Table of Content

Introduction **1**

1 Fundamental Concepts **3**
1.1 Quantum Optics . . . 3
1.1.1 Quantization of the Electromagnetic Field . . . 4
1.1.2 States of the Electromagntic Field . . . 7
1.1.3 Continuos-mode Field Operators . . . 8
1.1.4 Light-Matter Interaction . . . 9
1.2 Open Quantum System . . . 10
1.2.1 System-Plus-Reservoir Approach . . . 10
1.2.2 Master Equation . . . 12
1.2.2.1 Master Equation of the Damped Harmonic Oscillator . . . 14
1.2.3 Heisenberg-Langevin Equation of Motion . . . 17
1.2.3.1 Heisenberg-Langevin Equation for an Harmonic Oscillator . 18
1.3 Laser . . . 20
1.4 Linewidth of a Cavity QED Laser . . . 20
1.4.1 Model of a Laser on a Cavity QED system . . . 20
1.4.2 Steady state solutions of the Mean Field Equations of a Cavity QED Laser . . . 23
1.4.3 Phase Diffusion Linewidth . . . 24

2 Model of a Laser on a Waveguide QED **27**
2.1 Open Quantum System Approach . . . 27
2.1.1 System of interest . . . 27
2.1.2 Reservoir . . . 29
2.1.2.1 Pumping System . . . 29
2.1.2.2 Free Electromagnetic Field . . . 30
2.2 The Interaction Hamiltonian of the Total System . . . 30
2.2.1 Discrete Model . . . 30
2.2.2 Continuous Model . . . 31

3 Heisenberg-Langevin Equation of Motion of a Waveguide QED system Laser **33**
3.1 Heisenberg Equation of the total system operators . . . 33
3.1.1 Solutions of Heisenberg Equations of the Reservoir operators . . . 36
3.2 Deduction of the Heisenberg-Langevin equations of motion . . . 36
3.2.1 Deduction of the atomic coherence operator Heisenberg-Langevin Equation of Motion . . . 37

3.2.2 Deduction of the atomic inversion operator Heisenberg-Langevin Equation of Motion . . . 38
3.2.3 Deduction of the Continuous bosonic Heisenberg-Langevin Equation of Motion . . . 41
3.3 Heisenberg-Langevin Equations of Motion of the Atom-Field System operators 41
3.3.1 Equations of Motion in the Rotating Frame . . . 42

4 Linewidth of a Waveguide QED system Laser 43
4.1 Mean-Field Equation of Motion . . . 43
4.1.1 Steady State Solutions in the Mean Field . . . 44
4.2 Electric Field Equation of Motion in the Heisenberg Picture . . . 45
4.2.1 Steady-state Solution of the Mean-field equation of the Electric Field operator . . . 45
4.2.2 Electric Field Operator above the threshold . . . 47
4.3 The Spectrum . . . 48
4.3.1 Wiener-Khintchine Theorem . . . 48
4.3.2 Laser Power and Spectrum . . . 50

5 Waveguide Phase Equation Method for the Linewidth 55
5.1 Waveguide Phase Equation . . . 56
5.1.1 Derivation of the Phase Equation . . . 56
5.2 Fluctuating terms of the Phase . . . 59
5.2.1 Phase Correlations . . . 59
5.3 Coherence Function . . . 61
5.3.1 Derivation of the Coherence Function . . . 61
5.4 Spectrum of the Phase Equation Method . . . 62
5.4.1 Wiener-Khintchine Theorem . . . 63
5.4.2 Spectrum and Power in the Phase Equation Method . . . 63
5.4.2.1 Generalized Good Cavity Regime . . . 63
5.4.2.2 Generalized Bad Cavity Regime . . . 66

Conclusions 68

Bibliography 70

ANNEXES 73

Annex A Derivation of the Phase Fluctuations 73

Annex B Equations of motion of Atomics Opertors 74
B.1 Equation of Motion of Coherence Operator . . . 74
B.2 Equation of Motion of Inversion Operator . . . 74

Annex C Rotating frame variable change 76

Annex D Schrödinger, Heisenberg and Interaction Pictures 78
D.1 Interaction Picture . . . 78
D.2 Schrödinger Picture . . . 78
D.3 Heisenberg Picture . . . 79

D.4 Electric Field 79

Annex E Noise Operator Two times Correlation **83**
E.1 Gaussian Noise 83
E.2 Fluctuation Operator two times correlation 86
E.2.1 Coherence Noise Operator correlation 87
E.2.2 Bosonic Noise Operator Correlation 88
E.3 Summary of Correlations 88

Annex F Heisenberg-Langevin Equations for the reservoir operators in a Laser on a waveguide QED **91**
F.1 Heisenberg Equations of the Reservoir Operators 91
F.1.1 Solutions of Heisenberg Equations 91
F.2 Derivation of the Equations of Motion of the Atomic Operators 92
F.2.1 Equation of Motion of Inversion 92
F.2.1.1 Evolution generated by the reservoir R 92
F.2.1.2 Evolution generated by the reservoir $\mathcal{P}$ 97
F.2.2 Equation of Motion of Coherence 101
F.2.2.1 Evolution generated by the reservoir R 102
F.2.2.2 Evolution generated by the reservoir $\mathcal{P}$ 104