Human Capital and Patterns of Growth

TESIS PARA OPTAR AL GRADO DE MAGISTER EN ECONOMIA

Alumno: Pedro Antonio Schilling Ferraz
Profesores Guía: Luis Felipe Céspedes, Eduardo Engel

Abril 2024

PATTERNS OF GROWTH AND HUMAN CAPITAL

PEDRO SCHILLING

Abstract. This paper addresses economic growth and its determinants, proposing an endogenous growth model. It examines the role of educational levels in early stages of development in closing gaps and achieving sustainable growth, considering a potential poverty trap. A trade-off between educational levels and net wages funded by labor taxes is highlighted. The model involves investors choosing technologies, leading to four possible equilibriums: permanent growth, steady state, steady state with low initial capital, and a cycle of permanent fluctuations. The importance of education in avoiding poverty traps is emphasized, but it is cautioned that the optimal choice of educational level may depend on short or long-term preferences of policymakers.

"For if you suffer your people to be ill-educated, and their manners to be corrupted from their infancy, and then punish them for those crimes to which their first education disposed them, what else is to be concluded from this, but that you first make thieves and then punish them."

– Sir Thomas More, *Utopia*

1. Introduction

Many countries have failed to fully develop after a high growth initial stage. In preliminary levels of development, they were able to reduce the gaps with the countries in the frontier, but then failed to maintain their growth rate to continue closing such gaps, and instead remained stable in a certain distance.

This is typically addressed as the *Middle Income Trap*. However, we need to be careful with this definition because the classification of middle income country may depend on the measure used. There are at least two natural ways to classify a country on their GDP per capita: The first one depends on thresholds fixed in a certain level, and it is used by the World Bank. The second one is based on the percentage of the GDP per capita of the country relative to the GDP per capita of the USA, at the same year. The latter is often called Catch Up Index (CUI) and is the one we find more insightful because it shows a measure consistent with growth and development gaps: the frontier is not fixed at a certain GDP per capita but is moving forward with the growth of developed countries. According to Agénor (2017), there are three big categories: Low Income (CUI $<$ 22%); Middle Income (22% $\leq$ CUI $\leq$ 55%) and High Income (55% $<$ CUI).

The inconsistency in both definitions can lead to gray areas in the literature. Pruchnik and Zowczak (2017) argue that of all the countries in the world, 60.2% have been classified as middle income in at least one plausible definition of "middle income", in contrast with the 48.4% in the World Bank. To resolve this problem, they introduce a broader concept, known as the *Convergence Trap*: This is, the

Date: November, 2023

Pedro Schilling (pschilling@fen.uchile.cl)
Departamento de Economía, Facultad de Economía y Negocios, Universidad de Chile .

situation where a country's GDP per capita level fails to converge toward one of a more *advanced* economy.

The Convergence Trap is an example that growth is not a continuous or simple path. It has a lot of details, and it is a complex matter that may depend on the context of each individual country. Following the work of Nelson and Phelps (1966), one of the key issues that countries in the Convergence Trap face is that their Human Capital level is not enough to adopt the technologies of the productive frontier, leading those countries to fail in making the leap towards development.

In this work, we will try to approach the development problem following Nelson and Phelps insights of education and Human Capital, but with a temporal ingredient: raising the education level does not have an immediate impact on product, but it is rather deferred. We construct a simple overlapping generations model with endogenous growth, in which the country's educational level in early stages (which is taken as given and defines the level of Human Capital) can either aid or hinder the goal of achieve permanent growth: Given certain parameters of the model, economies fail to make the leap towards becoming *advanced* economies, becoming trapped in a steady state[1]. Since the education level is financed through a labor tax, we observe a natural trade-off: higher education levels correspond to lower net wages. We also assume that education does not affect the product in preliminary stages of development: The Human Capital is only used under the *advanced* technology. Therefore, there will be generations paying for education without receiving any benefit from it. Finally, we inquire about optimal education levels that allow economies with the same parameters to undergo structural changes towards more *advanced* economies.

The mechanism operating in our model is quite simple: there will be investors who, in a coordinated manner, make the decision on the technology production of the final good, choosing between two possible options: A *primitive* technology, which is a simple one sector economy with a production function with constant returns to scale and does not depend on the level of education; or an *advanced* technology, which in turn is a two sector economy, with intermediate and final product firms, whose productivity depends on the level of education via Human Capital. This implies that during the *primitive* face, the agents spends resources in education, but that effort it is not translated into product. This addresses the assumption that education spending does not have an immediate impact in the economy's production, but the effects are deferred. The investors will opt for the technology that yields the highest possible return rate.

We assume that the relevant measure of education is the percentage of the wage that it is spent in education, and not the the value in terms of product. Because we are interested in the effort that economies put in education, it was a more natural way to approach the problem.

To simplify the dynamics, we assume that the educational level is held fixed, and does not change in time. This will lead to four equilibriums given the parameters of the model: The first one is permanent growth: the economy transitions from a *primitive* to *advanced* technology smoothly; the second is a steady state: irrespective of the initial capital, the economy converges to a steady state level and thus fails to trigger perpetual growth dynamics, falling into a convergence trap; the third equilibrium depends on the initial capital: if is low, the economy converges to a steady state even though transitioning could allow the economy to grow indefinitely;

[1]This is conceptually very simillar to the idea of Zilibotti (1994), where he constructs a model where different patterns of growth arise depending on certain thresholds.

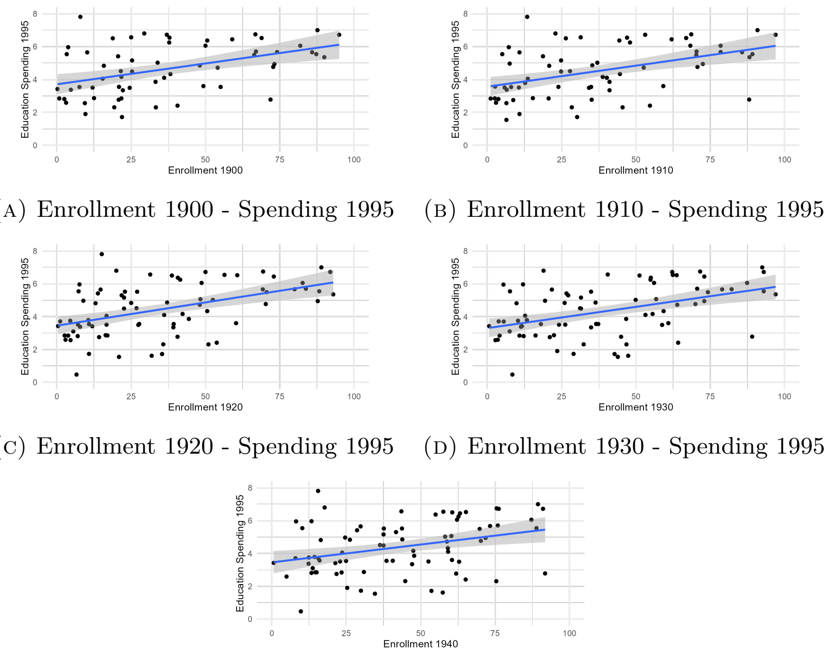
and finally, we have a cycle of permanent fluctuation, where investors switch to *advanced* technology, but the wages under that technology are not high enough to cover the capital level. This implies a decline in capital, and thus, an increase in the interest rate in the *primitive* sector, creating incentives to return to *primitive* technology. Note that three out of these four outcomes have a poverty trap dynamic, preventing the economy from growing indefinitely.

The key parameter of the model is the level of education: Given some parameter conditions, there exists a value in which the economy achieve an equilibrium path that escape the convergence traps and transition towards *advanced* technology. However, we will see that the optimal education level may be contingent on the decision maker's views of the future. If she prioritizes short-term considerations (for example, an electoral cycle), the chosen level of education could be insufficient to enable the economy to transition. Therefore, the choice of education could perpetuate convergence traps instead of aiding in overcoming them.

The remainder of the article is organized as follows. Section 2 presents motivational evidence and discusses related literature. Section 3 outlines the basic model, and Section 4 characterizes the equilibrium. In Section 5, we delve into the solution of the central planner and the optimal education level. Finally, Section 6 provides concluding remarks.

2. Evidence and Related Literature

2.1. **Evidence.** The main assumption of the analysis is that education level in initial stages of growth is a key variable for the development of countries, and through it, the gaps between developed and developing countries can be narrowed. Firstly, because there is little to no data in the early 1900 about education spending, we will investigate the correlation between primary enrollment rate during the 1900-1940 period with high educational spending in the year 1995. We argue that because the preferences are persistent, it is likely that in early 1900 economies with higher enrollment invested more in education. The data used is from the World Bank, and from Benavot and Riddle (1988). These latter data have been employed in other studies, such as Acemoglu et al. (2014).



(A) Enrollment 1900 - Spending 1995 (B) Enrollment 1910 - Spending 1995

(C) Enrollment 1920 - Spending 1995 (D) Enrollment 1930 - Spending 1995

(E) Enrollment 1940 - Spending 1995

Figure 1. Correlation Enrollment Early 1900's and Spending 1995

Figure 1 displays correlation graphs for enrollment in the decades between 1900 and 1940 and education expenditure for 1995. In all these specifications, the correlation is positive and statistically significant, suggesting that preferences for higher education in an economy persist over time. In Annex 1, we provide a table that also includes correlations between enrollment in the same decades but for different years in education spending. Overall, the pattern remains consistent. Because preferences over education are persistent, it is likely to believe that countries with a stronger inclination towards education also invested more in education, in the early 1900.

Now we will proceed to conduct a more systematic study to see if countries with higher primary education percentages at the beginning of the century tend to close the gaps with countries at the frontier decades later. For this purpose, we construct a dummy variable $I_{>60\%}$, which equals 1 if the country has an average enrollment between 1900-1940 greater than 60%. Due to attrition issues, we assume that countries unable to report a percentage for those years should have an average enrollment below 60%, as they had no record of enrolled children. This leads us to believe that education could not have been a priority for them at the beginning of the century.

The models we will study have different dependent variables and can be summarized in the following regression:

$$Y_i \quad = \quad \alpha_1 I_{>60\%} + X_i'\beta + u_i$$

Where Y_i represents possible dependent variables: growth rate between 1950 and 1995 ($\hat{g}_{1950-95}$), the distance to the frontier measured as a percentage of the United States' per capita GDP (y/y_{USA})[2], the Economic Complexity Index of 1995 (ECI) which is a rank of countries based on how diversified and complex their export basket is, and the Economic Complexity Outlook Index (COI), which serves as a measure of how many complex products are near a country's current set of productive capabilities. The latter two variables come from the Economic Complexity Atlas of Harvard University[3]. X is a set of control variables (per capita GDP at the beginning of the sample, state capacity[4], distance to the frontier at the beginning of the sample, population growth between the beginning and end of the sample, average latitude of the country[5], and finally, the Democracy Index[6]). The results are shown in Table 1.

Columns 1 and 2 have the growth between 1950 and 1995 as the dependent variable. Our variable of interest, $I_{>60\%}$, does not have a statistically significant coefficient. This may be because countries can be in distinct stages of development, resulting in both types of technologies leading to high or low growth. Columns 3 and 4 have the distance to the frontier in 1995 as the dependent variable, and our variable of interest $I_{>60\%}$ is positive and statistically significant. This could be because countries with higher education managed to transition from *primitive* to *advanced*, overcoming diminishing returns and narrowing the gap. Columns 5 and 6 show the Economic Complexity Index. Note that the variable of interest $I_{>60\%}$ is significant in one model and has the correct sign in both. Thus, as countries transition from a *primitive* to an *advanced* economy, they complexify their economy because their production process requires a greater variety of goods than before. Finally, in columns 7 and 8 the dependent variable is Complexity Outlook Index (COI). As explained, this index tries to measure the difference between countries in the variety of the products produced, comparing countries with similar capital levels. A higher level of this index is interpreted as a higher distance to the productive frontier. Note that in both specifications, the variable of interest has a negative sign and in the second one is statistically significant, which means that the country with higher education is closer to the productive frontier.

[2]All GDP variables where obtain from the World Bank.

[3]https://atlas.cid.harvard.edu/

[4]The state capacity index was obtained from Hanson and Sigman (2021)

[5]Constructed as the mean latitude of the cities in each country, pondering by population.

[6]This Index comes from the Polity Project

TABLE 1. Primary Education Regressions

	Var. Dep.							
	$\widehat{g}_{1950-1995}$		$\frac{y}{y_{USA}}_{1990}$		ECI_{1995}		COI_{1995}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$I_{>60\%}$	0.043	−0.246	0.303***	0.127*	0.472*	0.140	−0.599*	−0.739**
	(0.245)	(0.261)	(0.052)	(0.067)	(0.239)	(0.267)	(0.330)	(0.347)
$GDPpc_{1950}$	−0.254***	−0.243**			0.339***	0.445***	0.268**	0.268*
	(0.077)	(0.112)			(0.080)	(0.119)	(0.110)	(0.154)
SCI_{1960}		0.622***		0.105***		0.394**		0.490**
		(0.148)		(0.032)		(0.152)		(0.197)
$Dist_{1950}$			0.262***	0.567***				
			(0.036)	(0.117)				
ΔPop_{60-95}	−0.016	0.135	−0.035**	0.001	−0.191***	−0.039	−0.324***	−0.214*
	(0.063)	(0.083)	(0.015)	(0.018)	(0.064)	(0.087)	(0.088)	(0.113)
Lat	0.009***	0.007**	0.001**	0.001**	0.012***	0.011***	0.006	0.005
	(0.003)	(0.003)	(0.001)	(0.001)	(0.003)	(0.003)	(0.004)	(0.004)
Dem	0.040***	−0.004	0.009***	0.0003	0.029**	0.003	0.026	0.004
	(0.014)	(0.016)	(0.003)	(0.004)	(0.015)	(0.017)	(0.020)	(0.022)
Observations	112	90	112	90	106	86	106	86
R^2	0.239	0.299	0.763	0.799	0.672	0.689	0.347	0.434
Residual Std. Error	0.623 (df = 106)	0.577 (df = 83)	0.133 (df = 106)	0.127 (df = 83)	0.607 (df = 100)	0.591 (df = 79)	0.835 (df = 100)	0.767 (df = 79)
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$ Standart Errors in Parenthesis.							

Finally, the model suggests that in countries where policymakers are focused on the short term rather than considering longer time horizons, they are likely to choose a suboptimal level of education for the economy. This is because they would be maximizing the consumption of present generations rather than the present value of consumption overall. Measuring long-term thinking in political intentions is challenging; we will attempt to examine whether countries with higher political stability choose higher levels of education, as the model predicts. This aligns with the argument presented by Besley et al. (2013), who propose that state capacity is built on several factors, including political volatility.

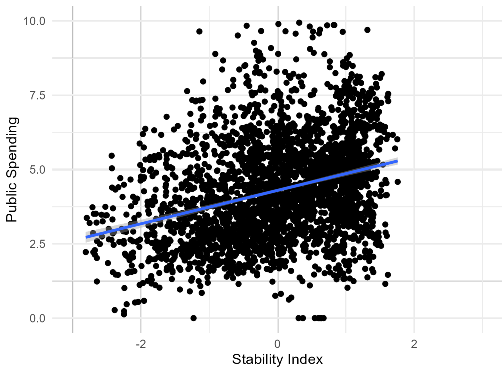


FIGURE 2. Stability Index - Public Spending in Education

Figure 2 depicts the correlation between the World Bank's political stability index and public spending on education, measured by the same institution for a panel of 111 countires between 1996 and 2015. The first variable reflects perceptions of political stability, and therefore, a higher index indicates greater perceived stability. The relationship is positive and statistically significant. Thus, there may be a connection between these two variables, possibly driven by the channel that this work aims to emphasize: political stability translated into long-term public policies aimed at the productive development and technological diversification of a country.

TABLE 2. Primary Education Spending and Political Stability

	Var. Dep.	
	e_it	
	(1)	(2)
Stability_{it}	0.264***	0.734***
	(0.078)	(0.055)
SCI_{it}	0.675***	
	(0.089)	
$Gini_{it}$	−0.001	
	(0.006)	
COI_{it}	0.027	
	(0.052)	
Observations	791	791
R^2	0.252	0.182
Residual Std. Error	1.280 (df = 786)	1.337 (df = 789)
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standart Errors in Parenthesis

This, of course, could be representing another correlation: countries with higher political stability have better institutions, which in turn could result in a greater education spending. To rule out this option, we present in the Table 2 a regression between education spending and political stability in the same time period, controlling by state capacity, inequality and the complexity index. Note that political stability still strongly correlates with the level of education spending, which is what our model predicted.

2.2. **Related Literature.** The study is connected to a broad variety of literature. Firstly, models of poverty traps within theoretical frameworks of endogenous growth have been previously explored. Zilibotti (1994), Acemoglu et al. (2006), Kitagawa and Shibata (2005), and Fukuda (2008) discuss poverty traps arising from frictions or agency problems, either in the capital market or the labor market.

Secondly, the model's relation to the convergence trap opens the door to a substantial body of empirical literature that has delved into studying the determinants of such traps. Works such as Agénor (2017), Yang (2019), Doner and Schneider (2016), and Felipe et al. (2012) explore probable causes and solutions for this trap, characterizing it as an empirical regularity. Agénor, for instance, classifies determinants of the trap and reviews the arguments for each. In his words, "These arguments include diminishing returns to physical capital, depletion of cheap labor, imitation gains, insufficient quality of Human Capital, poor enforcement of contracts and intellectual property protection, distorted incentives and misallocation of talent, lack of access to advanced infrastructure, and lack of access to financing, especially in the form of venture capital".

Thirdly, the study connects with the role of Human Capital in economic growth. Uzawa (1965) and Nelson and Phelps (1966) are seminal papers in this matter. Lucas (1988) formalized the concept of Human Capital. Tran-Nam et al. (1995) and Gaumont and Leonard (2010) specifically examine the dynamics of Human Capital in overlapping generations models. Also, the classic model by Romer (1990) links the importance of Human Capital to economic growth.

Lastly, we can draw connections to the literature on productive public spending, where government interventions through taxes can impact an economy's growth pattern. Glomm and Ravikumar (1997) propose an endogenous growth model where the government plays a productive role in a model of overlapping generations. Childs and Russell (2016) study the relationship between state capacity and the quality of education in the United States, finding that improvements in state capacity had positive implications for educational outcomes.

3. Model

3.1. **Agents.**

The model is based on the work conducted by Iwaisako (2002). The economy is a simple overlapping generations model with two periods. In each period, a generation of constant mass L is born, and each individual lives for two periods, referred to as *young* in the first period and *old* in the second period. In the first period, each individual offers an inelastic unit of labor and divides the labor income between first-period consumption and savings. Additionally, a labor tax of e is imposed, which will finance education. Therefore, the net wage per period for everyone is $w_t(1-e)$. In the second period, each individual consumes their savings and the interest earned. We assume that agents have a separable utility function, particularly a logarithmic one. Furthermore, we assume the existence of a financial market where agents can borrow and save at a future interest rate r_{t+1}[7]. The problem of the agents can be summarized as follows:

$$\begin{aligned}
\max \quad & \ln c_{N,t} + \frac{1}{1+\rho} \ln c_{O,t+1} \\
sa \quad & c_{N,t} + s_t = w_t(1-e) \\
& c_{O,t+1} = s_t \cdot r_{t+1}
\end{aligned}$$

Where c_i represents consumption for the individual in the stage of life $i \in \{N, O\}$, s corresponds to savings, $w(1-e)$ represents the net wage and ρ represents the subjective discount rate. Given the labor income and the rationally anticipated interest rate, the consumer's problem has the following standard solution for savings[8]:

$$s_t = \frac{w_t(1-e)}{2+\rho}$$

Finally, this overlapping generations model could also have a different interpretation: a working economic agent earns income only by supplying labor inelastically, and an investor agent earns income by lending their capital to firms. This interpretation will be used in the analysis because is a natural way of approaching the problem.

3.2. **Technology.**

In this economy, there are two available production technologies: *primitive* and *advanced.* Under *primitive* technology, the production function does not depend on education and is represented as a Cobb-Douglas function with constant returns to scale:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \quad 0 \leq \alpha \leq 1$$

Where Y_t is the output, K_t is the capital and L_t is the number of employed workers at time t.

The *advanced* technology exhibits increasing returns to scale. [9] And because introducing increasing returns of scale may have undetermined equilibriums, we follow the same logic as Benhabib and Farmer (1994): The *advanced* technology will face a two-stage production process. In the first stage, there are firms producing different intermediate goods with the same increasing returns to scale production

[7] Agents have perfect foresight

[8] All the derivations of the model can be found in the Appendix of the paper.

[9] There are various justifications for increasing returns to scale in technologically more advanced economies. For more details, refer to the works of Arthur (1989).

function. In the second stage, firms producing the final good use all intermediate goods in their production process. The production technologies for the final and intermediate goods would be as follows:

$$Y_t = \left[\int_0^1 X_t(i)^\lambda di\right]^{\frac{1}{\lambda}}, \quad 0 < \lambda < 1$$

$$X_t(i) = K_t(i)^a (H(e)L_t(i))^b, \quad a \geq 1 \ b > 0$$

Where Y_t is the same final good as in the *primitive* technology, $X_t(i)$ is the intermediate good, $K_t(i)$ is the capital used by the firm i, $L_t(i)$ of workers used by the firm i, and $H(e)$ represent the Human Capital in the economy. Additionally, note that the goods are complements when λ tends to 0, and they are substitutes when λ tends to 1. Note that Human Capital depends on the level of education. Finally, we assume that when transitioning from one technology to another, the same capital is used. This could be thought of as follows: as investors own the capital, they will lend it to firms that will use it with different production technologies.

3.3. **Production Process.**

3.3.1. *Primitive Technology.* From now on, we will normalize the price of the final good ($p_y = 1$). Under the *primitive* technology, the objective function of firms would be $\Pi^P = AK_t^\alpha L^{1-\alpha} - w_t^p L_t - r_t^p K_t$, where w^p and r^p represent the wage and the relevant interest rate under *primitive* technology at time t. Thus, we know that the results of profit maximization will be the following expressions (using per capita capital notation, $k = K/L$):

$$\begin{aligned} r_t^p &= \alpha A k_t^{\alpha-1} \\ w_t^p &= (1-\alpha) A k_t^\alpha \end{aligned}$$

This are the standart results derived from the Cobb-Douglas production function.

3.3.2. *Advanced Technology.* Under the *advanced* technology, the organization of the industry in this case is different. Since there are two production stages, we will start by solving the second stage through backward induction. The competitive behavior of firms producing the final good will lead to the following objective function:

$$\Pi_t = Y_t - \int_0^1 p_t(i) X_t(i) di$$

where $p_t(i)$ is the price of input i. Obtaining the first-order conditions, we find that the conditioned demand for each intermediate good takes the following form:

$$p_t(i) = Y_t^{1-\lambda} X_t(i)^{\lambda-1}$$

Now, with the demand for each intermediate good, we will proceed to solve the problem of firms in this sector. The profit function in this case takes the following form:

$$\Pi_t(i) = p_t(i) X_t(i) - w_t^A L_t(i) - r_t^A K_t(i)$$

Where w_t^A and r_t^A represent the wage and the interest rate under *advanced* technology at time t. Note that since $\lambda \neq 1$, intermediate goods are differentiated, and firms have a certain degree of monopolistic power. Substituting both the expression for the price and the production technology of intermediate goods, and assuming concavity of the benefits function[10], we will have that the first-order conditions for the firm are:

$$
\begin{aligned}
r_t^A &= \lambda a p_t(i)\frac{X_t(i)}{K_t(i)} \\
w_t^A &= \lambda b p_t(i)\frac{X_t(i)}{L_t(i)}
\end{aligned}
$$

As all firms producing intermediate goods have the same technology, we will have that $X_t(i) = X_t = Y_t$, implying $K_t(i) = K_t$, $L_t(i) = L_t$, and $p_t(i) = p_t$. Also, since profits are zero in the final good sector, we have that $Y_t = p_t X_t$, so $p_t = 1$ and $Y_t = K_t^a(H(e)L_t)^b$. Assuming $a = 1$[11], we will have (expressing in terms of per capita capital $k = K/L$):

$$
\begin{aligned}
r_t^A &= \lambda(H(e)Lt)^b \\
w_t^A &= \lambda b k_t(H(e)L_t)^b
\end{aligned}
$$

Finally, since the firm has increasing returns to scale, the obtained profits are positive and equal to $\Pi = (1-\lambda(1+b))X_t$. Additionally, as we assume that investors own the capital, profits are distributed as dividends to these investors. Thus, the relevant interest rate for investors is the following:

$$
\hat{r}_t^A = r_t^A + \frac{\Pi_t}{K_t}
$$

Replacing with the production technology, we will have that the relevant prices for households will take the following form:

$$
\begin{aligned}
\hat{r}_t^A &= (1-\lambda b)(H(e)L_t)^b \\
w_t^A &= \lambda b k_t(H(e)L_t)^b
\end{aligned}
$$

Here, the λ parameter is key to understand the relationship between the distribution of the resources under the *advanced* technology. In the production process, λ captures the level of substitution of the goods. But λ also can be interpreted as the fraction of the product given to the workers. As we shall see, a balance in the distribution of the resources between employees and investors is needed for the economy to grow permanently.

3.3.3. *Technology Choice.* The technology chosen by the investors will depend on which one yields them a higher return. Thus, we will have that the production function for the final good will take the following form:

$$
\text{Technology} = \begin{cases} \textit{Primitive} & \text{if } r_t^P > \hat{r}_t^A \\ \textit{Advanced} & \text{if } r_t^P \leq \hat{r}_t^A \end{cases}
$$

Therefore, we will have a threshold capital level where the economy transitions from a *primitive* to an *advanced* structure. This level of capital is found when the interest rate equals in both types of technology, i.e., $r_t^P = \hat{r}_t^A$. Substituting the expressions for the interest rate and solving for k, we will have the economy's transition threshold level:

[10] This implies a parameter condition, $\lambda(a+b) \leq 1$

[11] This assumption will help to make the results clearer.

$$k^u(e) = \left(\frac{\alpha}{(1-b\lambda)}\right)^{\frac{1}{1-\alpha}} \left(\frac{A}{(H(e)L)^b}\right)^{\frac{1}{1-\alpha}}$$

Note that the threshold depends on the country's level of education: with higher education, the threshold capital will shift to the left, because the higher education translates into higher Human Capital, which leads to a higher productivity of the capital, leading to a higher interest rate under the *advanced* technology, thus making more attractive this production process rather than the *primitive* one.

4. Competitive Equilibrium and Dynamics

4.1. **Competitive Equilibrium.** The capital market in equilibrium requires that investment be equal to net savings, which is the savings of the *young* minus the dissaving of the *old*. Since the *old* consume all their income in the second period, savings will be defined solely by the decisions of the *young*. Thus, the market equilibrium condition is as follows:

$$k_{t+1} = s_t = \frac{w_t(1-e)}{2+\rho}$$

Note that the wage will depend on the type of technology chosen by investors. In the case of *primitive* technology, we must substitute with w_t^p, resulting in the following:

$$k_{t+1} = \frac{1-\alpha}{2+\rho}(1-e)Ak_t^{\alpha}$$

And, in the case of *advanced* technology, the dynamic of the capital is:

$$k_{t+1} = \frac{\lambda b}{2+\rho}(1-e)(H(e)L_t)^b k_t$$

4.2. **Equilibrium Dynamics.** From the equations presented above, the dynamics of the economy can be characterized by the following expression that depends on the threshold:

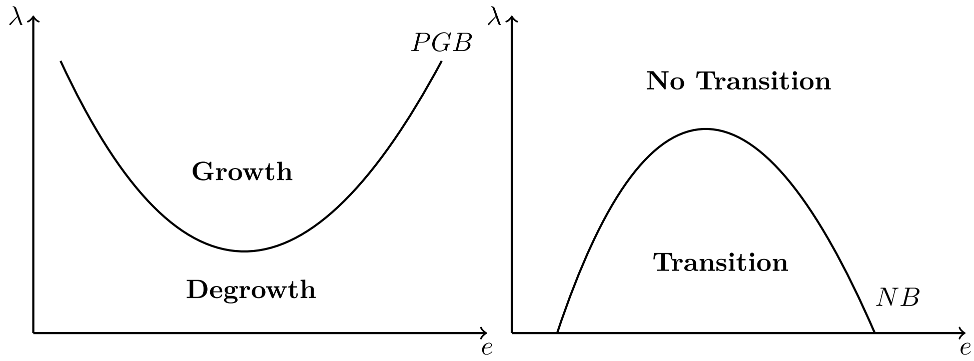
$$k_{t+1} = \begin{cases} \frac{1-\alpha}{2+\rho}(1-e)Ak_t^{\alpha} & \text{if } k_t < k^u(e) \\ \frac{\lambda b}{2+\rho}(1-e)(H(e)L)^b k_t & \text{if } k_t \geq k^u(e) \end{cases}$$

To analyze how the parameter values determine the properties of the model's dynamics, we can use conditions to divide the $e-\lambda$ space into certain regions. We will see that there are 4 possible growth patterns depending on the region we are in: Permanent Growth, Permanent Fluctuation Cycle, Convergence Trap, and a Steady State.

Firstly, we will examine the conditions that must be met for there to be permanent growth in the case of *advanced* technology. The condition is that the capital growth rate must be greater than 1, i.e., $k_{t+1}/k_t = \lambda b L^b(1-e)/(2+\rho) > 1$. Rearranging:

$$\lambda > \frac{2+\rho}{b(H(e)L)^b(1-e)}$$

The name of this condition is the *Permanent Growth Boundary* or PGB. In the region above this line, permanent growth exists. In the region below, it is not possible. Under the assumption that investors receive all the benefits from the firm, λ not only represents the degree of competition in the intermediate goods market but also the percentage of the final good that is distributed to workers. An increase in λ makes the income of labor higher. This line has a U-shape because if there is no education, the productivity level is very low, and therefore to sustain the growth workers must save a great part of their wage, requiring a very high λ. But also, if the level of tax imposed is too high, workers have a lower net wage, so it is also necessary for them to have a high λ, because they need to save more to sustain growth.



(A) *Permanent Growth Boundary* (B) *Neoclassical Boundary*

FIGURE 3. Lines Separators

Secondly, let's look at the conditions of $e - \lambda$ in which a steady state cannot exist under *primitive* technology. We define the steady state as k^{SS}. Substituting $k_{t+1} = k_t = k^{SS}$ into the dynamics of *primitive* technology, we have $k^{SS} = [(1-\alpha)A(1-e)/(2+\rho)]^{1/(1-\alpha)}$. The condition for adopting *advanced* technology before the economy converges to a steady state in *primitive* technology is $k^u < k^{SS}$. Substituting the terms above, we have:

$$\lambda < \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(1-e)(H(e)L)^b}$$

We will call this line the *neoclassical boundary* (NB). This line divides the space into two regions. In the upper region, the economy converges to a steady state under *primitive* technology. In the lower region, the economy will not converge to a steady state. In contrast with the PGB, the NB line has an inverted U-shape because low levels of education lower the productivity of the *advanced* technology, which in turns reduces the incentives of the investors to make the switch. For them to be willing to transition, they must be able to appropriate a large part of the production of the final good, i.e., with a low λ. Note that if taxes are too high, the steady state of the economy will be very low. Therefore, for investors to make the change before the steady-state level, they need to have a higher share of the product, and therefore, a lower λ. In Figure 3, it is shown how these two curves look.

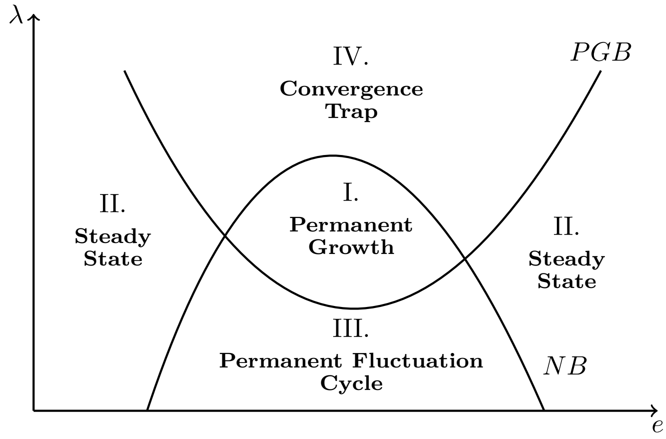


FIGURE 4. Regions plane $e - \lambda$

These conditions of the two lines in the $(e - \lambda)$ space can be seen in Figure 4. The PGB and NB form 4 regions. Let's see what happens in each region. In region I, the economy starts from any level of capital towards a path of permanent growth. The parameters allow the economy to save enough to enhance growth, and investors receive enough return under *advanced* technology to make the switch. In region II, economies converge to a steady state under *primitive* technology. This is, workers do not save enough to assure permanent growth, and investors does not have enough incentives to make the switch between technologies. In region III, all economies exhibit cycles of permanent fluctuation. That is, investors transition to an *advanced* economy, but with this modern technology, wages are not high enough to continue growing, so they decrease to a level of capital where the interest rate is higher again in *primitive* technology, restarting the cycle and fluctuating permanently between technologies. In the case of region IV, all economies have convergence traps depending on their initial capital stock. As λ is high, wages for workers under *advanced* technology are high, potentially allowing growth if the economy were in that technology. However, they can be so high that it would not be convenient for investors to switch technologies, getting stuck at the steady state level.

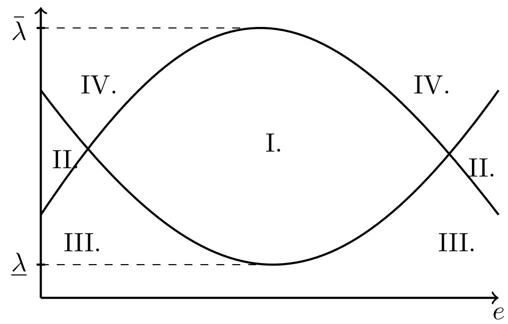
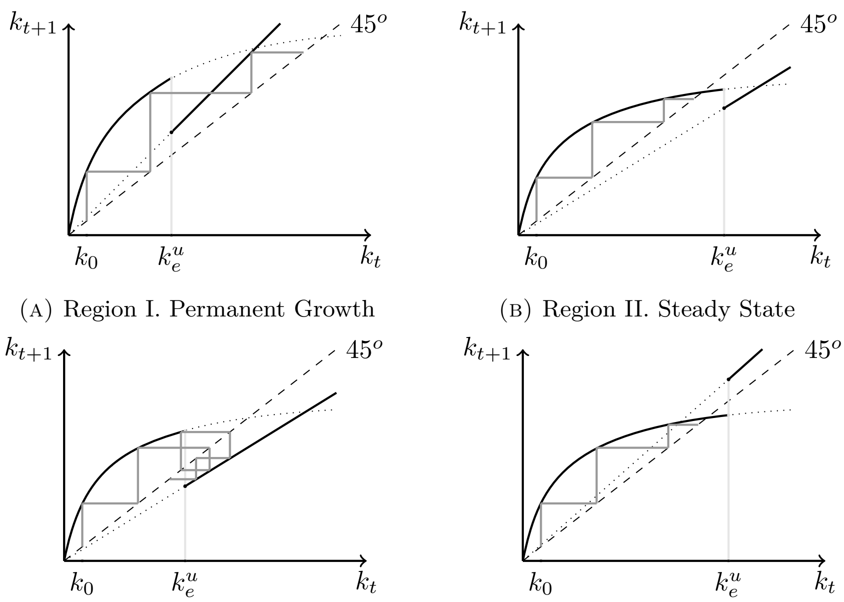


FIGURE 5. Regions plane $e - \lambda$

Notice that in the previous figure, there are parameter regions where education can never play a role in helping countries escape poverty traps. These regions are characterized by λ. If it is very high, investors receive very little of the output under *advanced* technology, so regardless of the productivity of it, they have no incentives to make changes. If it is very low, workers are not earning enough to sustain capital under *advanced* technology, so the economy declines and reverts to the *primitive* production process. Therefore, we will focus on the relevant cases, where $\lambda \in [\underline{\lambda}, \bar{\lambda}]$. This interval is best reflected in Figure 5. Thus, the parameter area analyzed would be all possible combinations of $e - \lambda$ that fall within that region.



(A) Region I. Permanent Growth (B) Region II. Steady State

(C) Region III. Permanent Fluctuation (D) Region IV. Convergence Trap

FIGURE 6. Policy Functions for all four Regions

The policy functions for all four regions are depicted in the Figure 6, and the regions are summarized in the following proposition:

Proposition 1. *This economy has a variety of possible growth patterns depending on the values of* $b\lambda$ *and* e*, as follows:*

(i) *If*

$$\lambda > \frac{2+\rho}{b(H(e)L_t)^b(1-e)} \quad \textit{and} \quad \lambda < \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(H(e)L_t)^b(1-e)}$$

permanent growth is sustained (Region I, Figure 2).

(ii) *If*

$$\lambda < \frac{2+\rho}{b(H(e)L_t)^b(1-e)} \quad \textit{and} \quad \lambda > \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(H(e)L_t)^b(1-e)}$$

the economy will converge to a unique Steady State under primitive technology (Region II. Figure 2).

(iii) *If*

$$\lambda < \frac{2+\rho}{b(H(e)L_t)^b(1-e)} \quad \textit{and} \quad \lambda < \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(H(e)L_t)^b(1-e)}$$

then a cycle of permanent fluctuation emerges (Region III. Figure 2).

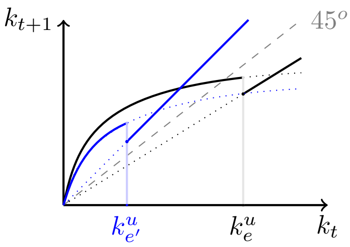
(iv) *If*

$$\lambda > \frac{2+\rho}{b(H(e)L_t)^b(1-e)} \quad \textit{and} \quad \lambda > \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(H(e)L_t)^b(1-e)}$$

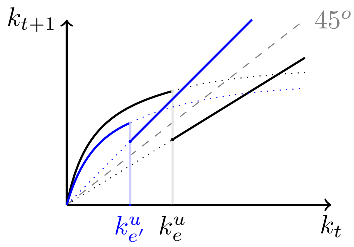
then a convergence trap arises, and the economy will converge to this trap when the initial capital level is less than $k^u(e)$ (Region IV. Figure 2).

4.3. **Role of Education.** Human Capital can play a decisive role in cases where permanent growth is not achievable. Depending on the situation, adjusting the tax rate could facilitate moving from Regions II, III, and IV to Region I, thus achieving sustained growth. However, it's important to note that education spending does not have a direct impact on output. To influence the decision of investors, there must be a certain threshold surpassed for transitioning from convergence traps or steady states to permanent growth.

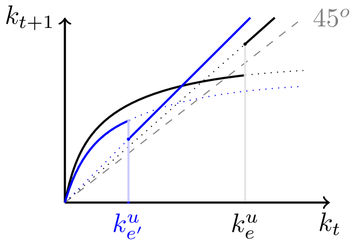
There is a significant trade-off when analyzing the level of education in the economy. A higher level of education leads to increased productivity under *advanced* technology. Consequently, investors would prefer to switch technologies earlier due to the higher return rate associated with the change. In Figure 7 it is shown how higher education can attract the investors to switch from poverty traps to permanent growth.



(A) Region II. Steady State



(B) Region III. Permanent Fluctuation



(C) Region IV. Convergence Trap

FIGURE 7. Policy Function Switch with $e' > e$

However , the raise in education has a counterpart in the capital level of the steady state. Because it is financed with labor taxes, a higher level of education reduces the total savings in the economy, implying a lower level on the steady state of the economy. Note that the raise in education level causes that both the threshold and the steady state to move to the left, so the effects of a raise in the level of education are unclear. This is shown in the Figure 8, where the policy function does not improve despite the higher effort in education.

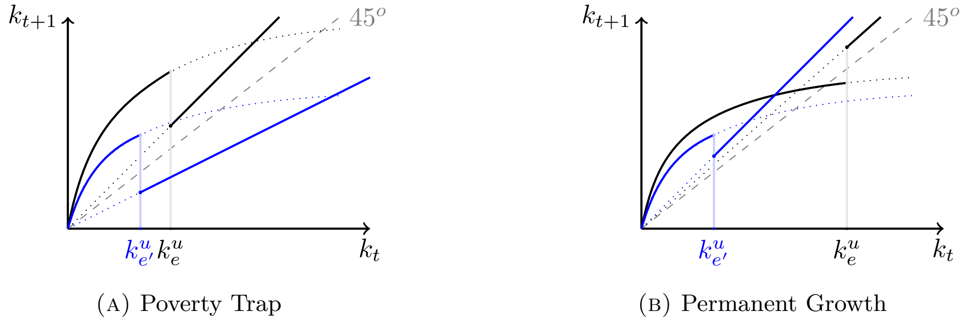


FIGURE 8. Policy Function Switch with $e' > e$

In some cases, the reduction on the education level has an impact via savings that compensate the decrease in productivity that potentially could help the economy on bypassing the poverty trap: because the effect in the steady state is greater than the effect on the threshold, the economy is able to make the transition towards the *advanced* technology.

Additionally, note that given the structure of the model, a corner solution is never desirable, at least from a development perspective. The goal is always to maintain a minimum level of taxes since having $e = 0$ would eliminate the *advanced* sector; and with $e = 1$, savings would be zero, not allowing growth at all.

As final comment, note that education could help the economy to transition from a simple economy (i.e., a simple production function) towards a more sophisticated one (one with two sectors, and intermediate goods).

5. Optimal Level of Education and Centralized Solution

As we saw in the model, Human Capital ($H(e)$) can be a key variable to a country that want to escape the poverty trap. To explore the consequences of different choices of the education level and its impact on the growth dynamics, we will engage in simulations. As in the model, we will assume that the level of education is decided before starting the model's dynamics and then held fixed. Following the example of Acemoglu et al. (2006), we will seek to find the education level that maximizes the discounted present value of total consumption with a discount factor of $\beta = 1/(1+\rho)$. That is, we will maximize $\sum_{j=0}^{T} \beta^j C_j$, where $C_j = c_{0,t} + c_{1,t}$. In the case of the central planner, besides the level of education, she will decide the threshold capital where the transition to an *advanced* economy occurs. Adding this decision, the planner incorporates the effect of the change of technology in the consumption trajectory, in contrast with the market solution where the technology depends on the investors.

To perform the simulation, we will assign certain functional forms to productivity, choose values for the parameters, and select the initial capital level. In Table 3, you will find all the summarized simulation information:

Table 3. Simulation

Parameter	Value
ρ	0.1
L	1.7
A	1
α	0.36
λ	0.35
b	1
$H(e)$	$17 \cdot e$
k_0	0.001
T	15
T_m	5

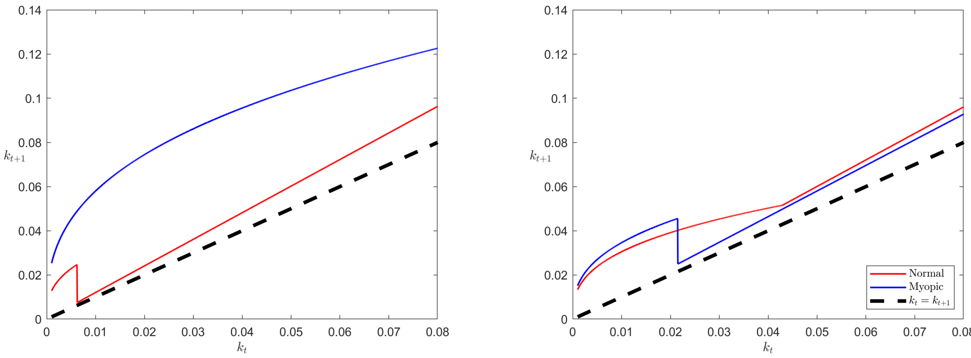
Next, we solve first the decentralized problem and then the social planner's problem. These problems will be solved in a two-stage manner: In the first stage, the economy selects a level of education which cannot be changed; In the second one, the economy follows the path determined by the initial conditions and the level of education. To obtain the optimal level of education, we need to calculate all the possible patterns of development that the economy could follow with every possible level of education, and then choose the one that maximize our objective function. We can think about it as if there exists a decision maker choosing that level of education.

In the planner's problem, at the first stage we need to add the choice of the threshold capital. This means that the problem has another dimension of complexity: Not only she needs to calculate the patterns of growth for all the possible levels of education to a certain threshold; she needs to be done for all possible thresholds.

To have a simpler problem, for education we take a grill of 100 points, as the level of the labor tax in percentages (%). And in the planner's case, we assume that education needs to be financed by the labor tax, prices are obtained in the market, and lastly we reduce the universe of possible capital thresholds to three: when the return rate is equal in both technologies ($r^p = r^a$); when the wage is equal in both technologies ($w^p = w^a$); and when the product is equal in both technologies ($y^p = y^a$).

Finally, we thought that it could be interesting to study what happens if the objective function changes, and instead of taking into account an infinite horizon for the periods, it takes just a limited number. We present the solutions to this idea as *myopic* when the objective function take a finite number of periods, and *normal* when it takes into consideration the lower number of periods in which adding more time does not change the decision. With this, we simplify the problem without losing the insights of the infinite time consideration.

Both market and planner's policy functions are summarized in Figure 9. Note that there is a difference in the policy functions between the decentralized solution and the planner's solution.

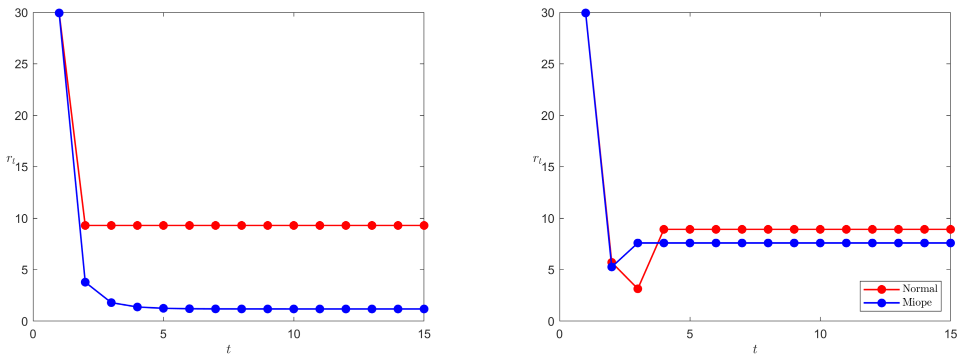


(A) Decentralized Case (B) Central Planner Case

Figure 9. Policy Functions Practical Exercise

The policy function in the *normal* decentralized case it's as usual: There is a threshold in the level of capital (where the return rate of *advanced* capital is equal to the *primitive* one), and finally, they achieve perpetual growth because at that level of education we satisfy the *Permanent Growth* and the *Neoclassical Boundary* Conditions. In the *myopic* case, the decentralized solution does not consider any level of education as desirable: This is because the education is financed by a tax, so increasing the level of education has an impact on the net wage, decreasing the consumption both the *young* and *old* present and near present generations. The *myopic* decision maker, seeing this situation, decide that to maximize the consumption of the present generations, a 0% level of tax is required, leaving the country with no options to develop and leave the poverty trap.

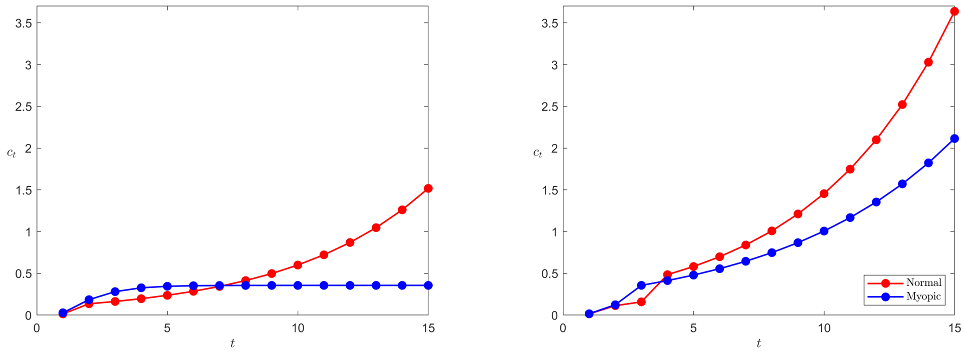
In the case of the *normal* central planner, note that the policy function has not the same shape as the decentralized one. In fact, the planner, even if she chooses the same level of education as the market, maintain the *primitive* production function longer changing to the *advanced* technology just when the wages are equal between technologies. This is because she takes advantage of the marginal productivity of labor that the *primitive* technology has. So, she waits a little longer with a higher wage, accumulating more capital and when the wages are equal, she switches. The cost of this strategy is that the return of the investment to the older generation is smaller than the market strategy, as we can see in Figure 10. But with the accumulation of capital, the savings channel outperforms the rate channel on increasing consumption, and increases the velocity in which capital is accumulating.



(A) Decentralized (B) Planner

Figure 10. Return Rate Trajectory

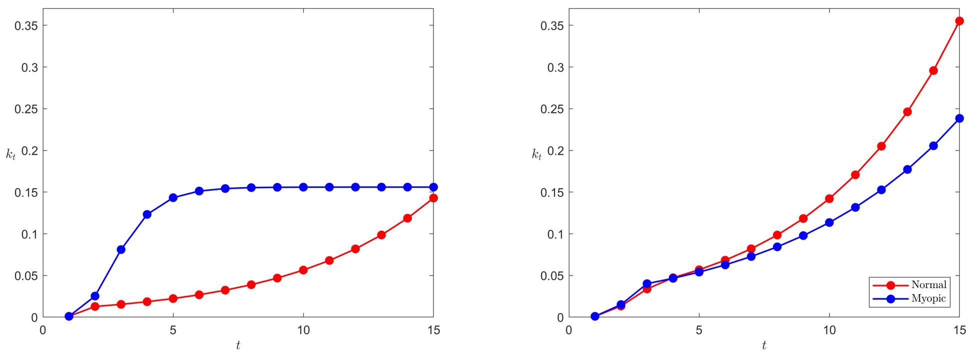
In the *myopic* central planner's case, note that the form of the policy function is not the same as the *normal* case. This is because the strategy followed is not the one that maximize capital accumulation as the previous one: Because the planner is *myopic*, wants to maximize the consumption of a finite number of generations. To do so, the planner then wants to switch not when the wages are equal, but when the products are equal between technologies. With this, the planner is not considering one generation over the other, but rather the overall of consumption and saving of this economy. This leads to a lower level of education, because of the same reason of the decentralized case: a higher labor tax implies a lower save and thus, a lower product in the next period. But, in other hand, the planner does consider a level of education that is consistent with perpetual growth as desirable, because it can set the capital threshold.



(A) Decentralized (B) Planner

FIGURE 11. Consumption Trajectory

In the Figure 11, we plot the consumption trajectory for both planner and decentralized cases, including *normal* and *myopic* time horizons. Here, it is materialized the intuition given above. Firstly, note that the consumption in the planner case is higher in almost all periods of the economy. Secondly, the *myopic* trajectory has a better performance on the first periods, but then it is dominated by the *normal* case, highlighting that in the latter there is a sacrifice of present consumption in pursuit of the future generation's consumption.



(A) Decentralized (B) Planner

FIGURE 12. Capital Trajectory

In Figure 12, it is shown the capital trajectory for all cases studied. It is interesting because it shows that an economy with a large stock of capital does not necessarily consume in a prominent level. The trajectory of the *normal* case is always dominated by the *myopic*, but that capital is not traduced effectively into welfare, because the productivity of that capital is low, considering that the marginal productivity of capital is decreasing under the *primitive* technology. In the *normal* case, a lower capital has higher productivity, allowing to a higher consumption even with a lower capital. And, thinking in distance to the productive frontier, note that at the same level of capital, the *normal* market case is in their productive frontier, in the sense that is a more complex economy. This means that the gap between the two paths are not by accumulation of capital, but by complexity, consumption and product.

Finally, we want to emphasize the effect of the *myopic* decision maker in the selection of education. In the decentralized case, the trajectory of the *myopic* economy stuck in the poverty trap. This can be interpreted as if the decision maker was a political agent, which wants to maximize the consumption of the current generations rather than consider all generations after them, as if it was a way of winning the election to maintain its power. But if the same political agent is thinking in long term, then the economy could avoid that trap by investing in education.

6. Conclusion

In this work, we have provided a model considering Human Capital with endogenous choice of technology. Human Capital depends on the education level, which in turn depends on the labor tax. Depending on a combination of parameters, this can lead to four different patterns of growth: One is a permanent growth equilibrium, where the economy can transition from the *primitive* to the *advanced* technology smoothly, and 3 different poverty traps. We highlight the level of education to escape all the forms of this poverty traps. Higher education increases the productivity in the *advanced* sector, which incentivizes the investors to make the leap towards a more complex economy, achieving permanent growth.

Some insights of the model are that not always more physical capital is better. It could be worth to invest more in Human Capital and improve the productivity and complexity of the production sector in the economy, rather than have a great amount of capital with almost no marginal productivity.

When analyzing the optimal level of education, we studied roughly 4 combinations divided in two planes: In one hand, social planner/market plane; in the other, *myopic*/*normal* plane. The planner outperforms the market solution in almost every scenario; and interestingly, the *myopic* decision is different to the *normal* one. The intuition is that when maximizing present value of consumption for a sufficiently low finite number of periods, then the optimal level of education is rather 0. Making the analogy, if political agents have short run preferences, then the level of education chosen is low because it means that there are low taxes, and more immediate consumption. But overall, this leads to a pattern of underdevelopment. This implies that a political implementation of education is difficult: Even if people are concerned about it, there exists motives to the politicians not to implement such type of reforms, or at least, not to do it in a manifest way.

In future work, it is worth a question on the political economy behind the construction of the labor tax. Besley et al. (2013) has a model in which the state capacity (the capacity of the state to collect taxes) is determined endogenously by the parameters of the model. An interaction between this work and development could shed key insights about the problems that economies that struggle to develop face.

Also, it could be interesting adding the Human Capital dimension to a model with Convergence Trap rather than a poverty trap, as in Acemoglu et al. (2006). Even if the work presented here have useful insights about the Human Capital development idea, the insights about the channel in which Human Capital helps escape the Trap are not very smooth, and it could be better in a model that is actually measuring a distance to the frontier.

References

Acemoglu, D., Gallego, F. A., and Robinson, J. A. (2014). Institutions, Human Capital, and Development. *Annual Review of Economics*, 6(1):875–912. _eprint: https://doi.org/10.1146/annurev-economics-080213-041119.

Acemoglu, D., Zilibotti, F., and Aghion, P. (2006). Distance to Frontier, Selection, and Economic Growth. *Journal of the European Economic Association*, 4(1):37–74. Publisher: Oxford University Press.

Agénor, P. (2017). Caught in the Middle? The Economics of Middle-Income Traps. *Journal of Economic Surveys*, 31(3):771–791.

Arthur, W. B. (1989). Competing Technologies, Increasing Returns, and Lock-In by Historical Events. *The Economic Journal*, 99(394):116–131. Publisher: [Royal Economic Society, Wiley].

Benavot, A. and Riddle, P. (1988). The Expansion of Primary Education, 1870-1940: Trends and Issues. *Sociology of Education*, 61(3):191–210. Publisher: [Sage Publications, Inc., American Sociological Association].

Benhabib, J. and Farmer, R. E. A. (1994). Indeterminacy and Increasing Returns. *Journal of Economic Theory*, 63(1):19–41.

Besley, T., Ilzetzki, E., and Persson, T. (2013). Weak States and Steady States: The Dynamics of Fiscal Capacity. *American Economic Journal: Macroeconomics*, 5(4):205–235. Publisher: American Economic Association.

Childs, J. and Russell, J. (2016). Improving Low-Achieving Schools: Building State Capacity to Support School Improvement Through Race to the Top. *Urban Education*, 52.

Doner, R. F. and Schneider, B. R. (2016). THE MIDDLE-INCOME TRAP: More Politics than Economics. *World Politics*, 68(4):608–644. Publisher: [Cambridge University Press, Trustees of Princeton University].

Felipe, J., Abdon, A., and Kumar, U. (2012). Tracking the Middle-Income Trap: What is it, Who is in it, and Why? *SSRN Electronic Journal*.

Fukuda, S.-i. (2008). Knightian uncertainty and poverty trap in a model of economic growth. *Review of Economic Dynamics*, 11(3):652–663.

Gaumont, D. and Leonard, D. (2010). Human capital, externalities and growth in an overlapping generations model. *Research in Economics*, 64(3):186–200.

Glomm, G. and Ravikumar, B. (1997). Productive government expenditures and long-run growth. *Journal of Economic Dynamics and Control*, 21(1):183–204.

Hanson, J. K. and Sigman, R. (2021). Leviathan's Latent Dimensions: Measuring State Capacity for Comparative Political Research. *The Journal of Politics*, 83(4):1495–1510. Publisher: The University of Chicago Press.

Iwaisako, T. (2002). Technology choice and patterns of growth in an overlapping generations model. *Journal of Macroeconomics*, 24(2):211–231.

Kitagawa, A. and Shibata, A. (2005). Endogenous growth cycles in an overlapping generations model with investment gestation lags. *Economic Theory*, 25(3):751–762.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.

Nelson, R. R. and Phelps, E. S. (1966). Investment in Humans, Technological Diffusion, and Economic Growth. *The American Economic Review*, 56(1/2):69–75. Publisher: American Economic Association.

Pruchnik, K. and Zowczak, J. (2017). Middle-income trap: Review of the conceptual framework. Working Paper 760, ADBI Working Paper.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, 98(5):S71–S102. Publisher: University of Chicago Press.

Tran-Nam, B., Truong, C. N., and Tu, P. N. V. (1995). Human capital and economic growth in an overlapping generations model. *Journal of Economics*, 61(2):147–173.

Uzawa, H. (1965). Optimum Technical Change in An Aggregative Model of Economic Growth. *International Economic Review*, 6(1):18–31. Publisher: [Economics Department of the University of Pennsylvania, Wiley, Institute of Social and Economic Research, Osaka University].

Yang, F. (2019). The impact of financial development on economic growth in middle-income countries. *Journal of International Financial Markets, Institutions and Money*, 59:74–89.

Zilibotti, F. (1994). *Endogenous growth and underdevelopment traps: A theoretical and empirical analysis.* phd, London School of Economics and Political Science.

Appendix

Appendix 1. .

Table 4. Correlations: % Enrolled 1900-1940 vs. Education Expenditure 2000-2010

Variable	Ed. Sp. 1995	Ed. Sp. 2000	Ed. Sp. 2010	Ed. Sp. 2020
Enroll. 1900	0.437***	0.264*	0.372***	0.180
Enroll. 1910	0.390***	0.153	0.340***	0.079
Enroll. 1920	0.370***	0.259**	0.392***	0.177
Enroll. 1930	0.409***	0.252**	0.357***	0.193*
Enroll. 1940	0.427***	0.251**	0.265**	0.049

Note: * $p < 0.1$, **, $p < 0.05$, *** $p < 0.01$

Appendix 2.

Proposition 2. *The saving solution to the consumer's problem is* $s_t = \frac{w_t(1-e)}{2+\rho}$

Proof. The consumer's problem is the following:

$$\begin{aligned} \max \quad & \ln c_{0,t} + \frac{1}{1+\rho} \ln c_{1,t+1} \\ sa \quad & c_{0,t} + s_t = w_t(1-e) \\ & c_{1,t+1} = s_t \cdot r_{t+1} \end{aligned}$$

Replacing the expressions of the consumption, we simplified the problem to:

$$\max \quad \ln(w_t(1-e) - s_t) + \frac{1}{1+\rho} \ln(s_t \cdot r_{t+1})$$

The First Order Condition is:

$$-\frac{1}{w_t(1-e) - s_t} + \frac{1}{1+\rho}\frac{1}{s_t} \quad = \quad 0$$

Solving to s_t, we have that:

$$s_t = \frac{w_t(1-e)}{2+\rho}$$

□

Proposition 3. *The return rate and the wage under the advanced technology are:*

$$\begin{aligned} w_t^a &= b\lambda k_t \left(H(e)L\right)^b \\ \hat{r}_t^a &= (1 - \lambda b) \left(H(e)L\right)^b \end{aligned}$$

Proof. As this is a two-sector model, we are going to solve with retroactive induction. The problem for the final good firms is:

$$\Pi_t^f = \left(\int_0^1 X_t(i)^\lambda di\right)^{\frac{1}{\lambda}} - \int_0^1 p_t(i) X_t(i) di$$

The First Order Conditions with respect to the intermediate good i, gives us the demand for that good:

$$\begin{aligned} p_t(i) &= \frac{1}{\lambda}\left(\int_0^1 X_t(i)^\lambda di\right)^{\frac{1-\lambda}{\lambda}} \lambda X_t(i)^{\lambda-1} \\ p_t(i) &= Y_t^{1-\lambda} X_t(i)^{\lambda-1} \end{aligned}$$

The intermediate firms anticipate the demand, and incorporate the response of the final good firm in the maximization problem:

$$\begin{aligned} \Pi_t(i) &= p_t(i)X_t - w_t^a L(i) - r_t^a K_t(i) \\ &= Y_t^{1-\lambda} K_t(i)^\lambda \left(H(e)L(i)\right)^{\lambda b} - w_t^a L(i) - r_t^a K_t(i) \end{aligned}$$

The firm of intermediate goods takes Y_t as given, following the work of Benhabib and Farmer (1994). The Firs Order Conditions are:

- For w_t^a:

$$\begin{aligned} w_t^a &= \frac{b\lambda Y_t^{1-\lambda} K_t(i)^\lambda \left(H(e)L(i)\right)^{b\lambda}}{L(i)} \\ w_t^a &= \frac{b\lambda p_t(i)X(i)}{L(i)} \end{aligned}$$

- For r_t^a:

$$\begin{aligned} r_t^a &= \frac{\lambda Y_t^{1-\lambda} K_t(i) \left(H(e)L\right)^{b\lambda}}{K_t(i)} \\ r_t^a &= \frac{\lambda p_t(i)X_t(i)}{L_t(i)} \end{aligned}$$

Because all intermediate firms face the same demand and have the same technology, the problem is symmetric and $L(i) = L$, $K_t(i) = K_t$ and $X_t(i) = X_t$. This implies:

$$Y_t = \left(\int_0^1 X_t(i)^\lambda di\right)^{\frac{1}{\lambda}} = \left(\int_0^1 X_t^\lambda di\right)^{\frac{1}{\lambda}} = X_t$$

Then, the price for the intermediate good is $p_t = 1$. Replacing in the expressions of the wage and interest rate:

$$\begin{aligned} w_t^a &= \frac{b\lambda X_t}{L} \\ w_t^a &= b\lambda k_t (H(e)L)^b \end{aligned}$$

$$\begin{aligned} r_t^a &= \frac{\lambda K_t \left(H(e)L\right)}{K_t} \\ r_t^a &= \lambda \left(H(e)L\right) \end{aligned}$$

Finally, because of the increasing return production function of the intermediate firm, the benefits in that sector are positive. Assuming that there are no resources wasted, we assume that the firm pays dividends to the capital owners. Then, the return rate of capital is:

$$\hat{r}_t^a = r_t^a + \frac{\Pi}{K_t}$$

The expression for the intermediate good firm benefits is:

$$\Pi_t = (1 - \lambda(1+b))K_t\,(H(e)L)^b$$

So, the relevant prices of the economy can be expressed as:

$$\begin{aligned} w_t^a &= b\lambda k_t\,(H(e)L)^b \\ \hat{r}_t^a &= (1-\lambda b)\,(H(e)L)^b \end{aligned}$$

□

Proposition 4. *The threshold capital where the economy changes from primitive to advanced technology is:*

$$k^u = \left(\frac{\alpha}{(1-b\lambda)}\right)^{\frac{1}{1-\alpha}} \left(\frac{A}{(H(e)L)^b}\right)^{\frac{1}{1-\alpha}}$$

Proof. From the equality of the return rates, we have:

$$\begin{aligned} r_t^p &= \hat{r}_t^a \\ \alpha A k^{\alpha-1} &= (1-\lambda b)\,(H(e)L)^b \\ k^u &= \left(\frac{\alpha}{(1-b\lambda)}\right)^{\frac{1}{1-\alpha}} \left(\frac{A}{(H(e)L)^b}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

□

Proposition 5. *The NBL curve is given by the following condition:*

$$\lambda < \frac{1}{b} - \frac{\alpha(2+\rho)}{b(H(e)L)^b(1-e)}$$

The PGB curve is given by the following condition:

$$\lambda > \frac{2+\rho}{b(H(e)L)^b(1-e)}$$

Proof. For the NBL, we need to know the conditions where the steady state under the *primitive* technology is after the threshold. So, the curve is given by the following:

$$\begin{aligned} k^{SS} &> k^u \\ \left(\frac{(1-\alpha)(1-e)}{2+\rho}\right)^{\frac{1}{1-\alpha}} &> \left(\frac{A\alpha}{(H(e)L)^b(1-\lambda b)}\right)^{\frac{1}{1-\alpha}} \\ 1-\lambda b &> \frac{A\alpha(2+\rho)}{(1-\alpha)(1-e)(H(e)L)^b} \\ \lambda &< \frac{1}{b} - \frac{A\alpha(2+\rho)}{b(1-\alpha)(1-e)(H(e)L)^b} \end{aligned}$$

On the other hand, we need a condition that under *advanced* technology the economy can in fact growth indefinitely. This is:

$$\begin{aligned} k_{t+1} &> k_t \\ \frac{(H(e)L)^b\,k_t(1-e)\lambda b}{2+\rho} &> k_t \\ \lambda &> \frac{2+\rho}{b(H(e)L)^b(1-e)} \end{aligned}$$

□