

Contents

1. Introduction	1
1.1. Objectives	2
1.1.1. Main objectives	2
1.1.2. Specific objectives	2
1.2. Methodology	3
1.3. Thesis content	3
2. Background	5
2.1. Closed Polygons	5
2.2. Polygon Meshes	5
2.2.1. Basic Definitions	5
2.2.1.1. Manifold Meshes	6
2.2.1.2. Orientation of a Mesh	7
2.2.2. Mesh Representations	7
2.2.2.1. Vertex-Vertex	7
2.2.2.2. Face-Vertex	8
2.2.2.3. Halfedge	9
2.3. Geometry Algorithms	10
2.3.1. Math Background	10
2.3.1.1. Cross Product	10
2.3.2. Points Related Algorithms	11
2.3.2.1. Point inside Polygon	11
2.3.2.2. Point inside Edge	12
2.3.2.3. Point inside Circle	13
2.3.3. Edge Related Algorithms	13
2.3.3.1. Edge intersection	13
2.3.3.2. Edge intersection with Quadrilateral	15
2.3.3.3. Edge intersection with Circle	16
2.3.4. Polygon Related Algorithms	18
2.3.4.1. Polygon intersection with Circle	18
2.3.4.2. Area of a Polygon	19
2.3.4.3. Centroid of a Polygon	19
2.3.4.4. Getting minimum angle of a Polygon	20
2.4. Data Structures	20
2.4.1. Quadtrees	20
2.4.2. Generalized Quadtrees	22
2.4.3. KD-Trees	23

3. Related Work	25
3.1. Polygon Meshing Algorithms	25
3.1.1. Geometric Meshes based on Quadtrees	25
3.1.2. Geometric Meshes based on Voronoy Cells	26
3.1.3. Geometric Meshes based on Centroid Voronoi Tessellation (CVT) . .	27
3.2. Mesh Visualizers	27
3.3. Polygon Mesh quality metrics	28
3.3.1. Scale Dependent measures	28
3.3.2. Scale Invariant measures	28
3.4. Polygon Clipping Algorithms	29
3.4.1. Greiner Hormann Algorithm	29
3.4.1.1. Phase 1: Searching intersections	30
3.4.1.2. Phase 2: Marking entry and exit points	30
3.4.1.3. Phase 3: Constructing the clipped polygon	31
3.4.1.4. Disadvantages of the algorithm	32
3.4.2. Extended Greiner Hormann Algorithm	33
3.4.2.1. Classification of Polygonal Chains	33
4. Design	35
4.1. Mesh Generation Process	35
4.2. Analysis of possible solutions	36
4.3. Proposed software architecture	36
4.3.1. General Architecture	36
4.3.2. General Implementation Choices	37
4.4. Process Design	37
4.4.1. User draws a contour geometry in our Application	38
4.4.2. User uploads Contour Geometry	39
4.4.3. User uploads Mesh by off File	39
4.4.4. Quadtree refining process	41
4.4.5. Quality refining process	42
4.4.6. Quality inspection	44
4.4.7. Exporting	45
4.5. Model Design	46
4.5.1. Geometry and Selector Region	46
4.5.2. Polygon in detail	47
4.5.3. Clipping Algorithms	47
4.5.4. Criteria	48
4.5.5. Quality Refining Algorithms	50
4.5.6. Division Algorithms	50
4.5.7. Mesh	51
4.5.8. HalfEdges	52
4.5.9. Storages	52
4.5.10. Tree	53
4.5.11. React frontend Modeling	54
4.6. Experimental Design	56

5. Implementation	57
5.1. Mesh Representation	57
5.1.1. Vertices	57
5.1.1.1. Vertex representation	57
5.1.1.2. Vertex Storage	58
5.1.2. Edges	60
5.1.2.1. Edge representation	60
5.1.2.2. Edge Storage	61
5.1.3. Polygons	63
5.1.3.1. Polygon representation	63
5.1.3.2. Polygon Storage	64
5.1.4. Halfedge Connectivity	65
5.1.4.1. Halfedge definition	65
5.1.4.2. Creating halfedges for a polygon mesh	66
5.1.4.3. Applying operations to polygons	68
5.1.4.4. Obtaining the neighbors of a polygon	68
5.1.4.5. Add a Polygon to the Mesh keeping while maintaining connectivity	69
5.2. Web Application Views	70
5.2.1. Integrating PixiJS	71
5.2.2. Geometry creation and initial panel	73
5.2.3. Quadtree Refining Panel	76
5.2.4. Refining Panel	79
5.2.5. Quality Component	81
5.3. Algorithms Implementation	81
5.3.1. Clipping Algorithms	81
5.3.1.1. Sutherland Hodgman Algorithm	82
5.3.1.2. Extended Greiner Hormann Algorithm	84
5.3.1.3. Calculating intersections	87
5.3.1.4. Determining the orientation of polygon chains	89
5.3.1.5. Classifying intersections	91
5.3.1.6. Marking Intersections Chains	92
5.3.1.7. Building the <i>Entering</i> and <i>Exiting</i> lists	93
5.3.1.8. Traversing the lists	96
5.3.1.9. Complexity Analysis	99
5.3.2. Point insertion in Tree Data Structures	100
5.3.2.1. Half Point Division Algorithm	101
5.3.2.2. Arbitrary Point Division Algorithm	104
5.3.3. Generating the Initial Mesh	107
5.3.3.1. Generating new polygons from a contour geometry	108
5.3.3.2. Obtaining the possible problematic points	108
5.3.3.3. Fixing polygons consistently with problem points	110
5.3.4. Refinement Algorithms	112
5.3.4.1. Tree Refinement	112
5.3.4.2. Splitting Longest Edge	114
5.3.4.3. Centroid	117
5.3.4.4. Centroid with Replication	118

6. Results	121
6.1. Time Analysis	121
6.1.1. Extended Greiner Hormann Algorithm	121
6.1.2. Initial meshes creation time	122
6.2. Initial Mesh Generation	124
6.2.1. Initial Meshes	124
6.3. Quality improvements	126
6.3.1. Initial Meshes	127
6.3.2. Quality Refinements to Bad Polygons	128
6.3.2.1. Centroid refinement to Initial Meshes	129
6.3.2.2. Centroid Replication refinement to Initial Meshes	130
6.3.2.3. Splitting Longest Edge refinement to Initial Meshes	132
6.4. Successive quality refinements	134
6.4.1. Declaring an upper limit to the area of the polygons	134
6.4.2. Upper limit equal to average mesh area	134
6.4.2.1. Results obtained by centroid algorithm	135
6.4.2.2. Results obtained by centroid replicate algorithm	136
6.4.2.3. Results obtained by Splitting Longest Edge algorithm	137
6.4.2.4. Results obtained by Quadtree Refining algorithm	138
6.4.3. Upper limit equal to one tenth of the average area	138
6.4.3.1. Results obtained by centroid algorithm	139
6.4.3.2. Results obtained by centroid replicate algorithm	140
6.4.3.3. Results obtained by Splitting Longest Edge algorithm	141
6.4.3.4. Results obtained by Quadtree Refining algorithm	142
6.4.4. Declaring an upper limit to the maximum edge length of the polygons	143
6.4.5. Upper limit equal to average edge length	143
6.4.5.1. Results obtained by Splitting Longest Edge algorithm	144
6.4.5.2. Results obtained by Quadtree Refining algorithm	145
6.4.6. Upper limit equal to one half of the average edge length	145
6.4.6.1. Results obtained by Splitting Longest Edge algorithm	146
6.4.6.2. Results obtained by Quadtree Refining algorithm	147
6.5. Comparison of quality metrics between different levels of refinement	148
6.5.1. Results of imposing a upper limit to the maximum area	148
6.5.1.1. Limit equal to the average area of the geometric mesh	149
6.5.1.2. Limit equal to $\frac{1}{10}$ of the average area of the geometric mesh	150
6.5.1.3. Analysis of metric results	150
6.5.2. Results of imposing a upper limit to the maximum length	151
6.5.2.1. Limit equal to the average length of the geometric mesh	151
6.5.2.2. Limit equal to $\frac{1}{2}$ of the average length of the geometric mesh	152
6.5.2.3. Analysis of metric results	152
6.6. Comparing meshes with Triangle	152
6.6.1. Comparing meshes	153
6.6.1.1. Maximum area equal to 1979 area units	153
6.6.1.2. Maximum area equal to 198 area units	157
6.6.1.3. Maximum length equal 30 length units	158
6.6.1.4. Maximum length equal 15 length units	159
6.6.2. Comparison in quality metrics	159

6.6.2.1.	Initial Mesh: Quadtree with Mid Point strategy - Maximum area equal to 198 area units	160
6.6.2.2.	Initial Mesh: KD-tree - Maximum area equal to 198 area units	161
6.6.2.3.	Initial Mesh: Quadtree with Mid Point strategy - Maximum length equal to 15 length units	162
6.6.2.4.	Initial Mesh: KD-tree - Maximum length equal to 15 length units	163
7.	Conclusions	164
7.1.	Application	164
7.2.	Results of Experiments	165
7.2.1.	Quadtree results	165
7.2.2.	KD-tree results	165
7.2.3.	Comparison with Triangle	166
	Bibliography	168
	Appendix A. .OFF file format	171