

# Table of Content

<b>Introduction</b>	<b>1</b>
Summary of the Chapters . . . . .	3
<b>1 Sample-Driven Optimal Stopping: From the Secretary Problem to the i.i.d. Prophet Inequality</b>	<b>5</b>
1.1 Summary of Results and Overview of the Chapter . . . . .	7
1.2 Preliminaries . . . . .	10
1.2.1 $p$ -DOS with Known Values. . . . .	10
1.2.2 $p$ -DOS with Adversarial Values. . . . .	11
1.2.3 Dependent Sampling . . . . .	12
1.3 Known Values . . . . .	12
1.3.1 Linear Programming Formulation . . . . .	13
1.3.2 Limit Problem . . . . .	17
1.3.3 Structure of Optimal Solution . . . . .	18
1.3.4 Finding the Optimal Thresholds . . . . .	21
1.4 Adversarial Values . . . . .	24
1.4.1 Factor Revealing LP . . . . .	24
1.4.2 The Limit Problem and its Solution . . . . .	26
1.4.3 Solving for Different Values of $p$ . . . . .	27
1.4.4 Connection Between the Sampling Models . . . . .	36
1.5 On Multiple-Choice $p$ -DOS Problems . . . . .	38
1.5.1 Relation Among Guarantees for Different $p$ on a Given Independence System $(S, \mathcal{I})$ . . . . .	40
1.5.2 Better Guarantees for $p$ -DOS on Special Types of Independence Systems	41
1.5.3 Limiting Problem as $p \rightarrow 1$ and Consequences for the Matroid Secretary Problem (MSP) . . . . .	43
1.6 Proofs of Section 1.3 . . . . .	45
1.6.1 Coupling Argument for Monotonicity . . . . .	45
1.6.2 Convergence of $\mathbf{E}(\text{ALG}_N^*(Y))$ to $\text{CLP}_p$ . . . . .	45
1.6.3 Monotonicity of $\sum_{j=\ell}^k \binom{j-1}{\ell-1} (1-t)^{j-\ell} t^\ell$ . . . . .	46
1.6.4 Concavity of $F_k(t)$ in Each Variable . . . . .	47
1.7 Proofs of Section 1.4 . . . . .	48
1.7.1 Derivation of $\text{SDLP}_{h,N}$ . . . . .	48
1.7.2 Solution of $\text{SDRP}_p$ for $p < 1/e$ . . . . .	49
1.7.3 Details on Numerical Bounds . . . . .	50

1.7.4	Proof of Theorem 1.13 . . . . .	51
<b>2</b>	<b>Selecting the Best with Samples</b>	<b>56</b>
2.1	Summary of Results . . . . .	58
2.2	Further related literature . . . . .	60
2.3	Model and definitions . . . . .	61
2.4	The Optimal Algorithm . . . . .	62
2.5	Computation of the time thresholds . . . . .	73
2.6	Numerical experiments . . . . .	74
2.6.1	Experimental setup . . . . .	76
2.6.2	Experimental results . . . . .	76
<b>3</b>	<b>Fairness in Online Selection: The Multi-Color Secretary Problem</b>	<b>78</b>
3.1	Summary of Results . . . . .	79
3.2	Illustrative Example . . . . .	80
3.3	Related Work . . . . .	80
3.4	Preliminaries . . . . .	82
3.5	Optimal Online Algorithm . . . . .	83
3.5.1	The Algorithm . . . . .	83
3.5.2	Competitive Ratio . . . . .	84
3.6	Fairness . . . . .	90
3.7	Empirical Evaluation . . . . .	93
3.8	Sample-Driven Multi-Color Secretary Problem . . . . .	94
3.9	Conclusion and Open Problems . . . . .	96
<b>4</b>	<b>Optimal Item Pricing in Online Combinatorial Auctions</b>	<b>99</b>
4.1	Context and Related Work . . . . .	100
4.2	A Technical Highlight and Additional Results . . . . .	101
4.3	Model . . . . .	102
4.4	Main Result: A $1/(d + 1)$ -approximation for Random Valuations . . . . .	103
4.5	Efficient and Sample-Based Computation . . . . .	107
4.5.1	Proof of Theorem 4.7 . . . . .	108
4.6	Deterministic Single-minded Valuations . . . . .	113
4.6.1	Matching in Graphs: $d = 2$ . . . . .	113
4.6.2	Hypergraph Matching: $d > 2$ . . . . .	115
4.7	Conclusion and Future Directions . . . . .	117
4.8	Bounds Using an Optimal Solution of LP . . . . .	118
	<b>Bibliography</b>	<b>121</b>