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# MÉTODO DE OPTIMIZACIÓN DE ENVOLVENTE ECONÓMICA SUAVIZADA PARA BLOCK Y PANEL CAVING

TESIS PARA OPTAR AL GRADO DE MAGISTER EN MINERÍA

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# RESUMEN

La formulación de un proyecto minero busca obtener una respuesta robusta que recomiende o no ejecutar el proyecto. Dentro de esta formulación se ubica la planificación estratégica donde se definen las reservas mineras contenidas en la envolvente económica. Específicamente para los métodos block y panel caving, la envolvente económica se calcula en dos pasos: el primer paso consiste en determinar un nivel (en la coordenada z) para ubicar el footprint, y luego se diseña la forma del footprint.

La envolvente económica debe respetar ciertos criterios que restringen su forma. Los principales criterios evitan los eventos geomecánicos no deseados. El radio hidráulico crítico asegura un área mínima para propiciar el hundimiento. La forma perimetral suavizada del footprint evita la concentración excesiva de esfuerzos inducidos en un área acotada. También, una diferencia controlada de altura entre columnas contiguas aminora la dilución de las leyes y mejorar el control de tiraje entre puntos de extracción. Estas restricciones hacen que la forma de la envolvente sea suavizada. Pero hasta el momento, no hay una metodología que integre estas restricciones dentro del calculo de evolvente y que además entregue un resultado económicamente optimo y con forma suavizada a la vez. Generalmente requieren que un ingeniero defina la forma del footprint o que un algoritmo suavice automáticamente una envolvente pre calculada, resultando subóptimo.

En este trabajo se desarrolla un método de optimización de envolvente económica con forma suavizada. Para ello se extiende el problema de límite de pit final, agregando nuevos conjuntos de arcos de precedencias e invirtiendo el rajo. El algoritmo desarrollado para el método open pit originalmente considera restricciones de forma mediante arcos de precedencia entre los bloques para imponer el ángulo de talud global a la envolvente resultante. Para el método propuesto en este trabajo, las precedencias de ángulo de talud se usan para controlar la diferencia de altura entre columnas contiguas. La precedencia vertical entre bloques permite definir columnas de extracción. Además, se agrega un nuevo conjunto de precedencias en un plano horizontal para suavizar el contorno del footprint. la envolvente económica es considerada como un rajo invertido, donde la superficie ahora será la base de la envolvente económica (footprint) y los bloques en el fondo del rajo serán los bloques en lo más alto de las columnas de extracción.

El método propuesto fue puesto a prueba en un modelo de bloques que contiene un cuerpo mineral poco regular. El resultado del método propuesto se contrasta con un resultado obtenido con un caso base inspirado en el método más implementado en la industria minera, conocido como PCBC. El método PCBC logra un resultado con valor máximo, pero no suavizado, requiriendo que se eliminen columnas de la envolvente. Este paso procura conservar la mayor cantidad de columnas de extracción, pero respetando los criterios de diseño, resultado un valor económico subóptimo. En cambio, el método propuesto logra un resultado suavizado y con un valor final levemente mayor.

En conclusión, se logra demostrar que el algoritmo de límite de pit final puede ser adaptado para resolver envolventes económicas optimizadas para los métodos block y panel caving, siempre y cuando se formule correctamente las restricciones de precedencia. Esto permite ampliar el uso del algoritmo para otros métodos de explotación.

# ABSTRACT

The formulation of a mining project seeks to obtain a robust response that recommends or not to execute the project. Strategic planning defines the mineral reserves contained in the economic envelope. Specifically for the block and panel caving methods, the economic envelope is calculated in two steps: the first step consists of determining a level (in the z coordinate) to locate the footprint, and then the shape of the footprint is designed.

The economic envelope must respect certain criteria that restrict its shape. The main criteria to avoid undesirable geomechanical events. The critical hydraulic radius ensures a minimum area to promote subsidence. The smoothed perimeter shape of the footprint avoids excessive concentration of induced stresses in a limited area. Also, a controlled height difference between close draw columns minimizes grade dilution and improves draw control between drawpoints. These constraints produce a smooth economic envelope. But so far, there is no methodology that integrates these constraints into the envelope calculation and delivers an economically optimal and smoothed shape at the same time. They generally require an engineer to define the footprint shape or an algorithm to automatically smooth a pre-calculated envelope, resulting in a suboptimal result.

In this work, an economic smoothed envelope optimization method is developed. The final pit boundary problem is extended by adding new sets of precedence arcs and inverting the pit. The algorithm developed for the open pit method originally considers shape constraints through precedence arcs between blocks to impose a global slope angle to the resulting envelope. For the method proposed in this work, the slope angle precedence arcs are used to control the height difference between contiguous columns. The vertical precedence arcs between blocks allows the definition of draw columns. In addition, a new precedence arcs set is added in a horizontal plane to smooth the footprint contour. The economic envelope is considered as an inverted pit, where the surface of the ultimate pit corresponds to the base of the economic envelope (footprint) and the blocks at the bottom of the ultimate pit corresponds to the top blocks on the draw columns.

The proposed method was tested on a block model containing an unregular ore body. The result of the proposed method is contrasted with a result obtained with a base case inspired by the most widely implemented method in the mining industry, known as PCBC. The PCBC method achieves a result with maximum value, but not smoothed, requiring draw columns to be removed from the envelope. This step seeks to conserve as many draw columns as possible, but respecting the design criteria, resulting in a suboptimal economic value. In contrast, the proposed method achieves a smoothed result with a slightly higher final value.

In conclusion, it is possible to demonstrate that the final pit limit algorithm can be adapted to solve optimized economic envelopes for block and panel caving methods, as long as the precedence restrictions are correctly formulated. This allows extending the use of the algorithm for other mining methods.

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Dedico este trabajo a mis hijas y esposa, quienes me acompañaron y brindaron amor a lo largo de este proceso. También, a mi bisabuelo Guillermo Maya, el cual me relato sus vivencias en la minería y me motivo a seguir su grandiosa carrera.

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# 1. Introducción

## 1.1 Motivación de la tesis

El primer paso de la planificación de una mina a explotar por block o panel caving es definir la envolvente económica, la cual equivale al recurso que será arrancado por la actividad minera. Esta es una etapa crítica en el proceso de planificación, ya que condiciona el resto de las etapas.

Debido a la naturaleza del hundimiento se debe considerar un conjunto de parámetros geotécnicos y comportamientos físicos para lograr una planificación robusta. En el caso de una envolvente económica de los métodos block y panel caving la forma del nivel basal (footprint) y la relación entre alturas de las columnas de extracción contenidas dentro del footprint deben respetar ciertas restricciones de forma. Estas restricciones de forma tienden a suavizar la figura de la envolvente económica.

El algoritmo mayormente utilizado para definir envolvente económica requiere que un ingeniero diseñe la forma final del footprint, logrando una envolvente económica subóptima. Para la definición de envolvente económica, al igual que en otros métodos, existen varios algoritmos que ayudan al ingeniero a encontrar una solución. Dentro de las alternativas de cálculo, ninguna ofrece una solución óptima y que a su vez respete las restricciones de forma.

Es por ello que este trabajo se enfoca en proponer una metodología para calcular una envolvente económicamente optima y que incorpore explícitamente las restricciones de forma dentro de este cálculo.

## 1.2 Objetivos

### 1.2.1 Objetivo general

Desarrollar un método que permita calcular una envolvente económica optimizada y que respete restricciones de forma para los métodos block y panel caving.

### 1.2.2 Objetivos específicos

- Identificar aquellos criterios geomecánicos que siempre sean considerados en un proyecto minero explotado por el método block o panel caving, independiente de las condiciones del macizo rocoso.

- Desarrollar o adaptar una metodología que permita incorporar los criterios geomecánicos identificados para obtener una envolvente económica con valor óptimo y forma suavizada.

- Cuantificar el rendimiento de la metodología propuesta, contrastando su resultado con el método PCBC.

## 1.3 Alcances

- Solo se desarrollará la etapa de estimación de reservas, sin considerar la etapa previa de estimación de recursos ni la etapa posterior de estrategia de extracción.

- El método de estimación de envolvente económica implementado en el caso base será inspirado en el propuesto por Diering (2000).

- La etapa de diseño del footprint, del método implementado en el caso base, solo considerará criterios para el diseño del contorno del footprint, dejando fuera el diseño de la red de galerías que conforman el nivel de hundimiento y nivel de extracción.

- La columna de extracción de mineral será discretizada a una columna conformada por bloques del modelo de bloques. Quiere decir que una columna contiene bloques con la misma coordenada X e Y.

## 1.4 Estado del arte

Actualmente existen varios métodos para definir una envolvente económica para los métodos block y panel caving. El más utilizado en la industria minera es el propuesto por Diering (2000) nombrado como PCBC y que además esta implementado en un software comercial de planificación minera. El método PCBC, en una primera instancia, ayuda a definir un nivel para ubicar el footprint y luego redefine la altura de las columnas de extracción de mineral. Este método requiere que el ingeniero escoja finalmente el nivel y el diseño del contorno del footprint, siendo una etapa manual que no garantiza que el valor económico obtenido sea el óptimo.

El método PCBC no es el único implementado en un software comercial, Arancibia & Soto (2018) exponen un nuevo módulo de planificación llamado CaveLogic$^{TM}$ el cual calcula una envolvente económica de manera automática. En primera instancia, el algoritmo identifica un nivel para ubicar el footprint, luego agrega columnas con valor máximo, sin restricciones de forma, y finalmente se suavizan mediante algoritmo genético. El resultado es una envolvente suavizada, pero con un valor económico subóptimo.

Noriega et al. (2020) desarrollan una metodología de dos pasos mediante modelos matemáticos BIP. El primer paso busca un nivel para ubicar el footprint y agrega columnas de extracción que maximizan los recursos mineros. El segundo paso redefine la altura de las columnas y decide qué columna es incorporada en la envolvente. Este método logra envolventes aceptables en cuerpos minerales con forma suavizada.

Otros métodos de estimación de envolvente para block y panel caving se basan en el cálculo de límite de pit final para open pit. Elkington et al. (2012) maximiza la cantidad de metal sobre la ley de corte para definir un cuerpo 3D basado en restricciones de forma. Estas restricciones de forma tratan de abordar los criterios de diseño, pero no se da información sobre el cómo formularlas. Luego, Vargas et al. (2015) y Julio et al. (2015) utilizan como base el cálculo de un pit final invertido que considera ángulos de talud verticales. Pero el enfoque de ambos trabajos es distinto al de obtener una envolvente económica optima y suavizada. Vargas et al. (2015) evalúan la variabilidad del tamaño de las reservas y el nivel de footprint según la incertidumbre geológica, mientras que Julio et al. (2015) buscan una estimación aproximada de la mejor ubicación de una mina subterránea por block/panel caving en el contexto de transición de minería cielo abierto a subterránea.

Finalmente, otro método, más bien para redefinir envolvente económica, es presentado por Rubio (2002) que incorpora el concepto de costo de oportunidad para calcular la altura de extracción optima, pero primero se debe contar con una envolvente económica y definir algunos parámetros.

Los métodos actuales para calcular una envolvente económica en los métodos de extracción block y panel caving requieren más de un paso, además no garantizan que el valor económico sea optimo ni que la forma sea operativamente valida. El método propuesto en este trabajo aborda este problema, incorporando explícitamente restricciones de forma en el calculo de envolvente optimizada.

## 1.5 Estructura de la tesis

El presente trabajo inicia con el capítulo de introducción, que explica el problema a resolver, el objetivo general y específicos, además de los alcances y una revisión crítica del estado del arte. Finalmente contiene la bibliografía utilizada en este capítulo.

El segundo capítulo contiene el artículo científico " A fast method to find smooth economic envelopes for block and panel caving mines”. Este artículo fue enviado a Resources Policy (ver Anexo B).

El siguiente capítulo contiene las conclusiones generales del trabajo, una sección de trabajos futuros y otra de recomendaciones.

Finalmente se encuentran los anexos A y B. El anexo A contiene el C.V. del estudiante y el anexo B contiene el documento que acredita el estado de publicación del articulo científico.

# 2. A fast method to find smooth economic envelopes for block and panel caving mines

**Estado documento:** En revisión

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**3 ideas claves**

- *Optimización de envolvente económica*
- *Restricciones de forma mediante arcos de precedencia*
- *Métodos de explotación block caving y panel caving*

**Contribución de los autores:**

Francisco identifica el problema a resolver, hace revisión del estado del arte, implementa el método propuesto en el trabajo, realiza experiencia numérica, como también el análisis de resultados y conclusiones. Nelson ayuda a proponer una solución al problema, además de apoyar en la redacción de resumen, introducción, metodología y experiencia numérica. Gonzalo aporta a la introducción y el estado del arte. René aporta en la introducción.

## 2.1 Abstract

The first step in the planning of a mine is the determination of the economic envelope, which is the volume that maximizes the total economic value and aims to encapsulate the whole size of the mine; thus, it has a considerable impact in the design and planning of the mining project.

The most common approach to calculate an economic envelope in block and panel caving is to fix the floor and then select the set of columns in the block model that have a positive economic value. This method is very fast and effective but may generate footprint that are difficult to design and operate as they do not consider any geometrical constraint of the shape of the optimal envelope. In this paper, we present a novel method that it incorporates geometrical constraints related to the connectivity and smoothness of the outline of the envelope, and the continuity and smoothness of

the columns that are obtained. Our method is based on the same algorithm used to compute the ultimate pit, i.e., the economic envelope of open pit mines. Therefore, it is very efficient and ensures to find the optimal solution for the set of constraints that are considered.

We test the method in a mine, comparing the economic envelopes of standard approach and our proposed method and show that the geometry obtained are operational and generate economic value that are close to the on calculated using the standard approach. That is, the loss of value due to the geometrical constraints is small compared to the gains in design and operability.

## 2.2 Introduction

Panel and block caving are massive underground mining methods. Panel and block caving are massive underground mining methods. After the undercutting of the orebody, the rock mass caves and ore flow to drawpoints due to gravity. This allows for higher productivities and lower operation costs compared to other underground methods.

Figure 1 presents the main elements of a panel/block caving mine. As the figure shows, extraction of the material is performed in drawpoints situated in the production level, which is located below the mineral reserve. The ore is then transported to underneath transportation levels for its extraction outside of the mine.

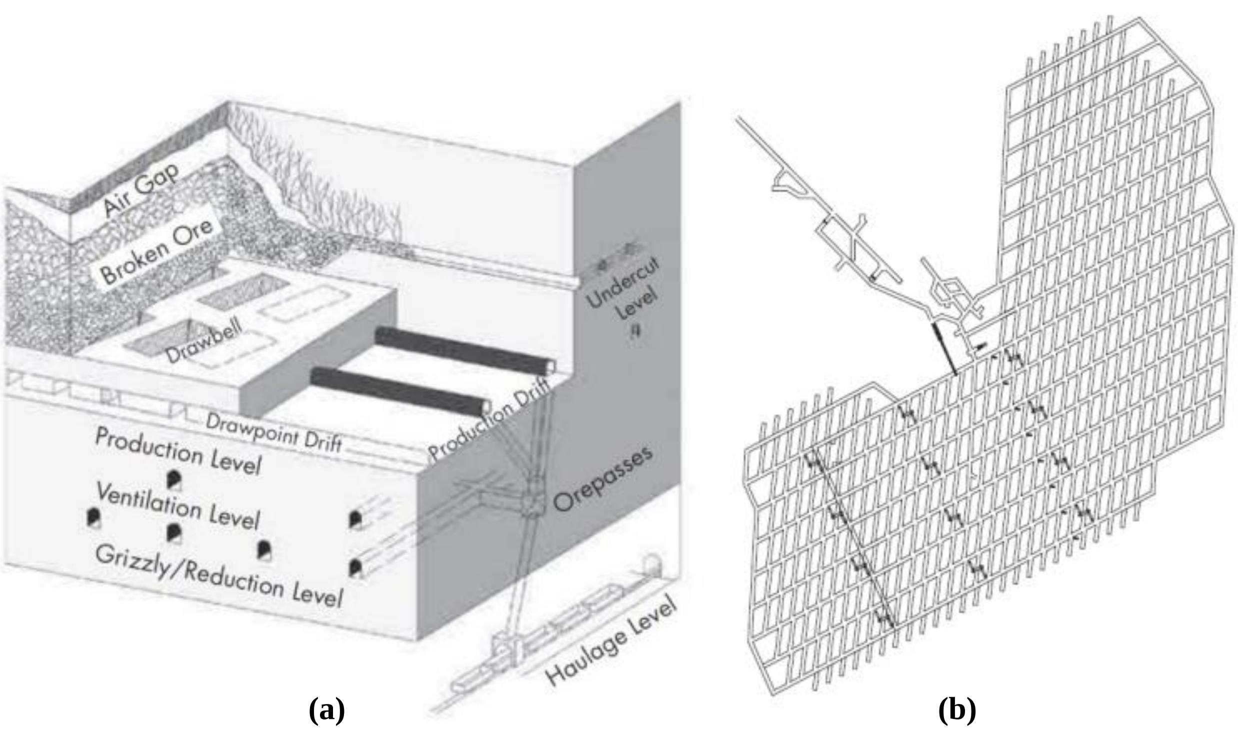


Figure 1 – Main elements of the layout of a panel caving mine (Brannonet al., 2011). a) Necessary infrastructure for the extraction method. b) A footprint layout on draw level.

The first step in planning a panel or block caving mine is determining its economic envelope, which can be defined as the set of blocks that maximizes the economic value fulfilling operational constraints given by the mining system. This is a critical step in the planning process, as it determines all the other stages.

The determination of the economic envelope in a panel or block caving mine can be decomposed into selecting the elevation of the footprint, determining the footprint outline, and limiting the height of the columns of mineral to be extracted. This is in fact the general procedure implemented in current software solutions.

The resulting footprint must comply with several geotechnical and geometrical considerations (Figure 2). First, the footprint must exclude portions that do not comply with a minimum critical hydraulic radius that facilitates a steady caving process. This radius is commonly estimated through Laubscher's stability graph and depends on the adjusted rock mass rating of the orebody (Laubscher, 1993, 1994, 2000).

Also, the footprint outline must be smooth to avoid stress concentrations in particular zones of the mine. This induced stress (or abutment stress) are generated during cave propagation in the caving front (Rojas et al., 2000; Quiñones et al., 2014; Shen et al., 2016; Viegas et al., 2018; Xia et al., 2021; Fan et al., 2020; Simanjuntak et al., 2020) or during broken column draw in the extraction level (Orellana et al., 2014; Shea et al., 2018; Sahupala et al., 2008; Kamp, 2022). Cave geometry, cave front shape and extraction level design -along with undercutting rate and strategy, and geomechanical support design- are critical parameters to control the induced stress in the mine infrastructure during the undercutting process (Brown, 2007; Holder et al., 2020; Simanjuntak et al., 2020; Landeros, 2022; de Beer, 2022; Cuello & Newcombe 2018). High induced stresses have generated damage, loss of reserves, additional development and fortification costs, decreases in productivity, and delays in several caving operations such as DOZ (Sahupala et al., 2008), Esmeralda (Orellana et al., 2014), Henderson (Shea et al., 2018), Premier (Barlett et al., 2000) and New Afton (Kamp, 2022).

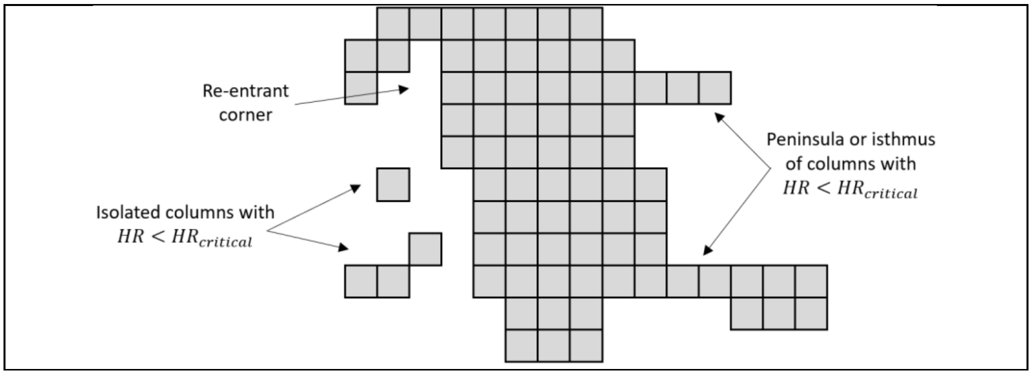


Figure 2. Features to avoid in the form of a footprint (plan view).

To obtain a favourable caving shape and decrease the effects of the abutment stress in the production level, the current practice in block caving optimization is to manually smooth the footprint outline. This process is usually made by the mining engineer and has received very little attention in the literature. Some examples of this procedure can be found in several works in block/panel caving optimization (Gantumur et al., 2016; Villa et al., 2022; Villa, 2014; Noriega et al., 2018) but the criteria used to define the outline are not discussed.

Another relevant geometric property of the economic envelope is a regular height of draw for close columns (Diering, 2000, 2004). This property facilitates an even draw during the mining operation. An even draw is desirable since enables the interaction of drawpoints which is a key phenomenon to control dilution entry (Castro & Paredes, 2014) and has been integrated into the mixing algorithm proposed by (Laubscher, 1994, 2000). Besides dilution control, there are different operational problems related to irregular draw policies, such as high-induced stress and hang-ups (Rubio & Diering, 2004; Nezhadshahmohammad et al., 2019; Laubscher, 1994, 2000; Brown, 2007). In particular, in presence of fine material, irregular draw can increase the mud-rush potential and/or inrush of fine material (Laubscher, 2000; Butcher et al., 2005; Hekmat et al., 2018; Castro et al., 2018). In the extraction level, damage during ore draw can occur due to stagnant zone in broken material, uneven draw, slow draw rate, or draw strategy where the extraction direction converges to a small area (Richardson, 1981; Pierce, 2019; Castro et al., 2020). In traditional caving envelope optimization, a post-processing step is used to regularize uneven draw columns (Diering, 2000, 2004) with a maximum difference in height between neighbors. For instance, El Teniente mine imposes a maximum difference of 40 meters between columns to avoid geomechanical and operational problems derived from uneven draw policies (Bustamante et al., 2021).

Given the importance of these geometrical properties in cave mining, in this work we propose a novel methodology to calculate an operational economic envelope. Our method includes constraints to obtain a smooth footprint outline and a regular caving profile in terms of the heigh of draw of adjacent columns. To the best of our knowledge, this is the first optimization model for block and panel caving mines that explicitly include these geometric constraints in the formulation and can be solved efficiently with an off-the-shelve solver.

## 2.3 Literature Review

In this section we review the relevant for two relevant topics for our work: methods to obtain an economic envelope in block/panel caving and modelling of geometrical properties in mining optimization.

### 2.3.1 Economic Envelope Optimization in Block/Panel Caving

Several works have proposed methodologies to find the economical envelope in block and panel caving mines. (Diering 2000, 2010), proposed an iterative algorithm that finds an optimal footprint elevation, with a manual smoothing procedure to obtain a regular outline and a final post-processing step to obtain an even height of draw. This algorithm is implemented in Geovia PCBC, and since it serves as comparison for our work, its description is deferred to Section 2.4.2.

Rubio (2002) proposed an algorithm to redefine the mining reserves by optimizing the height of draw. From an initial footprint shape, elevation, and mine design, the draw for each column is optimized considering the opportunity cost and the opening of new drawpoints according to a predefined undercutting direction.

Arancibia & Soto (2018) proposed a new long-term planning module for Maptek Vulcan called CaveLogic$^{TM}$. This module, based on a genetic algorithm, defines the footprint elevation and shape automatically. They include two geometrical requirements for the footprint: a regular outline and a smooth height of draw. A regular outline is achieved by a marching squares algorithm (Lorensen & Cline, 1987) and a smooth height of draw is obtained using image processing and geomorphology principles. However, there are not further comments on the actual implementation of these algorithms to solve the problem.

A novel methodology was proposed by (Noriega et al., 2018; Noriega et al., 2020). Since the actual profit perceived by the extraction of each drawpoint depends on the mining period, the authors aim to integrate the production scheduling problem into the economic envelope optimization. A two-step process is implemented. The first step uses an integer program to aggregate blocks into production units in the extraction level, and into mining units across the column defined for each production unit. This aggregation process reduces the number of variables for the second step. The second model, derived from (Nezhadshahmohammad et al., 2017), defines a production schedule considering capacity, mining advancement, continuous and even draw, maximum draw rates, among other constraints. As a result, the algorithm can quantify the value of the economic envelope considering a potential production schedule.

The idea of approximating the economic envelope of panel and block caving mines by means of an inverted open pit has also been used. Elkington et al. (2012) proposed that slope constraints can model the height of draw difference between adjacent columns. Julio et al. (2015) used inverted ultimate pits to have a rough estimation of the best location of a panel/block caving mine, in the context of transitioning from open pit to underground mining. Vargas et al. (2015) also approximates the economic envelope as an inverted pit, in this case to evaluate the variability the on the footprint size and elevation with regards to geological uncertainty. However, because the emphasis of those works was different, both works use the approximation as a pit as an estimation of the size and value of the envelope, i.e., they do not model the connectivity or smoothness of the solutions

### 2.3.2 Geometric properties in mining optimization

A common method to model geometric properties in mining optimization is to impose precedence constraints between blocks. For instance, the ultimate pit must satisfy a slope angle which is modelled by precedence arcs for a given set of angles and directions. The algorithms proposed by Lerchs & Grossmann (1965) and Hochbaum (Hochbaum & Orlin, 2013; Hochbaum, 2001, 2008) exploit the mathematical structure of the optimization model -with precedence arcs as the only constraint- to solve large instances of the ultimate pit problem (Morales et al., 2022). These precedence constraints are also ubiquitous in production scheduling optimization to control the mining sequence fulfilling the slope constraint in every period (Jélvez et al., 2020; Chicoisne et al., 2012; Osanloo et al., 2008; Nancel-Penard et al., 2022).

Beyond the slope angle constraint, other works have included additional geometric properties to the ultimate pit problem. For instance, Cullenbine et al., (2011) impose a precedence constraint to obtain a partial operational pit bottom. Nancel-Penard & Morales (2022) included several geometric constraints to obtain an operational pit's bottom. Bai et al. (2018) proposed an algorithm to obtain pushbacks with geometrical constraints applying several morphological operators.

In underground methods, precedence constraints are also used to model geometric properties. For instance, in Sublevel Stoping optimization, Bai et al. (2013) aim to determine the optimal shape of an underground stope. They transform the traditional cartesian coordinate system of the block model into a cylindrical coordinated system. This allows them to impose precedence constraints from the outer layer of the stope to a vertical raise. This transformation results in an ultimate pit problem which can be solved efficiently. Nelis et al. (2016) adds additional precedence constraints to improve the shape and geomechanical stability of the resulting stopes. We believe that our proposed method could be used to emulate their approach, without the need to change the coordinated system.

The importance of maintaining an even draw has been recognized in the production scheduling and draw control optimization for block and Panel Caves (Diering, 2012; Rahal et al., 2003; Smith & Rahal, 2001). In these works, the objective is to define a draw strategy that maximizes NPV and complies with several constraints such as maximum number of active drawpoints, maximum number of opened drawpoints, mining direction, continuous draw of opened points, among others. Even draw is commonly imposed as a maximum difference in draw tonnage or extraction height between adjacent drawpoints (Dirkx et al., 2019; Nezhadshahmohammad et al., 2017; Noriega et al., 2020), which turns into a smooth caving profile. Precedence constrains are also used to control the mining advancement direction and shape of the undercut front (Pourrahimian et al., 2013; Khodayari & Pourrahimian 2019) to avoid stress concentrations in the production level (Simanjuntak et al., 2020; Cuello & Newcombe, 2018; Beard & Brannon, 2018; Purba & Moss, 2020)

In this work we impose precedence constraints to model the geometric properties of an operational caving envelope. Given the geomechanical constraints of a block or panel cave, three sets of precedence arcs are implemented: vertically, to control the connectivity within each column; diagonally, to model a regular height of draw; and horizontally, to model a smooth footprint outline. The proposed optimization model and the strategy to impose these constraints are detailed in Section 2.4.3.

## 2.4 Methodology

This section presents the details of the different algorithms used for the computation of economic envelopes. We start describing the economic evaluation of individual blocks, including Laubscher's vertical mixing method which we used for diluting the grades. Then, we present the footprint finder algorithm and our proposed method. Finally, we indicate how these algorithms are utilized to produce several envelopes for comparison.

The proposed methodology computes an economic envelope by applying a special instance of the ultimate pit, which is constructed to obtain a smooth envelope for block and panel caving. In the proposed method, the pit is inverted so that the surface of the ultimate pit corresponds to the floor of the caving envelope, and the bottom of the pit corresponds to the highest blocks to be extracted in the caving envelope. Blocks with the same $x, y$ coordinates located between the floor and ceiling of the caving envelope are considered a *caving column*.

Notice that we assume that the level of the floor is given in advance. This assumption is not an issue, because the method is very fast; thus, many potential levels can be evaluated quickly.

### 2.4.1 Economic evaluation of individual block

We adhere to the method used by PCBC software (Diering, 2000, 2010) and follow the steps depicted in Figure 3, which shows a block $i$ at the different stages of its evaluation. First, the in-situ grades of the blocks are adjusted by a dilution method. Second, the new grades are used to determine a nominal economic value. Then, draw rates are used to estimate an extraction period. Finally, the economic value is computed by discounting the opportunity cost. The details of each step follow.

Figure 3 - Steps for economic evaluation of blocks. $g_i$ represents the grade of block, $w_i$ is the tonnage of the block, $\tilde{g}_i$ is the grade corrected by dilution, $\tilde{v}_i$ is the nominal economic value of the block. $t_i$ is the time required to extract the block at given vertical draw rates. Finally, $v_i$ is the adjusted economic value of the block.

**Laubscher method for vertical dilution.** The method proposed by Laubscher superimposes a deformed block column on top of the in-situ block column. Deformed blocks influence the grade of the in-place block by area weighting. The main parameter of this method is the percentage of dilution entry. The lower its value, the larger the dilution effects in block grades.

**Nominal economic value of a block.** The nominal economic value is computed using the following formula

$$\tilde{v}_i = pR\tilde{g}_i w_i - cw_i$$

Where $p$ is the price (USD/ton), $R$ is the metallurgical recovery (percentage), $\tilde{g}_i$ is the diluted grade of the block (percentage), $w_i$ is it tonnage (ton), and $c$ (USD/ton) is the total cost of mining and processing, without considering mining developments.

**Extraction period of a block.** To compute the updated economic value $v_i$, it is necessary to know the extraction period of the block. For this, a draw rate $d$ (t/m2/day) is used. Thus, if the blocks have an area $a$, then the time required to extract the $j$ the block in a column is $w_i/da$ and the time (in days) needed to reach block $i$ starting from the floor of the column is

$$t_i = \sum_{j<i} w_i$$

**Discounted economic value.** Finally, the discounted economic value of the block is calculated from the nominal value and using a daily discount rate $r$ and the extraction period $t_i$.

$$v_i = \frac{\tilde{v}_i}{(1+r)^{t_i}}$$

Once the blocks in the model have been evaluated, we consider two routines to compute the economic envelope: the PCBC Algorithm and "Smooth Envelope Optimizer".

### 2.4.2 PCBC algorithm to compute the economic envelope

Given a block $i$ in the floor, let us denote as $i+d$ the block that is located We implemented the PCBC algorithm as described in Diering (2000, 2010). The algorithm starts with a set of blocks with economic values, a $z$ coordinate of the floor of the envelope, a maximum column height $H$ and a development cost $D$.

The economic $v_i$ were generated as described in Section 3.1, i.e., applying a dilution algorithm and considering the opportunity cost due to maximum draw rates.

exactly $d$ levels above $i$.The algorithm works as follows. For each $i$ in the floor:

1. The algorithm computes the cumulative values $V_i(d) = \sum_{j=0}^{d} v_{i+j}$ for $d = 0, 1, \ldots, H-1$
2. The algorithm calculates $d^*$ such as $V_i(d^*)$ is maximum.
3. If $V_i(d^*) - D > 0$, then the blocks $i, i+1, \ldots, i+d^*$ belong to the economic envelope (and blocks $i+d^*+1, \ldots, i+H-1$ do not).

The procedure ensures that the set of selected blocks in a column is connected in the vertical direction. Moreover, it makes sure that the economic value of each column is not only positive, but enough to pay its development costs.

### 2.4.3 Smooth envelope optimization

Our method constructs an auxiliary instance of the ultimate pit problem, in which the pit is inverted so that the surface of the pit corresponds to the floor of the block/panel caving economic envelope.

In the ultimate pit problem, there are two main elements: the block model and overall slope angles, which are represented as arcs that encode precedence constraints. If $i, j$ are two blocks, then an arc $i \rightarrow j$ is used to encode that $j$ must be extracted if block $i$ is extracted. Thus, in the case of open pit mining, an arc $i \rightarrow j$ is created if block $j$ lies within the overall slope angle of block $i$.

We adapt the block model and precedence arcs to be used for block and panel caving. For the blocks, we keep their coordinates and use the same economic values utilized in the PCBC algorithm. Moreover, we subtract the development cost $D$ necessary to extract a mineral column from the block located at its base.

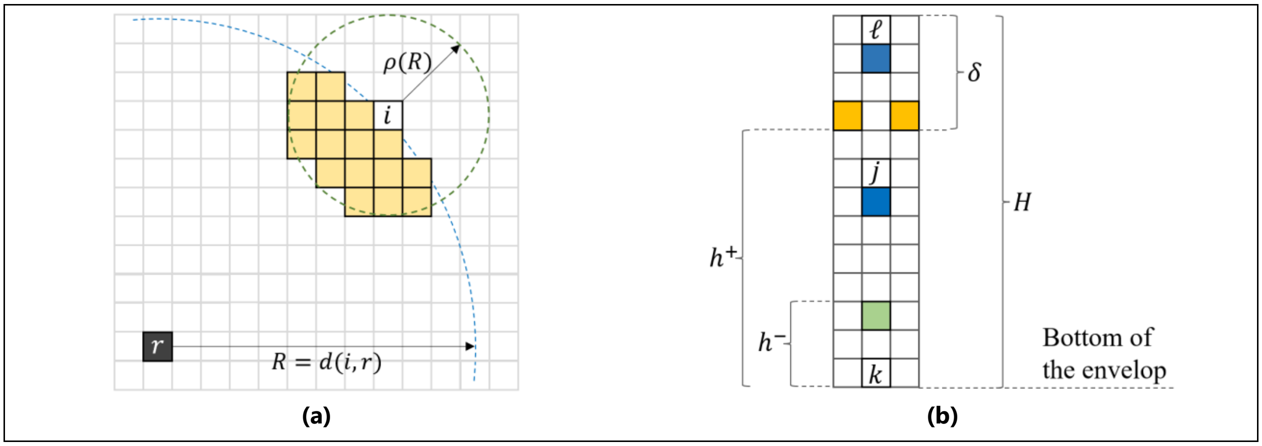
In terms of the precedence arcs, we separate these in three groups, described next.

**Horizontal arcs.** These are used for controlling the horizontal connectivity and smoothness of the economic envelope. To construct these precedence arcs, we need to introduce some notation and concepts.

For blocks $i, j$ in the floor of the envelope, we denote as $d(i, j)$ the Euclidean distance between these blocks and denote the disk of center $i$ and radius $r$ as $D(i, r) = \{j : d(i, j) \leq r\}$.

We consider also $R \subset F$ be a set of blocks called "initial points" and denote as $r_i \in R$ the initial point that is nearest to $i$ and $d_i = d(i, r_i)$. We introduce a "shape function" $\rho(\cdot)$, that depends on a distance. Therefore, if $i, j$ are blocks in the floor, Arc $i \rightarrow j$ is created if $j \neq i$ and $j$ is in the intersection of the disks $D(c_i, d_i)$ and $D(c_i, \rho(d_i))$. If several initial points exist, we consider the closest initial center to create the arcs. In this case, the envelope may consist of at most $|R|$ connected parts. Conversely, to ensure that the economic envelope is fully connected it suffices to select a single initial point.

Figure 4.a illustrates the concepts introduced above in the case of one initial point $r$ (in black) and an arbitrary block $i$, which is located at a distance $R$ from $r$. The predecessors of block $i$ are painted in yellow and correspond to those in the intersection of disk $D(r, R)$, marked in blue, and $D(i, \rho(R))$, marked in green. From a topological point of view, $\rho(R)$ controls the number of nodes and their arcs within an acyclic directed graph.



(a) (b)

Figure 4 – Conceptual description of horizontal and vertical precedences arcs. a**)** Plan view of precedences arcs for the blocks at the bottom of the envelope: An *initial point* $r$ (black), a block $i$ (white), and the horizontal predecessors of $i$ (yellow). b) Side view of vertical and diagonal precedences for connectivity and vertical smoothing of the envelope. Block $j$ is any block in the column (except the one at the bottom) and has a vertical predecessor colored blue. Block $\ell$ is a block above the interaction height $h+$. It has a vertical predecessor (in blue), and diagonal precedessors (orange), which are those located $\delta$ meters below and in neighboring columns. Block $k$ is block at the bottom and has one predecessor (green block).

**Vertical arcs**. Arcs are used to ensure the vertical connectivity and the minimum height of the columns that are selected for extraction.

The vertical connectivity is guaranteed by the creation of arc $i \rightarrow j$ if block $j$ is located immediately below block $i$. (The only exception for this is if $i$ is in the floor of the envelope).

We also consider a *minimum column height* $h^-$. That is, we enforce that any column selected as a part of the envelope must be at least $h^-$ meters high. This is achieved by creating a precedence arc $i \rightarrow j$ between a block $i$ in the floor and the block $j$ located exactly $h^-$ meters above, in the same column.

An example of vertical arcs is shown in Figure 4.b, with the vertical predecessor of a block $j$ (not in the bottom) colored in blue, and the vertical predecessor of block $k$ at the bottom colored in green.

**Diagonal arcs.** These arcs are used to ensure the vertical smoothness of the economic envelope. For this, we consider a *maximum interaction height* $h^+$ and a maximum difference $\delta$ and require that any two selected columns that are next to each other and have a height of at least $h^+$ must have a difference in height of at most $\delta$ meters.

To model this, an arc $i \rightarrow j$ is added if (a) $i$ and $j$ are both above $h^+$, (b) they belong to neighbor columns and (c) $j$ is $\delta$ meters below $i$. This is illustrated Figure 2 (Right), which presents a column and two others next to it. Diagonal predecessors of $j$ are colored in orange. (In this example, we have $\delta$ corresponding to the height of three blocks).

### 2.4.4 Economic envelopes

Using the algorithms described in the previous section, we produced four different economic envelopes for comparison. These are summarized in Figure 5, which represents the steps required to generate the envelopes. Boxes represent steps in the computation and they also specify the type of agent in parenthesis: “User” steps are manual processes or decisions made by the mine planner, while “Algorithm” steps are calculated by a computer algorithm. The green icons indicate the  our different envelopes.

**PCBC Raw Envelope.** This envelope is ideal from the economic point of view, but not necessarily apt for designing purposes, i.e., not operational. We obtain it by applying the PCBC method directly. (See 2.3.1 for a detailed description of the algorithm.)

**PCBC Smoothed Envelope.** This envelope is obtained manually. For this, the user must consider the design parameters and the PCBC Raw Envelope and manually selects the columns to be included in the footprint, to comply with the geometrical constraints necessary for an operational block/panel cave. Additionally, diagonal constraints are applied to smooth the column height. A version of this process is described in Diering (2000).

**Hybrid Smooth Envelope**. This envelope is generated by the proposed algorithm using the PCBC Raw Envelope as an input. The user specifies a single initial point within the Raw Envelope and $\rho(d_i)$. Then our proposed algorithm computes a smooth envelope. This process is no fully automated.

**Automated Smooth Envelope.** This economic envelope is determined by applying the algorithm for all possible initial points and then selecting the one that produces the highest economic value. This process does not require the PCBC Raw Envelope as an input.

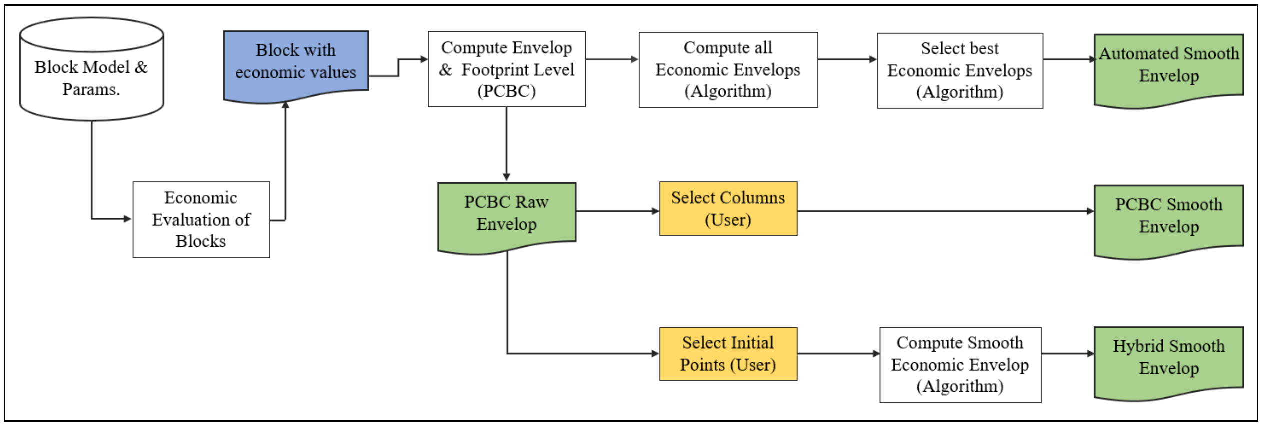


Figure 5 - Procedures for generating economic envelopes. Document shapes represent data or intermediate results. (Green documents are envelopes for comparison). Boxes represent manual or algorithmic steps (orange steps are manual, executed by the user).

## 2.5 Numerical experiences

In this section, we introduce the datasets (block models) and numerical experiments.

### 2.5.1 Block models and technical parameters

Table 1 summarizes the most relevant properties of the "D" block model used in the numerical experiments. It corresponds to a copper porphyry.

Table 1 - Summary of block models “D”

| Block model | “D” |
|---|---|
| Dimensions (NX x NY x NZ) | 46 x 34 x 27 |
| Block Sizes (DX x DY x DZ) | 25 x 25 x 15 |
| Number of Blocks (#) | 16,439 |
| Average Copper Grade (%) | 0.585 |

The technical parameters used to evaluate the blocks and determine the envelope are presented in Table 2.

Table 2 - Design and Planning Parameters

| Block model | “D” |
|---|---|
| $H$, Maximum column height (m) | 345 |
| $h^-$, Minimum column height (m) | 150 |
| $h^+$, Maximum interaction height (m) | 240 |
| Metallurgical Recovery (%) | 82 |
| Drawrate (t/m$^2$/day) | 0.84 |
| Percentage of Entry of Dilution (%) | 70 |
| Hydraulic Radius (m) | 25 |
| Mining Cost (USD/ton) | 9.5 |
| Processing Cost (USD/ton) | 16 |
| Development Cost (USD/m$^2$) | 5,800 |
| Discount Rate (%) | 12 |
| Price (USD/lb) | 2.15 |

### 2.5.2 Calibration of parameters

This set of experiments is designed to show the sensibility of the results of the algorithms in terms of the shape function $\rho(\cdot)$ and the maximum interaction height $h^+$. The values selected in this section and then used for the remaining experiments.

#### *2.5.2.1 Shape function $\rho$*

In this work, we consider constant shape functions, i.e., we assume that $\rho(d) = K$ for some constant $K$ in meters. This parameter controls the shape of the contour of the plan of the envelope, as smaller values of $K$ can be used to have more acute contours. A value $K = 0$ and no horizontal precedences arcs results in the PCBC Raw Envelope.

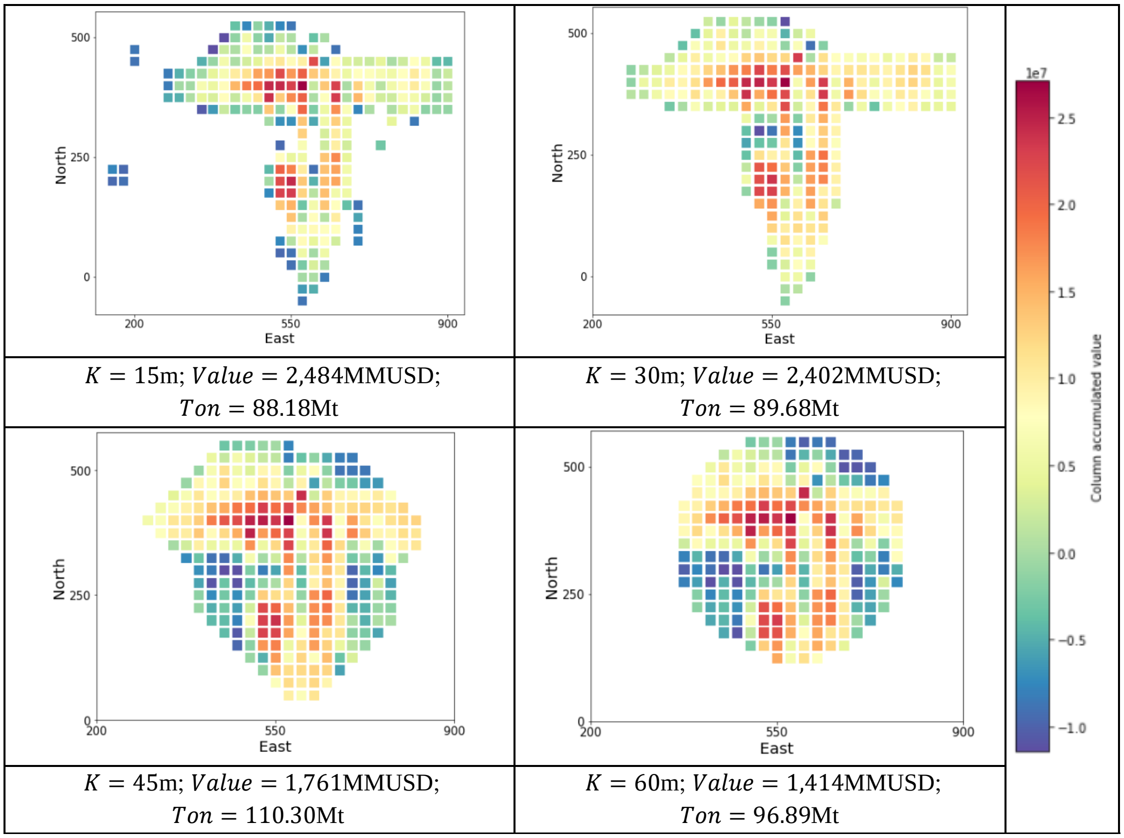
The results for different values of K are presented in Figure 6. For high values of K, the envelope becomes over-smoothed. Smoothness is achieved by incorporating columns of marginal economic value, i.e., a very smooth envelope does not contain precisely an optimal value. For this reason, we selected a value of K=30m for the rest of the paper. This value provides the smoothest and most economical horizontal shape at the same time.

#### *2.5.2.2 Maximum interaction height $h^+$ and maximum height difference $\delta$*

In this section we evaluate the impact of the maximum interaction height $h^+$ and maximum height difference $\delta$. These parameters control the smoothness of the columns selected for extraction, as it determines the point (height) from where the diagonal precedences arcs are created, and $\delta$ determines the “slope” angle that is allowed.

To evaluate these parameters, we considered $h^+$ as a fraction of the maximum column height. Similarly, we utilized the side of the block dimension $s$ to express the variability of the maximum height difference. Thus, for example $\delta/s = 1$ corresponds to a 1-to-1 slope, i.e., a maximum of 1 block of difference, while $\delta/s = 3$ allows 3 blocks of difference between contiguous columns.

The smoothness of the columns is more influenced by the interaction height $h^+$ than by the maximum height difference $\delta$. A low interaction height $h^+$ allows a higher degree of freedom between heights of adjoining columns. In our case, $\delta/s = 1$ and $h^+/H = 0.7$ are chosen, i.e., a conservative envelope is chosen.



$K$ = 15m; $Value$ = 2,484MMUSD; $Ton$ = 88.18Mt

$K$ = 30m; $Value$ = 2,402MMUSD; $Ton$ = 89.68Mt

$K$ = 45m; $Value$ = 1,761MMUSD; $Ton$ = 110.30Mt

$K$ = 60m; $Value$ = 1,414MMUSD; $Ton$ = 96.89Mt

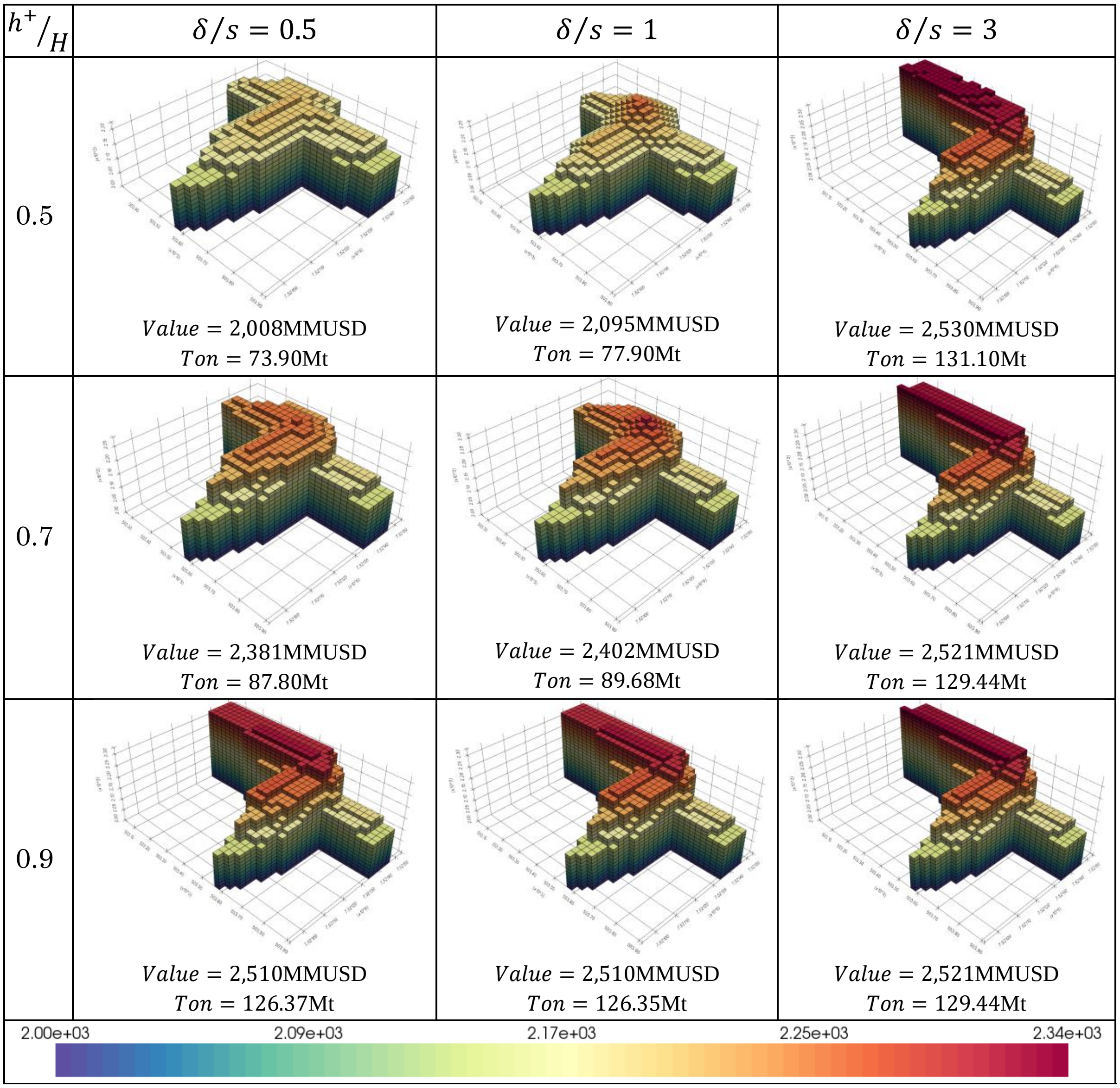
Figure 6 – Plan view of impact of the shape function on the contour of optimal envelopes. The colors scale represents the accumulated value of each draw column.

### 2.5.3 Comparison of envelopes

The PCBC Raw envelope is able to accumulate the maximum values in each column as it has no constraints (Table 3) and the location of the columns does not respect a smooth form (Figure 8).

Table 3 - Summary of results of block model "D"

| | Economic Value (MMUSD) | Total Tonnage (MTons) | Waste Tonnage (MTons) | Mineral (MTons) | Average Grade (%) |
|---|---|---|---|---|---|
| PCBC Raw | 2,737 | 128,575 | 28,302 | 100,273 | 1.59 |
| PCBC Smooth | 2,392 | 99,347 | 17,957 | 81,390 | 1.67 |
| Hybrid Smooth | 2,402 | 89,676 | 15,112 | 74,564 | 1.77 |
| Automated Smooth | 2,402 | 90,437 | 15,873 | 74,564 | 1.76 |



| $h^+/H$ | $\delta/s = 0.5$ | $\delta/s = 1$ | $\delta/s = 3$ |
|---|---|---|---|
| 0.5 | $Value = 2{,}008$MMUSD<br>$Ton = 73.90$Mt | $Value = 2{,}095$MMUSD<br>$Ton = 77.90$Mt | $Value = 2{,}530$MMUSD<br>$Ton = 131.10$Mt |
| 0.7 | $Value = 2{,}381$MMUSD<br>$Ton = 87.80$Mt | $Value = 2{,}402$MMUSD<br>$Ton = 89.68$Mt | $Value = 2{,}521$MMUSD<br>$Ton = 129.44$Mt |
| 0.9 | $Value = 2{,}510$MMUSD<br>$Ton = 126.37$Mt | $Value = 2{,}510$MMUSD<br>$Ton = 126.35$Mt | $Value = 2{,}521$MMUSD<br>$Ton = 129.44$Mt |

Figure 7 – Isometric view of economic envelopes for different values of the maximum interaction height ($h^+$) and the maximum height difference ($\delta$) in terms of the total column height $H$ and the side of the block $s$. The colors scale represents the height of the columns.

The Automatic Smooth envelope is obtained by using the initial point that results in the envelope with the highest economic value.

For the smooth PCBC envelope, a design that seeks stability and the incorporate of the largest number of columns is proposed. In addition, a 75-meter-wide column between panels is considered (see Figure 8). These columns are located in places with few columns. In the smooth hybrid envelope, a single point of origin was used, based on the location of the column with the highest economic value obtained in PCBC Raw.

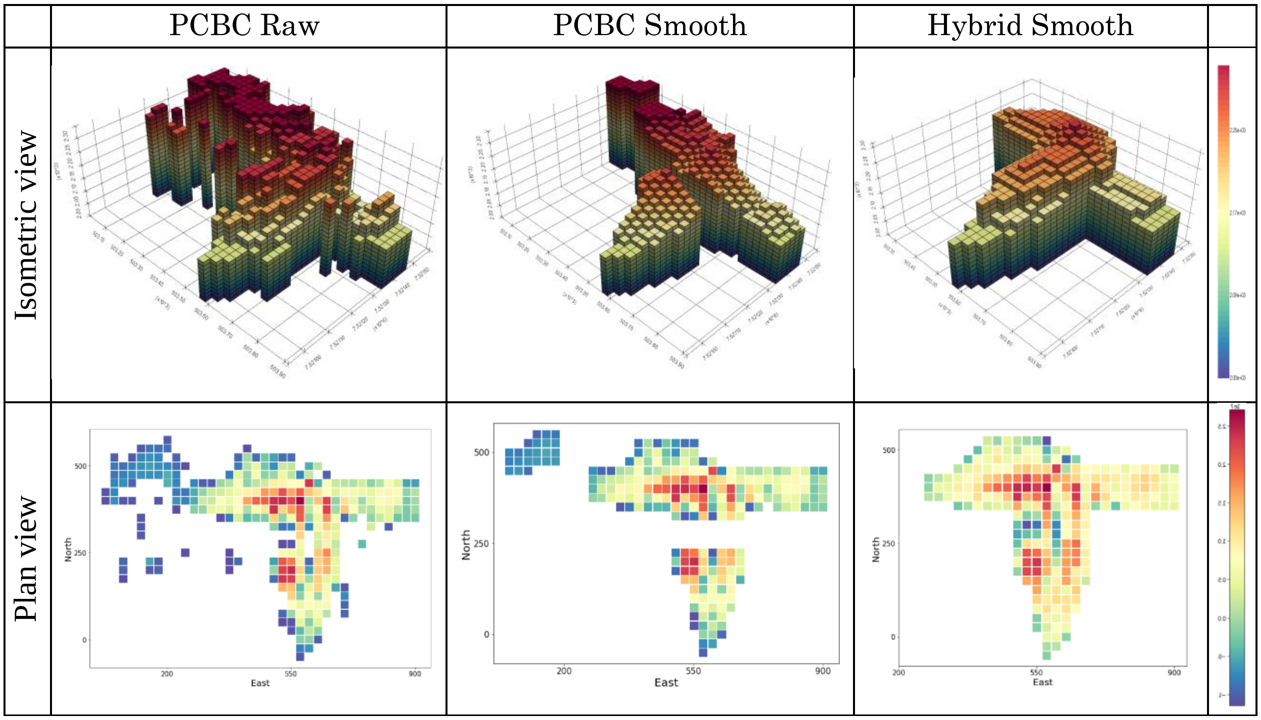


Figure 8 - Comparison of economic envelopes. In the isometric view, the colors scale represents the height of the columns and plan view the colors scale represents the accumulated value of each draw column.

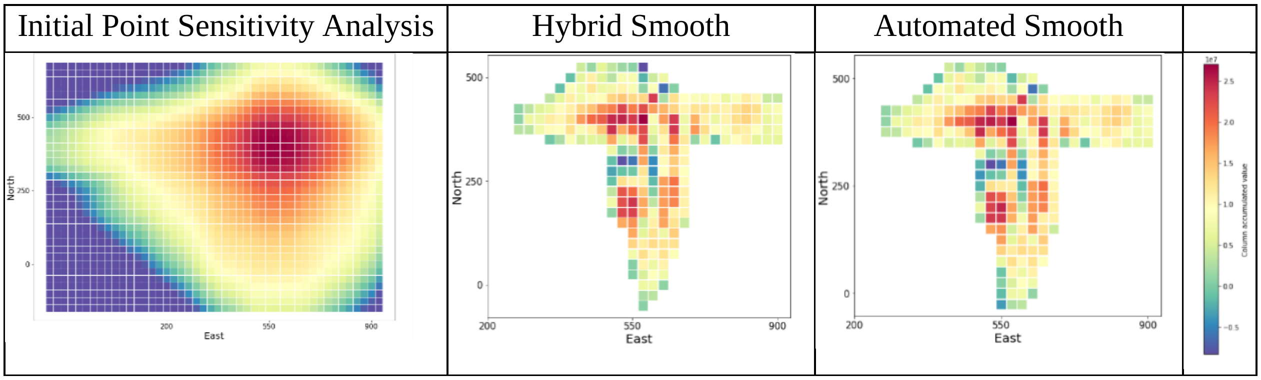


Figure 9 - Selection of best initial point. Plan views with colours scale for economics values. Warm colour for high economics values of envelopes in Sensitivity Analysis and accumulated value of draw column in Hybrid and Automated Smooth.

The Hybrid Smoot Envelope results in a design similar to the PCBC Smooth but with a higher economic value. The main difference is that our method omits the columns to the northwest of the PCBC Raw envelope and adds low value columns to smoothed the central area of the envelope (Figure 8).

Figure 9 shows the result of a sensitivity analysis of the initial point. Each point of the sensitivity analysis represents the spatial location of the initial point and its colour represents the total value of the resulting economic envelope (warm colour for high economics values).

The sensitivity analysis shows that the highest value economic envelopes are obtained when the initial point is in the high-grade zone (highest economic value columns within the PCBC Raw envelope). The Hybrid Smooth and Automatic Smooth envelopes have contiguous starting points. This suggests that the optimal starting point can be found in the columns with the highest value within the PCBC Raw envelope.

## 2.6 Conclusions

The final pit optimization algorithm can be successfully adapted to determine economic envelopes in other export methods, if care is taken to formulate adequate precedence arcs that guarantee an envelope with a working design. In this case, an acyclic directed graph that varies shows good performance in the final result.

In our case, the proposed method exceeds the standard method in irregular mineral bodies, i.e., it computes a smoothed envelope with an optimal value without the need for an engineer to intervene in the design of the envelope. In addition, the height and shape parameters described in section 2.4.2.1 and 2.4.2.2 require a simple calibration to calculate a smooth, maximum value envelope.

The Automated Smooth envelope does not differ much from the Hybrid Smooth envelope. This hints at the possibility of silvering a way to find the initial point without the need to compute an envelope for all possible initial points.

# 3. Conclusiones generales, trabajos futuros y recomendaciones

## 3.1 Conclusiones generales

En este trabajo, se propuso un método de cálculo de envolvente económica suavizada para block/panel caving. Dentro del calculo de optimización se incorporan las restricciones de forma que se requieren para obtener un contorno de footprint y alturas de columnas de extracción suavizadas. Por lo tanto, se logra calcular una envolvente económicamente optima y que a su vez respeta restricciones de forma que la hacen operacionalmente valida. Esto gracias a la extensión del problema de pit final, agregando un nuevo conjunto de precedencias horizontales e invirtiendo el rajo.

Para el método propuesto, la definición de arcos de precedencias requiere ciertos parámetros que deben ser calibrados, mediante una iteración, para obtener el mejor resultado económico con forma aceptable. Para la generación de arcos horizontales, mientras mayor sea el valor del parámetro $K$ el contorno del footprint será mas suave. Además, mientras más niveles de la envolvente presenten esta restricción horizontal ($altura\ de\ interacción\ máxima, h^{+}$), y menor sea el parámetro $\delta$, la diferencia de alturas entre columnas de extracción estará más restringida. Por lo tanto, mientras menor sean $K$ y $h^{+}$, y mayor sea $\delta$, se obtendrá una envolvente con mayor valor económico, pero con contornos más agudos. Otro parámetro que controla el rendimiento de la metodología es el $punto\ de\ inicio$, que es requerido para definir las precedencias horizontales. Para obtener el mejor resultado se puede iterar cada $punto\ de\ inicio$ posible dentro del plano horizontal. También, presentó un buen rendimiento haber seleccionado como $punto\ de\ inicio$ la coordenada $x, y$ de la columna de extracción con mayor valor económico acumulado dentro de la envolvente $PCBC\ Raw$.

## 3.2 Trabajos futuros

En trabajos futuros se debe considerar la incorporación de pilares al cálculo de optimización. La definición de envolvente económica en cuerpos minerales muy irregulares, con calidades de rocas distintas o muy extensos, se requiere diseñar pilares entre paneles. El método propuesto no logra generar pilares entre paneles con un diseño aceptable.

Por otro lado, este método requiere la calibración manual de parámetros de forma. Esta calibración podría proponerse con un enfoque automático que apunte a encontrar resultados óptimos con menor tiempo de cómputo.

Finalmente, el método propuesto implementa un nuevo conjunto de arcos de precedencia para satisfacer las restricciones de forma en este trabajo. Sin duda, la formulación de un grafo, desde la topología de red, puede extender el algoritmo de cálculo de pit final para definir reservas económicas en otros métodos de explotación.

## 3.3 Recomendaciones

La revisión bibliográfica da cuenta del bajo aporte científico en la etapa de estimación de reservas en los métodos block y panel caving. La literatura existente para el método open pit, en esta materia, es mucho mayor y para el caso del presente trabajo sirvió de referencia. Esta apreciación puede servir como motivación en el avance del aporte científico en la estimación de reservas económicas en métodos subterráneos, que no tan solo calcule el resultado de una envolvente económica óptima, sino que además busque una secuencia óptima de arranque de las reservas.

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# Anexos

## Anexo A: Curriculum Vitae del alumno

1) Datos personales

| | |
|---|---|
| Nombre: Francisco Javier Saavedra del Pozo | Estado civil: Casado |
| Dirección: Rómulo J. Peña 170, Copiapó | Mail: francisco.saavedra@uda.cl |
| Fecha de nacimiento: 10 de diciembre 1990 | Teléfono celular: +56967883894 |
| Nacionalidad: Chilena | |

2) Formación profesional

| | |
|---|---|
| 2020 a la fecha | Estudio en postgrado en Universidad de Chile. "Magister en Minería". |
| 2017 (6 meses) | Estudios en postítulo en Universidad de Atacama. "Diplomado en Docencia e Investigación Universitaria". |
| 2016 (6 meses) | Estudios en postítulo en Escuela de Negocios Mineros, Universidad Católica del Norte. "Diplomado en Proyectos para la Minería (PMI)". |
| 2008 - 2013 | Estudios de pregrado en Universidad de Atacama. Especialidad Ingeniería Civil en Minas. |
| 2008 - 2011 | Grado de Licenciado en Ciencias de la Ingeniería en Universidad de Atacama. |

3) Experiencia profesional

| | |
|---|---|
| Marzo 2022 a la fecha | Académico en Departamento de Minas, UDA, Chile. Área de desempeño: Métodos de explotación, y planificación minera. Cursos dictados: Métodos de explotación (MIN183104), Métodos de explotación (GEO6105), Geología de Minas (GEO185201), Servicios Mineros (MIN183204), Preparación y Evaluación de Proyectos (MIN185101), Proyecto Minero Cielo Abierto (MIN6101), Electivo "Computación y Planificación Minera" (MIN6104). También, desempeñando tareas como Coordinador de Aseguramiento de la Calidad, Coordinador de Actualización Curricular y Coordinador de Investigación. |
| Mayo de 2020 a la fecha | Tesista en Laboratorio de Planificación Minera Delphos (AMTC, DIMIN U. de Chile). Área de investigación: Estimación de reservas económicas considerando criterios geomecánicos para métodos block y panel caving. |
| Septiembre de 2018 hasta marzo de 2020 | Coordinador Docente del Departamento de Minas, UDA, Chile. |

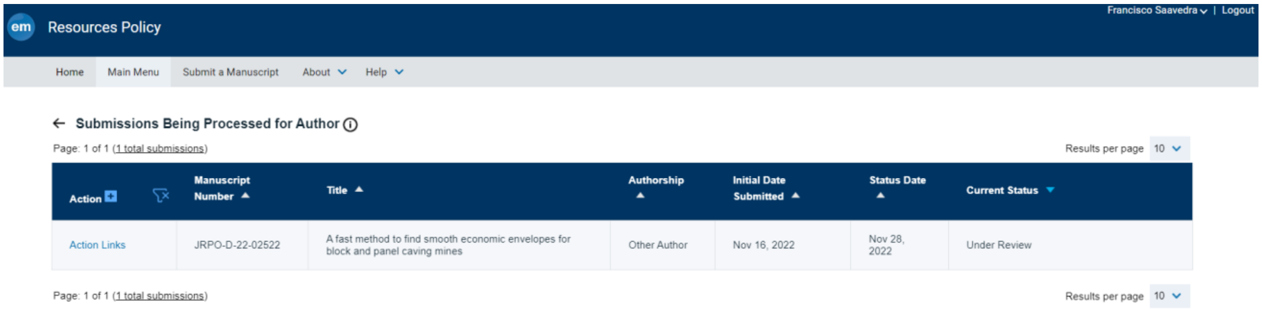
| | |
|---|---|
| Marzo de 2016 a diciembre 2017 | Coordinador de Plan Operacional Anual del Departamento de Minas, UDA, Chile. |
| Septiembre 2015 a marzo de 2020 | Académico en Departamento de Minas, UDA, Chile. Área de desempeño: Operaciones y métodos de explotación. Cursos dictados: Dibujo de Ingeniería (PCI2105), Introducción a la Minería (MIN181105), Perforación (MIN3206), Métodos de Explotación (MIN3201), Servicios Mineros (MIN4205), Electivo "Programación aplicada a la Minería" (MIN6104), Electivo "Modelamiento Estadístico aplicado a la Minería" (MIN6204). |
| Junio de 2014 hasta noviembre 2014 | Memorista en Mina Los Colorados, CAP. Desarrollando modelo estadístico para la estimación de consumo de combustible en flota de camiones de extracción. |

4) Otros antecedentes

| | |
|---|---|
| Participación Académica en Departamento de Minas, UDA. | • Miembro del Consejo Académico.<br>• Miembro de Comité de Acreditación.<br>• Miembro de Comité de Auscultación.<br>• Miembro del Comité de Pedagogía Curricular (COPECU).<br>• Apoyo en actividades de Vinculación con el Medio. |

# Anexo B: Estado de publicación

En el presente anexo se muestra la certificación del estado de publicación. En la pagina de la revista Resources Policy se indica que desde el 28 de noviembre del 2022 en articulo se encuentra en revisión.



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|---|---|---|---|---|---|---|
| Action Links | JRPO-D-22-02522 | A fast method to find smooth economic envelopes for block and panel caving mines | Other Author | Nov 16, 2022 | Nov 28, 2022 | Under Review |

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