Contents

Introduction **1**

1 Cosmological Perturbation Theory **6**
1.1 Background Dynamics . . . 7
1.2 Inflationary Backgrounds . . . 10
1.2.1 Single-field inflation . . . 10
1.2.2 Multi-field inflation . . . 12
1.3 Relativistic Perturbations . . . 15
1.3.1 Gauge Transformations . . . 17
1.3.2 Adiabatic and Isocurvature Perturbations . . . 19
1.4 Inflationary Perturbations . . . 20
1.4.1 Quantization . . . 23
1.4.2 The Power Spectrum . . . 25
1.4.3 Non-Gaussianities . . . 28

2 A Generalized Consistency Relation **34**
2.1 Review of the Consistency Relation Derivation . . . 35
2.2 A generalized Consistency Relation . . . 38
2.2.1 Case with $\epsilon \to 0$ . . . 39
2.2.2 Non-Gaussianity in ultra slow-roll inflation . . . 40
2.2.3 Case with $\epsilon \neq 0$ . . . 41
2.3 Analysis . . . 43
2.4 Discussion . . . 44

3 Vanishing of the Local non-Gaussianity **46**
3.1 Conformal Fermi Coordinates . . . 48
3.1.1 Central Geodesic . . . 48
3.1.2 Construction of the CFC Map . . . 49
3.1.3 Choosing the Conformal Scale Factor a_F . . . 51
3.1.4 CFC in Inflation . . . 51
3.1.5 Computation of Correlation Functions with CFC's . . . 53
3.1.6 Short Wavelength Modes in CFC . . . 55
3.2 Local non-Gaussianity in Single-field Inflation . . . 55
3.2.1 Further Developments . . . 56
3.2.2 Initial Conditions . . . 56
3.2.3 Computation of the Squeezed Limit . . . 57

3.2.4 Vanishing of Local non-Gaussianity . . . 59
3.2.5 On the Validity of CFC for non-Attractor Models . . . 61
3.3 Discussion . . . 61

4 A New Soft Theorem for Single-field Inflation 65
4.1 The General Picture . . . 67
4.2 Time Diffeomorphisms and FLRW Backgrounds . . . 67
4.3 Time Diffeomorphisms and Perturbations . . . 68
4.4 Modulation of Short Wavelengths in Comoving Coordinates . . . 72
4.5 Conformal Fermi Coordinates . . . 74
4.6 Observable bispectrum's squeezed limit . . . 76
4.6.1 Attractor backgrounds . . . 77
4.6.2 Non-attractor backgrounds . . . 77
4.6.3 Large observable non-Gaussianity? . . . 79
4.7 Discussion . . . 80

5 Inflation and Quantum Gravity 81
5.1 Super-Planckian Displacements in String Theory . . . 84
5.1.1 The Swampland Distance Conjecture . . . 85
5.1.2 A Geometrical Scalar Cutt-off . . . 89
5.1.3 A Multi-Field Lyth Bound . . . 90
5.2 Multi-field Inflation Overcomes the SDC . . . 92
5.2.1 Multi-Field Inflation . . . 92
5.2.2 Two-Field Inflation with Constant Turning Rates . . . 94
5.2.3 Perturbations . . . 95
5.2.4 Example: Inflation in Hyperbolic Spaces . . . 99
5.3 Geodesic Distances in Two-Field Models . . . 100
5.3.1 Example: Inflation in Hyperbolic Spaces . . . 101
5.3.2 Mixing Geodesic and Non-Geodesic Field Distances . . . 102
5.4 SDC, The Lyth Bound, and Non-Geodesic Motion . . . 103
5.5 Non-Gaussianity . . . 107
5.6 Discussion . . . 108

Conclusions 110

Bibliography 112

Appendices 126

A Details of computations in CFC 127
A.1 Map coefficients . . . 127
A.2 The 2-point correlation function for short modes . . . 128

B Temporal Diffeomorphism Generator 130

C Hyperbolic Coordinate Systems 132
C.1 Upper Half-Plane . . . 132
C.2 Poincaré Disk . . . 134

D Maximally Symmetric Geometries **135**
D.1 Planar Geometry 135
D.2 Spherical Geometry 136