

Table of content

1. Introduction	1
2. Basics of physical cosmology	3
2.1. Cosmological principle	3
2.2. Gravitation	4
2.2.1. Special relativity	4
2.2.2. General relativity	4
2.2.3. The FLRW metric	5
2.2.4. Einstein equations	5
2.3. An expanding universe	6
2.3.1. First evidence: The Hubble-Lemaître Law	6
2.3.2. Redshift, scale factor and Hubble rate	8
2.3.3. Friedmann Equations	10
2.4. Λ CDM model of the Universe	11
2.4.1. Cosmic inventory	11
2.4.2. Observational evidence	13
2.5. Cosmic distances	14
3. Cosmic structure formation	17
3.1. Preliminaries	18
3.1.1. Equations of motion	18
3.1.2. Density fluctuations	19
3.1.3. Initial Conditions	20
3.2. Physics of the cosmic structure	20
3.2.1. Linear perturbation theory	20
3.2.2. Correlation functions	21
3.2.3. Power spectrum	21
3.2.4. The cosmic density PDF	22
3.3. Observations of the cosmic structure	24
3.3.1. Galaxy density field	25
3.3.2. Galaxy bias	25
3.3.3. Higher-order statistics	26
3.3.4. Gravitational lensing	27
3.4. Sky surveys	29
3.4.1. The Dark Energy Survey	29
3.5. Density split statistics	30
3.5.1. Methodology	30

3.5.2. Shot-noise model 1: joint log-normal distribution	32
3.5.3. Shot-noise model 2: super-Poissonianity	32
3.5.4. Priors assigned to the shot-noise models	33
3.5.5. Results from DSS	33
4. Basics of probabilities	35
4.1. The basics of probabilities	36
4.1.1. Outcome spaces and events	36
4.1.2. Axioms of probability	36
4.2. Types of probabilities	38
4.2.1. Discrete probability distributions	38
4.2.2. Continuous probability distributions	39
4.3. Interpretation of the probabilities	41
4.3.1. Frequentist probability	41
4.3.2. Bayesian probability	42
4.3.3. Which probabilistic approach should we use?	44
5. Statistical inference	45
5.1. Law of large numbers	45
5.2. Point estimators	46
5.2.1. Method of moments	46
5.2.2. Method of least squares	48
5.2.3. Maximum likelihood method	50
5.2.4. Maximum a posteriori	51
5.2.5. Other point estimators	52
5.3. Interval estimators	52
5.3.1. Confidence Intervals	53
5.3.2. Credible Intervals	54
5.3.3. Other interval estimators	55
5.4. Discussion on selecting estimators	56
5.5. Sampling methods	57
5.5.1. Markov chains Monte Carlo	57
5.5.2. Sequential Monte Carlo	63
5.6. Errors and uncertainties	66
5.6.1. Measuring the systematic errors	70
6. Revision of the German tank problem	74
6.1. Presuppositions	74
6.2. Frequentist approach	75
6.3. Bayesian approach	78
6.4. Discussions and conclusion	82
7. Informed Total-Error-Minimizing (ITEM) Priors: Interpretable cosmological parameter constraints despite complex nuisance effects	84
7.1. Abstract	84
7.2. Introduction	85
7.3. Methodology	88

7.3.1.	Step 1: Obtain a set of data vectors that represent plausible realizations of the nuisance effect	90
7.3.2.	Step 2: Devise a family of nuisance priors among which to choose the ITEM priors	91
7.3.3.	Step 3: Determine the ITEM prior	92
7.4.	ITEM Priors in the Density Split Statistics	95
7.4.1.	Density Splits Statistics	96
7.4.2.	Modeling of the nuisance quantities in DSS	96
7.4.3.	Obtaining the ITEM priors on the Density Splits Statistics	100
7.5.	Discussions and conclusion	107
8.	Conclusions	110
9.	Glossary	112
Bibliography		114
Annexed A. Implementing the Sobol's quasi-random sampling		143
Annexed B. Extensions of the tank problem		144
B.1.	Cases with a larger sample	144
B.2.	Other prior distributions for the Bayesian approach	145
Annexed C. Summary of the DSS and its Stochasticity models		147
C.1.	α Model: Parametric model for non-Poissonianity	148
C.2.	r Model: correlation $r \neq 1$ between galaxy density and matter density	149