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ESSAYS ON LABOR MARKET FRICTIONS AND INTERNATIONAL FINANCE

TESIS PARA OPTAR AL GRADO DE
DOCTOR EN SISTEMAS DE INGENIERÍA

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RESUMEN DE LA MEMORIA PARA OPTAR
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ENSAYOS SOBRE FRICCIONES LABORALES Y FINANZAS INTERNACIONALES

Esta tesis es una colección de dos ensayos en temas macroeconómicos. Sus capítulos revisan tópicos relacionados con fricciones en el mercado laboral, fricciones en el mercado del crédito, cambio tecnológico, así como también modelo de pequeña economía abierta en el que se incluyen preferencias no homotéticas.

El Capítulo 1 revisa la hipótesis de shocks e instituciones del desempleo de largo plazo. Desarrolla la idea que trayectorias de desempleo heterogenéas pueden ser explicadas por un shock tecnológico común, propagado de manera diferente de acuerdo al grado de desarrollo que tenga el mercado del crédito. En términos empíricos se motiva la relación entre los tres ingredientes principales del modelo: desempleo, fricciones en el mercado del crédito y progreso tecnológico. En términos teóricos las fricciones, en ambos mercados, corresponden a fricciones de búsqueda y emparejamiento. El desempleo, es el resultante de la composición de dos objetos que responden endógenamente a cambios del mercado del crédito: duración del desempleo e incidencia al desempleo. El mensaje principal es que economías con un mal funcionamiento del mercado del crédito propagan el shock tecnológico principalmente a través de la duración del desempleo y en menor grado a través de la incidencia, lo que aumenta el efecto del cambio tecnológico sobre el desempleo. En contraposición, bajo un buen funcionamiento del mercado crediticio, el shock tecnológico afecta principalmente la incidencia en lugar de la duración del desempleo. Más aún, mayores fricciones crediticias inducen mayor desigualdad salarial, una distribución de edades de tecnología más heterogéneas y afecta el producto de la economía.

El Capítulo 2 analiza una pequeña economía abierta caracterizada por dos sectores, transables y no transables, además de fricciones nominales en el mercado del trabajo. El modelo incluye preferencias no homotéticas para capturar el hecho que en tiempos de buenos (malos) ingresos, los bienes producidos en el sector no transables sean más (menos) demandados que los bienes producidos en el sector transable. Con estas preferencias, la tasa marginal de sustitución queda dependiente del nivel total de consumo, además de permitir el análisis del efecto ingreso sobre bienes. Ambas características relevantes en el análisis de crisis financieras y ciclo económico en economías emergentes. El objetivo es estudiar cómo esta estructura de preferencias afecta la respuesta de política fiscal y de tipo de cambio durante ciclos de auge y caída en consumo. Los resultados teóricos y cuantitativos demuestran que la respuesta de una economía no-homotética amplifica el ciclo. Esta amplificación, obliga a racionalizar una respuesta de política mucho más agresiva de modo de hacer frente a los efectos negativos del ciclo. El mecanismo de esta amplificación, se basa en el mayor uso de la deuda con el fin de asignar consumo entre sectores, no sólo durante el ciclo sino también en el largo plazo.

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This dissertation is a collection of essays focusing on a broad sense Macroeconomics. Through its two chapters, this thesis reviews topics in labor and credit market frictions, capital-embodied technological change, as well as, non-homothetic preferences in small open economies with downward nominal wage rigidities.

In chapter 1, we revisited the shock-institution hypothesis of long-run unemployment. Through the chapter we find that heterogeneous long-run unemployment trajectories can be accounted for a common technological shock that is propagated differently by differences in the credit market functioning. We present empirical evidence that motivates the relationship between the three key ingredients of the model: labor market frictions, credit market frictions and capital embodied technical change. Frictions are modeled as in the Diamond-Mortensen-Pisarides framework and unemployment is the composite of two terms, the unemployment duration and unemployment incidence, both of which depend on credit market performance. The key implication of the model is that economies with a worse credit market absorbs the technological shock mainly through the labor market tightness, primarily affecting the unemployment duration. Moreover, the higher the credit market frictions the higher the wage inequality, and more heterogeneous the vintage technology distribution which affect the output of the economy.

In chapter 2, I analyze the effects of non-homothetic preferences in a small open economy with nominal frictions in the labor market. I present a standard small open economy model with two sectors, tradable and non-tradable, and whose friction is represented by downward nominal wage rigidity. I extend this environment with non-homothetic preferences to study how heterogeneous income elasticity among goods affect the policy response in a boom-bust cycle of consumption. Non-homothetic preferences ensure that the marginal rate of substitution depends on the level of total consumption in addition to allowing the analysis of income effects. Both features are especially important for the study of cycle and financial crises. This chapter shows theoretically and quantitatively an amplification of the boom-bust cycle in a non-homothetic economy. This amplification in turn, forces stronger policy responses to partially offset the adverse effects of the cycle. The underlying mechanisms is the greater use of debt in the non-homothetic economy to allocate consumption between sectors, not only during the cycle but also in the long term.

A mi esposa Gabriela y mi adorable hija Anita Milagros

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Introduction

This thesis is a compendium of two academic articles that is hoisted around the subfield of Macroeconomics. As pointed out by Mankiw (2006)

> *''the (macro) discipline has evolved through the efforts of two types of macroeconomist, those who understand the field as a type of engineering and those who would like it to be more of a science. Engineers are, first and foremost, problem solvers. By contrast, the goal of scientists is to understand how the world works ... Just as the world needs both scientists and engineers, it needs macroeconomists of both mindsets. ''*

The goal of this dissertation is precisely to be located in this route. The present doctoral project studies models that allow a better approach to reality and at the same time resolve the associated complexity that arises from them. The scope of the topics presented in the following chapters are broad in a macro sense. They range from a long-term perspective to a more focused look at cyclical behavior. They also include subjects related to a closed economy but also topics related to small open economies. We divide this thesis into two chapters.

Chapter 1 presents a closed economy characterized by two frictional markets, labor and credit, and an exogenous capital-embodied technical change. This chapter primarily deals with the interaction of this ingredients and its effect on long-run unemployment. The main hypothesis states that heterogeneous long-run unemployment trajectories can be accounted for a common technological shock that is propagated differently by differences in the credit market functioning

Both, labor and credit markets suffer from matching frictions and two-sided search. The theoretical framework to address the frictions in the labor market is the well known Diamond-Mortensen-Pissarides (DMP) model, work for which its authors won the Nobel prize in 2010. Less well known, however, is the fact that the same theoretical framework is used to study frictions concerning the credit market. This literature, first presented by Wasmer and Weil (2004) and then used by Petrosky-Nadeau (2013, 2014); Petrosky-Nadeau and Wasmer (2015), rationalize in a simple and tractable way the frictions presented in the credit allocation process. Our contribution to this environment lies in the analysis of the long-run unemployment and the interplay with technological progress.

Using data from EU KLEMS, OECD and the World Bank we show some evidence in the line of our hypothesis. In aggregate terms we find a significant negative correlation between our credit variable and unemployment[1]; a positive effect of technological change on unemployment; and a negative effect of the interaction term between credit and technology on unemployment. At industry level the data suggests that share of employment in ITC-Capital industries grew more in those high-credit countries than in low-credit ones. In terms of labor flows, the interaction term between credit and technology has a positive and significant correlation in hiring and separation rate.

We build a tractable vintage capital model with credit as well as labor market search and matching frictions. Unemployment in the model is a composite of two terms: unemployment duration and unemployment incidence, both objects dependent on technological progress and the credit market.

Our theoretical results suggest that an economy with a worse credit market absorbs the technological shock mainly by the adjustment of the labor market tightness leaving the economy in a more vulnerable scenario. These economies face a more plausible context in which the technological shock decreases the tightness of labor market increasing the unemployment duration. Conversely, an economy with a better credit market absorbs the technological shock mainly by the useful time-length of capital implying that the unemployment adjustment is leaded by the unemployment incidence and not by unemployment duration. As a result the relative change in unemployment is lower when the economy has a better credit market. Additionally, we also show that the technology gap induced by credit frictions affects wage inequality, the stationary density distribution of capital ages and the level of total output. The higher the credit market frictions the higher the wage inequality, and more heterogeneous the vintage distribution which in turns affect the output of the economy.

The second chapter examines the role of non-homothetic preferences in small open economy models. There is a long tradition of non-homothetic preferences in economics. Gorman (1965) and more clearly in Hanoch (1975) explores a generalization of the homogeneous $n-$goods Constant Elasticity of Substitution (CES) model to a non-homothetic and non-CES function using implicit separability of the direct utility (production) function. Sato (1975, 1977) derives a general class of CES functions which are non-homothetic and non-separable, that include the ordinary CES function as special case. All these papers have been studied on comparative statics basis, in partial-equilibrium, applicable to one-period models under certainty and competitive factor, or consumer good, markets. Recent developments, however, use this kind of non-homothetic CES preferences in a general equilibrium setting Comin et al. (2020); Hubmer (2020), where there exists good-specific non-homotheticity parameters that control the relative income elasticities.

The homotheticity assumption has two consequences which may be unwanted in some scenarios. First, the marginal rate of substitution, and therefore the relative prices, is not state dependent on the level of production or consumption. Second, all

[1]Our credit variable is rationalized by the ratio of private credit by deposit money banks and other financial institutions over GDP.

commodities are always treated as a normal goods, while under the non-homothetic CES function, income effects play a key role that can be studied. Both of these motivations are a fundamental insights of the second part of this project.

In chapter two I analyze how the exchange rate and fiscal policies change if heterogeneous income elasticities among goods are considered during boom-bust consumption episodes. For this purposes I build on a standard open-economy model with two sectors, tradable and non-tradable, and downward nominal wage rigidity, a pervasive feature of emerging economies. In this kind of models a well-know pecuniary externality arises from the combination of wage rigidity and a fixed nominal devaluation rate that translate into an involuntary unemployment.

I include to this environment non-homothetic preferences to capture that non-tradable goods have a higher income elasticity than tradables. The model shows an amplification in the responses of the economy respect to the canonical homothetic case in a boom-bust cycle. In the long-run, however, both economies present a similar pattern in aggregate variables but differ in an important dimension: the non-homothetic economy is characterized by a more heavy use of debt. The optimal first best full-employment exchange rate policy in this context is larger. In quantitative terms the currency devaluation required to undoes the externality is 40 % larger at the through of the bust. This policy ensures that real wages and the real exchange rate fall 37 % higher. When considered the second best fiscal policy, the non-homothetic economy shows a capital control rate 2 % higher at the boom and 1,5 % lower at the bust, whereas the unemployment rate shape is similar for both economies. In summary, adding a real-world feature to the standard model helps to understand wider macroeconomic fluctuations and so to rationalize stronger policy responses.

The contribution of this thesis lies in a policy view of the discipline. The three chapters answer different questions about economic aggregates based on the underlying idea: How the economy should be adjusted to reduce those negative effects derived from an exogenous shock, both in the long and short term.

Chapter 1

Vintage Capital, Credit Frictions and Labor Market Outcome

1.1. Introduction

How does the allocation of credit affect the long-run unemployment when the economy is hitting by an acceleration of the capital embodied technical change?. The recent breakneck technological change has stressed the role of technology adoption in shaping the unemployment rates among countries. The literature pushed that differences in labor market institutions are at the heart of the propagation mechanism to explain the heterogeneous evolution of unemployment. However, the heterogeneity in unemployment trajectories has not been fully explained by this mechanism. In this paper we lengthen the scope of this hypothesis and propose a search theoretical framework in which interactions across credit market and technological progress can explain different patterns in long-run unemployment.

On the one hand, a vast body of literature agrees that an increase in the rate of technological progress embodied in newly created jobs unambiguously increases unemployment in the long-run [cf. Aghion and Howitt (1994), Mortensen and Pissarides (1998), Mortensen and Pissarides (1999),Duernecker (2014), Hornstein et al. (2007)]. On the other hand, an important stream of the search literature examines credit market imperfections as an important contributor to the level and the persistence of unemployment as well as a driver of its cyclical volatility [cf. Acemoglu (2001), Dromel et al. (2010), Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014) Petrosky-Nadeau and Wasmer (2015)]. While much is known about the link between unemployment and each of these strands of the literature, less is known on how these two potential mechanisms interact each other and how this interaction affects the economy.

The aim of this paper is that. We explore the mechanism by which a common technology shock can be propagated differently to the labor market because of differences in the credit market functioning. We propose a simple theoretical search model to embed these two forces. We address how frictions in the allocation of credit shape technology adoption and how this pattern affects labor market outcomes. We argue that the ability of the credit market to allocates funds for the start-up of new technological productive projects, is essential to understand the labor market effects of technological progress.

This is important for the following reasons. First, several authors have pointed out to financial development as the main source in shaping technology adoption. The common view is that an efficient credit environment reduces financing constraints for entrepreneurs. With a higher openness in funds access, entrepreneurs can adopt the cutting edge technology (Cole et al., 2016), reach a higher economic growth (Rajan and Zingales, 1998) and the diffusion of technology-intensive capital accelerates (Comin and Nanda, 2019).

Second, we claim that the ability of a country to adopt new technologies in a context of faster arrival rate of technology progress, is crucial to the labor market performance. We thought this ability as the ease in which entrepreneurs find the necessary funds to start a new business idea speeding up the firm and job creation. Acemoglu (2001) compares the two distinct unemployment stories between the U.S. and Europe since 1970. The author argues that an economy with a more flexible credit market (U.S.), nimble in the provision of loans to new firms, can get a better response to new opportunities than an economy with less developed credit (Europe). As a result, the latter experienced a persistent adverse effect on unemployment.

In the same vein, Duernecker (2014) stresses the technological heterogeneity as a new dimension along which countries differ. The premise is that the size of the technology gap, resulting from a slack technology adoption process, is a key determinant of unemployment. In some sense, our paper is a micro foundation of the technology gap approach proposed by the author. These results suggest that the study of the interaction between the credit market and technological progress is important to understand the mechanisms by which the labor market outcomes are affected.

Third, high unemployment OECD countries have on average higher age of capital which is at odds with theoretical models. Search theoretical models of technology vintage agree that an increase in the rate of technology progress embodied in new created jobs unambiguously increase unemployment[1]. The underlying mechanism is the obsolescence feature of technology: the maximal age of jobs and labor market tightness are decreasing functions of technology progress. The lower these variables, the higher the unemployment rate.

[1] These models assume that once a vintage technology is installed its remain fixed until the age of destruction. However, when an upgrade to the leading-edge technology is allowed, the outcome depends on the size of the update costs. On the one hand, lower upgrading costs – zero in the limiting case – imply that all jobs in the economy benefit from technological progress. On the other hand, higher upgrading costs imply that only new jobs are benefited from technical change. But even in both cases, for quantitative reasonable parameters how technology is modeled does not matter for equilibrium outcome, and a positive correlation between technology growth and unemployment remains (Hornstein et al. (2005)).

In this kind of models, the maximal age of a job is equivalent to the profitable productive age of the oldest vintage technology. In other words, the maximal age at which the technology must be scrapped, since its productivity is lower than the cost of use it. This mechanism suggests that in the past decades of accelerated productivity growth embodied in new vintages, higher unemployment economies are those with lower scrapping age of technology and therefore with lower average age of technology. Figure 1.2 in the next section, however, show the opposite pattern and suggest that a common feature in this relationship is the functioning of the credit market. We return to this point later.

The contribution of this paper lies in the shocks-institutions hypothesis for long-run unemployment. According to this literature, different unemployment trajectories are accounted for by a common shock which is propagated in a different fashion due to institutional differences. We argue that not just matter labor institutions but also we emphasize that market interactions are at the heart of these large divergences in the unemployment evolution.

We show a tractable general equilibrium model that features technology progress in the form of capital-embodied technical change as well as search frictions in the labor and credit market . In doing so, this paper gives a simple way to understand the interplay of these forces but at the same time gives light about mechanisms that are present in the data but absent in the current theoretical approach. Additionally this paper try to give a first step in terms of international comparison using empirical measure of technology progress using data on quality-adjusted price of ICT-capital equipment.

The type of questions that we want to address are related to the following exercise. Imagine two economies that differ each other in the functioning of credit market. One of these two economies has a more developed credit market than the other. Starting from an initial steady state, the focus is on how different is the adjustment of these economies to the new steady state equilibrium when technology progress embodied in new capital vintage arises. The thesis is that credit frictions discourage the adoption of new technology and therefore reduces job creation, but at the same time lengthens the useful life of current jobs. How this effect interact is the mechanism that we want to explore.

For this purpose we work on a vintage capital model with labor and credit market frictions. Frictions in both markets are modeled using the workhorse Diamond-Moretensen-Pissarides (DMP) model extended to include credit frictions *à la* Wasmer and Weil (2004) and largely developed by Petrosky-Nadeau (2013) and Petrosky-Nadeau (2014).

Credit market is relevant because it allows entrepreneurs to find funds to install new capital and start a business. Due the frictions, searching for funds is a costly process that takes time and effort. The vintage capital building block closely follows Hornstein et al. (2007) where technological change embodied in new vintage of capital induces firm heterogeneity. However, unlike them in this paper firm heterogeneity is given in part for credit frictions.

Once a lender and an entrepreneur meet they act as a *joint-venture* in the costly search process for a suitable worker. When meet a worker, the production unit formed by joint-venture worker pair is called the *firm*. Both, the firm and the joint venture can separate for exogenous and endogenous reasons. When the firm suffers an exogenous separation shock that destroys the job, they may keep the loan relationship. This separation shock means that a production unit can transit back and forth between the labor market stage, as a joint venture, and in the production stage, as a firm. This features allows the existence of vintage capital distribution of joint ventures (vacant capital) and firms (matched capital)[2].

In the standard vintage capital model newly created production units embody the newest and more productive technology capital. When new vintages of capital enter the economy, older vintages become relatively less productive. Thus, new jobs are able to pay higher wages which, in turn, increase the outside option of workers employed in older vintages, making an upward wage pressure in those jobs.

There is a productivity threshold where the increasing wage makes an old vintage unprofitable. A faster rate of technology progress act as an obsolescence shocks that tends to shorten job duration, increasing unemployment incidence. However, these models assume that the capital acquisition is made through a perfectly competitive market, i.e., entrepreneurs always have access to the required funds to start a business. We relax these assumption and examine how the ease with which entrepreneurs access funds is relevant to understand the effect of technology progress on labor market.

We solve the model in a tractable and analytical fashion that let us analyze the equilibrium in the space defined by the labor market tightness and the useful time-length of a job. Our theoretical results proceed as follows.

Unemployment is a composite of two terms: unemployment duration, decreasing in the labor tightness, and unemployment incidence decreasing in the maximal life of a job. In line with previous literature, a capital-embodied technological shock reduces the useful life of capital (obsolescence effect) and has an ambiguous effect on labor tightness.

However, the propagation of technological shock differs considerably in the kind of credit market an economy has. The economy with a worse credit market absorbs the technological shock mainly by the adjustment of the labor market tightness leaving the economy in a more vulnerable scenario. Moreover, these economies face a more plausible context in which the technological shock decreases the labor tightness increasing the unemployment duration margin. As a result the unemployment rate increases.

Conversely, an economy with a better credit market absorbs the technological shock mainly by the useful time-length of capital. This imply that the unemploy-

[2]The distinction between vacant and matched capital is equivalent to say that the capital is idle or in operation.

ment adjustment of these economies is leaded by the unemployment incidence. The movement of the labor tightness in these economies has more chances to be increasing, lowering unemployment. Even if this is no the case the drop is lower than the drop in worsen credit economies. As a result the relative changes in unemployment is lower in the economy with a better credit market.

We also find that the technology gap induced by credit frictions affects wage inequality, the stationary density distribution of capital ages and the level of total output. The higher the credit market frictions the higher the wage inequality measured as the ratio between the maximum to a minimum wage. This result is also robust to an alternative measure of wage inequality, the mean-min wage ratio.

The aggregate output of the economy is also affected by credit frictions. Higher financial costs affect the density distribution of the matched and vacant capital. The distribution becomes more heterogeneous. Thus, an economy with higher financial cost will produce a longer time but with an on average less productive capital.

1.1.1. Related literature

This paper contribute to the large literature on unemployment determinants with focus on technology. Several theoretical studies propose a positive relation between technology progress and unemployment. Mortensen and Pissarides (1998) study how technological progress affect the equilibrium number of jobs, as main conclusion the authors stress that the effect of technology progress on unemployment depends on the updating cost of technology. When renovation costs are low, higher technology progress induces lower unemployment, but this response switches from positive to negative as the cost of updating existing technology rises above a unique critical level. Hornstein et al. (2005) calibrate a model of technological change and labor market frictions distinguishing two cases: a creative destruction economy where new technologies enter through new matches and upgrading economy where old technologies are replaced by new ones in the existing matches. These two polar cases are special instances of the updating cost theorized by Mortensen and Pissarides (1998). Hornstein et al. (2005) establish that for quantitative reasonable parameters how technology is modeled does not matter for equilibrium outcome, and a positive correlation between technology growth and unemployment remains. Postel-Vinay (2002) study the dynamics of technological unemployment and present a simple model of frictional labor market that capture the negative long-run unemployment effects of technical change through the job obsolescence, but also shown a positive short-run effect on employment. The intuition is the following: immediately after a sudden technological shock, agents start to offer less jobs because they anticipate the drop in profitability. The natural consequence of a lower availability of jobs is to make existing jobs more valuable postponing its destruction in the short-run. Prat (2007) and Pissarides and Vallanti (2007) unlike the previous papers focus on study the effect of disembodied technical change on unemployment and argue that disembodied technology is necessary for the model to match empirical evidence.

A closely related work in terms of the model are Hornstein et al. (2007) and Duernecker (2014) who try to explain the different labor market outcomes in Europe and U.S. since the 1970's. Hornstein et al. (2007) use a frictional labor market model with vintage capital and firm heterogeneity to address how capital-embodied technical change together with labor market institution influence labor market outcome. The authors demonstrate that capital-embodied technological change reduces labor demand, raises equilibrium unemployment and raises unemployment durations. The authors also demonstrate that these effects are exacerbated when technology interact with labor market institutions such as unemployment benefits, payroll taxes, and firing costs. In the model proposed by the authors, heterogeneity in productivity relies in the assumption on irreversible investment in new vintages of capital, however, our model endogenizes this investment through the presence of credit markets frictions. Duernecker (2014) motivated by technology gaps between countries, establishes that when there is capital-embodied technical change, the unemployment rate depends critically on how obsolete the installed capital stock is compared to the frontier. For this purpose the author builds a search and matching labor model with worker heterogeneity that closely follow Hornstein et al. (2005). The main finding shows that this channel accounts for about 70 % of the discrepancy between the behavior of unemployment rates in Europe and the United States.

All the literature mentioned above has the underlying assumption of a perfect credit market and, therefore, the credit channel is absent as a mechanism. A large and growing literature on the relation of credit market and unemployment has been developing in recent decade. Gatti et al. (2011) investigate in a panel of 18 OECD countries over the period 1980-2004 how labor and financial market features jointly affect the unemployment rate. The main message suggest that financial variables impact unemployment in a way that crucially depends on the labor market context. Increased market capitalization as well as decreased banking concentration, an scenario that we could named as a lower frictions, reduce unemployment if the level of labour market regulation, union density and coordination in wage bargaining is low. Increasing intermediated credit and banking concentration is beneficial for employment when the degree of labour market regulation, union density and coordination in wage bargaining is high. In another empirical paper, Belke and Fehn (2001) and Belke et al. (2002) explore the relationship of venture capital markets and unemployment in OECD countries. The main insight is that venture capital markets should help to alleviate financial frictions that are viewed as important obstacles against new firms and jobs creation. Overall venture capital investments and early stage venture capital investments in relation to GDP improve significantly labor-market performance. These effects are present in a wide array of different econometric specifications and they are in particular still prevalent when the standard institutional variables describing labor- and goods- market regulations are included in the panel regressions. Dromel et al. (2010) studied empirically the relationship between unemployment, labor market institution and credit market[3] finding that credit market constraints not only affect the level of unemployment but also its persistence.

Several theoretical papers address this relationship also. In their seminal paper Wasmer and Weil (2004) proposed a tractable framework to study the interaction

[3] In fact these authors use the same empirical measure of credit functionning used in this paper.

between credit market and labor market frictions. A novel treatment of the credit building-block was to apply the search and matching environment widely discussed in (Pissarides, 2000, ch. 1) to those frictions in the credit market. The authors demonstrate in first place that an increase in the credit market tightness, the theoretical device of credit frictions, lowering the labor market tightness. More important is that credit frictions amplify macroeconomic volatility through a financial accelerator. The magnitude of this general-equilibrium accelerator is proportional to the credit gap, defined as the deviation of actual output from its perfect credit market level. This framework has been widely extended by Petrosky-Nadeau and Wasmer (2013), Petrosky-Nadeau (2013), Petrosky-Nadeau (2014) and Petrosky-Nadeau and Wasmer (2015). In Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau (2014) for example, the authors studied the cyclical volatility of search and matching model of credit and labor market to respond in part to the volatility puzzle identified by Shimer (2005). They main point is that credit frictions create volatility by introducing an additional acyclical entry cost to procyclical job creation cost, what generates an increase in the elasticity of labor market tightness to productivity shocks.

In Petrosky-Nadeau (2013) the framework is extended to include heterogeneity in firm productivity to explain the sharp contraction in the aggregate output and employment and the increase in TFP that follow the financial crisis of 2008. According to the author, negative shocks to credit market destroy the least productive jobs and at the same time slow the job creation raising, consequently, the TFP and unemployment.

Petrosky-Nadeau (2013) shows a similar pattern in a relate model. However, the endogenous destruction decision in his model is drawn from an exogenous productivity distribution while in this paper is the result of the obsolescence feature of technology

Petrosky-Nadeau and Wasmer (2015) analyze de macro dynamics in a model of goods, labor and credit frictions motivated by mechanisms that endogenously generate large and persistent response to shocks. The main insight of the paper is that good market frictions are key to understand labor market dynamics and to generate large hump-shaped responses to productivity shocks. None of these extensions consider technical change embodied in new vintage of capital, its primarily focus is on the direct impact of credit market on labor market.

In this paper we study the interaction of these two well-studied building-blocks. Nevertheless, we are not the first to focus on the interaction of credit market imperfections and technology adoption and its effect on unemployment. In a closer paper in terms of the spirit of the mechanism, Acemoglu (2001) developed an hypothesis that credit market differences between US and Europe could be a factor to explain the opposite pattern in unemployment in these two regions. The main argument is that technology shocks open new investment opportunities that can be better used by those economies with a better credit market. In his model the author proposed a Walrasian framework where agents live for a period and are replaced by its offspring. Agents can become an entrepreneur, a worker or remain unemployed depending on the level of skills that owns. Individuals can accumulate wealth through a bequest left to their offspring and to become an entrepreneur individual needs a fixed amount of investment. In the equilibrium without credit frictions, an skill cut-off separate entrepreneurs from workers and these from being employed or unemployed. The model include an extreme form of credit market frictions where there is no borrowing. Therefore, in the equilibrium with credit frictions there exist another steady state equilibrium: only a fraction of the entrepreneurs with the necessary skills to become an entrepreneur will become one depending on the level of wealth that he can accumulate. With credit market frictions there exist an alternative equilibria with higher unemployment and lower wages, intuitively, because when only a few of the potential entrepreneurs have enough wealth, there is a limited demand for labor and this depresses wages.

We departure from that study in several ways. First, we explicitly model both the credit market and the labor market as non Walrasian markets. This allow us to directly get hold of the DMP framework to analyze unemployment. Additionally, as we mentioned above, we use this framework to explicitly include borrowing in the model. Second, we do not include any notion of wealth analysis. Although an important abstraction this choice allow us to remain the analysis in a tractable way. Third, although we also model technical change as an exogenous parameter, in the case of our model this artifact is directly translated in a form of capital-embodied technical change that allow us to better map with an empirical measure of technical progress.

1.2. Empirical Motivation

Based on macro-panel data we investigate empirically the impact of credit market and technological change on labor market outcomes. The analysis is carried out using data from the Capital Input File of the 2017 release of the EU KLEMS Growth and Productivity Accounts compiled by O'Mahony and Timmer (2009) and more recently updated by Van Ark and Jäger (2017).

This data contains information on investment and capital stock by industry and asset type. The type of capital we are interested in is computing equipment (IT), communication equipment (CT) and computer software and database (SoftDB). Unlike previous version of the EU KLEMS, the 2017 release relies on official ICT prices that are assumed to reflect quality adjusted price declines. That is, this is our empirical measure of the capital-embodied technical change [*c.f.* Jäger (2016) for the methodology of 2017 release. See Gordon (1990) and Cummins and Violante (2002) for quality adjusted price of equipment as a measure of technological change][4].

Data for countries labor market institutions were drawn from OECD.stat. The variables including in the sample are those seen in the literature as a mainly determinant of unemployment. The variables included are: employment protection legis-

[4] In fact as is pointed out by the author, recent evidence suggests that the official quality adjusted deflators underestimate the true price decline, even in the US. For more details see (Jäger, 2016, p.8)

lation (epl), product market regulation (pmr), output gap (outgap), labor-tax wedge (tw), unemployment benefits (arr), trade union density (tudens) and collective bargaining. From this source we extract the aggregate unemployment rate, employment by industry ISIC rev 4, the average duration of unemployment and the mean-to-average wage by country.

Our measure of the credit market is the ratio of private credit by deposit money banks and other financial institutions over GDP, developed by the World Bank. We see this variable as an indicator of the ability of an economy to allocate credit: the higher this measure the more flexible the credit market is in providing loans to take advantage of new investment opportunities. Under this view the higher this empirical ratio the lower the credit market frictions.

1.2.1. Unemployment, credit and capital-embodied technical change

We estimate the following equations for country i in period t, to test the effect of credit frictions and capital-embodied technical change on unemployment

$$Ur_{\text{i}t} = \phi Cre_{\text{i}t} + \alpha X_{\text{i}t} + \nu_{\text{i}} + \eta_t \tag{1.1}$$

$$Ur_{\text{i}t} = \beta Tec_{\text{i}t} + \alpha X_{\text{i}t} + \nu_{\text{i}} + \eta_t \tag{1.2}$$

$$Ur_{\text{i}t} = \phi Cre_{\text{i}t} + \beta Tec_{\text{i}t} + \alpha X_{\text{i}t} + \nu_{\text{i}} + \eta_t \tag{1.3}$$

$$Ur_{\text{i}t} = \phi Cre_{\text{i}t} + \beta Tec_{\text{i}t} + \gamma \hat{Cre}_{\text{i}t} x \hat{Tec}_{\text{i}t} + \alpha X_{\text{i}t} + \nu_{\text{i}} + \eta_t \tag{1.4}$$

$$Ur_{\text{i}t} = \phi Cre_{\text{i}t} + \beta Tec_{\text{i}t} + \gamma \hat{Cre}_{\text{i}t} x \hat{Tec}_{\text{i}t} + \nu_{\text{i}} + \eta_t \tag{1.5}$$

Where $UR_{\text{i}t}$ is the aggregate rate of unemployment, $Cre_{\text{i}t}$ is our empirical measure of credit market functioning and $Tec_{\text{i}t}$ is the quality-adjusted price index of the ICT-capital investment. The vector $X_{\text{i}t}$ includes variable of the labor market institutions often referred as a determinant of the unemployment. Additionally we include country and time fixed effects ν_{i} and η_t respectively, aiming to capture unobserved heterogeneity between countries for world trends and business cycle.

Equation (1.1) is our basic estimates that relate credit market with unemployment. Equation (1.2) is very similar to (1.1) although instead of credit as independent variable we use the empirical measure of technical change. Both equations show the baseline effect of each variable on unemployment rate, acting as a benchmark for the analysis. Equation (1.3) puts both variables in the same estimation to understand how is the direct effect of each key variable on unemployment. In equation (1.4) we added a key ingredient of the analysis: the interaction term between the functioning of the credit market and technological change. Through γ the model captures how the effect of technical change on unemployment depends on the credit market functioning. Important to note that the variables that interact are $\hat{Cre} := Cre_{\text{i}t} - \bar{Cre}$ and $\hat{Tec} := Tec_{\text{i}t} - \bar{Tec}$. In each case corresponds to deviations from the mean of each variable respectively.[5]

Dependent variable: Unemployment rate Ur_{it}					
	(1.1)	(1.2)	(1.3)	(1.4)	(1.5)
Credit (Cre)	-0.316		-0.101	-4.690**	0.173
	(-0.53)		(-0.18)	(-3.63)	(0.09)
Employment protection (Epl)	-1.462*	-0.491	-0.490	-0.330	
	(-2.23)	(-0.54)	(-0.54)	(-0.35)	
Product Market Regulation (Pmr)	0.358	0.532	0.524	0.211	
	(0.70)	(0.74)	(0.71)	(0.41)	
Union Density ($Udens$)	-0.00467	0.0802	0.0865	0.0914	
	(-0.08)	(1.15)	(1.19)	(1.35)	
Output Gap ($Outgap$)	-0.833***	-0.885***	-0.885***	-0.867***	
	(-10.21)	(-12.39)	(-12.28)	(-12.66)	
Technical Change (Tec)		0.00551	0.00540	0.0111*	0.00576
		(1.10)	(1.05)	(2.16)	(1.04)
Interaction Term ($\hat{Cre} \times Tec$)				-0.0426***	-0.0146
				(-4.35)	(-1.39)
N	498	327	321	321	446
Time dummies	YES	YES	YES	YES	YES
Country dummies	YES	YES	YES	YES	YES

t statistics in parentheses (robust std. error are considered)

* $p < 0{,}05$, ** $p < 0{,}01$, *** $p < 0{,}001$

Table 1.2.1 shows the basic results. Estimation of equation (1.1) shows that the effects goes in the same direction as in equation (14.a) in Dromel et al. (2010). However unlike these authors, our estimate of the effect of credit frictions on unemployment is no significant. Estimation of (1.2) shows that the effect of technical change is positive but no significant on unemployment. Estimation of (1.3) is consistent with the former two estimations.

Equation (1.4) is central in this analysis. This specification includes an interaction term between credit and capital-embodied technical change. The result shows the following interesting relationship: First, the negative effect of the credit variable becomes significant. In terms of the model that we present below, the lower the frictions in the credit market the lower the unemployment rate.

Second, the positive effect on unemployment of the capital-embodied technical change variable becomes significant. This result reflects that the higher the technological change embodied in new capital goods the higher the unemployment rate.

Third, the interaction term of these two variables negatively impacts the unem-

[5]As explained in Bassanini and Garnero (2013) using $Cre_{it}xTec_{it}$ biases the estimates of ϕ and β.

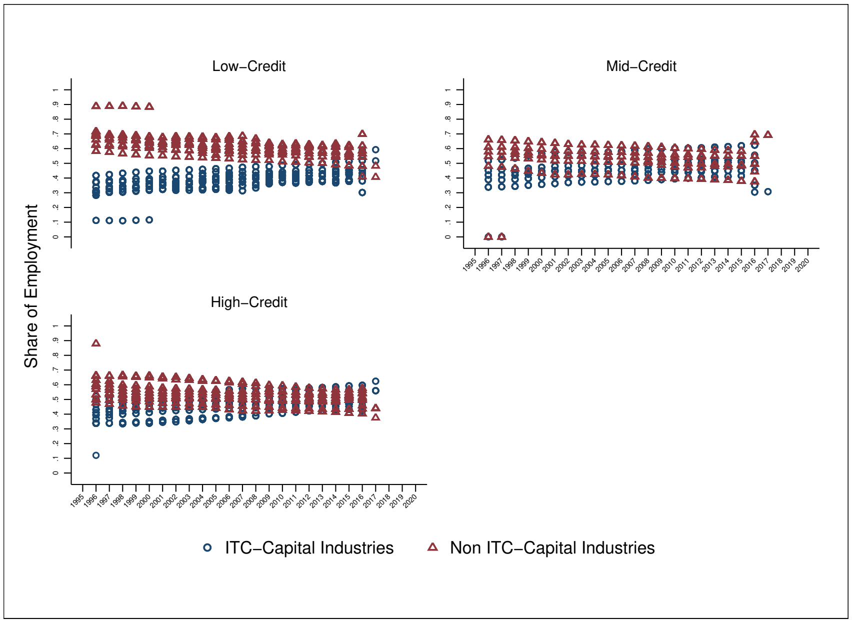


Figura 1.1: Average employment share trend between ITC-capital industries and non ICT-capital industries by type of credit countries.

ployment rate. Moreover, this effect is significant at one percent. That is, the positive effect on unemployment of capital-embodied technical change can be reversed by the credit variable. In other words, having a good credit indicator variable mediate the increase in unemployment induced by technical change.

1.2.2. Effect by Industries

If the story of the aggregate data is correct we should expect the evolution of the employment share in ICT-capital dependent industries to be higher in countries with higher credit rates. For this purpose we classify countries in three categories according to the empirical measure of credit: *Low-credit*, *Medium-credit* and *High-credit* countries. This classification is based on the average credit measure of a specific country respect to the credit measure of the average country on the sample[6].

At the same time we also classify industries in two categories according to the level of investment in ICT-capital: *ICT-capital* industries and *non ICT-capital* industries. This classification is based on the average ICT-investment by industries. We choose the top third on this measure[7]. Then we compute the employment share of the two type of industries and we study its evolution by type of country as shown in figure 1.1.

[6] For this procedure we compute an average country in terms of credit, unemployment and other related variables

[7] An alternative procedure is identify the industries with higher average ICT investment by country. Then to count the ones that are repeated as a higher ICT investors and finally choose the top third. Both procedures deliver the same result with an exception of the Agriculture industries.

Figure 1.1 shows that on average the share of employment in ICT-capital intensive industries has increased in the last 25 years. However, this increase has not been homogeneous for all countries. The increase in the employment share in *ICT-capital* industries is higher (lower) in those countries with higher (lower) values in the credit variable. Middle credit countries have a similar pattern as High credit countries. but its dispersion is slightly higher.

1.2.3. Labor-Market Flows

In this subsection we study the effect of credit market frictions and capital-embodied technical change on labor market flows. To this end, we use the harmonized data panel constructed by Bassanini and Garnero (2013). This data contains information on gross worker flows for 24 OECD countries and 23 business-sector industries at 2-digit level of the ISIC rev. 4 classification. The aim is to study which are the flows that are most affected by the interaction of accelerated technological change and credit market frictions.

The dataset contains flows regarding hiring rates (hr), separation rate (sr), job-to-job separations (j2j), job-to-jobless separations (j2jl), same sector separations (ssr) and other sector separations (os), among others. We include to this dataset the EU KLEMS and credit variables. Important to note that the flow data is defined in terms of one-year transitions which impose some limitation in the analysis, although there is an important contribution in terms of comparability. Nevertheless, one-year transitions are typically used in the analysis of gross job flows and in the literature on reallocation and efficiency.

In this data total worker reallocation is defined as the sum of total hiring and total separation.The main equation estimated has labor market institutions, the ratio of private credit by deposit money banks and other financial institutions over GDP and ICT investment quality adjusted price, as well as the interaction of the latter two as independent variables.

$$WF_{\mathrm{i}jt} = \alpha X_{\mathrm{i}t} + \phi Cre_{\mathrm{i}t} + \beta Tec_{\mathrm{i}jt} + \gamma \hat{Cre}_{\mathrm{i}t} \times Tec_{\mathrm{i}jt} + \nu_{\mathrm{i}} + \eta_t + \varepsilon_j \tag{1.6}$$

Where $WF_{\mathrm{i}jt}$ refers to the different worker flows mentioned above for the country i in industry j in the year t and ε_j capture the fixed effect of industries. The fact that the empirical measure of credit is defined at the aggregate level while data on worker flows are measured at the industry level allows to evaluate how its impact is likely to differ across industries. Table (1.2.3) summarize the results.

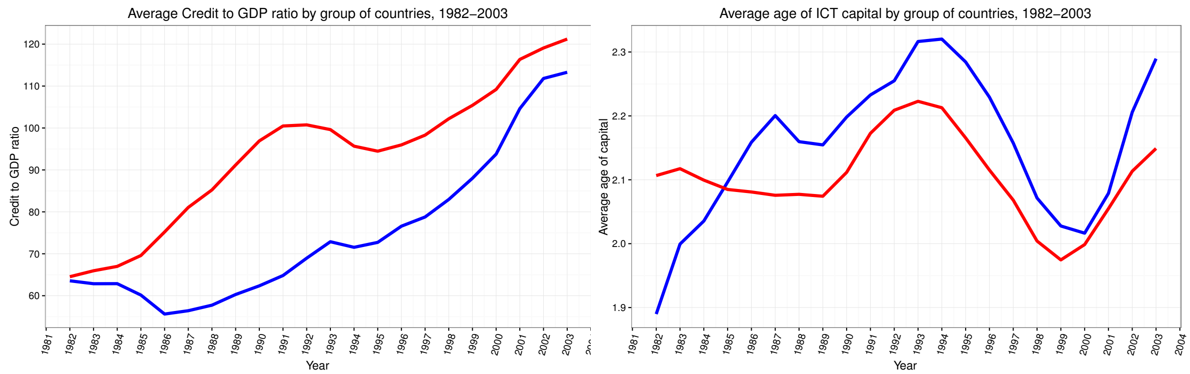
	(1) real	(2) hr	(3) sr	(4) j2j	(5) j2jl	(6) ssr	(7) os
Empl. Protection	-2.055	-1.769	-0.283	0.727	-0.0718	-1.698*	2.451**
	(-1.08)	(-1.50)	(-0.29)	(0.69)	(-0.09)	(-2.14)	(3.21)
Replacement Rate	0.165*	0.109*	0.0562	-0.146**	0.0861	-0.0805*	-0.0952***
	(2.24)	(2.40)	(1.25)	(-3.21)	(1.93)	(-2.02)	(-3.85)
Union Density	-0.182	-0.185	0.000155	0.0604	-0.0635	0.187*	-0.0826
	(-0.93)	(-1.57)	(0.00)	(0.48)	(-0.71)	(2.06)	(-1.07)
Tax Wedge	0.264**	0.113*	0.152*	-0.0244	0.137**	-0.0559	0.0220
	(2.65)	(1.96)	(2.34)	(-0.41)	(2.60)	(-1.32)	(0.62)
Prod. Market Reg.	-1.442***	-1.255***	-0.193	-0.457*	-0.0342	-0.353*	-0.112
	(-3.31)	(-5.65)	(-0.70)	(-2.32)	(-0.18)	(-2.46)	(-1.13)
Credit	9.023***	3.349*	5.520***	6.053*	0.676	1.106	5.546***
	(3.60)	(2.33)	(3.69)	(2.52)	(0.31)	(0.76)	(3.53)
Technology	0.000413	-0.000383	0.000996	0.00947**	-0.000462	0.000797	0.00857***
	(0.09)	(-0.14)	(0.34)	(2.69)	(-0.18)	(0.40)	(4.34)
$\hat{Cre} \times Tec$	0.0584***	0.0232*	0.0341***	-0.00383	0.0100	-0.00728	0.00594
	(3.93)	(2.48)	(4.09)	(-0.22)	(0.75)	(-0.83)	(0.63)
N	1561	1561	1561	1054	1054	1026	1026
Time dummies	YES	YES	YES	YES	YES	YES	YES
Country dummies	YES	YES	YES	YES	YES	YES	YES
Industry dummies	YES	YES	YES	YES	YES	YES	YES

t statistics in parentheses. * $p < 0{,}05$, ** $p < 0{,}01$, *** $p < 0{,}001$
* $p < 0{,}05$, ** $p < 0{,}01$, *** $p < 0{,}001$

As shown in table (1.2.3), the measure of credit market functioning has a positive and significant effect in almost each worker flow computed, with an exception of job-to-jobless and same sector separations. Technical change for its part has a small, but significant positive effect on job-to-job transition and other sector transitions. The interaction between these two variables, however, impact positive and significant hiring and separation rate, and consequently, the total reallocation of workers. This evidence suggests that while capital-embodied technical change is not a direct determinant of worker transitions in the labor market, in interaction with a well-behaved credit market can affect significantly the reallocation process of workers mainly through the separation rate between workers and employers.

1.2.4. Age of capital and credit market

Using data from the Capital Input Files of the 2009 release of the EU KLEMS Growth and Productivity Accounts compiled by O'Mahony and Timmer (2009) we compute the average age of the installed aggregate ICT capital stock (computer, software and communications) for a panel of 20 OECD countries. The idea underlying



(a) (b)

Figura 1.2: Average credit and average age of capital by group of country
Panel (a) Credit to GDP. Panel (b) Average age of ICT capital. In blue: OECD high unemployment. In red: OECD low unemployment

this computation is the following. In countries with high and increasing investment rates, firms are replacing a substantial part of their existing capital stock with new and technologically more advanced capital. Thus, the average age of the installed capital will be lower and therefore, the age at which technology is discarded is lower than in a country with stagnant and low investment rates. As noted by Colecchia and Schreyer (2002), the distinction of ICT-capital is important for three reasons: i) is viewed as an important factor that has been driving growth in some countries; ii) ICT-capital is often embodied in other ICT and non ICT-capital goods; iii) for ICT assets the rate of return tends to be higher than for other assets. In turn this translates into a faster obsolescence and a decreasing quality-adjusted price, a notion related with growth in capital-embodied technical change. With these points in mind is plausible to map the age of ICT-capital to the age of capital-embodied technology. With the average age of capital variable plus the unemployment rate series and the private credit to GDP ratio series, figure 1.2 shows two main observations.

First, panel (b) shows that countries with higher average unemployment rates are those with higher average age of ICT capital. This observation implies that higher unemployment countries have higher scrapping age of technology, remark that goes against the theoretical mechanism described in vintage models. As we saw above, sme authors argue that financial frictions play a key role in the adoption process of new technology putting on the stage an additional ingredient in the relationship between technology and unemployment. This fact motivates the second observation of figure 1.2. Panel (a) shows the same countries that have higher average unemployment rates and higher ages of ICT capital, have lower average credit-to-GDP ratio. These two facts suggest that the theoretical mechanism by which technological progress affects the labor market must be reviewed and that, credit frictions seems to play a key role in this process.

Taking stock, the facts suggested by the data in terms of our model are: First, a capital-embodied technical change shock increases the unemployment rate. This effect is heterogeneous across economies and mediated by credit market. The lower the frictions in the credit market the lower the positive effect of the technological shock on unemployment. This evidence is suggested by aggregate and dis-aggregate data.

Second, the interaction among capital-embodied technical change and credit market development mainly affects the total reallocation flow of workers. The main channel is through the separation rate and the hiring rate.

Third, in a period of fast capital-embodied technical change, higher unemployment economies are those with lower average credit to GDP ratio and higher average age of capital.

1.3. Vintage Model of Technical change and Frictional Markets

Time is continuous and exists three types of agents: entrepreneurs, financiers and workers. Entrepreneurs have ideas but they do not have funds to acquiring capital nor are they able to operate machines. In this setup capital stock and machines are used interchangeably since both refers to the technology installed. Financiers, have funds but no ideas, and workers can operate the technology but no have funds and no have ideas.

Additionally, we distinguish three stages in the economy: credit, labor and production stage. In the credit market stage an entrepreneur who wants to start a business looks for funds and installs the cutting-edge technology embodied in the new capital that he acquires. When an entrepreneur meets a financier they sign a debt contract and together act as a joint venture.

As a joint venture, the entrepreneur-financier pair engage in search process for a worker in the labor market. Finally, when a joint venture meets a worker they negotiate a wage contract and the production stage begins. The joint venture-worker pair is what we call a firm.

1.3.1. Production technology

The firm produces an homogeneous output trough a production function that allows both disembodied and embodied productivity improvements. As in Hornstein et al. (2007) we propose a set-up where the level of disembodied technology, $A(t)$, grows at a rate φ for all production units in the economy. The level of technical change that is embodied in the capital stock grows at a rate ψ in terms of efficiency units embodied in new machines. Therefore, let the output produced at time t by a production unit of age a with disembodied technology $A(t)$ that has $k(a,t)$ efficiency units of capital as

$$y(a,t) = A(t)k(a,t)^{\alpha} = \mathrm{e}^{\varphi t}[\mathrm{e}^{\psi(t-a)}\mathrm{e}^{-\delta a}]^{\alpha} \tag{1.7}$$

Where δ is the physical depreciation and α is the capital's share of output. Rearranging terms, equation (1.7) can be reduced to $e^{gt-a\Phi}$ where $g \equiv \varphi + \psi\omega$ is the growth rate of technological progress and $\Phi \equiv \omega(\psi + \delta)$ is the effective capital depreciation rate that includes physical and obsolescence depreciation. To focus on stationary equilibrium we render the model stationary by dividing all growing variables by e^{gt}, the common growth factor productivity at the frontier. This procedure let us to focus on the steady state of the normalized economy. Accordingly, the normalized output is given by

$$y(a) = e^{-a\Phi} \tag{1.8}$$

Equation (1.8) states that the normalized output is a decreasing function of both, the capital age of a production unit and the effective depreciation rate of the economy.

1.3.2. Matching

The meeting process is random. Let define the credit market tightness as the ratio of entrepreneurs looking for funds to financiers searching for suitable projects, $\phi = \mathbb{E}/\mathbb{B}$, describe the transition probabilities for the agents in this stage.

Let $m_c(\mathbb{B}, \mathbb{E})$ the credit market matching function homogeneous of degree one such that

$$\begin{aligned}\frac{m_c(\mathbb{B}, \mathbb{E})}{\mathbb{E}} &= m_c(\phi^{-1}, 1) \equiv z(\phi) \qquad z'(\phi) < 0 \\ \frac{m_c(\mathbb{B}, \mathbb{E})}{\mathbb{B}} &= m_c(\phi, 1) \equiv \phi z(\phi)\end{aligned}$$

$z(\phi)$ is the contact rate at which an entrepreneur meet a financier. The negative relationship with the credit market tightness implies a congestion externality to others entrepreneurs in the race for funds. Similarly $\phi z(\phi)$ is the probability that a financier meet an entrepreneur. That is an increasing function of the credit market tightness, since bankers benefit from the relative increase of entrepreneurs relative to them.

Vacancy heterogeneity

We allow for separation shocks that destroy an ongoing relationship between a joint venture and a worker without breaking the financing match. This shock forces the joint venture to get involved in a new costly search process in the labor market. This feature means that in the vacancy pool converges capital of different ages.

The source of this heterogeneity is due to the fact that not only newly created joint ventures are searching for workers, but also ongoing ones that suffered a bad shock in their employment relationship. Since vacant differs each other by the age of capital, define $v(a)$ as the measure of vacant joint venture of age a, such that the total number of vacancies is given by $v = \int_0^\infty v(a)\mathrm{d}a$. Let $m_l(u, v)$ the labor market matching function homogeneous of degree one that,

$$\frac{m_l(u,v)}{v} = m_l(\theta^{-1}, 1) \equiv q(\theta) \qquad q'(\theta) < 0$$
$$\frac{m_l(u,v)}{u}\frac{v(a)}{v} = m_l(\theta, 1)\frac{v(a)}{v} \equiv \theta q(\theta)\frac{v(a)}{v}$$

$q(\theta)$ is the rate at which an entrepreneur meets a worker and $\theta q(\theta)v(a)/a$ is the rate at which a worker meets a joint venture of age a. The latter depends on the density function of vacancies across ages.

1.3.3. Value Function

Let E_i the asset value equation for entrepreneurs participating in stage $\mathrm{i} = \{c, l, g\}$. Each subscript denote credit, labor and production good respectively.

$$(r-g)E_c = -\mathrm{d} + z(\phi)\left[E_l(0) - E_c\right] \tag{1.9}$$

$$(r-g)E_l(a) = \text{máx}\left\{q(\theta)\left[E_g(a) - E_l(a)\right] + \lambda\left[E_c - E_l(a)\right] + E_l'(a), (r-g)E_c\right\} \tag{1.10}$$

$$\begin{aligned}(r-g)E_g(a) = \text{máx}\Big\{&\mathrm{e}^{-a\Phi} - w(a) - \rho(a) + \sigma\left[E_l(a) - E_g(a)\right] + \\ &\lambda\left[E_c - E_g(a)\right] + E_g'(a), (r-g)E_c\Big\}\end{aligned} \tag{1.11}$$

Equations (1.9)-(1.11) state as follows: an entrepreneur searches for funds at a flow cost d and meet a financier at a rate $z(\phi)$ in which case adopts the leading-edge technology and installs the newest capital of age 0.

Once capital is installed an entrepreneur searches for a worker in the labor market stage. This meeting is successful at a rate $q(\theta)$ and allows the entrepreneur to starts the production. It is also possible that an exogenous destruction shock λ affects the financing contract forcing the entrepreneur to restart a fund search. In either case, the flow $E_l(a)$ stops receiving. Additionally, the entrepreneur makes capital losses from changes in the value of the job due to capital aging $E_l'(a)$.

The value of an entrepreneur in the production stage depends on their normalized productivity $\mathrm{e}^{-a\Phi}$ and the associated costs: wage rate $w(a)$ and repayment flow to the financier $\rho(a)$, both as a function of the age of capital. There is an exogenous separation shock σ that hit to the worker-entrepreneur pair that forces for a new labor search while keeping the debt contract. This shock means that in the labor stage not only newly created production units are looking for workers but also those that suffers this exogenous separation shock as we commented above. Like in (1.10) is still present the exogenous shock that can break up the contract debt and there is capital looses from advancing age.

Expressions (1.10) and (1.11) are maximization decisions that highlights the endogenous separation in both the labor stage and the production stage. The former between an entrepreneur and a lender, the latter between the worker, the lender and the entrepreneur. These separations are endogenous in the sense that is a rationale choice given the economy dynamics and the underlying state variable: the age of capital. In this model since vacancies are heterogeneous respect to age of capital not only production units eventually becomes obsoletes but also vacant joint ventures. As we shall see this impose two destruction threshold.

Accordingly, let B_{i} the asset value for financier participating in stage $\mathrm{i} = \{c, l, g\}$

$$(r-g)B_c = -k + \phi z(\phi)\left[B_l(0) - B_c\right] \tag{1.12}$$

$$(r-g)B_l(a) = \text{máx}\left\{-\gamma + q(\theta)\left[B_g(a) - B_l(a)\right] + \lambda\left[B_c - B_l(a)\right] + B_l'(a), (r-g)B_c\right\} \tag{1.13}$$

$$(r-g)B_g(a) = \text{máx}\left\{\rho(a) + \sigma\left[B_l(a) - B_g(a)\right] + \lambda\left[B_c - B_g(a)\right] + B_g'(a), (r-g)B_c\right\} \tag{1.14}$$

The intuition behind (1.12)-(1.14) is analogous. Note however, that equation (1.13) describe an important feature of the debt contract. The financier is who runs the expense flow γ of the labor search with the commitment that the entrepreneur returns a repayment flow $\rho(a)$ when he is in production. We make the assumption of full commitment through this paper, consequently, $\rho(a)$ is paid while the production and financing relationship last.

As the entrepreneurs, financiers can endogenously separate from the their respectively match in the labor and production stage, but as we see below its threshold coincide with the cut-off age of the entrepreneur.

Finally let U and $W(a)$ the workers value function to be unemployed and employed in a production unit with capital of age a respectively. These assets value are described as

$$(r-g)U = b + \theta q(\theta)\int_0^\infty \left[W(a) - U\right] g(a)\mathrm{d}a \tag{1.15}$$

$$(r-g)W(a) = \text{máx}\left\{w(a) + \sigma\left[U - W(a)\right] + \lambda\left[U - W(a)\right] + W'(a), (r-g)U\right\} \tag{1.16}$$

This equations are quietly standard but with a exception of the term $g(a) \equiv v(a)/v$, the density function for the vintage capital. Worker who finds a job does it in a specific vacant of age a, a process governed by the endogenous stationary distribution of vacant joint ventures.

1.3.4. Efficiency of cut-off separations

TBC (the aim of this subsection is to demonstrate that the cut-off separation for entrepreneur and bankers coincide, and therefore can be viewed as a joint decision)

1.3.5. Surplus functions

All decisions are jointly determined by a surplus sharing rule. Note however the multiplicity of surplus relationships in the model: an entrepreneur has a job contract with the worker in the labor market stage and a debt contract with the financier. The latter, in turns, generates two different surplus: one for labor search and other for production stage.

To start with, we define the total surplus in the labor market phase of the economy. At this stage the three agents agree how to split the final output of the firm. Let the total labor market surplus be defined as $S_L(a) = E_g(a) - E_l + B_g(a) - B_l(a) + W(a) - U$. Since the entrepreneur meets a financier before than a worker we will name the entrepreneur-banker pair as a joint-venture and we will distinguish it by $F_{\mathrm{i}}(a) = E_{\mathrm{i}}(a) + B_{\mathrm{i}}(a)$ for i $\in c, l, g$. So the total labor market surplus is given by

$$S_L(a) \equiv F_g(a) - F_l(a) + W(a) - U \tag{1.17}$$

A fraction β of equation (1.17) goes to worker and a fraction $1-\beta$ goes to the joint-venture to be split it by its members. As is known in the literature the parameter $\beta \in (0,1)$ represents the bargaining power of worker. Therefore, it is true that,

$$\begin{aligned} \beta S_L(a) &= W(a) - U & (1.18)\\ (1-\beta) S_L(a) &= F_g(a) - F_l(a) & (1.19) \end{aligned}$$

The surplus between the entrepreneur and financier generated in the credit market stage is defined as

$$S_C(a) \equiv E_l(a) - E_c + B_l(a) - B_c \tag{1.20}$$

Free entry in the credit market implies that $E_c = 0 = B_c$, reducing equation (1.20) to $S_C(a) \equiv F_l(a)$. Let $\eta \in (0,1)$ the bargaining power of financiers over entrepreneurs. This implies that a fraction η of this relationship goes to the financier and a fraction $1-\eta$ goes to the entrepreneur,

$$\eta S_C(a) = B_l(a) \tag{1.21}$$
$$(1-\eta)S_C(a) = E_l(a) \tag{1.22}$$

Indeed we assume that η also is the fraction at which the remaining labor market surplus is split it by financier and entrepreneurs in the production stage. So it is true that

$$\eta(1-\beta)S_L(a) = B_g(a) - B_l(a) \tag{1.23}$$
$$(1-\eta)(1-\beta)S_L(a) = E_g(a) - E_l(a) \tag{1.24}$$

So far we are just described some useful relations between different surplus functions of the economy. Let us define an explicit expression for the labor market surplus split it by the joint venture and the worker,

$$(r-g)S_L(a) = \text{máx}\left\{e^{-a\Phi} + \gamma - (\sigma+\lambda)\,S_L(a) - q(\theta)(1-\beta)S_L(a) - (r-g)U + S'_L(a), 0\right\} \tag{1.25}$$

Equation (1.25) is obtained by combining (1.9) to (1.16) and using the free-entry equilibrium condition[8]. Solving the differential equation give us,

$$S_L(a) = \int_a^{\bar{a}} e^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s-a)}\left[e^{-s\Phi} + \gamma - (r-g)U\right] ds \tag{1.26}$$

The exit age $\bar{a}$ is the age at which the surplus $S_L(a)$ is maximized. The first-order condition to the problem of $\text{máx}_{\bar{a}}\, S_L(a)$ states,

$$e^{-\bar{a}\Phi} = (r-g)U - \gamma \tag{1.27}$$

Equation (1.27) implies that at the age of destruction the productivity of the firm must equate the outside option of the worker less the labor search cost flow. Inserting (1.27) in (1.26) we have an endogenous surplus function for the joint venture-worker pair in the labor market,

$$S_L(a;\bar{a},\theta) = \int_a^{\bar{a}} e^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s-a)}\left[e^{-s\Phi} - e^{-\bar{a}\Phi}\right] ds \tag{1.28}$$

[8] For a detailed derivation of this expression see the Appendix

1.3.6. Job destruction

Equation (1.27) describes the requirement that must be satisfied by the job at the date of destruction. Using equation (1.15) and the relation $\beta S_L(a) = W(a) - U$ we can get the endogenous job destruction condition,

$$e^{-\bar{a}\Phi} = b + \beta\theta q(\theta) \int_0^\infty S_L(a; \bar{a}, \theta) g(a) \mathrm{d}a \tag{JD}$$

Note that (JD) depends on a vintage distribution of vacant firm $g(a) \equiv (a)/a$. The job destruction condition requires that the oldest vintage in production must have a productivity at least equal to the outside option of the worker. This outside option is equal to the non-wage income flow plus the expected share of labor surplus that the worker achieves in a new match.

1.3.7. Joint venture destruction

In this model there exist an endogenous break up of joint ventures. Eventually their surplus goes to zero as their capital reaches the maximum age. As we see above the credit market surplus is defined as $S_C(a) \equiv E_l(a) + B_l(a) = F_l(a)$ and is given by

$$S_C(a) \equiv F_l(a) = \int_a^{\hat{a}} e^{-(r-g+\lambda)(s-a)} \left[-\gamma + q(\theta)(1-\beta) S_L(s; \bar{a}, \theta) \right] \mathrm{d}s \tag{1.29}$$

Where $\hat{a}$ is the age at which $S_C(a)$ is maximized. Solving the integral inside (1.28) and maximizing w.r.t. $\hat{a}$ yields a joint venture destruction condition,

$$\frac{\gamma}{q(\theta)} = (1-\beta) \underbrace{\int_{\hat{a}}^{\bar{a}} e^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s-\hat{a})} \left[e^{-s\Phi} - e^{-\bar{a}\Phi} \right] \mathrm{d}s}_{S_L(\hat{a}; \bar{a}, \theta)} \tag{JVD}$$

The detailed steps to get these expressions are described in the appendix. Equation (JVD) describes an implicit function of $\hat{a}(\bar{a}, \theta)$. The maximum attainable age of a vacant joint venture, $\hat{a}$, is such that equals the total search cost to the fraction of the labor market surplus accruing to the joint-venture once in production stage. Note that the expression of the right-hand side is the value of being in production until the age of destruction conditional of starting in age $\hat{a}$. Thus, given a pair $(\bar{a}, \theta)$ the age $\hat{a}$ represents the age at which the joint venture equal the labor search cost flow with the expected benefits of a successful match.

1.3.8. Job creation

The asset pricing equation that describes the value of posting a vacancy for the joint venture is given by $F_l(a) \equiv E_l(a) + B_l(a)$. Thus, solving for the differential equation of the sum of equations (1.10) and (1.13) gives the present value of a vacant joint venture. Zero-profit condition at the joint venture creation date implies that when the leading-edge technology is installed, creation costs are equal to the expected benefits such that $C(\phi) = F_l(0)$. Hence the job creation condition read as follow,

$$C(\phi) = \int_0^{\hat{a}} \mathrm{e}^{-(r-g+\lambda)s} \left[-\gamma + q(\theta)(1-\beta)S_L(s;\bar{a},\theta)\right] \mathrm{d}s \tag{JC}$$

Where $C(\phi) \equiv B_l(0) + E_l(0) = \frac{\mathrm{d}}{z(\phi)} + \frac{k}{\phi z(\phi)}$. This object comes directly from the free-entry condition $E_c = B_c = 0$. The left hand side of (JC), $C(\phi)$ represents the total financial costs involved in the creation of a joint-venture.

Condition (JC) has the following intuition. The joint venture between a financier and an entrepreneur will keep posting a vacancy to the point that financial creation costs equal the expected benefits of being in the production stage until the age $\hat{a}$. That is, jobs are created keep in mind that financial creation costs are at least equal to the expected share of labor surplus of the firm minus the search costs of the labor market.

1.3.9. Stationary distributions

Figure (1.3) describes the market flows of a production unit of age $a \in (0, \bar{a})$. In this section we need to solve explicit expression for matching probabilities in terms of endogenous variables. Let $\mu(a)$ and $v(a)$ the measure of matched and vacant capital of age a respectively. The inflow of new joint ventures into the economy is the fraction of entrepreneurs who have success in meeting a financier $\phi z(\phi)\mathbb{E} \equiv v(0)$. New created joint ventures install the leading edge technology and immediately enter to the vacancy pool. Thereafter, moves back and forth between the matched or vacant until it reaches the destruction age $\bar{a}$ or $\hat{a}$, weather matched or vacant.

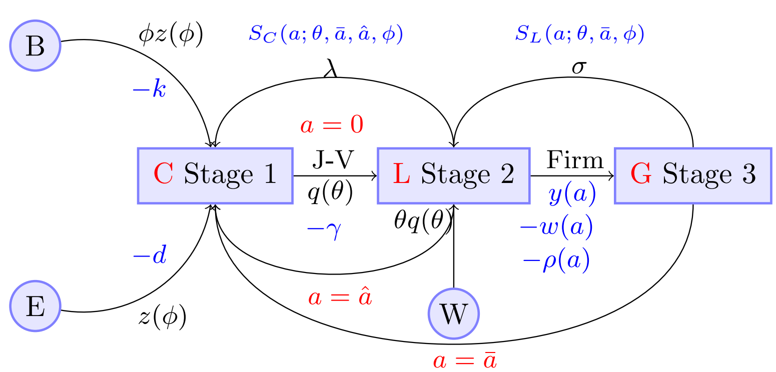


Figura 1.3: Flows of a machine of age a

$$\frac{v(a)}{v} = \frac{\sigma + \lambda + q(\theta)e^{-a(\sigma+\lambda+q(\theta))}}{(\sigma+\lambda)\hat{a} + \frac{q(\theta)}{\sigma+\lambda+q(\theta)}\left(1 - e^{-\hat{a}(\sigma+\lambda+q(\theta))}\right)} \tag{1.30}$$

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-a(\sigma+\lambda+q(\theta))}}{\bar{a} - \frac{1}{\sigma+\lambda+q(\theta)}\left(1 - e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)} \tag{1.31}$$

The density function of vacant capital, equation (1.30), is a decreasing function of the age $a \in [0, \hat{a}]$. A rise in $\hat{a}$ expands the support of the distribution. The density function of matched capital, equation (1.31), is an increasing function of $a \in [0, \bar{a}]$. A rise in $\bar{a}$ expands the support of the distribution. The appendix **??** shows te detailed steps to get this expressions.

1.4. Equilibrium characterization

1.4.1. Credit market equilibrium

Financiers and entrepreneurs split the surplus of their relationship according to a Nash-bargaining process. As a result the repayment flow from entrepreneur to worker, $\rho(a)$, is the $argmax E_l(a)^{1-\eta} B_l(a)^{\eta}$. The first order condition yields the usual sharing rule $\eta E_l(a) = (1-\eta) B_l(a)$. When the entrepreneur-financier match is formed the age of the capital is $a = 0$. The sharing rule states according with free entry conditions in the credit market, equations (1.9) and (1.12), an equilibrium value of the credit market tightness, as follow,

$$\phi^* = \frac{1-\eta}{\eta}\frac{k}{\mathrm{d}} \tag{1.32}$$

Like Wasmer and Weil (2004) and Petrosky-Nadeau (2013) this expression tells us that entrepreneurs and financier are attracted in fixed proportion keeping constant the equilibrium labor market tightness.

1.4.2. Equilibrium in $(\theta, \bar{a})$ space

A stationary equilibrium in this economy is given by 4-tuple $(\phi^*, \theta^*, \bar{a}, \hat{a})$ that are described by (JC), (JD) and (JVD) and (1.32). As we shall see this equilibrium together with the stationary distribution of vacant joint ventures can be expressed in a $(\theta, \bar{a})$ space.

Since $\hat{a}$ and $\bar{a}$ have a monotonic relationship, both can be mapped into the same space. Additionally, ϕ is complete determined by parameters as in equation (1.32), so job creation and destruction curves are depicted in the following form.

Lemma 1.1 (Shape of Job Creation): Job Creation Condition is an upward sloping curve in $(\theta, \bar{a})$ space.

a) *When $\bar{a} \to \infty$, θ approaches to the asymptote $\theta^{max} = q\left(\frac{\bar{r}C(\phi,\gamma)}{1-\bar{r}C(\phi,\gamma)}\frac{\bar{r}+\sigma}{1-\beta}\right)^{-1}$. Where $C(\phi,\gamma) \equiv C(\phi) + \frac{\gamma}{r-g+\lambda}$ and $\bar{r} \equiv r - g + \sigma + \lambda + \Phi$.*

b) *When $\theta \to 0$ (or $q(\theta) \to \infty$), $\hat{a}$ approaches to $\bar{a}^{min}$ defined implicitly by the following function.*

$$C(\phi) = \int_0^{\hat{a}(\bar{a}^{min})} \mathrm{e}^{-\bar{r}a}\left[1 - \mathrm{e}^{\Phi a}\left(\gamma + \mathrm{e}^{-\Phi\bar{a}^{min}}\right)\right] \mathrm{d}a$$

The intuition behind the upward sloping of the job creation is the following: the higher the maximal age of a job $\bar{a}$, the higher the expected profits for firms. The zero-profit condition establishes that more firms enter the market. However, to hold the equilibrium at the value of the financial entry cost $C(\phi)$, the probability of meet workers $q(\theta)$ must goes down and consequently the labor market tightness θ goes up. Part a) of lemma 1.1 establishes the existence of a positive value of the hiring rate. Vacant firms need a non-zero probability of hiring workers as incentive to recover the expected value of capital investment and financial costs. This is true even when the useful life of the matched capital is infinite. Part b) establishes that in absence of labor market frictions the age of capital reach a minimum. The minimum age of capital acts as a mechanism to meet the free entry condition. For some entrepreneurs useful life of capital may be too short to recover the financial costs. Expression in part b) of lemma 1.1 explicitly shows $\hat{a}$ as a function of $\bar{a}$, nevertheless, Lemma 2 shows that when $q(\theta) \to \infty$, $\hat{a} = \bar{a}$, so we can substitute $\hat{a}(\bar{a}^{min})$ simply by $\bar{a}_{min}$.

Lemma 1.2 (Shape of Joint Venture Destruction): Joint venture destruction condition is an upward sloping curve in the $(\theta, \bar{a})$ space and defines the maximal age of vacant firm as $\hat{a}(\theta, \bar{a})$.

a) *When $\theta \to 0$ ($q(\theta) \to \infty$), $\hat{a} = \bar{a}$.*

b) Given θ, $\hat{a}$ and $\bar{a}$ have an increasingly monotonic relationship. The higher θ the higher the distance between $\bar{a} - \hat{a}$

c) *When $\bar{a} \to \infty$, $\hat{a}$ approaches to:*

$$\hat{a} = \frac{-\log\left[\frac{\gamma}{\tilde{q}(\theta^{\text{máx}})}(\hat{r} + \Phi)\right]}{\Phi}$$

Where $\tilde{q}(\theta) \equiv q(\theta)(1-\beta)$; $\hat{r} \equiv r - g + \sigma + \lambda + q(\theta^{max})(1-\beta)$

The JVD condition states that the searching costs in the labor market equals the expected value of being in production. This condition describes the maximal age at which a vacant capital equal the costs of searching for a worker and the surplus obtained of the match.

For a given pair $(\bar{a}, \theta)$ there is an unique $\hat{a}$ that equal cost with benefits. As the job creation curve, the higher $\bar{a}$, the higher the total surplus in production and the higher the value of post a vacancy. Vacant firms are willing to wait longer to meet a worker, which translates into an increase in $\hat{a}$. Consequently θ has to augment and the probability to meet a worker has to fall to hold the equilibrium at the searching cost γ.

For the other parts of the lemma 1.2 consider first that always is true that $\hat{a} \leq \bar{a}$. Part $a)$ tells us that in absent of labor market frictions there is no distinction between the scrapping ages of matched and idle capital, since firms can immediately find a worker in case of receiving a separation shock. In other words there no exist vacant capital. Part $b)$ however, shows that as labor friction increases more valuable for a firm is the match with a worker and more difficult is to find a worker for a joint venture, rising the distance $\bar{a} - \hat{a}$.

Another way to give an intuition is the following. A higher θ induces a lower hiring probability $q(\theta)$. The harder to find a worker the higher the distance between the maximal age of the joint venture and the maximal age of a firm. This higher difference reflects that labor market frictions affect the distribution of vacant and matched capital.

Part $c)$ establishes that even when the life-span of capital goes to infinity, the life-span of vacant capital reach a maximum. This maximum depends on the maximal value of the labor market tightness as stated in lemma 1.1. The existence of a minimum hiring rate, given by $q(\theta^{max})$, impacts the effective discount rate of the labor searching cost, imposing a threshold on the scrapping age of capital for a vacant firm. Since, there is a lower but positive probability to meet a worker, there is a cut-off where joint ventures optimally choose scrapping their idle capital, even when $\bar{a}$ goes to infinity. This is a rational choice since the entrepreneurs, and lenders, could take advantage of the non-zero probability to find a worker and produce with a cutting-edge technology.

Lemma 1.3 (Shape of Job Destruction): Job Destruction Condition is a downward sloping curve in $(\theta, \bar{a})$ space.

Figure 1.4 depicts the equilibrium as the intersection of the job creation curve (JC) and the job destruction curve (JD). Given the equilibrium pair $(\theta^*, \bar{a^*})$ is associated an equilibrium maximal age of the joint venture $\hat{a}^*(\theta^*, \bar{a^*})$. The upward shape and convexity of (JVD) and (JC) curves reflects the effect of an increase in labor market tightness on the separation of the maximal age of the joint venture (vacant firm) and the maximal age of the firm (production firm).

In the following section we show some static comparative to the equilibrium and establish some useful corollaries and implications to aggregate variables.

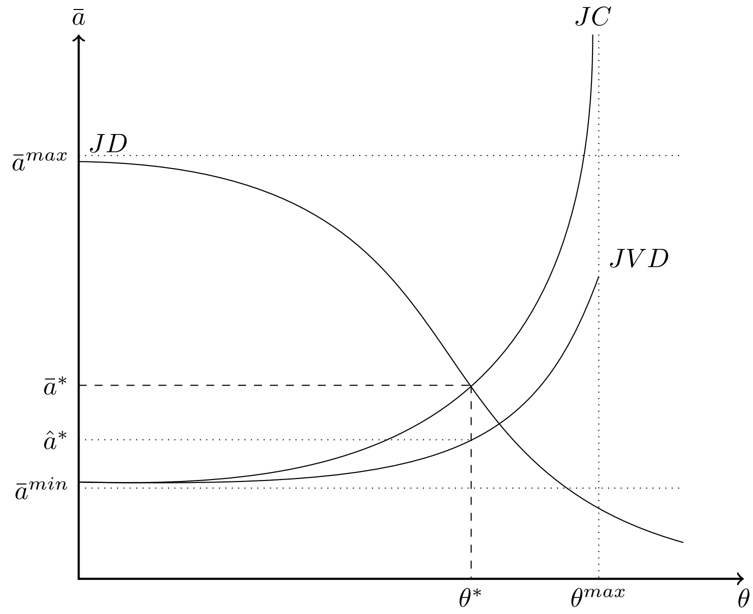


Figura 1.4: Stationary Equilibrium

1.5. Comparative Statics

This section presents some qualitative results. First, we describe comparative statics with regard to the rate of growth of capital-embodied technical change and comparative statics with respect to financial creation cost. Then we proceed to relate these comparative statics with proposition and predictions about unemployment, heterogeneity, wage inequality and total output.

Lemma 1.4 (Capital-embodied technical change): A rise in the rate of capital embodied technical change ψ induces a downward movement of both, the job creation and job destruction curves, lowering the maximal duration of a job, $\bar{a}^*$ and has ambiguous effect on θ^*.

This result is in line with Hornstein et al. (2007), the proof has the same steps as their article. The ambiguity of the movement of labor market tightness has to do with the strength of the effect of technological progress on the job creation curve. An increase in the rate of technical change imposes an *obsolescence effect* to older vintage since the leading-edge technology arrives a more rapid pace. At the same time, the technological change lowers the labor market tightness since more jobs are destroyed augmenting the pool of unemployed workers. Our interest, however, is focused on how credit frictions interact with this effect on shaping labor market outcome. To track this, the following lemma shows us the effect of an increase of credit cost.

Lemma 1.5 (Financial Creation Cost): A rise in the total financial cost of creating a joint venture $C(\phi)$ shifts the job creation curve upward and does not shift the job destruction curve. As a result, this movement induces a rise in $\bar{a}^*$ and a fall in θ^*.

Lemma 1.5 states that the higher the financial creation costs the lower the labor market tightness, and the larger the life-span of matched capital. This result highlight the *credit effects* of technology. An increase in financial creation costs limit the access to the necessary funds to capital investment augmenting the scrapping age of current technology. However, the labor market tightness fall as a response of lower job creation. The proof of this lemma is by simple inspection of the job creation and job destruction equations. In order to keep the equality in the face of an increase in $C(\phi)$, the right hand side rises through an increase in $\bar{a}$ for any given value of θ. Thus, an upward movement of the job creation is produced.

1.5.1. Unemployment

The steady state unemployment rate is computed equating the flows in and out of unemployment, as follow

$$\theta q(\theta) = (\sigma + \lambda)\mu + \mu(\bar{a}) \tag{1.33}$$

Let the identity of total employment, $\mu = 1 - u$ and substituting in (1.33),

$$\frac{u}{1-u} = \underbrace{\frac{1}{\theta q(\theta)}}_{\text{Unemployment duration}} \underbrace{\left[(\sigma + \lambda) + \frac{\mu(\bar{a}, \theta)}{\mu}\right]}_{\text{Incidence}} \tag{1.34}$$

Equation (1.34) decomposes unemployment rate in two components: the unemployment duration and the unemployment incidence. A decline in θ^* decreases the flow probability that an unemployed worker meet a joint venture increasing unemployment. A rise in $\bar{a}^*$ increases the useful life of a job lowering unemployment.

Therefore, higher financial creation costs $C(\phi)$ introduces two opposites forces on unemployment. An upward force through unemployment duration and a downward force through the unemployment incidence. The upward force on unemployment works in the same spirit as in Wasmer and Weil (2004). Frictions in the credit market depress the labor market tightness rising the average duration of unemployment. The downward force on unemployment comes from that financial costs lengthen the choice of endogenous separation, lowering the incidence on unemployment. We called the latter as the *technology-credit channel* of unemployment.

Using equation (1.31) for the the density of employed machines of age a and solving for u, we get an expression for unemployment as follow,

$$u = \frac{1+\sigma\left(\frac{\bar{a}}{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}} - \frac{1}{\sigma+\lambda+q(\theta)}\right)}{\theta q(\theta)+\sigma\left(\frac{\bar{a}}{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}} - \frac{1}{\sigma+\lambda+q(\theta)}\right)+1} \tag{1.35}$$

Proposition 1.1 (Unemployment): A rise in financial creation cost $C(\phi)$ leads to an increase in the unemployment duration and a decrease in the incidence on unemployment. The total effect induces an increase in the unemployment rate.

Proposition 1.1 states that when facing an increase in the cost of financing, the upward force on unemployment induced by a depressed labor market tightness is higher than the downward force leaded by the age destruction threshold. In the appendix we show formally how unemployment moves to changes in financial cost.

1.5.2. Wage inequality

A measure of wage inequality can be obtained through the model. For this purpose we use the standard approach of search literature to determine wage through a Nash bargaining solution of a bilateral problem between the joint-venture and the worker. Using the relationship stated in (1.19) the wage equation is such that satisfies $W(a) = U + \beta S_L(a)$. Substituting $W(a)$ from (1.16) and $S_L(a)$ from (1.25) and rearranging terms, we get the wage as

$$w(a) = (1-\beta)(r-g)U + \beta\left\{e^{-a\Phi} + \gamma - q(\theta)(1-\beta)S_L(a)\right\} \tag{1.36}$$

Hornstein et al. (2002) propose the maximum wage differential as a measure of wage inequality. The analysis highlighted the role of two forces shaping wage inequality: productivity differences across vintages and the Nash sharing rule. On the one hand, technological heterogeneity imposes that a longer life-span of a job increases the distance between the highest and the lowest firm's productivity, rising wage inequality. On the other hand, lower frictions in the labor market increase firm's outside option moving the economy towards a competitive benchmark.

The quantitative analysis predicted by the authors finds just a slight increase in wage inequality. We revisited this measure in our context. The expression for the maximum wage differential in the model is given by

$$\frac{w(0)}{w(\bar{a}(C(\phi)))} = (1-\beta) + \beta\frac{[1-q(\theta(C(\phi))(1-\beta)S_L(0;\bar{a}(C(\phi)),\theta(C(\phi))]}{e^{-\bar{a}(C(\phi))\Phi}+\gamma} \tag{1.37}$$

Equation (1.37) explicitly shows $\bar{a}$ and θ as a function of financial creation costs. The larger the financial costs the larger the maximal age of a job and the higher the distance between the maximum and the minimum wage of the economy. In other words, given a growth rate of the capital-embodied technical change an economy with higher credit market frictions posses a higher wage inequality. We can state this feature in the following lemma

An alternative measure of wage inequality is the the minimum relative to average wage. This measure has the feature to have an explicitly empirical counterpart. In the model we can compute this expression as follows,

$$\frac{w(\bar{a})}{\mathbb{E}[w(a)]} = \frac{(1-\beta)(r-g)U + \beta\left[e^{-\bar{a}\Phi} + \gamma\right]}{\int \{(1-\beta)(r-g)U + \beta\left[e^{-a\Phi} + \gamma - q(\theta)(1-\beta)S_L(a;\bar{a})\right]\} \frac{\mu(a;\bar{a})}{\mu} \mathrm{d}a} \tag{1.38}$$

Equation (1.38) is an increasing function of $\bar{a}$ and θ. With this expression in hand the following lemma arises

Proposition 1.2 (Wage Inequality): The higher the financial creation cost $C(\phi)$ the higher the wage inequality implied by expression (1.37) and expression (1.38)

Lemma 1.2 states that whatever the measure of wage inequality, an increase in financial costs raises the the wage inequality implied by the model. The proof is in the appendix.

1.5.3. Heterogeneity

Rewrite expression (1.31) to explicitly include the dependence of the endogenous variables on financial creation costs.

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-a(\sigma+\lambda+q(\theta(C(\phi))))}}{\bar{a}(C(\phi)) - \frac{1}{\sigma+\lambda+q(\theta(C(\phi)))}\left(1 - e^{-\bar{a}(C(\phi))(\sigma+\lambda+q(\theta(C(\phi))))}\right)} \tag{1.39}$$

By lemma 1.5 an increase in the financial creation cost affects directly the density distribution of matched capital. The higher the maximal age of jobs the higher the support of the distribution of the matched capital. Thus, there are more vintages in the economy and the probability to meet an specific vintage is lower for all ages. In other words, an increase in the financial creation cost rises the vintage heterogeneity of the pool of capital that is currently operating[9].

[9]This assertion is equivalent to say that rise firm heterogeneity. Recall that a firm in the model are represented by a joint venture worker pair.

However, equation (1.39) is also function of the labor market tightness. Ceteris paribus, a decrease in the labor market tightness rises the effective discount rate of the rents generated by the match. The latter is a consequence of a higher hiring rate and therefore an increase in the outside option value of the joint venture. Thus, a decline in θ incentivizes the adoption of new capital changing the distribution of vintages in the economy. In relative terms, the lower the labor market tightness the higher the mass of younger vintages respect to their shape before the movement in θ.

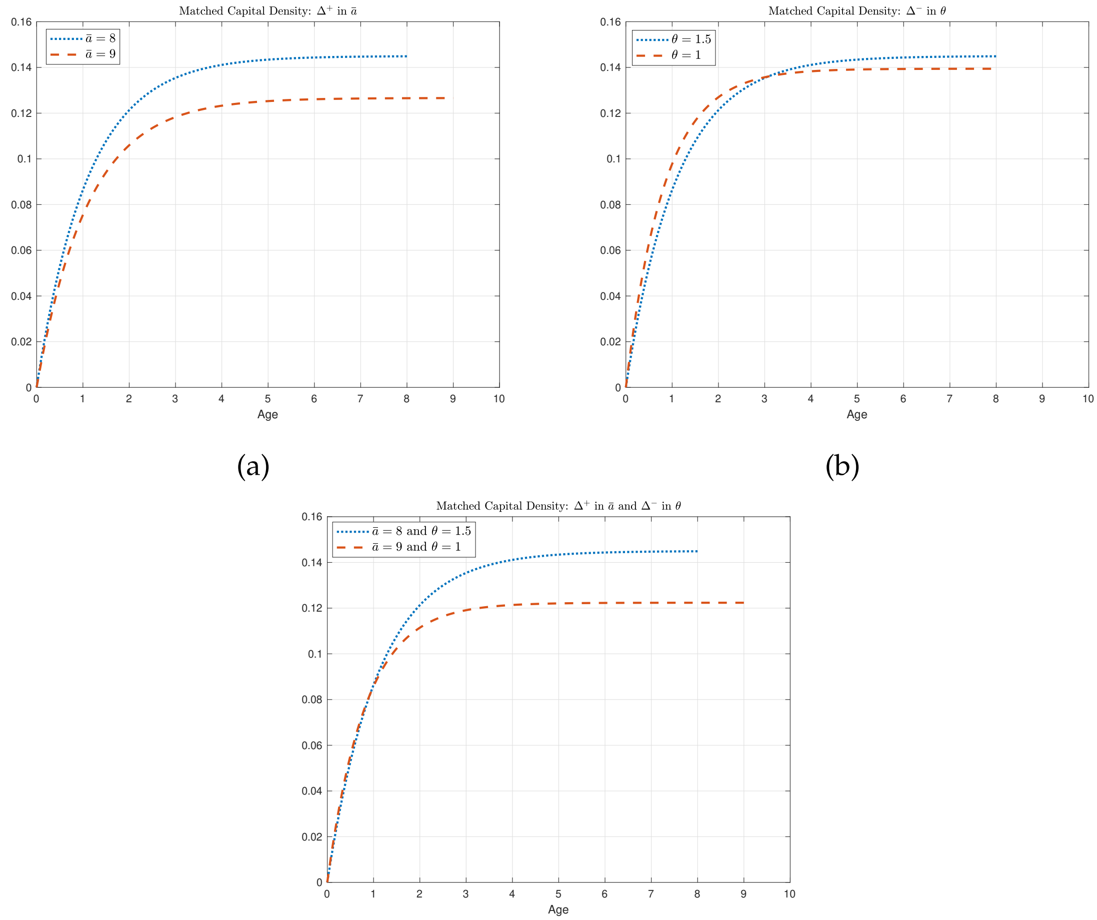
The density distribution of vacant capital (joint ventures) is also affected. The vacant density is decreasing and convex function of the age. However the same effects as the matched capital arises. An increase in $\hat{a}$ has the same effect on vacant-capital density as an increase in $\bar{a}$ has in the matched capital density. The higher $\bar{a}$ the higher the support of the vacant distribution. A decline in θ changes the shape of the curve in a single-crossing fashion. Younger vintages have more presence than older vintages respect to the shape before the movement in θ.

Lemma 1.6 (Vintage Heterogeneity): A rise in $C(\phi)$ enhances the support of the matched and vacant density distributions and lower the probability mass for all vintages. The vintage distribution is skewed towards older ages.

Proposition 1.6 states that the higher the financial costs the higher the heterogeneity of the density distribution of matched capital (employment). However, this movement affects mainly older relative to younger vintages. This lemma takes into account that the density distribution of matched capital has a support in $[0, \bar{a^*}]$, and the single-crossing property of $\frac{\mu(a)}{\mu}$ respect to θ. We show this in the appendix.

To gain intuition figure 1.5 represents these effects for a plausible parameterization. The figure shows the change in the matched capital density distribution to an increase (decrease) in $\bar{a}$ (θ). Note that the shape of the density function is asymmetric and goes to zero when the maximal age of a job is reached.

When $\bar{a}$ raises for a given θ, panel (a) shows that the density curve moves downward and the support enhances to the new level of $\bar{a}$. Conversely, when θ goes down for a given $\bar{a}$, panel (b) shows that the density curve moves backward producing a single-crossing of the curves. This movement implies that the density is biased to younger vintages. Panel (c) shows the movement of the density curve, to a movement in both an increase in the maximal age of a job $\bar{a}$ and a decrease in labor market tightness θ. This change represents the movement induced by an increase in the financial creation costs $C(\phi)$. As panel (c) shows, the density curve has a higher support and is biased to younger vintages. The effect in this parameterization is dominated by the movement of $\bar{a}$.



(a) (b)

(c)

Figura 1.5: Comparative statics in matched capital density distribution
Panel (a) shows the movement of the matched capital density distribution to an increase in $\bar{a}$. Panel (b) shows the movement of the matched capital density distribution to a decrease in θ. Panel (c) shows the joint movement of an increase in $\bar{a}$ and a decrease in θ.

1.5.4. Output

We can briefly comment about total output. To construct a measure of total output of the economy we aggregate individual output with regard the density distribution of matched capital.

$$Y_t = \mathrm{e}^{gt} \int_0^{\bar{a}} \mathrm{e}^{-a\Phi} \frac{\mu(a)}{\mu} \mathrm{d}a \tag{1.40}$$

.

Where $g \equiv (\varphi + \alpha\psi)$ and $\Phi \equiv \alpha(\psi + \delta)$. To obtain (1.40) simply use (1.7) and substitute in $Y_t = \int_0^{\bar{a}} A(t)k(t,a)^{\alpha}\mu(\bar{a})/\mu \mathrm{d}a$. Note that movements in $C(\phi)$ modify the output of an economy through changes in θ and $\bar{a}$.

Note in equation (1.40) that an increase in $\bar{a}$ increases the time-length of machines in production rising the output of the economy. However as wee seen in lemma 1.6 an increase in $\bar{a}$ enhances the support of the density of matched capital and skews the distribution to older vintages. Thus, an increase in the maximal age of jobs increases the time-length of production. Nevertheless, this production is made for less productive firms.

1.6. Discussion

How does these results interact?. Imagine that a capital-embodied technological shock is received by two economies that differ only in their financial creation costs. By lemma 1.4 a common technological shock produces an obsolescence effect that decreases the maximum useful time-length of machines and has an ambiguous effect on labor market tightness.

By lemma 1.5, in the $(\theta, \hat{a})$ space the equilibrium of the higher financial cost economy is located up and to the left of the equilibrium of the lower financial costs economy. In other words, the intersection between JD and JC equations for the higher financial costs economy, is closer to a flat portion of the job creation curve. Conversely, for the lower financial costs economy, the equilibrium is closer to the steepest portion of the job creation curve.

Where the equilibrium is located at the time of the technological shock is essential to understand the adjustment of unemployment. An economy with a worse credit market receives the technological shock in an initial equilibrium that leaves it vulnerable. When the equilibrium is located in the flat portion of the JC, the downward movement of JD and JC emphasize a response that is mainly leaded by the adjustment of labor market tightness, θ and to a lesser extent due to the fall of $\bar{a}^*$. Moreover, given the position of the equilibrium the downward movement of the (JC) and (JD) makes a drop in the labor market tightness more feasible. As a result, the adjustment to a shock of technological obsolescence is through an increase in the unemployment duration and consequently in an increase in unemployment.

When the equilibrium is located in the steepest portion of the JC the opposite occurs. The downward movement of JD and JC implies an adjustment of the equilibrium leaded by the maximal age of a a job, $\bar{a}^*$ and to a lesser extent due to the fall of θ^*. Moreover, given the position of the equilibrium the downward movement of the (JC) and (JD) makes a surge in the labor market tightness more feasible. Even if it falls, it does so to a lesser extent relative to the worse credit economy. As a result, the adjustment is through an increase in the incidence margin of unemployment, and the change in unemployment is moderated.

Note that in the worsen credit economy the useful time-length of vacant capital is closer to the useful time-length of matched capital. The opposite happens in the better credit economy. This fact is a consequences of the lack of liquidity in the economy that incentives to keep an old machine instead of upgrade to a new vintage.

To see these insights simply consider the slope of the job creation in these two portions of the curve. Then note by lemma 1.5 that higher financial creation cost induces an upward movement of job creation. Since job destruction does not move, this imply that the new intersection is always at a less steep point of job creation. Figure (1.6) describes these two economies.

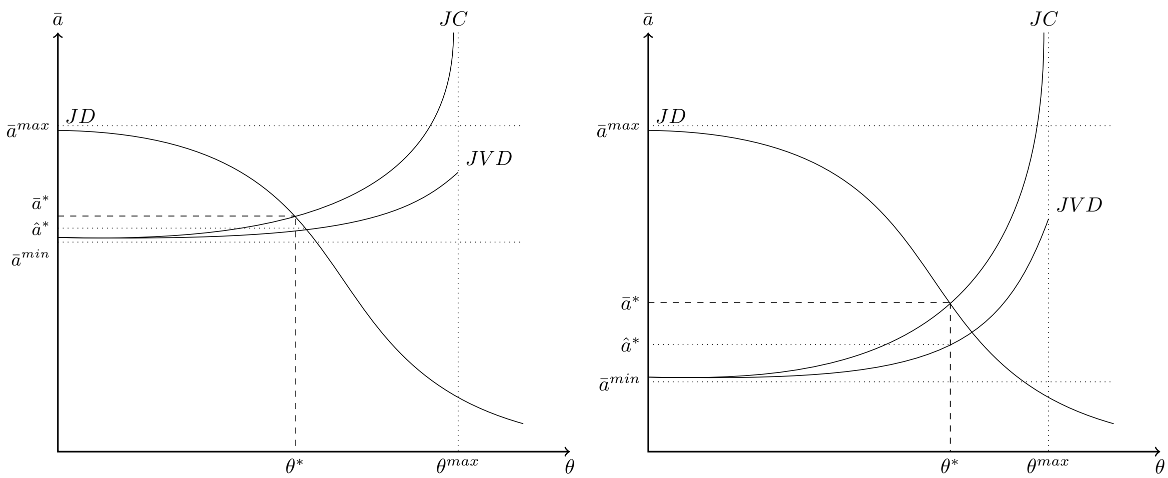


Figura 1.6: Initial Equilibrium

Note: The left (right) panel shows the initial equilibrium of a high (low) financial cost economy.

Although the initial position differs in both economies, nothing has been said about unemployment. As we shall see below the higher θ^* the lower the unemployment rate. At the same time the lower $\bar{a}^*$ the higher the unemployment rate. Therefore, in principle it is not clear which of these economies have a higher equilibrium unemployment. However, the technological shock propagates differently in both economy depending in how the adjustment of the equilibrium is. This propagation will affect directly the unemployment rate and other economics variables.

In addition to the initial position of equilibrium, changes in the credit market affect the economy. We can see by lemmas 1.6-1.2 that as the credit market worsens, the vintage distribution of matched capital becomes more heterogeneous, unemployment raises as well as wage inequality. The total output is also affected, useful life of machines is longer but produces with a lower productivity.

1.7. Conclusions

This paper explores the link between capital-embodied technical change, credit market frictions and long-run unemployment. The main argument is that difference in the long-run unemployment trajectories can be accounted for differences in the functioning of the credit market. A common capital-embodied technical change shock is propagated differently in economies with better credit markets than economies with worsen credit markets.

We show some evidence that supports this argument. We construct a panel of 20 countries using data from Capital Input File of the 2017 release of the EU KLEMS

Growth and Productivity Accounts, from OECD.stat for data on labor Markets, and from the World Bank for credit-to-GDP-ratio. We find: (i) a significant negative correlation between credit variable and unemployment; (ii) a positive effect on unemployment of the variable measure the capital-embodied technical change; (iii) a negative correlation on unemployment of the interaction term between credit and technology variables.

Making a zoom by industries the data suggests that share of employment in ITC-Capital industries grew more in those high-credit countries than in low-credit ones. We interpret this result as the ability of a better credit market to absorb a capital-embodied technical change shock and propagated to the economy. In terms of labor flows, the interaction term between credit and technology has a positive and significant correlation in hiring and separation rate, and consequently in total reallocation of workers. We interpret this result as the ability of a better credit market to stimulate endogenous separations as well as the outflow rate of unemployment.

To theoretically test these insights, we build a vintage capital model with credit as well as labor market frictions. The model has an exogenous rate of capital-embodied technical change and a vintage capital production technology combined with search frictions in the labor and the credit market. The main feature of the model is the obsolescence effect of technology that imposes movements in the useful time-length of capital (and jobs) $\bar{a}$, and the labor market tightness θ. Our contribution lies in the interplay between frictions in both market and this obsolescence effect.

We solve the model in a tractable and analytical fashion that let us analyze the equilibrium and comparative statics in $(\bar{a}, \theta)$ space. Our theoretical results proceed as follows. Unemployment in the model is an object compounds of two terms: unemployment duration and unemployment incidence, the latter decreasing in the labor market tightness and the former decreasing in the maximal life of a job. A capital-embodied technological shock produces an obsolescence effect (reduces $\bar{a}$) but has an ambiguous effect on θ.

The propagation of technological shock differs considerably in the kind oh credit market an economy has. An economy with a worse credit market absorbs the technological shock mainly by the adjustment of the labor market tightness leaving the economy in a more vulnerable scenario. Moreover, these economies face a more plausible context in which the technological shock decreases the labor tightness increasing the unemployment duration margin of the unemployment rate.

Conversely, an economy with a better credit market absorbs the technological shock mainly by the useful time-length of capital. This imply that the unemployment adjustment of these economies is leaded by the unemployment incidence and not by the unemployment duration. The movement of the labor tightness in these economies has more chances to be increasing and therefore unemployment decreasing. Even if this is no the case the drop is lower than the drop in worsen credit economies. As a result the relative changes in unemployment is lower when the economy has a better credit market.

Chapter 2

Exchange Rate Policy and Nominal Wage Rigidity with Non-homothetic Preferences

2.1. Introduction

The composition of goods that households consume vary as their income varies (Attanasio and Pistaferri, 2016; Aguiar and Bils, 2015). However, theoretical research in international economics and finance has largely ignored this real-world feature of consumers' demands. The macroeconomic analysis typically relies on the assumption of constant elasticity of substitution (CES) and the same income elasticity of demand across goods, i.e. a homothetic CES aggregator. By doing so, income determinants of relative prices are neglected, which limits the scope for monetary and fiscal policy analysis and ultimately the understanding of aggregate consumption and unemployment fluctuations.

This paper seeks to fill this gap in the context of a small open economy. I study the optimal exchange rate policy that the authority has to implement in order to moderate the negative effects of an economic boom-bust cycle. Preferences are non-homothetic with constant elasticity of substitution among goods and the economy is subject to nominal frictions. Comparing this environment with an otherwise standard setting with homogeneous income elasticities across goods shows important differences in the magnitude of the crisis, amplification mechanisms and, therefore, the type of policy intervention needed.

These insights are particularly important in the context of emerging and developing economies which are especially exposed to international swings and usually experience great income variations. As documented by Reinhart and Rogoff (2011) and Jordà et al. (2013), crises in open emerging countries are preceded by boom-bust credit consumption episodes, where credit expansions tend to be followed by deep recessions and deep exchange rate depreciation.

Additionally, the homotheticity assumption has two consequences which may be unwanted in some scenarios. First, the marginal rate of substitution, and therefore the relative prices, is not state dependent on the level of production or consumption. Second, all commodities are always treated as a normal goods, while under the non-homothetic CES function, income effects play a key role that can be studied. I consider that both of these features are a fundamental insights during financial distress.

In order to analyze the mechanism I build a small open economy model with two sectors, tradable and non-tradable, and downward nominal wage rigidity. Downward nominal wage rigidity is an important and prevalent feature of emerging economies, which can give rise to a pecuniary externality if is coupled with a more rigid nominal exchange rate regime (Schmitt-Grohé and Uribe, 2016). During a boom in aggregate demand wages rise, putting the economy in a vulnerable situation when the contractionary phase of the cycle starts. Downward nominal wage rigidity within a fixed exchange rate regime prevents real wages from falling to the level consistent with full employment. Conversely, an adjustable exchange rate could reach the first best allocation by devaluating the currency and thus lowering real wages, an optimal policy in terms of the labor market outcome and unemployment.

I consider a simple departure from the canonical environment by introducing heterogeneous income elasticity of demand across goods. I accomplish this task through a non-homothetic CES aggregator as in Comin et al. (2020) and recently studied in international finance by Rojas and Saffie (2020). The model assumes that non-tradable goods have a higher income elastic demand than tradables, thus, in periods of low income non-tradable goods are considerably less consumed than tradable goods.

The theoretical exercise proceeds in two steps. First, I analyze graphically the theoretical equilibrium conditions in the labor–real price of non-tradables space. As will be seen later, the former is a sufficient variable to represent non-tradable production, and the latter constitutes the real exchange rate of the economy. In this space I perform a comparative statics analysis of the response of the economy to a boom-bust cycle under non-homothetic and homothetic preferences. In particular, I examine two type of exchange rate regimes: the first-best full-employment exchange rate policy, and a currency peg regime. Main theoretical results are that: (1) non-homothetic preferences amplify the boom and the bust of the cycle; (2) the first-best exchange rate policy response necessary to eliminate inefficiencies produced by wage rigidity is much higher under non-homotheticity; and (3) when preferences are non-homothetic and the currency is pegged, the adjustment of the economy induces more involuntary unemployment in the non-homothetic case relative to the homothetic specification.

Second, I perform a quantitative exercise and calibrate the model as in Schmitt-Grohé and Uribe (2016) adding the non-homothetic parameterization. I simulate the economy in a boom-bust cycle and in the in the long-run for the full-employment exchange rate policy. The results confirm the amplification properties of the non-homothetic environment. In particular in a boom-bust cycle I find for the non-homothetic economy that: (1) the drop in the real exchange rate and real wages at the bust is around 37 % higher; (2) the annualized currency devaluation at the through of the crisis is about 40 % higher; Regarding the long-run behavior: (3) both economies, homothetic and non-homothetic presents a similar behavior in aggregate variables. However, the non-homothetic economy makes a greater use of debt; and (4) the debt distribution features lower dispersion and has more probability mass at higher values. Then I simulate the economy to analyze the second best fiscal policy in a boom-bust cycle. I find that: (5) capital control rate in the boom is 2 % higher at the boom and 1,5 % lower at the bust. This stronger macroprudential response in the debt tax results in an unemployment evolution in the cycle similar to the homothetic economy.

The main mechanism behind these quantitative results is the use of the debt. Debt, as usual, is an instrument to allocate intertemporal consumption, but also is used to allocate intratemporal consumption between tradable and non-tradable sector. Market clearing conditions in both markets imply that debt is used to regulate traded consumption. Thus, in booms debt is cut since households prefer non-tradable goods, and in bust debt is issued to increase te consumption of tradables. In fact in booms (busts) the rise (the fall) in domestic absorption of tradable goods is lower (larger) than the expansion (contraction) in the tradable supply. This results in a higher deterioration in the trade balance during booms and a higher improvement in the trade balance during bust for a non-homothetic economy.

Studying this context is important because the amplification mechanism relies on the dependence on the level of the composite consumption index. As we will see below, when we work with the standard homothetic CES aggregator, a change in the price of non-tradables and consequently in the real exchange rate depends exclusively on the relative absorption of tradables in terms of non-tradables. Thus, the total level of the composite consumption good does not affect the economy adjustment in a boom-bust cycle. In other words, in an homothetic economy the size of the consumption boom and the ensuing recession period has no effect on the amplification of the crisis and the optimal response of the economy. However, evidence suggests that in periods of economic expansion, larger non-traded macro fluctuations in emerging economies are indeed associated with consumption booms (Mendoza and Terrones, 2008)

Conversely, the response of a non-homothetic economy is proportional to the composite consumption, creating a dependence of the economic response to the state in which the consumption index is found. As I discuss later on, this may of be interest in the current lock-down context of Covid-19 where the non-tradable consumption was strongly restricted. Finally, this mechanism helps to unfold the the potential link between income inequality and financial crises.

The rest of the paper is organized as follows: The following subsection presents the contribution of the paper to the related literature. In section 2 the model is presented and the competitive equilibrium conditions are derived. Section 3 and 4 show respectively theoretically an analytical example solution of the equilibrium. In particular, in section 3 I derive some graphical properties of the equilibrium that allows me to study theoretically how the adjustment of the economy is achieved in different policy scenarios. In section 4, the mechanisms are shown through an analytical application of the model. The quantitative evaluation is shown in section 5 for the optimal full-employment policy. In section 6 I explore the optimal second best fiscal policy through capital control rate. Finally a brief conclusion is presented in section 7.

2.1.1. Related literature

A remarkably recent and emerging literature has highlighted the role of heterogeneous income elasticity of demands across goods in two key aspects of macroeconomic activity: structural change (Comin et al., 2020) and labor shares (Hubmer, 2020). Comin et al. (2020) show strong evidence of non-homotheticity across time, income levels, and countries, attributing an important role of non-homotheticity to explain reallocation of resources across sectors. Hubmer (2020) studies non-homothetic demands and labor shares. The author documents that high-income households spend relatively more on labor-intensive goods and services. This fact, interpreted as non-homothetic preferences is an important force driving the long-run behavior of labor shares.

This paper takes the non-homothetic setting of this literature and inserts it in a small open economy model to analyze optimal policy in financial crisis. In an ongoing work, Rojas and Saffie (2020) apply this specification in a Fisherian deflation model in the spirit of Bianchi (2011) and Mendoza (2010). In this class of models, a pecuniary externality arises because a movement in the relative price of non-tradables, endogenous to the model but exogenous to the households, affects the value of output (in terms of tradables), and household's borrowing capacity through a collateral constraint. During booms the value of the collateral rises, which relaxes the debt constraint, stimulating aggregate demand. When the bust arrives, it pushes down the value of collateral and the collateral constraint binds, unleashing a financial crisis. In this crisis agents deleverage, pushing down the price of the collateral, which again causes a massive sale of assets, a rapid contraction in demand and a current account surpluses. The whole process ends up in a sudden stop.

The analysis presented in this paper is different in at least two dimensions. First, the type of frictions studied here has to do with downward nominal rigidity and no with financial frictions. This is important because this kind of frictions allows to study the labor market and namely on involuntary unemployment, an important motivation for the analysis. Second, I analyze two types of exchange rate regimes: the first-best full-employment exchange rate, and the currency peg. The former is analyzed in both, theoretical and quantitative terms, while the latter is analyzed only theoretically. In a sense, this paper is a complementary view of the setup in Rojas and Saffie (2020) focused on macro stabilizing monetary policy.

This paper also contribute to the price rigidity literature. One way to model price rigidity is the staggered price setting framework commonly used in monetary economics (Calvo, 1983; Yun, 1996). This environment is characterized by two-sector economy, bidirectional price rigidity, and monopolistic competition in intermediate sector. The Calvo's assumption to setting-up the price makes that only a fraction of firms can re-optimize their price in each period. This feature induces an output loss due to price dispersion. As in this paper, this New-Keynesian open economy model, uses optimal exchange-rate policy to prevent the spreading of external crisis to the non-traded sector. Price rigidity is also modeled as a downward nominal rigidity, a prevalent feature in emerging economies. In Schmitt-Grohé and Uribe (2016), downward nominal wage rigidity combined with a fixed exchange rate create a pecuniary externality that prevents the economy adjustment to the first best. I extend this analysis by including heterogeneous income elasticity through non-homothetic preferences, which helps to understand wider macroeconomic fluctuations and so to rationalize stronger policy responses.

Related with the two building-blocks mentioned above, Ottonello (2020) puts together financial frictions due to a collateral constraint and nominal friction due to a downward nominal wage rigidity. The scope of the analysis, as in this paper, is to study the optimal policy response of the economy subject to the trade-off of its implementation. On the one hand, currency devaluation offsets nominal frictions inefficiencies and acting as buffer against unemployment. A welfare improving force (Schmitt-Grohé and Uribe, 2016). On the other hand, currency devaluation decreases the value of debt, tighten the collateral constraint, and potentially induces a sudden stop. A welfare worsen force.

2.2. A Model of Nominal Wage Rigidity and Non-Homothetic Preferences

This section develops a model of a small open economy with downward nominal wage rigidity, two types of consumption goods, tradable and non-tradable, and heterogeneous income elasticities across sectors. The model closely follows Schmitt-Grohé and Uribe (2016), extended with non-homothetic preferences as a device to study that non-tradable goods are relatively more income elastic than tradable goods. The non-homothetic building block is as in Rojas and Saffie (2020).

2.2.1. Households

Household preferences are described by a constant-relative-risk-aversion utility function

$$\mathbf{E_0} \sum_{t=0}^{\infty} \beta^t \left[U \left(c_t \right) \right] \tag{2.1}$$

where c_t denotes the total consumption, $\beta \in (0,1)$ denotes the subjective discount factor, and the utility function U is strictly increasing and strictly concave. The representative agent consumes a composite of tradable and non-tradable goods implicitly defined by:

$$1 = \left[a \left(c_t \right)^{\varepsilon_T (1+\eta) - 1} \left(c_t^T \right)^{-\eta} + (1-a) \left(c_t \right)^{\varepsilon_N (1+\eta) - 1} \left(c_t^N \right)^{-\eta} \right]^{-\frac{1}{\eta}} \tag{2.2}$$

where ε_{i} denotes the parameter that controls the income elasticity of demand across good $\mathrm{i} = \{T, N\}$, tradable and non-tradable. The non-homothetic specification highlights for a parameterization in which $\varepsilon_N > \varepsilon_T$, that is, a higher income elasticity of non-tradable goods. Since (2.2) represents a non-homothetic CES aggregator, the elasticity substitution between tradables and non-tradables is given as usual by the expression $\frac{1}{1+\eta}$. To ensure all the properties of a CES aggregator the parameterization assumes that $\varepsilon_{\mathrm{i}} > \frac{1}{1+\eta}$, and in this example $a \in (0,1)$. Denotes as c_t^T and c_t^N the tradable and non-tradable consumption respectively. Household receives a stochastic endowment of tradable goods y_t^T whose nominal price is denoted by P_t^T. Additionally, households trade a non-state-contingent one-period bond denominated in terms of the tradable goods. Let d_{t+1} the level of debt issued in t and due in period $t+1$. The households budget constraint is given by

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T \mathrm{d}_t = \left(1 + \tau_t^y\right) \left(P_t^T y_t^T + W_t h_t + \Pi_t \right) + \frac{P_t^T \left(1 - \tau_t^{\mathrm{d}}\right) \mathrm{d}_{t+1}}{1 + r_t} \tag{2.3}$$

where P_t^N is the nominal price of non-tradable goods, W_t is the nominal wage rate, h_t refers to hours worked, Π_t nominal profits from the ownership of firms, and r_t is the interest rate on debt. τ_t^y is an income subsidy rate and τ_t^{d} denotes the tax rate on external debt acquired in t, the capital control rate. The tradable output y^T and the interest rate r_t are stochastic variables subject to aggregate socks.

The model assumes that the law of one price hold for tradables. Let P_t^{T*} the foreign currency price of tradables. The law of one price implies

$$P_t^T = P_t^{T*} \mathcal{E}_t$$

where $\mathcal{E}_t$ denotes the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. Normalizing $P_t^{T*} = 1$, results that the nominal price of tradables equals the nominal exchange rate $P_t^T = \mathcal{E}_t$. Additionally, to prevent Ponzi schemes the household is subject to the natural debt limit.

$$\mathrm{d}_{t+1} \leq \bar{\mathrm{d}} \tag{2.4}$$

Denote the prices expressed in terms of tradables in lowercase, we can to rewrite the budget constraint as follows

$$c_t^T + p_t^N c_t^N + \mathrm{d}_t = \left(1+\tau_t^y\right)\left(y_t^T + w_t h_t + \pi_t\right) + \frac{\left(1-\tau_t^{\mathrm{d}}\right)\mathrm{d}_{t+1}}{1+r_t} \tag{2.5}$$

Households supply inelastically $\bar{h}$ hours to the labor market each period, but may not be able to sell all of them, which gives rise to the constraint

$$h_t \leq \bar{h} \tag{2.6}$$

The representative agent chooses the stochastic sequences $\left\{c_t^T, c_t^N, c_t, \mathrm{d}_{t+1}\right\}$ subject to (2.2), (2.4), (2.5) and (2.6) given d_0 and the sequence $\left\{p_t^N, w_t, r_t, h_t, \pi_t, \tau_t^{\mathrm{d}}, \tau_t^y, y_t^T\right\}$. The first-order conditions associated with this problem are the budget constraint (2.5) and the following set

$$\begin{aligned}
p_t^N\left(c_t^T, c_t^N\right) =& \frac{1-a}{a}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta}(c)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} && (2.7)\\
\lambda_t =& U_{c^T}\\
\lambda_t \frac{1-\tau_t^{\mathrm{d}}}{1+r_t} =& \beta \mathbb{E}_t \lambda_{t+1} + \mu_t && (2.8)\\
\mu_t \geq& 0\\
\mu_t\left(\mathrm{d}_{t+1} - \bar{\mathrm{d}}\right) =& 0
\end{aligned}$$

These set of optimal conditions differ from those in Schmitt-Grohé and Uribe (2016) in the term $(c)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}$ in the right hand side of (2.7). This equation establishes that the relative price of non-tradables is a function of the relative absorption of tradables in terms of non-tradables, but also is a function of the composite consumption good index.

2.2.2. Firms

Non-tradable goods are produced by a representative firm using labor as the only input of a neoclassical production technology:

$$y_t^N = F\left(h_t\right) \tag{2.9}$$

Where $F\left(\right)$ is strictly increasing and strictly concave production function. Profits are given by $\Pi_t = P_t^N F\left(h_t\right) - W_t h_t$, and the optimal condition state that the quantity of labor demanded by the firm is given by $P_t^N F'\left(h_t\right) = W_t$. Dividing both side by $P_t^T = \mathcal{E}_t$ yields

$$p_t^N = \frac{W_t/\mathcal{E}_t}{F'(h_t)} \tag{2.10}$$

2.2.3. Downward Nominal Wage Rigidity and Exchange Rate Policy

Nominal wage rigidity is modeled by the inclusion of a lower bound in the growth rate of nominal wages

$$W_t \geq \gamma W_{t-1} \tag{2.11}$$

where $\gamma > 0$ denotes the degree of downward wage rigidity. In real terms, equation (2.11) can be stated as

$$\frac{W_t}{\mathcal{E}_t} \geq \gamma \frac{W_{t-1}}{\mathcal{E}_t}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_{t-1}} \Rightarrow w_t \geq \gamma \frac{w_{t-1}}{\varepsilon_t} \tag{2.12}$$

where $\varepsilon \equiv \mathcal{E}_t/\mathcal{E}_{t-1}$. I follow Benguria et al. (2020) and state the following assumption about the twofold authority objectives. On the one hand, the authority seeks an exchange rate policy as stable as possible. On the other hand, is aware of the unemployment and is able to depreciate the nominal exchange rate in order to reduce it. To capture this feature I introduce a technological parameter $\phi \in [0,1]$, that models the weight that the authority gives to the employment care relative to exchange rate stabilization. Let w_t^{full} the full-employment real wage in t, thus, the general rule to address the trade-off between exchange rate stability and unemployment is given by:

$$\varepsilon_t = \begin{cases} 1 & \text{if} \quad \frac{w_t^{full}}{w_{t-1}} \geq \gamma \\ \left(\frac{w_t^{full}}{w_{t-1}}\right)^{-\phi} & \text{if} \quad \gamma^{\frac{1}{1-\phi}} \leq \frac{w_t^{full}}{w_{t-1}} < \gamma \\ \gamma^{\frac{-\phi}{1-\phi}} & \text{if} \quad \frac{w_t^{full}}{w_{t-1}} < \gamma^{\frac{1}{1-\phi}} \end{cases} \tag{2.13}$$

Recall from (2.12) that full-employment is achieved for the the policy family ε_t that meet $w_t^{full} \geq \gamma \frac{w_{t-1}}{\varepsilon_t}$. The first and second line in (2.13) correspond to cases in which the economy achieves full-employment equilibrium. In particular, the first line considers the case in which the current full-employment real wage w_t^{full} is larger than the previous w_{t-1}. When this is the case the downward wage constraint is not binding and no adjustments are needed to achieve full employment. The second line shows the case when the current full-employment real wage w_t^{full} is lower than w_{t-1}. Imposing $\varepsilon_t = 1$ implies the emergence of involuntary unemployment. However, setting $\varepsilon_t = \left(\frac{w_t^{full}}{w_{t-1}}\right)^{-\phi} > 1$ as in (2.13) restores full-employment. The third line establishes the case in which involuntary unemployment and nominal devaluation coexist[1].

This exchange rate rule has two interesting properties. First, it is a more general specification that includes as a particular case, exchange regimes studied in the literature. Second, allows studying the trade-off between unemployment and stability policy. As an extreme case when $\phi = 1$, the authority is only interested in full-employment being irrelevant the third line in (2.13). Thus, when $\phi = 1$ the full-employment exchange rate policy coincides with the one studied in Schmitt-Grohé and Uribe (2016). When $\phi = 0$ the authority is only care about to keep the exchange rate fixed, that is, a currency peg regime. Let $\tilde{\gamma} \equiv \gamma^{\frac{1}{1-\phi}}$, the effective downward wage rigidity is given by:

$$w_t \geq \tilde{\gamma}\frac{w_{t-1}}{\varepsilon_t} \tag{2.14}$$

The effective rigidity parameter $\tilde{\gamma}$ embodied the nominal rigidity and the exchange rate policy arrangement. This kind of friction implies the appearance of involuntary unemployment, $\bar{h} - h_t$. Employment and wages satisfy the following slackness condition:

$$\left(\bar{h} - h_t\right)\left(w_t - \tilde{\gamma}\frac{w_{t-1}}{\varepsilon_t}\right) = 0 \tag{2.15}$$

This condition states that in periods of full employment, $\bar{h} = h_t$, the wage constraint is not binding such that $W_t > \tilde{\gamma} W_{t-1}$.

2.2.4. The Government

The government set τ_t^y so as to balance its budget period by period and to rebate any revenue (or deficit) by capital control rate. Given τ_t^{d}, the government budget satisfies,

$$\tau_t^y\left(y_t^T + w_t h_t + \pi_t\right) = \tau_t^{\mathrm{d}}\frac{\mathrm{d}_{t+1}}{1+r_t} \tag{2.16}$$

2.2.5. Equilibrium Conditions

In equilibrium non-traded goods production clears in each period. Formally,

$$c_t^N = y_t^N \tag{2.17}$$

[1]More detailed derivation of (2.13) can be found in the appendix of Benguria et al. (2020)

Combining (2.17), (2.9), (2.5) and firm's profit condition we can express the equilibrium expression for consumption in tradables goods,

$$c_t^T = y_t^T + \frac{\mathrm{d}_{t+1}}{1+r_t} - \mathrm{d}_t \tag{2.18}$$

A competitive equilibrium is defined as a set of stochastic processes $\left\{c_t^T, h_t, w_t, \mathrm{d}_{t+1}, \lambda_t, \mu_t\right\}$satisfying the followings equations given $\left\{y_t^T, r_t\right\}$, the initial conditions $w_{-1}$and d_0 and the exchange rate $\varepsilon_t \equiv \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$

$$\begin{aligned}
c_t^T &= y_t^T + \frac{\mathrm{d}_{t+1}}{1+r_t} - \mathrm{d}_t && (2.19)\\
\mathrm{d}_{t+1} &\leq \bar{\mathrm{d}} && (2.20)\\
\lambda_t &= U_{c_t}\left(\cdot\right)\frac{\partial c_t}{\partial c_t^T} && (2.21)\\
\lambda_t \frac{1-\tau_t^{\mathrm{d}}}{1+r_t} &= \beta \mathbb{E}_t \lambda_{t+1} + \mu_t && (2.22)\\
\mu_t &\geq 0 && (2.23)\\
\frac{1-a}{a}\left(\frac{c_t^T}{F\left(h_t\right)}\right)^{1+\eta}\left(c\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} &= \frac{W_t/\mathcal{E}_t}{F'\left(h_t\right)} && (2.24)\\
\mu_t\left(\mathrm{d}_{t+1} - \bar{\mathrm{d}}\right) &= 0 && (2.25)\\
w_t &\geq \tilde{\gamma}\frac{w_{t-1}}{\varepsilon_t} && (2.26)\\
h_t &\leq \bar{h} && (2.27)\\
\left(\bar{h} - h_t\right)\left(w_t - \tilde{\gamma}\frac{w_{t-1}}{\varepsilon_t}\right) &= 0 && (2.28)\\
\tau_t^y\left(y_t^T + w_t h_t + \pi_t\right) &= \tau_t^{\mathrm{d}}\frac{\mathrm{d}_{t+1}}{1+r_t} && (2.29)
\end{aligned}$$

Later in section 5 I characterize quantitatively the economic dynamics in a boom-bust cycle under the first best policy through an optimal exchange rate ($\phi = 0$). This policy assumes there are no income taxes and no external debt taxes, i.e. $\tau_t^y = \tau_t^{\mathrm{d}} = 0$[2]. Additionally, in the following sections I analyze theoretically the currency peg without capital control. In this case the assumption is that $\tau_t^y = \tau_t^{\mathrm{d}} = 0$ and $\varepsilon_t = 1$. Thereby, the model presented so far corresponds to the most general specification.

[2]In section 6, Future Work, I present the case of the second best Ramsey optimal capital control with a fixed exchange rate regime. This policy assumes the opposite, debt and income taxes take positive values and the currency is peg, so $\varepsilon_t = 1$.

2.3. Equilibrium Analysis

The main goal in this section is to theoretically analyze how the equilibrium under two different exchange rate policies differs from the canonical case. For this purpose I begin analyzing graphically how the equilibrium is presented.

I analyze a boom-bust episode in a (c^N, p^N) space, plotting the supply and demand curves for non-traded goods. Since labor is the only input in the non-tradable production function, the interest variable in the horizontal axis is labor h. The supply curve equation is given by equation (2.10), a positive slope curve in (h, p^N) space. The demand curve is given by equation (2.7), and differs to the homothetic case on the dependence on composite consumption good.

To make a comparison, I analyze a situation in which both economies start with the same initial equilibrium. To do this, first note that demand curve with homothetic preferences, as I call P^H curve, is a monotonic decreasing function. Then, Lemma 2.1 establishes the condition under which the demand curve in the non-homothetic case, P^{NH}, is also monotonic decreasing function. Moreover, this lemma shows that both curves are convex and therefore the equilibrium is unique, at least for plausible parameterization parameters.

Lemma 2.1 (Negative slope and convexity of non-homothetic demand of non-tradables): The non-homothetic demand curve of non-tradables is a monotonically decreasing and convex function of labor, h, if $1/(\varepsilon_N - \varepsilon_T) > \xi_{C,C^N}$, where $\xi_{C,C^N} = \frac{\partial C}{\partial C^N}\frac{C^N}{C}$.

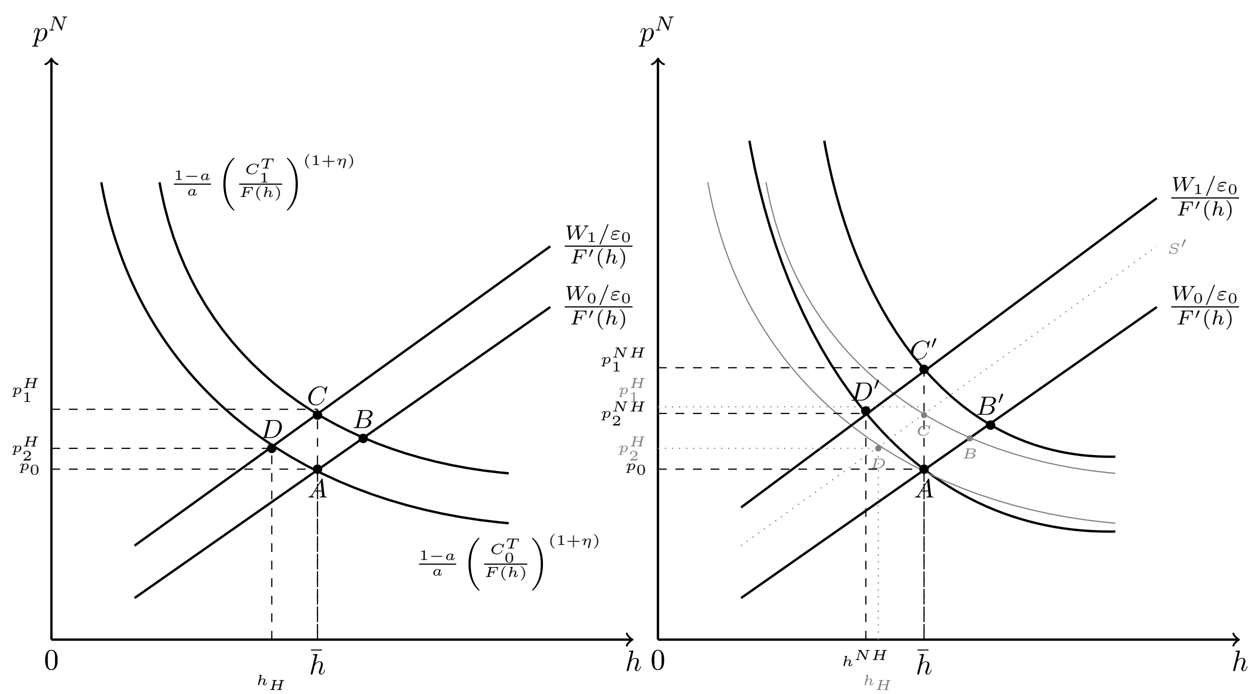
As we will see in the quantitative analysis this condition is met for plausible parameterization. Lemma 2.1 lets draw both curve as a decreasing function in (h, p^N) space. Regarding with the shape of these curves the following lemma establishes an interesting property.

Lemma 2.2 (Properties of the non-homothetic demand curve of non-tradables): To the left of the equilibrium non-homothetic demand curve goes above the homothetic demand. To the right of the equilibrium, the non-homothetic demand goes below the homothetic curve.

Lemma 2.2 states that at some value of h the $P^{N,NH}$ curve goes up to infinity whereas the $P^{N,H}$ curve remains with a finite value. At the same time, for large value of h both curves move closely. Figure 1 graphically shows these results. Panel (a) describes the results obtained by Schmitt-Grohé and Uribe (2016) reproduced in this model establishing $\varepsilon_N = \varepsilon_T = 1$. The figure plots the equilibrium as the intersection between the demand and supply curves of non-tradables goods. Point A shows the equilibrium with full employment $\bar{h}$ and relative price of non-tradables equals to p_0^H[3]. When a positive shocks arrives, either as an increase in tradable goods or a fall in the interest rate, the absorption of tradable increases from c_0^T to c_1^T shifting

[3]The superscript H refers to homothetic case.

the demand curve upward and to the right, changing the equilibrium to the point B. Instantaneously, as the economy is in full-employment, supply curve adjust shifting to upward and to the left, as the nominal wage raises from W_0 to W_1. This change put the new equilibrium in point C, keeping the full-employment value of $\bar{h}$ and increasing the relative price of non-tradables to p_1^H. When the positive shock vanishes, the bust part of the cycle arrives, and the absorption of tradables falls, say to the pre-shock level c_0^T, contracting the demand curve. As nominal wages are downward rigid, the supply curve remains unaltered an the intersection between demand a supply is reached at point D, with $h < \bar{h}$ and relative price of non-tradable equal to p_2^H, such that $p_0^H < p_2^H < p_1^H$. Nominal wage rigidity creates a labor market friction that put the economy in a vulnerable situation when the boom goes. Wages remain artificially high affecting the adjustment of firms, which are forced to raise the price of non-tradable in a inefficient way creating unemployment. This dynamic is replicated with gray in panel (b) of figure 1, however, I add in solid black lines the equilibrium when preferences are non-homothetic. It is possible to set the conditions under which non-homothetic preferences share the same equilibrium allocation as the homothetic case. This is represented in the figure by the same initial equilibrium assignment at point A, that is. the intersection of supply curve and demand curve, both with solid black lines. The subsequent equilibrium movements derived by the boom-bust cycle are represented by B', C', and D' points respectively. Next, I will describe how two possible monetary policies behave when preferences are non-homothetic.



(a) (b)

Figura 2.1: Equilibrium with Homothetic and Non-Homothetic preferences

2.3.1. Currency Peg $\phi = 0$

Recall that the objective of a desired policy is to return the economy to point A. This can be achieved lowering the real wage $w_t \equiv W_t/\varepsilon_t$, either by decreasing nominal wage W_t or by augmenting ε_t, that is, by a devaluation of the nominal exchange rate. However, since wages are downward rigid, the monetary authority will want to increase ε_t.

Currency peg means that the nominal exchange rate is fixed, in this setup implies that $\phi = 0$ and therefore, $\varepsilon_t = 1$ for all t in equation (2.13). Thus, the economy can not return to the initial equilibrium in point A, and the final result is the one described at point D' (D respectively), that is with involuntary unemployment and a higher relative price of non-tradables. As shown in panel (b) the equilibrium described by D' is more harmful than the one described by D. Since $h^{NH} < h^{H}$, the unemployment $\bar{h} - h^{\mathrm{i}}$ with $\mathrm{i} = \{NH, H\}$, is greater when preferences are non-homothetic. The shifting of the demand curve in a boom-bust episode is more pronounced in the non-homothetic case and the slope is steepest at the top of the curve. These two facts implies that the point D' is above D, with a greater relative price of non-tradable $p_2^{NH} > p_2^{H}$. A peg economy, for instance those group of peripheral European countries or Argentina before 2002, would suffer great damage in the face of a financial crisis by adopting this type of policy if indeed the income elasticity in the consumption of non-tradables is greater than the income elasticity of tradable goods, as the evidence seem to suggest.

2.3.2. Optimal Exchange Rate Policy $\phi = 1$

The opposite to the currency peg is the full flexible exchange rate. In this setup the optimal exchange rate is the one that ensures full-employment, that is, induce a movement in the supply curve that return to the point A where $h = \bar{h}$ and the relative price of non-tradable is p_0. Applying the optimal exchange rate in point D' implies that the devaluation necessary to return to point A involves a higher increase in the exchange rate ε_t than the one necessary to reach the same result in CES aggregator case. Thus, countries that implement a full employment exchange rate not taking into account that non-tradable goods are more income elastics than tradables goods may implement a sub optimal policy that partially generate unemployment spreading the crisis started in tradable sector to the non-tradable.

2.4. Analytical Example

In this section I present an analytical example in a way to show how different are the policy responses if we consider non-homothetic CES preferences. Aggregate consumption implicit function and production technology are as the one described in The Model section. Suppose now, that the tradable endowment is constant over time such that $y_t^T = y^T$, and that $\beta(1+r) = 1$. Additionally, suppose that the economy had been at full employment $\bar{h} = 1 = h$ prior to period 0 and that, $\mathrm{d}_{t+1} = 0$, $c_t^T = y^T$, and $c_t^N = 1$ for any t lower than period 0, and take $\tau_t^y = \tau_t^{\mathrm{d}} = 0$, i.e., there is not capital controls policy. Consider the following temporary change of the interest rate: $r_t = r$ in $t < 0$; $r_t = \underline{r}$ in $t = 0$; and $r_t = r$ for $t > 0$. In this terms, the only source of uncertainty is revealed at $t = 0$ and from then on there exists perfect foresight by the economic agents.

To characterize the equilibrium conditions stated in equations (2.19)-(2.28) I assume that $\mathrm{d}_t < \bar{\mathrm{d}}$, therefore the slackness condition for the debt limit implies that $\mu_t = 0$, and constant debt from period 1 on, i.e., $\mathrm{d}_t = \mathrm{d}_1$ for $t \geq 1$. The Euler equation (2.22) considering (2.21) in this example is

$$U'(c_t)\frac{\partial c_t}{\partial c_t^T} = \beta(1+r_t)U'(c_{t+1})\frac{\partial c_{t+1}}{\partial c_{t+1}^T}$$

For simplicity I take utility function to be logarithmic, this implies that in period $t = 0$,

$$c_1 = \beta(1+\underline{r})c_0\Xi \tag{2.30}$$

Where $\Xi \equiv \frac{\frac{\partial c_1}{\partial c_1^T}}{\frac{\partial c_0}{\partial c_0^T}} < 1$,since as will see $c_0 > c_1$. Recall that equilibrium tradable consumption is given by $c_t^T = y_t^T + \frac{\mathrm{d}_{t+1}}{1+r_t} - \mathrm{d}_t$ in equation (2.19), which under the assumption of this example implies that, $c_0^T = y^T + \frac{\mathrm{d}_1}{1+\underline{r}}$ and $c_1^T = y^T + \frac{\mathrm{d}_2}{1+r} - \mathrm{d}_1$. Since by the assumption stated above $\mathrm{d}_t = \mathrm{d}_1$ for $t \geq 1$, tradable consumption also met that $c_1^T = c_t^T$. We can rewrite expression in (2.30) as

$$c_1\left(y^T + \frac{\mathrm{d}_1}{1+r} - \mathrm{d}_1, 1\right) = \frac{1+\underline{r}}{1+r}c_0\left(y^T + \frac{\mathrm{d}_1}{1+\underline{r}}, 1\right)\Xi \tag{2.31}$$

Hence, there is a debt level in period 1, say $\mathrm{d}_1^* > 0$, that solves equation (2.31). Now, let's turn to wages in period $t = 0$, and $t \geq 1$. From equation (2.24) wages are given by
$\frac{1-a}{a}\left(\frac{c_t^T}{c_t^N}\right)^{1+\eta}(c)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}F'(h_t) = W_t/\mathcal{E}_t$, which in this example means that wages are given by

$$w_0 = \frac{1-a}{a}\left(y^T + \frac{\mathrm{d}_1}{1+\underline{r}}\right)^{1+\eta}\left(c_0\left(y^T + \frac{\mathrm{d}_1}{1+\underline{r}}, 1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} \tag{2.32}$$

$$w_1 = \frac{1-a}{a}\left(y^T + \frac{\mathrm{d}_1}{1+r} - \mathrm{d}_1\right)^{1+\eta}\left(c_1\left(y^T + \frac{\mathrm{d}_1}{1+r} - \mathrm{d}_1, 1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} \tag{2.33}$$

From expressions in (2.32) and (2.33) we obtain that $w_0 > w_1$. So, the positive shock in period 0 induced by the reduction in the interest rate from r to $\underline{r}$, produced an increase in consumption and wages in $t=0$. After the shock, wage and consumption fall.

Lets take first, the optimal full employment policy exchange rate. Equation (2.26) states that any exchange rate satisfying $\varepsilon_t \geq \gamma \frac{w_{t-1}}{w_t}$ ensures full-employment. Following Schmitt-Grohé and Uribe (2016) we take $\gamma = 1$ and the particular policy $\varepsilon = \frac{w_{t-1}}{\omega\left(c_t^T\right)}$, where $\omega\left(c_t^T\right)$ is the full employment wage rate for a given c_t^T defined as $\omega\left(c_t^T\right) \equiv \frac{1-a}{a}\left(\frac{c_t^T}{F\left(\bar{h}\right)}\right)^{1+\eta}\left(c\left(c_t^T, F\left(\bar{h}\right)\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} F'\left(\bar{h}\right)$. The optimal full employment exchange rate in this context is given by

$$\varepsilon^{NH} = \frac{w_0}{w_1} = \frac{\left(y^T + \frac{\mathrm{d}_1}{1+\underline{r}}\right)^{1+\eta}\left(c_0\left(y^T + \frac{\mathrm{d}_1}{1+\underline{r}}, 1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}}{\left(y^T + \frac{\mathrm{d}_1}{1+r} - \mathrm{d}_1\right)^{1+\eta}\left(c_1\left(y^T + \frac{\mathrm{d}_1}{1+r} - \mathrm{d}_1, 1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}} \tag{2.34}$$

The superscript NH comes from non-homothetic. Recall that if we chose $\varepsilon_N = \varepsilon_T = 1$ the model return to the standard homothetic CES aggregator. Since $c_0 > c_1$ and $\varepsilon_N > \varepsilon_T$ the expression $(c_0/c_1)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}$ is greater than one implying that $\varepsilon^{NH} > \varepsilon^H$. In words, this example shows that the optimal policy exchange rate required to keep full employment when preferences are non-homothetic involves a currency devaluation greater than the one required in the standard case.

Lets analyze the case when the exchange rate is fixed. In this context $\varepsilon_t = 1$ for all t implying by condition (2.26) that $w_1 = w_0$. The complementary slack equation (2.28) on its part establishes that unemployment arises, in such a way that $\bar{h} > h_1$. These conditions implies that

$$\frac{1-a}{a}\left(\frac{c_1^T}{h_1^\alpha}\right)^{1+\eta}\left(c_1\left(c_1^T, h_1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} = \frac{1-a}{a}\left(c_0^T\right)^{1+\eta}\left(c_0\left(c_0^T, 1\right)\right)^{(1+\eta)(\varepsilon_N-\varepsilon_T)} \tag{2.35}$$

Note that when $\varepsilon_N = \varepsilon_T = 1$, $h_{1,H}^\alpha \equiv F\left(h_{1,H}\right) = \frac{c_1^T}{c_0^T} < 1$. As is mentioned above, the subscript H refers to homothetic. When $\varepsilon_N > \varepsilon_T$, non-homotheticity implies that $F\left(h_{1,NH}\right) = \frac{c_1^T}{c_0^T}\frac{c_1\left(c_1^T,h_1\right)}{c_0\left(c_0^T,1\right)}$. Since $c_0\left(c_0^T,1\right) > c_1\left(c_1^T,1\right) > c_1\left(c_0^T,h_1\right)$ the expression $\frac{c_1\left(c_1^T,h_1\right)}{c_0\left(c_0^T,1\right)} < 1$ implying that $F\left(h_{1,H}\right) > F\left(h_{1,NH}\right)$. The properties of the production function ensures that $h_{1,H} > h_{1,NH}$ and therefore the unemployment $1-h_{1,NH} > 1-h_{1,H}$. In words, this example shows that when the currency is peg and preferences are non-homothetic, the pecuniary externality derived by downward nominal wage rigidity it is much more harmful for employment than in the traditional homothetic case.

2.5. Quantitative Evaluation

In this section I characterize aggregate dynamics under full-employment optimal exchange rate. The scope is to compare the model's quantitative predictions in a standard homothetic set-up versus the non-homothetic specification. Specifically, using a calibrated version of the model we compare aggregate dynamics in a boom-bust episode, first moment and volatility response associated with the optimal first-best policy.

2.5.1. Calibration

The economy is based on Schmitt-Grohé and Uribe (2016), thus I follow their calibration strategy and parameterization to highlights the difference between the two economies. I assume a CRRA functional form for the period utility function, and standard CES aggregator function to analyze homothetic case. The model is calibrated at a quarterly frequency to match Argentinean data, an emerging country widely studied in the International Finance literature. This strategy allows a direct comparison of the quantitative results with the previous literature.

Table 2.1: Baseline Parameters for Calibration

Parameter	Value	Description
σ	5	Inverse of intertemporal elasticity of substitution
η	1	Parameter that govern elasticity of substitution $\frac{1}{1+\eta}$
a	0,26	Share of tradables
β	0,9375	Subjective discount factor
α	0,75	Labor share in non-tradable
γ	0,99	Degree of downward nominal wage rigidity
ε_N	3,5	Parameter related to Income elasticity of non-tradables goods
ε_T	1	Parameter related to Income elasticity of tradables goods

Table (2.1) shows the baseline parameters values. The value of $\eta = 1$ ensures a value for intratemporal elasticity of substitution $\frac{1}{1+\eta}$ equal to 0,5, grater than the one used in Schmitt-Grohé and Uribe (2016) equals to 0,44, but useful for the computation of the implicit nonlinear function for consumption. Gonzalez-Rozada and Neumeyer (2003) estimates the elasticity of substitution in the demand for non-tradable goods in Argentina between the range of 0,4 and 0,48, leaving the calibration of this parameter very close to the upper range of this estimate.

The parameters related to non-homothetic setup are take it from Comin et al. (2015) and used in Rojas and Saffie (2020). The value of ε_T is set to 1, and the value of ε_N is set to 3,5 such the difference between the income elasticity of tradables and non-tradables is 0,24, according to their empirical work. The rest of the parameters are chosen as in Schmitt-Grohé and Uribe (2016): The coefficient of relative risk aversion σ is set to 5 a little higher than 2, the most common value used in the literature, but coincident with the value of the intertemporal elasticity of substitution, $1/\sigma$, equal to 0,21 estimated by Reinhart and Végh (1995) . The parameter a set to

0,26 match the share of traded output in total output. The parameter γ , governing the downward wage rigidity is set to 0,99 as the most conservative value reported by Schmitt-Grohé and Uribe (2016), and the exogenous shocks $\left(y_t^T, r_t\right)$ are estimated by a bi-variate $AR\,(1)$process through OLS using data for Argentina between 1983 : $Q1$ to 2001 : $Q4$. For simplicity, I take this estimation as given using the results of the authors that are kindly posted on the authors website. For robustness I made my own computation of this process reaching similar outcomes.

2.5.2. Full-Employment Exchange Rate Policy

The full-employment exchange rate policy undoes all the negative effects of nominal frictions, returning the economy to the first-best outcome where $h = \bar{h}$ (point A in figure 2.1). Therefore, solve for this policy is the same as solve the Pareto Optimal allocation. The value function that describes this problem is given by:

$$V\left(y^T, r, \mathrm{d}\right) = \max_{c^T, \mathrm{d}'} \left\{ U\left(c\left(c^T, 1\right)\right) + \beta \mathbb{E} V\left(y'^T, r', \mathrm{d}'\right) \right\} \tag{2.36}$$

$$s.t.\; c^T = y^T + \frac{\mathrm{d}'}{1+r} - \mathrm{d}$$

$$\mathrm{d}' \leq \bar{\mathrm{d}}$$

As discussed above the full-employment policy implies that the weight given to the exchange rate stabilizing target is zero, that is $\phi = 1$. From (2.26), any exchange rate policy satisfying $\varepsilon_t \geq \gamma \frac{w_{t-1}}{w_t^f\left(c^T, F\left(\bar{h}\right)\right)}$, where w_t^f represents the full-employment wage, ensures full employment for all periods. According with (2.13) and that $\phi = 1$, I choose the full-employment policy that minimizes movement in the devaluation rate, i.e., $\varepsilon_t = \max\left\{1, \gamma \frac{w_{t-1}}{w_t^f\left(c^T, F\left(\bar{h}\right)\right)}\right\}$.

I solve the dynamic problem (2.36) by value function iteration, with a 201 grid points for debt , 21 discretization points for the stochastic process of tradable endowment and 11 points for the interest rate process. Then, I simulate the model to analyze the economic response in a boom-bust cycle, and to examine the level and volatility of aggregates.

Boom-bust dynamics

I evaluate the quantitative response of the model in a boom-bust cycle starting at period t according to the following definition. The boom is characterized by a movement in tradable output that starts at or below trend in period t, and at least one standard deviation above trend in period $t+10$. In the bust, tradable output is one standard deviation below trend in period $t+20$. The model is simulated by 100,000 periods and then selected all the windows that coincides with the boom-bust definition.

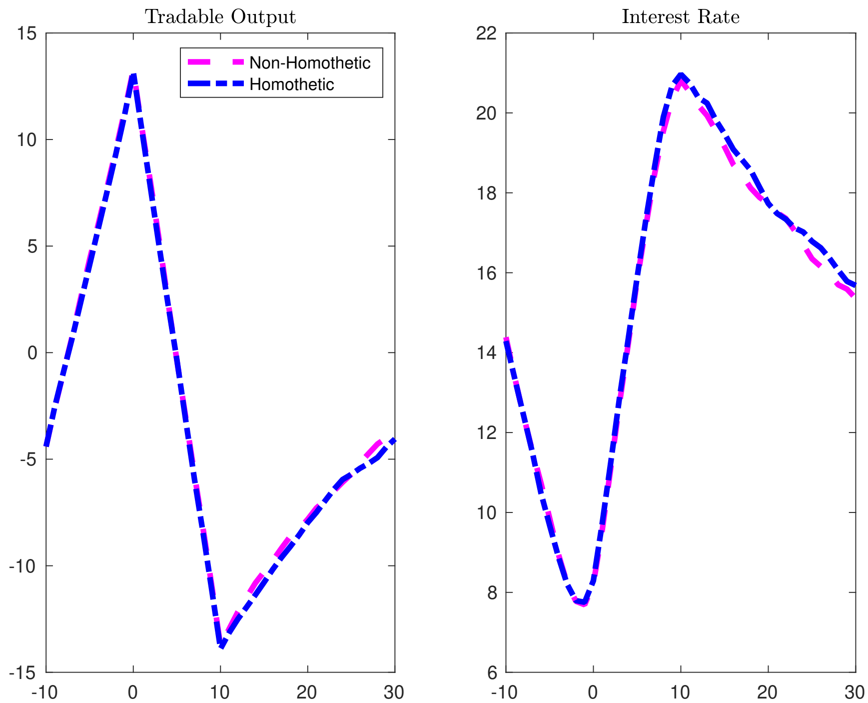


Figura 2.2: Negative shocks during a crisis

Figure 2.2 shows the shape of the tradable output and interest rate during a boom-bust cycle. The homothetic and non-homothetic economies have a similar shape as a result of the estimation of a bi-variate, first-order, auto-regressive process. However, the recovery phase of the boom-bust cycle driving process is slightly higher for the non-homothetic case.

When full-employment exchange rate policy is in action, the currency devaluation ensures that involuntary unemployment remains at zero at all periods. As expected the right panel of the first row in figure 2.3 shows that at all periods the two economies experience full-employment.

The dynamic differs in others variables. As we sketch in the equilibrium analysis in figure 2.1, the needed fall in the relative price of non-tradables – the real exchange rate -, to return to full employment equilibrium is greater in the non-homothetic than in the homothetic economy (second row, right panel). The percentage deviation from the trend in the boom is 62,87 % for the non-homothetic economy and 46,78 % for the homothetic[4]. However, in the bust the non-homothetic economy deviates from the trend in $-74{,}86\,\%$ versus the respective deviation of the homothetic economy by $-54{,}32\,\%$. The non-homothetic economy responds 16,09 % higher at the boom and 20,54 % lower at the through of the bust. The amplification mechanism implies a decrease in the real exchange rate 36,63 % higher for the non-homothetic economy.[5].

[4]Series are represented by percentage deviation from the trend.

[5]That is, $(62{,}87\,\% - (-74{,}86\,\%)) - (46{,}78\,\% - (-54{,}32\,\%)) = 36{,}63\,\%$

A similar pattern follows the real wage (second row, left panel). Non-homothetic economy amplifies the endogenous response of real wages in the boom as well as in the bust. Consistently with figure 2.1, the only way that the real wage downward adjusts in the model is through an increase in the nominal exchange rate. Therefore, the figure suggests that the currency devaluation rate needed to achieve the full-employment outcome is significantly higher in the non-homothetic economy. To see this, note that at the boom, the real wage in the non-homothetic economy deviates from the trend by 62,86 % and then goes down at the through deviating from the trend in $-74{,}86$. That is, the total decrease of wages to ensure full employment is 137,72 % in terms of deviation from the trend. In the homothetic economy this number is 101,2 %. I will return to this point later.

The left panel of the third row shows the tradable absorption. Both economy have a similar pattern in these series, however, the non-homothetic economy shows a slightly higher deviation at the peak of the boom and the through of the bust. In the boom the non-homothetic curve deviates form the trend by 26,03 % versus the homothetic economy which deviates by 23,39 %. The same behavior acts at he bust. The non-homothetic economy deviates by $-31{,}04$ % and the homothetic economy in $-27{,}16$ %. Thus, the amplification of the series by including heterogeneous income elasticities among goods is only around 3 %.

The composite consumption index show a clearly different pattern among these two economies (third row, right panel). The non-homothetic economy shows a much stronger consumption smoothing pattern than the homothetic. The deviation of the trend at the boom and the bust is respectively by 2,16 % and $-2{,}55$ %, whereas the homothetic economy is much higher by 7,44 % and $-8{,}51$ % respectively. This different pattern unfold an important behavior of the non-homothetic economy: a sharply smooth consumption pattern. However, note that y-axis values are in both economy considerably lower than the other variables.

The debt-to-output ratio (fourth row, right panel) and the trade-balance-to-output ratio (fourth row, left panel) present similar shapes but with different scales. In the boom, debt-to-output ratio series in the non-homothetic economy goes below the same homothetic case series. The opposite occurs in the bust. At the boom the deviation from the trend is by $-26{,}66$ % and $-19{,}98$ for non-homothetic and homothetic economy respectively, and in the bust, the deviation from the trend are 33,23 % and 22,21 % respectively. That is, the debt is almost 7 % lower at the boom and almost 10 % higher at the bust in the non-homothetic specification. The trade-balance-to-output ratio in the non-homothetic economy deviates by $-1{,}63$ % lower and 3,13 % higher than homothetic economy in the boom and bust respectively. In this way, the response of all variables except the composite consumption index, is amplified in greater or lesser degree by considering heterogeneous income-elasticity among goods.

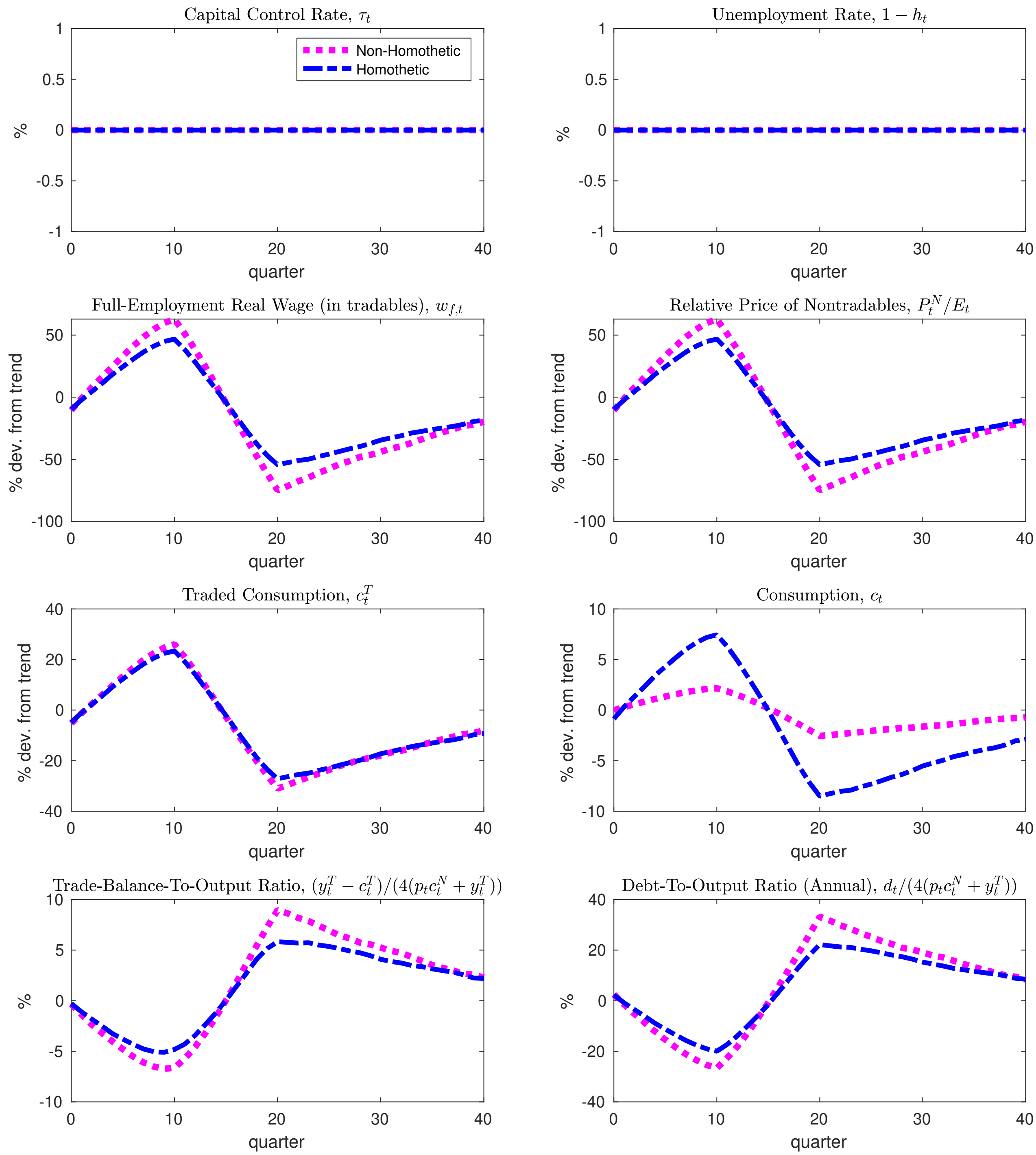


Figura 2.3: Full Employment Exchange Rate Policy

Economic responses to a boom-bust cycle with optimal full-employment exchange rate policy. For real wages, relative price of non-tradables, traded consumption and composite consumption index the figure plots percentage deviations from the trend. For trade-balance-to-output ratio, debt-to-output ratio, as well as unemployment and capital controls the figure plots percentages. The dotted pink line plots the non-homothetic economy and the dashed-dot line plots homothetic economy.

The pattern showed in figure (2.3) is consistent with the higher income-elasticity of non-tradable goods in the non-homothetic economy. Let analyze the boom in this economy. When the economy is richer households want to substitute tradable consumption for non-tradable consumption. Since non-tradable sector supply is fixed by the amount of labor, which in turn is at full-employment, implies that the labor market clearing put an extra upward pressure on the relative price of non-tradables (second row, right panel). Precisely, the fact that the price of non-tradable raises more with non-homothetic preferences, is the result of a stronger preference for these goods when income is higher. The real wages of the non-tradable sector reflected in a one to one mapping the raise of the price of non-tradable (second row, left panel).

Being richer, households have preferences to consume more non-tradables relative to tradable consumption. Since the exogenous driving forces are the same for both economies (figure 2.2) the only way to adjust tradable consumption is by the use of debt, as is clear from equation (2.18). This is the reason why the debt-to-output ratio goes below for the non-homothetic economy (fourth row, right panel).

Lower debt , d_t, implies that the tradable consumption must be higher in the non-homothetic economy, which is the case of traded consumption (third row, left panel). However, since the specification of the income-elasticity of tradable is equal for the two economies, this translate into a similar shape of the evolution of tradable consumption, but with differences at the peak of the boom and the through of the bust. That is, in the non-homothetic economy household have a similar pattern of tradable consumption in the boom (bust) but a lower (higher) use of debt. Finally, another consequence of the use of the debt as a mechanism to allocate basket consumption is the result of a sharply consumption smoothing in a boom-bust cycle.

Slightly higher tradable consumption at the peak in the non-homothetic specification, implies a slightly lower trade balance, since tradable output is equal for both economies (fourth row, left panel). This is nothing more than a consequence of the use of the debt.[6]

Nominal Devaluation in a Boom-bust cycle

Figure 2.4 plots the dynamic of the annualized devaluation rate required to eliminate the inefficiencies originated by nominal frictions. At the beginning of the bust, the homothetic economy devaluates their currency at an annualized rate of about 38 % and keeps it relatively constant until the end of the bust. In the recovery phase returns to the levels before the cycle.

In the non-homothetic economy, at the beginning of the bust the annual devaluation rate increases about 40 % and then raises to 87 % at the through of the bust. Then, at the end of the bust, returns to the pre-cycle trend following a similar shape as the homothetic economy.

To explain the intuition of this result, note that real wage is a function of tradable

[6]From equation (2.18) can be stated as $y_t^T - c_t^T = \mathrm{d}_t - \mathrm{d}_{t+1}/(1+r)$

consumption directly by the relative consumption term between tradable and non-tradable, but also indirectly through the total composite consumption index. The latter is a key insight in this pattern since in bad times, when tradable output is lower, the consumption index reinforces the negative effect of the downturn putting an extra downward force on real wages. As a result, the nominal devaluation required to reach the full-employment in the bust is markedly higher than the homothetic economy. Indeed, on the through of the crisis, the authority needs a devaluation rate about 40 % larger than the standard homothetic model.

Figura 2.4: Devaluation Rate
Annual devaluation series to keep full-employment equilibrium. Pink dashed line shows the homothetic case and blue dash-dotted line shows the non-homothetic specification.

First and second moment

Table 2.2 shows the long-run mean and volatilities the macroeconomic variables of interest. These series are constructed by simulated the optimal full-employment policy response of the economy.

The table shows a similar pattern in most of the aggregate variables. The composite consumption index has a slightly higher mean in the non-homothetic economy with a slightly lower variability. The long-run trade-balance-to-gdp ratio is 1,67 % higher in the non-homothetic economy but with higher standard deviation. Real wages, traded output and the annualized interest rate are practically the same in both economy. Traded output and interest rate share the same stochastic processes in both economies, and the long-run real wages are set roughly 1,5 in both economies.

Table 2.2: First and Second Moment

Variable	Mean		Std. Deviation	
	Hom. Pref.	N-Hom. Pref.	Hom. Pref.	N-Hom. Pref.
Unemployment rate, u_t	0	0	0	0
Consumption, c_t	0,93	0,98	0,08	0,03
Trade balance-to-GDP, $\frac{y_t^T - c_t^T}{y_t^T + p_t^N c_t^N}$	8,4	10,07	8	9,88
Real wage, $\frac{W_t}{E_t}$	1,5	1,45	0,78	0,88
Traded output, y_t^T	1,01	1,01	0,12	0,12
Annualized interest rate, r_t	13,3	13,55	7,8	7,84
External Debt, d_t	6,1	6,42	0,28	0,08
Debt-to-output, $\frac{\mathrm{d}_t}{4\left(y_t^T + p_t^N c_t^N\right)}$	58,3	66,73	26,6	32,89
Devaluation rate, $\varepsilon_t = \text{máx}\left\{1, \gamma \frac{w_{t-1}}{w_t^{full}}\right\}$	1,0715	1,1023	0,12	0,28

The long-run behavior of the debt is different in both specification. The external debt in the non-homothetic economy is marginally higher than the homothetic economy (6,42 versus 6,1 respectively) but with a considerably lower standard deviation (0,08 and 0,28 respectively). That is, most of the time the external debt is at its long-term value.

The long-term debt-to-output-ratio is about 9 % higher in the non-homothetic economy. The debt is more heavily used in this economy as a mechanism to substitute consumption among goods, not just to ride out the cycle but also in the long-run.

Debt distribution

How does the external debt distribution change with heterogeneous income elasticities?. Figure 2.5 shows the ergodic external debt distribution in both economies. The solid (blue) line plots the homothetic economy debt distribution, and the (red) dotted line plots the non-homothetic economy debt distribution.

We have just seen in table 2.2 that the average value of external debt is a bit higher for the non-homothetic economy. Debt distribution in non-homothetic economy is much more condensed at a higher values of debt. This behavior is consistent with the lower standard deviation of the long-run mean. In bad times non-tradable consumption is reduced toughly, and therefore the tradable consumption increases. To do this transition the economy makes a more heavy use of the debt, made possible by the commitment technology that force to honor the debt.

2.5.3. Discussion

What can we learn from the model? the main message is that heterogeneous income elasticities among goods amplify the dynamic of the variables in a boom-bust cycle. The mechanism behind this amplification is the debt adjustment of the economy. That is, debt in this model has two purposes: (1) an instrument to allocate *intertemporal* consumption, the standard consumption smoothing role of debt in

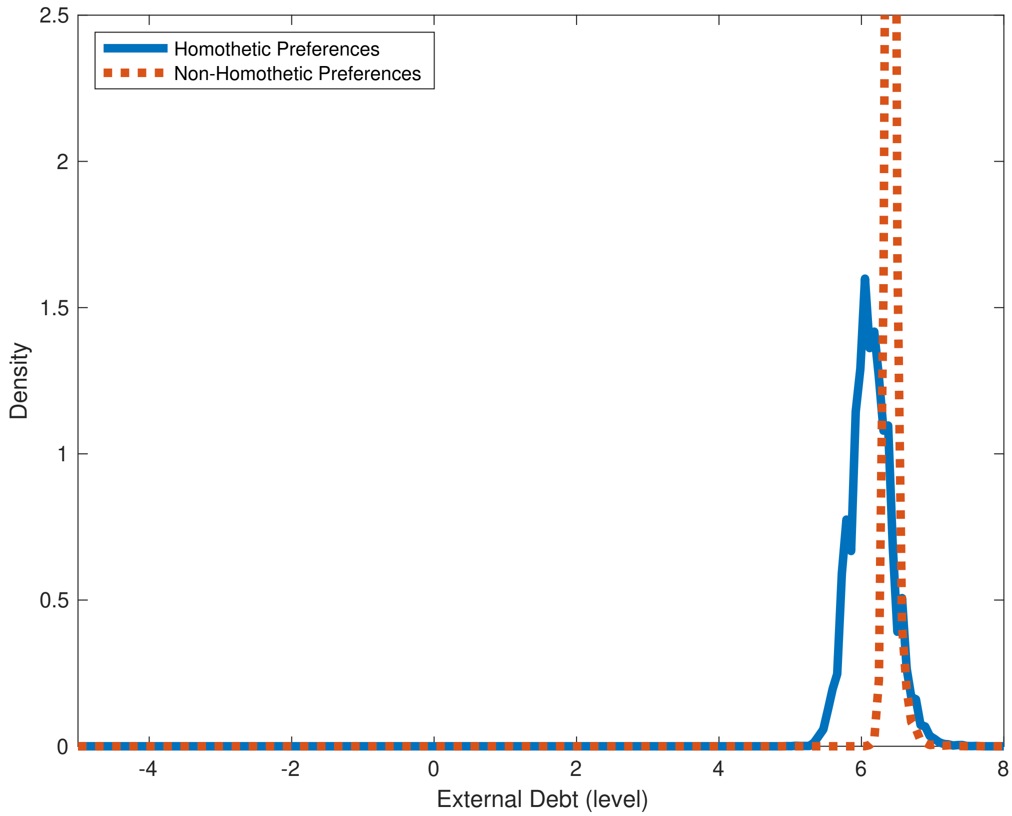


Figura 2.5: External debt distribution

the literature. (2) as an instrument to allocate *intratemporal* consumption. Since preferences are non-homothetic, the composition of the consumption basket differs in good and bad times. Changes in the composition of consumption basket are in turn enabled by the debt.

In particular the behavior of the economy in terms of the trade balance has a central role when preferences are non-homothetic. As I mentioned above, the traded consumption is quite similar in both economies, with a slightly higher (lower) value at the boom (bust) for non-homothetic economy. However, for this economy the current debt is less used in good (bad) times. How is it possible that similar exogenous driving forces, lead a similar tradable consumption if the current debt is lower in the non-homothetic economy?

To answer this question, inspects the equilibrium condition of the tradable goods market. By equation (2.19), since the current debt is lower in the non-homothetic economy, the only way to adjust the tradable consumption to be similar in both economies is through the debt issued today and to be paid tomorrow. Thus, what is important in the adjustment is the change in the debt position, that is the trade balance.

To see this more clearly, equation (2.19) can be stated as $y_t^T - c_t^T = \mathrm{d}_t - \mathrm{d}_{t+1}/(1 + r_t)$. The right hand side is the trade balance definition which equals the change in the debt position. That is, the similar pattern of traded consumption and the higher smoothing shape of the composite consumption index is the result of a heavy use of the current account. An interesting insight is the similar behavior in the long-run for both economies. First moments in both economies show the same behavior in the full-employment exchange rate policy. As in a boom-bust cycle, this pattern is achieved by a more heavy use of debt as fraction of output.

These results may be of interest in the current global pandemic context. Lockdowns led to a sudden halt in total consumption particularly in non-tradable sector (Campello et al., 2020). Although this picture does not correspond to a usual boom-bust cycle, not considering the drop in the level of consumption and only caring about relative consumption could lead to incomplete conclusions about the effects of the crisis. More generally, the amplification mechanism of non-homothetic preferences establishes a potential tie between financial crises and income inequality. The most unequal an economy is, the larger the amplification of the crises.

2.6. Capital Control Rate: The Ramsey Problem

This section analyzes quantitatively the second best instrument for a government intervention: a capital control rate. For this purpose, the model considered here assumes that the tax rate on debt and income, τ_t^y and τ_t^{d}, are different from zero. In the same way, the economy is characterized by a currency peg, which implies that the nominal devaluation policy is fixed to $\varepsilon_t = 1$. The debt tax distorts the interest rate of the economy. The goal of any kind of policy in this environment is to break the transition of the crisis from tradable to the non-tradable sector. In other words, any policy aims to mitigate the movement of the curves presented in section 3, returning the real wage around to its original level.

The government rises the cost of external debt through a debt tax during expansion. This tax smooths the movement of non-tradable demand and the growth of nominal wage. Thus, through capital controls tax, the government affects employment in the non-traded sector by augmenting the intertemporal price of tradables.

The Ramsey's optimization problem consists in choosing the path of $\left\{\tau_t^y, \tau_t^{\mathrm{d}}\right\}$ that maximize (2.1) subject to (2.2) and the set of equilibrium conditions stated in (2.17) and (2.19)-(2.29). As is common in the characterization of these problems, I solve a less constrained problem by dropping some constraints, and then show that the solution satisfies the more general formulation. The Ramsey problem can be written as

$$
\begin{aligned}
V\left(y_t^T, r_t, \mathrm{d}_t, w_{t-1}\right) &= \underset{c_t^T, h_t, \mathrm{d}_{t+1}}{\text{máx}} \left\{U\left(c_t\left(c_t^T, F\left(h_t\right)\right), F\left(h_t\right)\right) + \beta \mathbb{E}_t V\left(y_{t+1}^T, r_{t+1}, \mathrm{d}_{t+1}, w_t\right)\right\} \\
s.t. \qquad c_t^T &= y_t^T + \frac{\mathrm{d}_{t+1}}{1+r_t} - \mathrm{d}_t \\
\mathrm{d}_{t+1} &\leq \bar{\mathrm{d}} \\
w_t &\geq \gamma w_{t-1} \\
h_t &\leq \bar{h}_t \\
\frac{w_t}{F'\left(h_t\right)} &= \frac{1-a}{a}\left(\frac{c_t^T}{F\left(h_t\right)}\right)^{1+\eta}\left(c_t\left(c_t^T, F\left(h_t\right)\right)\right)^{(1+\eta)(\varepsilon_N - \varepsilon_T)}
\end{aligned}
$$

and the optimal capital control policy must satisfy the following

$$
\tau_t^{\mathrm{d}} = 1 - \left(1+r_t\right)\beta \frac{\mathbb{E}_t\left[U'\left(c_{t+1}\right)\frac{\partial c_{t+1}}{\partial c_{t+1}^T}\right]}{U'\left(c_t\right)\frac{\partial c_t}{\partial c_t^T}} \tag{2.37}
$$

Expression in (2.37) has a known result: the larger the expected growth in consumption the larger the tax on the external debt. This result comes from the fact that what matters for capital control scheduling is the ratio between the expected marginal utility of tradable consumption among two consecutive periods.

The right hand side in (2.37) encompasses the marginal utility of tradable consumption, which differs in the homothetic and the non-homothetic economy. Therefore, the capital control rate embodied the mechanism underlying different income elasticities across goods. As mentioned above, this policy affects the intertemporal allocation of tradable consumption by distorting the interest rate.

The intuition is the following: in a boom, the contribution of tradable consumption to composite consumption index is lower in the non-homothetic economy.

This means that the expected marginal utility of tradables is lower for this economy relative to the homothetic one. Since the boom implies a expected growth in consumption, and the marginal utility of tradables is decreasing in consumption, the negative term in the right hand side of (2.37) is lower for non-homothetic economy. Therefore, this economy displays a higher capital control rate during booms. Conversely, in the bust, the share of tradable consumption in the composite consumption index is higher in the non-homothetic economy. Thus, in this economy, the capital control rate is lower than the homothetic economy.

Figure (2.6) shows the response of the Ramsey economy in a boom-bust cycle. Both economies are characterized by a currency peg regime. That is the reason why the devaluation rate has no movements in the cycle (left panel first row). However, these economies have a fiscal policy instrument to lessen the effects of the boom-bust cycle: a capital control rate.

The capital control rate is a debt tax acting in a macroprudential fashion. In booms the tax debt increases to curb capital inflow preventing the boom of tradable consumption, and consequently, restrain the raise in the demand of non-tradables. During the bust, the debt tax is reduced to support, and even subsidize, traded consumption.

As noted in the first row right panel, the non-homothetic economic requires a higher increases in the capital control rate than the homothetic economy. That is, at the boom the non-homothetic capital control rate is about 2 % higher (5,7 % versus 5,7 %). Conversely, in the bust the non-homothetic capital control rate of is 1,5 % lower than the homothetic economy. This pattern is coincident with the desire of households to consume more (less) non-tradables during boom (bust). The fiscal policy instrument is stronger when preferences are non-homothetic.

In fact, as shown in the second row right panel, traded consumption is lower at the boom and higher in the bust or the non-homothetic economy. However, in both economies the tradable absorption is sharply stable along the cycle. The unemployment rate is very similar in both economies characterized by the optimal fiscal policy (second row left panel).

2.7. Conclusions

This paper studies how the exchange rate policy changes when heterogeneous income elasticity among goods interacts with nominal frictions. For this purpose I embed non-homothetic preferences in an otherwise small open economy model with two sectors, tradable and non-tradable, and downward nominal wage rigidity. In this class of models a pecuniary externality arises when downward wage rigidity and a rigid nominal exchange rate converge. This externality manifests itself as an increase in involuntary unemployment.

The present study shows an amplification of the pecuniary externality when preferences are non-homothetic. High income elasticity of non-tradable sector induces a disproportionately consumption of these goods during booms. Market clearing in

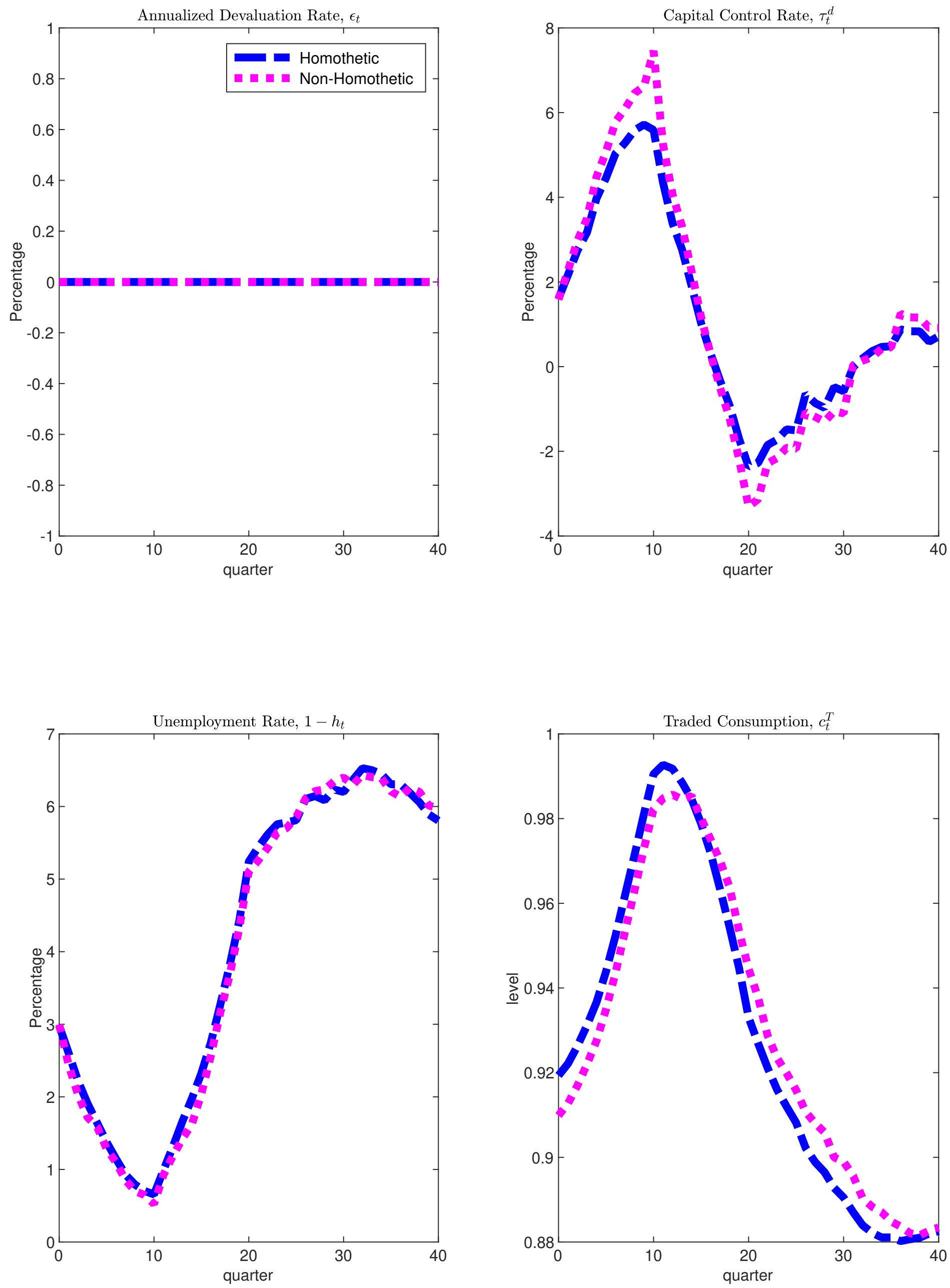


Figura 2.6: Ramsey Economy

this sector triggers a higher non-tradable equilibrium price, which in turn leads an increase in nominal wages higher than the canonical case. In the bust, relative consumption turns towards the tradable sector and non-tradable demand curve adjusts to a higher price of non-tradables. Thus, involuntary unemployment is higher when preferences are non-homothetic.

The above implies that the currency devaluation required to achieve the full-employment equilibrium is higher in the non-homothetic economy. This result is tie with the larger increase in the nominal wages. The way to return to full-employment equilibrium is by lowering the real wage. Since wages are nominal rigid, the authority must incurs in a higher currency devaluation.

The model is calibrated as in Schmitt-Grohé and Uribe (2016) to analyze the quantitative properties of non-homothetic preferences. This exercise is used to quantify the first best full-employment exchange rate policy. I find that in a boom-bust cycle the drop in the real exchange rate and real wages (from the peak of the boom to the through of the bust) in a non-homothetic economic is around 37 % higher and the associated annualized currency devaluation at the through of the crisis is 40 % higher. In the long-run both economies present a similar first and second moments of aggregate variables with exception of the debt. In the long-run, the debt distribution in the non-homothetic economy is located to the right of the debt distribution of the homothetic economy. This imply that debt-to-output ratio has a higher mean value for this economy. Finally, when considered the second best fiscal policy, the non-homothetic economy shows a capital control rate 2 % higher at the boom and 1,5 % lower at the bust, whereas the unemployment rate shape is similar for both economies.

Future robust analysis remain to be done. In the appendix I present the model when the parameter $\phi \in (0, 1)$ and the economy has debt in foreign currency. This case is important since it establishes a trade off in the exchange rate policy. On the one hand, currency devaluation helps the full employment goal of the authority. On the other hand, it shrinks the borrowing capacity of the economy, as well as undermines the stabilizing objective of the monetary authority.

In summary these results suggest that non-homothetic preferences lead a greater response from the economy. Including a real-world feature, this study finds a wider macroeconomic fluctuation that is important to rationalize in order to get stronger policy responses. This may be of interest in the current lock-down context of Covid-19, where non-tradable consumption has been the most affected. Additionally, a potential link arise between financial crisis and income inequality.

Conclusion

Throughout the preceding chapters we study the response of a frictional economy to exogenous shocks that disturb its steady state. The frictions that we consider in each case, however, as well as the environment needed for their analysis, differ in each chapter. In this way, we claim that this thesis is broad in a macroeconomic sense.

Chapter 1 presents a closed economy characterized by two frictional markets, labor and credit, and an exogenous capital-embodied technical change. This chapter primarily deals with the interaction of this ingredients and its effect on long-run unemployment. The main hypothesis states that heterogeneous long-run unemployment trajectories can be accounted for a common technological shock that is propagated differently by differences in the credit market functioning. Empirical motivation suggests that a better credit market economy absorbs technological progress in such a way that unemployment is less affected. We find evidence that points out to this line at the aggregate level, by industry and in terms of flows. The vintage capital model with DMP frictions in both markets, provides an endogenous response of the economy to an exogenous capital-embodied technical change. In turn, this response depends on the state of credit market.

The key insight of the model is that economies with a worse credit market absorbs the technological shock mainly through the labor market tightness, primarily affecting the unemployment duration. Moreover, the higher the credit market frictions the higher the wage inequality, and more heterogeneous the vintage technology distribution which affect the output of the economy.

Chapter 2 analyzes the effects of non-homothetic preferences in a small open economy with nominal frictions in the labor market. I present a standard small open economy model with two sectors, tradable and non-tradable, and whose friction is represented by downward nominal wage rigidity. I extend this environment with non-homothetic preferences to study how heterogeneous income elasticity among goods affect the policy response in a boom-bust cycle of consumption. Deviate from the homotheticity assumption in preferences ensures that the marginal rate of substitution depends on the level of total consumption in addition to allowing the analysis of income effects. Both features that are especially important for the study of financial crises.

The punchline of the model is that non-homothetic preferences amplifies the adverse effects of a boom-bust cycle by linking the response of the economy to the level of aggregate consumption. In this sense, the policy response of the monetary and fiscal authority should be greater than the case of homothetic economy to counteract the amplification. The quantitative exercise suggests that to reach the first-best full-employment policy relative to the canonical case, the authority must to: (i) devalue the nominal exchange rate at an annual rate 40 % higher. In this way, (ii) the drop in the real exchange rate and real wages is about 37 % higher. With this policy, the economy ensures that the market clearing condition reaches full employment. (iii) During booms the use of the debt is lower and higher in the bust. A remarkable result in terms of long-run behavior is that (iv)) non-homothetic economy has a higher use of debt. In the case of a peg currency policy, (v) the second-best fiscal policy is higher in booms and lower in bust, to basically get the same unemployment pattern.

The goal of future work is to examine a more realistic feature of the model that include a trade off between policies. In the appendix I present the model when the authority objective is twofold, that is considering in the calibration that parameter $\phi = 1$, and the economy has debt in foreign currency. On the one hand, currency devaluation helps the full employment goal of the authority. On the other hand, it shrinks the borrowing capacity of the economy, as well as undermines the stabilizing objective of the monetary authority. Additionally, a labor sector reallocation can be included in the analysis by considering that tradable sector produce by a production function that need labor as input.

The contribution of this thesis lies in a policy view of the discipline. The two chapters answer different questions about economic aggregates based on the underlying idea: How the economy should be adjusted to reduce those negative effects derived from an exogenous shock, both in the long and short term.

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Annexes A

Annexed to Chapter 1

A.1. Useful Derivations

This appendix outlines the steps and details of the key equations of the main text.

A.1.1. Labor Market Surplus

Let first to get the value functions of the joint-venture $F_l(a)$ and $F_g(a)$ as (1.10)+(1.13) and (1.11)+(1.14) respectively and using the free entry condition in the credit market.

$$(r-g)F_l(a) = \text{máx}\left\{-\gamma + q(\theta)\left[F_g(a) - F_l(a)\right] - \delta F_l(a) + F_l'(a), 0\right\} \tag{A.1}$$

$$(r-g)F_g(a) = \text{máx}\left\{\mathrm{e}^{-a\Phi} - w(a) + \sigma\left[F_l(a) - F_g(a)\right] - \delta F_g(a) + F_g'(a), 0\right\} \tag{A.2}$$

Labor-market surplus between the joint-venture and the worker is defined as $S_L(a) \equiv F_g(a) + W(a) - F_l(a) - U$. Using equations (A.1), (A.2), (1.16) and (1.15) we get,

$$\begin{aligned}(r-g)S_L(a) = {} & \text{máx}\left\{\mathrm{e}^{-a\Phi} - w(a) + \sigma\left[F_l(a) - F_g(a)\right] - \delta F_g(a) + F_g'(a), 0\right\} \\ & + \text{máx}\left\{w(a) + \sigma\left[U - W(a)\right] + \delta\left[U - W(a)\right] + W'(a), (r-g)U\right\} \\ & - \text{máx}\left\{-\gamma + q(\theta)\left[F_g(a) - F_l(a)\right] - \delta F_l(a) + F_l'(a), 0\right\} \\ & - (r-g)U \end{aligned} \tag{A.3}$$

$$(r-g)S_L(a) = \text{máx}\left\{\mathrm{e}^{-a\Phi} + \gamma - (\sigma+\delta)\,S_L(a) - q(\theta)(1-\beta)S_L(a) - (r-g)U + S_L'(a), 0\right\} \tag{A.4}$$

Solving the differential equation

$$S_L(a) = \int_a^{\bar{a}} \mathrm{e}^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s-a)} \left[\mathrm{e}^{-s\Phi} + \gamma - (r-g)U\right] \mathrm{d}s \tag{A.5}$$

The maximal age at destruction $\bar{a}$ maximize the joint surplus between the joint-venture and the worker such that $\bar{a} = \arg \underset{\bar{a}}{\text{máx}} S_L(a)$.

Let $\hat{r} := r - g + \sigma + \lambda + q(\theta)(1-\beta)$. Computing the integral term, the above problem can be described as:

$$\begin{aligned}
S_L(a) &= \int_a^{\bar{a}} \mathrm{e}^{-\hat{r}(s-a)} \left[\mathrm{e}^{-s\Phi} + \gamma - (r-g)U\right] \mathrm{d}s \\
&= \mathrm{e}^{\hat{r}a} \int_a^{\bar{a}} \left[\mathrm{e}^{-s(\hat{r}+\Phi)} + \gamma \mathrm{e}^{-\hat{r}s} - (r-g)U\mathrm{e}^{-\hat{r}s}\right] \mathrm{d}s \\
&= \mathrm{e}^{\hat{r}a} \left[\frac{\mathrm{e}^{-s(\hat{r}+\Phi)}}{-(\hat{r}+\Phi)} + \gamma \frac{\mathrm{e}^{-\hat{r}s}}{-\hat{r}} - (r-g)U\frac{\mathrm{e}^{-\hat{r}s}}{-\hat{r}}\right]_a^{\bar{a}} \\
&= \mathrm{e}^{\hat{r}a} \left[\frac{\mathrm{e}^{-\bar{a}(\hat{r}+\Phi)} - \mathrm{e}^{-a(\hat{r}+\Phi)}}{-(\hat{r}+\Phi)} - ((r-g)U - \gamma)\left\{\frac{\mathrm{e}^{-\bar{a}\hat{r}} - \mathrm{e}^{-a\hat{r}}}{-\hat{r}}\right\}\right] \\
&= \left[\frac{\mathrm{e}^{-a\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}}{(\hat{r}+\Phi)} - ((r-g)U - \gamma)\left\{\frac{1 - \mathrm{e}^{-\hat{r}(\bar{a}-a)}}{\hat{r}}\right\}\right]
\end{aligned} \tag{A.6}$$

Note from the latter that evaluated at $\bar{a}$ implies that $S_L(\bar{a}) = 0$. To get $\bar{a}$ we derived the first-order condition of the problem stated above. $\frac{\partial S_L(a)}{\partial \bar{a}} = 0$

$$\begin{aligned}
\frac{\partial S_L(a)}{\partial \bar{a}} &= \left[\frac{\mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}(\hat{r}+\Phi)}{(\hat{r}+\Phi)} - ((r-g)U - \gamma)\frac{\mathrm{e}^{-\hat{r}(\bar{a}-a)}\hat{r}}{\hat{r}}\right] \\
&= \left[\mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi} - ((r-g)U - \gamma)\mathrm{e}^{-\hat{r}(\bar{a}-a)}\right] = 0
\end{aligned}$$

First-order condition implies

$$\mathrm{e}^{-\bar{a}\Phi} = (r-g)U - \gamma \tag{A.7}$$

Substituting (A.7) in (A.5) we get an endogenous expression for the labor market surplus:

$$S_L(a; \bar{a}, \theta) = \int_a^{\bar{a}} \mathrm{e}^{-(r-g+\sigma+\delta+q(\theta)(1-\beta))(s-a)} \left[\mathrm{e}^{-s\Phi} - \mathrm{e}^{-\bar{a}\Phi}\right] \mathrm{d}s \tag{A.8}$$

A.1.2. Some computation about labor market surplus

Expanding the integral of (A.8) using the definition of $\hat{r}$

$$
\begin{aligned}
S_L(a;\bar{a},\theta) &= \mathrm{e}^{\hat{r}a}\int_a^{\bar{a}}\left[\mathrm{e}^{-s(\hat{r}+\Phi)}-\mathrm{e}^{-\hat{r}s-\bar{a}\Phi}\right]\mathrm{d}s \\
&= \mathrm{e}^{\hat{r}a}\left[\frac{\mathrm{e}^{-s(\hat{r}+\Phi)}}{-(\hat{r}+\Phi)}-\frac{\mathrm{e}^{-\hat{r}s-\bar{a}\Phi}}{-\hat{r}}\right] \\
&= \mathrm{e}^{\hat{r}a}\left[\frac{\mathrm{e}^{-\bar{a}(\hat{r}+\Phi)}-\mathrm{e}^{-a(\hat{r}+\Phi)}}{-(\hat{r}+\Phi)}-\frac{\mathrm{e}^{-\bar{a}\hat{r}-\bar{a}\Phi}-\mathrm{e}^{-a\hat{r}-\bar{a}\Phi}}{-\hat{r}}\right] \\
&= \left[\frac{\mathrm{e}^{-a\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}}{\hat{r}+\Phi}-\frac{\mathrm{e}^{-\bar{a}\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}}{\hat{r}}\right]
\end{aligned}
\tag{A.9}
$$

A.1.3. Job Destruction

Using equation (1.15) and the surplus condition (1.18) we can express (A.7) as

$$
\mathrm{e}^{-\bar{a}\Phi} = b-\gamma+\beta\theta q(\theta)\int_0^{\infty} S_L(a;\bar{a},\theta)g(a)\mathrm{d}a \tag{JD}
$$

A.1.4. Joint-Venture Destruction

The joint-venture can be destroyed endogenously by obsolescence, i.e., when the surplus of an entrepreneur-financist pair goes to zero. The surplus in the credit market stage using the free entry condition is defined as $S_C(a) \equiv E_l(a) + B_l(a) = F_l(a)$. Therefore using equation (A.1) the credit market surplus solve the following

$$
S_C(a) \equiv F_l(a) = \int_a^{\hat{a}} \mathrm{e}^{-(r-g+\lambda)(s-a)}\left[-\gamma+q(\theta)(1-\beta)S_L(s;\bar{a},\theta)\right]\mathrm{d}s \tag{A.10}
$$

Where $\hat{a}$ is the age at which $S_C(a)$ is maximized. Let $\tilde{r} := r-g+\lambda$ and using the expand version of the labor market surplus (A.9) in (A.10)

$$
\begin{aligned}
S_C(a) &= \int_a^{\hat{a}} \mathrm{e}^{-(\tilde{r})(s-a)}\left[-\gamma+q(\theta)(1-\beta)\left\{\frac{\mathrm{e}^{-s\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-s)-\bar{a}\Phi}}{\hat{r}+\Phi}-\frac{\mathrm{e}^{-\bar{a}\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-s)-\bar{a}\Phi}}{\hat{r}}\right\}\right]\mathrm{d}s \\
&= \mathrm{e}^{a\tilde{r}}\left[\int_a^{\hat{a}} -\gamma\mathrm{e}^{-s\tilde{r}}+q(\theta)(1-\beta)\int_a^{\hat{a}}\mathrm{e}^{-s\tilde{r}}\left\{\frac{\mathrm{e}^{-s\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-s)-\bar{a}\Phi}}{\hat{r}+\Phi}-\frac{\mathrm{e}^{-\bar{a}\Phi}-\mathrm{e}^{-\hat{r}(\bar{a}-s)-\bar{a}\Phi}}{\hat{r}}\right\}\right]\mathrm{d}s \\
&= \mathrm{e}^{a\tilde{r}}\left[\int_a^{\hat{a}} -\gamma\mathrm{e}^{-s\tilde{r}}+q(\theta)(1-\beta)\int_a^{\hat{a}}\left\{\frac{1}{\hat{r}+\Phi}\left[\mathrm{e}^{-s(\tilde{r}+\Phi)}-\mathrm{e}^{s(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)}\right]+\frac{1}{\hat{r}}\left[\mathrm{e}^{-s\tilde{r}-\bar{a}\Phi}-\mathrm{e}^{s(\hat{r}-\tilde{r})-}\right.\right.\right.
\end{aligned}
$$

Solving the integrals by blocks:

$$
\begin{aligned}
[A] &\equiv -\gamma \int_{a}^{\hat{a}} \mathrm{e}^{-s\tilde{r}} \mathrm{d}s \\
&= -\gamma \frac{\mathrm{e}^{-\hat{a}\tilde{r}} - \mathrm{e}^{-a\tilde{r}}}{-\tilde{r}}
\end{aligned}
$$

And the second block

$$
\begin{aligned}
[B] \equiv\ & q(\theta)(1-\beta) \left\{ \frac{1}{\hat{r}+\Phi} \left[\frac{\mathrm{e}^{-\hat{a}(\tilde{r}+\Phi)} - \mathrm{e}^{-a(\tilde{r}+\Phi)}}{-(\tilde{r}+\Phi)} - \frac{\mathrm{e}^{\hat{a}(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)} - \mathrm{e}^{a(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)}}{\hat{r}-\tilde{r}} \right] \right\} \\
& - q(\theta)(1-\beta) \left\{ \frac{1}{\hat{r}} \left[\frac{\mathrm{e}^{-\hat{a}\tilde{r}-\bar{a}\Phi} - \mathrm{e}^{-a\tilde{r}-\bar{a}\Phi}}{\tilde{r}} - \frac{\mathrm{e}^{\hat{a}(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)} - \mathrm{e}^{a(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)}}{\hat{r}-\tilde{r}} \right] \right\}
\end{aligned}
$$

Therefore the surplus relationship in the credit market is as follow:

$$
S_C(a) = \mathrm{e}^{a\tilde{r}} \left[[A] + [B] \right] \tag{A.11}
$$

To obtain the maximal age of a vacant joint-venture $\hat{a}$, differentiate the latter equation w.r.t $\hat{a}$ and equating to zero to get the first-order condition:

$$
\frac{\partial S_C(a)}{\partial \hat{a}} = \mathrm{e}^{a\tilde{r}} \left[\frac{\partial A}{\partial \hat{a}} + \frac{\partial B}{\partial \hat{a}} \right] = 0 \tag{A.12}
$$

Note that

$$
\begin{aligned}
& \frac{\partial A}{\partial \hat{a}} = -\gamma \mathrm{e}^{-\hat{a}\tilde{r}} \\
\text{and} \quad & \frac{\partial B}{\partial \hat{a}} = q(\theta)(1-\beta) \left\{ \frac{1}{\hat{r}+\Phi} \left[\mathrm{e}^{-\hat{a}(\tilde{r}+\Phi)} - \mathrm{e}^{\hat{a}(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)} \right] - \frac{1}{\hat{r}} \left[\mathrm{e}^{-\hat{a}\tilde{r}-\bar{a}\Phi} - \mathrm{e}^{\hat{a}(\hat{r}-\tilde{r})-\bar{a}(\hat{r}+\Phi)} \right] \right\}
\end{aligned} \tag{A.13}
$$

Inserting in (A.12) and dividing by $\mathrm{e}^{-\hat{a}\tilde{r}}$ we get

$$\frac{\partial S_C(a)}{\partial \hat{a}} = \mathrm{e}^{a\tilde{r}} \left[-\gamma + q(\theta)(1-\beta) \underbrace{\left\{ \frac{\mathrm{e}^{-\hat{a}\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-\hat{a})-\bar{a}\Phi}}{\hat{r}+\Phi} - \frac{\mathrm{e}^{-\bar{a}\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-\hat{a})-\bar{a}\Phi}}{\hat{r}} \right\}}_{S_L(\hat{a};\bar{a},\theta,\Omega)} \right] = 0$$

$$\Rightarrow \quad \frac{\gamma}{q(\theta)} = (1-\beta) S_L(\hat{a};\bar{a},\theta;\Omega)$$

Therefore the destruction condition of a vacant Joint-Venture is given by

$$\frac{\gamma}{q(\theta)} = (1-\beta) \int_{\hat{a}}^{\bar{a}} \mathrm{e}^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s-\hat{a})} \left[\mathrm{e}^{-s\Phi} - \mathrm{e}^{-\bar{a}\Phi} \right] \mathrm{d}s \qquad \text{(JV-D)}$$

A.1.5. Job Creation

From equation (A.1) we can obtain the present value of a vacant joint-venture

$$F_l(a) = \int_a^{\hat{a}} \mathrm{e}^{-(r-g+\lambda)(s-a)} \left[-\gamma + q(\theta)(1-\beta) S_L(s;\bar{a},\theta) \right] \mathrm{d}s \qquad \text{(A.14)}$$

Since there exist free-entry in the credit market stage this implies that when the leading-age technology is installed the benefits are equal to entry cost. This mean that the job creation condition is given by $C(\phi) = F_l(0)$ as follow.

$$C(\phi) = \int_0^{\hat{a}} \mathrm{e}^{-(r-g+\delta)s} \left[-\gamma + q(\theta)(1-\beta) S_L(s;\bar{a},\theta,\Omega) \right] \mathrm{d}s \qquad \text{(JC)}$$

A.1.6. Vintage Distribution

To obtain matching probabilities in terms of endogenous variables, we apply the steps present in () . Let $\mu(t,a)$ measure of matched capital (firms or employment) of vintage a matched at t, and let $v(t,a)$ the measure of unmatched capital (joint-venture) at t. This measure evolves in a short time interval as

$$\mu(t+\Delta) = \mu(t,a-\Delta) - \mu(t,a-\Delta)\Delta\sigma - \mu(t,a-\Delta)\Delta\lambda + v(t,a-\Delta)\Delta q(\theta)$$

Subtracting $\mu(t,a)$ in both sides and dividing by Δ

$$\frac{\mu(t+\Delta) - \mu(t,a)}{\Delta} = \frac{\mu(t,a-\Delta) - \mu(t,a)}{\Delta} - (\sigma+\lambda)\mu(t,a-\Delta) + v(t,a-\Delta) q(\theta)$$

Taking $\text{lím}_{\Delta\to 0}$

$$\frac{\mathrm{d}\mu\left(t,a\right)}{\mathrm{d}t}=-\frac{\mathrm{d}\mu\left(t,a\right)}{\mathrm{d}a}-\left(\sigma+\lambda\right)\mu\left(t,a\right)+v\left(t,a\right)q\left(\theta\right)$$

We are interested in stationary distribution. Thus $\frac{\mathrm{d}\mu(t,a)}{\mathrm{d}t}=0$

$$\frac{\mathrm{d}\mu\left(t,a\right)}{\mathrm{d}a}=-\left(\sigma+\lambda\right)\mu\left(t,a\right)+v\left(t,a\right)q\left(\theta\right) \tag{A.15}$$

The measure of unmatched capital evolves

$$v\left(t+\Delta,a\right)=v\left(t,a-\Delta\right)+\mu\left(t,a-\Delta\right)\Delta\sigma-v\left(t,a-\Delta\right)\Delta\lambda-v\left(t,a-\Delta\right)\Delta q\left(\theta\right)$$

Applying the same steps as before we obtain:

$$\frac{\mathrm{d}v\left(t,a\right)}{\mathrm{d}a}=-\left(\sigma+q\left(\theta\right)\right)v\left(t,a\right)+v\left(t,a\right)\sigma \tag{A.16}$$

Consider the inflow of new joint ventures into the economy is the fraction of entrepreneurs who have success in meeting a financier $\phi z(\phi)\mathbb{E}\equiv v(0)$. This measure is constant so $\phi z(\phi)\mathbb{E}=\mu\left(a\right)+v\left(a\right)$ for all $a\in[0,\bar{a})$. Using this relation in (A.15) and (A.16)

$$\begin{aligned}\frac{\mathrm{d}\mu\left(t,a\right)}{\mathrm{d}a}&=-\left(\sigma+\lambda\right)\mu\left(t,a\right)+\left(\phi z(\phi)\mathbb{E}-\mu\left(a\right)\right)q\left(\theta\right)\\ \left(\sigma+\lambda+q\left(\theta\right)\right)&=q\left(\theta\right)\phi z(\phi)\mathbb{E}-\frac{\mathrm{d}\mu\left(t,a\right)}{\mathrm{d}a}\end{aligned} \tag{A.17}$$

Solving the differential equation (A.17) we get

$$\mu\left(a\right)=\frac{q\left(\theta\right)\phi z(\phi)\mathbb{E}}{\sigma+\lambda+q\left(\theta\right)}\left(1-\mathrm{e}^{-a(\sigma+\lambda+q(\theta))}\right) \tag{A.18}$$

Applying for (A.16) we obtain

$$v\left(a\right)=\frac{\phi z(\phi)\mathbb{E}}{\sigma+\lambda+q\left(\theta\right)}\left(\sigma+\lambda+q\left(\theta\right)\mathrm{e}^{-a(\sigma+\lambda+q(\theta))}\right) \tag{A.19}$$

Note that $\mu=\int_0^{\bar{a}}\mu\left(a\right)\mathrm{d}a$ and $v=\int_0^{\hat{a}}v\left(a\right)\mathrm{d}a$. Integrating (A.18) and (A.19) we get:

$$\mu = \frac{q\left(\theta\right)\phi z(\phi)\mathbb{E}}{\sigma+\lambda+q\left(\theta\right)}\left[\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q\left(\theta\right)}\right] \tag{A.20}$$

$$v = \frac{\phi z(\phi)\mathbb{E}}{\sigma+\lambda+q\left(\theta\right)}\left[\left(\sigma+\lambda\right)\hat{a} - \frac{q\left(\theta\right)\left(1-\mathrm{e}^{-\hat{a}(\sigma+\lambda+q(\theta))}\right)}{\sigma+\lambda+q\left(\theta\right)}\right] \tag{A.21}$$

Finally we get the densities for matched and vacant capital dividing (A.18) by (A.20) and (A.19) by (A.21), as follows.

$$\frac{\mu\left(a\right)}{\mu} = \frac{1-\mathrm{e}^{-a(\sigma+\lambda+q(\theta))}}{\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}}$$

$$\frac{v\left(a\right)}{v} = \frac{\sigma+\lambda+q\left(\theta\right)\mathrm{e}^{-a(\sigma+\lambda+q(\theta))}}{\left(\sigma+\lambda\right)\hat{a} + \frac{q(\theta)\left(1-\mathrm{e}^{-\hat{a}(\sigma+\lambda+q(\theta))}\right)}{\sigma+\lambda+q(\theta)}}$$

A.2. Proofs of Lemmas and Propositions

A.2.1. Lemma 1: Shape of job creation

Proof of Lemma 1.1. Upward sloping JC curve in $(\theta, \bar{a})$ space.

To prove that is an upward sloping curve, we establish that the RHS of (JC) is increasing in $\bar{a}$ and decreasing in θ.

- *Increasing in $\bar{a}$*: Note that the only function that depends on $\bar{a}$ is $S_L(a;\bar{a},\theta)$. Solving the integral associated we get,

$$S_L(a;\bar{a},\theta) = \left[\frac{\mathrm{e}^{-a\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}}{\hat{r}+\Phi} - \frac{\mathrm{e}^{-\bar{a}\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-a)-\bar{a}\Phi}}{\hat{r}}\right]$$

$$= \mathrm{e}^{-a\Phi}\left[\frac{1-\mathrm{e}^{-(\hat{r}+\Phi)(\bar{a}-a)}}{\hat{r}+\Phi}\right] - \mathrm{e}^{-\bar{a}\Phi}\left[\frac{1-\mathrm{e}^{-\hat{r}(\bar{a}-a)}}{\hat{r}}\right] \tag{A.22}$$

  Where as earlier $\hat{r} := r - g + \sigma + \lambda + q(\theta)(1-\beta)$ and $\tilde{r} := r - g + \lambda$. From the latter equation is clear that the surplus is an increasing function of $\bar{a}$. Thus, RHS of (JC) is increasing in $\bar{a}$.

- *Decreasing in θ*: Rewritten job creation condition using the expression in (1.28)[1],

$$C(\phi) = -\gamma\int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s}\mathrm{d}s + \int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s} q(\theta)(1-\beta)\left\{\int_s^{\bar{a}} \mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))(\tilde{a}-s)}\left[\mathrm{e}^{-\tilde{a}\Phi} - \mathrm{e}^{-\bar{a}\Phi}\right]\mathrm{d}\tilde{a}\right\}\mathrm{d}s \tag{A.23}$$

  Let $a_1 = \tilde{a} - s$. Note that if $\tilde{a} = s \Rightarrow a_1 = 0$ and $\tilde{a} = \bar{a} \Rightarrow a1 = \bar{a} - s$

$$C(\phi) = -\gamma\int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s}\mathrm{d}s + \int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s} q(\theta)(1-\beta)\left\{\int_0^{\bar{a}-s} \mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1}\left[\mathrm{e}^{-(a_1+s)\Phi} - \mathrm{e}^{-\bar{a}\Phi}\right]\mathrm{d}a_1\right\}\mathrm{d}s$$

$$C(\phi) = -\gamma\int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s}\mathrm{d}s + \mathrm{e}^{-\bar{a}\Phi}\int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s}\left\{\int_0^{\bar{a}-s} q(\theta)(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1}\left[\mathrm{e}^{\Phi(\bar{a}-a_1-s)} - 1\right]\mathrm{d}a_1\right\}\mathrm{d} \tag{A.24}$$

  Define $f(a_1,\theta) = q(\theta)(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta)a_1}$ and $h(a) = \mathrm{e}^{\Phi(\bar{a}-a_1-s)} - 1$. We have to demonstrate that $\frac{\partial}{\partial\theta}\left[\int_0^{\bar{a}-s} f(a_1,\theta)h(a_1)\mathrm{d}a_1\right] < 0$, so we proceed in two steps:

  a. To demonstrate that $\frac{\partial f(a_1,\theta)}{\partial\theta} < 0$ for $a_1 < a^* = 1/q(\theta)(1-\beta)$.

$$\frac{\partial f(a_1,\theta)}{\partial\theta} = q'(\theta)(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1} + q(\theta)(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta)a_1}(-q'(\theta)(1-\beta)a_1)$$

$$= \underbrace{q'(\theta)}_{<0}\underbrace{(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1}}_{>0}\underbrace{[1-q(\theta)(1-\beta)a_1]}_{>0 \text{ if } a_1<\frac{1}{q(\theta)(1-\beta)}} \tag{A.25}$$

[1] For a practical reason, in this part of the proof, consider $\hat{r} := r - g + \sigma + \lambda$

b. To demonstrate that $\frac{\partial}{\partial\theta}\left[\int_0^{\bar{a}-s} f(a_1,\theta)\mathrm{d}a_1\right] < 0$ for all $a_1 \in (0, \bar{a}-s)$

$$\begin{aligned}\int_0^{\bar{a}-s} q(\theta)(1-\beta)\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1}\mathrm{d}a_1 &= q(\theta)(1-\beta)\left\{\frac{\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))a_1}}{-(\hat{r}+q(\theta)(1-\beta))}\Big|_0^{\bar{a}-s}\right\} \\ &= q(\theta)(1-\beta)\left\{\frac{\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))(\bar{a}-s)}}{-(\hat{r}+q(\theta)(1-\beta))}\right\} \\ &= \frac{q(\theta)(1-\beta)}{\hat{r}+q(\theta)(1-\beta)}\left\{1-\mathrm{e}^{-(\hat{r}+q(\theta)(1-\beta))(\bar{a}-s)}\right\}\end{aligned}$$

Differentiating w.r.t. θ and let $\tilde{q}(\theta) = q(\theta)(1-\beta)$

$$\begin{aligned}&\frac{\partial}{\partial\theta}\left\{\int_0^{\bar{a}-s} f(a_1,\theta)\mathrm{d}a_1\right\} \\ &= \frac{\tilde{q}'(\theta)(\hat{r}+\tilde{q}(\theta)) - \tilde{q}'(\theta)\tilde{q}(\theta)}{[\hat{r}+\tilde{q}(\theta)]^2}\left[1-\mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}\right] + \frac{\tilde{q}(\theta)}{\hat{r}+\tilde{q}(\theta)}\left[-\mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}(-(\bar{a}-s)\tilde{q}'\right. \\ &= \underbrace{\frac{\tilde{q}'(\theta)\hat{r}}{[\hat{r}+\tilde{q}(\theta)]^2}\left[1-\mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}\right]}_{<0} + \underbrace{\frac{\tilde{q}(\theta)}{\hat{r}+\tilde{q}(\theta)}\left[-\mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}(-(\bar{a}-s)\tilde{q}'(\theta))\right]}_{<0} < 0\end{aligned} \tag{A.26}$$

So, let $a^* \equiv \frac{1}{q(\theta)(1-\beta)}$ as in (a) and note for simple inspection that $h(\cdot)$ is a positive and decreasing function of a_1

$$\frac{\partial}{\partial\theta}\int_0^{\bar{a}-s} f(a_1,\theta)h(a_1)\mathrm{d}a_1 = \int_0^{a^*} f_\theta(a_1,\theta)h(a_1)\mathrm{d}a_1 + \int_{a^*}^{\bar{a}-s} f_\theta(a_1,\theta)h(a_1)\mathrm{d}a_1 r \tag{A.27}$$

$$< \int_0^{a^*} f_\theta(a_1,\theta)h(a^*)\mathrm{d}a_1 + \int_{a^*}^{\bar{a}-s} f_\theta(a_1,\theta)h(a^*)\mathrm{d}a_1 \tag{A.28}$$

$$= h(a^+)\int_0^{\bar{a}-s} \underbrace{f_\theta(a_1,\theta)\mathrm{d}a_1}_{<0 \; by \; (b)} < 0 \tag{A.29}$$

The first inequality in (A.28) comes from part (a) of the proof stated in (A.25): $f_\theta < 0$ in $a_1 \in (0, a^*)$ and $h(\cdot)$ evaluated at a^* attain its lowest value. The second inequality in (A.29) comes from part (b). Since JC curve is increasing in $\bar{a}$ and decreasing in θ, it has a positive sloping in $(\theta, \bar{a})$ space ■.

Shape of JC when $\bar{a} \to \infty$

Let $\tilde{q}(\theta) \equiv q(\theta)(1-\beta)$; $\hat{r} \equiv r - g + \sigma + \lambda$; $\tilde{r} \equiv r - g + \lambda$ and solving the integral inside the brackets in equation (A.24)

$$
\begin{aligned}
C(\phi) &= -\gamma\frac{1-\mathrm{e}^{-\hat{a}\tilde{r}}}{\tilde{r}} + \mathrm{e}^{-\bar{a}\Phi}\int_0^{\hat{a}} \mathrm{e}^{-\tilde{r}s}\tilde{q}(\theta)\left\{\frac{\mathrm{e}^{\Phi(\bar{a}-s)} - \mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}}{\hat{r}+\tilde{q}(\theta)+\Phi} - \frac{1-\mathrm{e}^{-(\hat{r}+\tilde{q}(\theta))(\bar{a}-s)}}{\hat{r}+\tilde{q}(\theta)}\right\}\mathrm{d}s \\
&= -\gamma\frac{1-\mathrm{e}^{-\hat{a}\tilde{r}}}{\tilde{r}} + \mathrm{e}^{-\bar{a}\Phi}\int_0^{\hat{a}} \tilde{q}(\theta)\left\{\frac{1}{\hat{r}+\tilde{q}(\theta)+\Phi}\left[\mathrm{e}^{-s(\tilde{r}+\Phi)+\bar{a}\Phi} - \mathrm{e}^{s(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta))}\right]\right. \\
&\qquad\qquad \left. - \frac{1}{\hat{r}+\tilde{q}(\theta)}\left[\mathrm{e}^{-\hat{r}s} - \mathrm{e}^{s(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta))}\right]\right\}\mathrm{d}s \\
&= -\gamma\frac{1-\mathrm{e}^{-\hat{a}\tilde{r}}}{\tilde{r}} + \int_0^{\hat{a}} \tilde{q}(\theta)\left\{\frac{1}{\hat{r}+\tilde{q}(\theta)+\Phi}\left[\mathrm{e}^{-s(\tilde{r}+\Phi)} - \mathrm{e}^{s(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta)+\Phi)}\right]\right. \\
&\qquad\qquad \left. - \frac{1}{\hat{r}+\tilde{q}(\theta)}\left[\mathrm{e}^{-\hat{r}s-\bar{a}\Phi} - \mathrm{e}^{s(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta)+\Phi)}\right]\right\}\mathrm{d}s \\
&= -\gamma\frac{1-\mathrm{e}^{-\hat{a}\tilde{r}}}{\tilde{r}} + \tilde{q}(\theta)\left\{\frac{1}{\hat{r}+\tilde{q}(\theta)+\Phi}\left[\frac{\mathrm{e}^{-\hat{a}(\tilde{r}+\Phi)}-1}{-(\tilde{r}+\Phi)} - \frac{\mathrm{e}^{\hat{a}(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta)+\Phi)} - \mathrm{e}^{-\bar{a}(\hat{r}+\tilde{q}(\theta)+\Phi)}}{\tilde{q}(\theta)}\right]\right. \\
&\qquad\qquad - \frac{1}{\hat{r}+\tilde{q}(\theta)}\left[\frac{\mathrm{e}^{-\hat{r}\hat{a}-\bar{a}\Phi}-\mathrm{e}^{\bar{a}\Phi}}{-\hat{r}} - \frac{\mathrm{e}^{\hat{a}(\sigma+\tilde{q}(\theta))-\bar{a}(\hat{r}+\tilde{q}(\theta)+\Phi)} - \mathrm{e}^{-\bar{a}(\hat{r}+\tilde{q}(\theta)\cdot}}{\tilde{q}(\theta)}\right.
\end{aligned}
\tag{A.30}
$$

When $\bar{a} \to \infty$.

$$
C(\phi) = -\gamma\frac{1-\mathrm{e}^{-\hat{a}\tilde{r}}}{\tilde{r}} + \tilde{q}(\theta)\left\{\frac{1}{\hat{r}+\tilde{q}(\theta)+\Phi}\frac{1-\mathrm{e}^{-\hat{a}(\tilde{r}+\Phi)}}{\tilde{r}+\Phi}\right\} \tag{A.31}
$$

Shape of JC when $\theta \to \infty$

■ Return to lemma

A.2.2. Lemma 2: Shape of Joint Venture Destruction

Proof of Lemma 1.2.Shape of Joint Venture Destruction

Note from equation (JVD) can be written as

$$
\begin{aligned}
\frac{\gamma}{q(\theta)} &= (1-\beta)S_L(\hat{a};\bar{a};\theta) \\
&= (1-\beta)\left[\frac{\mathrm{e}^{-\hat{a}\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-\hat{a})-\bar{a}\Phi}}{\hat{r}+\Phi} - \frac{\mathrm{e}^{-\bar{a}\Phi} - \mathrm{e}^{-\hat{r}(\bar{a}-\hat{a})-\bar{a}\Phi}}{\hat{r}}\right]
\end{aligned}
$$

When $\bar{a} \to \infty$

$$
\begin{aligned}
&\frac{\gamma}{q(\theta)} = (1-\beta)\left[\frac{\mathrm{e}^{-\hat{a}\Phi}}{\hat{r}+\Phi}\right] \\
\Rightarrow &\frac{\gamma}{q(\theta)}\frac{r-g+\sigma+\lambda+q(\theta)(1-\beta)+\Phi}{1-\beta} = \mathrm{e}^{-\hat{a}\Phi} \\
\Rightarrow &\gamma\left[\frac{r-g+\sigma+\lambda+\Phi}{q(\theta)(1-\beta)}+1\right] = \mathrm{e}^{-\hat{a}\Phi} \\
\Rightarrow &\theta = q^{-1}\left\{\frac{\gamma}{\frac{\mathrm{e}^{-\hat{a}\Phi}}{\gamma}-1}\frac{r-g+\sigma+\lambda+\Phi}{1-\beta}\right\} && \text{(A.32)} \\
& && \text{(A.33)}
\end{aligned}
$$

■ Return to Lemma

A.2.3. Lemma 6: Vintage Heterogeneity

Proof of Lemma 1.6.Vintage Heterogeneity

Recall first the expression of vintage distribution:

$$
\frac{\mu(a)}{\mu} = \frac{1-\mathrm{e}^{-a(\sigma+\lambda+q(\theta))}}{\bar{a}-\frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} \qquad a \in [0,\bar{a})
$$

Note that both θ and $\bar{a}$ are function of $C(\phi)$. Since the maximal age of a job is given by $\bar{a}$ is clear that an increase in this variable increases the support of the distribution. Let us analyze what happens with a movement of θ. The proof consists in analyze the single-crossing properties of the vintage distribution respect of θ. Just for notation let define $\tilde{q}(\theta) \equiv \sigma+\lambda+q(\theta)$.

$$
\begin{aligned}
\frac{\partial^2\frac{\mu(a)}{\mu}}{\partial\theta\partial a} = \mathrm{e}^{-a\tilde{q}(\theta)}\left\{\overbrace{\frac{q'(\theta)\left[-\left(1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}\right)\right]}{\tilde{q}(\theta)^2}}^{(A)}+\overbrace{\frac{\bar{a}q'(\theta)\,\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}}^{(B)}\right\}\tilde{q}(\theta)+\overbrace{\frac{q'(\theta)\,\mathrm{e}^{-a\tilde{q}(\theta)}}{\bar{a}-\frac{1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}}}^{(C)}-\dots \\
\dots\underbrace{-\frac{aq'(\theta)\,\mathrm{e}^{-a\tilde{q}(\theta)}\tilde{q}(\theta)}{\bar{a}-\frac{1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}}}_{(D)}
\end{aligned}
$$

Considers that $q'(\theta)<0$. Thus, expressions in (A) and (D) are positive while (B) and $(C)$are negative. Let's first look at the term outside the bracket, $(C)$and (D)

$$-q'(\theta)\left[\frac{a\mathrm{e}^{-a\tilde{q}(\theta)}\tilde{q}(\theta)-\mathrm{e}^{-a\tilde{q}(\theta)}}{\bar{a}-\frac{1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}}\right]\iff -q'(\theta)\left[\frac{\mathrm{e}^{-a\tilde{q}(\theta)}\left(a\tilde{q}(\theta)-1\right)}{\bar{a}-\frac{1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}}\right]$$

That is, the expression is positive is if $q(\theta) > 1/a - (\sigma+\lambda)$. Since the age $a \in [0, \bar{a})$ and $(\sigma+\lambda)$ is a small number, this condition is satisfies for plausible values of the equilibrium. In words, this condition states that the joint venture requires a minimum hiring rate. Now turn to the term inside the bracket (A) and (B):

$$-q'(\theta)\left\{\frac{1-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}}{\tilde{q}(\theta)}-\bar{a}\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}\right\}\iff -q'(\theta)\left\{\frac{1}{\tilde{q}(\theta)}-\mathrm{e}^{-\bar{a}\tilde{q}(\theta)}\left[\frac{1}{\tilde{q}(\theta)}+\bar{a}\right]\right\}$$

Thus, the expression is positive if and only if $\mathrm{e}^{\bar{a}\tilde{q}(\theta)} \geq 1+\tilde{q}(\theta)\bar{a}$. Note that for a sufficiently small x, $\mathrm{e}^{x} \approx 1+x$ in neighbor of 0. So, the expression is positive and therefore $\frac{\partial^2 \frac{\mu(a)}{\mu}}{\partial\theta\partial a} > 0$ and satisfy the single-crossing property.

■ Return to Lemma 6

A.2.4. Lemma 7: Unemployment

Proof of Lemma 1.1.Unemployment

Let analyze the equation (1.34):

$$\frac{u}{1-u}=\frac{1}{\theta q(\theta)}\left(\sigma+\lambda+\frac{\mu(\bar{a})}{\mu}\right)$$

By lemma 1.5 we know that an increase in $c(\phi)$ reduces θ and raises $\bar{a}$.

$$\underbrace{\frac{\partial\frac{1}{\theta q(\theta)}}{\partial C(\phi)}\left(\sigma+\lambda+\frac{\mu(\bar{a})}{\mu}\right)}_{(+)}+\left(\underbrace{\underbrace{\frac{\partial\frac{\mu(\bar{a})}{\mu}}{\partial\theta}}_{(+)}\underbrace{\frac{\partial\theta}{\partial C(\phi)}}_{(-)}+\underbrace{\frac{\partial\frac{\mu(\bar{a})}{\mu}}{\partial\bar{a}}}_{(-)}\underbrace{\frac{\partial\bar{a}}{\partial C(\phi)}}_{(+)}}_{(-)}\right)\underbrace{\frac{1}{\theta q(\theta)}}_{(+)} \tag{A.34}$$

a) Showing that $\frac{\partial \frac{\mu(\bar{a})}{\mu}}{\partial \theta} > 0$,

$$\frac{\partial \frac{\mu(\bar{a})}{\mu}}{\partial \theta} = \frac{\overbrace{\bar{a}q'(\theta)\,\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}^{(-)}}{\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} + \cdots$$

$$+ \frac{\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\left\{\frac{\overbrace{q'(\theta)\left[-\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\right]}^{(+)}}{(\sigma+\lambda+q(\theta))^2} + \frac{\overbrace{\bar{a}q'(\theta)\,\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}^{(-)}}{\sigma+\lambda+q(\theta)}\right\}}{\left(\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}\right)^2} > 0 \tag{A.35}$$

To be positive it must be fulfilled that positive terms are higher than negative terms

$$\frac{\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\frac{q'(\theta)\left[-\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\right]}{(\sigma+\lambda+q(\theta))^2}}{\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} > \frac{\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\frac{\bar{a}q'(\theta)\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}}{\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} + \cdots$$

$$\cdots + \bar{a}q'(\theta)\,\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}$$

Rearranging terms

$$\frac{-q'(\theta)\left(1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)^2}{(\sigma+\lambda+q(\theta))^2} > \bar{a}^2 q'(\theta)\,\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))} \tag{A.36}$$

The above is always true considering that $q'(\theta) < 0$. So the left hand is positive while the right hand is negative.

b) Showing that $\frac{\partial \frac{\mu(\bar{a})}{\mu}}{\partial \bar{a}} < 0$

$$\frac{\partial \frac{\mu(\bar{a})}{\mu}}{\partial \bar{a}} = \frac{\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\,(\sigma+\lambda+q(\theta))}{\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} - \frac{\left(-\left[1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}\right]\right)^2}{\left(\bar{a} - \frac{1-\mathrm{e}^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}\right)^2} < 0 \tag{A.37}$$

To be negative it must be fulfilled that negative terms are higher than positive terms

$$
\begin{aligned}
\frac{e^{-\bar{a}(\sigma+\lambda+q(\theta))}\left(\sigma+\lambda+q\left(\theta\right)\right)}{\bar{a}-\frac{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} &< \frac{\left(-\left[1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right]\right)^2}{\left(\bar{a}-\frac{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}\right)^2} \\
e^{-\bar{a}(\sigma+\lambda+q(\theta))}\left(\sigma+\lambda+q\left(\theta\right)\right) &< \frac{\left(-\left[1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right]\right)^2}{\bar{a}-\frac{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}}{\sigma+\lambda+q(\theta)}} \\
e^{-\bar{a}(\sigma+\lambda+q(\theta))}\left(\sigma+\lambda+q\left(\theta\right)\right) &< \frac{\left(1+e^{-2\bar{a}(\sigma+\lambda+q(\theta))}-2e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)\left(\sigma+\lambda+q\left(\theta\right)\right)}{\bar{a}\left(\sigma+\lambda+q\left(\theta\right)\right)-\left(1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)} \\
3e^{-\bar{a}(\sigma+\lambda+q(\theta))}\left(\sigma+\lambda+q\left(\theta\right)\right) &< \frac{\left(1+e^{-2\bar{a}(\sigma+\lambda+q(\theta))}\right)\left(\sigma+\lambda+q\left(\theta\right)\right)}{\bar{a}\left(\sigma+\lambda+q\left(\theta\right)\right)-\left(1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)} \\
3e^{-\bar{a}(\sigma+\lambda+q(\theta))} &< \frac{\left(1+e^{-2\bar{a}(\sigma+\lambda+q(\theta))}\right)}{\bar{a}\left(\sigma+\lambda+q\left(\theta\right)\right)-\left(1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}\right)}
\end{aligned}
$$

The above is always true considering that the right hand has a 1 in the numerator, the expression $\exp(-\bar{a}(\sigma+\lambda+q(\theta)))$ is small and $\bar{a} \in (0, \infty)$.

c) To analyze the effect on unemployment let $\chi \equiv 1+\sigma\left(\frac{\bar{a}}{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}}-\frac{1}{\sigma+\lambda+q(\theta)}\right)$ in equation (1.35) in the main text.

$$
u = \frac{\chi}{\theta q\left(\theta\right)+\chi}
$$

We can express the change of unemployment to changes in $C(\phi)$ as follows,

$$
\begin{aligned}
\frac{\partial u}{\partial C\left(\phi\right)} &= \frac{\frac{\partial \chi}{\partial C(\phi)}\left(\theta q\left(\theta\right)+\chi\right)-\left(\frac{\partial \theta q(\theta)}{\partial C(\phi)}+\frac{\partial \chi}{\partial C(\phi)}\right)\chi}{\left(\theta q\left(\theta\right)+\chi\right)^2} \\
&= \frac{\frac{\partial \chi}{\partial C(\phi)}\theta q\left(\theta\right)-\frac{\partial \theta q(\theta)}{\partial C(\phi)}\chi}{\left(\theta q\left(\theta\right)+\chi\right)^2}
\end{aligned}
$$

We know that $\frac{\partial \theta q(\theta)}{\partial C(\phi)} < 0$ so $-\frac{\partial \theta q(\theta)}{\partial C(\phi)} > 0$. Let's analyze $\frac{\partial \chi}{\partial C(\phi)}$. A simple argument to state that $\frac{\partial \chi}{\partial C(\phi)} > 0$ is simply to note that χ acts as the inverse of $\frac{\mu(\bar{a})}{\mu}$. To see this, note that we can describe the fraction of endogenous job destruction as follows

$$
\frac{\mu\left(\bar{a}\right)}{\mu} = \frac{1}{\frac{\bar{a}}{1-e^{-\bar{a}(\sigma+\lambda+q(\theta))}}-\frac{1}{\sigma+\lambda+q(\theta)}} \tag{A.38}
$$

Compares equation (A.38) and the definition of χ . Therefore as is shown in equation (A.34) , since $\frac{\partial \frac{\mu(\bar{a})}{\mu}}{\partial C(\phi)} < 0$ it must follows that $\frac{\partial \frac{1}{1+\sigma\left(\frac{\mu(\bar{a})}{\mu}\right)}}{\partial C(\phi)} > 0$. Thus, immediately is true that $\frac{\partial u}{\partial C(\phi)} = \frac{\frac{\partial \chi}{\partial C(\phi)}\theta q(\theta) - \frac{\partial \theta q(\theta)}{\partial C(\phi)}\chi}{(\theta q(\theta)+\chi)^2} > 0$.

■ Return to Lemma 7

A.2.5. Lemma 8: Wage Inequality

Proof of Lemma 1.2.Wage Inequality

To proof that the maximal wage differential is increasing in $C(\phi)$ note first that the denominator in equation (1.37) is decreasing in $\bar{a}$. Next, observe that the labor market surplus function evaluated at $\bar{a}$ is as follows

$$S_L(0; \bar{a}, \theta) = \int_0^{\bar{a}} e^{-(r-g+\sigma+\lambda+q(\theta)(1-\beta))(s)} \left[e^{-s\Phi} - e^{-\bar{a}\Phi} \right] ds \tag{A.39}$$

The above equation is clearly increasing in $\bar{a}$. The higher $\bar{a}$ the higher the surplus. Since a decrease in θ only affects the effective discount rate through an increase in $q(\theta)$, the effect of $\bar{a}$ dominates. Since denominator is decreasing and numerator is increasing the max-min wage is increasing in $C(\phi)$.

■ Return to Lemma 8

Annexes B

Annexed to Chapter 2

This appendix contains the proofs of propositions reported in the paper.

B.1. Proofs of Lemmas

B.1.1. Lemma 1: Negative slope and convexity of non-homothetic demand of non-tradable

Proof of Lemma 2.1.Negative slope of non-homothetic demand of non-tradables

To prove that the curve is monotonically decreasing we have to state that $\frac{\partial P^{NH}}{\partial h} < 0$ given the fact that $P^H = \frac{1-a}{a}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\eta}(c)^{(1+\eta)(\varepsilon_N-\varepsilon_T)}$ is continuous in the domain of h.

$$\frac{\partial P^{NH}}{\partial h} = \frac{1-a}{a}(1+\eta)F'(h)\left(\frac{c^T}{F(h)}\right)^{1+\eta} c^{(\varepsilon_N-\varepsilon_T)(1+\eta)}\left[(\varepsilon_N-\varepsilon_T)\frac{\partial c}{\partial F(h)}c - \frac{1}{F(h)}\right] \tag{B.1}$$

Thus, the condition to establishes that $\frac{\partial P^{NH}}{\partial h} < 0$ is

$$\frac{1}{\varepsilon_N-\varepsilon_T} > \xi_{c,F(h)} \tag{B.2}$$

where $\xi_{c,F(h)} \equiv \frac{\partial c}{\partial F(h)}\frac{c}{F(h)}$ is the elasticity of total consumption respect the consumption of non-tradables such that,

$$\xi_{c,F(h)} = \frac{\eta(1-a)F(h)^{-\eta}}{a\left[\varepsilon_T(1+\eta)-1\right]c^{-(\varepsilon_N-\varepsilon_T)(1+\eta)}(c^T)^{-\eta} + (1-a)\left[\varepsilon_N(1+\eta)-1\right]} \tag{B.3}$$

. Condition in (B.2) is fulfilled for plausible parameterization.

To prove convexity we have to demonstrate that $\frac{\partial^2 P^{NH}}{\partial h^2} > 0$. For convenience rewrite (B.1) as follows,

$$\frac{\partial P^{NH}}{\partial h} = \frac{P^{NH}(1+\eta)}{F(h)} F'(h) \left[(\varepsilon_N - \varepsilon_T)\xi_{c,F(h)} - 1 \right] \tag{B.4}$$

Let define $\Phi \equiv \frac{P^{NH}(1+\eta)}{F(h)} > 0$ and $\Omega \equiv F'(h) \left[(\varepsilon_N - \varepsilon_T)\xi_{c,F(h)} - 1 \right] < 0$. Then to proof convexity we need that

$$\frac{\partial^2 P^{NH}}{\partial h^2} = \frac{\partial \Phi}{\partial h} \times \Omega + \frac{\partial \Omega}{\partial h} \times \Phi > 0 \tag{B.5}$$

Basically, we have to proof that $\frac{\partial \Phi}{\partial h} < 0$ and $\frac{\partial \Omega}{\partial h} > 0$

The first part:

$$\frac{\partial \Phi}{\partial h} = \frac{1-a}{a}(c^T)^{1+\eta} \frac{F'(h)}{F(h)^{3+\eta}} (c)^{(\varepsilon_N - \varepsilon_T)(1+\eta)} \left\{ (\varepsilon_N - \varepsilon_T)(1+\eta)\xi_{c,F(h)} - (2+\eta) \right\} \tag{B.6}$$

Since condition (B.2), we have that

$$(\varepsilon_N - \varepsilon_T)(1+\eta)\xi_{c,F(h)} < 1+\eta < (2+\eta) \tag{B.7}$$

which implies that $\frac{\partial \Phi}{\partial h} < 0$

The second part:

$$\frac{\partial \Omega}{\partial h} = F''(h) \left[(\varepsilon_N - \varepsilon_T)\xi_{c,F(h)} - 1 \right] + (\varepsilon_N - \varepsilon_T) \frac{\partial \xi_{c,F(h)}}{\partial h} F'(h). \tag{B.8}$$

Since $F(h)$ is a neoclassical production function that satisfies Inada conditions, it is enough to demonstrate that $\frac{\partial \xi_{c,F(h)}}{\partial h} > 0$. The condition for this to happen is

$$c^{(\varepsilon_N - \varepsilon_T)(1+\delta)} a \left[\varepsilon_T(1+\eta) - 1 \right] (c^T)^{-\eta} \left\{ (\varepsilon_N - \varepsilon_T)(1+\eta)\xi_{c,F(h)} \frac{1}{F(h)^{1+\eta}} - \eta \right\} > \eta(1-a) \left[\varepsilon_N(1+\eta) - 1 \right] \tag{B.9}$$

The expression inside brackets is positive given the condition in (B.2) and the fact that the labor force is bounded by 1. For the parameterization used this condition is satisfied. Then $\frac{\partial \xi_{c,F(h)}}{\partial h} > 0$ and the proof is complete.

■ Return to Lemma 1

B.1.2. Lemma 2: Properties of the non-homothetic demand curve of non-tradable

Proof of Lemma 2.2.Properties of the non-homothetic demand curve of non-tradables

To prove the lemma we analyze the slope of both curves at the common equilibrium point $(1, P)$. That is, we want to study the slope ratio among curves evaluated a the equilibrium point. The starting point is that both curves intersect at the point $(1, P)$. (we know that this point exist by by lemma (2.1). Let the slope of the non-homothetic curve as follows,

$$\frac{\partial P^{NH}}{\partial h}|_{(1,P)} = -P(1+\delta)\frac{F'(h)}{F(h)}\left[1-(\varepsilon_N-\varepsilon_T)\xi_{c,F(h)}\right] \tag{B.10}$$

Let slope of the homothetic curve,

$$\frac{\partial P^{H}}{\partial h}|_{(1,P)} = -P(1+\delta)\frac{F'(h)}{F(h)} \tag{B.11}$$

Dividing the above expressions

$$\frac{\frac{\partial P^{NH}}{\partial h}}{\frac{\partial P^{H}}{\partial h}} = 1-(\varepsilon_N-\varepsilon_T)\xi_{c,F(h)} < 1 \tag{B.12}$$

Since curves are downward sloping, convex and monotonically decreasing the proof is complete.

■ Return to Lemma 2

B.2. Foreign currency debt and $\phi \in (0,1)$

In this extension I consider that the traded output y_t^T is internationally exported at a price P_t^X. I assume that the law of one prices holds for the traded goods that are exported, so $P_t^X = \mathcal{E}_t P_t^{X^*}$. Here, $P_t^{X^*}$ represents the international price of the exported tradables expressed in terms of foreign currency. Denote the terms of trade of the economy as p_t^x which is defines as the export-to-import ratio,

$$p_t^x \equiv \frac{P_t^{X^*}}{P_t^{T^*}} = \frac{P_t^X}{P_t^T} = \frac{P_t^X}{\mathcal{E}_t} \tag{B.13}$$

Recall the equation (2.3) in the main text with $\tau_t^y = \tau_t^{\mathrm{d}} = 0$. In this environment this equation is,

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T \mathrm{d}_t = P_t^X y_t^T + W_t h_t + \Pi_t + \frac{P_t^T \mathrm{d}_{t+1}}{1 + r_t} \tag{B.14}$$

Using equilibrium conditions, $c_t^N = A_t F(h_t)$ and the profits definition in the non-tradable sector[1] $\Pi_t = P_t^N A_t F(h_t) - W_t h_t$ and dividing by P_t^T the market clearing equilibrium in tradable sector is

$$c_t^T = p_t^x y_t^T + \frac{\mathrm{d}_{t+1}}{1 + r_t} - \mathrm{d}_t \tag{B.15}$$

To analyze the trade off in the exchange rate policy I assume that the economy is subject to an external borrowing constraint as follows,

$$\mathrm{d}_{t+1} \leq \kappa p_t^x y_t^T \tag{B.16}$$

Equation (B.16) can be interpreted as a collateral constraint where $\kappa > 0$ is a parameter that regulate the borrowing limit of the economy. The idea is that in the event of default, lenders can recoup a fraction κ of the tradable output. Note that if the authority devaluates the currency with the aim to reduce unemployment, also reduce the borrowing limit and also the ability to allocate intratemporal consumption. To see this note by equation (B.13) that p_t^x is smaller as $\mathcal{E}_t$ increases. The assumption is that individuals see the constraint but do not internalize it in aggregate terms.

[1] A more complex alternative is to rule out the existence of endowments in the tradable sector and establish a production function that uses labor, as the non-tradable sector. Although this alternative allows a richer analysis in terms of the reallocation of resources between sectors, it makes a bit more complex the solution.