approach’ attribute while the ‘risk identification’ and ‘risk analysis’ clusters build up the ‘definition’ attribute. Besides that, the ‘process’ attribute contains standardization, level of approval and risk workshop, with risk identification, risk analysis and preparation, forming the three key elements of risk workshop. The ‘technique’ attribute is divided into qualitative and quantitative that is commonly used within the division of the international engineering sector. The first three qualitative and quantitative techniques are the most commonly used to perform risk assessment [11].

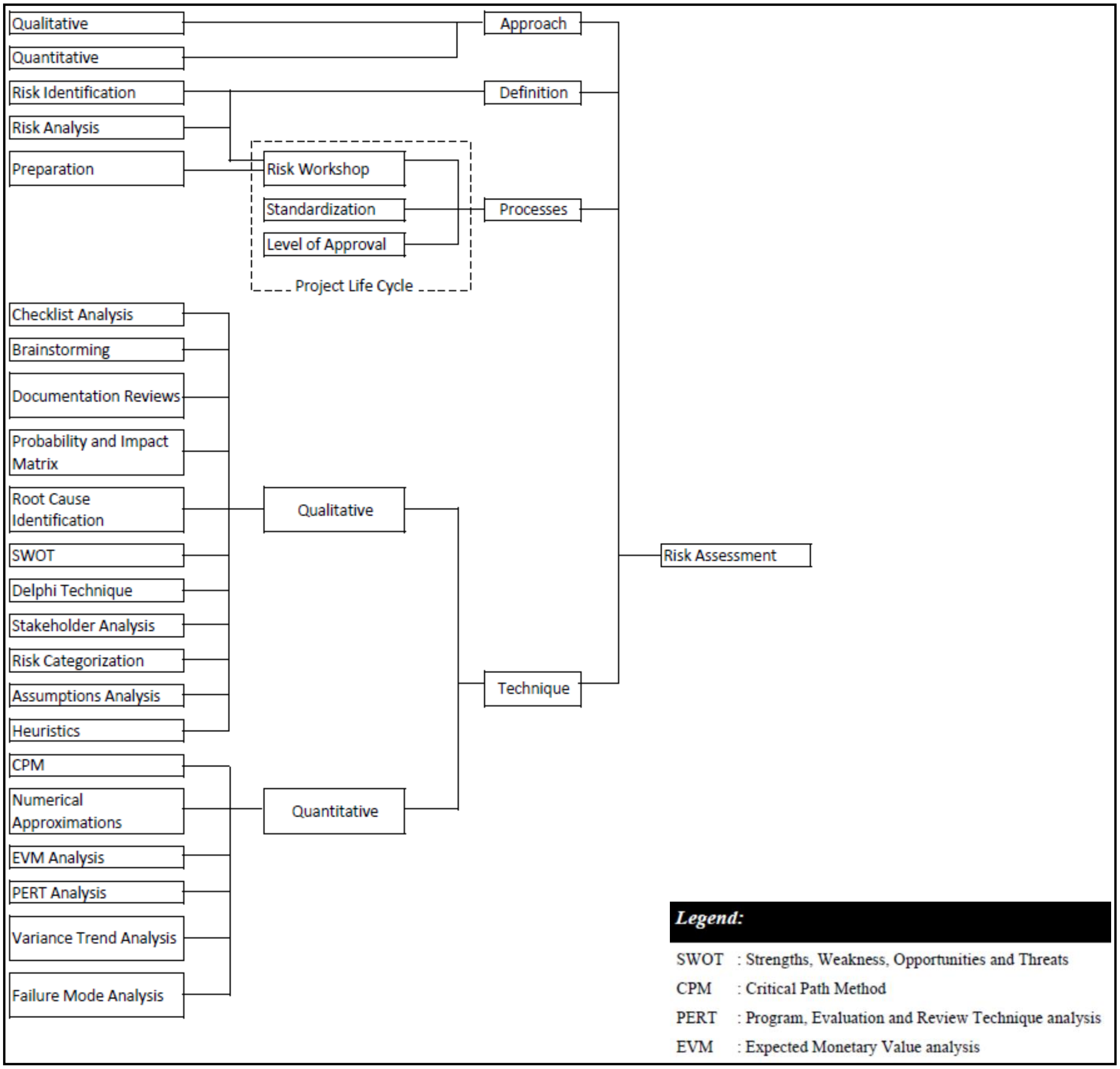


Figure 2 Results of the risk assessment using 'dendogram' method (Muller, 2009)

A complete risk assessment process is likely to consist of five steps:

1. Identification of the risk that is to be analyzed.
2. A qualitative description of the problem and the risk – why it might occur, what you can do to reduce the risk, probability of the occurrence etc.
3. A quantitative analysis of the risk and the associated risk management options that is available to determine or find an optimal strategy for controlling and hereby solving the risk problem.
4. Implementing the approved risk management strategy.
5. Communicating the decision and its basis to various decision-makers [18].

As it was presented above, there are various definitions of the risk assessment process, but all of them have a part in common and there no so different between each other. Based on them, the risk assessment process was presented considering two following steps:

- Risk management plan including identification, estimation, evaluation, control and monitoring of risk.
- Risk analysis techniques including the presentation of the most known quantitative and quantitative techniques, as well as selection of one using for the real case study.

3.1. Risk management plan

Risk management is an important business practice that helps businesses identify, evaluate, track, and mitigate the risks present in the business environment. Risk management is practiced by the business of all sizes; small businesses do it informally, while enterprises codify it.

A risk management plan is a written document that details the organization's risk management process. This process starts by creating a team of stakeholder across the organization to review potential risks to the organization. This stakeholder team should include senior management, the compliance officer, and any department managers.

A generic risk management process as shown in Figure 3 comprising five tasks:

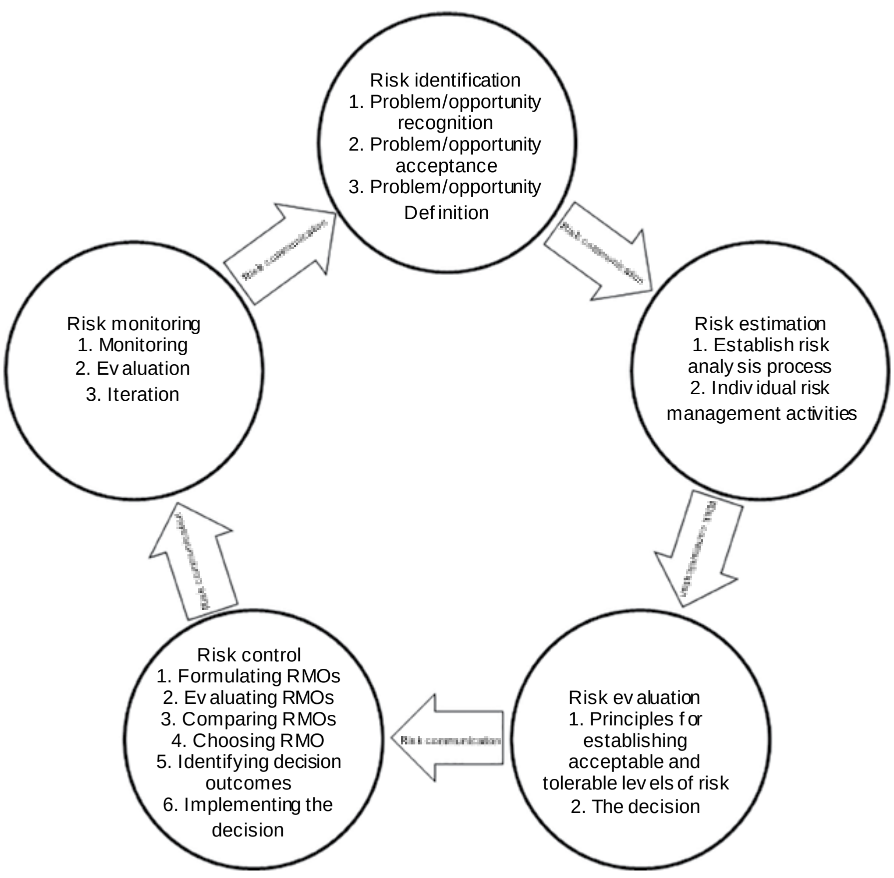


Figure 3 A generic risk management process comprising five tasks. (Yoe, 2019)

It is important to note that the risk management plan should be easily understandable in broader terms during its implementation. As well as with a comprehensive approach on behalf of the customer and in the course of the development of a strategic plan.

3.1.1. Identification of risk

Identification of risk, which is to identify the risks that the business is exposed to in its operating environment. There are many different types of risks – legal risks, environmental risks, market risks, regulatory risks, and much more. It is important to identify as many of these risk factors as possible.

Risk identification process consisting of the following steps was identified [19]:

- Identify the trigger event.
- Identify the hazard or opportunity for uncertain gain.
- Identify the specific harm or harms that could result from the hazard or opportunity for uncertain gain.
- Specify the sequence of events that is necessary for the hazard or opportunity for uncertain gain to result in the identified harm(s).
- Identify the most significant uncertainties in the preceding steps.

Charles Yoe in his book "Principles of Risk Analysis" proposes to think about the risk as a problem. As shown in Figure 4 below the problem is defined as a three-part process.

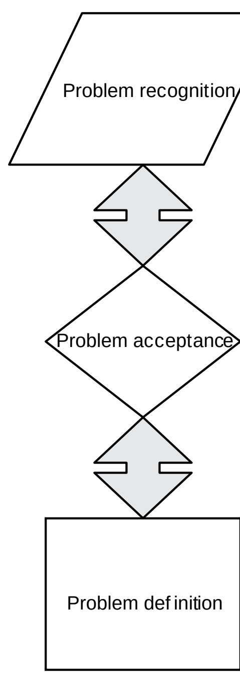


Figure 4 Problem identification steps. (Yoe, 2019)

In the copper concentrate sale, among the other, there can be recognized the following problems:

- TCRCs mean Treatment Charge (TC) and Refining Charge (RC) are commonly used in the terms of purchase for copper concentrate or nickel ore for refining. They are amounts designed to cover refining costs, i.e. the remuneration received by a smelter for processing smelting material and extracting metals. Copper concentrate contracts may define a purchase price based on the LME price at a certain date, minus the TC or RC being used at the time.
- Gold and silver refine charges (Au & Ag RCs) refer to processes used to extract and separate the precious metals in mined material.
- Purity of copper concentrate means raw materials in copper smelting; copper concentrates have a copper content of about 23 - 30% by weight. Copper concentrates are made mostly from sulfide ores, but includes also gold and silver, treated as premium by-products, as well as molybdenum, arsenic, lead, antimony etc.
- Customer financial position means the current balances of the recorded assets, liabilities and equity of an organization. This information is recorded in the balance sheet, which is one of the financial statements.
- Quotation period (QP) means the month prior to the month of scheduled delivery, irrespective of physical delivery.

Form the above-mentioned five risks, two first of them are totally out of control of a mining company, where else three last ones can be controlled or, at least, party controlled.

3.1.2. Estimation of risk

The scope of the risk must be determined. It is also important to understand the link between the risk and different factors within the organization. To determine the severity and seriousness of the risk it is necessary to see how many business functions the risk affects. There are risks that can bring the whole business to a standstill if actualized, while there are risks that will only be minor inconveniences in the analysis. In a manual risk management environment, this analysis must be done manually. When a risk management solution is implemented one of the most important basic steps is to map risks to different documents, policies, procedures, and business processes. This means that the system will already have a mapped risk framework that will evaluate risks and let you know the far-reaching effects of each risk.

Estimating risks is the assessor's job. It cannot be done without direction and guidance from the risk manager. Risk managers have an important, but limited, role in the science-based risk assessment process. The risk manager's positive decision-making role is found in the risk estimation activities (see Figure 5) that help describe the world as it actually is. That role includes establishing the organization's risk analysis process and managing individual risk management activities.

There are two groups of activities in the risk estimation part of risk management, as seen in Figure 5. The first, developing a risk analysis process, consists of one-time or periodic activities required to establish and maintain the risk analysis process. The other, individual risk management activities, consists of duties that recur in every risk management activity.

These activities are shown in the figure.

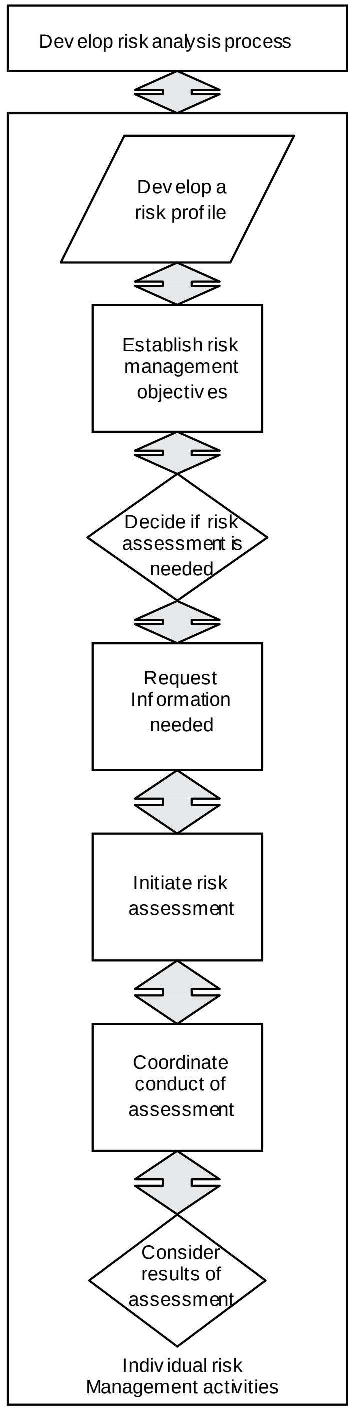


Figure 5 Risk evaluation process (Yoe, 2019)

3.1.3. Evaluating of risk

Risks need to be ranked and prioritized. Risk management solutions have different categories of risks, depending on the severity of the risk. A risk that may cause some inconvenience is rated lowly, risks that can result in catastrophic loss are rated the highest. It is important to rank risks because it allows the organization to gain a holistic view of the risk exposure of the whole organization. The business may be vulnerable to several low-level risks, but it may not require upper management intervention. On the other hand, just one of the highest-rated risks is enough to require immediate intervention.

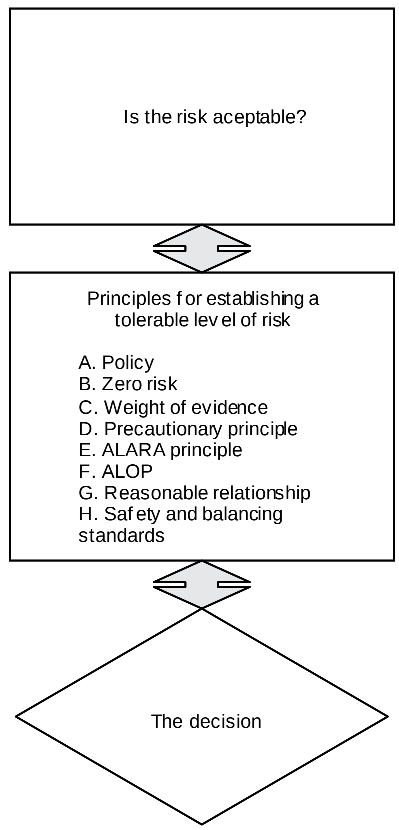


Figure 6 Risk evaluation steps. (Yoe, 2019)

It is conceptually possible to take steps to reduce an unacceptable level of risk to an acceptable level. More often than not, however, unacceptable risks are managed to tolerable levels. A tolerable risk is a non-negligible risk that has not yet been reduced to an acceptable level, think of it as a subset of unacceptable risk. The risk is tolerated for one of three reasons. We may be unable to reduce the risk further; the costs of doing so are considered excessive; or the magnitude of the benefits associated with the risky activity are too great to reduce it further. A tolerable risk is not an acceptable risk, but its severity has been reduced to a point where it is tolerated [19].

As it is mentioned by Yoe in Principles of Risk Analysis. Decision Making Under Uncertainty, page 70 “There is no magic bullet to be found in this section. Deciding whether an assessed risk is acceptable or not and determining a tolerable level of risk for risks that cannot be rendered acceptable are fundamentally searches for subjective targets. Does the risk manager seek the highest possible level of protection, a desirable level of protection, an achievable level of protection, or something that is practical (implementable) or affordable? Does equity matter? Must there be a consistent level of protection, or is the economic efficiency of a level of protection more important? There is no one to answer that will satisfy everyone. Therefore, the process by which this decision is reached may be as important as the decision rule that is used to reach it. To determine an acceptable or tolerable level of risk, managers must take into account the scientific evidence, the uncertainty, and the values evident in their objectives and constraints. Several principles used by risk managers are reviewed here briefly.”

3.1.4. Controlling of risk

Every risk needs to be eliminated or contained as much as possible. This is done by connecting with the experts of the field to which the risk belongs. In a manual environment, this entails contacting each and every stakeholder and then setting up meetings so everyone can talk and discuss the issues. The problem is that the discussion is broken into many different email threads, across different documents and spreadsheets, and many different phone calls. In a risk management solution, all the relevant stakeholders can be sent notifications from within the system. The discussion regarding the risk and its possible solution can take place from within the system. Upper management can also keep a close eye on the solutions being suggested and the progress being made within the system. Instead of everyone contacting each other to get updates, everyone can get updates directly from within the risk management solution.

Risk control is a term of art used to avoid greater confusion with the risk management strategies described previously. It may be misleading to suggest that we can control some risks. It may be more honest to say that we struggle to manage them. However, calling this risk management activity “risk management” might cause even greater confusion. Therefore, to be forewarned not to interpret control too literally in the current context, risk treatment is a synonym. The basic tasks during this risk control phase of the manager’s tasks are shown in Figure 7 [19].

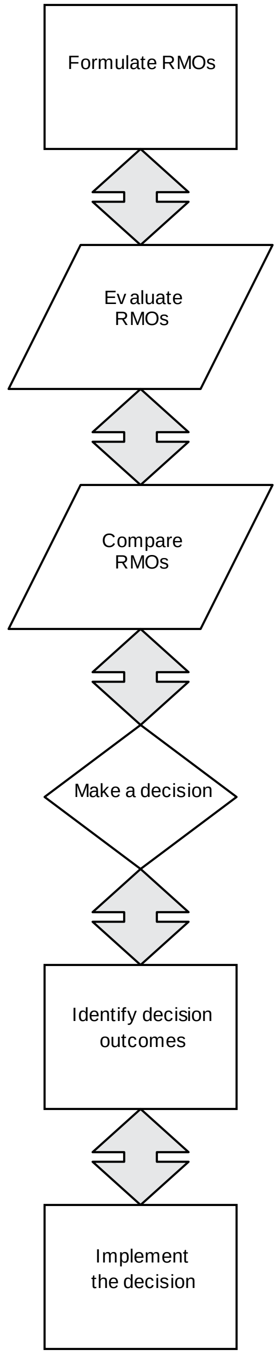


Figure 7 Risk controlling steps. (Yoe, 2019)

3.1.5. Monitoring of risk

Not all risks can be eliminated – some risks are always present. Market risks and environmental risks are just two examples of risks that always need to be monitored. Under manual systems, monitoring happens through diligent employees. These professionals must make sure that they keep a close watch on all risk factors. Under a digital environment, the risk management system monitors the entire risk framework of the organization. If any factor or risk changes, it is immediately visible to everyone.

Computers are also much better at continuously monitoring risks than people. Monitoring risks also allows a business to ensure continuity.

It is important to provide feedback to the organization and its stakeholders on how well they are achieving their objectives. Risk managers are responsible for monitoring the outcomes of their decisions to see if they are working. Actually, three distinct things may be monitored in any given situation. These are decision information, decision implementation, and decision outcomes.

Some risk management decisions may not yield immediately observable outcomes. Actions taken now to change conditions in the distant future are not observable, for example, measures taken to ameliorate effects of future sea level change. Some risk problems are so uncertain that the risks themselves may be considered speculative. It is difficult to observe the reduction of risks of rare events. In these kinds of situations, it may be important to monitor information to see if data gaps is being filled [19].

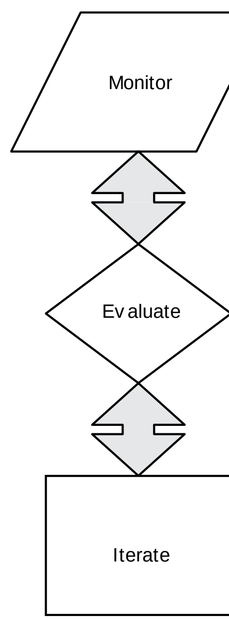


Figure 8 Risk monitoring steps. (Yoe, 2019)

3.2. Qualitative and quantitative analysis techniques

3.2.1. Qualitative analysis techniques

Qualitative analysis is described as the identification process produces a well-documented description of project risks. As analysis begins, it helps to organize and stratify the identified risks. By using the information for conducting risk identification plus the outputs from risk identification, it is possible to begin a basic analysis of the risks identified [14].

Qualitative risk analysis is an analytical method that does not identify and evaluate risks with numerical and quantitative ratings. Qualitative analysis involves a written definition of the uncertainties, an evaluation of the extent of the impact (if the risk ensues), and countermeasure plans in the case of a negative event occurring. A qualitative risk tools, among the other, are as follows:

- Documentation Reviews,
- Strengths, Weakness, Opportunities and Threats (SWOT),
- Judgment of specialists and experts (Delphi Technique).

Documentation Reviews

The documentation reviews technique is usually used in the initial stage of identifying risks for any projects by looking into lesson learned documentations of past projects. In some business units of this sector, the lesson learned documentations and other important documentations of past projects are stored in a database for easy reference. Some interviewees acknowledged the usefulness of this method but sometimes, it is not applied in projects because the project managers prefer to rely on his or her experience and also the incomplete and/or lack of information in some of the lesson learned documentations.

Strengths, Weakness, Opportunities and Threats (SWOT)

The next qualitative technique used is SWOT. This technique is applied to project milestones, which is part of its internal project methodology. It is used along with the brainstorming technique especially to identify the strengths and weaknesses of the project.

Delphi Technique

According to the interviewees, the Delphi technique is useful when it is used with the brainstorming method. This technique is useful to identify and analyze risks of the project when the project has external suppliers, customers, project teams, subject matter experts and other involved parties located in different geographical locations [11].

3.2.2. Quantitative analysis techniques

Quantitative risk analysis is the effort to examine risk and assign hard metric values to both the project risk as a whole and to the most significant risks (as established through risk qualification). Project managers conduct risk quantification to establish the odds of achieving project goals, to justify contingency reserves, to validate targets associated with the triple constraint, and to conduct in-depth "what-if" analyses.

In a perfect world, the pool from which quantitative risk information is drawn is deep and rich with data. It includes information from the previous processes discussed here as well as any statistical data repositories existing within the organization. To augment those data, project managers use a variety of tools, including expert interviews, expected monetary value, decision tree analyses, program evaluation and review technique (PERT) assessments, sensitivity analysis, and simulations [14].

Quantitative risk assessment is a risk model built using simulation or deterministic statistics to assign numerical values to risk. Inputs that are mostly assumptions and random variables are fed into a risk model or any given range of input, the model generates a range of output or outcome. The model is analyzed using graphs, scenario analysis, and/or sensitivity analysis by risk managers to make decisions to mitigate and deal with the risks. A qualitative risk tools, among the others, are as follows:

- Critical Path Method (CPM),
- Program, Evaluation and Review Technique (PERT) Analysis,
- Computer simulation (Monte Carlo Method).

Critical Path Method (CPM)

The Critical Path Method is one of the commonly used quantitative techniques especially for time and schedule risks of the project. This is one of the many techniques that are used in the risk workshop.

Program, Evaluation and Review Technique (PERT) Analysis

The PERT analysis is used for similar purposes as the CPM technique, focusing on time and schedule risks. It is used to determine the worst case and best-case scenario of the project duration. However, this technique is not commonly used but it is valuable to be applied in order to obtain improved accuracy on the probability and likelihood of results from the experts, according to some of the interviewees [11].

Monte Carlo simulation

This technique not only considers cost and schedule risk for individual activities, but also for the entire project. In many cases, there is the temptation to assume that all project risks must be accounted for in the worst case. The Monte Carlo analysis technique, however, takes a more holistic approach. As such, the total project cost risk and the total project schedule risk are usually expressed as a cumulative probability distribution of total project cost and total project schedule, respectively. Such distribution information can be used to reflect project risk by computing the probability that the project will be accomplished within particular cost or schedule targets. It can also be used to assess what level of funding or schedule would be required to guarantee success.

A computer is necessary to use this technique because the analysis requires repetitive computations [14].

3.2.3. Qualitative and quantitative approaches

The most commonly used risk identification and analysis techniques are divided into qualitative and quantitative approaches as shown in Figure 9.

Qualitative Techniques	Quantitative Techniques
• Documentation Reviews (e.g., lesson learned documentation of past projects) • Brainstorming • Root Cause Identification (e.g., Ishikawa fishbone) • Strengths, weakness, opportunities and threats (SWOT) • Delphi technique • Checklist analysis • Assumptions analysis • Risk categorization (e.g. Risk breakdown structure) • Probability and impact Matrix • Heuristics (Rule of thumb)	• Critical Path Method (CPM) • Program, evaluation and review technique (PERT) analysis • Expected monetary value analysis (e.g. Decision tree analysis) • Sensitivity Analysis • Variance trend analysis • Numerical approximations • Monte Carlo analysis

Figure 9 The most commonly used qualitative and quantitative techniques (Muller, 2009)

Since many indications were highlighted that the tendency of using qualitative approach is higher than quantitative approach to assess risk due to the present of experience risk manager, sometimes, the availability of time and budget. As well as appropriateness of those approaches in order to develop effective risk responses, the authors are expecting the results of this study to skew towards the qualitative approach and limited data will be collected about quantitative approach. It is expected that four out of the six boxes will be filled, referring to the matrix shown in Figure 10.

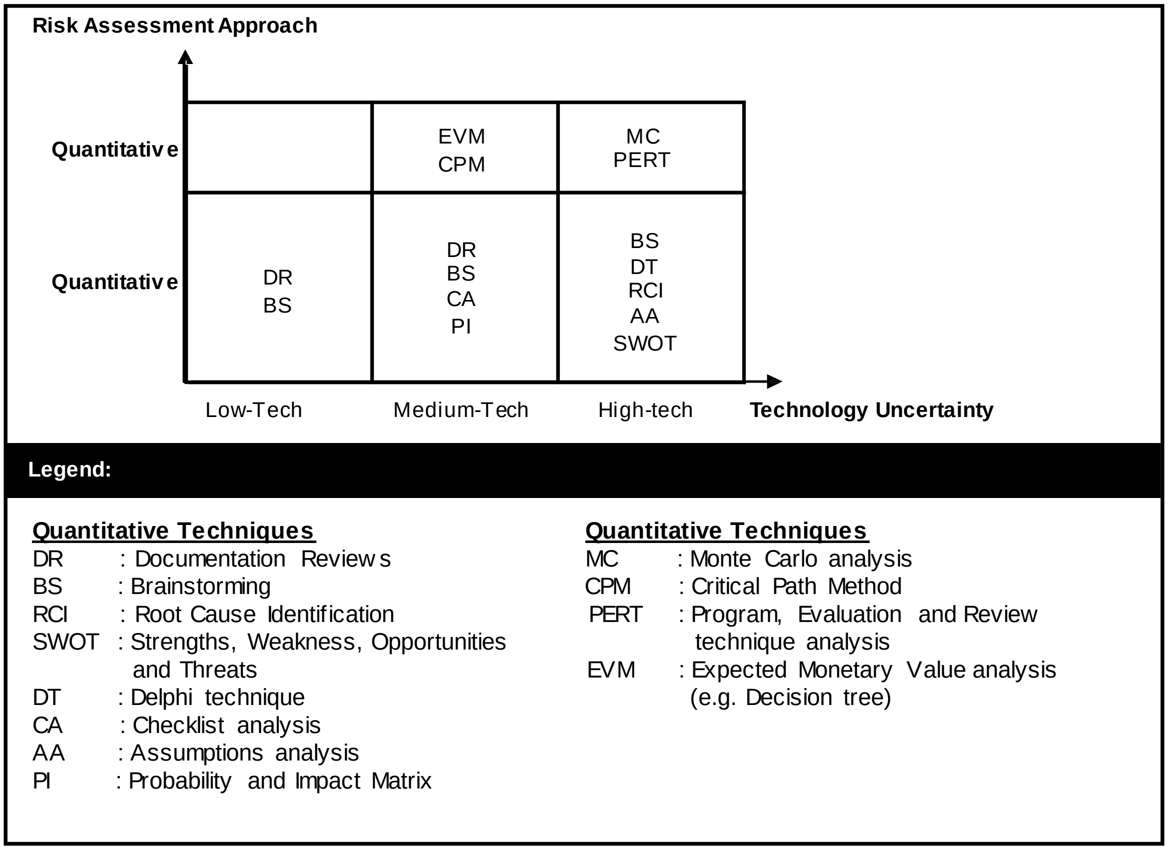


Figure 10 Qualitative and quantitative analysis techniques matrix (Muller, 2009)

Considering the above-mentioned definitions and approaches of the analysis techniques, the Monte Carlo simulation, as high-tech quantitative one, was chosen to perform the copper concentrate sale model presented in the following section.

4. ANALYSIS MODEL BASED ON MONTE CARLO SIMULATION

The main structure of a risk analysis model is similar to a deterministic single value rate of return model except that each variable in the analysis model is represented by a probability distribution function. The objective is to calculate the combined impact of the variability and uncertainty in the models parameters to determine a total probability distribution of the model. Then the resulting point estimate is transformed into an interval estimate illustrated in terms of a probability distribution. The technique used in this thesis is a Monte Carlo simulation, which involves a random sampling method concerning each different probability distribution selected for the actual model set-up. These distributions are defined hundreds or even thousands of different scenarios. Each probability distribution is sampled in a manner it reproduces the original shape of the distribution meaning that the actual model outcome reflects the probability of the values occurrence [3].

The MCS is a numerical technique used to replace uncertain point estimates of parameters and values in models and calculations with probability distributions. That represent the natural variability and knowledge uncertainty in those inputs. The MCS samples an individual value from each probability distribution in the model. These values then replace the point estimates in the model's equations and calculations. Thus, the model's calculation can be completed and outputs can be produced. This simulation is repeated the desired number of times to generate a distribution of output values [19].

The term "Monte Carlo" was first introduced by von Neumann and Ulam during World War II, as code name for the secret work at Los Alamos where the allied forces combined tried to discover the atom bomb. It was suggested by the gambling casinos at the city of Monte Carlo in Monaco to get some "advertising". The work at Los Alamos involved direct simulation of behavior concerned with random neutron diffusion in fissionable material.

The MCS is now one of the most powerful and commonly used technique for analyzing complex problems. The different types of applications can be found in many fields from radiation transport to river basin modeling. Furthermore, it is not only on stochastic processes the MCS is applicable, but also at deterministic problems, this method is usable. There are three major points suggesting MCS instead of traditional simulation methods:

1. In MCS, time does not play as substantial role as it does in stochastic simulation in general.
2. The observations in the MCS, as a rule, are independent. In simulation, however, the experiment with the observations is over time so, as a rule, these are serially correlated and hence dependent of each other.
3. In MCS, it is possible to express the responses in a rather straightforward manor by simple functions of the stochastic input variables. In simulation, the response is often a complicated one and can only be expressed explicitly by computer programs [3].

The MCS is a popular simulation technique that enables analysts to propagate the uncertainty in a decision problem and produce a numerical description of the range of potential model outputs. These output distributions can be subjected to statistical analysis to inform decision-making.

This process can be used to replace point estimates in any kind of model. Easy-to-use commercial software has made the method popular to use in spreadsheet models. Thus, the process can be used in any spreadsheet model where one or more model inputs are uncertain, that is, subject to natural variability or a matter of some knowledge uncertainty. This makes it a widely applicable tool for assessing risks. Its use is not restricted to spreadsheet models, however. It can be employed in virtually any quantitative model.

MCS has its strengths and weaknesses presented below:

Strengths:

- Widespread applicability.
- Natural and physical systems too complex to analytically assess the effects of uncertainty can be assessed by describing input uncertainties and running simulations that sample the inputs to represent possible outcomes.
- Can examine complex situations that are difficult to understand and solve by other means.
- Models are relatively simple to develop.
- They can represent virtually any influences or relationships that arise in reality.
- Can accommodate a wide range of distributions in an input variable, including empirical distributions derived from observations of real phenomena.
- Large amounts of data that can be generated lend themselves readily to sensitivity analysis to identify strong and weak influences on outputs.
- Commercially available software makes it relatively easy to apply this numerical technique to any spreadsheet model.

Weaknesses:

- Solutions are not exact and their usefulness may depend on the number of iterations or simulations completed
- It is not a transparent process
- The process is so easy that analysts may overlook analytical solutions in favor of a simulation [19].

The Monte Carlo simulation begins by replacing parameters in a model with distributions that represent natural variability or knowledge uncertainty in the model inputs. MCS is, in essence, a sampling method. Given a probability distribution, MCS randomly selects a value from that distribution. In a model with many probability distributions, a random value is selected from each distribution in the model.

The programmed calculations are executed using the sampled values and outputs are generated [19].

Inputs for this method include:
1. Knowledge of the source and nature of the uncertain inputs.
2. Monte Carlo software that can produce the required output.
3. Preparation of a model with uncertain inputs.
4. Probability distributions to represent the uncertainty and knowledge of those distributions
5. Running of Monte Carlo simulation.
6. Outputs of Monte Carlos simulation.

The above-mentioned inputs were presented in the following sections.

4.1. Source and nature of the uncertain inputs

The data collection and analysis is divided into three sub-categories; approach and type of data collection, data collection and data correlation analysis. The first sub-category covers the descriptions of data in this thesis and the type of data to be collected. Therefore, different qualitative data collection methods are viewed in order to select the most appropriate method for this thesis.

Data collection and preparation

Form the copper mining company's historical sale data by client was downloaded based on copper concentrate sold between 2015 and 2020. This date includes the following information:
- Dry metric ton sold.
- Copper treatment changers (Cu TCs).
- Copper refine charges (Cu RCs).
- Silver refine charges (Ag RCs).
- Gold refine charges (Au RCs).
- Credit risk ranking by client from 1 to 4, where 1 means client with very reliable financial performance and 4 client with not reliable financial performance.
- Penalties.

Form historical data was selected six mayor clients considering three mayor traders and three mayor smelters. For the purpose of the thesis, the selected clients were named as follows:

Traders: "A"," AA", "AAA"
Smelters: "B", "BB", "BBB"
Rest of the clients was grouped together and named "Other".

For each selected client, the data was divided in seven separated Excel spreadsheets including the following information:
- Dry metric ton sold.
- Copper treatment changers (Cu TCs).
- Copper refine charges (Cu RCs).
- Silver refine charges (Ag RCs).
- Gold refine charges (Au RCs).

- Penalties.

Correlation matrix

The correlation coefficient is a measure of the strength and direction of the relationship two variables. It can take on any values between -1.0 and +1.0. The correlation coefficient can be decomposed into its direction or sign (positive or negative relationship between two variables) and the magnitude or strength of the relationship (the higher the absolute value of the correlation coefficient, the stronger the relationship).

The correlation coefficient can be computed in serval ways. The first approach is to manual compute the correlation coefficient *r* of the pair of variables *x* and *y* using:

$$r_{x,y} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum y_i^2 - (\sum y_i)^2}}$$

Formula 1 Formula to compute the correlation coefficient.

The second approach is to use Excel's *CORREL* function. For instance, if the 10 data for *x* and *y* are listed in cells A1:B10, then the Excel function to use is *CORREL(A1:A10,B1:B10)* [15].

The second approach was used to compute the correlation coefficient between selected clients' Cu TCs. From each of seven separated Excel spreadsheets by client, was generated pivot table to present the average Cu TCs by year. Then, the results from all seven-pivot tables were consolidated in one table as below:

	A	AA	AAA	B	BB	BBB	Other
2015		93.20	89.10		107.00	107.00	89.39
2016	92.00	93.85	98.03	97.35	97.35	107.00	95.20
2017	75.33	90.81	92.68	97.35	93.54	107.00	81.56
2018	53.90	83.56	81.98	88.18	82.25	87.38	68.63
2019	43.13	62.90	78.47	81.09	77.04	81.53	70.09
2020	35.88	67.95	71.40	62.00	62.00	72.53	63.96

Table 1 The average Cu TCs by year.

Using the formula "correl" there was created the following correlation matrix between each of the clients' Cu TCs:

	A	AA	AAA	B	BB	BBB	Other
A	1	0.91	0.99	0.87	0.94	0.96	0.97
AA		1	0.88	0.82	0.87	0.92	0.82
AAA			1	0.93	0.87	0.96	0.92
B				1	0.98	0.93	0.79
BB					1	0.95	0.90
BBB						1	0.92
Other							1

Table 2 Correlation matrix between each of the clients' Cu TCs

However, the correlation coefficients were inconsistent, therefore the MCS software adjusted the coefficients by the average of 0.07 (worst case of 0.22) to allow the simulation to continue. The correlation between each of client was included in the assumptions' distribution presented in the Section 4.5 Monte Carlo simulation below.

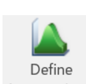
4.2. Monte Carlo software

Oracle Crystal Ball tool was selected to realize copper concentrate sale model simulation. Oracle Crystal Ball is the leading spreadsheet-based application for predictive modeling, forecasting, simulation, and optimization. It gives unparalleled insight into the critical factors affecting risk, which support the right tactical decisions to reach objectives and gain a competitive edge—even in an uncertain market.

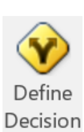
Oracle Crystal Ball is a spreadsheet-based application for predictive modeling, forecasting, simulation, and optimization. It gives unparalleled insight into the critical factors affecting risk.

Crystal Ball is an easy-to-use simulation program that helps you to analyze the risks and uncertainties associated with your Microsoft Excel spreadsheet models. This Spotlight is a quick overview of how Crystal Ball is used Excel models are deterministic, which means that the inputs are fixed (one value to one cell).

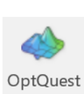
The first step to using Crystal Ball is to determine which model inputs are uncertain (refer Section 4.1 above), which can be called assumptions, and then define decisions and, at the end, find the optimal solution. Therefore, to run the Monte Carlo simulation with Crystal Ball software, there were used the following comments:



Define Assumption – to define the selected cells as assumptions by choosing from a gallery of probability distribution types. Assumptions are the uncertain variables in the in the spreadsheet model.



Define Decision – to define the selected cells as decision variables in the spreadsheet model.



OptQuest – to search for and find optimal solutions to the simulation model.

OptQuest is an optimization tool that runs with Crystal Ball. As an add-in to Crystal Ball, OptQuest enhances simulation models by automatically searching for and finding optimal solutions.

It can also use OptQuest for portfolio analysis, where you determine an investment strategy that optimizes profit while taking into consideration the uncertainty around the annual returns of each asset. Other applications for optimization include workforce planning of employees, configuration of machines for production scheduling, and the location of facilities for distribution, and tolerances in manufacturing design.

Before using OptQuest, first it is needed to create a Crystal Ball model. Once the model is a valid representation of the problem, system, or process, it is needed to use Crystal Ball to define the decision variables. Decision variables are variables in the model, such as, how much rent to charge or how much money to invest in a mutual fund (see below).

The software samples a random value from each input distribution and runs the model using those values. After repeating the process a number of times (typically 1,000 to 10,000), the simulation estimates probability distributions for the uncertain outputs of the model using the random sample of output values. A larger number of samples takes longer to compute for a complex model but enables a more accurate estimate of the resulting distributions, including their mean (expected value), standard deviation, median, or percentiles.

4.3. Model with uncertain inputs

Modeling is the process of producing a model. The model is a representation of the construction and working of some system of interest. A model is similar to, but simpler, than the system it represents. One purpose of the model is to enable the analyst to predict the effect of changes to the system. On the one hand, the model should be a close approximation to the real system and incorporate most of its salient features. On the other hand, it should not be too complex to understand and experiment with it. The good model is a judicious tradeoff between realism and simplicity [8].

In general, copper smelting business consists of the gross profit elements, among the other, copper TCRCs, gold and silver RCs, free metals, by-product credits, metal premiums and penalties. Therefore, mining clients, both smelters and traders, put the biggest pressure and are focused only on those elements, especially the copper TCRCs.

The uncertainty of the value of the copper TCRCs was a motivation to develop the copper concentrate sale's analysis model. The analysis model considers historical copper treatment charges and refining charges, gold and silver refining charges, as well as paid penalties in order to optimize annual future sales distribution.

The company uses, without Risk Analysis, this distribution: Trader A, AA, AAA of six lots each, Smelter B and BB of five lost each, Smelter BBB of 25 lots and Other clients of 30 lots. This distribution was defined by on historical performance of sales based short-, mid- and long-term sale contracts, as well as based on the experience of the company experts' judgment. This company distribution is amounted to US$ 109,651,518. The object of the analysis is to optimize annual future sales distribution by minimizing the final value of total copper TCRCs, gold and silver RCs, as well as penalties (Minimal Final Value or MFV) paid to clients considering the historical ranges of copper TCRCs and gold and silver RCs, as well as penalty paid in the past.

To calculate the Minimal Final Value the following formula was established:

$$min \in [\sum(y_1)x_1 - \propto x_1]$$

Formula 2 MFV formula calculation.

Where x_1 is the sum of Cu TCRCs, Au RCs and Ag RCs less α, which is percentage of the probability of non-payment based on historical performance of each client.

The following conversions were used:

Cu TCs amount US$ = Cu TCs x Dry Metric Ton (DMT)
Cu RCs amount US$ = Cu RCs x Cu concentrate lbs
Ag RCs amount US$ = Ag RCs x Ag oz in Cu concentrate
Au RCs amount US$ = Au RCs x Au oz in Cu concentrate
Penalty amount US$ = Penalty amount US$
Payment risk = % of the probability of non-payment

where:
1 shipment = 1 lot = 10,100 DMT = 6,063,212 Cu concentrate pounds
Total annual production is assumed of 843,000 DMT.
Ag oz in Cu concentrate pound = 0.445%
Au oz in Cu concentrate pound = 0.018%
Penalty amount = probability based penalty paid in the past

Payment risk based on expert judgment using on the Credit risk ranking which included payments delays as follows:

- 0% for clients with ranking 1 or 2, where 1 and 2 mean client with reliable financial performance,
- 1% for clients with ranking 3, where 3 means client with less reliable financial performance,
- 5% for clients with ranking 4, where 4 means client with not reliable financial performance.

Having the above-mentioned data and assumptions, it was started the creation of the analysis model in the following way.

Firstly, in the Excel spreadsheet was create a table with the assumptions, which are uncertain inputs. Therefore, these assumptions includes copper TCRCs, gold and silver RCs, as well as penalties using the weight average values based on historical information. In the Table 3 below presents the results.

Client	Cu TCs	Cu RCs	Ag RCs	Au RCs	Penalty
A	59.56	0.060	0.35	4.00	-
AA	87.80	0.088	0.35	-	-
AAA	86.06	0.086	-	5.00	-
B	84.41	0.084	0.50	5.00	-
BB	85.00	0.085	0.50	5.00	-
BBB	85.95	0.086	0.40	5.00	-
Other	82.37	0.082	-	-	-

Table 3 Assumptions (uncertain inputs) in the analysis model.

Afterwards, there was created the decisions variables. The object of this thesis is to obtain the optimal copper concentrate sale distribution, therefore as a starting point there was used the mining company distribution presented in the Table 4.

Client	Lots
A	6
AA	6
AAA	6
B	5
BB	5
BBB	25
Other	30
	83

Table 4 Initial lots assigned to each client.

Next, using the conversions, assumptions and decisions presented above, the analysis model was developed as presented in Table 5. Each of the column has the following calculation:

- First two columns maintain information presented in the Table 4 above.
- The column number 3 includes the amount of the copper TCs for each client calculated as copper TC value (form assumption table 3) multiply by a number of lots (from the column number 2) multiply by 10,100 ton (DMT conversion).
- The column number 4 is the recalculation of the number of lots in the column number 2 into the pounds multiplying by the number of lots by 10,100 ton (DMT conversion) and 600.32 (pound conversion).
- The column number 5 includes the amount of the copper RCs calculated as copper RC value (form assumption table 3) multiply by the column number 4.
- The column number 6 is the calculation of silver ounces in copper pounds included in one lot from the column number 4 multiplying by 0.445% (silver ounces conversion).
- The column number 7 includes the amount of the silver RCs calculated as silver RC value (form assumption table 3) multiply by the column number 6.
- The column number 8 is the calculation of gold ounces in copper pounds included in one lot from the column number 4 multiplying by 0.018% (gold ounces conversion).
- The column number 9 includes the amount of the gold RCs calculated as gold RC value (form assumption table 3) multiply by the column number 8.

- The column number 10 includes historical information of paid penalties to each of client.
- The column number 11 includes percentage of the payment risk based on expert judgment based on the Credit risk ranking.
- The column number 12 is a sum of the columns number 3, 5, 7, 9, 10 and 11.

1	2	3	4	5	6	7	8	9	10	11	12
Client	Lots	Total TCs USD	Cu conc pounds	Total Cu RCs USD	Ag oz	Total Ag RCs USD	Au oz	Total Au RCs USD	Penalty USD	Payment risk	Total
A	6	3,609,538	36,379,274	2,186,549.8	161,848	56,646.83	6,578	26,313.42	-	1%	**5,820,258**
AA	6	5,320,650	36,379,274	3,194,082.1	161,848	56,646.83	6,578	-	-	0%	**8,571,379**
AAA	6	5,215,146	36,379,274	3,130,746.2	161,848	-	6,578	32,891.78	-	1%	**8,294,996**
B	5	4,262,831	30,316,062	2,559,054.6	134,873	67,436.70	5,482	27,409.82	-	0%	**6,916,732**
BB	5	4,292,699	30,316,062	2,576,984.9	134,873	67,436.70	5,482	27,409.82	-	0%	**6,964,531**
BBB	25	21,702,546	151,580,310	13,028,430.0	674,367	269,746.81	27,410	137,049.08	-	0%	**35,137,772**
Other	30	24,959,415	181,896,371	14,983,587.3	809,240	-	32,892	-	-	5%	**37,945,852**
	83	**69,362,824**	**503,246,628**	**41,659,434.9**	**2,238,899**	**517,914**	**91,001**	**251,074**	**-**		**109,651,518**

Table 5 Analysis model.

Finally, having the analysis model prepared, there was created the cell with the Minimal Final Value formula uses to improve the economic value related to the copper concentrate sale.

4.4. Choosing the Probability Distribution

The human striving of predicting a future outcome has been a wanted skill for many decades. Uncertainty and variability satisfies our inability to be able to predict the future. This means that if it were able to determine these two components, it would be possible to predict the future outcome [3].

Therefore, it is recommended to present briefly descriptions of the two concepts of variability and uncertainty.

The variability is the effect of a given chance. It is a function of the given model system. This variable is the most difficult to explain as it is not possible to reduce by this thesis or further measurements. However, it is possible to minimize the variability by changing the physical modeling system. One of the best-known case examples of a variable experiment is the tossing of a coin. The normal prediction of a coin toss is a probability 50% for heads and 50% for tails. However, when making the experiment it is not possible to predict whether you achieve heads or tails in the next toss due to the coins inherent randomness.

It is therefore impossible to get a zero contribution from variability as designing any modeling systems. However, there are possibilities of controlling the variability by altering the whole modeling system. Different sources of variability can be distinguished e.g. inherent randomness of nature: the chaotic and unpredictable nature of natural processes and human behavior (behavioral variability): 'non-rational' behavior, discrepancies between what people say and what they actually do (cognitive dissonance) [3].

The uncertainty is related to many aspects of modelling, policy analysis for example, limited, and inaccurate data, measurement error, incomplete knowledge, limited understanding, imperfect models, subjective judgement, ambiguities, etc.

The uncertainty is the modelers' lack of knowledge concerning the parameters that characterizes the modeling system (defined as the level of ignorance). This factor has the possibility of being reduced by further studies or measurements. The uncertainty is further by definition subjective since it is a function of the modelers' level of knowledge [3].

The probability distributions are in the further all-continuous distributions, hence they will be referred to as distributions only. There are five types in use Uniform, Triangular, PERT, Normal and Erlang distribution. The research in the application of various probability distributions is an ongoing topic in the Ph.D. study entitled "Decision Support and Risk Assessment for Transportation Projects" [3].

To give a brief overview of the various impacts together with the assignment of probability distributions and their nature of uncertainty, see Figure 11.

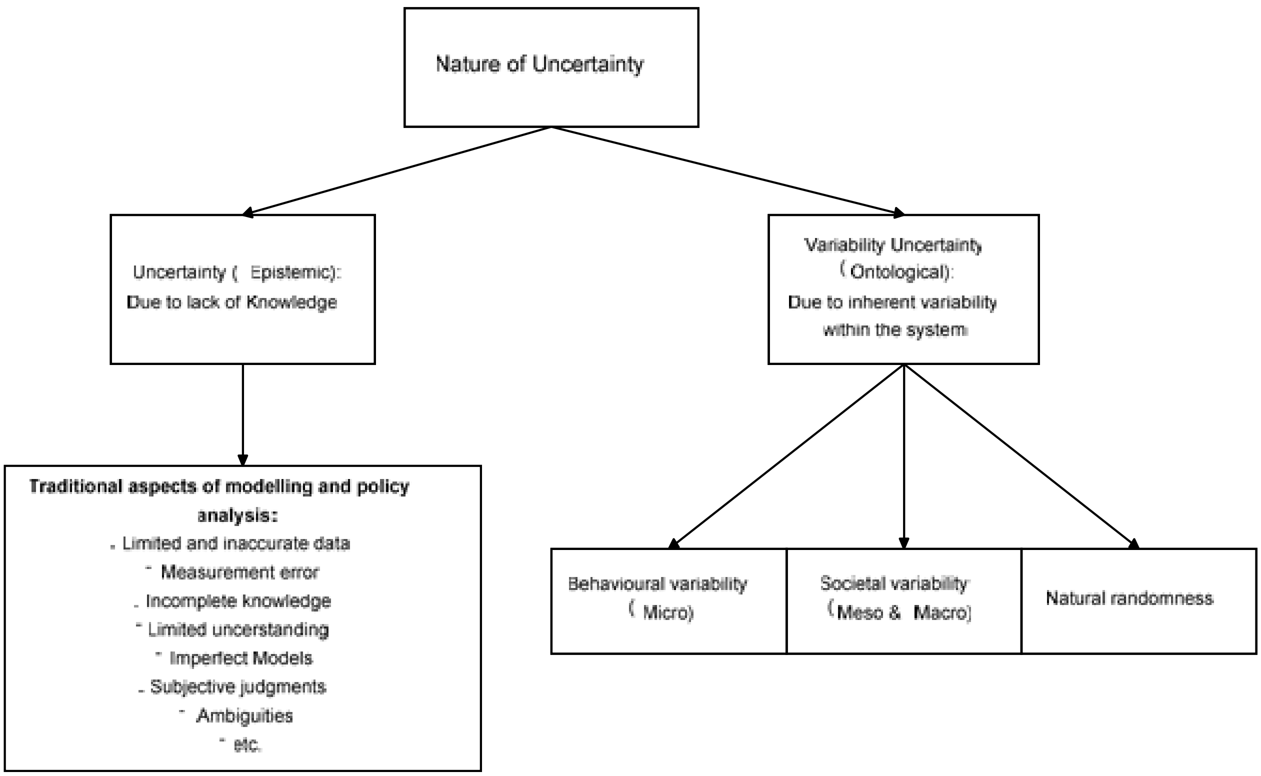


Figure 11 Overview of applied uncertain impacts within the Risk Analysis framework (Bang Salling, 2007)

One of the main advantages of separating the uncertainty and variability is that the total uncertainty of a model system does not show the actual source of the uncertainty. The information corresponding to the two sources implied in the total uncertainty is of relevance towards the decision makers in a given situation. If a result shows that the level of uncertainty in a problem is, this means that it is possible to collect further information. Thereby, it reduces the level of uncertainty, which enables us to improve our estimate. On the other hand, if the total uncertainty is nearly all due to variability, it is proven a waste of time to collect further information. The only way to improve and hereby reduce the total uncertainty would be to change the whole modeling system [3].

For the purpose of this risk analysis, there were used Normal and Triangular probability distributions described below.

4.4.1. The Normal probability distribution

The Normal distribution is an extremely important probability distribution in many fields.

It is a family of distributions of the same general form, differing in their location and scale parameters: the mean and standard deviation, respectively. The standard normal distribution is the normal distribution with a mean of zero and a standard deviation of one (the green curve in Figure 12). It is often called the bell curve because the graph of its probability density resembles a bell.

The probability density function of the normal distribution with mean μ and variance σ^2 (equivalently, standard deviationσ) is an example of a Gaussian function, [3]

$$f(x;\mu;\rho) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot \exp\left(-\frac{(x-\ \mu)^2}{2\sigma^2}\right)$$

Formula 3 Normal probability distribution.

If a random variable *x* has this distribution, it is used $X \sim N(\mu\,, \sigma^2)$. If $\mu =\ 0$ and $\sigma = 1$, the distribution is called the *standard normal distribution* and the probability density function reduces to, $f(x) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2}\right)$.

Some different types of probability density function of the normal distribution with various parameter values are shown in Figure 12.

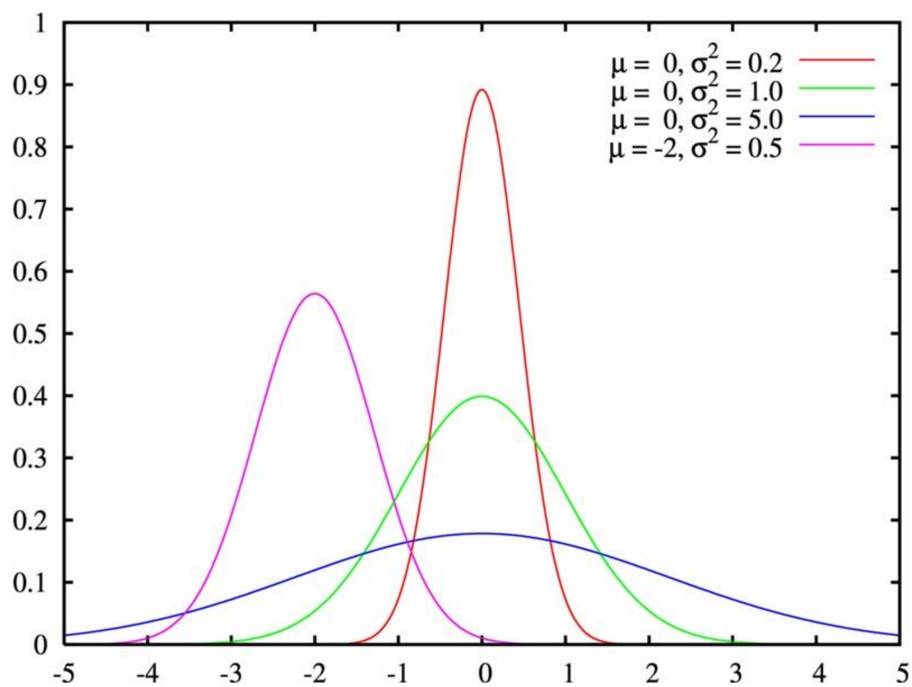


Figure 12 Illustration of a normal distribution with mean μ and std. deviation σ (Wikipedia, 2006)

4.4.2. The Triangular probability distribution

The Triangular distribution is typically used as a subjective description of a population for which there is only limited sample data. It is based on knowledge of the minimum and maximum and an inspired guess (referred to as the Most Likely value ML - mode) as to what the modal value might be, see Figure 13. Despite being a simplistic description of a population, it is a very useful distribution for modeling processes, where the relationship between variables is known, but data is scarce. The Triangular distribution has a lower limit *a*, mode *c* and upper limit *b*.

$$f(x|a,b,c) = \begin{cases} \dfrac{2 \cdot (x-a)}{(b-a) \cdot (c-a)} & \text{for } a \leq x \leq b, \\ \dfrac{2 \cdot (b-x)}{(b-a) \cdot (b-c)} & \text{for } c \leq x \leq b, \end{cases}$$

Formula 4 Triangular probability distribution

The triangular distribution or in an enhanced version; the Trigen-distribution, allows the upper and lower boundaries to be skewed. The Trigen-distribution further offers the analyst the possibility of choosing a confidence interval, where the upper and lower boundaries can be exceeded within a predefined percentage [3].

An illustration of the triangular distribution is shown in Figure 13.

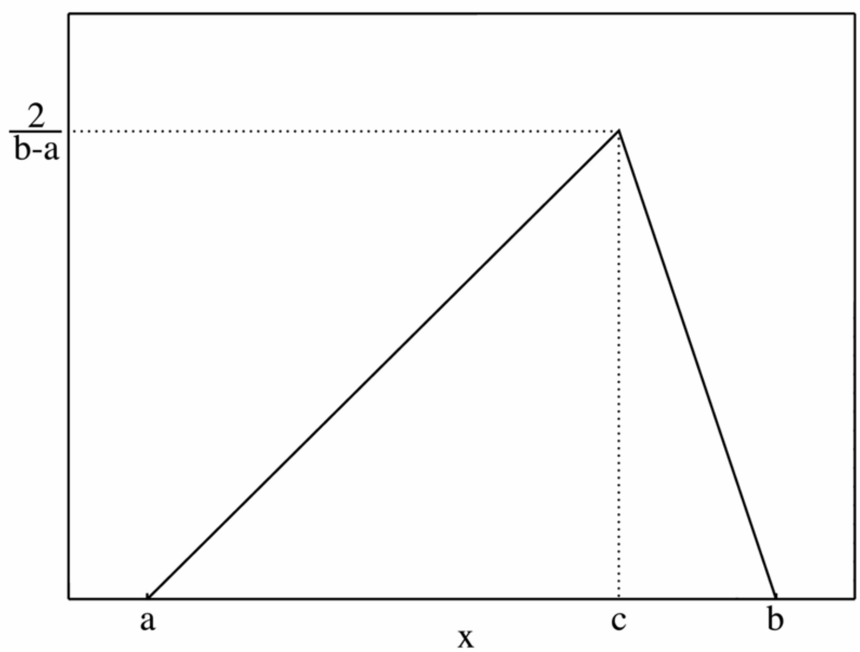


Figure 13 Illustration of a triangular distribution within the interval of [a ; b] with the mode c (Wikipedia, 2006).

4.5. Monte Carlo Simulation

It is now possible to give a short seven-step overview of how a Monte Carlo simulation works containing a method of Monte Carlo sampling.

1. Determine the uncertain inputs.
2. Add a suitable probability distribution to each selected parameter.
3. Generate for each individual probability distribution a random value by a sampling method.
4. Define decision variables, i.e. minimum and maximum lots can we sold to each client.
5. Prepare OptQuest to enhance simulation model by automatically searching for and finding optimal solution.
6. Review the new total probability distribution and MFV.

Coming up next in the description of each step taken and realized.

4.5.1. Determine the uncertain inputs

As described in Section 4.3. above, in the Excel spreadsheet was create a table with the assumptions, which are uncertain inputs. Therefore, these assumptions includes copper TCRCs, gold and silver RCs, as well as penalties using the weight average values based on historical information. In the Table 6 below presents the results.

Client	Cu TCs	Cu RCs	Ag RCs	Au RCs	Penalty
A	59.56	0.060	0.35	4.00	225,480
AA	87.80	0.088	0.35	-	-
AAA	86.06	0.086	-	5.00	-
B	84.41	0.084	0.50	5.00	-
BB	85.00	0.085	0.50	5.00	-
BBB	85.95	0.086	0.40	5.00	-
Other	82.37	0.082	-	-	-

Table 6 Average values based on historical data.

4.5.2. Add a probability distribution to each selected parameter

As presented in the step 1, copper TCRCs, gold and silver RCs, as well as penalties for each client are uncertain inputs in the model. Therefore, it was used the table of the critical values Dn,α as described in Kolmogrov-Smirnov (K-S) test. Kolmogrov-Smirnov test is a nonparametric test of the equality of continuous one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution (one-sample K–S test), or to compare two samples (two-sample K–S test). For this analysis one-dimensional probability distributions for 0.5 per number of sample as presented in the Table 7 below. If lower than in K-S table used Normal or Triangular probability distributions.

SAMPLE SIZE (N)	LEVEL OF SIGNIFICANCE FOR D = MAXIMUM [$F_0(X)$ - $S_n(X)$]				
	.20	.15	.10	.05	.01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.733
5	.446	.474	.510	.565	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.410	.490
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.392
17	.250	.266	.286	.318	.381
18	.244	.259	.278	.309	.371
19	.237	.252	.272	.301	.363
20	.231	.246	.264	.294	.356
25	.210	.220	.240	.270	.320
30	.190	.200	.220	.240	.290
35	.180	.190	.210	.230	.270
OVER 35	$\frac{1.07}{\sqrt{N}}$	$\frac{1.14}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

Table 7 The critical values $D_{n,\alpha}$ as described in Kolmogorov-Smirnov test (http://people.cs.pitt.edu/~lipschultz/cs1538/prob-table_KS.pdf

4.5.3. Generate for each individual probability distribution

Using the function Define assumption, for each of the uncertain input was defined the probability distribution. Starting with the selection of the probability distribution Normal or Triangular, as described in the step 2, as presented in the Figure 14.

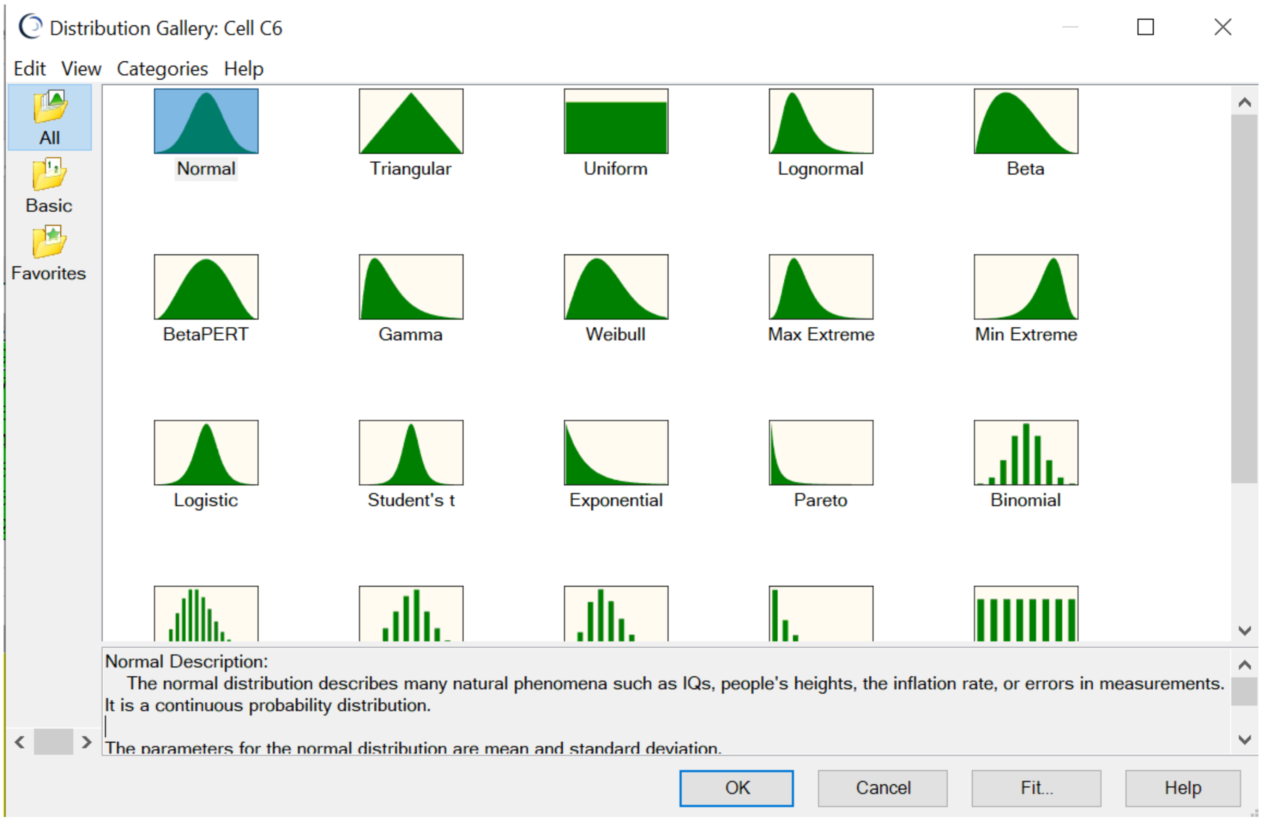


Figure 14 Probability distribution gallery in Define Assumption function

As described in the section 4.1 above, using the formula “correl“ there was created the following correlation matrix between each of the clients’ Cu TCs:

	A	AA	AAA	B	BB	BBB	Other
A	1	0.91	0.99	0.87	0.94	0.96	0.97
AA		1	0.88	0.82	0.87	0.92	0.82
AAA			1	0.93	0.87	0.96	0.92
B				1	0.98	0.93	0.79
BB					1	0.95	0.90
BBB						1	0.92
Other							1

Table 8 Correlation matrix between each of the clients’ Cu TCs

These correlations were included into the Define Assumption function to be considered once the simulation is started as presented in Figure 15.

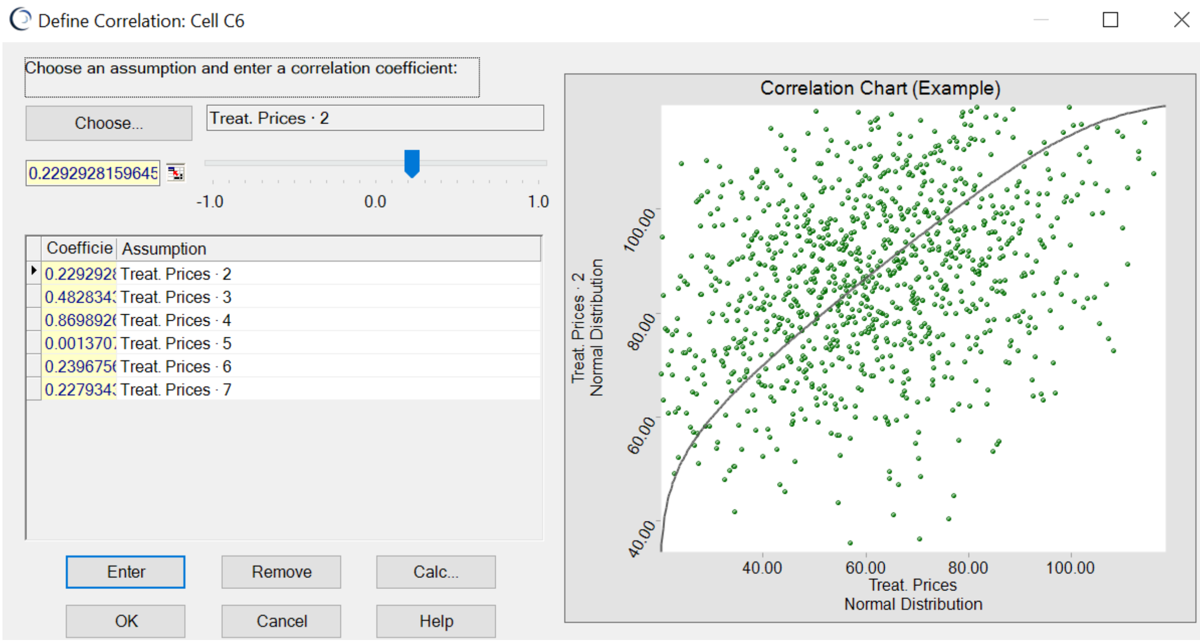


Figure 15 Correlation in Define Assumption function.

Finally, there were set the limits of minimum and maximum range of uncertain inputs based on expert judgment using the historical date. The Figure 16 is the presentation of the probability distribution defined for one of the client.

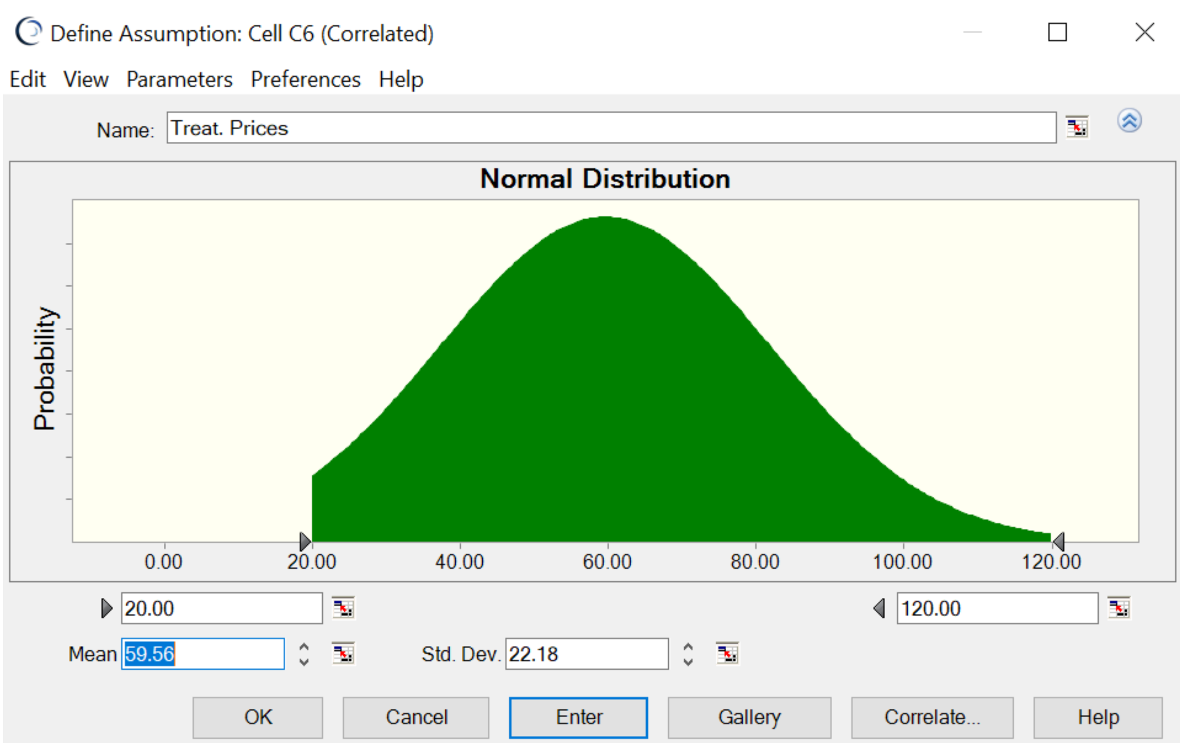


Figure 16 An example of Define Assumption set for one of the client.

The data with variable, was used the Define Assumption function are automatically highlighted in green by Crystal ball software (Table 8). However, where the data was not variable (one fixed value) the values were assume based on historical information (white cells in the Table 9).

Client	Cu TCs	Cu RCs	Ag RCs	Au RCs	Penalty
A	59.56	0.060	0.35	4.00	-
AA	87.80	0.088	0.35	-	-
AAA	86.06	0.086	-	5.00	-
B	84.41	0.084	0.50	5.00	-
BB	85.00	0.085	0.50	5.00	-
BBB	85.95	0.086	0.40	5.00	-
Other	82.37	0.082	-	-	-

Table 9 Probability distributions by client.

4.5.4. Define decision variables

Decisions preparation is based on the number of lots to sale to each client, where in total 83 lots are available and maximum 50% of total lots (maximum 41 lots) can be sold to one client.

For traders the maximum total lots was assumed at 15 lots. It was defined that the numbers of lots can be discrete only by 1.00, not decimals are accepted.

Considering the above-mentioned constraints, the decision variables by number of lots were as follows:
Trader A >= 0 lot and <= 15 lots
Trader AA >= 0 lot and <= 15 lots
Trader AAA >= 0 lot and <= 15 lots
Smelter B >= 0 lot and <= 41 lots
Smelter BB >= 0 lot and <= 41 lots
Smelter BBB >= 0 lot and <= 41 lots
Other >= 0 lot and <= 41 lots

Based on this, the Define Decision variable function was used. As presented in the Figure 17 presenting a set of the constraints for one of the client, Trader A.

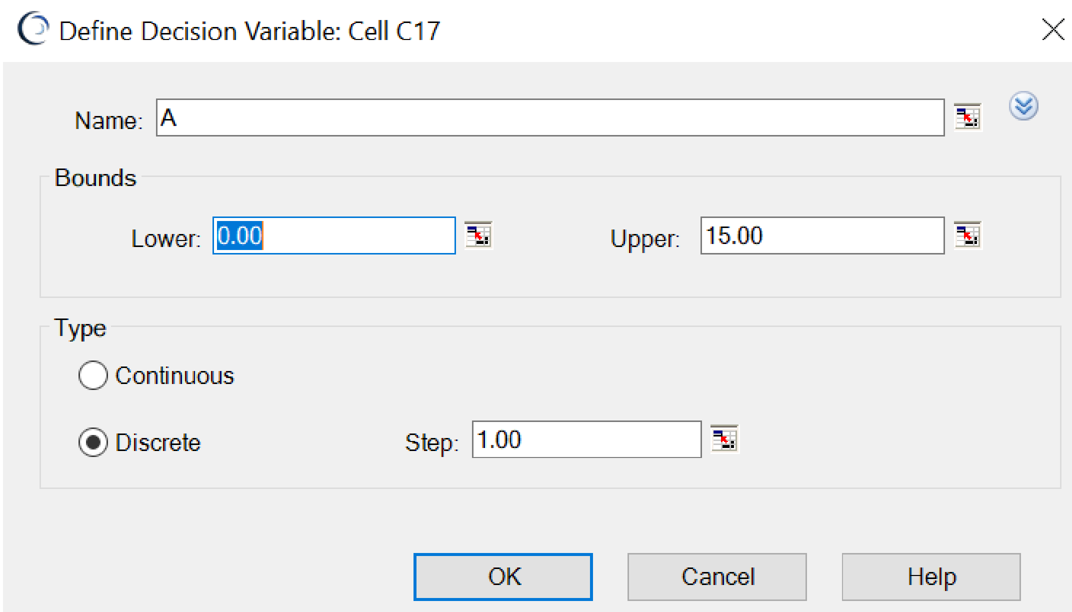


Figure 17 Define Decision variable function for one of the client.

The decision variable for which was used the Define Decision variable function are automatically highlighted in yellow by Crystal ball software (Table 10).

Client	Lots
A	6
AA	6
AAA	6
B	5
BB	5
BBB	25
Other	30
	83

Table 10 Define Decision variable function applied before running the simulation.

4.5.5. Prepare OptQuest to enhance simulation model

The almost final step to run MCS, is to set the OptQuest function. As presented in the Figure 18, the first activity is to define the objective, which is to find MFV.

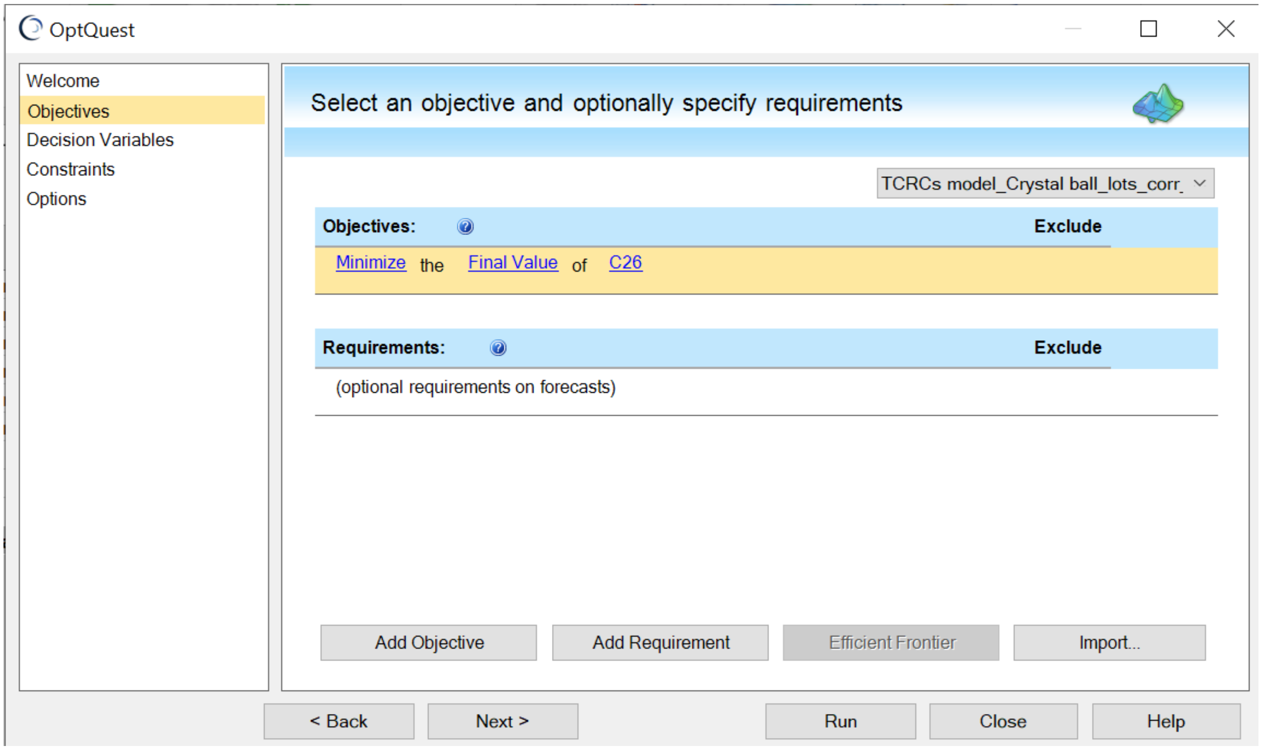


Figure 18 OptQuest function settings.

Next activity is to set the Decision variable, which automatically is copied once used Define Decision variable function as presented in the Figure 19.

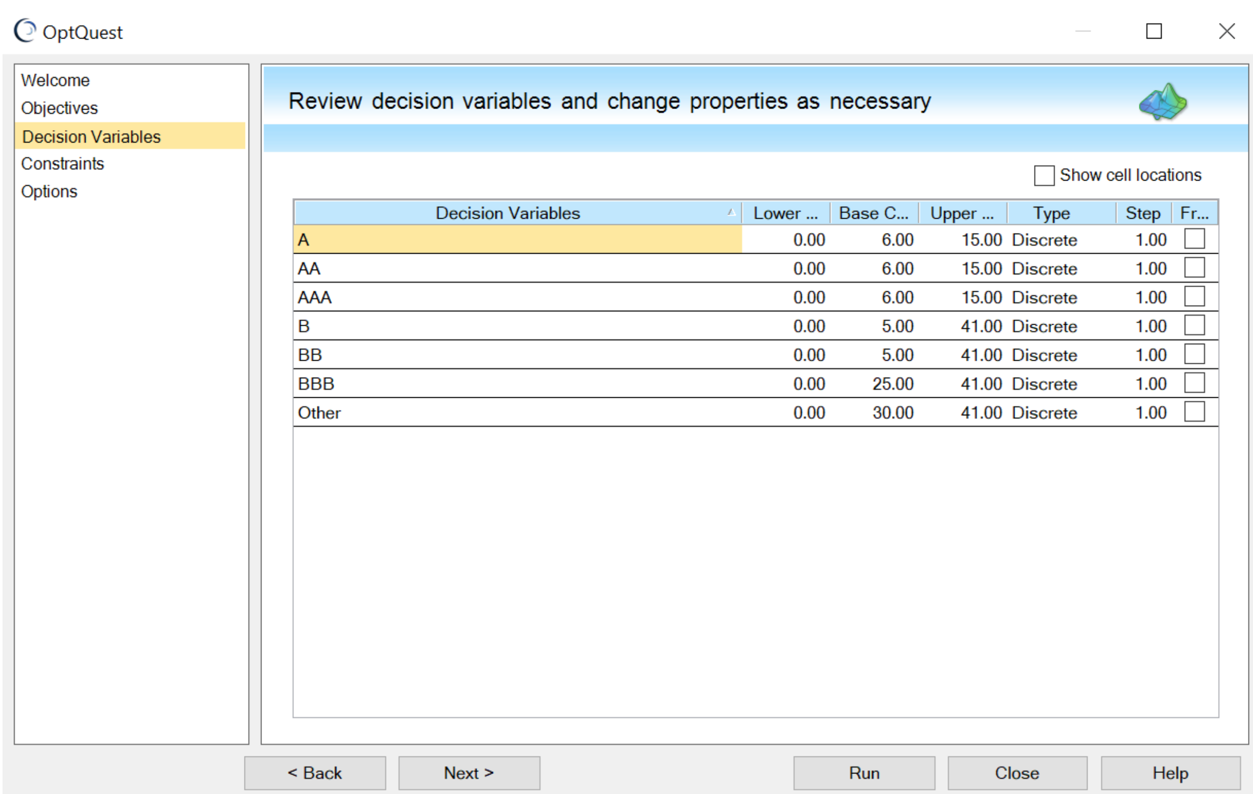


Figure 19 Decision variable copied by Define Decision variable function.

Afterward, the constraints have to been set. In total there were defined 15 different constraints considering minimum and maximum lots of each client and total lots available to be sold. In the Figure 20 are presented the first eight constraints.

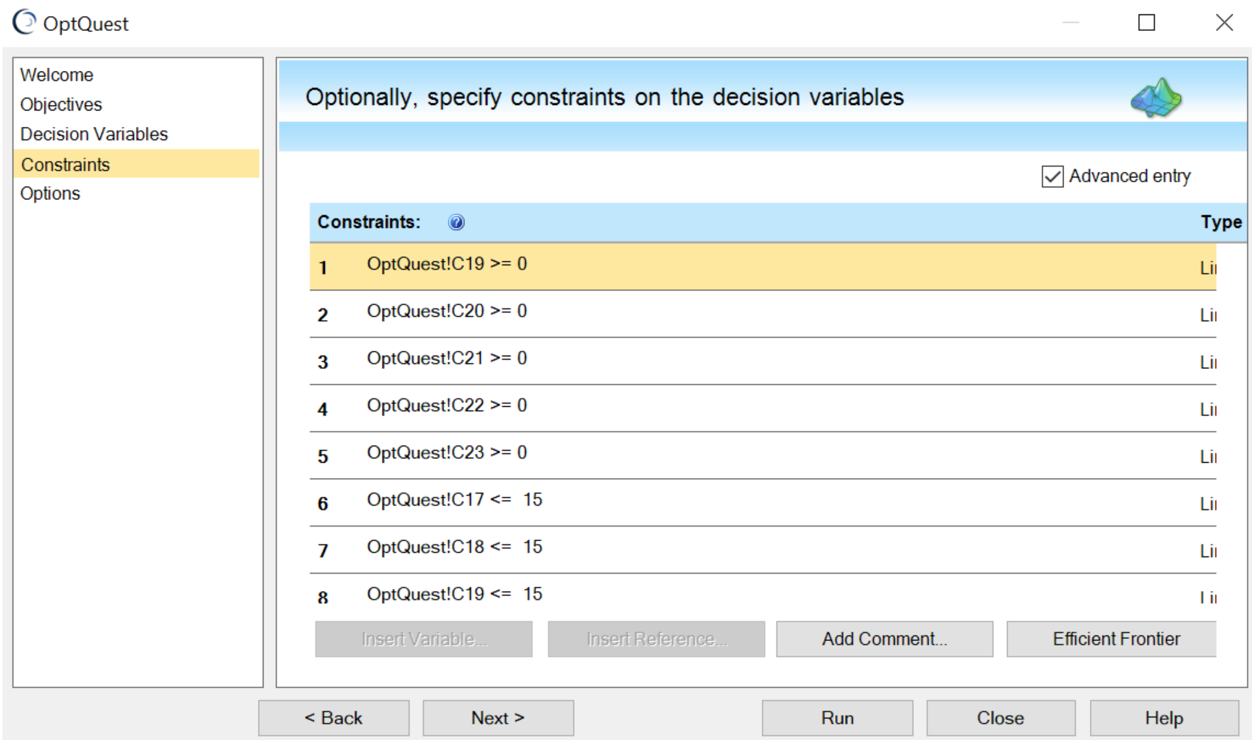


Figure 20 First eight constraints defined.

Final activity is to set the Options as presented in the Figure 21. These Options include as follows:

- Optimization control, which was set on 10,000 trails based on the bench market practice.
- Type of optimization set as the stochastic one.
- While running to show only target forecast windows and
- Decision variable cell was chosen to automatically set to best solution.

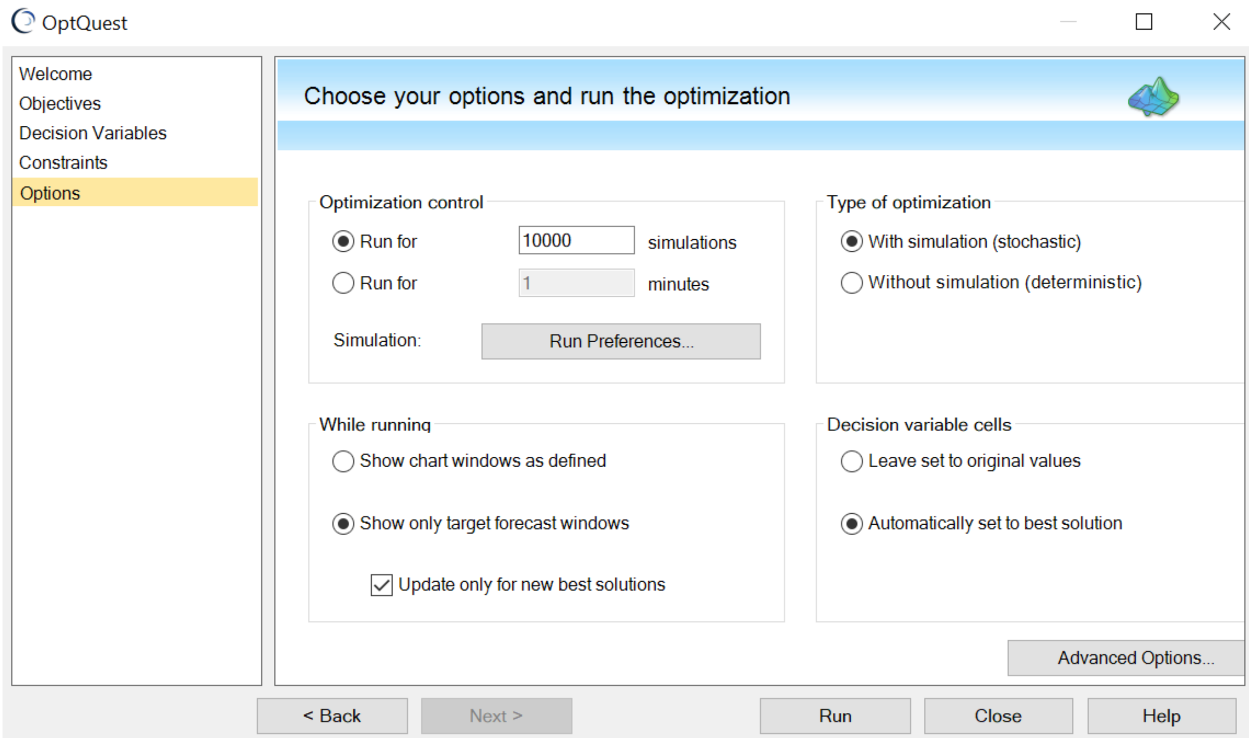


Figure 21 Options setting.

With all of the abovementioned setting, QptQuest was run.

4.5.6. Review the new total probability distribution of MFV.

After the completely 10,000 trails were run, the OptQuest presented the new total probability distribution of MFV as presented in the Figure 22, together with the analysis report.

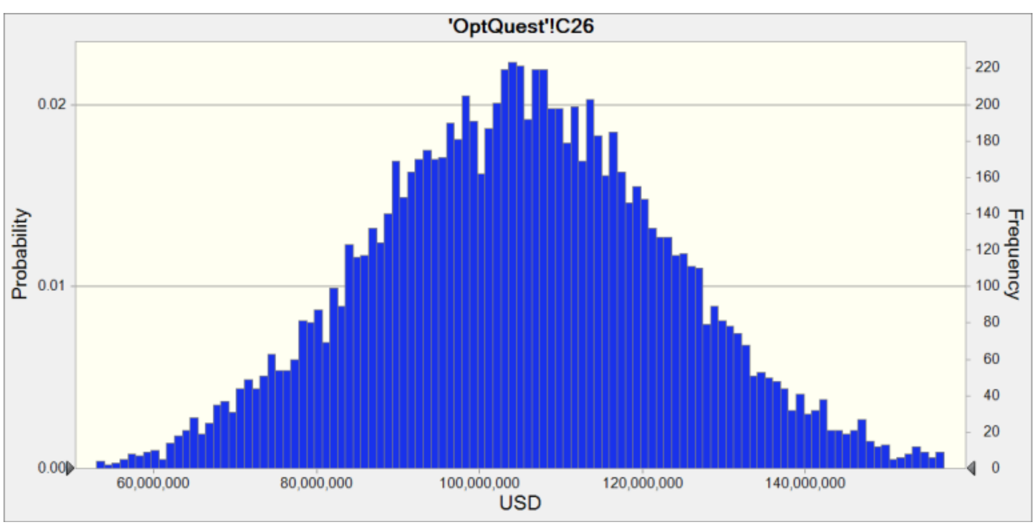


Figure 22 New total probability distribution of MFV

4.6. Monte Carlo simulation outputs

The outputs of the MCS are distributions of values calculated by the models. These distributions can include the actual input values used in the calculations, intermediate calculations or model outputs. These distributions can be analyzed using statistical technique to support decision-making process [19].

After 10,000 trails were evaluated in 12 minutes, the objective of the final value of TCRCs was improved from US$ 109,651,518 to US$ 87,524,294 a change of 20.1%.

The best solution amounted to US$ 87,524,294 includes the lots distribution as follows:

- Trader A 15 lots
- Trader AAA 15 lots
- Smelter B 12 lots
- Other 41 lots

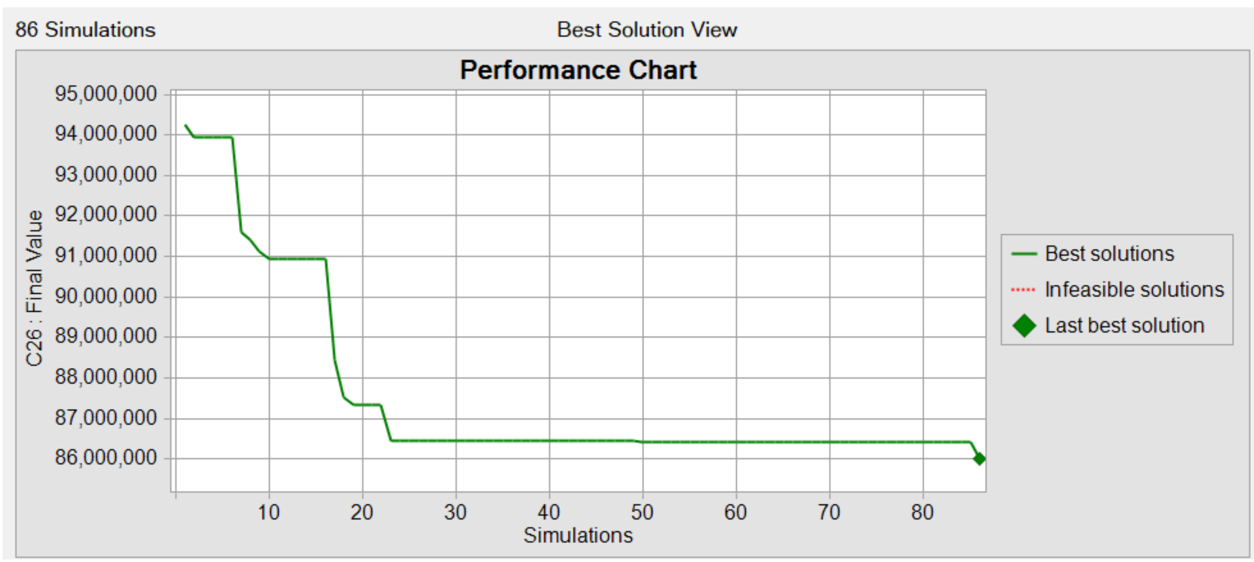


Figure 23 OptQuest performance chart (first 86 trials).

5. PROPOSAL OF COPPER CONCENTRATE SALE MODEL

Implementing an RMO (Risk Mitigation Options) means acting on the decision that was made. It requires risk managers to identify and mobilize resources necessary to actualize the RMO. Implementing a decision will very often expand the definition of who is a risk manager.

Implementation may require the cooperation of many people outside the relatively small circle of people who have worked on a risk management issue [19].

The Monte Carlo process samples individual values from a probability distribution so that they can be used to characterize the range of potential outputs in uncertain situations. This two-step process requires the generation of a simple random number and a means to convert that number to a useful value from the chosen distribution. A calculation-intensive numerical process has become immensely popular and easy to use with advances in personal computers and spreadsheet software.

The Monte Carlo process can be added to a wide variety of model structures. When probabilistic methods like that are added to scenario-structuring tools, like event trees, for instance, they create a powerful bundle of tools called probabilistic scenario analysis (PSA) [19].

PSA refers to a bundle of tools and techniques that make use of scenarios, probabilities, and probabilistic analysis. Most quantitative risk assessments are likely to be some form of probabilistic scenario analysis. Deterministic quantitative risk assessments would be the exception. Many decisions are hard because they are complex. They involve inherent uncertainty and have conflicting objectives. The various stakeholders with an interest in the decision are likely to hold many different perspectives. Scenarios can address these aspects of a decision problem.

A scenario is literally an outline or synopsis of a play. Scenarios can be used to describe the present or the past. They are most often used to describe possible futures.

Risk scenarios are readily used to provide the answers to our four informal risk assessment questions [19]:

1. What can go wrong?
2. How can it happen?
3. What are the consequences?
4. How likely is it?

Sensitivity analysis is the study of how the variation in a risk assessment output can be apportioned, qualitatively or quantitatively, to different sources. Complex risk assessments may have dozens of input and output variables that are linked by a system of equations and calculations. Risk assessors need to consider how sensitive a model's output, a risk characterization, or other important assessment outputs are to changes or estimation errors that might occur in model inputs, model parameters, assumptions, scenarios, and the functional forms of models. This information must then be effectively conveyed to risk managers so they can explicitly consider its significance for their decision-making.

Some risk assessment outputs and the decisions that rely on them may be sensitive to changes in assumptions and input values. However, it is not always immediately obvious which assumptions and uncertainties most affect outputs, conclusions, and decisions. The purpose of sensitivity analysis is to systematically find this out [19].

A good sensitivity analysis increases the assessor and manager's confidence in the risk assessment model and its predictions. It provides a better understanding of how model outputs respond to changes in the inputs. Because risk assessments can be qualitative or quantitative, sensitivity analysis can likewise be qualitative or quantitative. In a qualitative sensitivity analysis, the assessor identifies the uncertainties affecting the assessment and forms a judgment of their relative importance. A quantitative sensitivity analysis quantifies the variation in model outputs that is caused by specific model inputs and the model structure [19].

The decision model schematic was presented in the figure below:

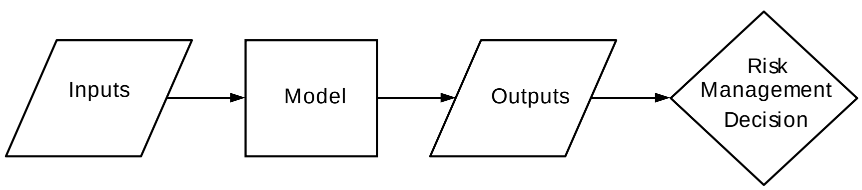


Figure 24 Decision model schematic (Yoe, 2019)

The Figure 15 presents the central elements of a decision process. There are the characteristics of the decision problem, which can be divided into the complexity of the decision task and the complexity of the decision context, the decision maker's goals and preferences, the decision strategy used, and the decision that is made.

The complexity of the decision task refers to the number of alternatives from which to choose and the number of attributes upon which to judge. The complexity of the decision context is affected by such things as the nature of the trade-offs among the alternatives, the range of attribute metrics (alternatives with more similar metrics tend to be more difficult for decision makers to differentiate), the quality of the alternatives, and their similarities or differences [19].

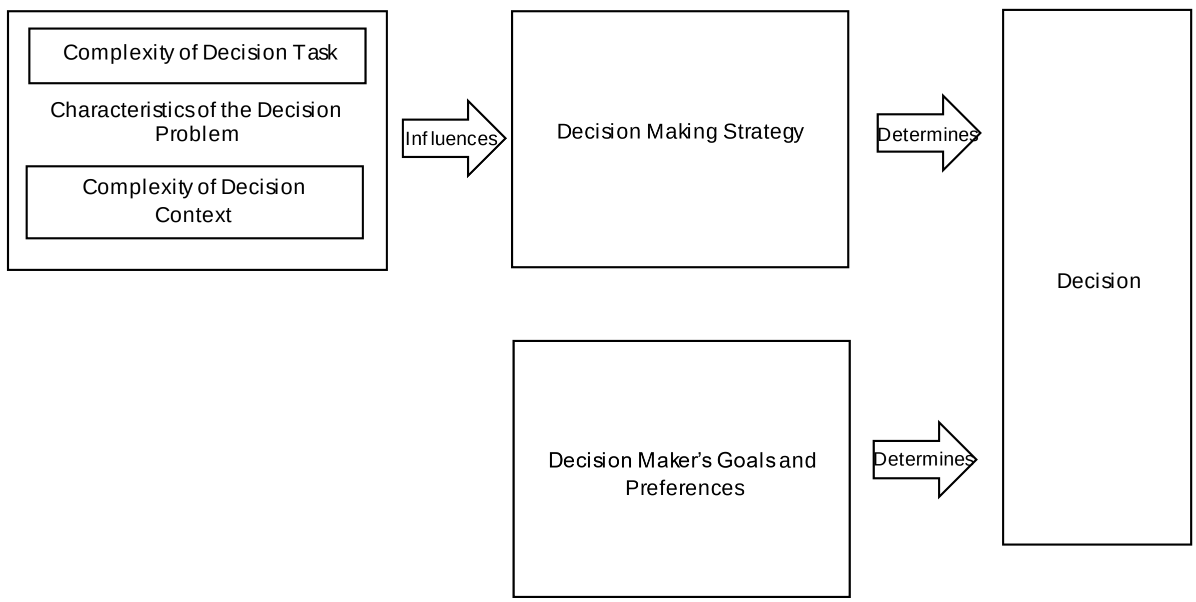


Figure 25 Central elements of a decision-making process. (Yoe, 2019)

Decision making strategies vary by such characteristics as the amount of available information that is processed, whether trade-offs are considered explicitly or not, whether decision makers process information by alternative or attribute, the use of weights and/or aspiration levels, whether alternatives are screened in or out, the use of quantitative or qualitative data, and the like. All of these elements contribute to the decision that is made. Risk analysis is evidence-based decision making under conditions of uncertainty [19].

Considering the Monte Carlo simulation, described above, the proposal and recommendation for the copper concentrate sale model is to sale 15 lots to trader A and trade AAA, 12 lost to smelter B and 41 lots to Other. This will incorporate greater agility and methodological flexibility to the execution of copper concentrate sale with more integrated vision to limit the risk of loss of cash flow and prepare the most efficient composition of sales.

6. CONCLUSION

In risk analysis, decision makers cannot count on the outcomes of their decisions. There is a range of outcomes and decision makers that need to understand. Risk is an important decision-making criterion and when risk is the only criterion, decisions are risk-based. When risk is an influential member of a set of decision criteria, decisions are risk-informed. In risk-based decision-making, risk estimates and risk narratives form the basis for decisions. Unacceptable levels of risk trigger action. Otherwise, risk-informed decision-making trades off levels of risk with other criteria to arrive at a decision [19].

Since most of the simulation models use random variables as input as randomized probability distributions the simulation output data themselves are random. Care must then be taken in drawing conclusions about the model's true characteristics both concerning the random variables but also the inter correlations [3].

Copper TCRCs are a keystone for pricing copper concentrates, which are the actual feedstock for copper smelters. The potential evolution of TCRCs is a question of both economic and technical significance for miners, as their value decreases the potential final selling price of concentrates. Additionally, copper miners' revenues are more narrowly related to the market price of copper, as well as to other technical factors such as ore dilution or the grade of the concentrates produced. Smelters, on the contrary, are hugely affected by the discount, which they succeed in getting when purchasing the concentrates, since that makes up the largest part of their gross revenue, besides other secondary sources. In addition, eventual differences between TCRCs may give commodity traders ludicrous arbitrage opportunities. In addition, differences between short- and long-term TCRCs agreements offer arbitrage opportunities for traders, hence comprising a part of their revenue in the copper concentrate trading business, as well copper price fluctuations and the capacity to make economically optimum copper blends [7].

The results obtained from the analysis model suggests that the annual optimal distribution is Trader A and Trader AAA of 15 lots each, Smelter B of 12 lots and Other clients of 41 lots. Having this optimal distribution, total copper treatment charges and refining charges, gold and silver refining charges, as well as paid penalties are amounted to US$ 87,524,294. However, the company uses this distribution Trader A, AA, AAA of six lots each, Smelter B and BB of five lost each, Smelter BBB of 25 lots and Other clients of 30 lots. This company distribution is amounted to US$ 109,651,518. Therefore, the difference gives the profit of US$ 22,002,650 lower cash outflows.

Based on the above mentioned result obtained by performing analysis model present in this thesis, recommended optimal copper concentrate sale distribution will incorporate greater agility and methodological flexibility to the execution of copper concentrate sale with more integrated vision to limit the risk of loss of cash flow and improve the economic value related to the copper concentrate sale.

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8. GLOSSARY

- Ag - Silver is a chemical element with the symbol Ag.
- Ag RCs - Silver refine charges.
- Au - Gold is a chemical element with the symbol Au.
- Au RCs - Gold refine charges.
- B/C-Rate - Benefit/Cost rate is an indicator showing the relationship between the relative costs and benefits of a proposed project, expressed in monetary or qualitative terms.
- CPM - Critical Path Method is one of the commonly used quantitative techniques especially for time and schedule risks of the project.
- Cu – copper is a chemical element with the symbol Cu.
- Cu RCs - Copper refine charges.
- Cu TCs - Copper treatment changers.
- DMT – dry metric ton.
- IRR – Internat Rate of Return is the annual rate of growth an investment is expected to generate.
- K-S - Kolmogrov-Smirnov test is a nonparametric test of the equality of continuous one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution (one-sample K–S test), or to compare two samples (two-sample K–S test).
- LME - London Metal Exchange.
- MCS – Monte Carl Simulation.
- ML - Most Likely value mode.
- Mo - Molybdenum is a silvery-white metal with the symbol Mo.
- NPV – Net Present Value is the difference between the present value of cash inflows and the present value of cash outflows over a period of time.
- NSR - Net Smelter Revenue method is commonly used to analyze the economic impact of the concentration grade of enriched minerals on mine revenue in the light of processing costs and metal market prices.
- PERT - Program Evaluation and Review Technique analysis is used for similar purposes as the CPM technique, focusing on time and schedule risks. It is used to determine the worst case and best-case scenario of the project duration.

- PSA - Probabilistic scenario analysis refers to a bundle of tools and techniques that make use of scenarios, probabilities and probabilistic analysis.
- QP - quotation period means the month prior to the month of scheduled delivery, irrespective of physical delivery.
- RA – Risk Analysis the process of assessing the likelihood of an adverse event occurring within the corporate, government, or environmental sector. Risk analysis is the study of the underlying uncertainty of a given course of action and refers to the uncertainty of forecasted cash flow streams, the variance of portfolio or stock returns, the probability of a project's success or failure, and possible future economic states.
- RMO - Risk Mitigation Options means acting on the decision that was made. It requires risk managers to identify and mobilize resources necessary to actualize the RMO.
- SWOT - Strengths, Weakness, Opportunities and Threats is qualitative risk tool. This technique is applied to project milestones, which is part of its internal project methodology. It is used along with the brainstorming technique especially to identify the strengths and weaknesses of the project.
- TCRCs - Treatment Charge (TC) and Refining Charge (RC) are commonly used in the terms of purchase for copper concentrate or nickel ore for refining. They are amounts designed to cover refining costs, i.e. the remuneration received by a smelter for processing smelting material and extracting metals.

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