Abstract: Recently, a novel high-starting torque scalar control scheme (HST-SCS) for induction motor(s) (IM) emerged. It expands the scalar control application field beyond centrifugal pumps, blowers, and fans, moving, for instance, some conveyor belts with nominal torque loading. This paper proposes a normalized model reference adaptive control (N-MRAC) applied to HST-SCS for IM. First, the proposal extends the MRAC, resulting in a class of nonlinear systems encompassing the IM dynamical model. It uses a normalized information vector, jointly with a direct control approach, reducing the trial and error adaptive controller tuning. Second, a properly designed N-MRAC is applied to regulate the starting stator current within the variable speed drive under investigation. As a result, the proposed methodology keeps the HST-SCS as a simple control scheme without needing variable observers or parameter estimators and employing tuning information only from the motor nameplate and datasheet. Test bench experiments with a 10 HP motor validate the proposal effectiveness.

Keywords: adaptive control; induction motors; nonlinear dynamical systems; variable speed drives

1. Introduction

IM feeds by variable frequency drive(s) (VFD) [1] have been widely used in variable speed applications. Compared to other electrical motors, IM have lower costs, higher efficiency, and lower maintenance, which explain their increasingly widespread use.

One of the most employed VFD is the scalar control scheme (SCS) [2,3]. It is used in low-performance applications, demanding a low starting-electromagnetic torque, up to 25% of the nominal torque. Examples of these applications are: blowers, fans, and centrifugal pumps [4]. Furthermore, it has the simplest control scheme of all the VFD, not needing parameters or variables estimation.

Efforts have been made to improve the SCS. The work of [5] analyses its slow and even oscillatory transient speed behavior and proposes two methods to mitigate them. Furthermore, the work by [6] proposes an optimal v/f ratio based on a reactive power estimate. On the other hand, the SCS has a steady-state speed accuracy of between 1% and 4% [2,4,5], which some works aim to improve i.e., in [7], after using closed-loop speed control, while [8] proposes a PI slip controller. The work by [9] proposes a rotor angular speed observer based on a spectral search method.

Other proposals to improve the steady-state speed-accuracy have developed slip speed estimators. Several examples of these are: the work by [10], based on an electromagnetic torque estimator, the proposal by [11] uses a gap power estimation, [12] utilizes a stator flux observer, and [13,14] employs the ratio between the rated slip and rated stator current. The proposals by [6,10–12] require IM parameters or variables estimation.
The above mentioned research improves several aspects of the standard SCS. Nevertheless, they do not address the reduced starting-electromagnetic torque issue described in the second paragraph, as in [15]. This method depends on a stator resistance estimator and a stator flux observer [15]. Later, the HST-SCS proposed in [16] avoids parameters and variable estimators. However, this last approach has a multiple-input and multiple-output (MIMO) controller. Hence, it shows speed and current chattering issues after switching the controller output vector voltage to the non-aligned scalar boost voltage. This issue is improved in [17] by including a scalar HST output that smoothly switches to the standard boost voltage curve of the SCS [2,14].

Hence, the viable solution [17] proposed an HST-SCS for IM, which adds to the standard SCS [2,3,14] the high starting torque capability while keeping a control scheme without using parameter estimates or variable observers. As a result, it expands the SCS applications beyond centrifugal pumps, blowers, and fans [2,14]. The HST-SCS move, for instance, loaded conveyor belts requiring 100% of the nominal torque at the start and no steady-state speed accuracy [17].

The HST-SCS [17] uses a normalized adaptive passivity-based controller (N-APBC) [18], based on the adaptive concepts given in [19]. It uses the adjustment setting of [16] that improves the APBC given in [18] by reducing trial and error tuning. Moreover, it normalized the APBC given in [16], decreasing its tuning dependency on the operational range and motor power.

In contrast to [17], this paper’s main contribution proposes a novel N-MRAC and its application to HST-SCS for IM. It extends the MRAC for a class of nonlinear systems, encompassing the IM dynamical model while using a normalized information vector, reducing trial and error adjustments. Later, an N-MRAC design regulates the IM starting stator current, keeping a simple control scheme. The solution only uses tuning information from the motor nameplate and datasheet, not needing variable observers or parameter estimators.

The paper is organized as follows. Section 2 describes the block diagram, control strategy, and method background of the HST-SCS. Later, Section 3 depicts the proposed N-MRAC and its application to HST-SCS, including its block diagram. Section 4 includes and discusses the comparative results obtained after testing both HST-SCS strategies. Finally, the conclusions present an appraisal of the main findings.

2. Preliminaries

Figure 1 describes the general block diagram of the HST-SCS, which is based on Figure 1 from [17] and is more detailed.

![Figure 1. General block diagram of the HST-SCS for IMs, based on [17].](image-url)
Here, the operator establishes the required rotor angular speed \( \omega_r^* \). Based on this, the method automatically defines the required angular electrical frequency \( \omega_e^* \) through the angle \( \rho = \int_0^t \omega_e^* \, dt \) and the needed stator voltage amplitude \( V_s^* \). This last is alternated using the the Park inverse transformation \( e^{-j\rho} \) [14,20], obtaining the two-phase alternating \( \alpha - \beta \) instantaneous voltage, i.e., \( u_{\alpha\beta}^* = e^{-j\rho} V_s^* \). Later, \( u_{\alpha\beta}^* \) is converted to the three-phase using the Clarke transformation \( T_{2\rightarrow3} \) [21,22], having \( u_s^* = [u_a^* \ u_b^* \ u_c^*]^T = T_{2\rightarrow3} u_{\alpha\beta}^* \). The following sections describe the details.

2.1. Required Angular Electrical Frequency \( \omega_e^* \)

The required angular electrical frequency \( \omega_e^* \) is computed as follows [17]:

\[
\omega_e^* = \left( \frac{p}{2} \right) \omega_r^* + \omega_{slip}, \text{ with } \omega_{slip} = \frac{j \beta}{2 \rho_{sn}} \left| V_{sdn} \right|
\]

\[
\omega_{slip} = \omega_{en} - \left( \frac{p}{2} \right) \omega_{rn}, \text{ with } \omega_{en} = \frac{4 \pi f_n}{p},
\]

and \( I_{sdn} = \sqrt{2} I_{sn} P_{F_n} \).

Many mentioned steady-state nominal variables are taken from the motor nameplate, such as the rated current per phase \( I_{sn} \), power factor \( P_{F_n} \), electrical frequency \( f_n \), number of poles \( p \), and rated rotor angular speed \( \omega_{rn} \). The variable \( \omega_{slip} \) is the slip speed. Furthermore, \( \omega_{slipn} \) is its nominal value calculated as the difference between the rated synchronous speed \( \omega_{en} \) and the rated rotor angular speed \( \omega_{rn} \). The variable \( I_{sdn} \) is the rated direct stator current component. Moreover, the measured direct stator current component \( I_{sd} \) is obtained from the Park transformation \( e^{j\beta} \) [3,14] and Clarke, after sensing the stator current \( i_s \).

The following section identifies the needed voltage calculation.

2.2. Needed Voltage Amplitude \( V_s^* \)

The HST-SCS applies the required stator voltage amplitude (2) next defined [17]:

\[
V_s^* = \begin{cases} 
V_{s0}^* \text{ (HST controller)} & \text{if } En = 1 \& V_{s0}^* < V_{s1}^* \\
V_{s1}^* = \sqrt{2} (P_1 \omega_e^* + V_{boost}) & \text{if } V_{s1}^* < V_{s1}^* < V_{s0}^* \\
V_{s2}^* = \sqrt{2} P_2 \omega_e^* & \text{if } V_{s1}^* < V_{s2}^* < V_{s0}^* \\
V_{s3}^* = \sqrt{2} V_{sn} & \text{if } V_{s1}^* < V_{s3}^* 
\end{cases} 
\]

Here, depending on the required angular electrical frequency \( \omega_e^* \), once the VFD is enabled (En=1), the needed stator voltage amplitude \( V_s^* \) swaps between the different voltage curves \( V_{s0}^*, V_{s1}^*, V_{s2}^*, \text{ and } V_{s3}^* \). This last, \( V_{s3}^* \), protects the IM, avoiding its operation over the nominal stator voltage \( V_{sn} \), taken from the nameplate. From zero to \( \omega_e^* \), it starts with \( V_{s0}^* \) acting as a starting adaptive-current source, initially using a N-APBC [17]. Later, as \( \omega_e^* \) increases, it switches to \( V_{s1}^* \), having the boost voltage \( V_{boost} \), adjusted from 3% to 50% of \( V_{sn} \). This curve applies under the cut angular frequency \( \omega_c \), tuned from 40% to 50% of \( \omega_{en} \) [14]. Finally, over the cut angular frequency \( \omega_c \), it applies \( V_{s3}^* \). Here, the proportional controller parameters \( P_1 \) and \( P_2 \) are computed as follows:

\[
P_1 = \left( \frac{V_{sn}}{\omega_{en}} - \frac{V_{boost}}{\omega_c} \right) \quad \text{and} \quad P_2 = \left( \frac{V_{sn}}{\omega_{en}} \right).
\]

2.3. Methodology Background

The HST-SCS relies on the steady-state IM equivalent circuit per phase. Here, on the one hand, it applies the Kirchhoff’s stator voltage law [3,17,21]. Then, it neglects the stator impedance voltage drop, simplifying the obtained equation as follows:

\[
\Phi_s = R_s i_s + j \omega_e^* L_d i_d + j \omega_e^* L_m i_m \Rightarrow |\Phi_s| \sim \left| \frac{I_s}{\omega_e^*} \right|.
\]
where $R_s$ and $L'_s$ are the stator resistance and leakage inductance, respectively. $L_m$ is the magnetizing inductance. Moreover, $i_s$ is the stator phase current, $u_s$ is the stator phase voltage, and $i_m$ is the magnetizing phase current. Based on (4), the voltage curves $V_{s1}^*$ and $V_{s2}^*$ aim to keep a constant stator flux magnitude $|\Phi_s|$ for the ranged $\omega^*$.

On the other hand, the voltage curve $V_{s0}^*$, does not use the stator flux magnitude dependence $|\Phi_s| \sim \frac{V_s}{\omega}$ (4), but its dependence on the magnetizing current, $|\Phi_s| = L_m |i_m|$ [17]. After having that $i_m = i_s - i_r$ and replacing the rotor current $i_r$ expression, we have [17]:

$$|\Phi_s| = L_m |i_m| = L_m |i_s - jL_m R_r \omega_{\text{slip}} + jL_r i_s|,$$

$$|\Phi_s| = L_m |i_s (\frac{R_r}{\omega_{\text{slip}}} + jL'_r) i_s|,$$

where $L'_r$ is the rotor leakage inductance. Considering this, the HST-SCS aims to keep a constant starting flux magnitude by fixing the stator phase current $i_s$ (5) at the start. As a result, it guarantees a high starting electromagnetic torque capability, as can be seen next, with $K_p$ a proportionality constant [17]:

$$T_{em} = 3 \left( \frac{p}{2} \right) |\frac{R_r}{\omega_{\text{slip}}} + jL_r|^2 |i_s|^2.$$

Remark 1. As shown in Figure 1, the proposed HST-SCS keeps a constant starting stator current direct component $I_{sd, \text{starting}}^*$ instead of the needed starting stator current $I_{s, \text{starting}}^*$. Moreover, it uses a N-APBC to regulate it [16,17].

The following section proposes using a N-MRAC instead of a N-APBC. Furthermore, it directly controls the starting stator current $i_{s, \text{starting}}$.

3. Proposal

In this section, based on the described IM dynamical d-q model [17], this paper extends an MRAC and applies it to HST-SCS for IM.

3.1. Proposed N-MRAC for Nonlinear Systems

Let us consider the following class of the nonlinear system:

$$\dot{y}(t) = a(a(y) + \beta b(y) u(t),$$

$$\dot{z}(t) = q(z, y).$$

(7)

Here, the output is $y(t) \in \mathbb{R}$, the control input is $u(t) \in \mathbb{R}$. The unknown parameters are $\beta \in \mathbb{R}$ and $\alpha \in \mathbb{R}^{1 \times m}$. The known nonlinear functions are $b(y) \in \mathbb{R}$ and $a(y) \in \mathbb{R}^m$. It is assumed that it has bounded-input bounded-output (BIBO) internal dynamics $z(t) \in \mathbb{R}^v$. The dimensions $m$ and $v$ are natural numbers.

The following Theorem 1 extends a direct MRAC [19] to nonlinear systems of the form (7).

Theorem 1. After considering the desired model reference output $y_m$, and applying the following N-MRAC to the nonlinear system (7), it guarantees that $\lim_{t \to \infty} e(t) = 0$, with $e(t) = y_m(t) - y(t)$:
\[
\dot{y}_m(t) = -A_m y_m(t) + B_m y^*(t), \quad \text{model reference}
\]
\[
u(t) = b(y)^{-1}(t) \dot{y}(t), \quad \text{adaptive controller}
\]
\[
\theta^T = \int_0^t \Gamma \Omega(t) \, d\tau, \quad \text{with } \theta^T(0) = 0, \quad \text{adaptive controller parameter}
\]
\[
\Omega(t) = 100 \begin{bmatrix}
a(y) & y(t) & y^*(t) \\
a_n & y_n & y_n^*
\end{bmatrix}, \quad \text{normalized information vector (8)}
\]

with \( \Gamma = \frac{\gamma 100}{1+100^2} \), normalized fixed gain
and \( B_m = S A_m \). with S a scaling factor

Here, \( y^*(t) \) is the desired trajectory. The normalization factors \( a_n, y_n, \) and \( y_n^* \) correspond to the maximum operational range of \( a(y), y(t), \) and \( (y^*(t)) \), respectively. The normalized information vector considers these factor and has a 100 range. Then, the fixed-gain \( \Gamma \in \mathbb{R}_+ \), having a form \( \frac{\gamma^2}{1+100^2} \) [19]—Section 5.5, with \( \gamma = 100 \) herein due to range of \( \Omega \), becomes \( \frac{\gamma 100}{1+100^2} \). It is conformed by the terms \( \frac{100}{1+100^2} \) for a fast tuning assuring a reasonable operating range and the design parameter \( \gamma \in [0.1, 10] \), which allows a fine-tuning adjustment. Moreover, the reference model input \( y^* \) and output \( y_m \) may be expressed in different units of measurement, to which the scaling factor \( S \) relates.

Appendix A describes the theoretical proof of the proposed N-MRAC. Moreover, the following section includes a design example of the proposed method applied to the HST-SCS of IM.

3.2. N-MRAC Applied to HST-SCS for IM

This section uses an N-MRAC as an alternative to the N-APBC proposed in [17] to directly controls the starting stator current \( i_{s,\text{starting}} \) as required by (5) and (6).

Figure 2 describes the general block diagram of the proposed HST-SCS, improved after using a N-MRAC and direct control of the stator current \( i_s \).

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**Figure 2.** General block diagram of the propose HST-SCS for IMs, remarking the differences in red.
The following subsections describe the details.

### 3.2.1. Calculus for the Angular Electrical Frequency $\omega^s$

Taken from a proven industry solution [14], here, the slip speed $\omega_{\text{slip}}$ depends on the rated current per phase $I_{\text{sn}}$, and the consumed stator current per phase $I_s$, instead of using the relations given in (1). Hence, the required angular electrical frequency $\omega^s$ computes as follows:

$$\begin{align*}
\omega^s &= \left( \frac{L_s}{L_m} \right) \omega^r + \omega_{\text{slip}}, \quad \text{with} \quad \omega_{\text{slip}} = \frac{I_{\text{sn}}}{L_m} \frac{L_m}{I_s}, \\
\omega_{\text{slip}} &= \omega_{\text{en}} - \left( \frac{L_s}{L_m} \right) \omega_{\text{en}}, \quad \text{with} \quad \omega_{\text{en}} = \frac{4\pi f_n}{p}.
\end{align*}$$

(9)

The nominal slip $\omega_{\text{slip}}$ is calculated as in (1). Please note that $I_{\text{sn}}$ and $I_s$ are RMS values used in (9). Hence, the proposal computes $I_s$ as the RMS of the instantaneous vector magnitude $I_{\text{sd}}$, considering it is a sinusoidal signal, and as a function of its instantaneous components as follows:

$$I_s = \frac{I_{\text{sd}}}{\sqrt{2}}, \quad \text{with} \quad I_{\text{sd}} = \sqrt{I_{\text{sd}}^2 + I_{\text{sq}}^2}$$

(10)

The following section describes the required stator voltage amplitude, which follows the same structure as [17] but with a N-MRAC instead of a N-APBC.

### 3.2.2. Proposed HST-SCS Voltage Amplitude $V^s$

The proposed HST-SCS considers that IM dynamical d-q model has the form of (7), where the output is $y(t) = I_{\text{dq}} \in \mathbb{R}$, the control input is $u(t) = V_{\text{sd}} \in \mathbb{R}$. The unknown parameters are $\beta = \frac{1}{\sqrt{L_m}}$ and $\alpha = [A_{1\text{upper}} \ 1 \ A_{2\text{upper}}] \in \mathbb{R}^3$. The known nonlinear functions are $b(y) = 1$ and $a(y) = \left[ I_{\text{sd}} \ I_{\text{sq}} \ \omega_{\text{q}} \ I_{\text{sd}} \ I_{\text{sq}} \ \omega_{\text{r}} \ I_{\text{sd}} \right]^T \in \mathbb{R}^5$. Moreover, the internal dynamics equals $z(t) = \left[ I_{\text{sd}} \ \Phi_{\text{rd}} \ \Phi_{\text{rq}} \ \omega_{\text{r}} \right]^T \in \mathbb{R}^4$, defined as follows:

$$q(z, y) = \begin{bmatrix}
A_{1\text{lower}} & l_{\text{sd}} & -\omega_{\text{q}}^2 I_{\text{sd}} + A_{2\text{lower}} \omega_{\text{r}}^2 & l_{\text{sq}} & \beta V_{\text{sd}} \\
C & [I_{\text{sd}} \ I_{\text{sq}}] & \Phi_{\text{rd}} - A_{1\text{upper}} & \Phi_{\text{rq}} - A_{2\text{upper}} \\
-\omega_{\text{r}}^2 & l_{\text{sd}} & l_{\text{sq}} & l_{\text{sd}} - l_{\text{sq}} & -I_{\text{sd}}
\end{bmatrix}.$$  

(11)

Here, the stator current direct component $I_{\text{sd}}$ is the variable to be controlled via the stator voltage direct component $V_{\text{sd}}$. $I_{\text{sq}}$ and $V_{\text{sq}}$ are the quadrature components of the stator current and the voltage, respectively. $\Phi_{\text{dq}}$ is the rotor flux direct component, and $\Phi_{\text{rq}}$ is the rotor flux quadrature component. The rotor angular speed $\omega_{\text{r}}$ is assumed to be equal $\omega^r$. Furthermore, the mechanical viscous damping is $b_{\text{m}}$, the motor-load inertia is $J$, and the load torque is $T_{\text{L}}$. Here, the vector $A_{1\text{upper}} \in \mathbb{R}^2$ and $A_{1\text{lower}} \in \mathbb{R}^2$ are upper and lower rows of the matrix $A_1$, respectively. The vector $A_{2\text{upper}} \in \mathbb{R}^2$ and $A_{2\text{lower}} \in \mathbb{R}^2$ are the upper and lower rows of the matrix $A_2$, respectively. Moreover, $C, d, A_1$, and $A_2$ are defined as follows [17]:

$$C = \begin{bmatrix}
-\frac{R_s}{L_m} & \omega_{\text{slip}} & -\frac{R_s}{L_m} & \frac{R_s}{L_m} & \frac{R_s}{L_m} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad d = \frac{R_s L_{\text{tr}}}{L_m},$$

$$A_1 = \begin{bmatrix}
-\frac{R_s}{L_m} & \omega_{\text{slip}} & -\frac{R_s}{L_m} & \frac{R_s}{L_m} & \frac{R_s}{L_m} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad A_2 = -\frac{R_s L_{\text{tr}}}{L_m} d \begin{bmatrix}
0 & 1 & 0 & 0 & 0
\end{bmatrix}.$$  

(12)

Furthermore, $\sigma$ is the leakage or coupling coefficient, given by $\sigma = 1 - L_m^2 / (L_s L_d)$, $R_s^*$ is the stator transient resistance, with $R_s^* = R_s + (L_m^2 / L_s)$.

After applying the N-MRAC (8) to the IM model, once considering its nonlinear parameters and functions $a, \beta, a(y)$, and $b(y)$, the HST controller conforming the voltage curve $V^*_{\text{sd}}$ of (2) takes the form:
The following section describes the HST-SCS configuration tested.
Figure 3. Test bench configuring. (a) Mirror-flipped picture of the test bench. (b) Block diagram of the test bench.

4.1. HST-SCS Experimental Setup

The experimental setup considers the standard HST-SCS from [14], the previous HST-SCS from [17], and the HST-SCS proposed herein for comparison purposes. Table 2 presents the tuning parameters.

For comparison purposes, with the existing HST-SCS proposed in [17], the following sections repeat the tests made in the manuscript [17]. Hence, the test starts at 0 s and results are shown for over 6 s. Moreover, the applied speed command considers the step wave
speed of 200 rpm, 100 rpm, 1500 rpm, 1300, 1100 rpm, at times 1 s, 1.4 s, 1.7 s, 4 s, and 5 s, respectively. Furthermore, the following section repeats the tests made in [17] for a starting load torque equals 16% of rated torque and 100% of rated torque.

Table 2. Tuning parameters for HST-SCS [17] versus proposed HST-SCS.

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4.2. Comparative Experimental Results and Discussion

4.2.1. Starting Load Torque equals 16% $T_{\text{nom}}$

The Prony brake, acting as a load, was set at 21 NM at the rated angular rotor speed of 1455 rpm. It results in a load torque equals 19.6 Nm (16% of the rated torque) at 200 rpm and 100 rpm, 21 Nm at 1500 rpm, 20.7 Nm at 1300 rpm, and 20.5 NM at 1100 rpm. Then, we developed the comparative experiment, having the previously described speed command considered in [17].

Figures 4–6 display standard SCS, previous HST-SCS [17], and the proposed HST-SCS experimental results, respectively. Each figure presents the oscilloscope waveforms of the required stator phase voltage amplitude $V^*_s$, the rotor angular speed $\omega_r$, and the consumed stator phase current $i_{*a}$ and $i_{*b}$. As described in Figure 3, the oscilloscope only has four measurement channels, with two of them displaying the signals $V^*_s$ and $\omega_r$ obtained from the OPAL-RT. The first one has a 200 V per division as indicated in the oscilloscope. The second one has a 1000 rpm per division, although the oscilloscope shows its unit in $V$.

Figure 4. Oscilloscope waveforms of $V^*_s$, $\omega_r$, and $i_a$ and $i_b$, for the standard HST-SCS [14].
As shown in Figures 4–6, and discussed in [17], the HST-SCS scheme has a peak of 45 A of starting current, lower than the 75 A of standard SCS. This last is characterized by applying the $V_{\text{boost}}$ voltage at $\omega_{\text{min}}$ instead of 0, such as the HST-SCS schemes. Thus, standard SCS has a rotor speed delay of 0.1 s. The HST-SCS methods start at 1.4 s, faster than the standard SCS, which begins at 1.5 s. These are the advantages of the HST-SCS strategies, having a faster speed response and a lower peak current consumption, when compared with standard SCS. Furthermore, the following section shows the additional advantages of starting with a load torque of 100% of the rated torque.

**Remark 2.** The obtained results validate in practice the N-MRAC proposed for nonlinear systems and applied to the HST-SCS for IM as an alternative to the N-APBC used in [17].
For all three methods, the voltage curves change smoothly. The adaptive starting voltage curves $V_{s0}^*$ switches to the boost voltages curves $V_{s1}^*$ around 1.65 s for the HST-SCS methods, and from $V_{s1}^*$ to the $V/f$ voltage curve $V_{s2}^*$ around 3 s, including the standard SCS. Moreover, the actual rotor speed follows the required rotor speed with a steady-state accuracy of 1.5% for speeds over 500 rmp. For lower speeds, the HST-SCS strategies show 1.0%, lower than the 1.3% of standard SCS.

Let us now discusses the advantages of the proposed HST-SCS (results are shown in Figure 6) over the previous HST-SCS [17] of Figure 5. As expected, the MRAC current controller shows a smoother response than APBC. Hence, the proposed HST-SCS has a flatter and lower required voltage $V_{s}^*$ and consumed currents $i_a$ and $i_b$. The peak current of the proposed HST-SCS is 36 A, lower than the 40 A of the previous HST-SCS [17].

4.2.2. Starting Load Torque equals 100% $T_{nom}$

The Prony brake applied 50 NM with the IM running at the rated angular rotor speed of 1455 rpm. This tuning results in a load torque, equals 49.2 Nm (the rated torque) at 200 rpm and 100 rpm, 50.6 Nm at 1500 rpm, 50.4 Nm at 1300 rpm, and 50.2 Nm at 1100 rpm.

Figures 7 and 8 show the previous HST-SCS [17] and proposed HST-SCS experimental results, respectively. Each figure presents oscilloscope waveforms of a-b line voltage and phase currents, during 100 ms around 4.2, and oscilloscope waveforms of $V_{s}^*$, $\omega_r$, and phase currents $i_a$ and $i_b$.

The accomplished experiments verified the results from [16,17] confirming that standard SCS cannot start with rated load torque. Here, Figures 7 and 8 show that, as expected, the IM with HST-SCS started loaded with a nominal torque load achieving the proposed goal. It was done after the HST-SCS applied a DC voltage magnetizing the IM, such as the field-oriented control (FOC). The previous (see Figure 7) and proposed HST-SCS (see Figure 8) have a step stator current reference at 1.4 s after enabling the VFD.

Moreover, there is no speed and current chattering issues reported in [16], improving the HST-SCS proposed in [16]. It was achieved after all voltage curves changed smoothly, when switching from the voltage $V_{s0}^*$ (adaptive starting voltage curve) to the $V_{s1}^*$ (boost voltage curve) around 2.3 s and from the $V_{s1}^*$ to the $V_{s2}^*$ ($V/f$ voltage curve) around 3.6 s.

There is a higher speed-response time at the beginning of the simulation. It results from the starting-adaptive controller action, which assures HST capability in this zone. Furthermore, the actual rotor speed follows the required rotor speed with a steady-state accuracy of 2%, as expected for a VFD with the SCS method. It is due to having an open-loop speed control. In contrast to SCS methods, as described in [17], closed-loop speed control FOC or DTC schemes have 0.01% steady-state speed accuracy, and sensorless strategies have 0.1%.

Let us now compare the results of the previous HST-SCS from [17], described in Figure 7, and the proposed HST-SCS shown in Figure 8. Again, as in previous subsection, the proposed current N-MRAC shows a smoother $V_{s}^*$ and $I_{s, starting}$ response than the N-APBC proposed in [17]. The proposal has a slightly lower DC starting current than the previous HST-SCS, 20 A versus 23 A, respectively, between 1.4 s and 2 s. Moreover, the proposed HST-SCS peak current is 48 A, lower than the previous HST-SCS 56 A. As a result, the proposed HST-SCS has lower integral absolute error (IAE) and integral square input (ISI) indexes than the previous HST-SCS proposed in [17]. These are 38.3 and $2.613 \times 10^5$ for the proposed HST-SCS and 43 and $2.618 \times 10^5$ for the previous one.
Figure 7. Previous HST-SCS [17] experimental results. (a) Oscilloscope waveform of a-b line voltage and phase currents, during 100 ms around 4.2 s. (b) Oscilloscope waveforms of $V_s^*$, $\omega_r$, and phase currents $i_a$ and $i_b$. 

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Figure 8. Proposed HST-SCS experimental results. (a) Oscilloscope waveform of a-b line voltage and phase currents, during 100 ms around 4.2 s. (b) Oscilloscope waveforms of $V_s^*$, $\omega_r$, and phase currents $i_a$ and $i_b$.

5. Conclusions

This paper proposes a novel N-MRAC for a class of nonlinear systems and its application to the HST-SCS for IM. It started expanding the direct approach of the MRAC strategy, considering a normalized information vector and fixed gains. Appendix A describes its theoretical proof, while experimental results validate, in practice, a design example applied to the HST-SCS of IM.

The proposed N-MRAC is an alternative solution to the N-APBC proposed in [17]. Future work should consider obtaining indirect and combined N-MRAC approaches.
The proposed HST-SCS with N-MRAC assures 100% of the starting electromagnetic torque surpassing the typical 25% of the standard SCS. It does it at the expense of adding a voltage curve at the start, based on an adaptive controller. Moreover, in the proposed method, there is no need for estimators of IM parameters or variables. The proposal only needs information from the IM nameplate and datasheet for its tuning. Hence, it still has a simple control scheme compared to the schemes of FOC and the direct torque control (DTC).

The comparative experiments carried out, considering the N-MRAC and the N-APBC HST-SCS [17], show that both strategies follow the required speed with high starting torque. However, the N-MRAC exhibits lower performance indexes.

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Abbreviations
The following abbreviations are used in this manuscript:

- IM: Induction motors.
- PC: Personal computer.
- HST: High starting torque.
- SCS: Scalar control scheme.
- APBC: Adaptive passivity-based controller.
- MRAC: Model reference adaptive controller.
- MIMO: Multiple-input multiple-output.
- BIBO: Bounded-input bounded-output.
- FOC: Field-oriented control.
- DTC: Direct torque control.
- RMS: Root mean square.
- PWM: Pulse width modulation.
- IAE: Integral absolute error.
- VFD: Variable frequency drives.
- ISI: Integral square input.

Main Notation
The following main notations are used in this manuscript:

- \( \omega^* \), \( \omega_r \), and \( \omega_e^* \): Required rotor, actual, and electrical angular speed.
- \( \omega_{rn} \) and \( \omega_{en}^* \): Rated rotor and required angular speed.
- \( \omega_{slip} \) and \( \omega_{slipn}^* \): Actual and rated angular slip speed.
- \( e^p \) and \( e^{-ip} \): Park transformation and its inverse.
- \( T_{3 \rightarrow 2} \) and \( T_{2 \rightarrow 3} \): Clarke transformation and its inverse.
- \( V_s^* \): Needed stator voltage amplitude.
- \( V_{sa}^* \) and \( V_{sb}^* \): Needed two-phase alternating instantaneous voltage in \( a-\beta \) coordinates.
- \( u_a^* \) and \( V_{sn} \): Needed three-phase alternating instantaneous voltage.
- \( V_{sn} \): Rated voltage per phase (RMS value).
\( I_{in} \) and \( I_{d} \) Rated and actual current per phase (RMS value).
\( I_{sd} \) and \( I_{sq} \) Actual stator current direct and quadrature component per phase (Instantaneous value).
\( i_s, i_r, \) and \( i_m \) Stator, rotor, and magnetizing instantaneous current.
\( P_{FN} \) Rated power factor.
\( f_n \) Rated electrical frequency in Hz.
\( p \) Number of poles.
\( En \) VFD enable.
\( V_{boost} \) boost voltage.
\( V_{s0} \) Starting voltage curve for HST.
\( V_{s1} \) Vboost voltage curve.
\( V_{s2} \) V/f voltage curve.
\( V_{s3} \) Weakening flux zone voltage curve.
\( P_1 \) and \( P_2 \) Controller parameters for \( V_{s1} \) and \( V_{s2} \), respectively.
\( \omega_c \) Cut angular frequency, switching point between \( V_{s1} \) and \( V_{s2} \).
\( R_s \) and \( R_r \) Stator and rotor resistance.
\( L_s' \) and \( L_r' \) Stator and rotor leakage inductance.
\( L_s, L_r, \) and \( L_m \) Stator, rotor, and magnetizing inductance.
\( \sigma \) Leakage or coupling coefficient, given by \( \sigma = 1 - L_m^2/(L_rL_s) \).
\( R_s' \) Stator transient resistance, with \( R_s' = R_s + (L_m^2 + R_r)/(L_s^2) \).
\( |\Phi_s| \) Stator flux magnitude.

**Appendix A**

**Proof of Theorem 1.** After, subtracting the nonlinear system Equation (7) from the model reference given in (8), adding and subtracting the term \( A_m y(t) \) into the right side of the obtained equation, considering that \( e(t) = y_m(t) - y(t) \), and making some algebraic arrangements, it is obtained that

\[
\dot{e}(t) = -Ke + b(\theta^T\Omega(t) - u(t)),
\]

where \( b \) and \( \theta^T = [A_{1upper} \ 1 \ A_{2upper}] \in \mathbb{R}^6 \) are the ideal controller fixed-parameter, with a positive \( \text{Sign}(b) \). All parameters are assumed to be unknown and constant, which may have a slow variation. The internal dynamics is BIBO and has no influence on the error equation.

Thus, substituting \( u(t) \) (8) into (A1), defining the controller parameters error \( \Phi = \theta^T\Omega_k - \theta(t) \) (which implies that \( \Phi(t) = -\dot{\theta}(t) \) (13)) as \( \theta^T\Omega_k \) is a constant vector, and that symbolically \( \Omega(t) = 100\Omega(t)/\Omega_m \) from (13), assuming \( \omega_r = \omega_s^* \), we obtain the following error model:

\[
\dot{e}(t) = -Ke + b\Phi(t)^T\Omega(t),
\]

\[
\Phi(t)^T = -\Gamma e(t)^{\Phi(t)}.
\]

This error model (A2) has the following associated Lyapunov function:

\[
V(e, \Phi) = \frac{1}{2}e(t)^2 + \text{Trace}(\frac{1}{2} b|\Phi(t)^T\Gamma^{-1}\Phi(t)).
\]

Taking the time derivative of this Lyapunov function we have that

\[
\dot{V}(e, \Phi) = e(t)\dot{e}(t) + \text{Trace}(|b|\Phi(t)^T\Gamma^{-1}\Phi(t)).
\]

Now, the time derivatives \( \dot{e}(t) \) and \( \Phi(t) \) from (A2) are substituted into this last expression. Moreover, the term \( e(t)b\Phi(t)^T\Omega(t) \) becomes \( \text{Trace}(be(t)^{\Phi(t)}\Omega(t)^{\Gamma^{-1}\Phi(t)}) \), after considering...
the two vector properties, where $a^T b = \text{Trace}(ab^T)$. Finally, rearranging and canceling terms due to $b = |b|$ after knowing that $\text{Sign}(b)$ is positive, we obtain

$$V(e, \Phi) = -Ke(t)^2. \quad (A5)$$

This last Equation (A5) shows the first time derivative of the Lyapunov function (A3) is negative semidefinite; thus, the autonomous system (A2) is stable. Moreover, as $e(t)$ and $\Phi(t)$ are stable, and $\Omega(t)$ is bounded, then $\dot{e}(t)$ is bounded. Integrating both sides of (A5) it can be concluded that $e(t) \in \ell^2$. Hence, according to Barbalat's Lemma, the error $e(t)$ is asymptotically stable, concluding the proof. 

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