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ANÁLISIS DE ESTABILIDAD DE LAS CALIFICACIONES DE RIESGO CREDITICIO DE CDOS SINTÉTICOS

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL INDUSTRIAL.

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RESUMEN DE LA MEMORIA
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El objetivo de este trabajo es analizar la estabilidad de las calificaciones de riesgo crediticio (*ratings*) de un tipo de derivados de crédito conocido como *synthetic Collateralized Debt Obligations* (CDO sintéticos).

Durante la crisis *subprime* gatillada el 2007, la mayoría de los derivados de crédito tipo CDO tuvo un muy mal desempeño. Debido a que cada CDO poseía una calificación de riesgo crediticio, este mal desempeño evidenció la falta de precisión de los ratings de las agencias calificadoras. En este contexto, este trabajo se enfoca en los CDO sintéticos, por dos motivos. Primero, pues ellos tuvieron un rol protagónico en la crisis *subprime* al transformarse en uno de los instrumentos favoritos de los especuladores para hacer "apuestas unidireccionales". Y segundo, dado que los CDO sintéticos se transaban en un mercado secundario no regulado, y poco transparente, esto los hace más interesantes como objetos de estudio.

Tradicionalmente, los ratings de las calificadoras de riesgo se han basado en un único estimador, sin considerar el error asociado con éste. Por ello, este trabajo analiza la estabilidad de las calificaciones de riesgo, mediante la estimación de intervalos de confianza y análisis de sensibilidad en función de los distintos parámetros considerados. Este trabajo utiliza la metodología de calificación de Moody's, una de las calificadoras de mayor participación de mercado, que emplea el concepto de pérdida esperada. En el desarrollo de los análisis, se consideró la información a la cual un inversionista habría tenido acceso previo a la crisis *subprime*. Los casos de estudio seleccionados corresponden a CDO sintéticos representativos del mercado global de riesgo de crédito.

Este trabajo concluye que el empleo de un solo valor como medida de riesgo de crédito para los CDO sintéticos es inadecuado. Los intervalos de confianza estimados para la perdida esperada contienen consistentemente más de un rating, es decir, contienen un margen de error significativo. Además, este trabajo revela que la información disponible previa a la crisis *subprime* habría permitido a inversionistas sofisticados haber detectado el peligroso margen de error asociado a los ratings. Por último, este trabajo pone de manifiesto la importancia de reconsiderar la estructura de los marcos regulatorios financieros que en la mayoría de los países se basan en ratings emitidos por calificadoras de riesgo, y por lo tanto, son inherentemente inestables.

Considerando la importancia de las conclusiones de este trabajo, sería interesante extender esta investigación a otras metodologías de calificación de riesgo y a otros tipos de derivados de crédito.

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INDEX OF CONTENTS

INDEX OF FIGURES

INDEX OF TABLES

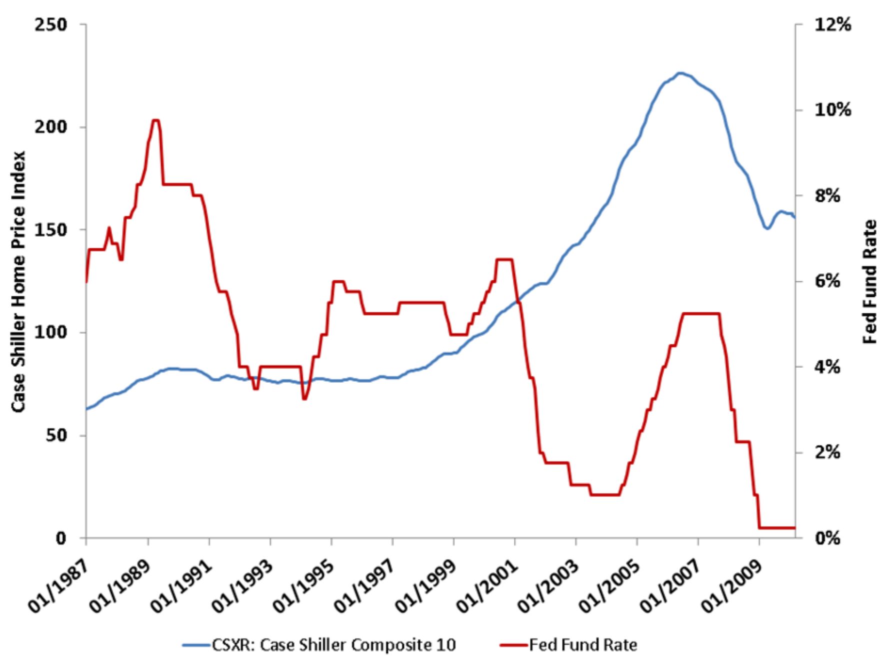
1. INTRODUCTION

The subprime crisis has been widely regarded as the most important financial disruption since the Great Depression. Chiefly, this crisis occurred because the U.S. housing prices experienced significant growth, which created a speculative bubble in the real estate market. This bubble burst in early 2007 causing a market meltdown which lasted until 2008 and beyond. This burst caused huge losses and even bankruptcies that affected many investors and financial institutions[1]. What were the factors that triggered this crisis? So far, among all of the possible causes that experts have identified, there seems to be consensus that two that played a key role were the Credit Rating Agencies (CRAs), and certain class of credit derivatives known as synthetic Collateralized Debt Obligations (CDOs).

1.1 The Subprime Crisis

The origins of the subprime crisis are to be found in the American residential real estate market. The residential real estate prices had a long period of low volatility but experienced a steady increasing trend starting in 2000's (see Figure 1). As Taylor (2007) argues, this increase in price was due to the fact that during the early 1980's there was an active monetary policy by the U.S. Federal Reserve (FED), which focused on two main goals: achieving inflation targets and stimulating the growth of the American economy. Nevertheless, starting around the 2000's the FED changed the policy rule, by lowering the interest rate compared to previous levels. Therefore, mortgage financing became cheaper which caused an increase in real estate loans demand.

Figure 1: Case-Shiller Home Price Index and Federal Fund Reserve Rate



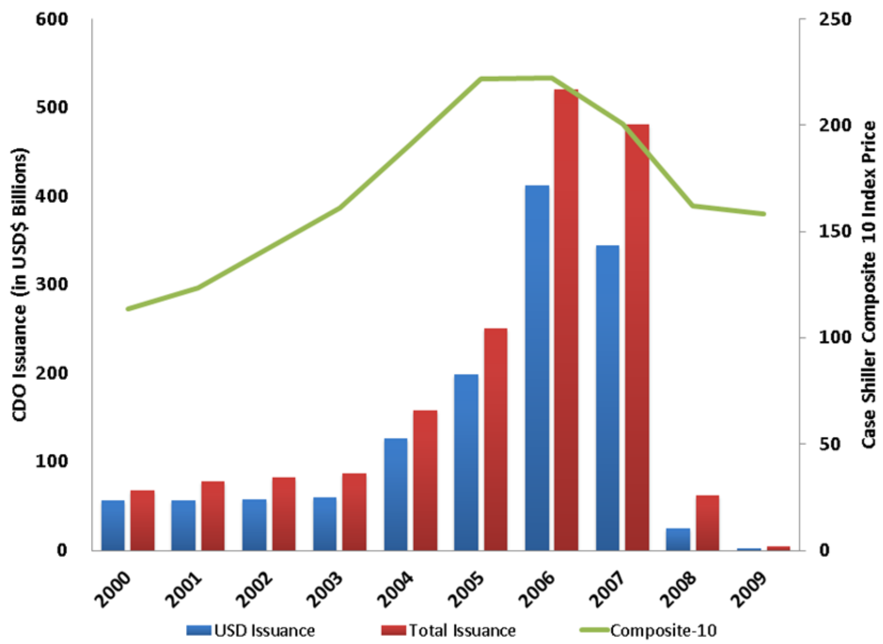
Source: Standard & Poor's, Federal Reserve Bank of New York

[1] See Appendix 1 for further details in bank losses.

Cheap funding was extended not only to the prime but also to the subprime segment that could now afford a mortgage. This, in turn, attracted numerous mortgage lenders, increasing loans demand and resulting in an upward price spiral. Prices grew more than 20% per year in certain parts of the country during that period. All this led to a house-price boom.

With housing demand expanding, investors and banks tried to make a profit from this boom. On the one hand, mortgage lenders needed to create space in their balance sheet to continue lending. To accomplish this goal, mortgage lenders packaged and removed all sort of mortgage credits from their balance sheets, and offered them to investors through different kinds of credit derivative products. On the other hand, institutional investors were attracted to these products, because they offered higher returns compared to securities with a "similar" risk profile. This fueled a vicious circle of lending and repackaging which resembled a snowball. For instance, the issuance of CDOs, one of the most popular credit derivatives, grew tremendously while the house financing conditions remained favorable (see Figure 2).

Figure 2: Global CDO Issuance and Case Shiller Composite 10 Index



Source: Standard & Poor's, Federal Reserve Bank of New York

However, when the FED started increasing interest rates back to its normal levels in late 2006, demand for houses fell quickly and so did the house prices. In this new scenario, homeowners with variable interest rate mortgages faced increasing monthly payments, while the prices of their houses were going down. Therefore, foreclosures and delinquency rates rose critically leaving many mortgages unpaid. Consequently, the cashflows linked to mortgages were interrupted, leaving many CDO investors exposed to losses. Finally, these mortgage-related losses, especially those arising from the subprime market, were spread out among many financial institutions by the CDOs, exacerbating a market meltdown[2].

1.2 The Role of the CRAs and the Synthetic CDOs

Throughout the subprime crisis the CRAs and synthetic CDOs played an important role. CRAs participated in most CDO transactions. In order to issue CDOs, mortgage lenders needed a rating and therefore the approval of the CRAs. The steady growth of the real estate market, in addition to the complexity of this credit derivative made investors relied blindly on the CRAs opinions to make investment decisions. However, the subprime crisis painfully exposed how unstable these ratings were as the CDO experienced substantial losses.

A synthetic CDO is a particular type of CDO structured with Credit Default Swaps (CDSs). CDSs behave essentially as insurance agreements. The difference between a CDS and a synthetic CDO is that the CDS references a single asset, whereas a synthetic CDO refers to a basket of securities. These instruments will be explained in more detail in Chapter 2.

Two important considerations regarding these credit derivatives are worth mentioning. First, CDSs and synthetic CDOs are normally traded over the counter (OTC), which means they lack the transparency that is typical of products traded in regulated exchanges. And second, since they are not subject to short selling restrictions[3], they afford an investor the opportunity to take a long or short position regarding the market. In summary, these instruments allow speculators to take one-way bets regarding the real estate market.

Figures 3 and 4 show the tremendous growth of both the CDS and the synthetic CDO markets. This growth was partly fueled by the need of banks to manage risk and partly by the desire of some investors to take speculative positions.

To sum up, a thorough understanding of the stability of synthetic CDO ratings issued by CRAs can shed light on one of the factors that triggered the subprime crisis. Moreover, getting a better grasp of the methodology behind these ratings can help to identify some critical weaknesses. This could be useful not only for future investors but also for financial regulators, central bankers and government agencies.

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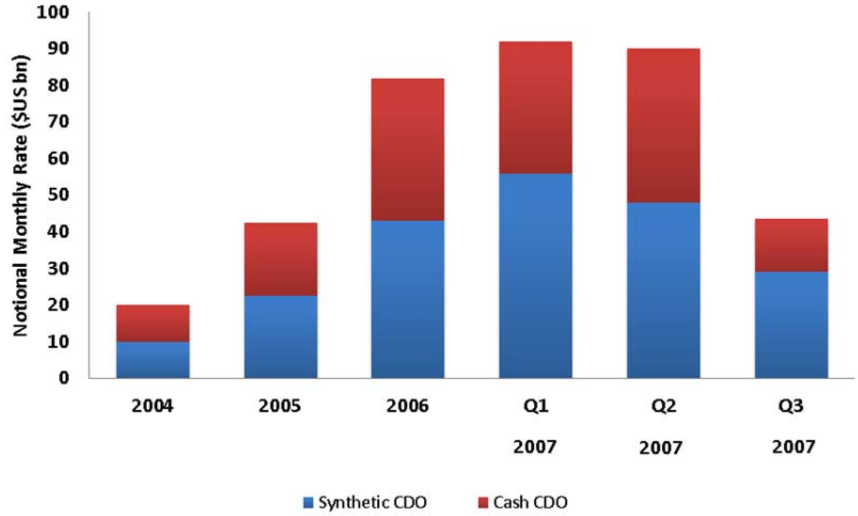
[2] Regarding to this, Warren Buffet a fairly well known American investor stated: “Derivatives are financial weapons of mass destruction”.

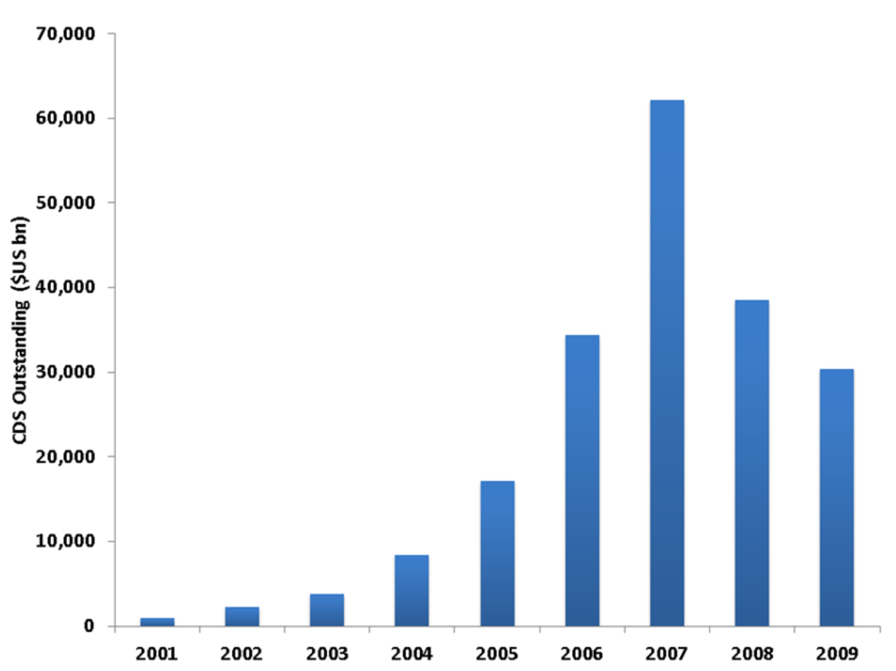
[3] Securities Act together with the Exchange Act encompass the definitions of security and short sale. By 2007, none of these definitions included neither the synthetic CDO nor the CDS. For further explanations, see Lufrano and Pekarek (2011)

Figure 3: Growth of CDO Issuance by Type



Source: Bank of International Settlements

Figure 4: Notional CDS Outstanding



Source: Bank of International Settlements

2. CREDIT DERIVATIVES: BASIC CONCEPTS

2.1 Credit Risk

Credit risk is the likelihood that a borrower will fail to meet on time his debt obligations with his counterparty, the lender. At the root of this concept is the ability or willingness of the borrower to meet his obligations. The borrower can experience financial stress, or deterioration of his solvency or liquidity profile. Alternatively, he can start a legal dispute about the contract or simply lack the willingness to pay. All this factors are collectively encompassed under the concept of credit risk.

The case in which the borrower fails to meet his debt obligation is called a default. The likelihood that this could happen is known as default probability. Under this scenario, the lender will experience a loss whose severity will depend on the percentage of debt that is recovered. The percentage of recovered amount is called recovery rate.

In order to better manage their credit risk firms can enter into contracts known as credit derivatives. Credit derivatives exchange credit risk between parties with a payoff from the seller of protection to the buyer of protection. This payoff depends on the creditworthiness of the securities that are referenced by the contract. As a result, companies are able to trade credit risk in the same way they trade market risk, but generally OTC. The remaining of the chapter describes in more details the most important credit derivatives products relevant to this study.

2.2 Credit Default Swap

A Credit Default Swap (CDS) is a type of credit derivative in which a protection buyer pays a protection fee (premium or spread) to a protection seller during a certain period of time. The CDS makes reference to specific security. If during the life of the CDS the referenced security defaults, the protection seller compensates the protection buyer. This compensation is normally proportional to the loss experienced by the reference security.

Unlike an insurance contract, the protection buyer is not required to own the referenced security in the CDS contract. This situation is known as a naked CDS position. Normally these contracts are traded OTC. The fee paid by the protection buyer is calculated as a number of basis points (bps) per annum times a specified notional amount. Figure 5 shows how the CDS works.

Figure 5: Diagram of the Credit Default Swap Model

Source: Own Elaboration

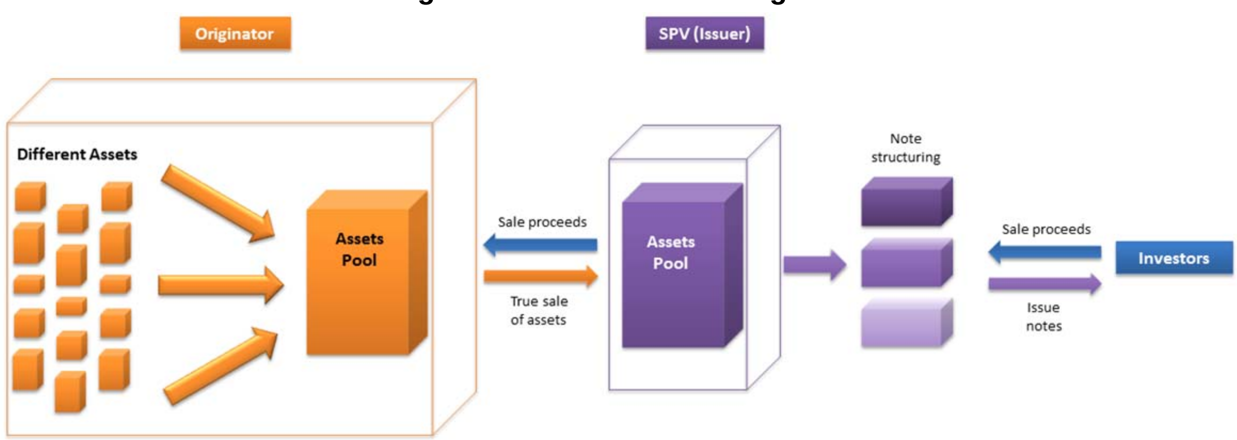
As an example, consider a CDS contract that references a specific security with a notional amount equal to a $100. The premium is 500 bps per annum. Therefore the annual protection premium paid by the protection buyer will be $100 x 500bps = $5. In case there is a credit event[4] (a default of the referenced security) the protection seller will compensate the protection buyer. Assume that the defaulted security is traded at 55 cents on the dollar. Then, the protection buyer will receive a payment equal to $100 x (100% - 55%) = $45.

2.3 Cash CDO

CDOs, in general, are a type of credit derivatives that link a pool of repackaged securities with several investors. Each one of these investors has a different seniority (priority) to receive the cashflows from the pool of securities. As the seniority of the investor increases, his risk profile becomes more conservative. On the other hand, as the seniority decreases, the risk profile of the investor becomes more speculative.

The mechanics of a basic cash CDO will be explained using Figures 6, 7, 8 and 9. First, a diversified pool of assets is placed in a special purpose vehicle (SPV). In principle, the assets in the SPV could be any debt securities. High-yield bonds and bank loans are the most commonly used securities. In reality, at the start of the transaction the SPV purchases the pool with the proceeds from issuing several notes. These notes have different seniorities depending on the priority to receive the cashflows from the SPV (see Figure 6).

Figure 6: Securitization Diagram



Source: Own Elaboration

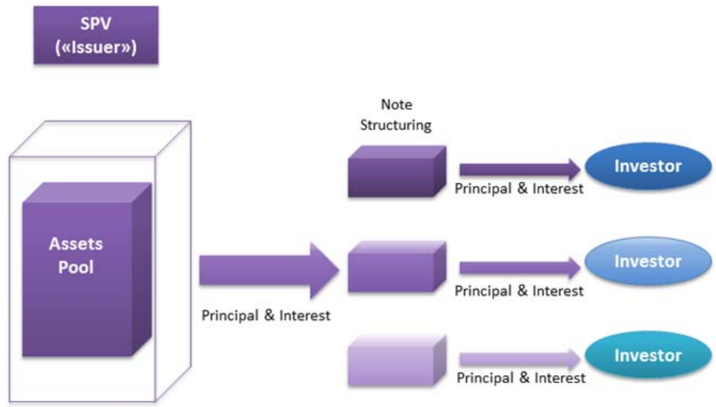
Figures 7 and 8 show the distribution of cashflows. Principal and interest payments originated from the assets in the pool, go first to pay principal and interest to the most senior investors and then sequentially to the subordinated investors. This pay-down schedule is known as waterfall or priority of payments.

Different investors are grouped into different tranches, according to their risk profile. The safest tranche is normally referred to as the senior tranche. The most risky position is referred to as the equity tranche. And the positions in the middle are known as mezzanine tranches.

[4] Appendix 2 explains in more detail the most common credit events used by the standard CDS agreements.

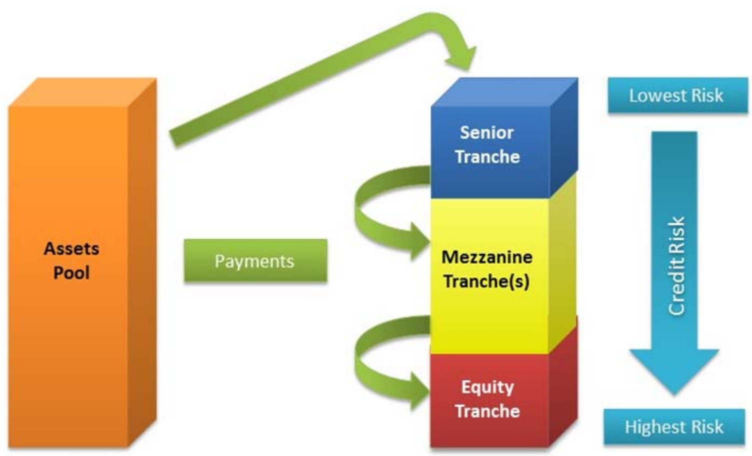
The risk of different tranches not only depends on the seniority, but also in the size of the tranches. As bigger the tranches below a given tranche, the safer it is. However, each one of these tranches can have a different size as long as the sum of their sizes is backed by the notional value of the underlying pool of securities.

Figure 7: After Securitization Diagram



Source: Own Elaboration

Figure 8: Diagram of Payments in the Cash CDO Model



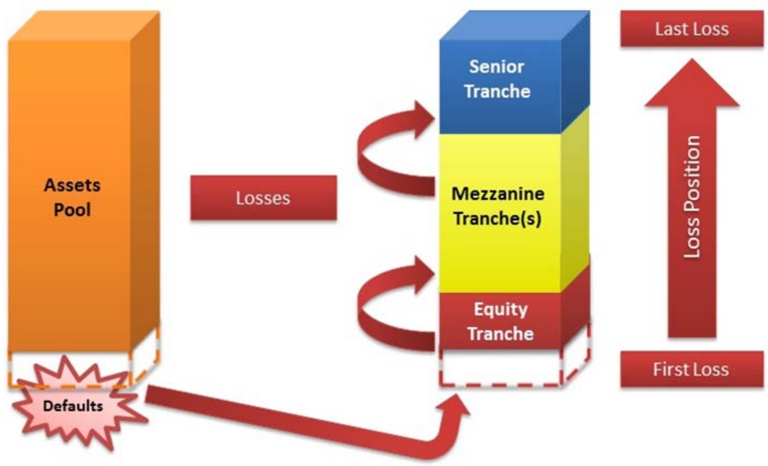
Source: Own Elaboration

In case the pool of assets experience defaults, the pay-down priority schedule applies. First, the senior tranche is paid. Then, the mezzanine tranches and finally, the equity. In other words, if the cashflows are impaired, the equity tranche will be the first to experience a loss. If the impairment of the cashflows is more severe, it could reach the mezzanine investors, and so on. In summary, the losses are applied in reverse order of priority (see Figure 9).

Broadly speaking, the process of repackaging securities, placing them in an SPV, and then issuing securities supported by the cashflows generated by SPV is known as securitization. The rationale for the securitization process is to redistribute the risk of the collateral pool according to the risk appetite of the risk investors. The senior investors take the least amount of risk, whereas the equity investors take the most risk. However, the total amount of risk remains the same. In addition, the securitization permits, in the case of placing illiquid and non-tradable securities into the SPV, the creation of more tradable and liquid securities. Finally, the SPV is bankruptcy remote. This means that if the originator of the assets were to file for bankruptcy, it cannot reclaim the assets that have been sold (true sale) to the SPV. Similarly, if the assets in the SPV default, the note holders do not have any claim against the originator.

From the point of view of the originator of the assets, the motivation for doing the securitization is to create space in his balance sheet and presumably, decrease the cost of capital or perhaps increase the lending capacity.

Figure 9: Diagram of Losses in the Cash CDO Model



Source: Own Elaboration

2.4 Synthetic CDO

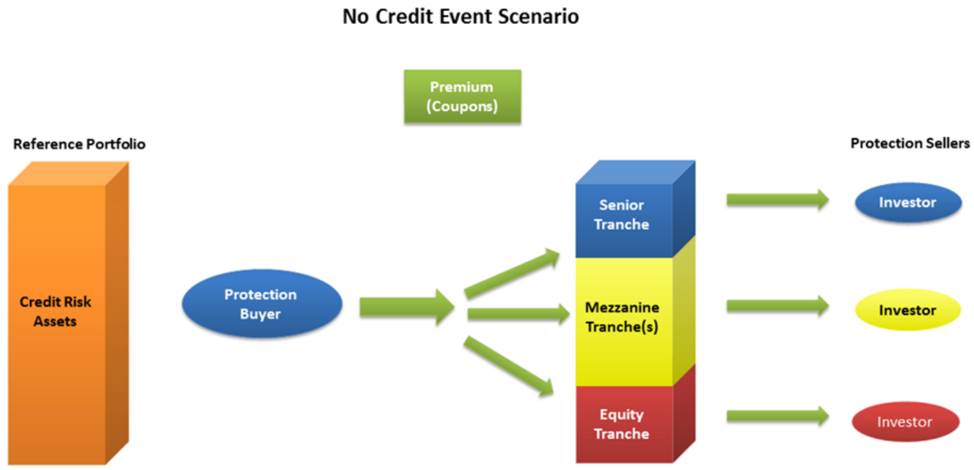
The synthetic CDO is used to transfer the credit risk in a similar way than a CDS contract, but it incorporates a sequential payment mechanism similar to that of a cash CDO. However, unlike a cash CDO that securitizes debt instruments by placing them into a SPV, the synthetic CDO is structured using an array of many CDS agreements without using a SPV. In this case, no notes are issued. Figures 10 and 11 show the mechanics of a synthetic CDO.

In a synthetic CDO, the protection sellers (investors) receive a fee according to their seniority. The lowest fee goes to the most senior investor. Besides, instead of referencing one security, the synthetic CDO references a diversified pool of securities.

If a default occurs in the pool, the losses (payment to the protection buyer) are assigned in reverse order of priority. In other words, when the first credit event occurs, the equity investor will compensate the protection buyer. As the number of defaults increases, eventually, the payment capacity of the equity will be exhausted and the mezzanine investor will start making payments. Finally, under an extreme scenario, the senior investor will have to compensate the protection buyers.

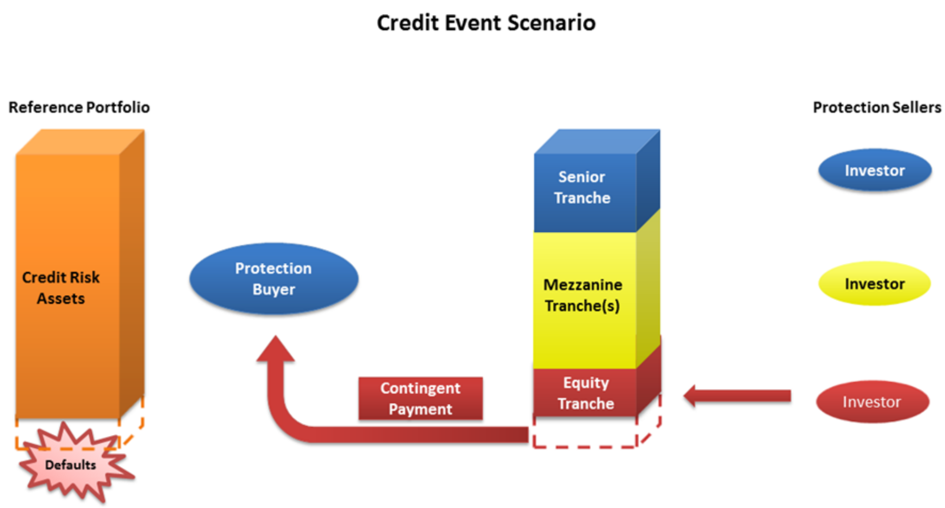
To sum up, at the root of the mechanics of the synthetic CDO, there are two potential sources of losses for the investor. First, he might be forced to compensate the protection buyer in the case of an event of default. And secondly, his premium will be decreased as a result of a reduction in the notional amount of the reference portfolio. This amount will be equal to the severity of the loss.

Figure 10: Mechanics of the Synthetic CDO in a No Credit Event Scenario



Source: Own Elaboration

Figure 11: Mechanics of the Synthetic CDO in a Credit Event Scenario



Source: Own Elaboration

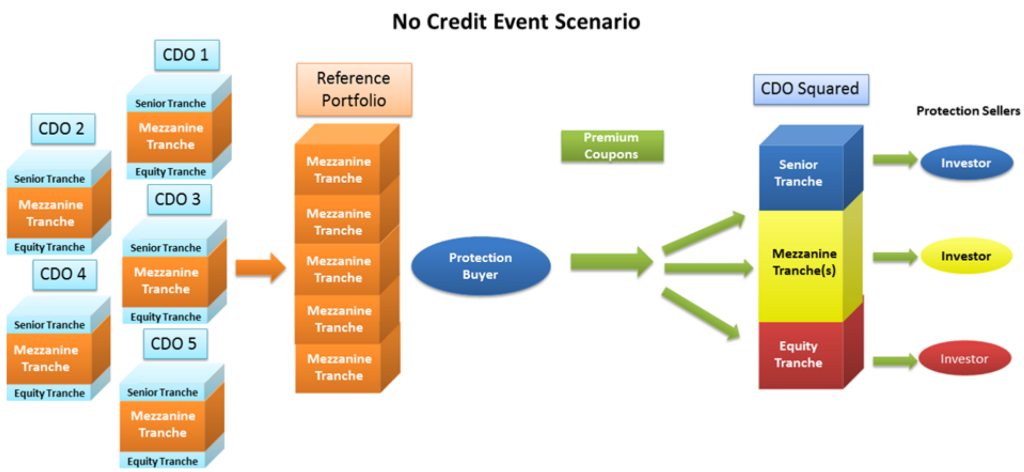
Finally, there are two types of synthetic CDOs, funded and unfunded. In a funded synthetic CDO, initially the investors make a payment equal to the notional amount of the notional securities. This payment, in turn, is used to buy a portfolio of risk free securities. The returns of these securities together with the premium fee paid by the protection seller go to the investor. If there is an event of default, a portion of the risk free securities is liquidated and the proceeds are used to compensate the protection buyer. On the other hand, in the unfunded case there is no initial payment. Thus, in case of a credit event, the protection seller compensates the protection buyer directly. Consequently, the protection buyer - at least in principle – is exposed to the credit risk of the protection seller.

2.5 Synthetic CDO-Squared

In principle, there are two types of CDO-squared: cash and synthetic. For the purpose of this study, we will concentrate on the synthetic CDO-squared.

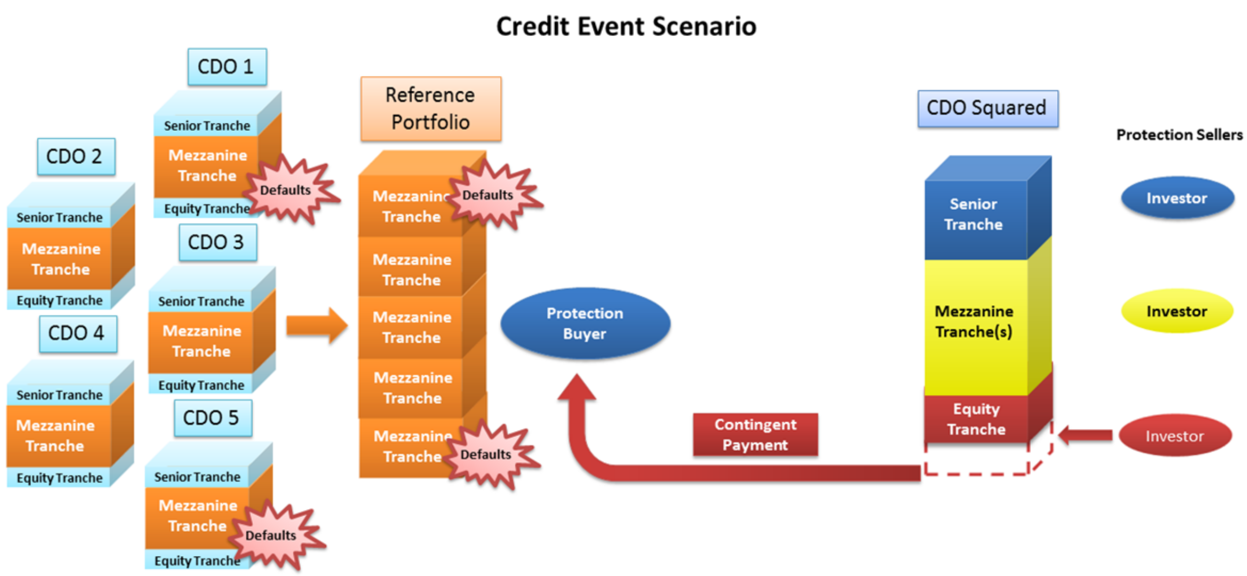
A synthetic CDO-Squared works very much like a conventional synthetic CDO except for one difference: the reference portfolio consists of a pool of mezzanine tranches from previous synthetic (unfunded) CDOs. Figure 12, 13 and 14 show a CDO-Squared example. Very much, like a regular synthetic CDO, once the referenced mezzanine tranches experience credit events, the losses to the investor will be assigned in reverse seniority order. In turn, the losses experienced by the mezzanine tranches will be dictated by the losses in the underlying reference portfolios of the "first order" synthetic CDOs.

Figure 12: Mechanics of the Synthetic CDO-Squared in a No Credit Event Scenario



Source: Own Elaboration

Figure 13: Mechanics of the Synthetic CDO-Squared in a Credit Event Scenario

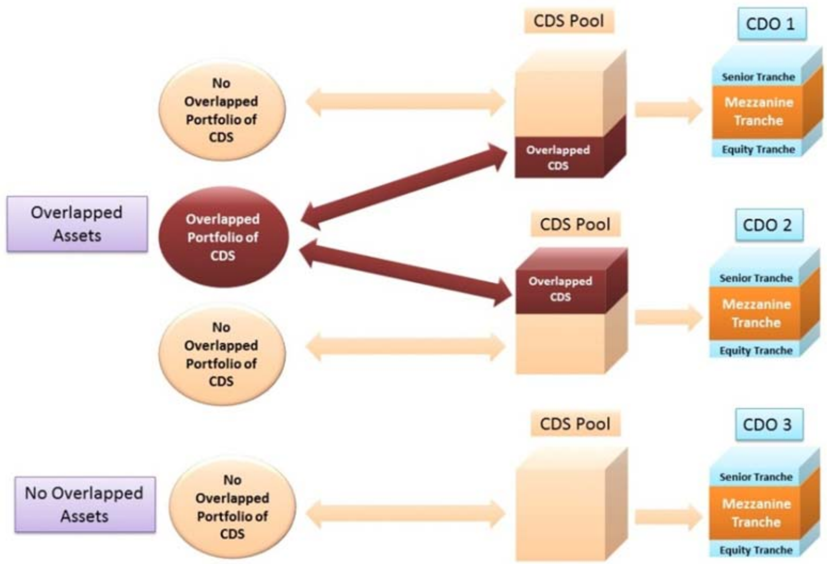


Source: Own Elaboration

An important feature of this type of CDO is the degree of overlap among the underlying pools referenced by the mezzanine tranches. If the degree of overlap is very high, the risk will be more concentrated and one default has the potential to affect many mezzanine tranches. In other words, the effect of a default in such a security could be magnified. This feature is displayed in Figure 14. Alternatively, an asset that is only present in one referenced pool will have a more reduced effect on the integrity of the CDO-squared.

In general, the number of mezzanine tranches referred to in a CDO-squared is much smaller than in a conventional synthetic CDO. Typically, in a conventional synthetic CDO the number of referenced securities will be between fifty and one hundred approximately. In a synthetic CDO-squared, the number of referenced mezzanine tranches is between five and ten.

Figure 14: Case of Overlap among the Underlying Pools Referenced by the Mezzanine Tranches.



Source: Own Elaboration

3. CREDIT RATING AGENCIES AND THEIR RATINGS

The CRAs are private firms that publish credit risk assessments of debt instruments issued in the fixed income market. The assessments of the CRAs are known as credit ratings and are based on the creditworthiness of the debt issuer.

In order to issue a rating, a CRA performs a qualitative and quantitative analysis. The qualitative analysis consists of a due diligence process that focuses on the issuer. This involves looking into several factors such as: management structure and experience of the issuer, operational risk, position within the market place, legal and environmental risk, among others. The outcome of this analysis might influence some of the input values to be employed in the quantitative analysis.

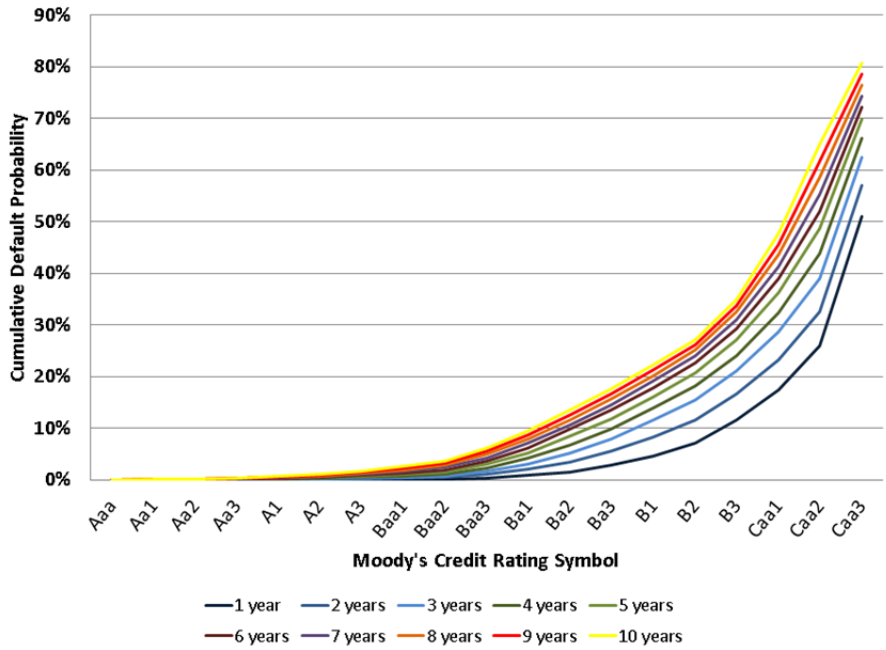
The quantitative analysis involves deciding which model to use along with the value of the key parameters and assumptions. Typically, the model is employed to analyze several scenarios under a variety of “stressful” conditions (stress tests). Finally, the rating is determined by some meaningful metric resulting from running the model. In general, the capacity of the debt to withstand stressful scenarios is proportional to the quality of the rating.

For all practical purposes, there are only three rating agencies: Moody’s, Standard & Poor’s (S&P) and Fitch. All three rating agencies employ 20 categories designated with different alphanumeric symbols (see Appendix 4). The top ten categories are referred to as investment grade, whereas the bottoms ten are known as speculative or non-investment grade.

The different CRAs assess the credit risk using a different focus and different methodologies. While Moody’s credit ratings measure the expected loss of a debt obligation, both S&P and Fitch assess the probability of default. Regardless of the different approach, investors and financial institutions usually consider the ratings by the different CRAs somewhat equivalent. However, in the context of synthetic CDOs, some authors argue that the rating of synthetic CDOs cannot be mapped onto another rating done by a different CRA. This is mostly due to the different rating methodologies. See Tavakoli (2008) and Lucas, Goodman, Fabozzi & Manning (2007). Figure 15 shows the different expected loss target that Moody’s uses for different ratings.

In the case of a CDO, the credit risk is different for different tranches. The senior tranche being the safest tranche offers the lowest yield and receives the highest rating among the CDO tranches. The equity tranche, in turn, being the riskiest tranche offers the highest yield and receives the lowest rating. In general, tranches at the top of the waterfall usually receive an investment grade rating, whereas those tranches at the bottom usually receive non-investment grade rating. During the life of the transaction, the CRAs normally upgrade or downgrade the rating of a given tranche depending on the evolution of the credit characteristics of the underlying pool.

Figure 15: Expected Loss Targets for Moody's Credit Ratings



Source: Moody's

3.1 A Brief History of the CRAs and the Ratings Market

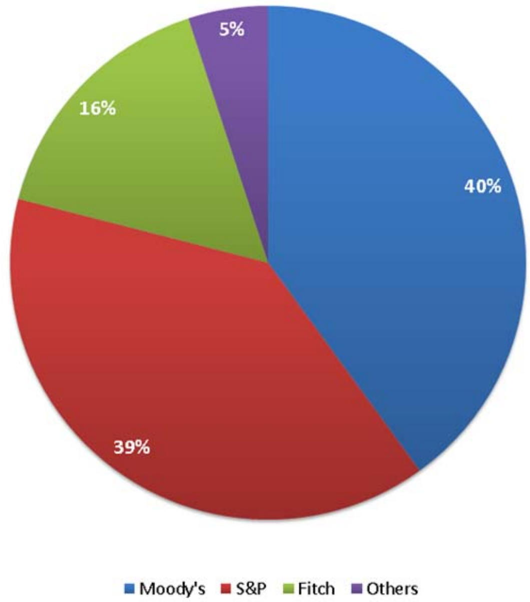
The first CRAs were created in 1850s to provide investors with information on the financial status of the railroad industry. In early 1900's, they rated a bond for the first time. And since then, CRAs have rated a wide variety of debt instruments.

In 1975, the SEC created the designation of Nationally Recognized Statistical Rating Organization (NRSRO). The rationale for this was to afford more credibility to the CRAs. In addition, rating started being used for regulatory purposes. However, no specific standards were specified. That came later in 2006, when the U.S. Congress passed the Rating Agency Act, that specifically spell out the requirements to become an NRSRO. Unfortunately, this piece of legislation established very high barriers to entry which has resulted in very few players in the credit rating market. At the time of the crisis, the ratings market was dominated by Moody's, S&P and Fitch[5] (see Figure 16 and 17).

As the Figure 17 shows, not only Moody's but also S&P has a significant participation in the rating market. Indeed, these two CRAs have almost the same participation in the CDO market. This is because issuers usually required two independent rating for regulatory purposes.

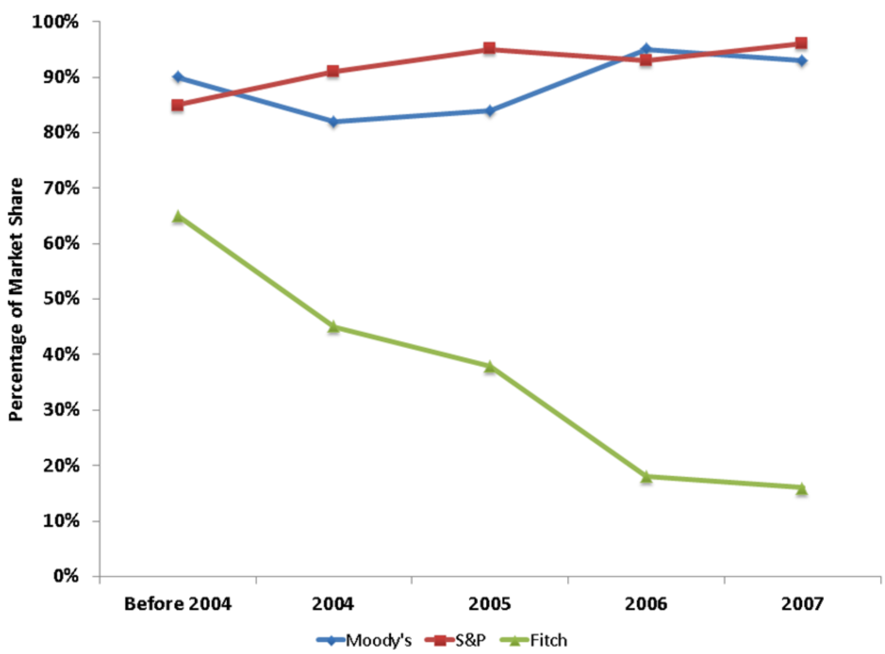
[5]By 2007, the others CRAs designated as NRSRO were: A.M. Best Company, Japan Credit Rating Agency, and Rating and Investment Information.

Figure 16: Global Market Ratings Participation 2007



Source: M.C. Rom, 2009

Figure 17: Market Share in CDO Ratings



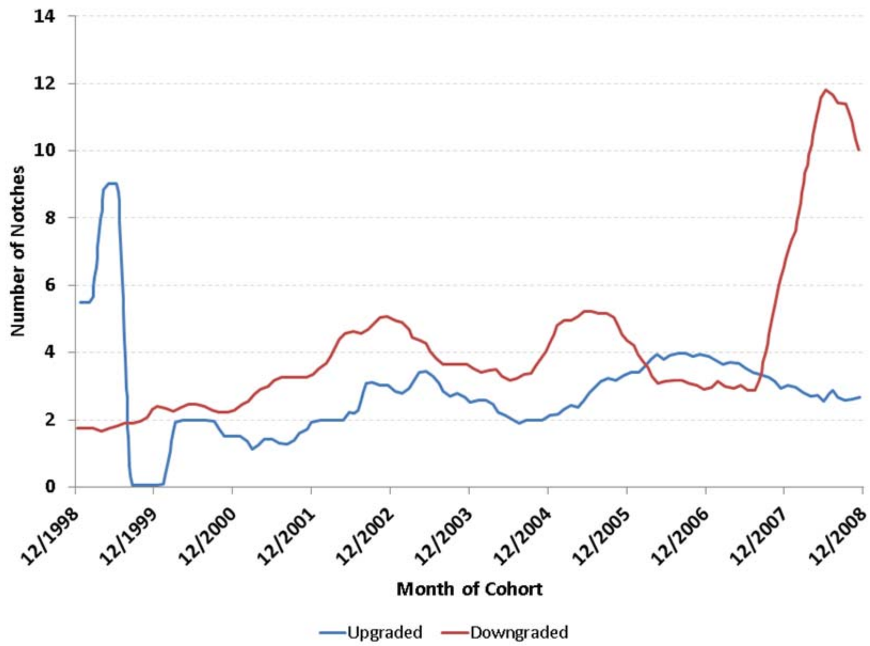
Source: A.K. Barnett, 2009

3.2 The Stability of CDO Ratings

CDO ratings proved to be extremely unstable during the financial crisis. For example, see Figure 18.This figure shows that the number of upgrades/downgrades was reasonable stable between late 1999 and early 2007. However, at the beginning of

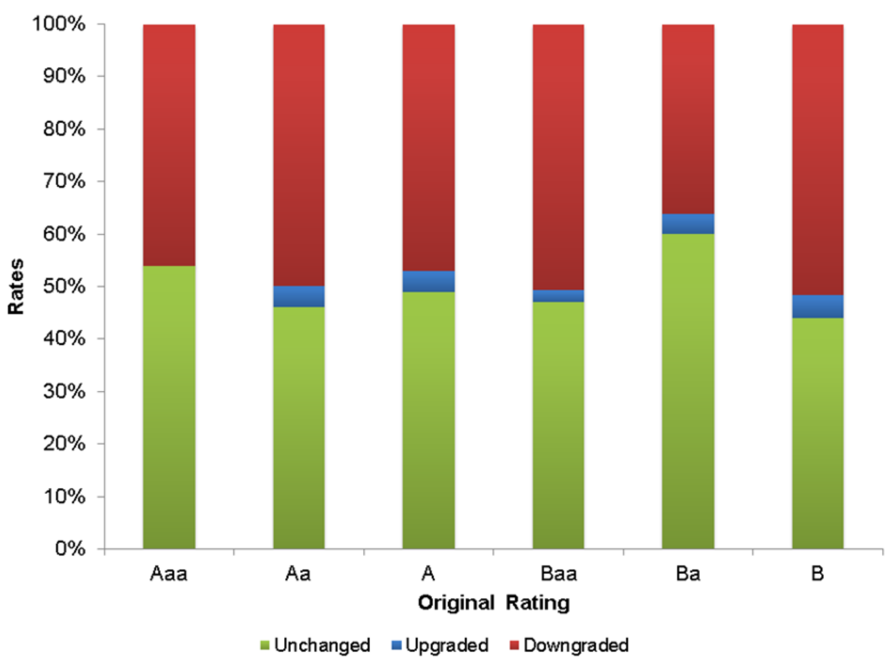
2007, the average number of downgrade notches increased dramatically. By 2008, this number has reached the value of twelve. Figure 19 shows, that roughly speaking, half of the ratings were changed. And the changes were overwhelmingly downgrades.

Figure 18: Average Number of Notches Upgraded and Downgraded



Source: Moody's Structured Finance Rating Transitions: 1983-2008.

Figure 19: Cumulative Upgraded and Downgraded Rates by Rating

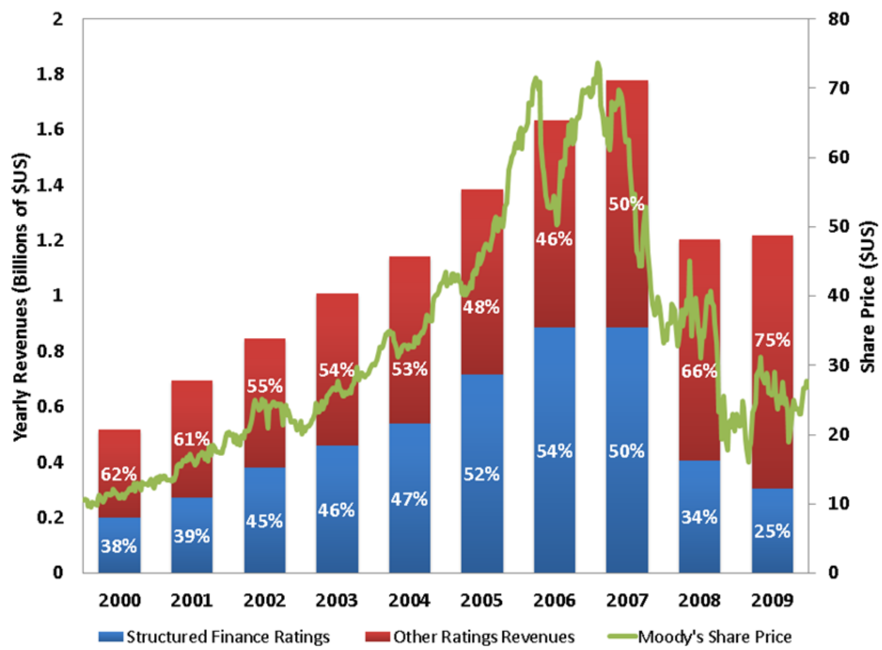


Source: Moody's Structured Finance Rating Transitions: 1983-2008.

The lack of stability of the CDO ratings amounts essentially to a failure of accuracy. According to Rom (2009), this failure, from a qualitative point of view, could be explained by several factors: misalignment of interest between CRAs and investors, lack of historic data and overworked rating analysts.

First of all, the business model of the rating agencies has incentives problems. The firms who are looking to have their debt rated, they themselves pay the CRAs. In short, it is an "issuer pays" compensation model where conflicts of interest arise. In the case of CDOs, these deals were highly profitable for CRAs. For this reason, the CRAs were encouraged to give the issuer the rating he was looking for. In case the CRAs made the deal harder to rate, they risked losing a lucrative client: twelve underwriters accounted for eighty percent of the transactions in the CDO market (according to the SEC). In essence, upsetting anyone of these twelve underwriters would translate into a significant market share loss. Figure 20 shows Moody's rating revenues composition and share price of the firm.

Figure 20: Moody's Rating Revenues & Share Price

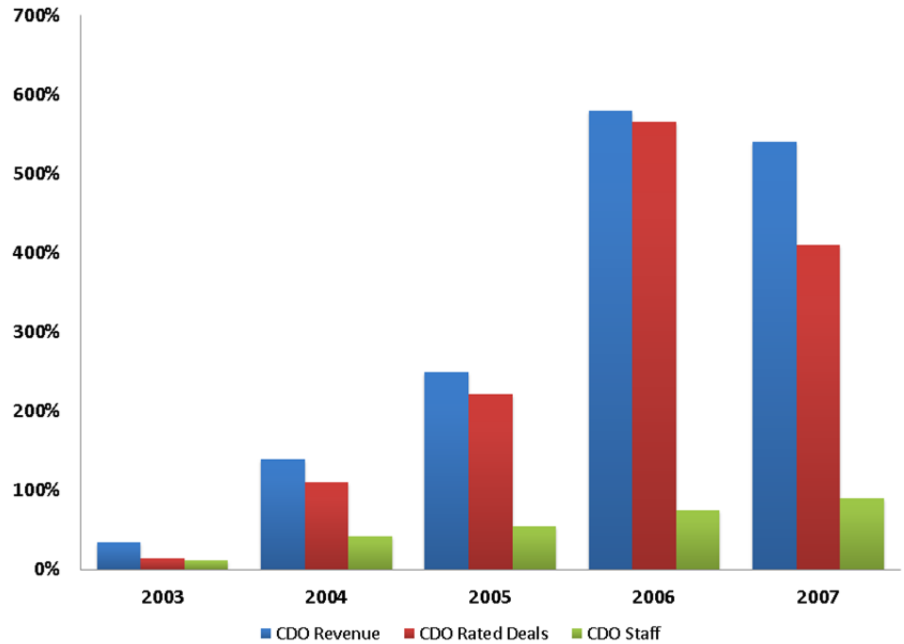


Source: Moody's Corporation Annual Reports: 2000-2009.

The second problem was the ignorance of the raters regarding the subprime market. Specifically, the CRAs did not have historical data regarding the performance of this segment. Furthermore, since the CDO market was growing very fast the CRAs did not have the time to perform a careful due diligence to verify the information provided by the issuers.

Finally, the CRAs were overwhelmed by the rapid growing of the CDO market. The agencies were understaffed and their employees overworked. The SEC issued a detail report recognizing this staffing problem. The revenue grew faster than the number of issues rated, but rating staff did not. Figure 21 shows that while the number of deals and revenues grew substantially, the number of analyst did not exhibit the same pattern.

Figure 21: Comparison of Growth Among CDO Revenues, CDO Rated Deals and CDO Staff



Source: M.C. Rom, 2009.

4. RELATED LITERATURE

Since 2007, many studies have tried to explain the subprime crisis. Some of these studies have focused in the role of the credit ratings applied in the context of synthetic CDOs and the validity of the models used by the CRAs. Within the framework of this study, the following papers offer some interesting considerations.

Cifuentes & Katsaros (2007) criticize the one-factor Gaussian copula method. Their study shows a conceptual flaw in the model which is manifested as a correlation smile. In principle, if one starts with the rating of a CDO tranche as an input variable, one can, through the application of the Gaussian copula model, estimate the correlation of the underlying pool of assets. The problem arises because when starting with different tranches, one obtains different estimates of the correlation of the underlying pool. On the other hand, the authors recognize another difficulty with the Gaussian copula: the implied default correlation of the assets depends on the default probability. In theory, this should not be the case.

Coval, Jurek & Strafford (2007) study the sensitivity of the expected loss of a CDO tranche and a CDO-Squared tranche as a function of the parameters of the underlying portfolio. The study concludes that among the CDO structures analyzed by them, the CDO-Squared tranches were more sensitive than the CDO tranches to both the correlation and the default probability. In addition, the study shows that tranches with lower seniority were more sensitive than more senior tranches. However, their study did not address the influence of the recovery rate in the results.

Lucas, Fabozzi, Goodman & Manning (2008), compared S&P's and Moody's synthetic CDO ratings on portfolios comprised of credits in the major CDS indices. The authors found that on portfolios equally rated by both CRAs, Moody's ratings tend to be higher. However, since Moody's usually rates underlying assets lower than S&P, Moody's ratings tend to be lower.

Meng & Sengupta (2010) perform an analytical study of the one factor Gaussian copula, considering the sensitivities of CDO tranches supported by an homogeneous portfolio. The authors derive an explicit formula for the tranche sensitivities to some parameters under certain simplifying assumptions. By considering the losses of the tranches, the resulting formula demonstrates how the losses of the equity tranches decrease as the correlation increases. Conversely, the model shows that the losses of the tranches above the equity, taken as a whole, behave in the opposite fashion.

Hull & White (2010) study the risk profile of tranches created from mortgages. Using several variations of the Gaussian copula model, they concluded that the ratings were estimated more accurately for senior tranches. However, the accuracy deteriorated significantly for the mezzanine tranches. Moreover, this conclusion was also valid for different subordination levels. In addition, the authors compare the expected loss criteria and the probability of loss criteria when specifying a credit rating. The expected loss criteria resulted in more conservative rating in the case of tranches with less seniority, but it was more results in more to be more conservative with tranches with less seniority, but more forgiving in the case of the most senior tranches.

5. METHODOLOGY

5.1 Introduction

The main purpose of this study is to analyze the stability of synthetic CDO ratings using the Moody's methodology as the framework of reference. The main reason for this choice is that Moody's is a very important CRA and its CDO rating methodology is the most well-known[6]. Indeed, considering Moody's market share of the CDO rating market - about 90% - this choice makes sense. In addition, the Moody's methodology, whatever its shortcoming, is very transparent and easy to replicate.

The methodology employed to analyze the stability of Moody's credit ratings for a synthetic CDO is divided in two parts. First, the effect of errors in the parameters that describe the assets is studied in a deterministic fashion. The purpose of this approach is to understand how errors in the assets parameters could result in a poor estimate of the synthetic CDOs ratings. The more sensitive the model to errors in the input parameters, the more accuracy is required to describe the underlying reference portfolio the synthetic CDO.

Secondly, the effect on the ratings of the uncertainty in the model parameters is studied with a stochastic approach. For a better understanding of this, the confidence interval of the output quantity that defines the ratings is estimated. Thus, a range of more likely scenarios for synthetic CDO ratings is obtained. Using this approach, it is possible to get a sense for the magnitude of the variation of the expected loss (in short, a range of the variability of possible upgrades and downgrades until the maturity of the CDO).

To sum up, both analysis shed light on Moody's synthetic CDO rating methodology. The approach followed in this study will help investors and financial institutions to get a better understanding of Moody's rating methodology. Furthermore, it will highlight what a skeptical investor could have done in order to check the robustness of Moody's ratings.

This chapter is organized as follows. The next section explains Moody's rating approach. The following section explains the approach taken to analyze the stability of synthetic CDO ratings. The final section, provide a detail explanation of the Moody's credit rating model. It also discusses the deterministic and stochastic approach undertaken to investigate the stability of the ratings.

5.2 Moody's Credit Rating Model

The Moody's credit rating model applied to synthetic CDOs is based on the expected loss concept. The expected loss is expressed as a percentage in reference to the notional amount of the tranche under study. Once the expected loss is estimated, a credit rating can be assigned to the tranche under consideration, using as a reference a benchmark table of expected losses for different time horizons[7].

[6] Fender & Kiff (2004)

[7] The complete table of idealized expected loss for each rating and each time horizon is in the Appendix F

In the case of synthetic CDOs tranches, as well a synthetic CDO-Squared tranches, the determination of the expected loss requires the determination of all the potential losses under all possible default scenarios of the underlying assets of the reference portfolio. In addition, this requires estimating the likelihood that each default scenario could occur.

In order to do this, the model takes into account both the characteristics of the underlying pool and the structure of the synthetic CDO. The pool characteristics are defined by the following parameters:

i. Average default probability of the assets
ii. Average recovery rate of the assets
iii. Average default correlation of the assets

The synthetic CDO structure is defined by the following parameters:

i. Number of assets in the portfolio
ii. Number of tranches
iii. Size of the tranches
iv. Maturity of the CDO.

The default probability, obviously, refers to the likelihood that an asset will default. The recovery rate, express as percentage, reflects the value of an assets once it has defaulted. Finally, the default correlation captures the tendency of the assets to default together. A more detailed description of these parameters is found later in this chapter.

In what follows, the default probability, the recovery rate, the correlation, and the number of assets in the reference portfolio will be denoted as $\boldsymbol{P}$, $\boldsymbol{\rho}$, $\boldsymbol{\alpha}$ and $\boldsymbol{N}$ respectively. As can be seen, even though the assets pool in general could be heterogeneous in nature, for the purpose of Moody's analysis, is treated as an homogeneous pool represented by its average characteristics.

The following sections explain the procedure used to estimate the probability of each default scenario as well as the loss for the tranche under consideration for each default scenario.

5.3 Synthetic CDO

5.3.1 Determination of the Probability of Each Default Scenario

Given $\boldsymbol{N}$ securities in the portfolio, let $\boldsymbol{j}$ be the number of securities that default $\boldsymbol{j = 0, 1, 2, \dots, N}$) (. The probability of occurrence of each default scenario, is denoted as $\boldsymbol{p_j}$, where $\boldsymbol{j = 0, 1, 2, \dots, N}$.

In order to estimate $\boldsymbol{p_j}$, Moody's relies on the one-factor Gaussian copula approach. The one-factor Gaussian copula approach is a numerical algorithm to generate samples of normally distributed random variables that have a given pair-wise correlation. According to Mackenzie (2008), the approach is known as 'one-factor' due to the fact that there is only one common factor that attempts to capture the common element to all the securities in the pool. This factor could be interpreted as the health of the economy and its breadth, in some sense, captures the way these assets could “default together”. ‘Gaussian’ refers to the use of a multi-dimensional normal standard distribution, and ‘copula’ means the model of how default risks occur together.

The one-factor Gaussian copula algorithm consists of the following steps:

a) Let $\boldsymbol{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N}$ be $\boldsymbol{N}$ independent and identically distributed (i.i.d.) standard normal random variables. Therefore, $\boldsymbol{\varepsilon_i}$ represents the independent factor corresponding to asset $\boldsymbol{i}$, that is:

$$\boldsymbol{\varepsilon_i \sim N(0,1)}$$

with $\boldsymbol{i = 1, 2, \dots, N}$.

b) Define $\boldsymbol{X_i}$ as

$$\text{(Equation 1)} \quad \boldsymbol{X_i = Z\sqrt{\rho} + \varepsilon_i\sqrt{1-\rho}}$$

where $\boldsymbol{Z}$ follows a $\boldsymbol{N(0,1)}$ but it is independent of all $\boldsymbol{\varepsilon_i}$, with $\boldsymbol{i = 1, 2, \dots, N}$. In Equation 1, $\boldsymbol{Z}$ represents the common factor of the portfolio. The default correlation $\boldsymbol{\rho}$ intents to capture the average pair-wise default correlation which is defined as:

$$\text{(Equation 2)} \quad \boldsymbol{\rho_{ij} = Corr(X_i, X_j) = E(X_i X_j)} \;\; \forall \boldsymbol{i, j = 1, 2, \dots, N}$$

In the case of an homogeneous portfolio (all assets have the same notional amount), $\boldsymbol{\rho}$ is defined as:

$$\text{(Equation 3)} \quad \boldsymbol{\rho = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \rho_{ij}}$$

c) Define the index variable $\boldsymbol{I_i}$ as

$$\text{(Equation 4)} \quad \boldsymbol{I_i} = \begin{cases} \boldsymbol{1}, & \boldsymbol{\Phi(X_i) \leq P} \\ \boldsymbol{0}, & \boldsymbol{\Phi(X_i) > P} \end{cases}$$

where $\boldsymbol{\Phi(X_i)}$ is the standard normal cumulative distribution function[8] with zero mean and variance equals to one, with $\boldsymbol{i = 1, 2, \dots, N}$. Equation 4 denotes a default with the value of 1 and a no default with a value of 0. In essence, we have $\boldsymbol{N}$ Bernoulli trials linked with a common correlation equals to $\boldsymbol{\rho}$. In summary, the number of defaults in the portfolio follows a correlated Binomial distribution.

d) Performing a Monte Carlo simulation by repeating steps a), b) and c) many times, we can estimate the probability $\boldsymbol{p_j}$ of each one of the default scenarios. More

---

[8] The normal cumulative function is defined as: $\boldsymbol{F(x \leq X) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(z-\mu)^2}{2\sigma^2}} z dz}, \;\forall \boldsymbol{x} \in (-\infty, +\infty]$

precisely, defining $\boldsymbol{S}$ as the number of simulations and $\boldsymbol{s_j}$ the number of scenarios where $\boldsymbol{j}$ securities default, $\boldsymbol{p_j}$ can be estimated as follows:

$$\text{(Equation 5)} \quad \boldsymbol{p_j = Prob\left(\sum_{i=0}^{N} I_i = j\right) \approx \frac{s_j}{S}}$$

with $\boldsymbol{j = 0, 1, 2, \dots, N}$, and where:

$$\text{(Equation 6)} \quad \boldsymbol{\sum_{j=0}^{N} s_j = S}$$

5.3.2 Computational Issues Regarding Moody's Approach

The estimation of the values of $\boldsymbol{p_j}$ requires Monte Carlo simulations. The difficulty of this approach relies in the necessity to generate a big number of values for the different random variables involved in the one-factor Gaussian copula approach.

The following example clarifies this issue. Considering $\boldsymbol{N}$ securities in the reference portfolio, this means that there are $\boldsymbol{N+1}$ values of random variables to be generated in each trial: $\boldsymbol{N}$ values for $\boldsymbol{\varepsilon_i}\ \forall \boldsymbol{i = 1, 2, \dots, N}$ and one value for $\boldsymbol{Z}$. A synthetic CDO usually has one hundred assets approximately. The typical number of simulations needed for a good estimate of the expected loss of a synthetic CDO tranche, is of the order of 10^5. Thus, 10^5 trials of one hundred and one assets simulated equals one hundred million one hundred thousand (~10^7) values of random numbers to be generated.

In order to do this, it is important to use an adequate random number generator. This random number generator needs to satisfy not only the i.i.d. condition of the random variables but also the tests of randomness. Considering these criteria, Ranq1 is an appropriate choice. The implementation of the Ranq1 in C++[9] is obtained from Press, Teukolsky, Vetterling & Flannery (2007). According to the authors, Ranq1 is one of the fastest random generators satisfying the tests of randomness.

4.3.3 Determination of the Expected Loss

Let $\boldsymbol{v}$ the notional amount of each security in the pool and recall that $\boldsymbol{\alpha}$ represents the recovery rate of those securities. The following expressions will be used to determine the expected loss of the different tranches under consideration.

Let $\boldsymbol{e}$ denote the equity tranche, let $\boldsymbol{M_e}$ be the size of the equity tranche and let $\boldsymbol{l_e(j)}$ be the loss (expressed as a percentage) experienced by the equity tranche when $\boldsymbol{j}$ assets default. Then, $\boldsymbol{l_e(j)}$ is defined as:

$$\text{(Equation 7)} \quad \boldsymbol{l_e(j) = Min\left(\frac{(1-\alpha)\times v \times j}{M_e}, 100\%\right)}$$

with $\boldsymbol{j = 0, 1, 2, \dots, N}$.

[9] C++ is an object oriented programming language, flexible and practical enough to implement computation routines for finance

Let $\boldsymbol{m}$ and $\boldsymbol{s}$ denote the mezzanine and senior tranches. Let $\boldsymbol{M_m}$ and $\boldsymbol{M_s}$ be the size of the tranches. Let $\widetilde{\boldsymbol{M}}_{\boldsymbol{m}}$ and $\widetilde{\boldsymbol{M}}_{\boldsymbol{s}}$ be sum of the notional amount of the tranches below $\boldsymbol{m}$ and $\boldsymbol{s}$ respectively. This sum is known as the subordination level of the tranche. Let $\boldsymbol{l_T(j)}$ be the loss (expressed as a percentage) experienced by the tranche $\boldsymbol{T}$ ($\boldsymbol{T} = \boldsymbol{m}$ or $\boldsymbol{s}$) when $\boldsymbol{j}$ assets default. Thus, $\boldsymbol{l_T(j)}$ is defined as:

$$\text{(Equation 8)} \quad \boldsymbol{l_T(j)} = \boldsymbol{Min}\left(\boldsymbol{Max}\left(\boldsymbol{0}, \frac{(\boldsymbol{1}-\boldsymbol{\alpha}) \times \boldsymbol{v} \times \boldsymbol{j} - \widetilde{\boldsymbol{M}}_{\boldsymbol{T}}}{\boldsymbol{M_T}}\right), \boldsymbol{100}\%\right)$$

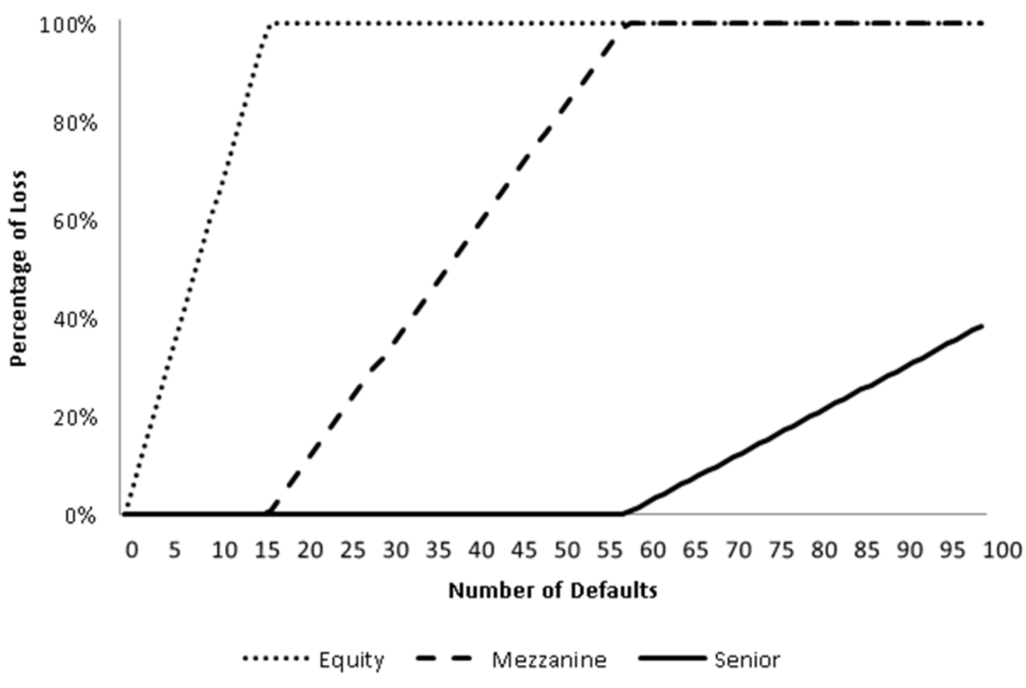
with $\boldsymbol{j} = \boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{N}$, and where the values of $\widetilde{\boldsymbol{M}}_{\boldsymbol{T}}$, with $\boldsymbol{T}$ denoted as $\boldsymbol{m}$ and $\boldsymbol{s}$, are defined as:

$$\text{(Equation 9)} \quad \widetilde{\boldsymbol{M}}_{\boldsymbol{m}} = \boldsymbol{M_e}$$

$$\text{(Equation 10)} \quad \widetilde{\boldsymbol{M}}_{\boldsymbol{s}} = \boldsymbol{M_m} + \boldsymbol{M_e}$$

As an example, Figure 22 compares the values of $\boldsymbol{l_e(j)}$, $\boldsymbol{l_m(j)}$ and $\boldsymbol{l_s(j)}$ under different number of defaults in the reference portfolio. For the purpose of this example was considered a synthetic CDO of three tranches: Equity, Mezzanine and Senior. The values of the parameters in this example are: $\boldsymbol{\alpha}$=40%, $\boldsymbol{v}$=1, $\boldsymbol{M_e}$=10, $\boldsymbol{M_m}$=25 and $\boldsymbol{M_s}$=60.

Figure 22: Example of Losses for Different Tranches



Source: Own Elaboration

Finally, the expected loss for any given tranche of the synthetic CDO is estimated as:

$$\text{(Equation 11)} \quad \boldsymbol{E(l_T) = \sum_{j=0}^{N} p_j \times l_T(j)}$$

Notice that in Equation 9, $\boldsymbol{T}$ could denote $\boldsymbol{e}$, $\boldsymbol{m}$ or $\boldsymbol{s}$. Now that the value of the expected loss has been estimated, the rating is obtained using the benchmark table provided by Moody's[10].

5.3.3 The Gauss-Hermite Quadrature Alternative

The Gauss-Hermite quadrature is an alternative to improve the time performance of the Monte Carlo simulation when using the one-factor Gaussian copula. This is a numerical integration formula, useful to approximate integrals of the form:

$$\int_{-\infty}^{+\infty} \boldsymbol{e^{-x^2} f(x) dx \approx \sum_{i=1}^{n} A_i f(x_i)}$$

where $\boldsymbol{f(x)}$ is a smooth function, and both $\boldsymbol{x_i}$ and $\boldsymbol{A_i}$ are the set of $\boldsymbol{n}$ abcissas and coefficients and determined by the quadrature[11]. Thus, the Gauss-Hermite quadrature can approximate the value of $\boldsymbol{p_j}$ as follows:

a) The asset $\boldsymbol{i}$ in the portfolio will default when:

$$\boldsymbol{\Phi(X_i) \leq P \;\forall i = 1, 2, \dots, N}$$

thus:

$$\boldsymbol{X_i \leq \Phi^{-1}(P)}$$

$$\Rightarrow \quad \boldsymbol{Z\sqrt{\rho} + \varepsilon_i\sqrt{1-\rho} \leq \Phi^{-1}(P)}$$

$$\Rightarrow \quad \boldsymbol{\varepsilon_i \leq \frac{\Phi^{-1}(P) - Z\sqrt{\rho}}{\sqrt{1-\rho}}}$$

b) For a given value of $\boldsymbol{Z}$, the likelihood of default for the asset $\boldsymbol{i}$ is $\boldsymbol{q(z)}$, which is defined as:

$$\boldsymbol{q(z) \equiv Prob(X_i \leq c \,/\, Z = z) = Prob\left(Z\sqrt{\rho} + \varepsilon_i\sqrt{1-\rho} \leq \Phi^{-1}(P) / Z = z\right)}$$

$$\Rightarrow \boldsymbol{q(z) = Prob\left(\varepsilon_i \leq \frac{\Phi^{-1}(P) - Z\sqrt{\rho}}{\sqrt{1-\rho}} / Z = z \right)}$$

---

[10] See Appendix 5.

[11] For a more detailed explanation of the Gauss-Hermite Quadrature, see Appendix 6.

(Equation 12) $$q(z) = \Phi\left(\frac{\Phi^{-1}(P) - Z\sqrt{\rho}}{\sqrt{1-\rho}} / Z = z\right)$$

With this definition, the variable I_i now can be expressed as a function of Z as:

(Equation 13) $$I_i(z) = \begin{cases} 1, & \text{with probability } q(z) \\ 0, & \text{with probability } 1 - q(z) \end{cases} \quad \forall i = 1, 2, \ldots, N$$

c) Then, the value of p_j with $j = 0, 1, 2, \ldots, N$, becomes:

$$p_j = Prob\left(\sum_{i=0}^{N} I_i = j\right) = \int_{-\infty}^{+\infty} Prob\left(\sum_{i=0}^{N} I_i(Z) = j \Big/ Z = z\right)\phi(z)dz$$

with $\phi(z)$ being the probability density function of the standard normal distribution. Thus, considering that the sum of I_i is the sum of Bernoulli trials, this summation is distributed as a Binomial of N trials and probability $q(z)$, hence:

$$Prob\left(\sum_{i=0}^{N} I_i(Z) = j \Big/ Z = z\right) = \binom{N}{j} q(z)^j \left(1 - q(z)\right)^{N-j}$$

which leads to:

$$p_j = \int_{-\infty}^{+\infty} \binom{N}{j} q(z)^j \left(1 - q(z)\right)^{N-j} \phi(z)dz$$

$$= \int_{-\infty}^{+\infty} \binom{N}{j} q(z)^j \left(1 - q(z)\right)^{N-j} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz$$

d) Changing the variable z^2 for $2y^2$, p_j can be rewritten as:

$$p_j = \int_{-\infty}^{+\infty} \binom{N}{j} q(2y)^j \left(1 - q(2y)\right)^{N-j} \frac{1}{\sqrt{\pi}} e^{-y^2} dy$$

Define $f(y)$ as:

$$f(y) = \binom{N}{j} q(2y)^j \left(1 - q(2y)\right)^{N-j} \frac{1}{\sqrt{\pi}} e^{-y^2}$$

the expression for p_j can be finally rewritten as:

$$\text{(Equation 14)} \quad \boldsymbol{p_j} = \int_{-\infty}^{+\infty} \boldsymbol{f(y)e^{-y^2}dy}$$

Therefore, the Gauss-Hermite quadrature can be used directly to obtain an estimate of the value of $\boldsymbol{p_j}$.

The advantage of this method compared to the Monte Carlo simulation approach utilized by Moody's is that it requires less computation time. This becomes especially important when trying to estimate the values for the expected loss for the most senior tranches. The senior tranches, since they are exposed to much lower credit risk, require a much bigger number of Monte Carlo simulations to generate enough loss scenarios for the senior tranches. This topic is treated in detailed in Appendix 7.

5.4 Synthetic CDO-Squared

In the case of a synthetic CDO-Squared the value of the expected loss is obtained in a different way in comparison with the synthetic CDO. As explained in Chapter 2, the synthetic CDO-Squared references a portfolio of mezzanine tranches from different synthetic CDOs. Therefore, the procedure to estimate the expected loss should consider the possible overlap among the underlying securities referenced by the mezzanine tranches.

In contrast with the case of the synthetic CDO, the estimation procedure to obtain the probabilities for each default scenario in the synthetic CDO-Squared will use Monte Carlo simulations. The reason for this choice is that the Gauss-Hermite quadrature implementation for the synthetic CDO-Squared is computationally very intensive.

The following sections explain the procedure used to estimate the probability of each default scenario for a synthetic CDO-Squared, as well as the loss for the tranche under consideration for each default scenario.

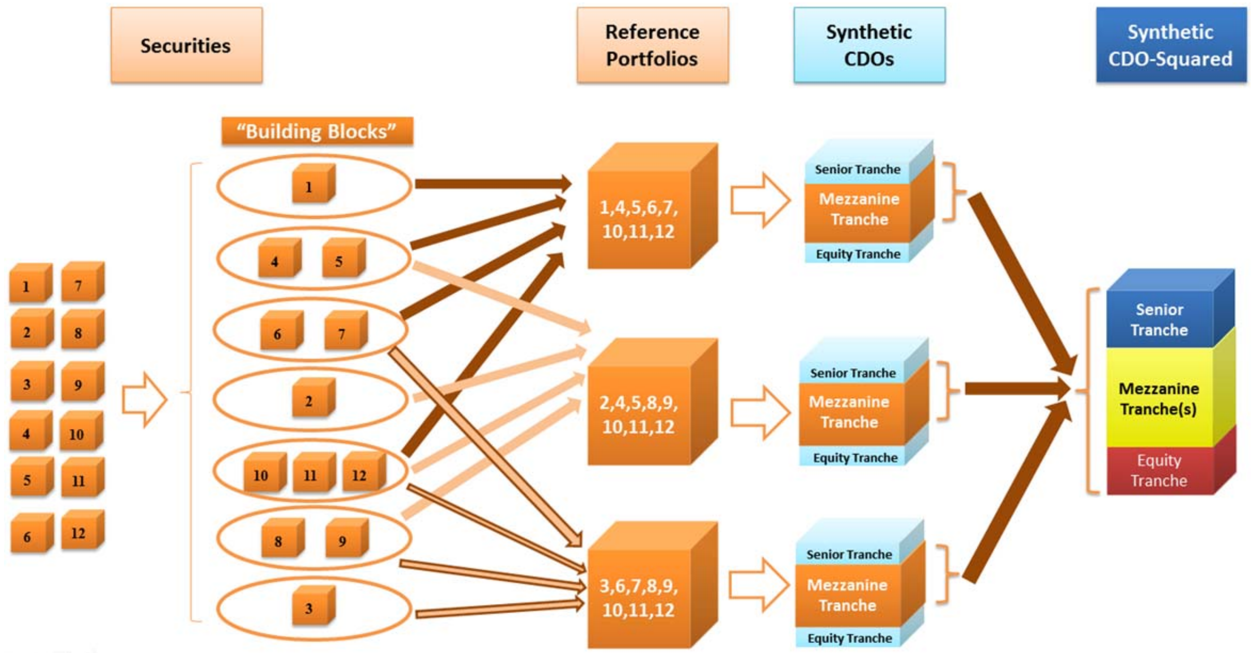
5.4.1 Overlap Characterization

The overlap characterization will be explained in reference to the synthetic CDO-Squared example shown in Figure 23.

Consider a set of $\boldsymbol{K}$ mezzanine tranches in the reference portfolio of a synthetic CDO-Squared. This means that there are $\boldsymbol{K}$ synthetic CDOs referencing $\boldsymbol{K}$ different portfolios. Let $\boldsymbol{N}$ be the total number of different securities contained in the $\boldsymbol{K}$ aggregate reference portfolios. Let $\boldsymbol{N_k}$ be the number of securities referenced by the synthetic CDO $\boldsymbol{k}$, with $\boldsymbol{k = 1, 2, \dots, K}$. In the case of the synthetic CDO-Squared shown in Figure 23, $\boldsymbol{K = 3}$, $\boldsymbol{N = 12}$ and $\boldsymbol{N_k = 8}$, with $\boldsymbol{k = 1, 2, 3}$.

Because these $\mathbf{K}$ portfolios can exhibit some degree of overlapping, a default of an underlying security can manifest itself in several of the $\boldsymbol{K}$ reference portfolios. For instance, consider the case of securities 1, 4 and 10 in Figure 23. Security 1 belongs only to the first underlying portfolio. Both, securities 4 and 10, belong to several portfolios. For instance, security 4 belongs to the underlying portfolio supporting synthetic CDO 1 and 2; whereas security 10 belongs to all three portfolios.

Figure 23: Diagram of Overlapping for a Synthetic CDO-Squared



Source: Own Elaboration

In order to take into account the overlap, define the index matrix $\boldsymbol{Y} = [y_{n,k}]_{N\times K}$ to indicate into which reference portfolio(s) each security belongs. Then, $\boldsymbol{Y}$ and $y_{n,k}$ are defined as:

$$\text{(Equation 15)} \quad \boldsymbol{Y} = \begin{bmatrix} y_{1,1} & \cdots & y_{1,K} \\ \vdots & \ddots & \vdots \\ y_{N,1} & \cdots & y_{N,K} \end{bmatrix}$$

$$\text{(Equation 16)} \quad y_{n,k} = \begin{cases} 1, \text{if asset } n \text{ belongs to portfolio } k \\ 0, \text{otherwise} \end{cases}$$

with $n = 1, 2, .., N$ and $k = 1, 2, ..., K$. Due to the fact that there are N_k assets in the reference portfolio of the synthetic CDO k, it follows that:

$$\text{(Equation 17)} \quad N_k = \sum_{n=1}^{N} y_{n,k}$$

with $k = 1, 2, ..., K$.

In the case shown in Figure 23, $\boldsymbol{Y}$ will be a matrix of twelve rows and three columns with the following values:

$$
\boldsymbol{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
$$

Consider the case of securities 1, 4 and 10 in Figure 23:

- For security 1, $\boldsymbol{y_{1,1} = 1}$ and $\boldsymbol{y_{1,2} = y_{1,3} = 0}$ since security 1 belongs only to the first reference portfolio.
- For security 4, $\boldsymbol{y_{4,1} = y_{4,2} = 1}$ and$\boldsymbol{y_{4,3} = 0}$ since security 4 belongs to both the first and second reference portfolios.
- For security 10, $\boldsymbol{y_{10,1} = y_{10,2} = y_{10,3} = 1}$ since security 10 belongs to all three reference portfolios.

Define $\boldsymbol{C}$ as the power set of the $\boldsymbol{K}$ portfolios. This means that $\mathbf{C}$ is the collection of all the possible subsets of different portfolios from the set of $\boldsymbol{K}$ portfolios. The number of possible subsets in $\boldsymbol{C}$ is equal to $\boldsymbol{2^K - 1}$. Let $\boldsymbol{c_{a,b}}$ denote a subset of $\mathbf{C}$. The subindex $\boldsymbol{a, b}$ means that portfolios $\boldsymbol{a}$ and $\boldsymbol{b}$ are the only portfolios included in subset $\mathbf{c_{a,b}}$. With reference to Figure 23, the seven following subsets are in $\boldsymbol{C}$:

$$
\boldsymbol{C = \{c_1, c_2, c_3, c_{1,2}, c_{1,3}, c_{2,3}, c_{1,2,3}\}}
$$

Define the index matrix $\boldsymbol{X} = \left[\boldsymbol{x_{n,c_{a,b}}}\right]_{\boldsymbol{N\times(2^K-1)}}$ to indicate into which subset $\boldsymbol{c_{a,b}}$ each security $\boldsymbol{n}$ belongs. Thus, $\boldsymbol{X}$ and $\boldsymbol{x_{n,c_{a,b}}}$ are defined as:

$$
\text{(Equation 18)}\quad \boldsymbol{X} = \begin{bmatrix} \boldsymbol{x_{1,c_1}} & \cdots & \boldsymbol{x_{1,c_{1,2,..,K}}} \\ \vdots & \ddots & \vdots \\ \boldsymbol{x_{N,c_1}} & \cdots & \boldsymbol{x_{N,c_{1,2,..,K}}} \end{bmatrix}
$$

$$
\text{(Equation 19)}\quad \boldsymbol{x_{n,c_{a,b}}} = \begin{cases} \boldsymbol{1, if\ y_{n,k} = 1} \ \textit{and}\ \boldsymbol{y_{n,j} = 0}\ \forall \boldsymbol{k = a, b,}\ \textit{and}\ \boldsymbol{j \neq k} \\ \boldsymbol{0}, \quad \textit{otherwise} \end{cases}
$$

with $\boldsymbol{n = 1, 2, .., N}$ and $\boldsymbol{c_{a,b}}$ any subset of $\boldsymbol{C}$. Similarly, the definition of $\boldsymbol{x_{n,c_{a,b}}}$ can be rewritten as:

$$
\text{(Equation 20)}\quad \boldsymbol{x_{n,c_{a,b}}} = \prod_{\boldsymbol{k}} \boldsymbol{y_{n,k}}
$$

With $\boldsymbol{k = a, b}$ and $\boldsymbol{n = 1, 2, .., N}$ and $\boldsymbol{c_{a,b}}$ any subset of $\boldsymbol{C}$. From this definition is easy to see that $\boldsymbol{x_{n,c_{a,b}} = 1}$ means that asset $\boldsymbol{n}$ belongs only to portfolios $\boldsymbol{a}$ and $\boldsymbol{b}$.

In the case of Figure 23, the matrix $\boldsymbol{X}$ takes the following values:

$$\boldsymbol{X} = \begin{bmatrix} x_{1,c_1} & x_{1,c_2} & x_{1,c_3} & x_{1,c_{1,2}} & x_{1,c_{1,3}} & x_{1,c_{2,3}} & x_{1,c_{1,2,3}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{12,c_1} & x_{12,c_2} & x_{12,c_3} & x_{12,c_{1,2}} & x_{12,c_{1,3}} & x_{12,c_{2,3}} & x_{12,c_{1,2,3}} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For instance, consider the case of securities 1, 4 and 10:

- Security 1 belongs only to the first reference portfolio. Specifically it is only contained in the portfolios of the subset $\mathbf{c_1}$. Thus, $\boldsymbol{x_{1,c_1} = 1}$ and $\boldsymbol{x_{1,c_2} = x_{1,c_3} = x_{1,c_{1,2}} = x_{1,c_{1,3}} = x_{1,c_{2,3}} = x_{1,c_{1,2,3}} = 0}$.
- Security 4 belongs to both the first and second portfolios. Specifically it is only contained in the portfolios of the subset $\boldsymbol{c_{1,2}}$. Thus, $\boldsymbol{x_{4,c_{1,2}} = 1}$ and $\boldsymbol{x_{4,c_1} = x_{4,c_2} = x_{4,c_3} = x_{4,c_{1,3}} = x_{4,c_{2,3}} = x_{4,c_{1,2,3}} = 0}$.
- Security 10 belongs to all three reference portfolios. Specifically it is only contained in the portfolios of the subset $\boldsymbol{c_{1,2,3}}$. Thus, $\mathbf{x_{10,c_{1,2,3}} = 1}$ and $\boldsymbol{x_{10,c_1} = x_{10,c_2} = x_{10,c_3} = x_{10,c_{1,2}} = x_{10,c_{1,3}} = x_{10,c_{2,3}} = 0}$.

Let the index variable$\boldsymbol{z_n}$ identify whether asset $\boldsymbol{n}$ defaults. Then, $\boldsymbol{z_n}$ is defined as:

$$\text{(Equation 21)} \quad \boldsymbol{z_n} = \begin{cases} \mathbf{1}, \textbf{ if asset } \boldsymbol{n} \textbf{ defaults} \\ \mathbf{0}, \qquad \textbf{otherwise} \end{cases}$$

with $\boldsymbol{n = 1, 2, .., N}$**.**

Let $\boldsymbol{j_k}$ be the number of defaults in the reference portfolio of the synthetic CDO $\boldsymbol{k}$. Then, $\boldsymbol{j_k}$ is defined as:

$$\text{(Equation 22)} \quad \boldsymbol{j_k} = \sum_{n=1}^{N} \sum_{c_{a,b} \in C} \boldsymbol{z_n} \times \boldsymbol{x_{n,c_{a,b}}}$$

with $\boldsymbol{k = 1, 2, .., K}$. This definition of $\boldsymbol{j_k}$ allows one to see the link between a defaulted security and its impact on the portfolios behind each synthetic CDO.

From the definition of $\boldsymbol{j_k}$ it is obvious that a specific default scenario for the synthetic CDO-Squared will depend on the value of $\boldsymbol{j_k}$ for all the synthetic CDOs, with $\boldsymbol{k = 1, 2, .., K}$. Therefore, let $\boldsymbol{\hat{j}}$ be a vector that specifies the number of defaults in each one of the $\boldsymbol{K}$ underlying portfolios. Then, $\boldsymbol{\hat{j}}$ is defined by a particular set of value for $\boldsymbol{j_k}$, with $\boldsymbol{k = 1, 2, .., K}$. Hence:

$$\hat{j} = \{j_1, j_2, j_3, \dots, j_{K-1}, j_K\}$$

Finally, let $\boldsymbol{J}$ be the set of all possible $\boldsymbol{\hat{j}}$'s.

Consider the example shown in Figure 23 and assume that only securities 1, 4 and 10 default. Thus, only $\boldsymbol{z_1, z_4}$ and $\boldsymbol{z_{10}}$ are equal to 1. Then, the values of $\boldsymbol{j_1, j_2}$ and $\boldsymbol{j_3}$ will be:

$$j_1 = z_1 \times x_{1,c_1} + z_4 \times x_{4,c_{1,2}} + z_{10} \times x_{10,c_{1,2,3}} = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$$

$$j_2 = z_4 \times x_{4,c_{1,2}} + z_{10} \times x_{10,c_{1,2,3}} = 1 \times 1 + 1 \times 1 = 2$$

$$j_3 = z_{10} \times x_{10,c_{1,2,3}} = 1 \times 1 = 1$$

Then, this set of values of $\boldsymbol{j_1, j_2}$ and $\boldsymbol{j_3}$ define one possible default scenario for $\boldsymbol{\hat{j}}$.

5.4.2 The Probability of Each Default Scenario

From the definition of $\boldsymbol{\hat{j}}$ it is possible to estimate the probability of each default scenario. Define $\boldsymbol{p_{\hat{j}}}$ as the probability of a specific default scenario $\boldsymbol{\hat{j}}$. A possible approach to estimate $\boldsymbol{p_{\hat{j}}}$ is by taking advantage of the definition of the subsets $\boldsymbol{c_{a,b}}$; notice that the "building blocks" supporting the reference portfolios of the synthetic CDOs are all disjoint. See Figure 23. This makes the application of the Gauss-Hermite quadrature straightforward.

However, although this approach is theoretically feasible, from a practical standpoint is computationally very intensive.

To highlight this difficulty of the Gauss-Hermite quadrature, define $\boldsymbol{p_{\hat{j}}}$ as:

$$p_{\hat{j}} = Prob(j_1, j_2, j_3, \dots, j_{K-1}, j_K)$$

This means that the estimation of $\boldsymbol{p_{\hat{j}}}$ requires the estimation of the conjoint probability $\boldsymbol{Prob(j_1, j_2, j_3, \dots, j_{K-1}, j_K)}$ for scenarios $\boldsymbol{j_1, j_2, j_3, \dots, j_{K-1}}$ and $\boldsymbol{j_K}$. This conjoint probability depends on which securities default in each subset $\boldsymbol{c_{a,b}}$. Thus, the Gauss-

Hermite quadrature approach requires the estimation of all the possible default combinations among the “building blocks”.

Let us clarify this concept in reference to the example shown in Figure 23. Each scenario of default $\hat{j}$ requires values for j_1, j_2 and j_3. Thus, the possible scenarios for $\hat{j}$ are obtained from all the possible default combinations among all the “building blocks”. In the example, the total number of combinations can be determined as follows:

- The “building blocks” linked to subsets c_1, c_2 and c_3 have one security each one. Therefore, each “building block” can experience two default scenarios: 0 or 1. Thus, there 2^3 possible scenarios.
- The “building blocks” linked to subsets $c_{1,2}$, $c_{1,3}$ and $c_{2,3}$ have two securities each one. Therefore, each “building block” can experience three default scenarios: 0, 1 or 2 defaults. Thus, there are 3^3 possible scenarios.
- The “building block” linked to subset $c_{1,2,3}$ has three securities. Therefore, each “building block” can experience four default scenarios: 0, 1, 2 or 3 defaults. Thus, there are 4 possible scenarios.
- Finally, the total number of combinations is $2^3 \times 3^3 \times 4 = 864$.

This number of combinations might not be computationally intensive since the example is simple and not very realistic. However, in the case of an actual synthetic CDO-Squared, N is in the order of 10^2 and K could be between 5 and 10. Obviously, the number of possible combinations makes the problem computationally untracktable with the Gauss-Hermite quadrature.

For this reason, a conventional Monte Carlo simulation will be used to estimate $p_{\hat{j}}$. The difference is that, after S simulations, the focus is on the occurrence of each $\hat{j}$ specific default scenario. Defining $s_{\hat{j}}$ as the number of times the $\hat{j}$ scenario occurs, we have that $p_{\hat{j}}$ can be estimated as:

$$\text{(Equation 23)} \quad p_{\hat{j}} \approx \frac{s_{\hat{j}}}{S} \quad \forall \hat{j} \in J$$

where:

$$\text{(Equation 24)} \quad \sum_{\hat{j} \in J} s_{\hat{j}} = S$$

5.4.3 The Expected Loss

Denote m_k be the mezzanine tranche of synthetic CDO k in the reference portfolio of the synthetic CDO-Squared. Similarly, as before, let M_m and $\widetilde{M}_m$ be the size and subordination of tranche m_k respectively. The loss experienced by tranche m_k, when j_k assets default, is determined by:

$$\text{(Equation 25)} \quad \tilde{l}_{m_k}(j_k) = Min\big(Max\big(0, (1-\alpha) \times v \times j_k - \widetilde{M}_m\big), M_m\big)$$

with $\boldsymbol{j_k = 0, 1, 2, \dots, N_k}$ and $\boldsymbol{k = 1, 2, \dots, K}$.

For the equity tranche of the synthetic CDO-Squared (denoted as $\boldsymbol{e^2}$), let $\boldsymbol{M_{e^2}}$ be size of the tranche. Define $\boldsymbol{l_{e^2}(\hat{j})}$ as the loss experienced by the equity tranche for a specific scenario of defaults $\boldsymbol{\hat{j}}$. Hence:

$$\text{(Equation 26)}\quad \boldsymbol{l_{e^2}(\hat{j}) = Min\left(\frac{1}{M_{e^2}}\sum_{k=0}^{K}\sum_{j_k\in\hat{j}}\tilde{l}_{m_k}(j_k)\,,100\%\right)}$$

For the mezzanine and senior tranches of the synthetic CDO-Squared (denoted as $\boldsymbol{m^2}$ and $\boldsymbol{s^2}$ respectively), let $\mathbf{M_{m^2}}$ and $\boldsymbol{M_{s^2}}$ be the size of the tranches $\boldsymbol{m}$ and $\boldsymbol{s}$ respectively. Let $\boldsymbol{\widetilde{M}_{m^2}}$ and $\boldsymbol{\widetilde{M}_{s^2}}$ be the subordinations of tranches $\boldsymbol{m}$ and $\boldsymbol{s}$ respectively. Let $\boldsymbol{l_{m^2}(\hat{j})}$ and $\boldsymbol{l_{s^2}(\hat{j})}$ be the loss experienced by tranches $\boldsymbol{m}$ and $\boldsymbol{s}$ for a specific scenario of defaults $\boldsymbol{\hat{j}}$. Therefore:

$$\text{(Equation 27)}\quad \boldsymbol{l_T(\hat{j}) = Min\left(Max\left\{0\%,\frac{1}{M_T}\left(\sum_{k=0}^{K}\sum_{j_k\in\hat{j}}\tilde{l}_{m_k}(j_k) - \widetilde{M}_T\right)\right\},100\%\right)}$$

with $\boldsymbol{j = 0, 1, 2, \dots, N}$ and $\boldsymbol{T}$ equals $\boldsymbol{m^2}$ or $\boldsymbol{s^2}$.

Finally the expected loss for a given tranche is estimated as:

$$\text{(Equation 28)}\quad \boldsymbol{E(l_T) = \sum_{\hat{j}\in J} p_{\hat{j}} \times l_T(\hat{j})}$$

with $\boldsymbol{T}$ equals to $\boldsymbol{e^2}$, $\boldsymbol{m^2}$ or $\boldsymbol{s^2}$.

5.5 Moody’s Values for the Pool Characteristics

As previously stated, the Moody’s analysis is based on the average value of the pool characteristics. The purpose of this section is to explain the rationale behind the Moody’s approach to estimate these values, rather than passing judgment on the validity of these choices.

5.5.1 Default Probability

The values for the default probability of the assets in the pool are obtained from Cantor, Hamilton & Tennant (2007). The authors estimated confidence intervals for corporate default rates by rating category and time horizon. For this purpose, the authors used Moody’s default data for corporate bonds covering the period from 1970 until 2006.

The authors employed a bootstrapping method to estimate the mean and standard deviation of an asset default probability, as a function of its rating and time to maturity. In addition, the authors assumed that the default rates followed a normal distribution, based on some empirical evidence provided by Cantor & Falkenstein (2001) and Stein (2006).

Table 1 and 2 show the relevant data. Notice that the table shows the cumulative default probability (not the marginal default probability) for a given time horizon and rating.

Table 1: Mean Values of Cumulative Default Probability for Different Ratings and Time Horizon

		Time Horizon (Years)									
		1	2	3	4	5	6	7	8	9	10
Ratings	Aaa	0.00%	0.00%	0.00%	0.03%	0.10%	0.17%	0.25%	0.33%	0.42%	0.52%
	Aa	0.01%	0.02%	0.04%	0.11%	0.18%	0.26%	0.34%	0.42%	0.46%	0.52%
	A	0.02%	0.10%	0.22%	0.34%	0.47%	0.61%	0.76%	0.93%	1.11%	1.29%
	Baa	0.18%	0.51%	0.93%	1.43%	1.94%	2.45%	2.96%	3.45%	4.01%	4.63%
	Ba	1.20%	3.22%	5.57%	7.95%	10.21%	12.23%	13.99%	15.69%	17.37%	19.10%
	B	5.24%	11.30%	17.04%	22.05%	26.79%	30.98%	34.76%	37.97%	40.91%	43.32%
	Caa-C	19.47%	30.51%	39.73%	46.94%	52.66%	56.84%	59.97%	63.29%	66.36%	69.25%

Source: Cantor, Hamilton & Tennant (2007)

Table 2: Standard Deviation Values of Cumulative Default Probability for Different Ratings and Time Horizon

		Time Horizon (Years)									
		1	2	3	4	5	6	7	8	9	10
Ratings	Aaa	n/a	n/a	n/a	0.03%	0.07%	0.10%	0.15%	0.21%	0.27%	0.34%
	Aa	0.01%	0.01%	0.02%	0.04%	0.06%	0.08%	0.11%	0.14%	0.16%	0.18%
	A	0.01%	0.02%	0.04%	0.06%	0.08%	0.11%	0.13%	0.16%	0.18%	0.21%
	Baa	0.03%	0.06%	0.10%	0.15%	0.19%	0.24%	0.29%	0.34%	0.40%	0.46%
	Ba	0.08%	0.19%	0.30%	0.42%	0.53%	0.64%	0.73%	0.83%	0.94%	1.04%
	B	0.18%	0.38%	0.56%	0.73%	0.90%	1.07%	1.24%	1.41%	1.60%	1.79%
	Caa-C	0.75%	1.18%	1.59%	1.96%	2.33%	2.69%	3.04%	3.50%	3.85%	4.62%

Source: Cantor, Hamilton & Tennant (2007)

The determination of the confidence intervals, once the mean and the standard deviation are known, coupled with the normality assumption, is straightforward. In addition, the time dimension effect is taken into account implicitly, as a result of working with cumulative default probabilities.

5.5.2 Recovery Rate

For the purpose of modeling the recovery rate, Moody's relies on a Beta distribution. The use of the Beta distribution is supported by both Moody's CDOROM and Moody's Loss-Given-Default (LGD). The CDOROM is the software tool provided by Moody's to investors, in order to calculate the expected loss of tranches of synthetic

CDOs. On the other hand, Moody's LGD refers to the forecasts of the losses experienced by investors at the resolution of the default event.

Obviously, the choice of a Beta distribution function will automatically limit the value of the recoveries to the interval 0 and 1. Besides, the Beta distribution affords enough modeling flexibility since one can generate different shapes by controlling the value of the mean and standard deviation.

The probability density function for the Beta distribution is defined as:

$$f(x,\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du}$$

where:

$$\alpha = \mu\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right) \qquad \beta = (1-\mu)\left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right)$$

with μ defined as the mean and σ^2 refers to the variance of the Beta distribution.

The values used by Moody's for μ and σ, are 50% and 26% respectively in the case bonds and loans. These values are obtained from Moody's Ultimate Recovery Database[12] for corporate family recovery rates[13] for the period since 1987 until 2006. This database includes the recovery values received by creditors at the resolution of default, which is consistent with Moody's LGD. The probability density function of the Beta distribution is shown in Figure 24.

The use of the Beta distribution to model the recovery rate has also been endorsed by a number of studies (see Appendix 8).

In addition, in the context of this study a second approach to estimate the value of the recovery rate will be used. This approach relies on a log-linear relationship[14] between the recovery rate and the default probability. The motivation behind this alternative approach is that the empirical data support the view that high probabilities of the default are normally associated with lower recovery rates.

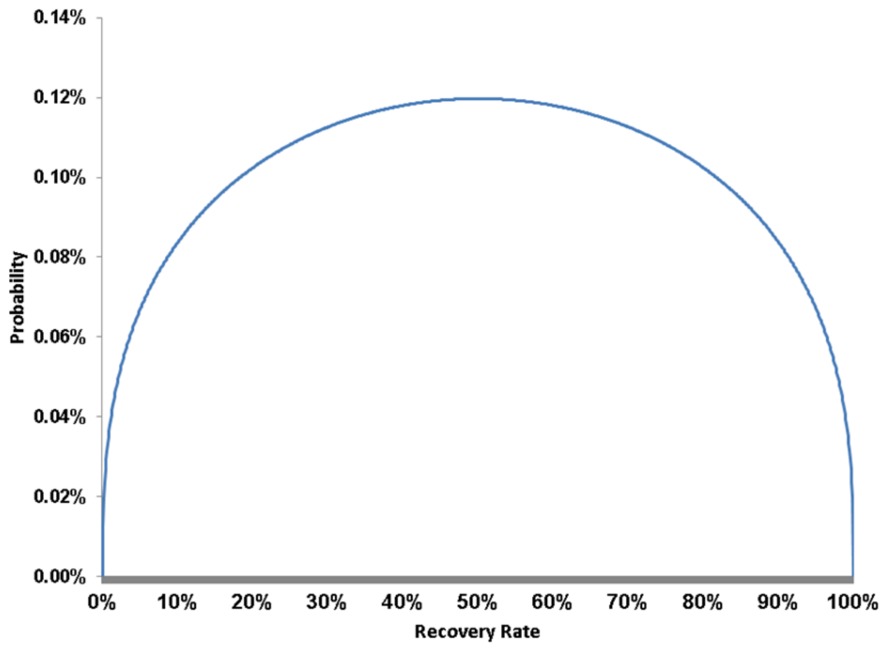
This relationship between the recovery rate and the default probability is also supported by the studies of Emery, Cantor, Keisman & Ou (2007). The following expression captures this idea:

(Equation 29) $\alpha = -0.11 \times ln\, p + 0.19$

[12] Emery, Cantor, Keisman & Ou (2007)
[13] The corporate family recovery rate is a measure of the value of an enterprise to be distributed among creditors due to a default resolution
[14] Emery, Cantor, Keisman & Ou (2007)

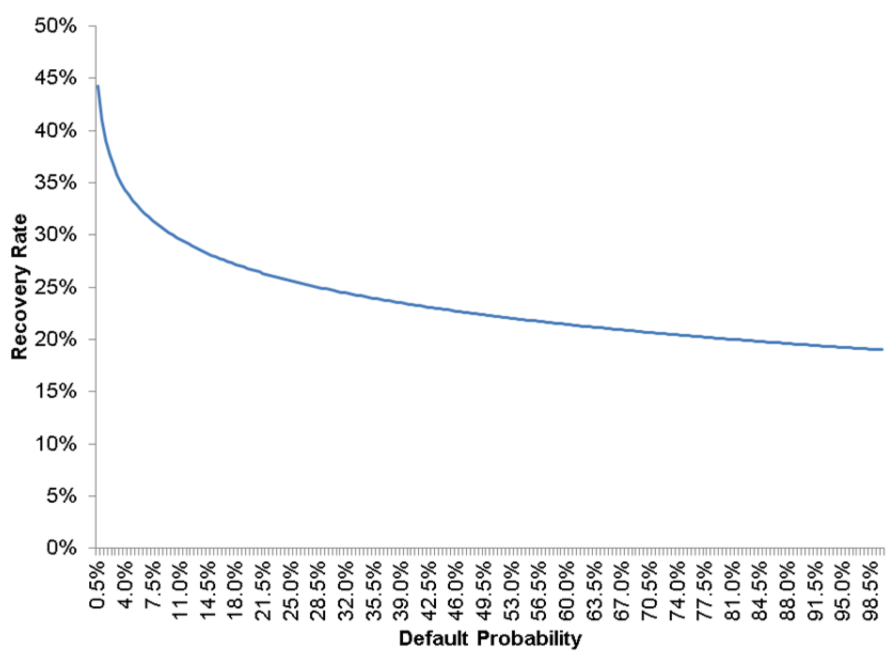
Although, the predictive power of this formula is not very high ($R^2 = 0.44$), it has the advantage that somehow, unlike the Beta, it captures the relationship observed between the recovery rate and the default probability (see Figure 25).

Figure 24: Density Probability Distribution of the Recovery Rate



Source: Own Elaboration

Figure 25: Relation between Probability Distribution and Recovery Rate (Equation 29)



Source: Own Elaboration

5.5.3 Default Correlation

The default correlation is notoriously difficult to estimate in comparison with the recovery rate and the default probability. Part of the difficulty is due to the fact that there is not enough data regarding default correlation values. The articles by Zhang, Zhu & Lee (2008) and Cifuentes & Katsaros (2007) support this view. This is chiefly because defaults do not happen that often. A second complication arises from the observation that default correlations are very time dependent. In addition to this difficulty, it should also be noticed that correlation values have perhaps received less attention since they are not part of any specific regulatory framework, such as Basel II.

Due to these difficulties, in general, practitioners rely on asset correlation values as a proxy for default correlation values.

The values of default correlation employed by Moody's are taken from the study by Zhang, Zhu & Lee (2008). The authors used data for U.S. public firms for the period between 1981 until 2006. Depending on the type of industry, these values fluctuate between 5% and 30%. Table 3 compares the values obtained by other previous studies.

Table 3: Comparison of Default Correlation from Other Studies

Study	Data Source	Default-Implied Asset
Gordy (2000)	Standard and Poor's	1.5%~12.5%
Cespedes (2000)	Moody's' Investor Service	0.1
Hamerle et al (2003)	Standard and Poor's 1982-1999	0.4%~6.04%
Frey et al (2001)	UBS	2.6%, 3.8%, 9.21%
Frey & McNeil (2003)	Standard and Poor's 1981-2000	3.4%–6.4%
Dietsch & Petey (2004)	Coface 1994–2001; Allgemeine Kredit 1997-2001	0.12%–10.72%
Jobst & de Servigny (2004)	Standard and Poor's 1981-2003	Intra 14.6%, inter 4.7%
Duellmann & Scheule (2003)	Deutsche Bundesbank 1987-2000	0.5%–6.4%
Jakubik (2006)	Bank of Finland 1988-2003	5.70%

Source: Zhang, Zhu & Lee (2008)

6. CASES TO BE ANALYZED

The analysis focuses on seven synthetic CDO structures that are interesting and/or representative of the U.S. market. The first one is the ABACUS transaction from Goldman Sachs. The next two synthetic CDOs are two synthetic CDS indices. The fourth one is a CDO-Squared called MIDGARD. And finally, theoretical synthetic CDO-Squared based on different synthetic CDS indices will be analyzed.

In what follows, the structures of these transactions are explained in detail.

6.1 Case #1: ABACUS

The ABACUS is a $2 billion synthetic CDO referencing mid-prime and subprime bonds backed by residential mortgages. ABACUS was structured by Goldman Sachs and issued in early 2007. This transaction did not receive media attention until 2010 when the SEC sued Goldman Sachs for fraud.

The SEC accusation was based on the fact that Goldman Sachs offered ABACUS to investors hiding the participation of Paulson & Co. in structuring the deal. Paulson & Co., an American hedge fund, not only participated in the selection of the ABACUS referenced securities but also took a short position against them through a series of CDS contracts. Therefore, Goldman Sachs was aware of Paulson & Co.'s pessimistic prescience about the referenced securities. However, Goldman Sachs did not inform investors in the deal of Paulson & Co. involvement.

By early 2008, almost the entire ABACUS reference portfolio had defaulted. This left Paulson & Co. and Goldman Sachs with $1.1 billion in profit and $15 million structuration fees respectively. However, the two main investors, ABN AMRO and IKB[15], lost together almost $1 billion. A brief description of the chain of events behind this transaction can be found in Appendix 9.

The structure of the ABACUS synthetic CDO is detailed in Table 4.

Table 4: ABACUS Transaction Details

Index Name	Time Horizon (in Years)	# of Corporate Names	Total Notional Amount	Tranche Structure		
				Tranche Name	Attachment/ Detachment Point	Moody's Ratings
ABACUS	4.2	90	US $2 billion	Super Senior	45% - 100%	N/A
				Class A	21% - 45%	Aaa
				Class B	18% - 21%	Aa2
				Class C	13% - 18%	Aa3
				Class D	10% - 13%	A2
				First Loss	0% - 10%	N/A

Source: ABACUS Pitch Book

[15] ABN AMRO is Dutch private and commercial bank; IKB is a German commercial bank.

The study will focus on Class A and Class D tranches of ABACUS.

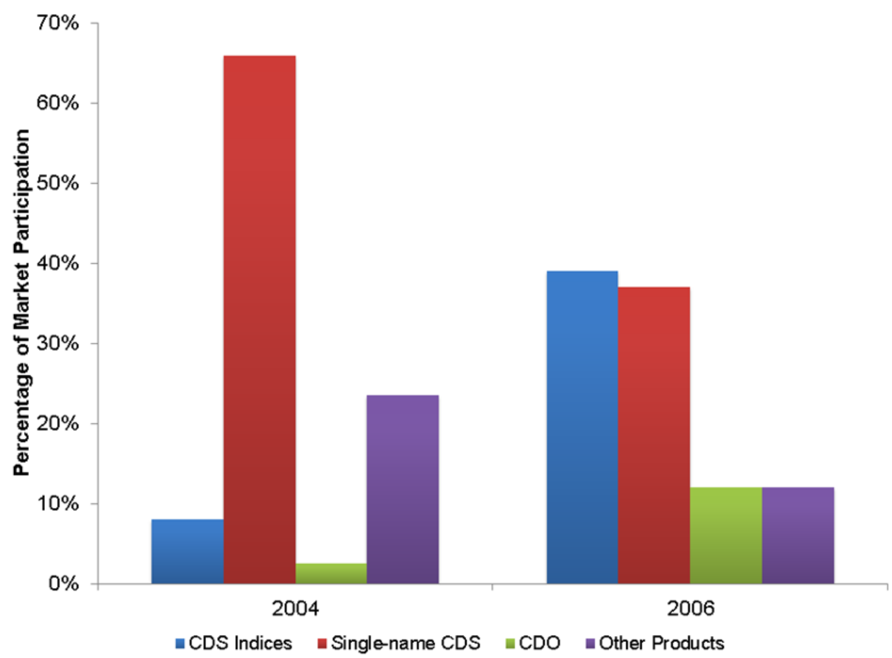
6.2 Cases #2 and #3: CDX Indices

The CDX indices are a part of the family of CDS indices released by Markit[16]. A CDS index is referred to the exchange of credit risk between the protection buyers of a basket of liquid CDS contracts and protection sellers. In general, a CDS index is a collection of CDS contracts that offers the opportunity to sell or buy protection on the reference portfolio of each index. In particular, CDS index tranches, such as the CDX indices, give investors the opportunity to take exposure to specific segments of the CDS index default distribution.

The growth of the CDS indices market was fueled by three main reasons. First, because CDS indices provide a diversified credit risk exposure in one single transaction. Second, CDS indices are traded as OTC products. This feature facilitates the trading of credit risk among investors, and the implementation of investment strategies. And finally, because CDS indices help the credit market to become more liquid, efficient and transparent.

Figures 25, 26 and 27 describe the growth of the CDS indices market. Figure 25 shows the market participation of CDS Indices. Figure 26 shows the issuance of CDS index tranches in the market shows the participation of the CDS indices in the credit derivative market. Figure 27 shows the notional amount of outstanding CDS indices in the market.

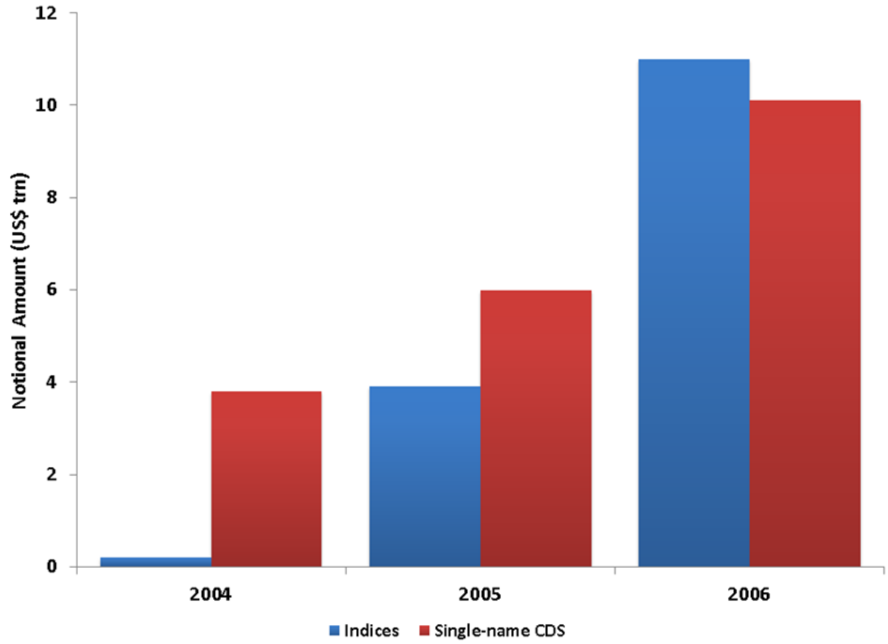
Figure 26: CDS Indices Credit Derivative Market Participation



Source: Bank for International Settlements

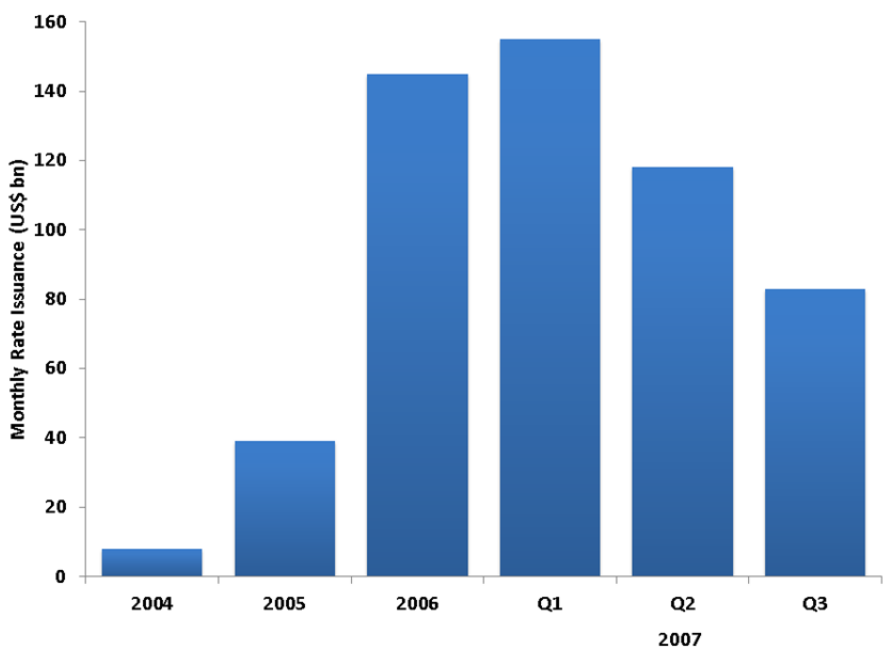
[16] Markit is a private firm that values CDS contracts and creates CDS information products. This firm has been releasing synthetic credit indices since its foundation in 2001.

Figure 27: Notional Amount Outstanding of CDS Indices and Single-Name CDS



Source: Fitch Ratings and Bank for International Settlements

Figure 28: Issuance of CDS Index Tranches



Source: Creditflux and Bank for International Settlements

In particular, the CDX indices are characterized by two main features. On the one hand, the securities in the reference portfolios are corporate names in North America or

Emerging Markets. On the other hand, the credit events that trigger the defaults are bankruptcy and failure to pay.

The CDX indices considered for the purpose of this study are two: the CDX North America Investment Grade (CDX.NA.IG, Case #2) and the CDX North America High Yield (CDX.NA.HY, Case #3).

The mechanics of the credit protection offered by the CDX indices works as an unfunded synthetic CDO, but adds some index features. In contrast with common synthetic CDO issues, the tranches of a CDX index are very liquid and are traded from a dealer's trading desk instead of being offered one to one to the investors. The reason for this is the high level of trading activity of the credits in the reference portfolio.

Another feature of the CDX indices is in the management of their reference portfolios. The reference portfolios are composed by different CDS contracts with a fixed premium. These CDS contracts are renewed every six months[17]. In case that the renovation changes the price of the credits in the portfolio, a compensation premium is exchanged between the buyers and sellers of protection in the indices. Due to this reason, the CDX indices are attractive for short-term trading and hedging. For a long-term investment in corporate credit risk, synthetic CDOs in the market are preferred. Table 5 summarizes the characteristics of these indices[18].

Table 5: CDX Indices Structures

Index Name	Assets Class	Time Horizon (Years)	# of Corporate Names	Tranche Structure	
				Tranche Name	Attachment/Detachment Point
CDX.NA.IG	North America Investment Grade	1,2,3,5,7 and10	125	Super Senior	30%-100%
				Senior 1	15%-30%
				Senior 2	10-15%
				Mezzanine 1	7%-10%
				Mezzanine 2	3%-7%
				Equity	0%-3%
CDX.NA.HY	North America High Yield	5	100	Super Senior	35%-100%
				Senior 1	25%-35%
				Senior 2	15%-25%
				Mezzanine	10%-15%
				Equity	0-10%

Source: Markit

The study will consider the CDX.NA.IG index with 2, 5 and 10 years horizon. This study will focus on Senior 1 and Mezzanine 2 tranches. In the case of the CDX.NA.HY, This study will focus on the Senior 1 and Mezzanine tranches.

[17] These renovations are exchanged every 20th of March, June, September and December
[18] For a detailed composition of both indices, see Appendix 10

6.3 Case #4: MIDGARD

MIDGARD is a $50 million synthetic CDO-Squared referencing a portfolio of mezzanine tranches of other synthetic CDOs. Those synthetic CDOs and are backed by corporate names. The MIDGARD was managed by Henderson Global Investors Limited. CALYON was the CDS counterparty on this transaction.

MIDGARD is an interesting structure to analyze since the credit rating of the mezzanine tranche was downgraded dramatically from Aaa (the original rating in 2005) to Baa3 (February of 2009). The chief reason for the downgrade was an "update" in the value of the parameters assumptions used in the synthetic CDO rating model. MIDGARD defaulted by March of 2009.

The structure of the MIDGARD transaction is detailed in Tables 6 and 7. Figure 28 shows diagram with the structure of the transaction. This study will focus on Super Senior and Class III tranches.

Table 6: MIDGARD Synthetic CDOs Portfolio Structure

Synthetic CDO-Squared	# of Different Securities	# of Synthetic CDOs	# of Assets per Synthetic CDO	Synthetic CDO Mezzanine Tranches	
				Tranche Name	Attachment-Detachment Point
MIDGARD	259	7	80	Senior	10.1%-100%
				Mezzanine	7.1%-10.1%
				Equity	0%-7.1%

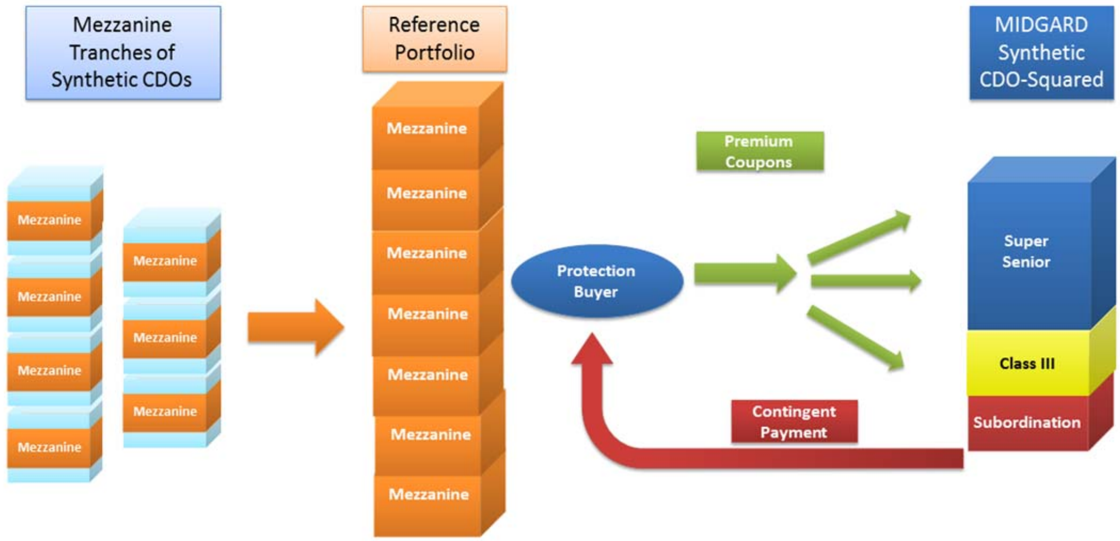
Source: MIDGARD Moody's Issue Report, 2005.

Table 7: MIDGARD Synthetic CDO-Squared Structure

Synthetic CDO-Squared	Synthetic CDO-Squared Tranche Structure		
	Tranche Name	Attachment-Detachment Point	Rating
MIDGARD	Super Senior	32%-100%	Aaa
	Class III	22%-32%	Aaa
	Subordination	0%-22%	N/A

Source: MIDGARD Moody's Issue Report, 2005.

Figure 29: MIDGARD Structure Diagram



Source: MIDGARD Issue Report, 2005.

The overlap of securities in the MIDGARD structure is described by the following conditions:

- The average overlap between synthetic CDO pools is on average 26%.
- The number of overlapped entities between any two synthetic CDOs is less than 30%.
- None of the securities underlying the synthetic CDOs is present in more than 4 reference portfolios.

As can be seen, the overlap among reference portfolios is not completely defined. For this reason, this study will rely on a particular structure that satisfies the above mentioned overlap conditions. This structure was determined by solving a linear optimization problem that includes the above mentioned overlap conditions as restrictions. This linear optimization problem is fully detailed in Appendix 11.

6.4 Cases #5 and #6: Theoretical CDO-Squared

Two theoretical synthetic CDO-Squared are considered. These two structures are based on both mentioned CDX indices and are intended to study the effect of the overlap in the rating stability. The theoretical structures are used because there is not a CDS index with a synthetic CDO-Squared structure.

The motivation behind the design of these three structures is based on the liquidity of the CDS market. Consider a group of N investors interested on betting against the price of N mezzanine tranches. Given the high liquidity of the CDS market, it could be possible for the N investors to somehow replicate the structure of the CDX indices in N different synthetic CDOs. Thus, the N investors obtain N identical mezzanine tranches with some degree of overlap. Then, these N mezzanine tranches can be referenced by the portfolio of a synthetic CDO-Squared.

There are two possible theoretical synthetic CDO-Squared to consider from both CDX indices. In the CDX.NA.IG and CDX.NA.HY indices there are two and one mezzanine tranches to consider respectively. The theoretical structures based on the CDX.NA.IG index, will be denoted as T.CDX.NA.IG. The theoretical structure based on the CDX.NA.HY will be denoted as T.CDX.NA.HY. Since the usual number of mezzanine tranches referenced by a synthetic CDO-Squared is between 5 and 10, the study will consider 7 tranches. The structure of the two theoretical structures is summarized in Tables 8,9,10 and 11. Figures 29 and 30 show a diagram of the structure involved for both theoretical structures.

Table 8: T.CDX.NA.IG Synthetic CDOs Portfolio Structure

Synthetic CDO-Squared	CDX Index Considered	# of Synthetic CDOs	Mezzanine Tranches Included	Synthetic CDO Tranche Structure	
				Tranche Name	Attachment/Detachment Point
T.CDX.NA.IG	CDX.NA.IG	7	Mezzanine 2	Super Senior	30%-100%
				Senior 1	15%-30%
				Senior 2	10-15%
				Mezzanine 1	7%-10%
				Mezzanine 2	3%-7%
				Equity	0%-3%

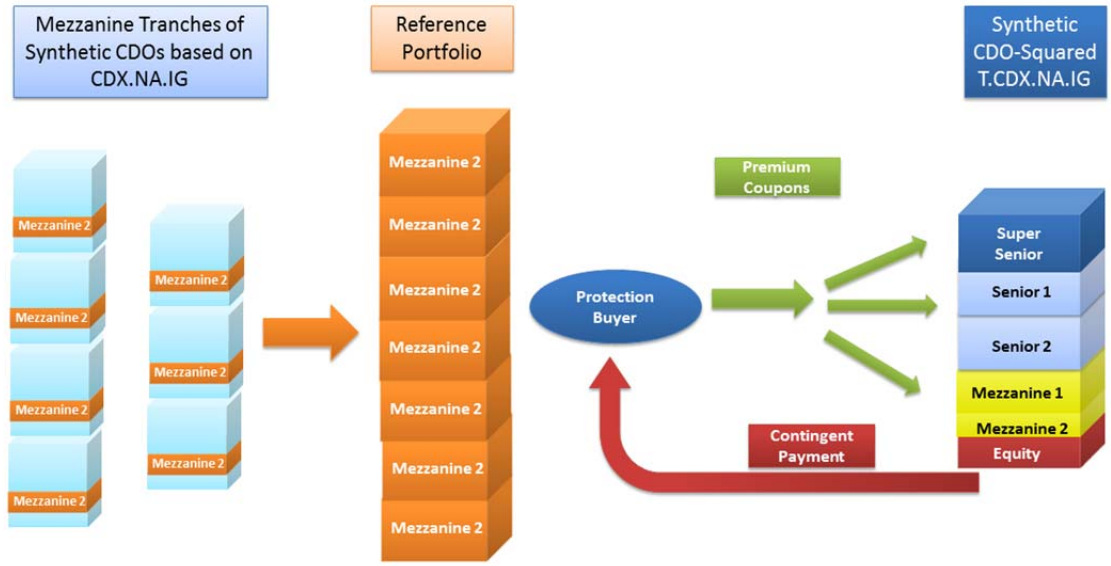
Source: Own Elaboration

Table 9: T.CDX.NA.IG Synthetic CDO-Squared Structure

Synthetic CDO-Squared	Synthetic CDO-Squared Tranche Structure	
	Tranche Name	Attachment/Detachment Point
T.CDX.NA.IG	Super Senior	30%-100%
	Senior 1	15%-30%
	Senior 2	10-15%
	Mezzanine 1	7%-10%
	Mezzanine 2	3%-7%
	Equity	0%-3%

Source: Own Elaboration

Figure 30: T.CDX.NA.IG Structure Diagram



Source: Own Elaboration

Table 10:T.CDX.NA.HY Synthetic CDOs Portfolio Structure

Synthetic CDO-Squared	CDX Index Considered	# of Synthetic CDOs	Mezzanine Tranches Included	Synthetic CDO Tranche Structure	
				Tranche Name	Attachment/Detachment Point
T.CDX.NA.HY	CDX.NA.HY	7	Mezzanine	Super Senior	35%-100%
				Senior 1	25%-35%
				Senior 2	15%-25%
				Mezzanine	10%-15%
				Equity	0-10%

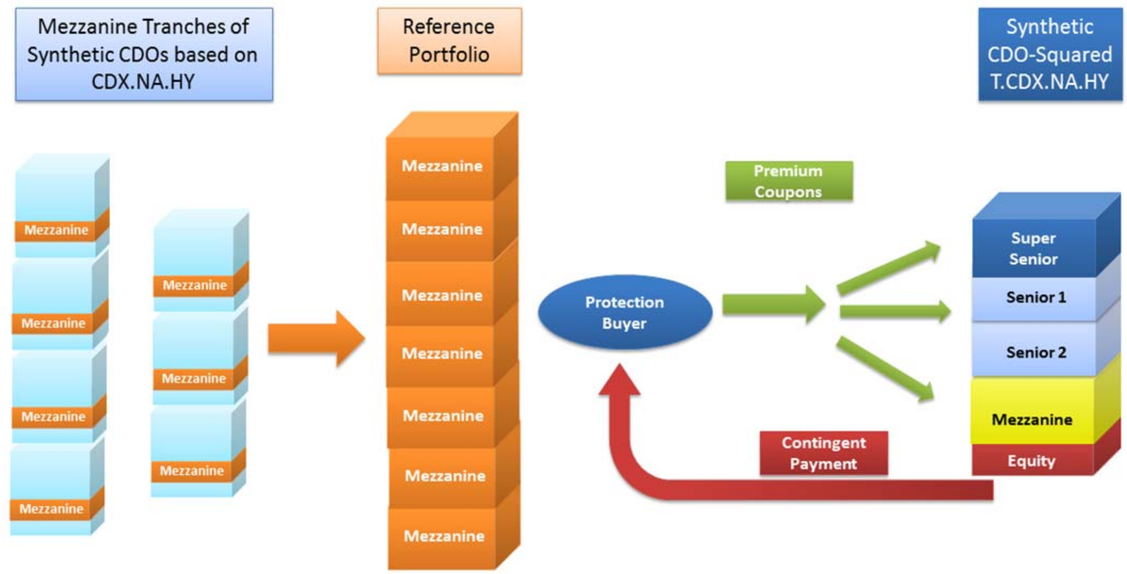
Source: Own Elaboration

Table 11: T.CDX.NA.HY Synthetic CDO-Squared Structure

Synthetic CDO-Squared	Synthetic CDO-Squared Tranche Structure	
	Tranche Name	Attachment/Detachment Point
T.CDX.NA.HY	Super Senior	35%-100%
	Senior 1	25%-35%
	Senior 2	15%-25%
	Mezzanine	10%-15%
	Equity	0-10%

Source: Own Elaboration

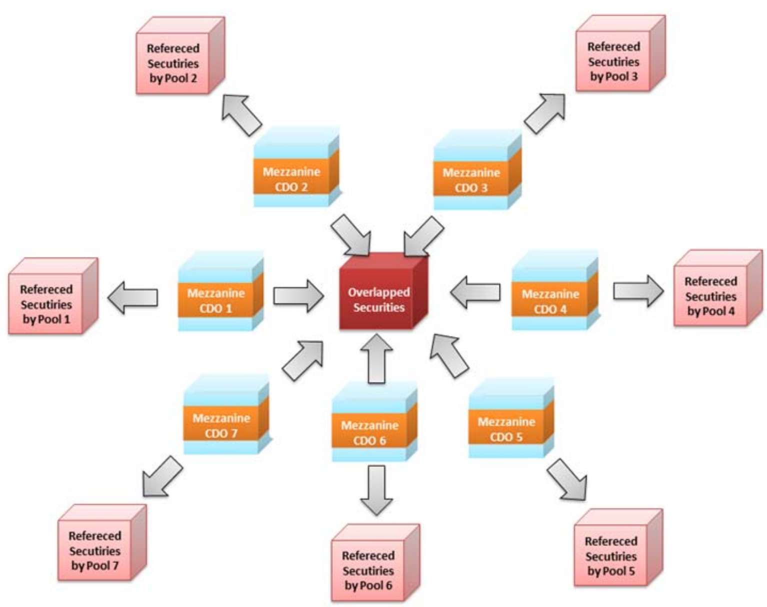
Figure 31: T.CDX.NA.HY Structure Diagram



Source: Own Elaboration

The effect of the degree of overlapping in the structure is analyzed with the following model. Each one of the synthetic CDOs will reference a pool of securities. A portion equals to 100 - x% of the pool of securities are exclusively referenced by the synthetic CDO, whereas x% are referenced by all of the synthetic CDOs. Similar to other synthetic CDO-Squared transactions, the maximum degree of overlapping consider in this study will be 30%. Figure 31 illustrates the above mentioned overlapping.

Figure 32: Overlap Diagram



Source: Own Elaboration

7. RESULTS

This chapter shows the results of the analysis of the different cases studied. For each case, there is an explanation regarding the specifics for the particular analysis undertaken[19].

7.1 Case #1: ABACUS

7.1.1 Confidence Intervals

In the beginning the ABACUS transaction was marketed with a set of credit ratings provided by Moody's. These ratings described the risk profile of ach one of the different tranches and were based, among other things, on the characteristics of the underlying portfolio. Using this information it is possible to estimate the values for the asset parameters that let one obtain the ratings of the ABACUS tranches.

The assets in the underlying portfolio were rated, on average, Baa. The average maturity was 4.2 years. Therefore, the default probability of the pool can be described using a normal distribution with a mean equals to 1.53% and a standard deviation equals to 0.16%[20].

By taking the value of the default probability equal to the mean of the distribution, one can estimate the pairwise values for default correlation and recovery rate that match the rating of the ABACUS tranches using the expected loss criteria. Figure 32 shows the different possible combinations of default correlation and recovery rate that were obtained.

There are 44 different pairs of default correlation and recovery rate values. These pairs cover the range from 23% to 55% for the default correlation, and from 37% to 74% for the recovery rate. The trend across the pairs is very clear: the higher the default correlation, the higher the recovery rate. This is because, in general, a higher default correlation increases the expected loss of the tranches. Thus, a higher recovery rate is required to decrease the expected loss.

For each one of the 44 pairs described in Figure 33, it is possible to estimate with a 95% confidence, an interval for the expected loss of a given tranche, using the probabilistic behavior of the default probability. Figures 34 and 35 show the confidence intervals obtained for two tranches: Class A and Class D respectively.

As can be seen, the confidence intervals are very stable for both tranches. In the case of the Class A tranche, the different confidence intervals are completely contained within the Aaa rating region. In the case of the Class D tranche, almost all the confidence intervals (except for the last one) contain three possible credit ratings: A1, A2 and A3. Therefore, with a 95% certainty, it can be stated that the rating of the Class A should have been Aaa, no matter what combination of default correlation/recovery

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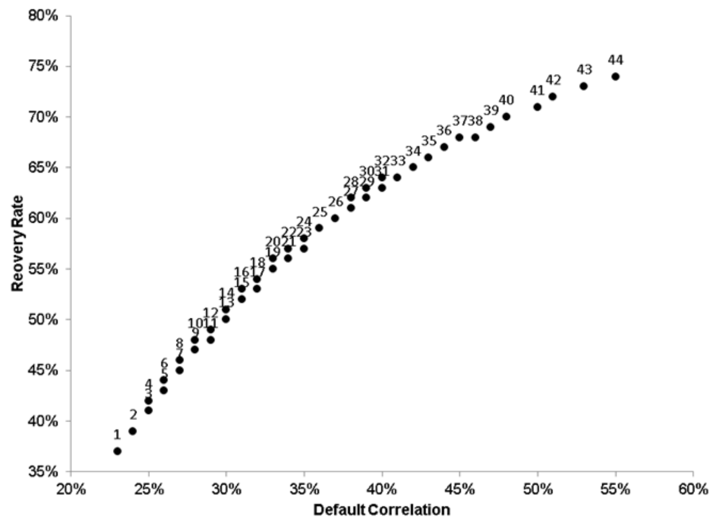
[19] In Appendix 13 are detailed the confidence intervals explained in this chapter.

[20] These values were obtained by a cubic spline interpolation of the values for the mean and standard deviation for different time horizons and ratings, from Cantor, Hamilton & Tennant (2007).

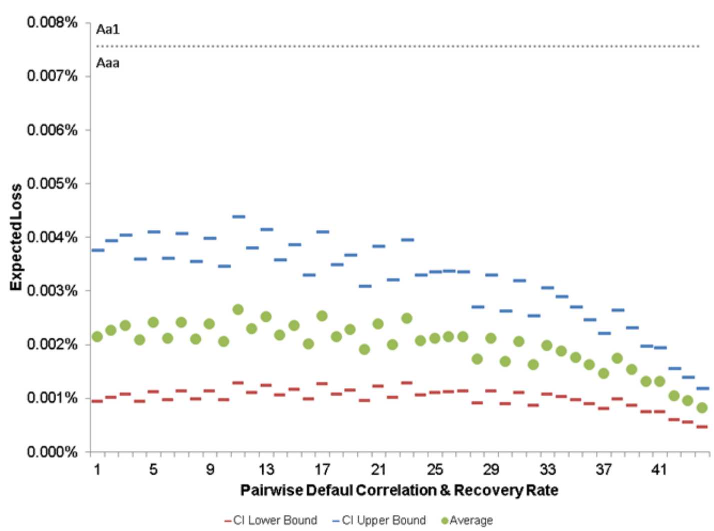
rate is used. On the other hand, for the Class D tranche for almost all combinations of default correlation/recovery rate the confidence interval spans the complete A region.

Figure 33: Combinations of Default Correlation and Recovery Rate that Match the ABACUS Tranches Ratings



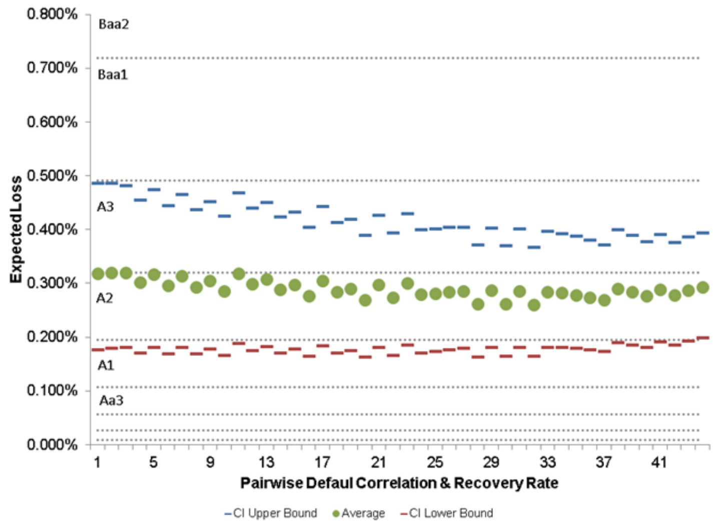
Source: Own Elaboration

Figure 34: ABACUS Class A Tranche Confidence Intervals for the Pairwise Default Correlation and Recovery Rate.



Source: Own Elaboration

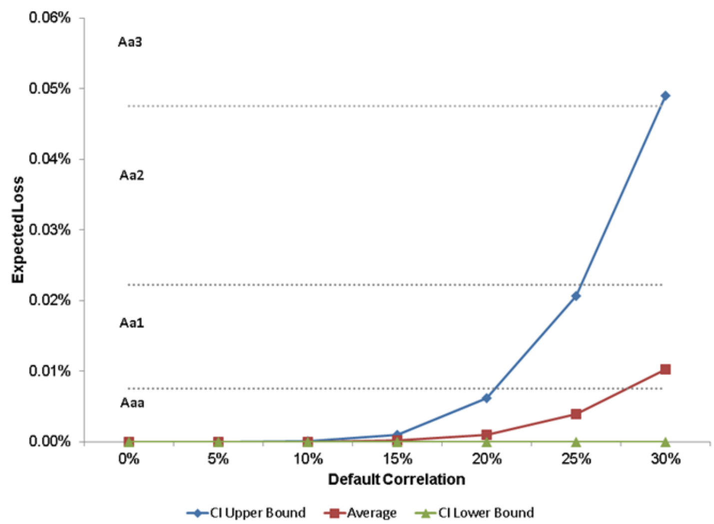
Figure 35: ABACUS Class D Tranche Confidence Intervals for the Pairwise Default Correlation and Recovery Rate.



Source: Own Elaboration

A more refined analysis can be performed using different alternative to characterize the recovery rate. The first possibility is to assume that the recovery rate follows a Beta distribution (with a mean equals to 50% and a standard deviation equals to 26%)[21]. Alternatively one can assume a recovery rate inversely proportional to the default probability according to Equation 29 from Chapter 5. Finally, the idea is to estimate a 95% confidence interval for the expected loss for both tranches. Figures 36 and 37 show the results for the Class A tranche. Figure 36 assumes a Beta distribution and Figure 37 uses Equation 29 from Chapter 5.

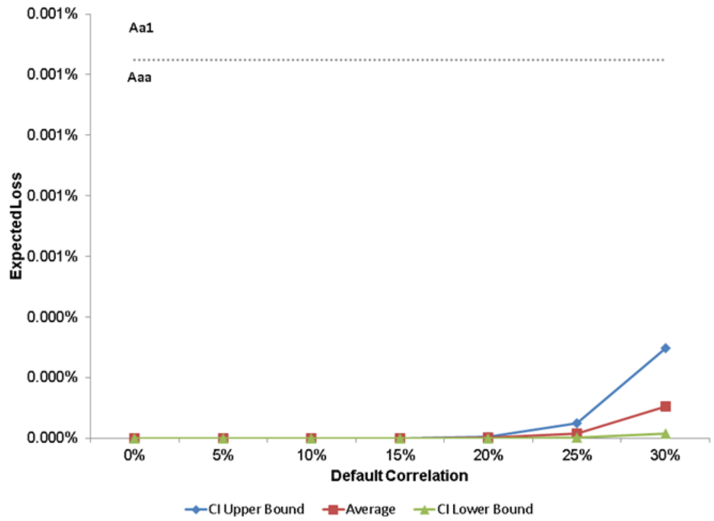
Figure 36: ABACUS Class A Tranche Confidence Intervals (Beta Distribution)



Source: Own Elaboration

[21] According to Emery, Cantor, Keisman & Ou (2007)

Figure 37: ABACUS Class A Tranche Confidence Intervals (Chapter 5, Equation 29).

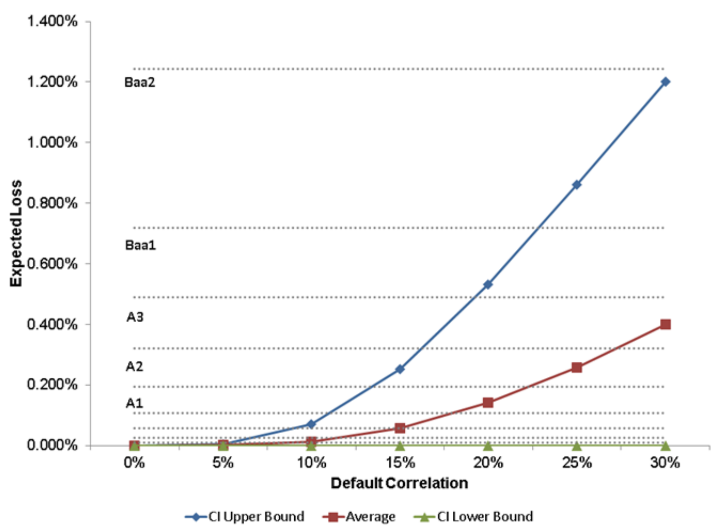


Source: Own Elaboration

Figures 36 and 37 show two clear trends. First, as expected, the simulation using a Beta distribution shows more variability in terms of the expected loss. And second, increasing the default correlation value results in more uncertainty regarding the expected loss. The reason is that higher values for the default correlation make the effect of each default more onerous. Finally, Figure 38 shows that for a default correlation of 30% (the value used by Moody's) the rating of the Class A tranche falls within the range Aaa /Aa3 (four notches).

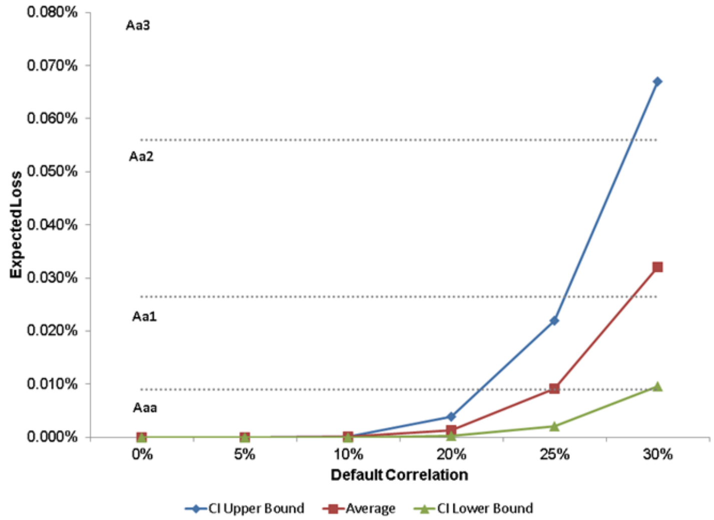
Figures 38 and 39 display the same results for Class D tranche.

Figure 38: ABACUS Class D Tranche Confidence Intervals (Beta Distribution)



Source: Own Elaboration

Figure 39: ABACUS Class D Tranche Confidence Intervals (Chapter 5, Equation 29).



Source: Own Elaboration

These two figures show the same trends show for the Class A tranche. The only difference is that the same tendencies appear more exacerbated. More to the point, the 95 % confidence interval for the rating of Class D is much wider, covering a range from Aaa/Baa2 (nine notches) in the case of a 30% of default correlation.

Comparing both approaches, the use of a Beta distribution for the recovery rate creates confidence intervals much more onerous. However, both approaches show some degree of instability for the credit rating of the ABACUS tranches.

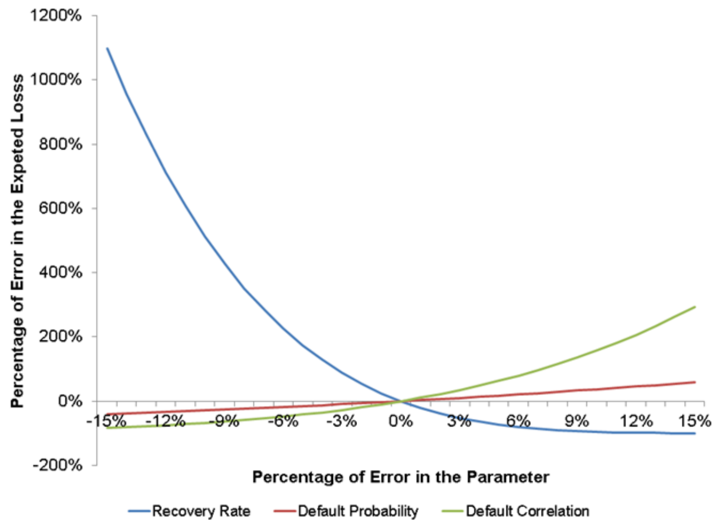
To summarize, for default correlation values higher than 15%-20% (the most likely values for a realistic point of view) there is an increasing uncertainty regarding the expected loss.

The uncertainty regarding the ratings uncovered by this analysis seems to be in tune with the empirical evidence. Initially Moody's rated the Class A tranche as Aaa. However, within six months of the closing date, in an embarrassing admission of inaccuracy Moody's was forced to downgrade the Aaa rating to Baa2. Four months later, the Baa2 became Caa.

7.1.2 Sensitivity Analysis to Errors in the Asset Parameters

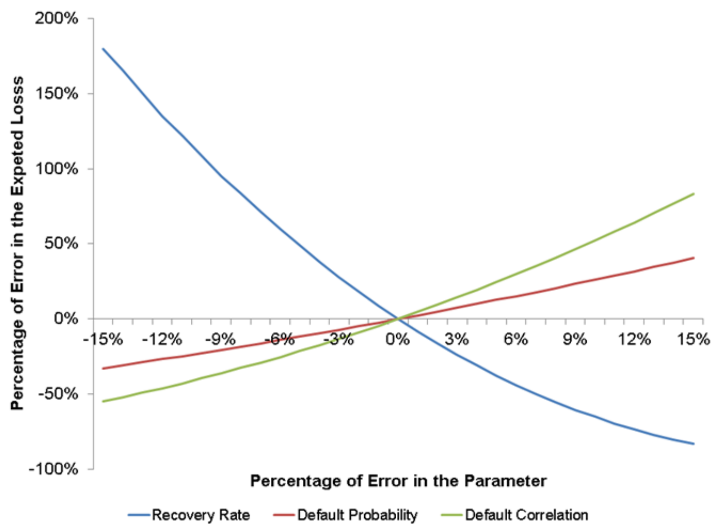
The degree of sensitivity of the expected loss of the Class A and Class D tranche to errors in the asset parameters is also analyzed. For this analysis a recovery rate equals to 50% is considered (this is the value typically used by Moody's for synthetic CDO tranches and used as the mean of the Beta distribution for the recovery rate). The corresponding value of the correlation is 30%. The sensitivity of the expected loss is explored by keeping two parameters constant while allowing the third to vary. Figures 40 and 41 display the sensitivity of the expected loss for Class A and Class D tranches respectively.

Figure 40: ABACUS Class A Tranche Parameter Sensitivity



Source: Own Elaboration

Figure 41: ABACUS Class D Tranche Parameter Sensitivity

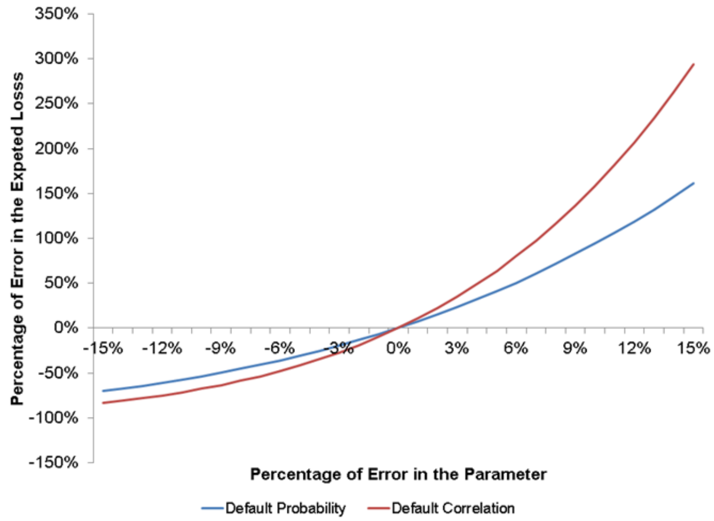


Source: Own Elaboration

Figures 40 and 41 are self-explanatory. They both show that an error in the recovery rate has the most influence in the expected loss when such error is negative. For positive errors, the default correlation becomes the most influential parameter. In addition, underestimating the recovery rate implies a positive error in the expected loss. On the other hand, a positive error in the recovery it leads to underestimate the expected loss. Both trends are intuitively correct. Errors in estimating the correlation parameters show a reverse trend compare to the recovery rate. This also makes sense intuitively.

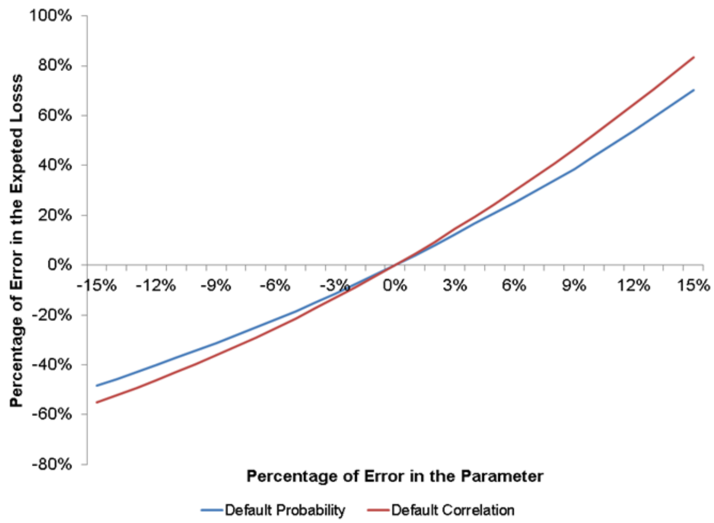
Figures 42 and 43 are analogous to Figures 40 and 41, except for the fact that the recovery rate is assumed to be determined by Equation 29 from Chapter 5. Therefore, only two sensitivity curves are included (with respect to an error in the default probability and in the default correlation).

Figure 42: ABACUS Class A Tranche Parameter Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 43: ABACUS Class D Tranche Parameter Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

The trends seem to be consistent with the behavior displayed in the previous two cases.

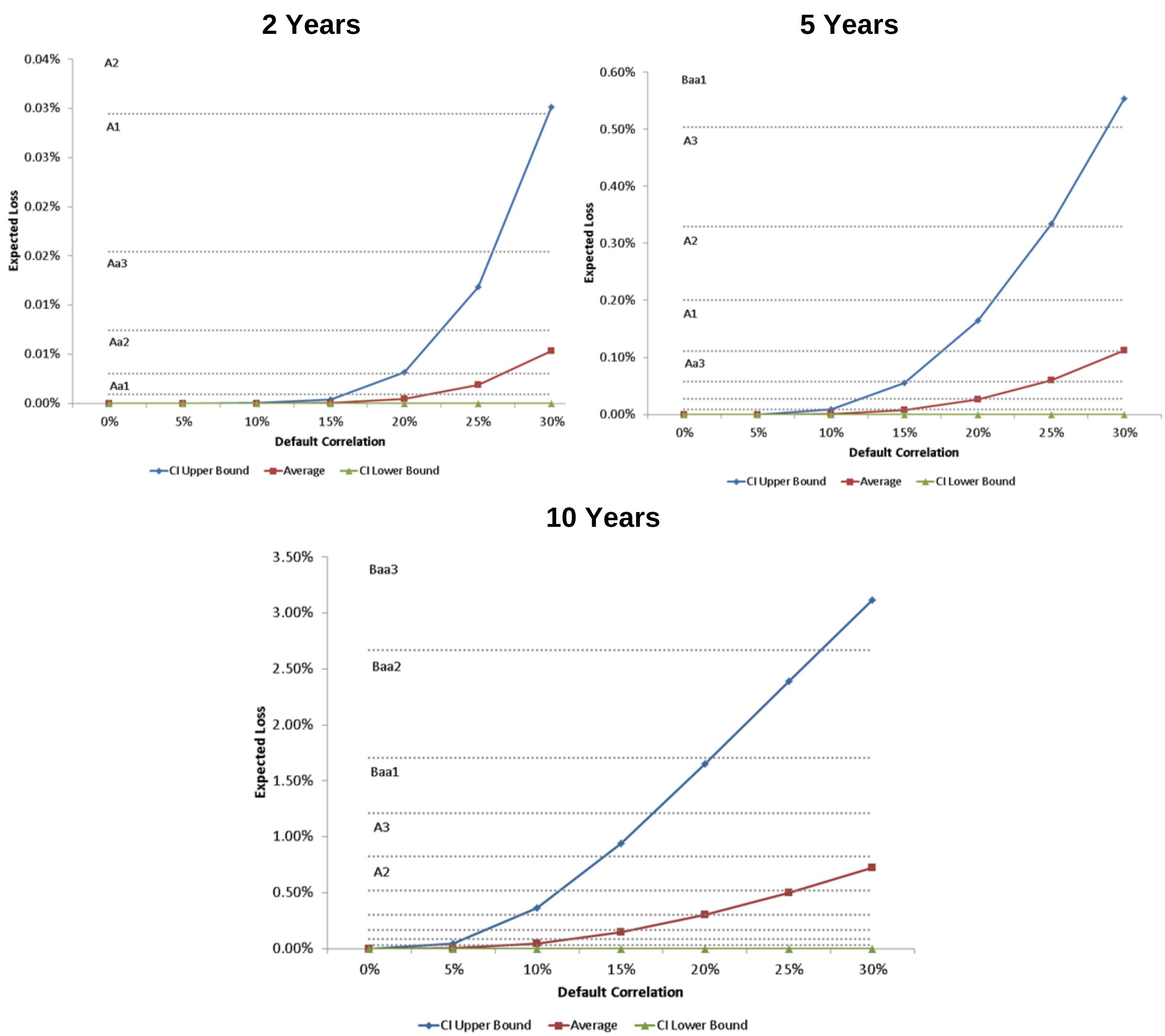
7.2 Cases #2 and #3: CDX Indices

7.2.1 Confidence Intervals for the CDX.NA.IG Index

The analysis of the CDX indices is similar to that of the ABACUS transaction. The main difference with the ABACUS transaction is that the tranches that compose these indices do not have a formal credit rating. Nevertheless, a rating can be estimated simply by estimating the expected loss associated with the tranche under consideration.

In the case of the CDX.NA.IG, the expected loss for the two tranches considered, and using a 95% confidence interval is estimated. Figures 44, 45 and 46 show the expected loss for the Senior 1 tranche, assuming the recovery rate follows a Beta distribution. For the different maturities (2, 5 and 10 years), the default probability of the pool rated on average Baa, can be described using a normal distribution with the respective different mean (0.51%, 1.94% and 1.63%) and standard deviation (0.03%, 0.19% and 0.46%).

Figures 44, 45 and 46: CDX.NA.IG Senior 1 Tranche Confidence Intervals (Beta Distribution), for 2, 5 and 10 Years Horizon Respectively

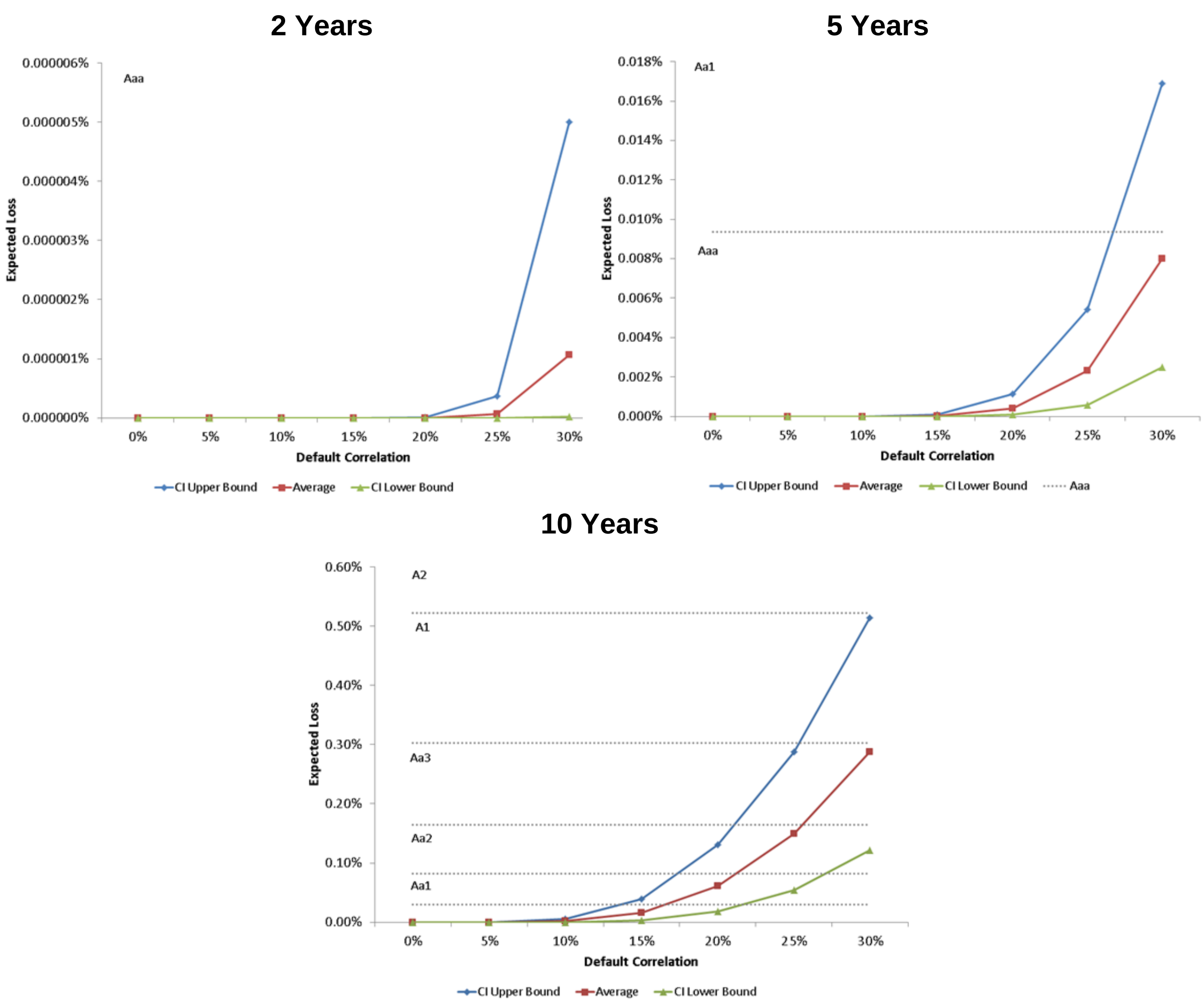


Source: Own Elaboration

The behavior shown by these figures is consistent with the trends detected for the ABACUS transaction. Namely, increasing values of the correlation are associated with increasing uncertainty in the ratings. Specifically, for default correlation values within the 25%-30% range (realistic values for most portfolios) the ratings can vary as much as ten notches (Aaa/Ba1). As expected, more variability in the ratings is associated with longer maturity horizons.

Figures 47, 48 and 49 show the case for the Senior 1 tranche with the recovery rate following Equation 29 from Chapter 5.

Figures 47, 48 and 49: CDX.NA.IG Senior 1 Tranche Confidence Intervals (Chapter 5, Equation 29), for 2, 5 and 10 Years Horizon Respectively



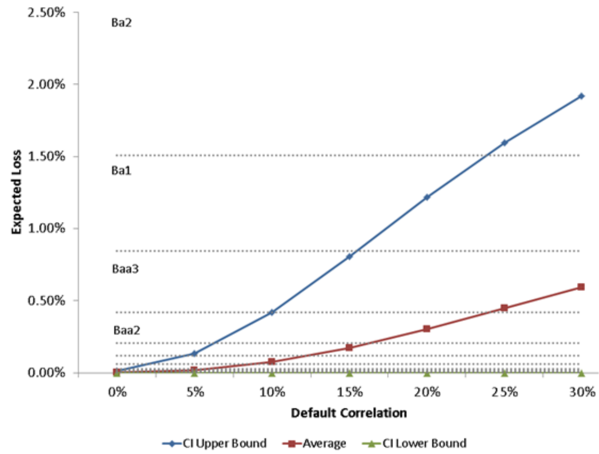
Source: Own Elaboration

These figures are very much in agreement with the trends displayed in the previous case (Figures 45, 46 and 47) except that the variability is much more bounded. However, even under these conditions there is a considerable variability in terms of the ratings (three notches for the 10 years maturity).

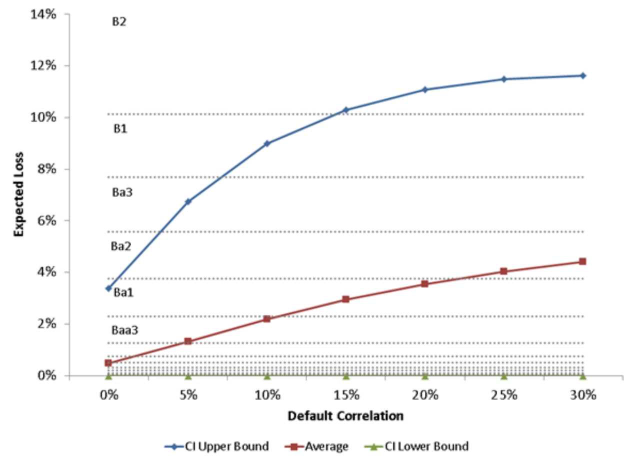
Figures 50, 51 and 52 show the case of the Mezzanine 2 tranche with the recovery rate distributed as a Beta.

Figures 50, 51 and 52: CDX.NA.IG Mezzanine 2 Tranche Confidence Intervals (Beta Distribution), for 2, 5 and 10 Years Horizon Respectively

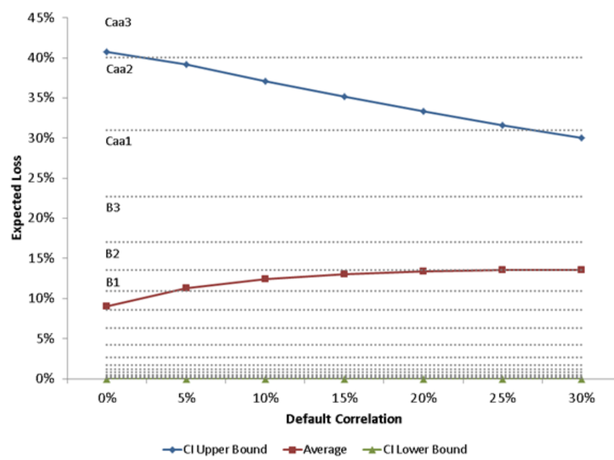
2 Years



5 Years



10 Years



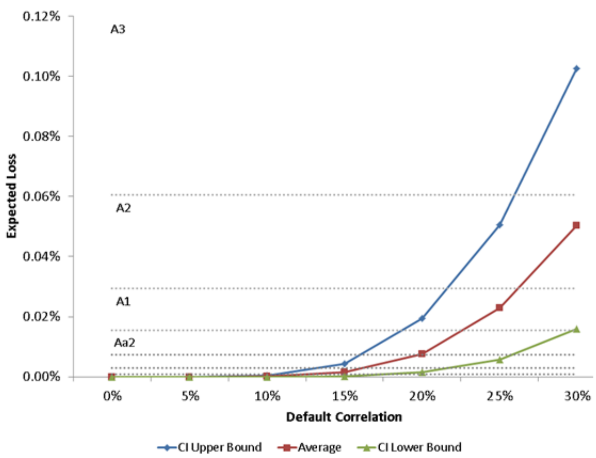
Source: Own Elaboration

Again, and broadly speaking, the trends displayed by these figures are in agreement with the previous findings. There is however one significant difference. The upper bound of the expected loss interval (blue line) becomes "inverted" for a ten year maturity. In essence, for longer horizons which in turn correspond to higher default probabilities, a higher default correlation benefits the tranche. A different way to look at this is to think that for longer horizons the behavior of the mezzanine tranche tends to resemble the behavior of an equity tranche. The fact that equity tranches improve their expected loss behavior for higher correlations has been well established by Meng & Sengupta (2009).

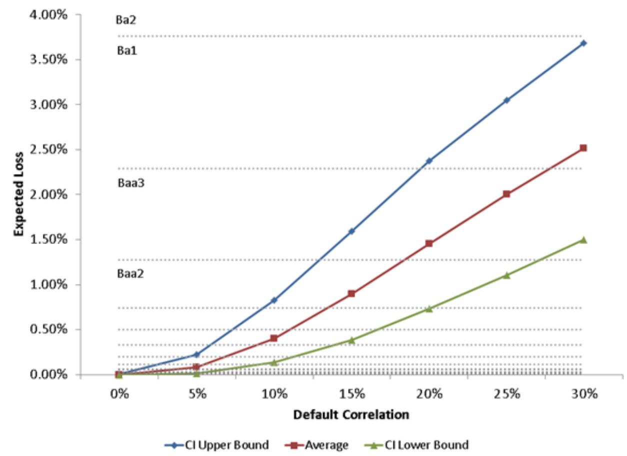
Figures 53, 54 and 55 show the case of the Mezzanine 2 tranche with the recovery rate following Equation 29 from Chapter 5.

Figures 53, 54 and 55: CDX.NA.IG Mezzanine 2 Tranche Confidence Intervals (Chapter 5, Equation 29), for 2, 5 and 10 Years Horizon Respectively

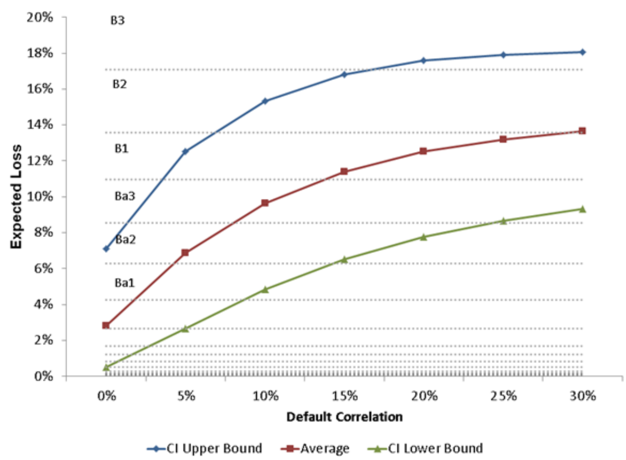
2 Years



5 Years



10 Years



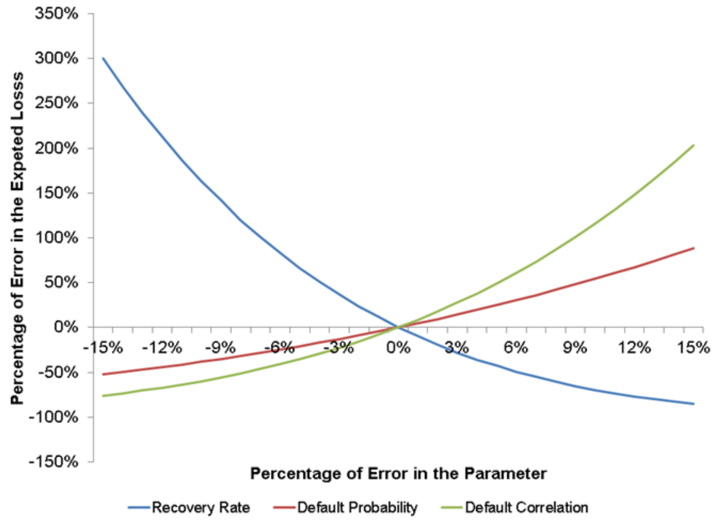
Source: Own Elaboration

The behavior shown in these figures is similar to that of Figures 50, 51 and 52 with the caveat that the benefits of high correlation values for longer horizons becomes less pronounced. The reason is straightforward: the longer the horizon, the higher the default probability, which in turn results in a low recovery value (more onerous for the mezzanine tranche) unlike the previous case.

7.2.2 Sensitivity Analysis to Errors in the Asset Parameters for the CDX.NA.IG Index

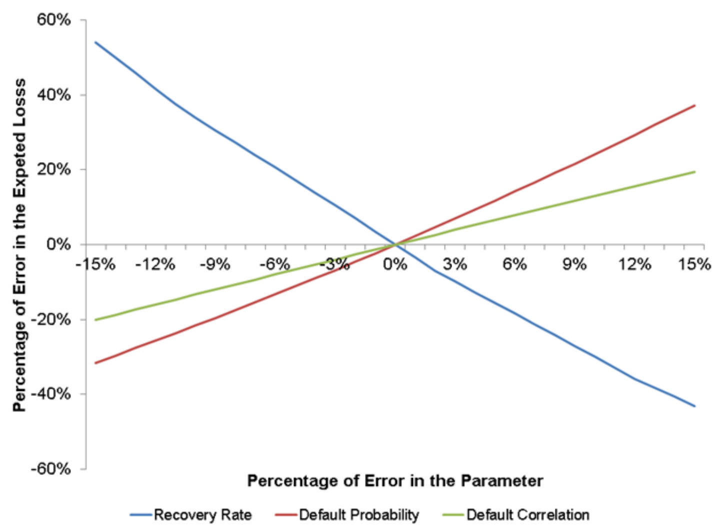
Finally, Figures 56 and 57 show the sensitivity of the expected loss for the Senior 1 and Mezzanine 2 tranches to errors in the asset parameters. In these cases, a 5 years maturity is considered, assuming the recovery rate is 50%, the default correlation is 30% and the default probability equals 1.94%.

Figure 56: CDX.NA.IG Senior 1 Tranche Sensitivity



Source: Own Elaboration

Figure 57: CDX.NA.IG Mezzanine 2 Tranche Sensitivity



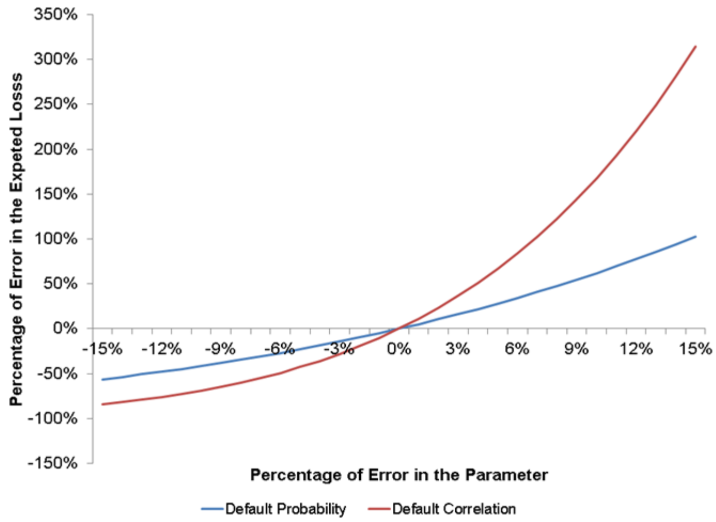
Source: Own Elaboration

Generally speaking, these figures are consistent with those of the ABACUS case. The fact that the default probability plays a more important role when one goes down in the capital structure is at least intuitively reasonable since lower seniority tranches are impacted by defaults much faster than more senior tranches.

Figures 58 and 59 show a similar sensitivity analysis but assuming now that recovery rate is given by Equation 29 from Chapter 5.

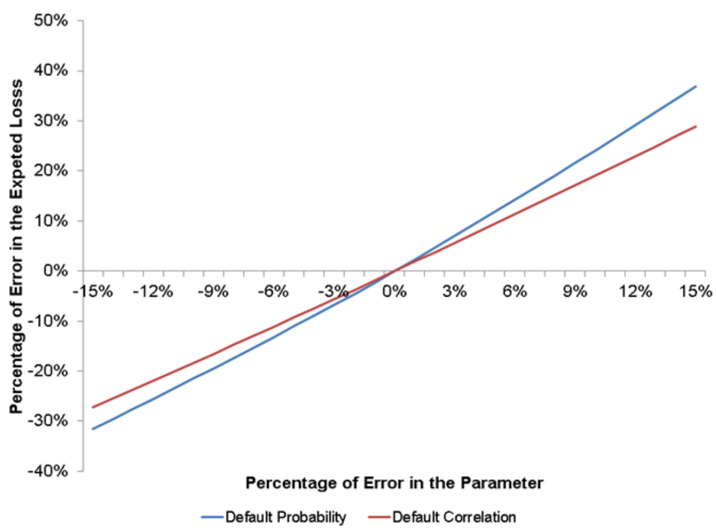
The results are analogous of Figures 42 and 43 in ABACUS, except that the Mezzanine 2 tranche is more sensitive to errors in the default probability.

Figure 58: CDX.NA.IG Senior 1 Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 59: CDX.NA.IG Mezzanine 2 Tranche Sensitivity (Chapter 5, Equation 29)



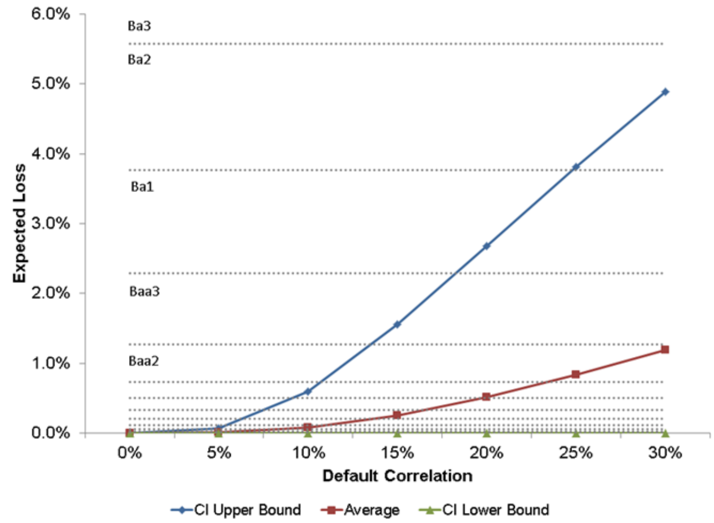
Source: Own Elaboration

7.2.3 Confidence Intervals for the CDX.NA.HY Index

In the case of the CDX.NA.HY there is only one maturity (five years). Figures 60 and 61 show the expected loss for Senior 1 and Mezzanine tranches for 5 years maturity, assuming that the recovery rate follows a Beta distribution. The default probability of the pool rated on average Ba, can be described using a normal distribution with a mean equals to 10.21% and standard deviation equals to 0.53%.

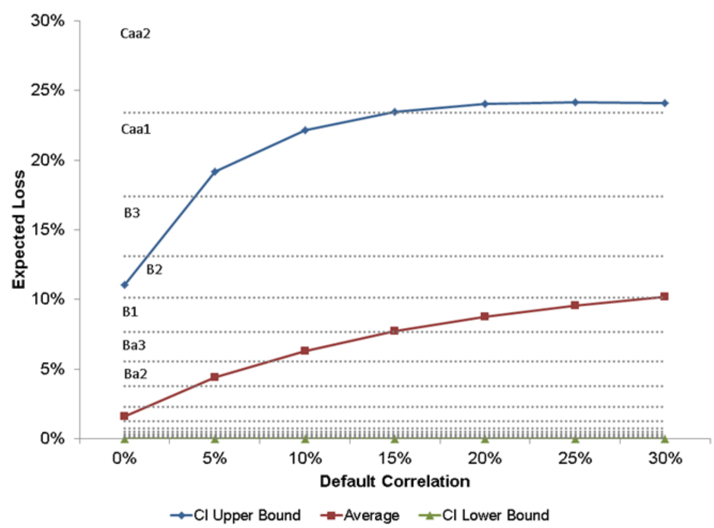
For correlation values around 15%, the confidence interval for the rating of the Senior tranche is fairly wide Aaa/Baa3 (nine notches). In the case of the Mezzanine tranche the rating could be anything between Aaa/Caa.

Figure 60: CDX.NA.HY Senior 1 Tranche Confidence Intervals (Beta Distribution)



Source: Own Elaboration

Figure 61: CDX.NA.HY Mezzanine Tranche Confidence Intervals (Beta Distribution)

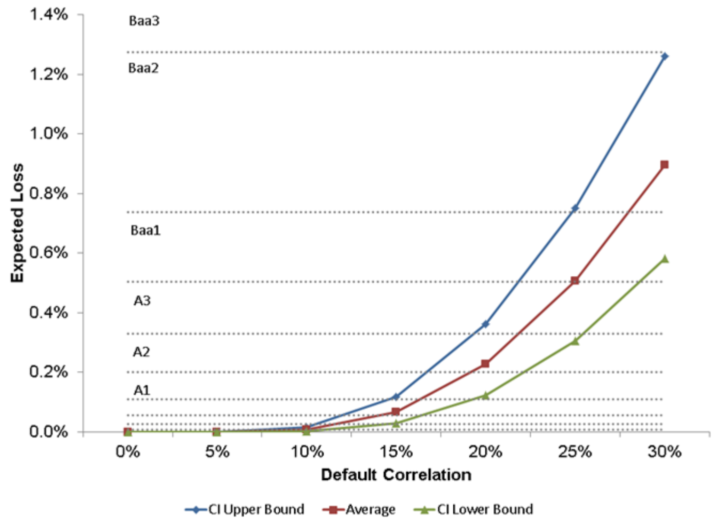


Source: Own Elaboration

Figures 62 and 63 show the expected loss for the Senior 1 and Mezzanine tranche with the recovery rate following Equation 29 from Chapter 5.

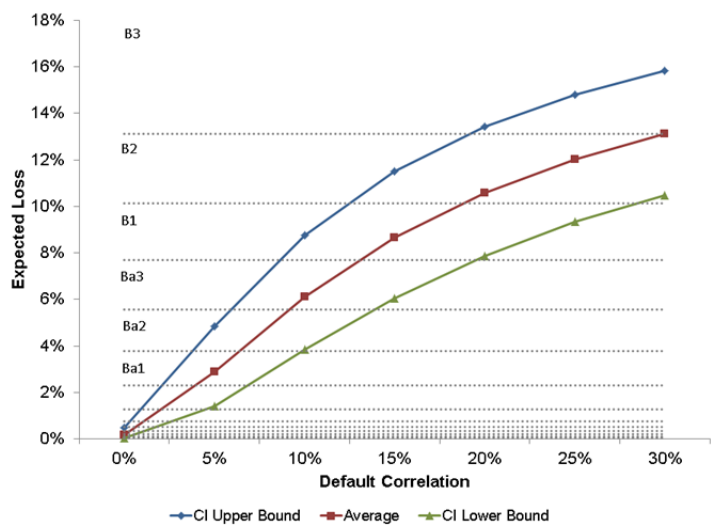
The behavior displayed in these figures is similar to the previous case except that it shows less variability.

Figure 62: CDX.NA.HY Senior 1 Tranche Confidence Intervals (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 63: CDX.NA.HY Mezzanine Tranche Confidence Intervals (Chapter 5, Equation 29)



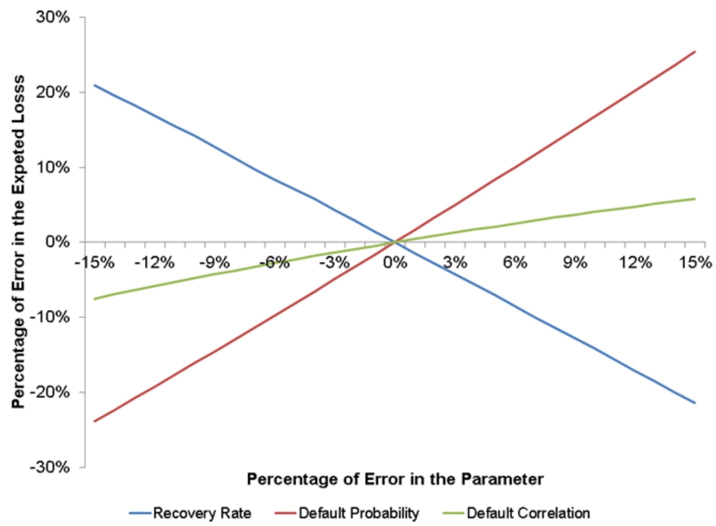
Source: Own Elaboration

7.2.4 Sensitivity Analysis to Errors in the Asset Parameters for the CDX.NA.HY Index

Figures 64 and 65 show the sensitivity of the expected loss for Senior 1 and Mezzanine tranches, assuming that the recovery rate recovery rate equals to 50% and a default. The default correlation is equal to 30% and the default probability is 10.21%.

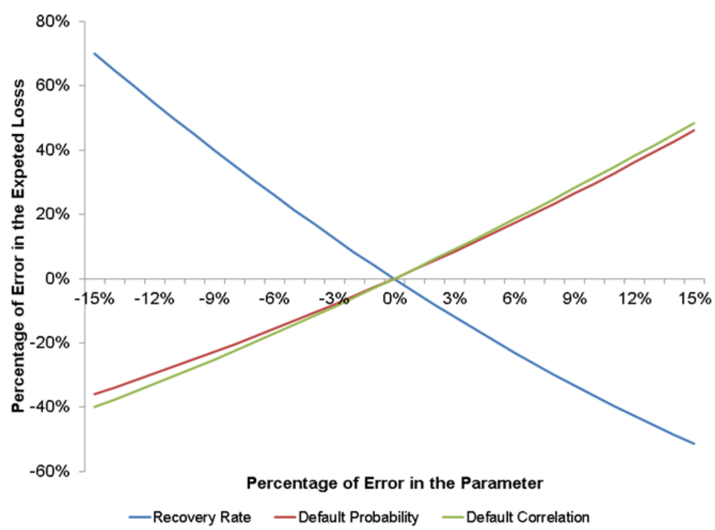
These figures are consistent with the findings explained in the context of the CDX.NA.IG.

Figure 64: CDX.NA.HY Senior 1 Tranche Sensitivity



Source: Own Elaboration

Figure 65: CDX.NA.HY Mezzanine Tranche Sensitivity

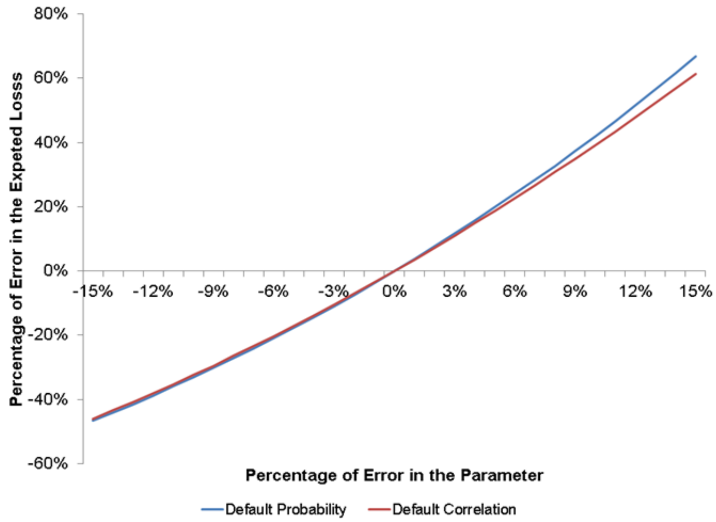


Source: Own Elaboration

Figures 66 and 67 show the sensitivity of the expected loss for the Senior 1 and Mezzanine tranches, assuming that the recovery rate is given by Equation 29 from Chapter 5.

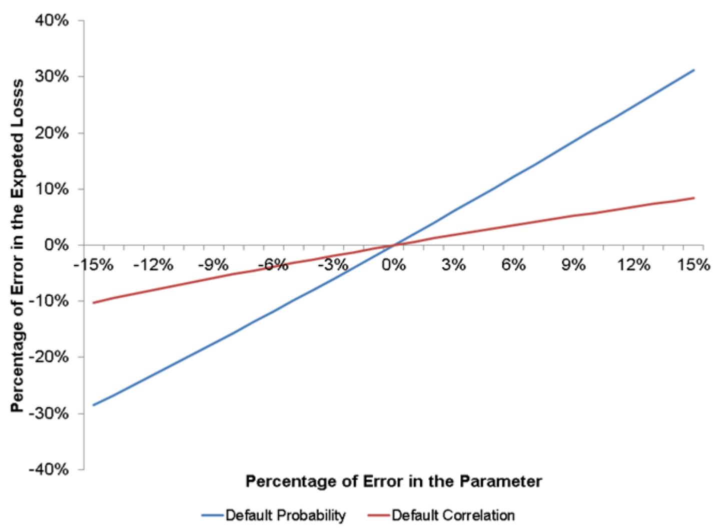
Again, and as expected, the tendencies are the same shown in the previous figures albeit less variability.

Figure 66: CDX.NA.HY Senior 1 Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 67: CDX.NA.HY Mezzanine Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

7.3 Case #4: MIDGARD

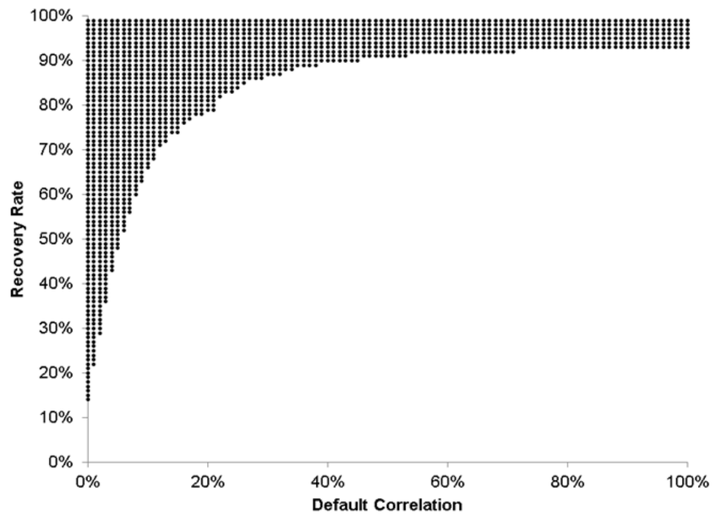
7.3.1 Confidence Intervals

Initially the Super Senior and the Class III (Mezzanine) tranches of the MIDGARD transaction were both rated Aaa by Moody's. Based on this information, it is possible to estimate the values for the asset parameters that let one obtain the ratings of the above-mentioned tranches.

The assets in the underlying portfolio were rated Baa2 with an average maturity equal to 5 years. Thus, considering the normal distribution assumptions, the default probability can be characterized using a mean equals to 1.94% and a standard deviation equals to 0.19%.

By setting the value of the default probability equal to the mean, one can estimate the pairwise values of the default correlation and the recovery rate that match the expected loss associated to the given ratings. The set of pairs that satisfy this criterion is shown in Figure 68 (black points).

Figure 68: Combinations of Default Correlation and Recovery Rate that Match the MIDGARD Tranches Ratings



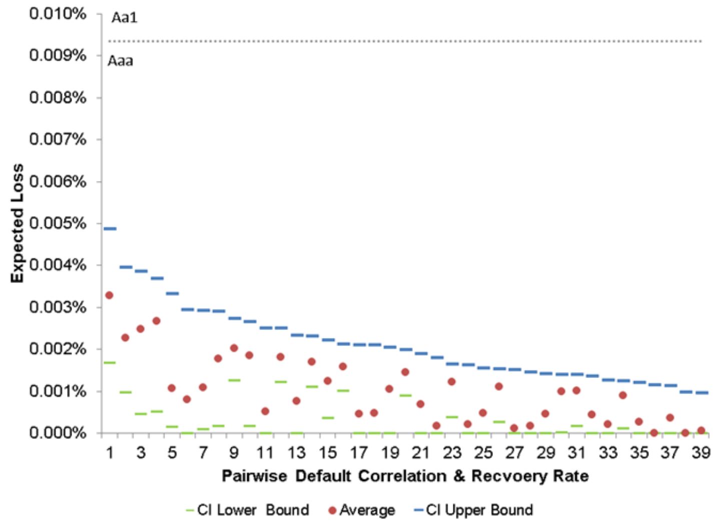
Source: Own Elaboration

It can be seen from the Figure 68 that higher correlation values are associated with higher recovery rates. This is analogous to the situation already described in the context of the ABACUS transaction. It might seem strange at first sight that the pair of feasible points is a region instead of curve. One reason is due to the fact that there are only two rated tranches (the size of the tranches is bigger than in a transaction with many tranches) and therefore, there is more amplitude to find the pair of values that accommodate the rating. Additionally, the fact that the two tranches have the same rating and a similar maturity contribute to increase the number of feasible combinations of default correlation and recovery rate.

Figures 69 and 70 show a subset of the pairs in Figure 68 that have the widest confidence intervals for the expected loss of the MIDGARD tranches. Figure 69 corresponds to the Super Senior tranche whereas Figure 70 corresponds to the Mezzanine tranche.

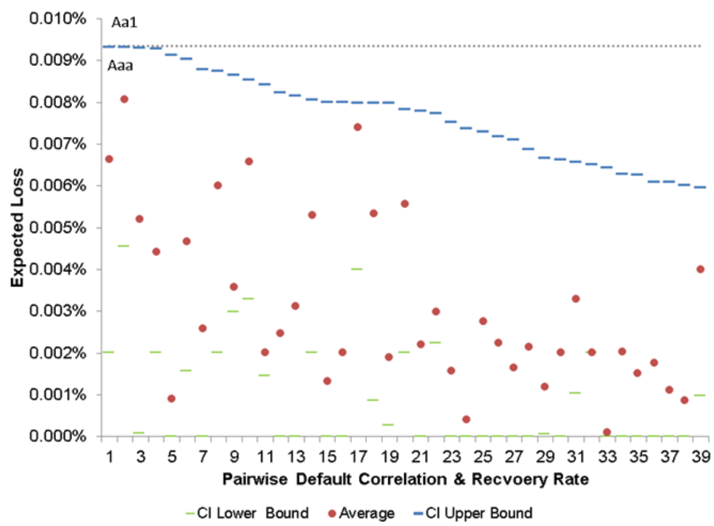
Regarding the Super Senior tranche, it can be safely said that it is comfortably contained within the Aaa region. In the case of the Mezzanine tranche is similar but the confidence intervals are more close to include more than one notch.

Figure 69: MIDGARD Super Senior Tranche Confidence Intervals for the Pairwise Default Correlation and Recovery Rate.



Source: Own Elaboration

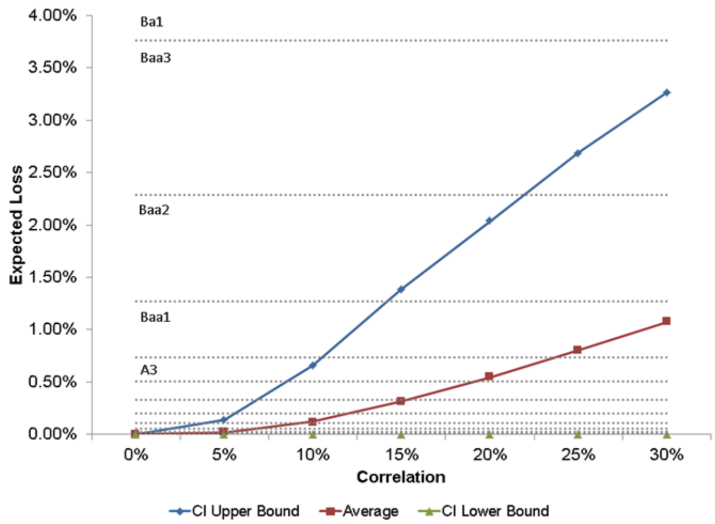
Figure 70: MIDGARD Mezzanine Tranche Confidence Intervals for the Pairwise Default Correlation and Recovery Rate



Source: Own Elaboration

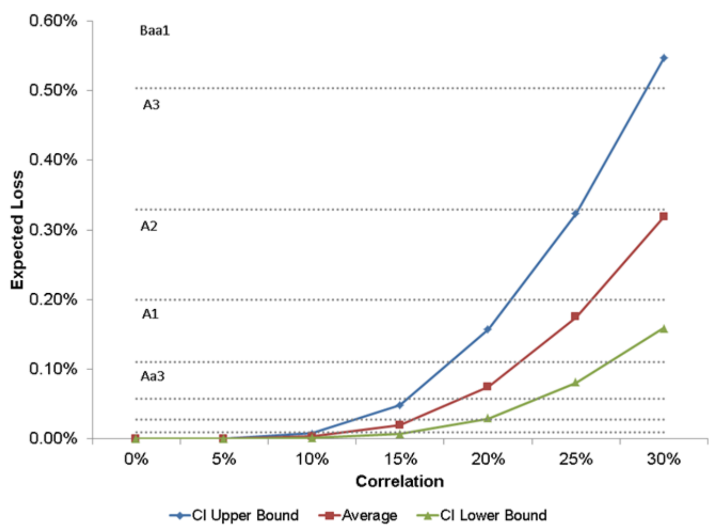
Figures 71 and 72 show the upper and lower bound with a 95% confidence interval for the expected loss in the case of the Super Senior tranche for different default correlation values. Figure 71 assumes that the recovery rate follows a Beta distribution. And in Figure 72, the recovery rate is given by Equation 29 from Chapter 5.

Figure 71: MIDGARD Super Senior Tranche Confidence Intervals (Beta Distribution)



Source: Own Elaboration

Figure 72: MIDGARD Super Senior Tranche Confidence Intervals (Chapter 5, Equation 29)

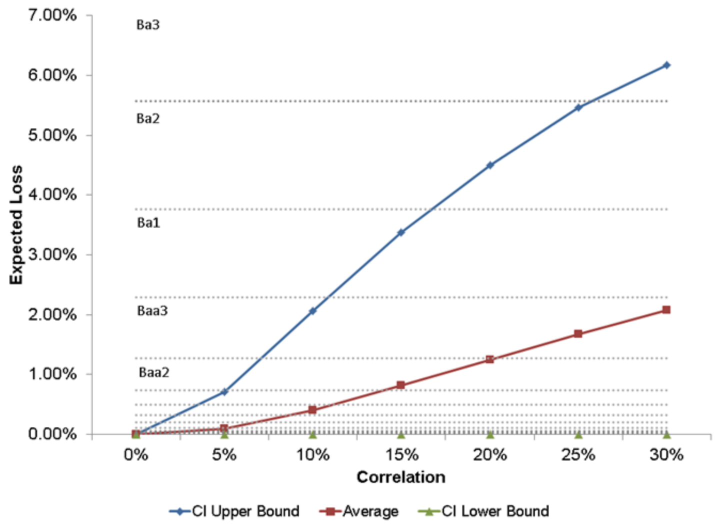


Source: Own Elaboration

Since correlation values are typically within the 15%-25% range, it is worth noting that the rating of this tranche could have been as low as Baa3 according to Figure 71... Moreover, even based on the center of the interval, the Aaa rating seems unwarranted for correlation values higher than 10%. Notice also that the Aaa rating even assuming that the recovery rate follows the behavior described by the Equation 29 from Chapter 5 seems undeserved, for default correlations higher than 15%.

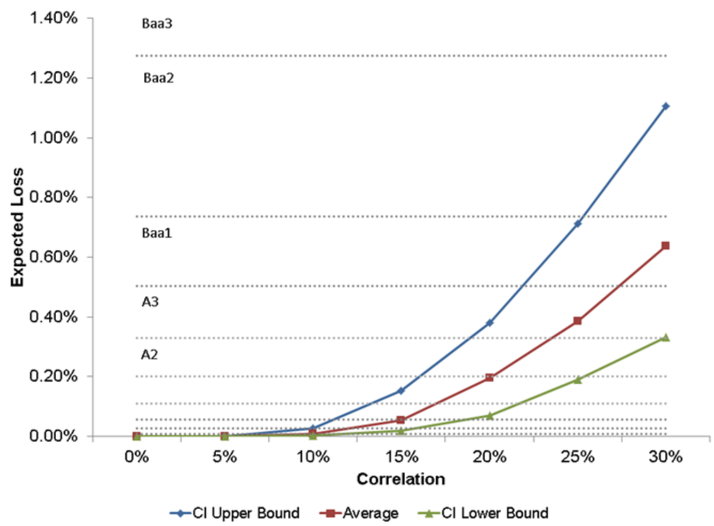
Figures 73 and 74 show the upper and lower bound for the 95% confidence interval for the expected loss in the case of the Mezzanine tranche for different default correlation values. Figure 73 assumes that the recovery rate follows a Beta distribution. And in Figure 74, the recovery rate is given by Equation 29 from Chapter 5.

Figure 73: MIDGARD Mezzanine Tranche Confidence Intervals (Beta Distribution)



Source: Own Elaboration

Figure 74: MIDGARD Mezzanine Tranche Confidence Intervals (Chapter 5, Equation 29)



Source: Own Elaboration

Generally speaking, the same observations applied to the case of the Super Senior are valid here. However, the weakness of the Aaa rating becomes more salient.

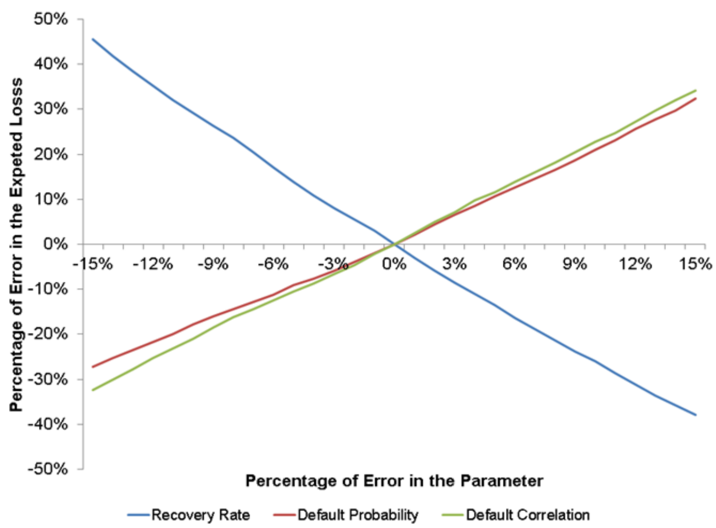
The results presented in the previous figures call into question pretty much the entire Moody's analysis. More to the point, even using the Moody's methodology it seems impossible to conclude comfortably that these tranches (especially the Mezzanine tranche) deserved an Aaa rating. One may speculate that the Moody's analysis was performed based on the oversimplifying assumption that MIDGARD was a regular CDO supported by seven assets (each one representing one mezzanine tranche of the synthetic CDOs).

7.3.2 Sensitivity Analysis to Errors in the Asset Parameters

Finally, Figure 75 and 76 show the sensitivity of the expected loss for the Super Senior and Mezzanine tranches assuming a mean recovery rate of 50% and a default correlation of 30%. The value for the default probability is 1.94%.

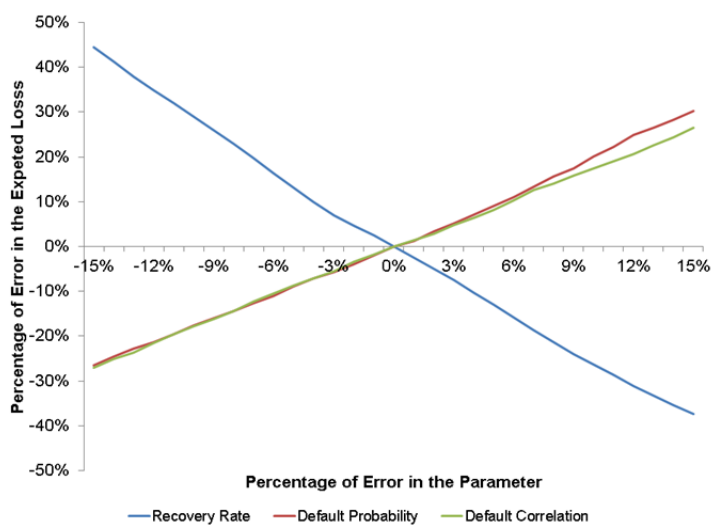
Broadly speaking, these figures are consistent with those of the ABACUS and CDX indices cases. The fact that the default probability plays a more important role when one goes down in the capital structure is at least intuitively reasonable since lower seniority tranches are impacted by defaults much faster than more senior tranches. However, compared with previous cases, the role of the default correlation is diminished.

Figure 75: MIDGARD Super Senior Tranche Sensitivity



Source: Own Elaboration

Figure 76: MIDGARD Mezzanine Tranche Sensitivity

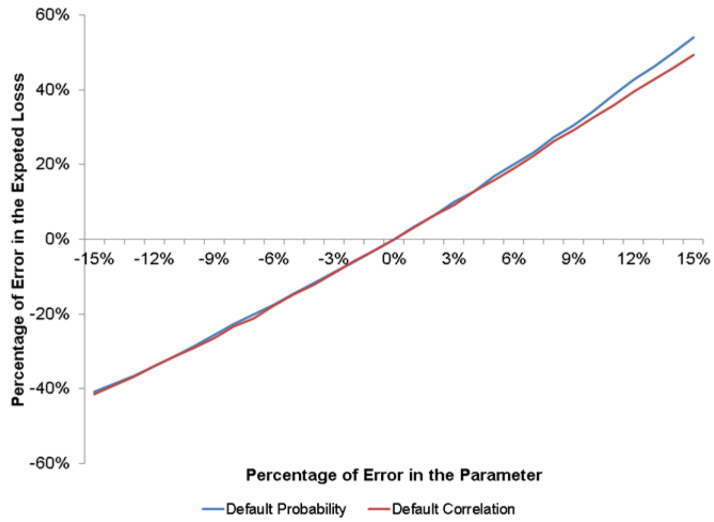


Source: Own Elaboration

Figures 77 and 78 show the sensitivity of the expected loss for the Super Senior and Mezzanine tranches, assuming that the recovery rate is given by Equation 29 from Chapter 5.

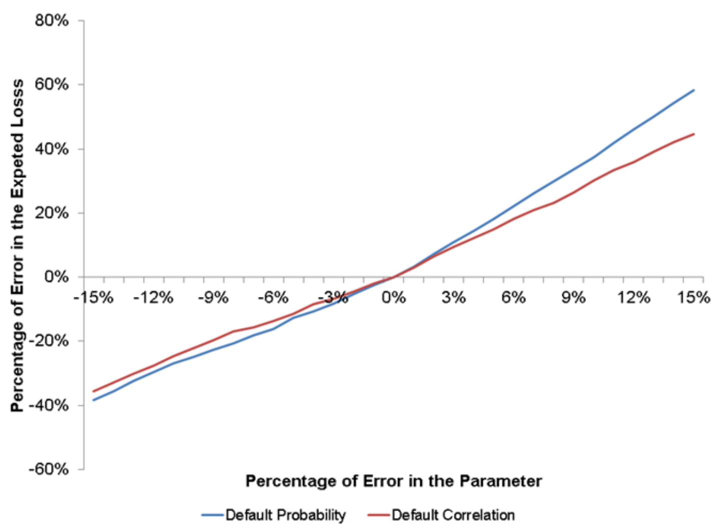
Again, and as expected, the tendencies are the same shown in the previous Figures 77 and 78. However, the variability is less manifest.

Figure 77: MIDGARD Super Senior Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 78: MIDGARD Mezzanine Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

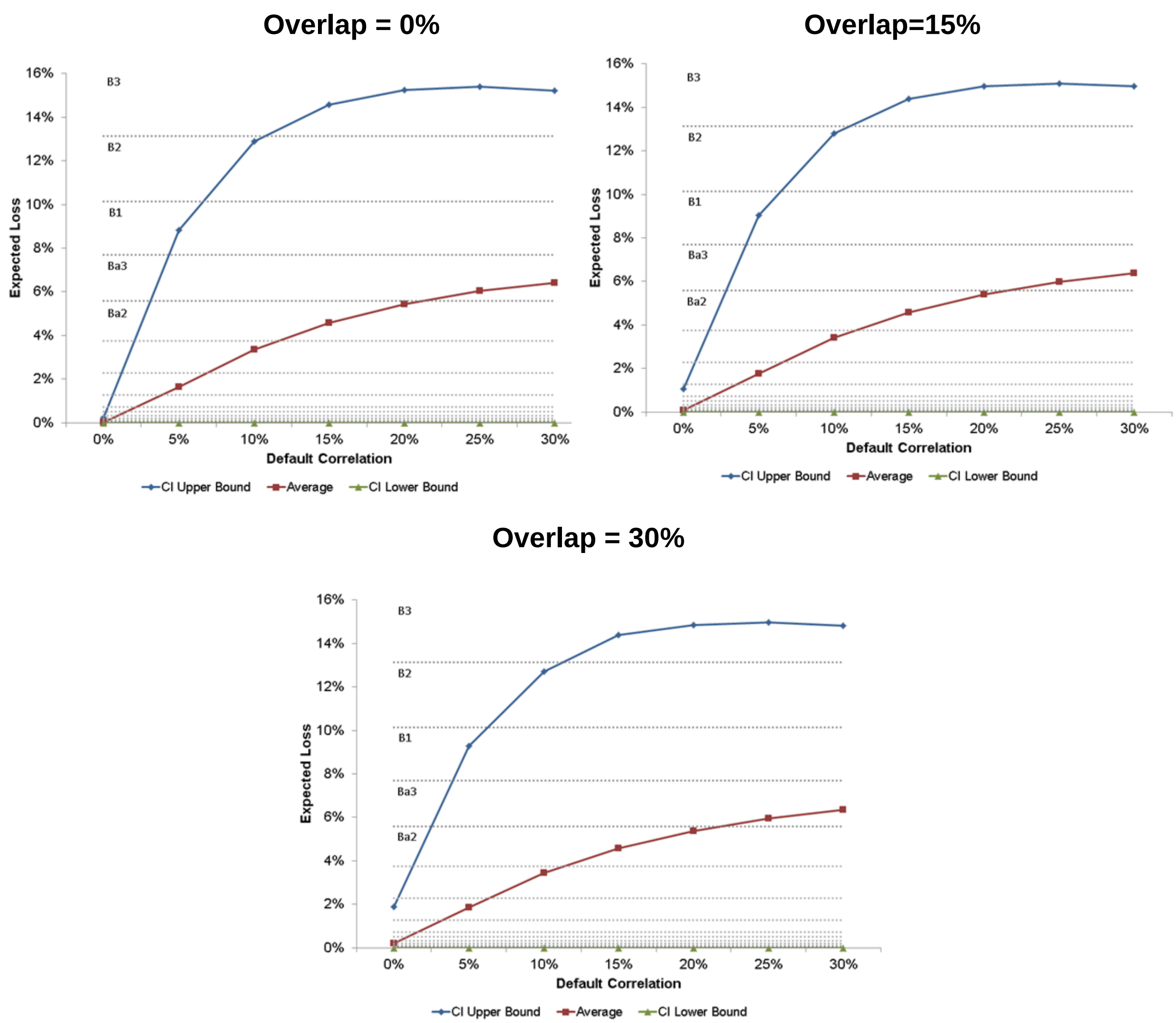
7.4 Cases #5 and #6: Theoretical CDO-Squared

The analysis of the theoretical synthetic CDO-Squared is similar to that of the indices. The main difference is in the incorporation of the degree of overlapping as another factor in the analysis. The effect of the overlap is studied by considering three levels: 0%, 15% and 30%.

7.4.1 Confidence Intervals for the T-CDX.NA.IG

In the case of T-CDX.NA.IG, a 95% confidence interval for the expected loss of the two tranches considered is estimated. Figures 79, 80 and 81 show the expected loss for the Senior 1 tranche, assuming that the recovery rate follows a Beta distribution, for a 5 years maturity and the three levels of overlap previously mentioned. The parameters for the default probability are identical to the case of the CDX.NA.IG.

Figure 79, 80 and 81: T-CDX.NA.IG Senior 1 Tranche Confidence Intervals (Beta Distribution) With 0%, 15% and 30% of Overlap Respectively



Source: Own Elaboration

The behavior shown by these figures is consistent with the trends detected for the CDX.NA.IG index. Namely, increasing values of the correlation are associated with increasing uncertainty in the ratings. Specifically, for default correlation values within the 15%-30% range (realistic values for most portfolios) the ratings can vary as much as fifteen notches (Aaa/B2). As expected, the squared nature of the T-CDX.NA.IG brings more variability to the resulting confidence intervals.

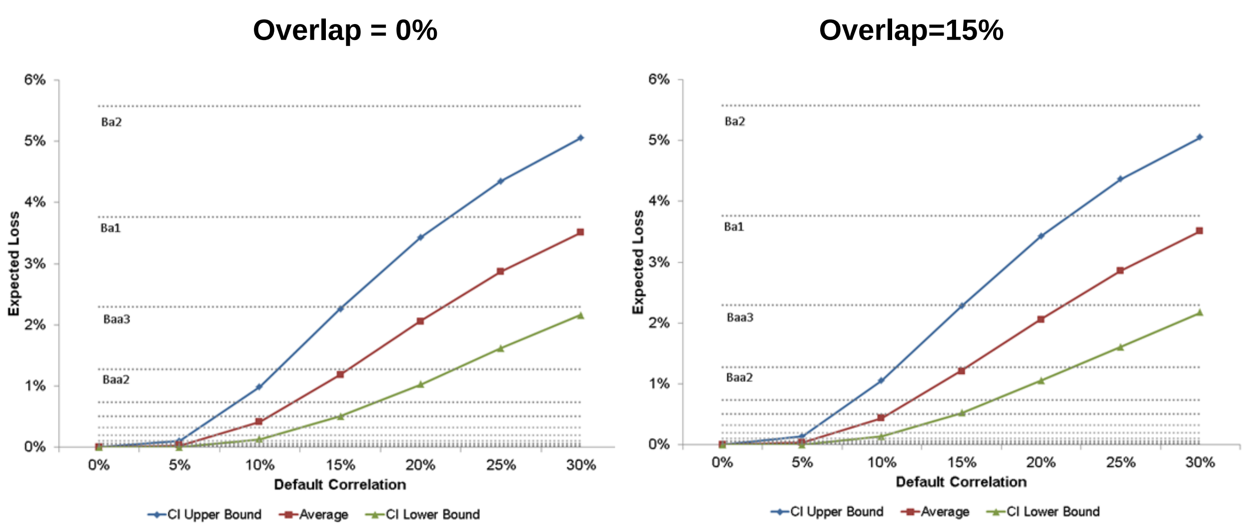
Additionally, in Figures 79, 80 and 81 it can be seen that an increment in the degree of overlapping has neither an important effect in the resulting confidence intervals nor on the average value. The reason for this effect is that the degree of diversification, even when is a 30% of overlap, is high enough to achieve all the benefits of diversification.

In a common synthetic CDO, the diversification of the pool of assets is dependent not only on the number of referenced securities but also on the default correlation. This means that when the default correlation is close to one, the pool of assets behaves like a one single asset (no diversification). On the other hand, a default correlation close to zero makes the diversification to be dependent only on the number of referenced securities.

In the case of a synthetic CDO-Squared, the degree of overlap among the securities referenced by the pool of mezzanine tranches is a new dimension of diversification. Namely, a 100% of overlap forces the pool of tranches to behave like one single tranche, whereas a 0% of overlap the pool of tranches behaves highly diversified. However, this new diversification dimension is limited by the number of different assets referenced by the mezzanine tranches. Over certain number of assets, the impact of the overlap among referenced securities in the expected loss is meaningless.[22]

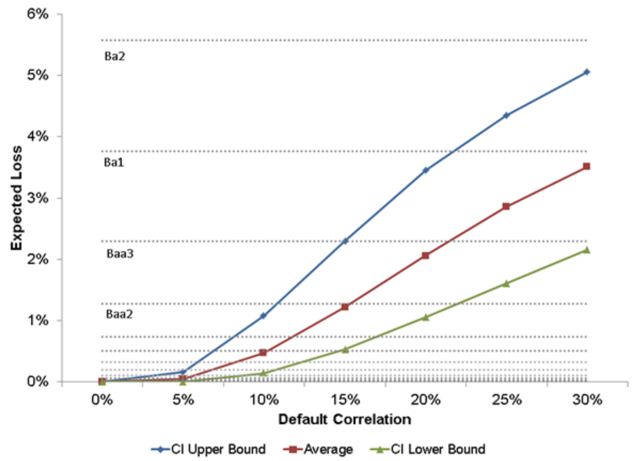
Figures 82, 83 and 84 show the same case for the Senior 1 tranche but with the recovery rate following Equation 29 from Chapter 5.

Figure 82, 83 and 84: T-CDX.NA.IG Senior 1 Tranche Confidence Intervals (Chapter 5, Equation 29) With 0%, 15% and 30% of Overlap Respectively.



[22] See Appendix 12 for a brief example of this situation.

Overlap = 30%



Source: Own Elaboration

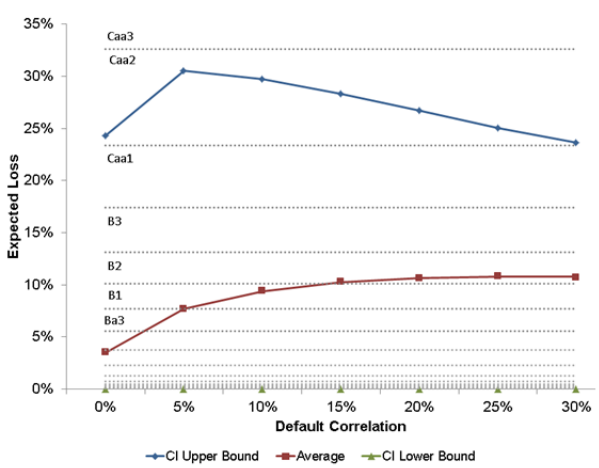
These figures are very much in agreement with the trends displayed in the previous case (Figures 79, 80 and 81) except that the variability is much more bounded. However, even under these conditions there is a considerable variability in terms of the ratings (three notches).

Figures 85, 86 and 87 show the case of the Mezzanine 2 tranche with the recovery rate distributed as a Beta.

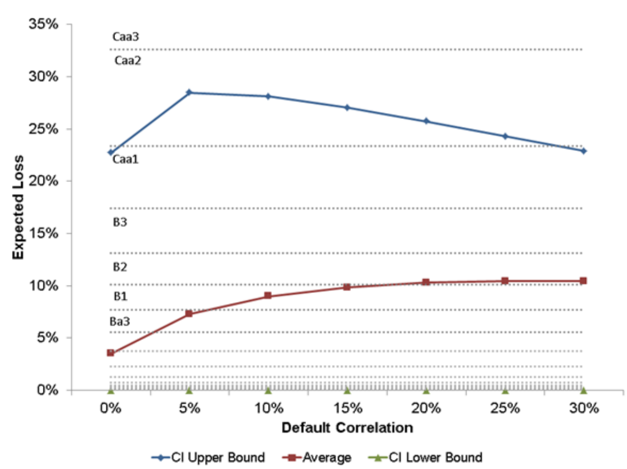
Again, and broadly speaking, the trends displayed by these figures are in agreement with the previous findings. Across all of the confidence intervals estimated, the ratings can vary as much as 17 notches. There is however one significant difference. The upper bound of the expected loss interval (blue line) becomes "inverted" for a default correlation over the 5%. This means that the mezzanine tranche resembles the behavior of the equity tranche.

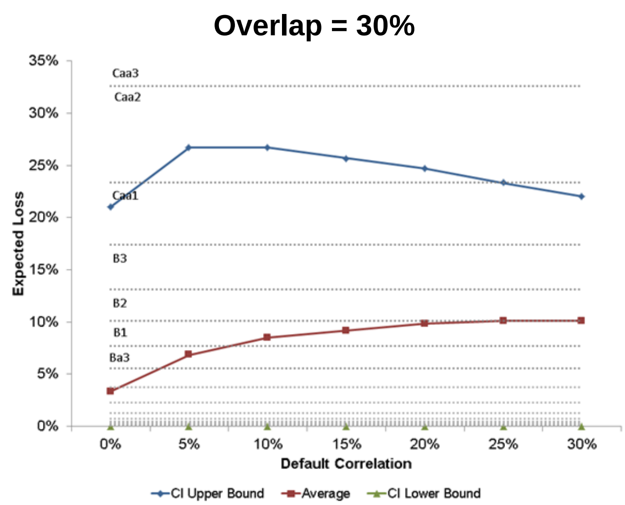
Figure 85, 86 and 87: T-CDX.NA.IG Mezzanine 2 Tranche Confidence Intervals (Beta Distribution) With 0%, 15% and 30% of Overlap Respectively.

Overlap = 0%



Overlap=15%



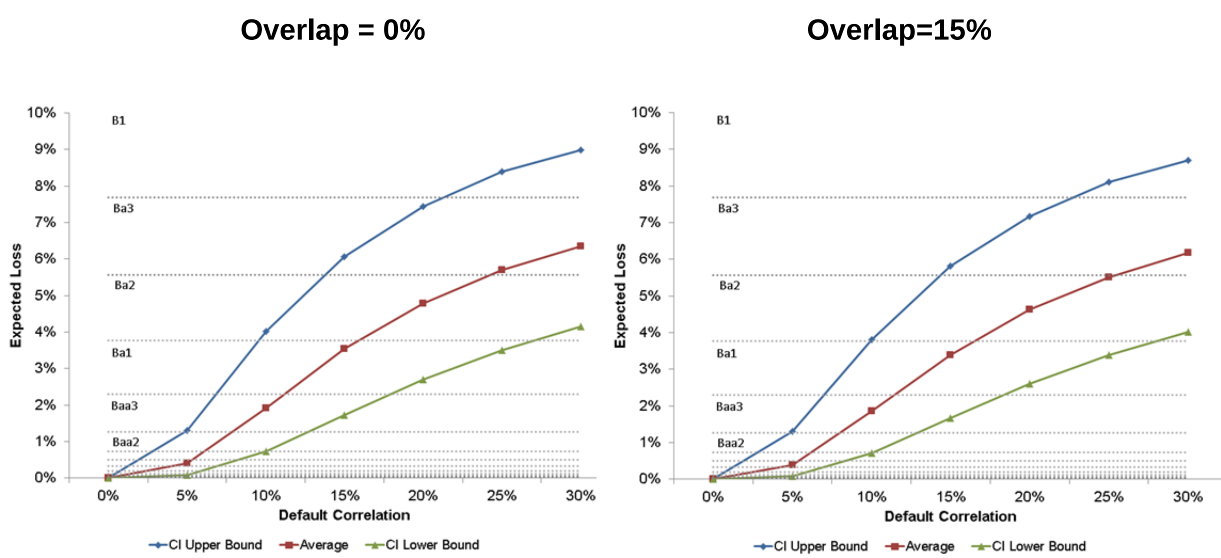


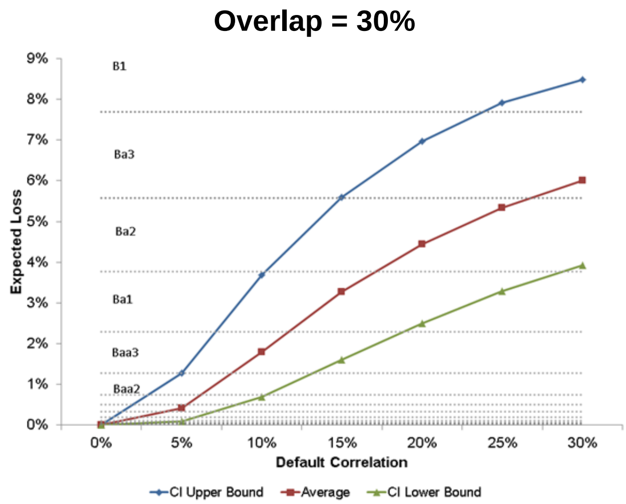
Source: Own Elaboration

Figures 88, 89 and 90 show the case of the Mezzanine 2 tranche with the recovery rate following Equation 29 from Chapter 5.

The behavior shown in these figures is similar to that of Figures 88, 89 and 90, with the caveat that there is no benefit for a higher level of default correlation. However, the ratings do not vary more than three notches.

Figures 88, 89 and 90: T-CDX.NA.IG Mezzanine 2 Tranche Confidence Intervals (Chapter 5, Equation 29) with 0%, 15% and 30% of Overlap Respectively.



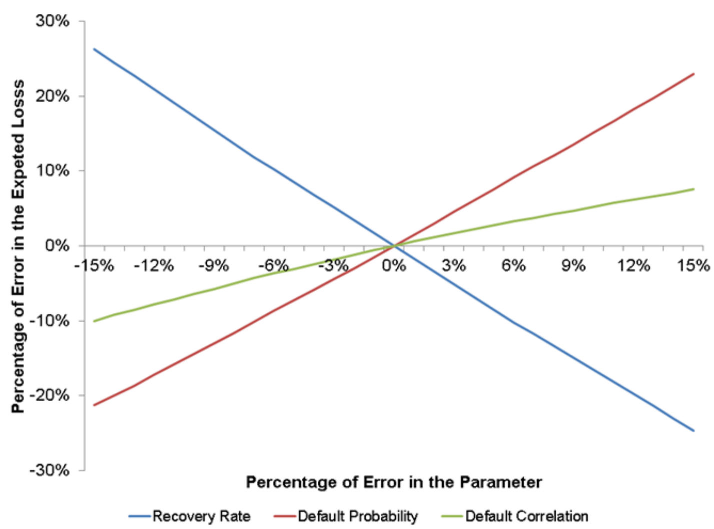


Source: Own Elaboration

7.4.2 Sensitivity Analysis to Errors in the Asset Parameters for the T-CDX.NA.IG

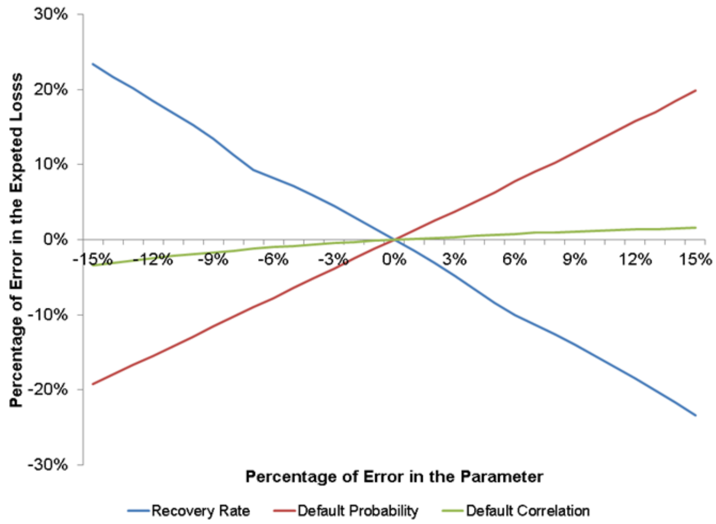
Figures 91 and 92 show the sensitivity of the expected loss for the Senior 1 and Mezzanine 2 tranches assuming a constant recovery rate equal to 50%, a default correlation equals to 30% and a default probability equals to 1.94%.

Figure 91: T-CDX.NA.IG Senior 1 Tranche Sensitivity



Source: Own Elaboration

Figure 92: T-CDX.NA.IG Mezzanine 2 Tranche Sensitivity

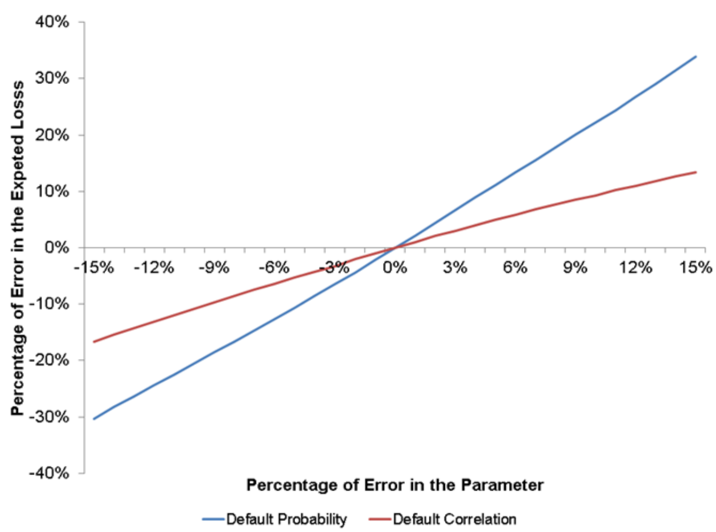


Source: Own Elaboration

The sensitivity of the expected loss is consistent with the previous cases of study. The lower the seniority of the tranche, the more important is the role played by the default probability. However, there is an important difference. Both tranches experience the same order of sensitivity to parameters in terms of underestimating and overestimating the expected loss errors. This type of behavior is consistent with certain characteristics that market participants have already detected in CDO-Squared structures, namely, the “speed” at which a tranche deteriorates, as a function of the number of defaults, is much faster in the CDO-Squared structures.

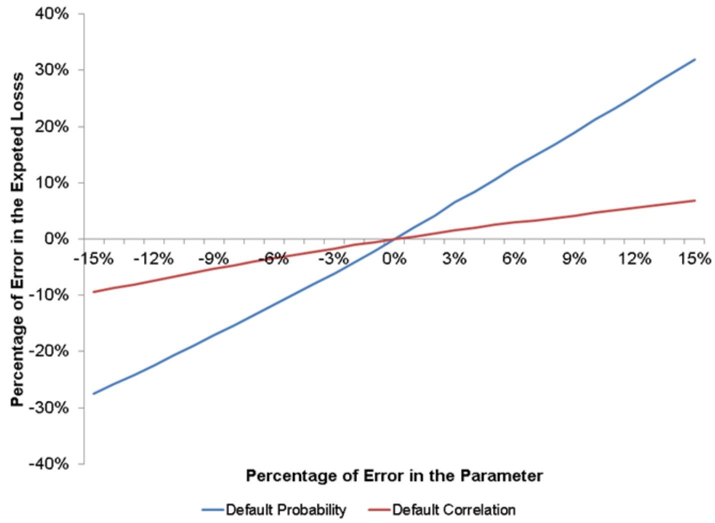
Figures 93 and 94 show the sensitivity of the expected loss for the Senior 1 and Mezzanine tranches, assuming that the recovery rate is given by Equation 29 from Chapter 5.

Figure 93: T-CDX.NA.IG Senior 1 Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 94: T-CDX.NA.IG Mezzanine 2 Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

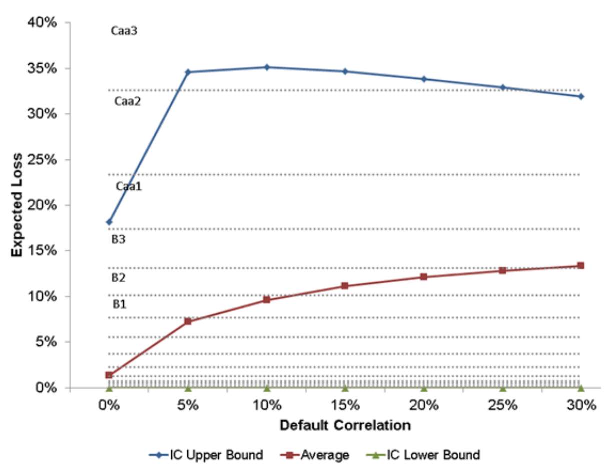
Again, and as expected, the tendencies are the same shown in the previous figures albeit with more variability. The expected loss is more sensitive to errors in the default probability than in the default correlation.

7.4.3 Confidence Intervals for the T-CDX.NA.HY

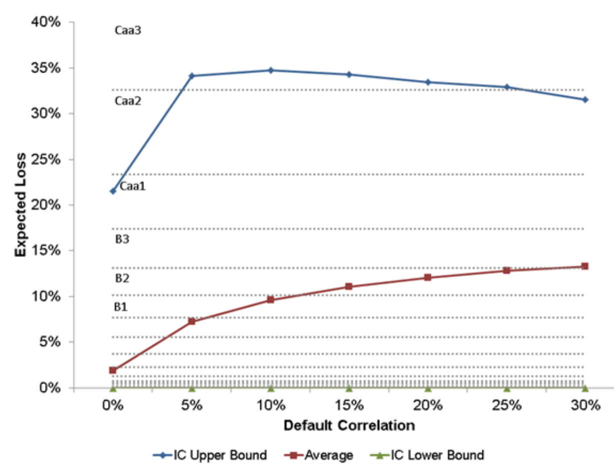
In the case of the T-CDX.NA.HY the conclusions are similar to the T-CDX.NA.IG. Figures 95, 96 and 97 show the expected loss for the Senior 1 tranche, assuming the recovery rate follows a Beta distribution, for a 5 years maturity and the three levels of overlap previously mentioned. The parameters for the default probability are identical to the case of the CDX.NA.HY.

Figures 95, 96 and 97: T-CDX.NA.HY Senior 1 Tranche Confidence Intervals (Beta Distribution) With 0%, 15% and 30% of Overlap Respectively.

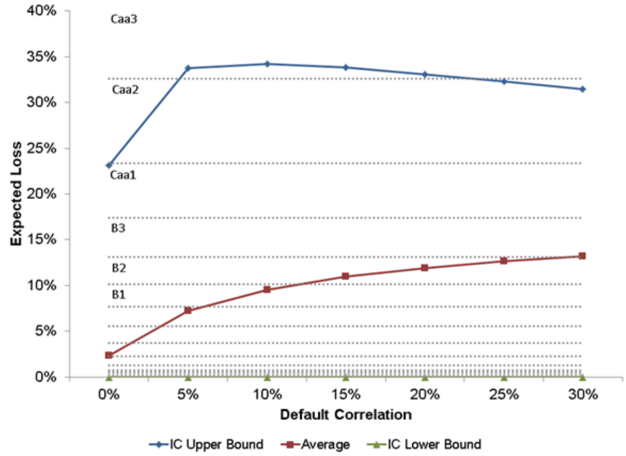
Overlap = 0%



Overlap=15%



Overlap = 30%



<u>Source:</u> Own Elaboration

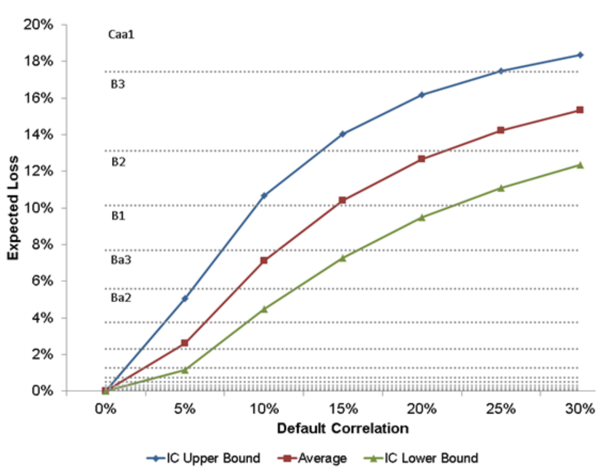
Once again, the trends displayed by these figures are consistent with the previous findings. The effects of the overlap are not relevant. Across all of the possible correlation values, the rating exhibits a high degree of variability (up to 18 notches).

Figures 98, 99 and 100 show the case of the Senior 1 tranche with the recovery rate following Equation 29 from Chapter 5.

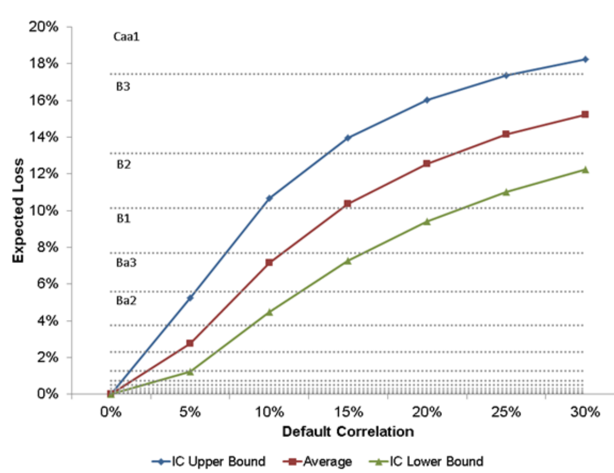
These figures are very much in agreement with the trends displayed in the previous case (Figures 95, 96 and 97) except that the variability is much more bounded. However, even under these conditions there is a considerable variability in terms of the ratings (three notches).

Figures 98, 99 and 100: T-CDX.NA.HY Senior 1 Tranche Confidence Intervals (Chapter 5, Equation 29) with 0%, 15% and 30% of Overlap Respectively.

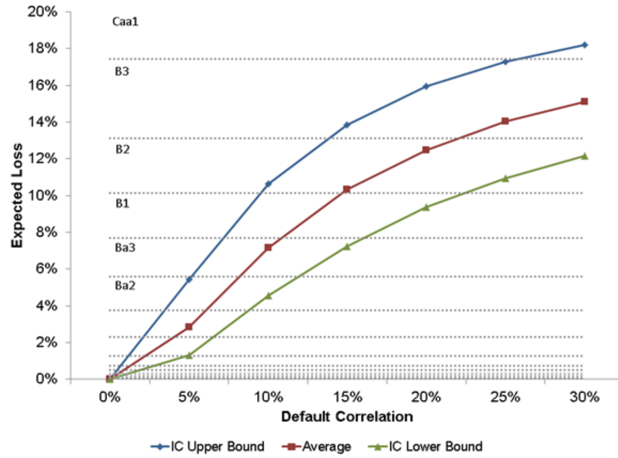
Overlap = 0%



Overlap=15%



Overlap = 30%



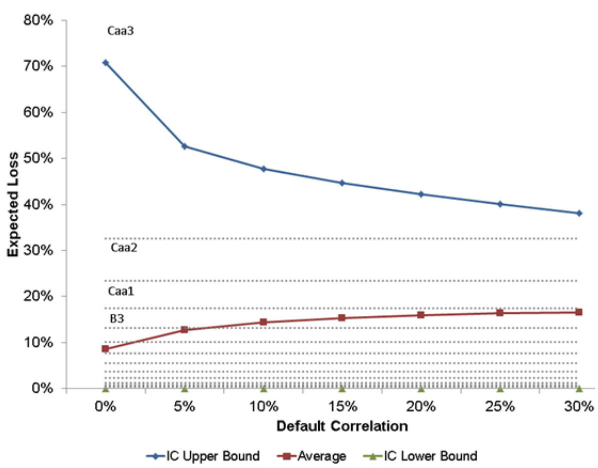
Source: Own Elaboration

Figures 101, 102 and 103 show the case of the Mezzanine tranche with the recovery rate distributed as a Beta.

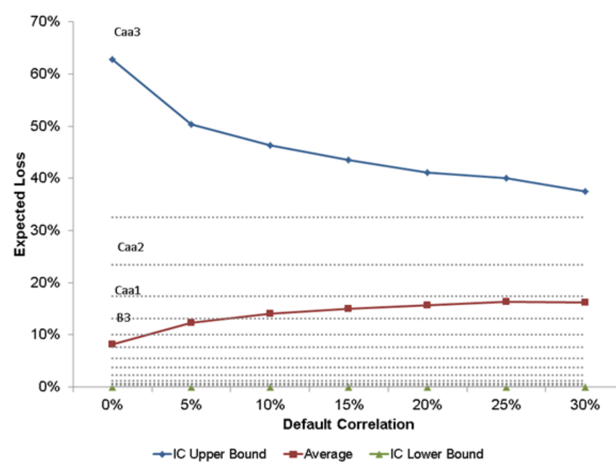
Again, and broadly speaking, the trends displayed by these figures are in agreement with the previous findings. Across all of the confidence intervals estimated, the ratings can vary as much as 18 notches. There is, however, one significant difference. The upper bound of the expected loss interval is decreasing in the default correlation. This means that the mezzanine tranche behaves very much like an equity tranche.

Figures 101, 102 and 103: T-CDX.NA.HY Mezzanine Tranche Confidence Intervals (Beta Distribution) with 0%, 15% and 30% of Overlap Respectively.

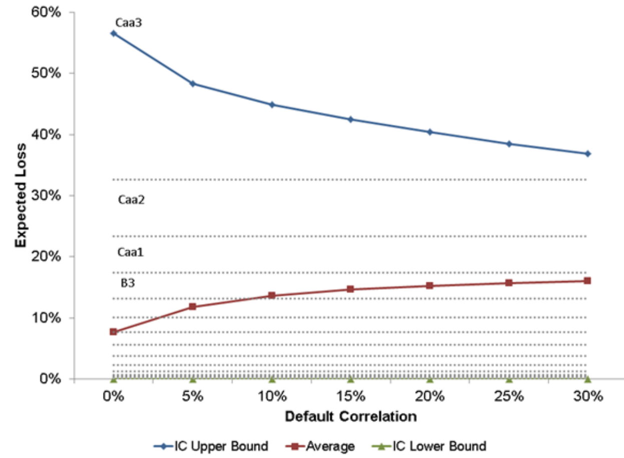
Overlap = 0%



Overlap=15%



Overlap = 30%



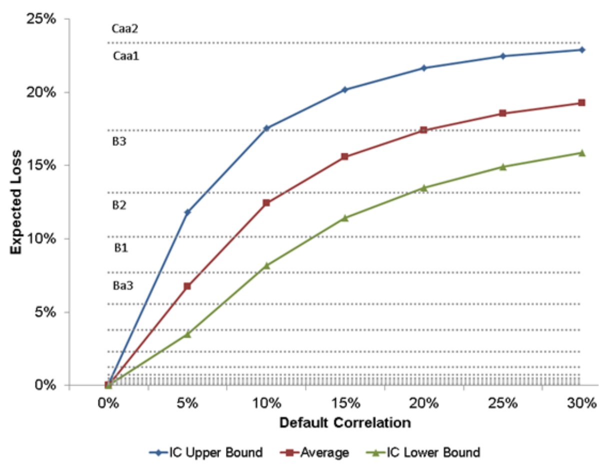
Source: Own Elaboration

Figures 104, 105 and 106 show the case of the Mezzanine tranche with the recovery rate following Equation 29 from Chapter 5.

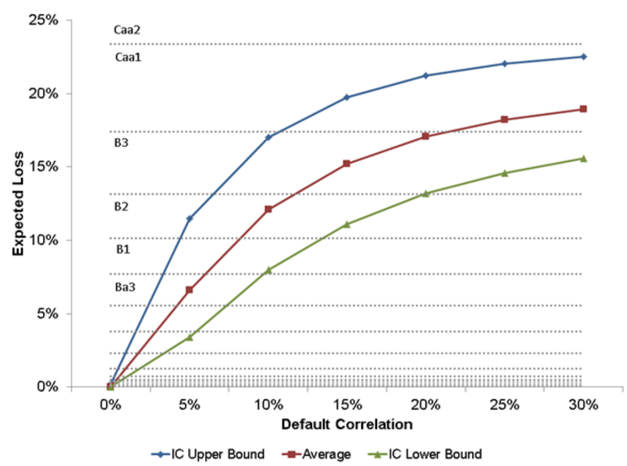
The behavior shown in these figures is similar to that of Figures 101, 102, 103 with the caveat that there is no benefit for a higher level of default correlation. However, the ratings do not vary more than three notches.

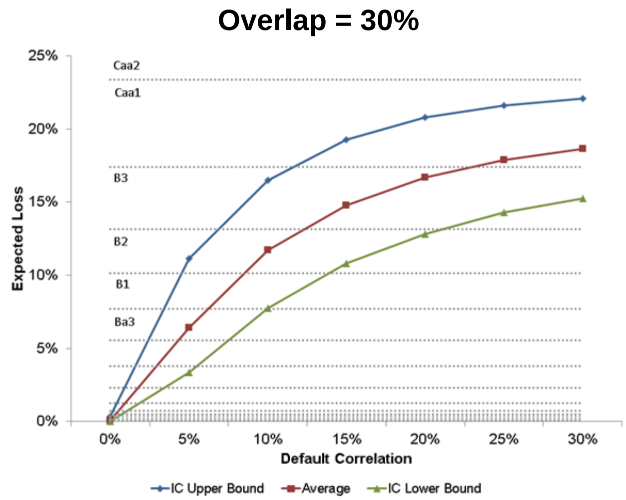
Figures 104, 105 and 106: T-CDX.NA.HY Mezzanine Tranche Confidence Intervals (Chapter 5, Equation 29) with 0%, 15% and 30% of Overlap Respectively.

Overlap = 0%



Overlap=15%



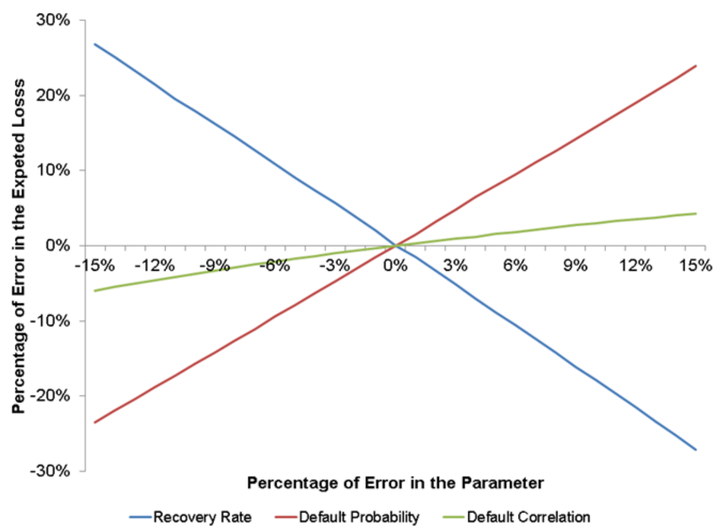


<u>Source:</u> Own Elaboration

7.4.4 Sensitivity Analysis to Errors in the Asset Parameters for the T-CDX.NA.HY

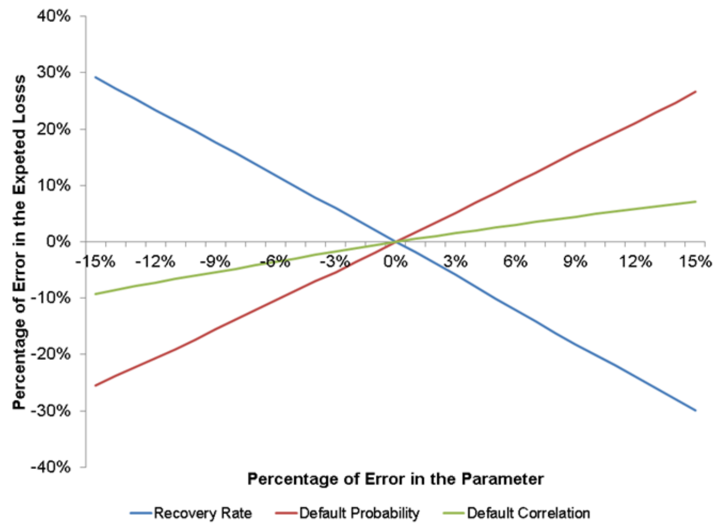
Figures 107 and 108 show the sensitivity of the expected loss for the Senior 1 and Mezzanine 2 tranches assuming a constant recovery rate equal to 50%, a default correlation equal to 30% and a default probability equal to 10.21%.

Figure 107: T-CDX.NA.HY Senior 1 Tranche Sensitivity



<u>Source:</u> Own Elaboration

Figure 108: T-CDX.NA.HY Mezzanine Tranche Sensitivity

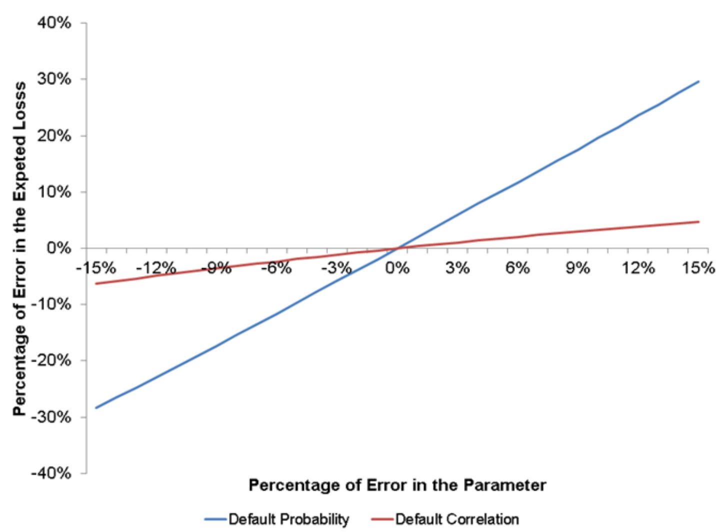


Source: Own Elaboration

These figures are consistent with the findings presented in the context of the T-CDX.NA.HY. However, in this case, the expected loss seems to be more sensitive to errors in the default probability than in the default correlation.

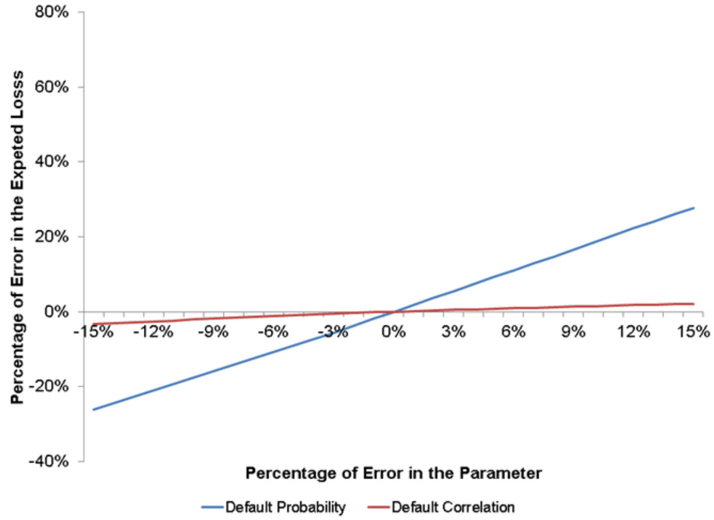
Figures 109 and 110 show the sensitivity of the expected loss for the Senior 1 and Mezzanine tranches, assuming that the recovery rate is given by Equation 29 from Chapter 5.

Figure 109: T-CDX.NA.HY Senior 1 Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Figure 110: T-CDX.NA.HY Mezzanine Tranche Sensitivity (Chapter 5, Equation 29)



Source: Own Elaboration

Again, and as expected, the tendencies are the same shown in the previous figures albeit with more variability. The expected loss shows relatively the same sensitivity to both the default probability and default correlation.

8. CONCLUSIONS

The conclusions of this study can be summarized as follows:

1) The amplitude exhibited by the confidence intervals for the expected loss of the different synthetic CDO tranches is significant. In fact, in many cases the intervals cover several rating notches. This shows that single-point estimators as a proxy for credit quality are very misleading. A better credit risk metric should include at least an assessment of the error in the single-point estimate of the rating.
2) The study shows that the lower the seniority of a synthetic CDO tranche, the higher the degree of instability of the credit ratings. This result is consistent with the risk nature of the different tranches as well as the priority of payments structure. The study also shows that for mezzanine tranches the ratings could vary by as much as 4 to 9 notches; whereas for senior tranches the variations were less significant, in the 2 to 4 notches range. This variations is even more for the case of synthetic CDO-Squared.
3) Even in the cases in which the recovery rate is assumed to be deterministic, the stochastic behavior of the probability of default is enough to bring significant variability to the expected loss computation and thus the ratings themselves.
4) Contrary to what is generally believed, namely that the degree of overlapping in a synthetic CDO-Squared has a major significance in the performance of the tranches of such CDO, this study has proved that this is not the case. More precisely, for overlapping levels less than 30% (the most typical cases), variation in the expected loss are minimal. The main reason for this is that, in general, the underlying portfolios are already highly diversified due to the high number of securities involved.
5) The study shows that the expected loss is much more sensitive to errors in the recovery rate. Whether the expected loss is more sensitive to the default probability than the default correlation will depend on the tranche structure of the synthetic CDO. In the case of a synthetic CDO-Squared, the expected loss is more sensitive to the default probability. In this sense, the focus of many regulatory frameworks which are specified in terms of the recovery rate and default probability seems to be reasonable for synthetic CDOs.
6) The study shows that it was possible for cautious investors to be skeptical about synthetic CDO tranches deals even before 2007. Since only sophisticated investors participated in synthetic CDO deals, one can expect them to have performed a risk analysis before entering into any these transactions. For example, the analysis shows that using public information available from the marketing material of the ABACUS and MIDGARD transactions was sufficient to detect the sensitivity of the ratings to the modeling assumptions. This effect was much more pronounced in the ABACUS case. These two transactions, as it is known already, resulted in major losses for most investors.

7) The instability of synthetic CDO credit ratings should be of interest to institutional investors. Normally, their portfolios credit risk exposure is subject to limits according to some benchmarks determined by ratings. Thus, surpassing these limits could be very costly: whenever a security is downgraded below certain level, these institutions are forced to sell. For these reasons, institutional investors should be very sensitive to the likelihood that their holdings could be downgraded.

It is important that future regulation based on credit ratings (at least in the case of synthetic CDOs) should move beyond the single-point estimators. Ideally, regulatory rules related to reserves percentages of holdings, margin accounts, collateral posting, etc., should somehow incorporate a measure of ratings stability.

Some suggestions for future studies are the following. The present study was largely based on Moody's methodology; it should be useful to carry out a similar effort based on the S&P and Fitch methodologies. Additionally, the extension of this study to cashflow CDOs could be very useful. More broadly, ratings stability studies should be extended to the entire gamut of structured products (securitization of student loans, credit card receivables, future flows, etc.) and other debt instruments that use ratings, such as bonds.

Finally, regulators should be able to identify ex-ante a level of accuracy for ratings such that no one ex-post should be surprised with the performance, bad or good, of a synthetic CDO. Will it be possible to meet this challenge? Only time will tell.

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10. APPENDIX

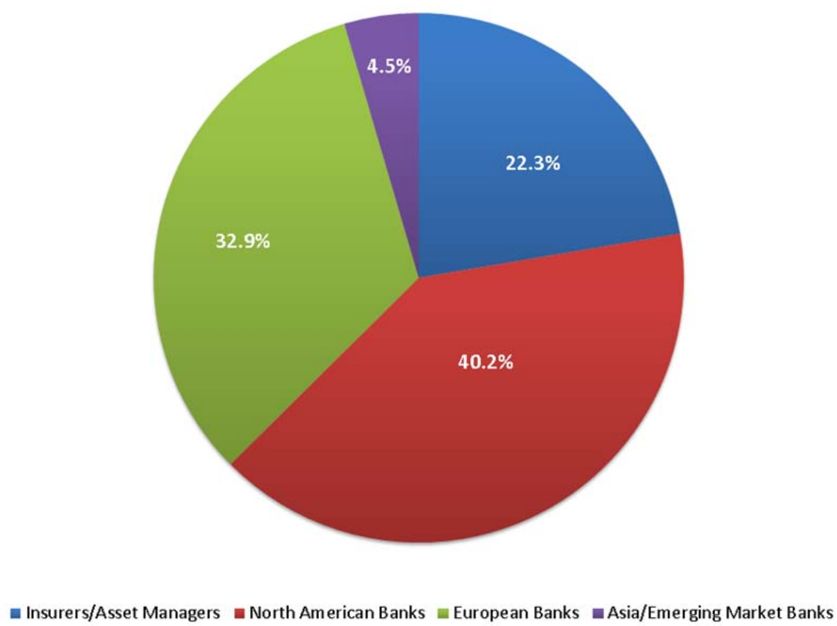
Appendix 1: How Much Did Banks and Insurance Companies Lose During the Subprime Crisis?

One of the major consequences of the subprime crisis was that many financial institutions experienced substantial losses. In some extreme cases, these institutions disappeared altogether (filed for bankruptcy). That was the case, for example, with Lehman Brothers and Bear Sterns. Others were merged with stronger institutions. For example, Merrill Lynch was absorbed by Bank of America. A third group was kept alive with significant help from the government (for example Citigroup, Fannie Mae and AIG).

Finally, several monolines (like MBIA, Ambac and FGIC) either went out of business or were intervened by insurance regulators.

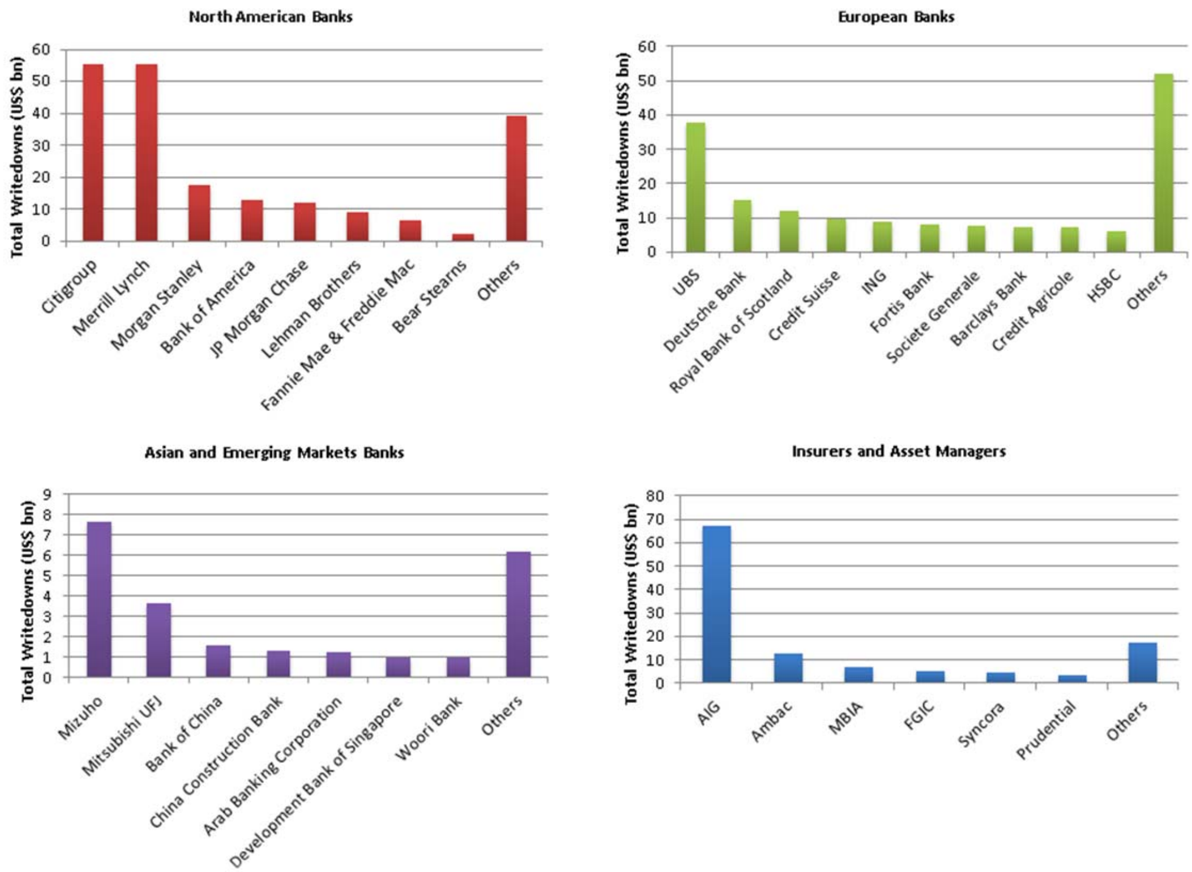
Figure A.10.1 shows the distribution of write-downs during the subprime crisis. Figure A.10.2 is a more detailed version of Figure A.10.1, which shows the losses suffered by the most important institutions.

Figure A.1.1: Distribution of Write-Downs



Source: Creditflux

Figure A.1.2: Detailed Distribution of Write-Downs



Source: Creditflux

Appendix 2: Credit Event Definitions

CDS contracts are structured with standard swap agreements provided by the International Swaps and Derivatives Association (ISDA). Under the ISDA agreement, the most commonly used credit events in the case regular CDS are: failure to pay, bankruptcy, restructuring or repudiation. When the CDS refers to a mezzanine tranche, like in the case of a synthetic CDO-Squared, the most typical credit events are: Failure to pay, principal write-down and rating downgrade. The table below describes in more detail these credit events.

Credit Event	Description
Failure to Pay	The failure of the reference entity to make payments due on any obligation before the expiration of any applicable grace period.
Bankruptcy	The dissolution or insolvency of a reference entity, the inability to pay debts, or the shift of control to a secured party, custodian, or receiver.
Restructuring	The reference entity or governmental authority changes an obligation by reducing the interest rate or the principal amount, postponing the payment of interest or principal, lowering the payment priority of the obligation, or changing the currency to one that is not permitted
Repudiation	The validity of an obligation is rejected either by the reference entity or a government authority. This event is mostly applicable to sovereign credits.
Principal Write-down (Loss Event)	Whenever any amount of principal with respect to any write-down reference obligation is permanently reduced due to the (loss event) allocation of losses, write-offs, charge-offs, defaults, or liquidations.
Rating Downgrade	The assignment of a below-CCC rating in combination with downgrade the postponement of interest for two or more periods.

Source: Handbook of Fixed Income Securities. Chapter 31: "Synthetic CDOs"

Appendix 3: Comparison between Cash and Synthetic CDOs.

Cash and synthetic CDO differ slightly in terms of both their mechanics and their purpose. The table below details some of these differences.

Characteristic	Typical Cash CDO	Typical Synthetic CDO
Collateral Pool	High-yield corporate bonds, leveraged loans, trust-preferred securities, emerging market debt and ABS	Credit default swap linked to a pool of balance-sheet assets (loans, senior or mezzanine structured finance) or to a reference pool of corporate credits (usually investment grade)
Size	US$200 million- US$600 million	$1 billion plus
Collateral Quality	Investment-grade or below investment-grade and even distressed collateral	Primarily investment-grade
Management	Typically managed	Typically static
Moral Hazard	Possible through the purchase of collateral designed to benefit one class of investors over others	Generally no due static nature of these transactions
Payment Frequency	Quarterly o semiannually	Quarterly
Legal final	Generally 12 years for transactions tied to corporate credits but as long as 30 years for transactions tied to ABS	For balance-sheet or arbitrage transactions linked to corporate debts, 4-6 years. For structured finance deals, 10-30 years
Expected Maturities	Generally 7-12 years for transactions tied to corporate debt credits depending on the payment priority of the investment	For arbitrage and balance-sheet transactions linked to corporates, 3-5 years. For structured finance deals, approximately eight years for senior debt and 15 years for subordinate debt
Ramp-Up Period	0-6 months	Generally, immediate to 1 month although arbitrage structured finance CDOs may have periods as long as a year
Prepayment Risk	Yes	Generally, no
Reinvestment Risk	Yes, for transactions with reinvestment periods	Generally no due to static nature of these transactions

Source: Handbook of Fixed Income Securities. Chapter 31: "Synthetic CDOs"

Appendix 4: Credit Ratings Description.

The CRAs ratings use different symbols to denote their ratings. The table below lists the symbols used by Moody's, S&P and Fitch. As can be seen, rated debt instruments can be broadly divided into two different categories: investment grade and high-yield.

Ratings			Description
Moody's	S&P	Fitch	
Aaa	AAA	AAA	Investment Grade
Aa1	AA+	AA+	
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non-Investment Grade (High-Yield)
Ba2	BB	BB	
Ba3	BB-	BB-	
B1	B+	B+	
B2	B	B	
B3	B-	B-	
Caa	CCC+		
Ca	CCC	CCC	
C	CCC-		
-		DDD	
-	D	DD	
-		D	

<u>Source:</u> Moody'S, S&P and Fitch.

Appendix 5: Moody's Expected Loss.

The following table lists the different values of Moody's expected loss, for different time horizons and credit rating.

	Time Horizon (Years)									
Rating	1	2	3	4	5	6	7	8	9	10
Aaa	0.0000%	0.0001%	0.0004%	0.0010%	0.0016%	0.0022%	0.0029%	0.0036%	0.0045%	0.0055%
Aa1	0.0003%	0.0017%	0.0055%	0.0116%	0.0171%	0.0231%	0.0297%	0.0369%	0.0451%	0.0550%
Aa2	0.0007%	0.0044%	0.0143%	0.0259%	0.0374%	0.0490%	0.0611%	0.0743%	0.0902%	0.1100%
Aa3	0.0017%	0.0105%	0.0325%	0.0556%	0.0781%	0.1007%	0.1249%	0.1496%	0.1799%	0.2200%
A1	0.0032%	0.0204%	0.0644%	0.1040%	0.1436%	0.1815%	0.2233%	0.2640%	0.3152%	0.3850%
A2	0.0060%	0.0385%	0.1221%	0.1898%	0.2569%	0.3207%	0.3905%	0.4560%	0.5401%	0.6600%
A3	0.0214%	0.0825%	0.1980%	0.2970%	0.4015%	0.5005%	0.6105%	0.7150%	0.8360%	0.9900%
Baa1	0.0495%	0.1540%	0.3080%	0.4565%	0.6050%	0.7535%	0.9185%	1.0835%	1.2485%	1.4300%
Baa2	0.0935%	0.2585%	0.4565%	0.6600%	0.8690%	1.0835%	1.3255%	1.5675%	1.7820%	1.9800%
Baa3	0.2310%	0.5775%	0.9405%	1.3090%	1.6775%	2.0350%	2.3815%	2.7335%	3.0635%	3.3550%
Ba1	0.4785%	1.1110%	1.7215%	2.3100%	2.9040%	3.4375%	3.8830%	4.3395%	4.7795%	5.1700%
Ba2	0.8580%	1.9085%	2.8490%	3.7400%	4.6255%	5.3735%	5.8850%	6.4130%	6.9575%	7.4250%
Ba3	1.5455%	3.0305%	4.3285%	5.3845%	6.5230%	7.4195%	8.0410%	8.6405%	9.1905%	9.7130%
B1	2.5740%	4.6090%	6.3690%	7.6175%	8.8660%	9.8395%	10.5215%	11.1265%	11.6820%	12.2100%
B2	3.9380%	6.4185%	8.5525%	9.9715%	11.3905%	12.4575%	13.2055%	13.8325%	14.4210%	14.9600%
B3	6.3910%	9.1355%	11.5665%	13.2220%	14.8775%	16.0600%	17.0500%	17.9190%	18.5790%	19.1950%
Caa1	9.5599%	12.7788%	15.7512%	17.8634%	19.9726%	21.4317%	22.7620%	24.0113%	25.1195%	26.2350%
Caa2	14.3000%	17.8750%	21.4500%	24.1340%	26.8125%	28.6000%	30.3875%	32.1750%	33.9625%	35.7500%
Caa3	28.0446%	31.3548%	34.3475%	36.4331%	38.4017%	39.6611%	40.8817%	42.0669%	43.2196%	44.3850%

Source: Moody's

Appendix 6: The Gauss-Hermite Quadrature

The numerical integration is used whenever is not analytically possible to obtain the value of a definite integral. In this context, the Gauss-Hermite quadrature is a numerical integration formula, useful to approximate integrals of the form:

$$\int_{-\infty}^{+\infty} e^{-x^2} f(x) dx \approx \sum_{i=1}^{n} A_i f(x_i)$$

where $f(x)$ is a smooth function, and both x_i and A_i are a set of n abscissas and coefficients and determined by the quadrature.

The numerical integration is the computation of the value of a definite integral. The numerical integration is useful when is not possible to obtain an analytical solution for a definite integral.

There are two groups of methods for numerical integration, the Newton-Cotes formulas and the Gaussian quadratures. The first ones, separate the interval of integration in a given number of equally spaced abscissas. Then, the area under the curve of the function is estimated in each interval. On the other hand, Gaussian quadratures locate the abscissas in determined positions to find the best possible accuracy for a given number of points to consider. The main difference between Newton-Cotes formulas and Gaussian quadratures is that the last group requires fewer evaluations of the function and can hand integrable singularities.

The Gauss-Hermite quadrature is a numerical integration method which is useful to estimate integrals of the form:

$$\int_{-\infty}^{+\infty} e^{-x^2} f(x) dx$$

where $f(x)$ is a smooth function. Considering $\mathbf{n}$ abscissas, the integral is estimated by:

$$\int_{-\infty}^{+\infty} e^{-x^2} f(x) dx \approx \sum_{i=1}^{n} A_i f(x_i)$$

where $f(x)$ is a smooth function, and both x_i and A_i are a set of n abcissas and coefficients and determined by the quadrature. The abscissas are calculated as the zeros of the orthogonal polynomials $\varphi_n(x)$ with respect to e^{x^2}, defined as:

$$\int_{-\infty}^{+\infty} e^{-x^2}\varphi_n(x)\varphi_m(x)dx \;\; m \neq n$$

where $\varphi_n(x)$ is defined with the following recurrence:

$$a_n\varphi_{n+1}(x) = (b_n + c_n x)\varphi_n(x) - d_n\varphi_{n-1}(x)$$

with:

$$\varphi_0(x) = 1;\; \varphi_1(x) = 2x; a_n = 1; b_n = 0; c_n = 2; d_n = 2$$

The coefficientsA_i are obtained by:

$$A_i = \frac{2^{n+1}n!\sqrt{\pi}}{[H_n'(x_i)]^2}$$

where $H_n'(x)$ is defined as:

$$H_n'(x) = 2n(-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$$

Appendix 7: Comparison between Monte Carlo Simulation and Gauss-Hermite Quadrature for a Synthetic CDO.

This section explains why the Gauss-Hermite quadrature outperforms the Monte Carlo simulation in the estimation of $\boldsymbol{p_j}$ for a synthetic CDO. The reason for this is that for a given level of accuracy, Gauss-Hermite quadrature is less computationally intensive than the Monte Carlo simulation.

i. **Estimation of $\boldsymbol{p_j}$**

The first step is to compare whether the Gauss-Hermite quadrature and Monte Carlo simulation can give similar results for $\boldsymbol{p_j}$.

In order to check the goodness of fit taking the result of the Monte Carlo as the reference values, there are two alternatives.

- The most common statistical method is the Chi-Square Goodness-Fit-Test, which allows the use of discrete and continuous probability distributions. Given expected values for a specified distribution, this method check whether the observed data fit the specified distribution. In order to do this, the method tests null hypothesis that the observed data fit the specified distribution, with an error of $\boldsymbol{\alpha}$%. The null hypothesis could be accepted or rejected under a $\mathbf{1}-\boldsymbol{\alpha}\%$ level of confidence depending on the value of the following statistic:

$$\chi^2_{k-1} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

  where $\boldsymbol{k}$ is the number of events, $\boldsymbol{O_i}$ is the observed frequency of event $\boldsymbol{i}$ and $\boldsymbol{E_i}$ is the expected frequency of event $\boldsymbol{i}$. For the purpose of this study, one might consider the values of $\boldsymbol{p_j}$ obtained from a Monte Carlo and the Gauss-Hermite quadrature as the estimated and observed frequencies respectively.

  The problem with this method is that the estimated values for $\boldsymbol{p_j}$ can be very close to zero. For a given set of pool characteristics, there are many default scenarios whose value of $\boldsymbol{p_j}$ is very close to zero. Due to this fact, the estimation of $\boldsymbol{\chi^2_{k-1}}$ can lead to floating point errors. For this reason the Chi-Square Goodness-Fit-Test is not ideal.

- Another alternative is the use of a linear regression to compare the estimation of $\boldsymbol{p_j}$ by both the Gauss-Hermite quadrature and the Monte Carlo simulation. The rationale behind the use of a linear regression is the following.
  It is well known that the use of a Monte Carlo simulation leads to an accurate estimation of the values of $\boldsymbol{p_j}$. Therefore, the estimation of $\boldsymbol{p_j}$ by the Gauss-

Hermite quadrature should explain this value substantially. Consider the following expression:

$$Y = \beta_0 + \beta_1 X$$

where:

$Y =$ The value of p_j estimated by using Monte Carlo simulation, for $j = 0, 1, 2, \dots, N$.

$X =$ The value of p_j estimated by using the Gauss-Hermite quadrature, for $j = 0, 1, 2, \dots, N$.

$\beta_0 =$ The intercept term, which is expected to be very close to 0

$\beta_1 =$ The regression coefficient, which is expected to be very close to 1.

In order to check whether $\beta_0 = 0$ and $\beta_1 = 1$, tests of linear restrictions can be used.

The use of linear regression to compare both approaches requires to estimate p_j for different values of P and ρ. For this purpose, consider the following synthetic CDO:

- The reference pool consists of 100 assets. Because this analysis focuses on p_j, neither the value of α nor the synthetic CDO tranche structure is required.
- For the securities in the reference portfolio, the study considers all the possible pairwise combinations of P and ρ from Tables 7.1 and 7.2.

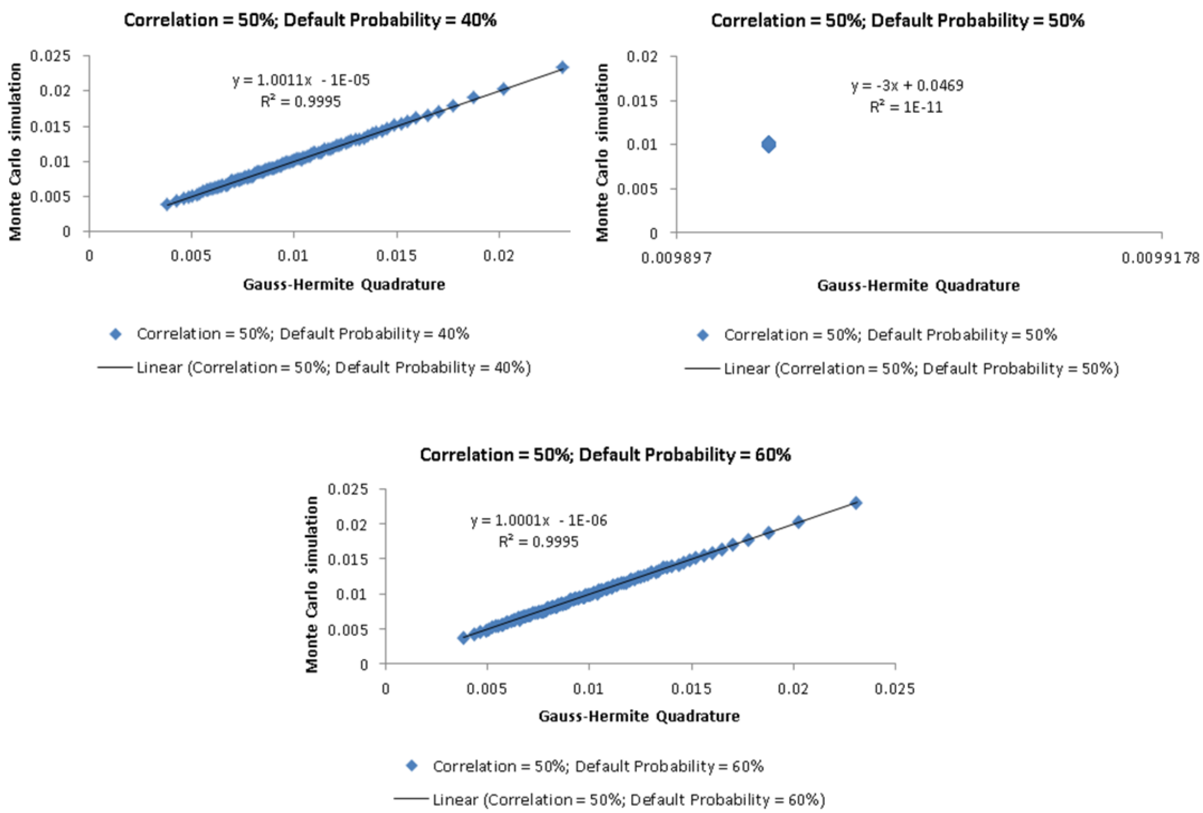
Tables A.7.1 and A.7.2: Considered Values for Default Probability and Default Correlation

Default Probability (P)%							
0.00001	0.0001	0.001	0.01	0.1	1	10	20
30	40	50	60	70	80	90	100

Default Correlation (ρ) %										
0	10	20	30	40	50	60	70	80	90	100

The result of this analysis is that both hypothesis of $\beta_0 = 0$ and $\beta_1 = 1$ are never rejected in any of the cases. The case of P and ρ equal to 50% is the only case that could not be checked with the linear regression. In this particular case the linear regression did not work since all of the p_j take the same value. Figures A.7.1, A.7.2 and A.7.3 show this situation more clearly.

Figures A.7.1, A.7.2 and A.7.3: Considered Values for Default Probability and Default Correlation



Source: Own Elaboration

The reason for this particular case is explained by the following analytical demonstration:

- Let $P = 0.5$ and $\rho = 0.5$. Therefore, the value of $q(z)$ becomes:

$$q(z) = Prob\left(\varepsilon_i \leq \frac{\Phi^{-1}(P) - Z\sqrt{\rho}}{\sqrt{1-\rho}} / Z = z \right) = Prob\left(\varepsilon_i \leq \frac{0 - Z\sqrt{0.5}}{\sqrt{1-0.5}} / Z = z \right)$$

$$= Prob(\varepsilon_i \leq Z / Z = z) = \Phi(z)$$

Because of this, p_j can be rewritten as:

$$p_j = \int_{-\infty}^{+\infty} \binom{N}{j} q(z)^j \left(1 - q(z)\right)^{N-j} \phi(z) dz = \int_{-\infty}^{+\infty} \binom{N}{j} \Phi(z)^j \left(1 - \Phi(z)\right)^{N-j} \phi(z) dz$$

- Due to the fact that $\boldsymbol{\Phi(z)}$ have values in the interval $[\mathbf{0},\mathbf{1}]$, $\boldsymbol{\Phi(z)}$ can be replaced by a variable $\boldsymbol{u}$ with values in the same interval. Thus:

$$\boldsymbol{\Phi(z) = u} \text{ with } \boldsymbol{u} \in [\mathbf{0},\mathbf{1}]$$

$$\Rightarrow \boldsymbol{\phi(z)dz = du}$$

then,

$$\boldsymbol{p_j = \int_{-\infty}^{+\infty} \binom{N}{j} q(z)^j \big(1 - q(z)\big)^{N-j} \phi(z) dz = \int_{0}^{1} \binom{N}{j} u^j (1-u)^{N-j} du}$$

- Considering the definition of the Gamma function for integer numbers as:

$$\boldsymbol{\Gamma(n) = (n-1)!}$$

then,

$$\boldsymbol{p_j = \int_{0}^{1} \frac{N!}{j!\,(N-j)!} u^j (1-u)^{N-j} du = \int_{0}^{1} \frac{\Gamma(N+1)}{\Gamma(j+1)\Gamma(N-j+1)} u^j (1-u)^{N-j} du}$$

$$\boldsymbol{= \frac{\Gamma(j)}{\Gamma(j+1)} \int_{0}^{1} u \; \frac{\Gamma(N+1)}{\Gamma(j)\Gamma(N-j+1)} u^{j-1} (1-u)^{N-j} du}$$

- Finally, the integral equals the mean of a Beta distribution of parameters $\boldsymbol{j}$ and $\boldsymbol{N-j+1}$, so:

$$\boldsymbol{p_j = \frac{\Gamma(j)}{\Gamma(j+1)} \; \frac{j}{(N-j+1)+j} = \frac{1}{j} \; \frac{j}{N+1}}$$

$$\boldsymbol{\Rightarrow p_j = \frac{1}{N+1} \; \forall j = 0, 1, 2, \ldots, N}$$

This result means that, when $\boldsymbol{P = 0.5}$ and $\boldsymbol{\rho = 0.5}$, all of the default scenarios are equiprobable. This result is consistent with the numerical results obtained with the Monte Carlo simulation.

ii. **Time Computation of $\boldsymbol{p_j}$**

Although both approaches yield similar estimates of $\boldsymbol{p_j}$, the Gauss-Hermite quadrature outperforms the Monte Carlo simulation in terms of time. The following table compares the computation time required for both approaches.

Monte Carlo		Gauss-Hermite	
# of Simulations		# of Abcissas	
10000	20000	100	150
1800	4200	20	30

Time in milliseconds

Appendix 8: The Beta Distribution for the Recovery Rate

The use of the Beta distribution for the recovery rate is supported by several studies. The Basel Committee on Banking Supervision (BCBS), for example, endorses this of using a model for the recovery rate, rather than practice of choosing one fixed value. Ideally, the model should take into account downturn economic conditions. With this approach banks are supposed to be less likely to underestimate the economic capital they require. In this sense, the BCBS express skepticism that the recovery rate cannot be obtained from historical data. The reason is that historical data can be unreliable when credit losses become higher.

Many studies have tried to explain the behavior of the recovery rates using different credit risk models. (Morozovskiy, 2004) categorizes these approaches in three groups:

- The first group consists of structural models. They consider the accounting concepts of assets and liabilities to check whether the firm defaults or not. In these models the dependence of the recovery rate on the default probability does not follow a Beta distribution.
- The second group is called the reduced form models. These models are based on the Poisson distribution. The recovery rate and the default probability are modeled independently.
- The third big group is known as the credit portfolio models. These models are based on the Value at Risk methodology to describe the default probability of the firm portfolio.

None of the mentioned groups leads to a Beta distribution to model the recovery rate. Supporting the use of a Beta distribution (Morozovskiy, 2004) develops a theoretical model that explains the Beta distribution for the recovery rate. The author considers a demand and supply framework where the remaining assets of a defaulted firm can be traded in the secondary market. The author concludes that in general, his model leads to quasi-Beta distributions. However, a Beta distribution can be obtained under certain parameters for the models he uses.

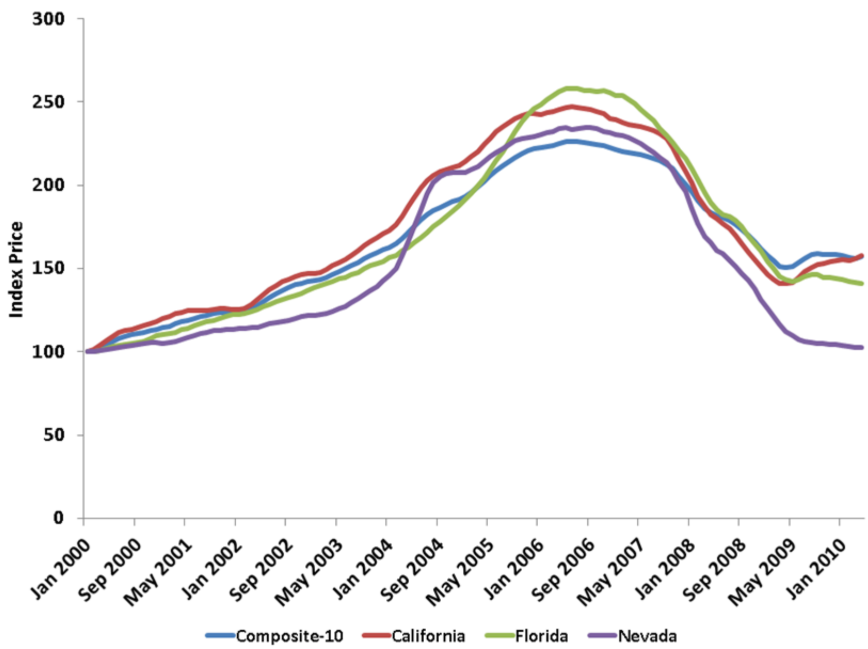
Other studies have used the Beta distribution for the recovery rate. For instance, both Unal & Madan (1998) and Gaspar & Slinko (2008) have used it when studying the pricing of risk in debt securities and credit risk models respectively. (Uhrig-Homburg & Schläfer, 2010) analyze the recovery rate behavior in CDS contracts with the Beta distribution. Finally, Gupton & Stein (2002) also use a Beta distribution in Moody's model for predicting the loss given default.

Appendix 9: The Chain of Events in Relation with the ABACUS Deal

Paulson & Co., one of the largest hedge fund in the world by early 2007, approached Goldman Sachs and express interest in taking a short position in a synthetic CDO agreement referencing a certain group of mortgage bonds. In order to do this, Paulson & Co. needed to buy protection on some reference portfolio.

To structure the deal, Paulson & Co. chose 123 subprime residential mortgage backed securities. These securities were rated as Baa2 by Moody's. The portfolio concentrated in adjustable rate mortgages with relative low borrower credit scores. The geographic distribution of those residential mortgages was highly concentrated in the states of Arizona, California, Florida and Nevada. The performance of the home price in these states can be seen in Figure A.9.1.

Figure A.9.1: Comparison of Home Prices in the States of California, Florida and Nevada With Respect to the Composite 10

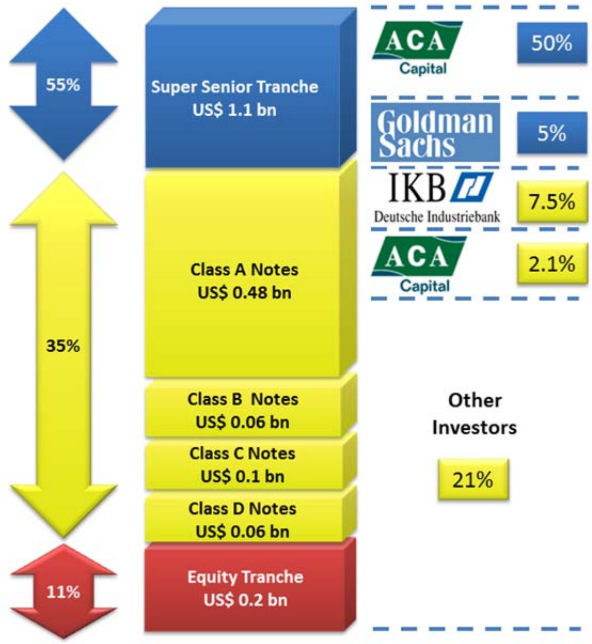


Source: Standard & Poor's

Next, a third party was brought into the ABACUS transaction. ACA Management, a collateral manager of CDOs, was asked by Goldman Sachs to make the final selection of the reference portfolios based on the initial names determined by Paulson & Co. According to SEC filings, ACA Management was misled to think that Paulson & Co. was interested in investing in the equity tranche of the ABACUS at issuance.

After several meeting among executives of the three involved firms, they mutually agreed on 92 residential mortgage backed securities to be included in the reference portfolio. Then, Goldman Sachs marketed ABACUS among investors without making any reference to the participation of Paulson & Co. in the portfolio selection. The initial participation of the different investors can be seen in Figure A.9.2.

Figure A.9.2: Initial Participation of Different investors in the ABACUS Transaction.



Source: Business Insider

ACA Capital, a parent company of ACA Management, together with IKB, a German commercial bank, agreed to sell credit protection on the ABACUS reference portfolio. ACA invested US$42 million in the Class A notes and another US$ 900 million in the Super Senior tranche. IKB invested US $150 million in the Class A notes. A remaining 5% of the Class A Notes was acquired by Goldman Sachs.

On the opposite side of the deal, Paulson & Co. was the big participant. Instead of investing in the equity tranche of the ABACUS, Paulson &Co. bought credit protection on the reference portfolio.

Lately, the Dutch bank ABN AMRO got involved in the transaction. Through a series of CDS contracts, ABN assumed the credit risk of Goldman Sachs and ACA Capital.

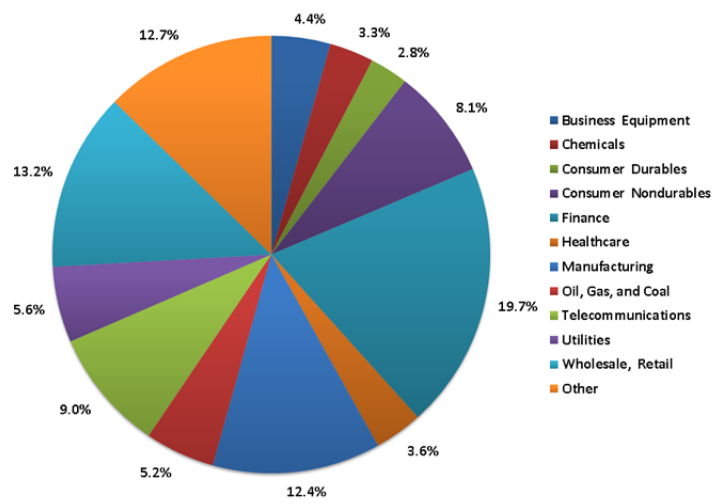
Finally, Paulson & Co. view of the collateral turned out to be right. By early 2008, almost the entire reference portfolio had defaulted. Therefore, the ABACUS tranches experienced heavy losses. IKB lost US$150 million and ABN other US$840 million. On the other hand, Paulson & Co. profited US$ 1.1 billion and Goldman Sachs pocketed US$ 15 million in structuration fees.

The SEC did not sue Goldman Sachs until 2010. After many sessions in court, the SEC and Goldman Sachs reached an agreement. Goldman Sachs paid a total amount of US$ 550 million in settlement charges. IKB and ABN received US$150 and US$100 million respectively in compensations. The remaining US$ 300 million were paid to the U.S. Treasury. In addition, Goldman Sachs acknowledged that not disclosing the name of Paulson & Co. in the marketing material had been a mistake.

Appendix 10: CDX Indices Composition

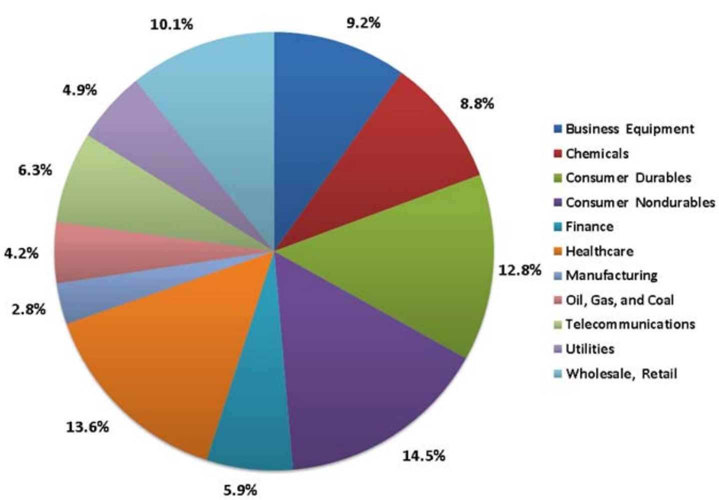
Since the inception of the two CDX indices in late 2004, the portfolio composition among industries has been very stable through time. The following figures show the composition of the CDX.NA.IG and CDX.NA.HY indices respectively.

Figure A.10.1: Composition of CDX.NA.IG



Source: Own Elaboration, Based on Longstaff and Myers 2009.

Figure A.10.2: Composition of CDX.NA.HY



Source: Own Elaboration, Based on Longstaff and Myers 2009.

Appendix 11: Linear Programming Solution for the MIDGARD Overlapping

To find out the actual structure of the MIDGARD CDO-Squared, based on the constraints described in the issue report of the transaction, the following linear optimization problem can be solved.

Sets

$$\boldsymbol{i} = \mathbf{1},..,\mathbf{560};\quad \boldsymbol{j} = \mathbf{1},..,\mathbf{7};\ \boldsymbol{k_j} = \{\hat{\boldsymbol{j}} > \boldsymbol{j}\ \backslash \hat{\boldsymbol{j}} = \boldsymbol{j} + \mathbf{1}, \boldsymbol{j} + \mathbf{2},..,\mathbf{7}\}$$

Decision Variables

$$\boldsymbol{x_{i,j}} = \begin{cases} \mathbf{1}, \textit{if asset } \boldsymbol{i} \textit{ belongs to CDO } \boldsymbol{j} \\ \mathbf{0}, \textit{otherwise} \end{cases} \quad \forall \boldsymbol{i}, \boldsymbol{j}$$

$$\boldsymbol{y_{i,j}} = \begin{cases} \mathbf{1}, \textit{if asset } \boldsymbol{i} \textit{ belongs to } \boldsymbol{j} \textit{ CDOs} \\ \mathbf{0}, \textit{otherwise} \end{cases} \quad \forall \boldsymbol{i}, \boldsymbol{j}$$

$$\boldsymbol{z_{i,j,k_j}} = \begin{cases} \mathbf{1}, \textit{if asset } \boldsymbol{i} \textit{ belongs to both CDOs } \boldsymbol{j} \textit{ and } \boldsymbol{k_j} \\ \mathbf{0}, \textit{otherwise} \end{cases} \quad \forall \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k_j}$$

Restrictions

The total number of assets per CDO should be equal to *80* $\Rightarrow \sum_{i=1}^{560} \boldsymbol{x_{i,j}} = \mathbf{80}\ \forall \boldsymbol{j}$

Overlap of the asset $\boldsymbol{i}$ among $\boldsymbol{j}$ CDOs $\Rightarrow \sum_{j=1}^{7} \boldsymbol{j} \times \boldsymbol{y_{i,j}} = \sum_{j=1}^{7} \boldsymbol{x_{i,j}}\ \forall \boldsymbol{i}$

The total number of independent referenced assets equals 259 $\Rightarrow \sum_{j=1}^{4} \boldsymbol{y_{i,j}} = \mathbf{259}\ \forall \boldsymbol{i}$

No asset $\boldsymbol{i}$ belongs to more than 4 CDOs $\Rightarrow \sum_{j=5}^{7} \boldsymbol{y_{i,j}} = \mathbf{0}\ \forall \boldsymbol{i}$

Overlapped asset $\boldsymbol{i}$ between a pair of CDOs $\Rightarrow \mathbf{2} \times \boldsymbol{z_{i,j,k_j}} \leq \boldsymbol{x_{i,j}} + \boldsymbol{x_{i,k_j}}\ \forall \boldsymbol{i}, \boldsymbol{j}$

The overlapped between any pair of synthetic CDOs is less or equal than 30%

$$\Rightarrow \sum_{i=1}^{560} \boldsymbol{z_{i,j,k_j}} \leq \mathbf{30}\% \times \mathbf{80}\quad \forall \boldsymbol{j}, \boldsymbol{k_j}$$

The average degree of overlapping among the seven synthetic CDOs is 26%: This condition was translated by the following two conditions:

Average overlapped assets should be greater or equal to 25.5%×80

$$\Rightarrow \frac{\mathbf{1}}{\mathbf{21}} \sum_{i=1}^{560} \sum_{j=1}^{6} \sum_{k_j=j+1}^{7} z_{i,j,k_j} \geq \mathbf{25.5}\% \times \mathbf{80}$$

Average overlapped assets should be less or equal to 26.5%×80

$$\Rightarrow \frac{\mathbf{1}}{\mathbf{21}} \sum_{i=1}^{560} \sum_{j=1}^{6} \sum_{k_j=j+1}^{7} z_{i,j,k_j} \leq \mathbf{26.5}\% \times \mathbf{80}$$

<u>Objective Function</u>

$$\underset{x_{i,j}, y_{i,j}, z_{i,j,k_j}}{max(or\ min)} \sum_{i=1}^{560} \sum_{j=1}^{6} \sum_{k_j=j+1}^{7} z_{i,j,k_j}$$

In this study it was considered the maximization of the objective function. However, there are no big differences in the results when using any of both objective functions.

Appendix 12: The Diversification Effect due to the Number of Securities in the Pool.

In a common synthetic CDO, the diversification of the pool of assets is dependent not only on the number of referenced securities but also on the default correlation. This means that when the default correlation is close to one, the pool of assets behaves like a one single asset (no diversification). On the other hand, a default correlation close to zero makes the diversification to be dependent only on the number of referenced securities.

To make this clear, consider the estimation of the expected loss of a synthetic CDO with the tranche structure indicated in the table below.

Tranche Name	Tranche Width
Senior	100% - X%
Mezzanine	X%
Equity	5%

In addition, for the asset parameters consider a default correlation equals to zero, a default probability equals to 20% and a recovery rate equals to 60%. Besides, consider values for the size of the Mezzanine from 10 % to 50%. Besides, consider the number of pooled securities to take values from 10 to 200.

Figure A.12.1 shows the diversification effect in the expected loss of the Mezzanine tranche due to the number of referenced securities. The trend is clear: no matter the width of the Mezzanine tranche, the diversification due to an increase in the number of securities is bounded over a certain number of referenced securities.

Figure A.12.1: Expected Loss of a Mezzanine Tranche for Different Width and Different Number of Securities.

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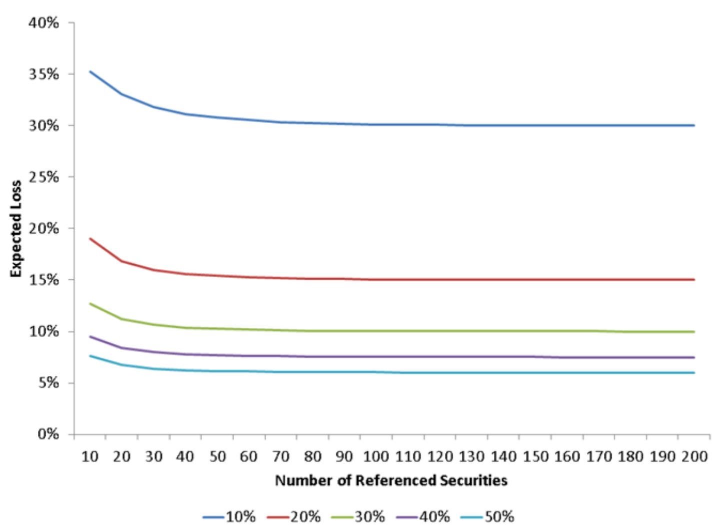

Source: Own Elaboration

Appendix 13: Detailed Confidence Intervals for the Cases of Study

ABACUS		Recovery Rate Following a Beta Distribution							
		95% Confidence Interval							
Tranche Name	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.000007%	0.000001%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.000076%	0.000008%	0.000000%	Aaa	Aaa	Aaa	0	0
	30%	0.000415%	0.000047%	0.000000%	Aaa	Aaa	Aaa	0	0
Class A	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000043%	0.000005%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000993%	0.000142%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.006165%	0.001029%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.020633%	0.003895%	0.000000%	Aa2	Aaa	Aaa	2	0
	30%	0.049048%	0.010260%	0.000000%	Aa3	Aa1	Aaa	2	1
Class B	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000004%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.001135%	0.000146%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.013822%	0.002264%	0.000000%	Aa1	Aaa	Aaa	1	0
	20%	0.056593%	0.011073%	0.000000%	Aa3	Aa1	Aaa	2	1
	25%	0.139667%	0.031497%	0.000000%	A1	Aa2	Aaa	2	2
	30%	0.261342%	0.066376%	0.000000%	A3	Aa3	Aaa	3	3
Class C	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000211%	0.000023%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.009954%	0.001540%	0.000000%	Aa1	Aaa	Aaa	1	0
	15%	0.061109%	0.011853%	0.000000%	Aa3	Aa1	Aaa	2	1
	20%	0.174571%	0.039984%	0.000000%	A2	Aa2	Aaa	3	2
	25%	0.344858%	0.090130%	0.000000%	A3	A1	Aaa	2	4
	30%	0.554390%	0.161668%	0.000000%	Baa1	A2	Aaa	2	5
Class D	**0%**	0.000003%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.005092%	0.000641%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.070776%	0.012694%	0.000000%	Aa3	Aa1	Aaa	2	1
	15%	0.253077%	0.056803%	0.000000%	A2	Aa3	Aaa	2	3
	20%	0.531799%	0.140381%	0.000000%	Baa1	A1	Aaa	3	4
	25%	0.860923%	0.257891%	0.000000%	Baa2	A2	Aaa	3	5
	30%	1.202065%	0.399461%	0.000000%	Baa3	A3	Aaa	3	6
First Loss	**0%**	11.466080%	6.938786%	0.812440%	B2	B1	Baa2	1	5
	5%	11.464605%	6.938583%	0.812440%	B2	B1	Baa2	1	5
	10%	11.441203%	6.934153%	0.812440%	B2	B1	Baa2	1	5
	15%	11.357738%	6.914799%	0.812440%	B2	B1	Baa2	1	5
	20%	11.195236%	6.870886%	0.812440%	B2	B1	Baa2	1	5
	25%	10.957096%	6.797513%	0.812440%	B2	B1	Baa2	1	5
	30%	10.647910%	6.693318%	0.812440%	B2	Ba3	Baa2	2	4

ABACUS		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
		95% Confidence Interval							
Tranche Name	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	30%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
Class A	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.000004%	0.000001%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.000050%	0.000015%	0.000001%	Aaa	Aaa	Aaa	0	0
	30%	0.000297%	0.000104%	0.000014%	Aaa	Aaa	Aaa	0	0
Class B	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000008%	0.000002%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.000199%	0.000059%	0.000005%	Aaa	Aaa	Aaa	0	0
	25%	0.001578%	0.000562%	0.000082%	Aaa	Aaa	Aaa	0	0
	30%	0.006650%	0.002716%	0.000557%	Aaa	Aaa	Aaa	0	0
Class C	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000004%	0.000001%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000259%	0.000076%	0.000007%	Aaa	Aaa	Aaa	0	0
	20%	0.002697%	0.000982%	0.000155%	Aaa	Aaa	Aaa	0	0
	25%	0.012207%	0.005162%	0.001178%	Aa1	Aaa	Aaa	0	0
	30%	0.035164%	0.016637%	0.004864%	Aa2	Aa1	Aaa	0	0
Class D	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000174%	0.000045%	0.000003%	Aaa	Aaa	Aaa	0	0
	15%	0.003839%	0.001328%	0.000184%	Aaa	Aaa	Aaa	0	0
	20%	0.021905%	0.009154%	0.002044%	Aa2	Aa1	Aaa	1	1
	25%	0.067045%	0.031972%	0.009551%	Aa3	Aa2	Aa1	1	1
	30%	0.146233%	0.076857%	0.027966%	A1	Aa3	Aa2	1	1
First Loss	0%	6.815370%	5.364225%	3.924648%	B1	Ba3	Ba2	1	1
	5%	6.815369%	5.364225%	3.924648%	B1	Ba3	Ba2	1	1
	10%	6.815315%	5.364211%	3.924648%	B1	Ba3	Ba2	1	1
	15%	6.814086%	5.363788%	3.924590%	B1	Ba3	Ba2	1	1
	20%	6.807381%	5.360968%	3.923956%	B1	Ba3	Ba2	1	1
	25%	6.788560%	5.351847%	3.921166%	B1	Ba3	Ba2	1	1
	30%	6.751211%	5.331785%	3.913626%	B1	Ba3	Ba2	1	1

CDX.NA.IG 5 Years Horizon		Recovery Rate Following a Beta Distribution							
		95% Confidence Interval							
Tranche Name	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000003%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000130%	0.000013%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.001167%	0.000128%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.004843%	0.000600%	0.000000%	Aaa	Aaa	Aaa	0	0
	30%	0.013361%	0.001830%	0.000000%	Aa1	Aaa	Aaa	1	0
Senior 1	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000137%	0.000012%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.008698%	0.000957%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.055904%	0.007537%	0.000000%	Aa2	Aaa	Aaa	2	0
	20%	0.164645%	0.026078%	0.000000%	A1	Aa1	Aaa	3	1
	25%	0.334522%	0.060574%	0.000000%	A3	Aa3	Aaa	3	3
	30%	0.553424%	0.112120%	0.000000%	Baa1	A1	Aaa	3	4
Senior 2	0%	0.000003%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.022226%	0.002236%	0.000000%	Aa1	Aaa	Aaa	1	0
	10%	0.232606%	0.031402%	0.000000%	A2	Aa2	Aaa	3	2
	15%	0.673281%	0.114092%	0.000000%	Baa1	A1	Aaa	3	4
	20%	1.232483%	0.247957%	0.000000%	Baa2	A2	Aaa	3	5
	25%	1.814056%	0.418221%	0.000000%	Baa3	A3	Aaa	3	6
	30%	2.368860%	0.609936%	0.000000%	Ba1	Baa1	Aaa	3	7
Mezzanine 1	0%	0.005302%	0.000449%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.383039%	0.046010%	0.000000%	A3	Aa2	Aaa	4	2
	10%	1.386030%	0.228844%	0.000000%	Baa3	A2	Aaa	4	5
	15%	2.524910%	0.517141%	0.000000%	Ba1	Baa1	Aaa	3	7
	20%	3.539335%	0.850894%	0.000000%	Ba1	Baa2	Aaa	2	8
	25%	4.364929%	1.191098%	0.000000%	Ba2	Baa2	Aaa	3	8
	30%	5.016553%	1.516433%	0.000000%	Ba2	Baa3	Aaa	2	9
Mezzanine 2	0%	3.371988%	0.476665%	0.000000%	Ba1	A3	Aaa	4	6
	5%	6.734140%	1.325091%	0.000000%	Ba3	Baa3	Aaa	3	9
	10%	8.984532%	2.194044%	0.000000%	B1	Baa3	Aaa	4	9
	15%	10.301811%	2.940518%	0.000000%	B2	Ba1	Aaa	4	10
	20%	11.064686%	3.546847%	0.000000%	B2	Ba1	Aaa	4	10
	25%	11.476732%	4.026054%	0.000002%	B2	Ba2	Aaa	3	11
	30%	11.606860%	4.395887%	0.000023%	B2	Ba2	Aaa	3	11
Equity	0%	59.293904%	30.456460%	2.800050%	Caa3	Caa2	Ba1	1	7
	5%	54.400032%	29.275876%	2.800050%	Caa3	Caa2	Ba1	1	7
	10%	50.106263%	27.881100%	2.800050%	Caa3	Caa2	Ba1	1	7
	15%	46.211125%	26.426499%	2.800050%	Caa3	Caa2	Ba1	1	7
	20%	42.685908%	24.965793%	2.800050%	Caa3	Caa2	Ba1	1	7
	25%	39.433897%	23.519399%	2.800034%	Caa3	Caa2	Ba1	1	7
	30%	36.411680%	22.094983%	2.799927%	Caa3	Caa1	Ba1	2	6

CDX.NA.IG 5 Years Horizon		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
Tranche Name	Default Correlation	95% Confidence Interval							
		Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	20%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	25%	0.000004%	0.000001%	0.000000%	Aaa	Aaa	Aaa	0	0
	30%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
Senior 1	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000001%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.000098%	0.000029%	0.000003%	Aaa	Aaa	Aaa	0	0
	20%	0.001131%	0.000416%	0.000075%	Aaa	Aaa	Aaa	0	0
	25%	0.016902%	0.008003%	0.002510%	Aa1	Aaa	Aaa	1	0
	30%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
Senior 2	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000001%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000535%	0.000151%	0.000013%	Aaa	Aaa	Aaa	0	0
	15%	0.008911%	0.003341%	0.000636%	Aaa	Aaa	Aaa	0	0
	20%	0.042897%	0.019225%	0.005412%	Aa2	Aa1	Aaa	1	1
	25%	0.237742%	0.131638%	0.055429%	A2	A1	Aa2	1	2
	30%	0.000101%	0.000019%	0.000000%	Aaa	Aaa	Aaa	0	0
Mezzanine 1	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000236%	0.000052%	0.000002%	Aaa	Aaa	Aaa	0	0
	10%	0.016673%	0.005790%	0.000890%	Aa1	Aaa	Aaa	1	0
	15%	0.104610%	0.046263%	0.012487%	Aa3	Aa2	Aa1	1	1
	20%	0.292239%	0.149866%	0.054412%	A2	A1	Aa2	1	2
	25%	0.887063%	0.540434%	0.267054%	Baa2	Baa1	A2	1	2
	30%	0.001860%	0.000456%	0.000010%	Aaa	Aaa	Aaa	0	0
Mezzanine 2	**0%**	0.004615%	0.000886%	0.000019%	Aaa	Aaa	Aaa	0	0
	5%	0.225714%	0.084290%	0.015770%	A2	Aa3	Aa1	2	2
	10%	0.826223%	0.403016%	0.133792%	Baa2	A3	A1	2	2
	15%	1.596084%	0.896387%	0.386300%	Baa3	Baa2	A3	1	2
	20%	2.371475%	1.455589%	0.730387%	Ba1	Baa3	Baa1	1	2
	25%	3.685579%	2.518230%	1.501565%	Ba1	Ba1	Baa3	0	1
	30%	0.051231%	0.017590%	0.001649%	Aa2	Aa1	Aaa	1	1
Equity	**0%**	30.542897%	24.388926%	18.417944%	Caa2	Caa2	Caa1	0	1
	5%	30.247861%	24.277668%	18.396942%	Caa2	Caa2	Caa1	0	1
	10%	29.429850%	23.846711%	18.238669%	Caa2	Caa2	Caa1	0	1
	15%	28.300989%	23.142948%	17.889341%	Caa2	Caa1	Caa1	1	0
	20%	27.017693%	22.265334%	17.380313%	Caa2	Caa1	B3	1	1
	25%	24.267038%	20.232599%	16.043895%	Caa2	Caa1	B3	1	1
	30%	5.211722%	3.459687%	1.826898%	Ba2	Ba1	Baa3	1	1

CDX.NA.HY		Recovery Rate Following a Beta Distribution							
		95% Confidence Interval							
Tranche Name	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000134%	0.000010%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.007487%	0.000740%	0.000000%	Aaa	Aaa	Aaa	0	0
	15%	0.044323%	0.005282%	0.000000%	Aa2	Aaa	Aaa	2	0
	20%	0.125892%	0.017220%	0.000000%	A1	Aa1	Aaa	3	1
	25%	0.254924%	0.038846%	0.000000%	A2	Aa2	Aaa	3	2
	30%	0.428450%	0.071302%	0.000000%	A3	Aa3	Aaa	3	3
Senior 1	**0%**	0.000002%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.069963%	0.006859%	0.000000%	Aa3	Aaa	Aaa	3	0
	10%	0.597468%	0.080163%	0.000000%	Baa1	Aa3	Aaa	4	3
	15%	1.551462%	0.256279%	0.000000%	Baa3	A2	Aaa	4	5
	20%	2.675431%	0.516601%	0.000000%	Ba1	Baa1	Aaa	3	7
	25%	3.812223%	0.833986%	0.000000%	Ba2	Baa2	Aaa	3	8
	30%	4.885342%	1.187249%	0.000000%	Ba2	Baa2	Aaa	3	8
Senior 2	**0%**	0.177721%	0.015449%	0.000000%	A1	Aa1	Aaa	3	1
	5%	2.689502%	0.415819%	0.000000%	Ba1	A3	Aaa	4	6
	10%	5.627772%	1.139807%	0.000000%	Ba3	Baa2	Aaa	4	8
	15%	8.003049%	1.930632%	0.000000%	B1	Baa3	Aaa	4	9
	20%	9.816756%	2.696662%	0.000000%	B1	Ba1	Aaa	3	10
	25%	11.201099%	3.410250%	0.000000%	B2	Ba1	Aaa	4	10
	30%	12.250474%	4.065471%	0.000000%	B2	Ba2	Aaa	3	11
Mezzanine	**0%**	11.021221%	1.582565%	0.000000%	B2	Baa3	Aaa	5	9
	5%	19.198764%	4.387833%	0.000000%	Caa1	Ba2	Aaa	5	11
	10%	22.171907%	6.321886%	0.000000%	Caa1	Ba3	Aaa	4	12
	15%	23.481062%	7.713374%	0.000000%	Caa2	B1	Aaa	4	13
	20%	24.022386%	8.759801%	0.000000%	Caa2	B1	Aaa	4	13
	25%	24.182623%	9.569022%	0.000000%	Caa2	B1	Aaa	4	13
	30%	24.095763%	10.204935%	0.000000%	Caa2	B2	Aaa	3	14
Equity	**0%**	80.736939%	46.900314%	4.960956%	Caa3	Caa3	Ba2	0	7
	5%	74.074204%	45.090386%	4.960956%	Caa3	Caa3	Ba2	0	7
	10%	69.070287%	43.321322%	4.960956%	Caa3	Caa3	Ba2	0	7
	15%	64.854239%	41.629119%	4.960956%	Caa3	Caa3	Ba2	0	7
	20%	61.103978%	40.001951%	4.960956%	Caa3	Caa3	Ba2	0	7
	25%	57.671744%	38.425799%	4.960956%	Caa3	Caa3	Ba2	0	7
	30%	54.469307%	36.888398%	4.960956%	Caa3	Caa3	Ba2	0	7

CDX.NA.HY		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
		95% Confidence Interval							
Tranche Name	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.000013%	0.000005%	0.000001%	Aaa	Aaa	Aaa	0	0
	15%	0.000438%	0.000198%	0.000061%	Aaa	Aaa	Aaa	0	0
	20%	0.003163%	0.001658%	0.000672%	Aaa	Aaa	Aaa	0	0
	25%	0.011577%	0.006711%	0.003213%	Aa1	Aaa	Aaa	1	0
	30%	0.029378%	0.018296%	0.009806%	Aa2	Aa1	Aa1	1	0
Senior 1	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000191%	0.000060%	0.000008%	Aaa	Aaa	Aaa	0	0
	10%	0.017043%	0.007927%	0.002588%	Aa1	Aaa	Aaa	1	0
	15%	0.119156%	0.066771%	0.030249%	A1	Aa3	Aa2	1	1
	20%	0.362409%	0.226840%	0.122430%	A3	A2	A1	1	1
	25%	0.751090%	0.506302%	0.305133%	Baa2	Baa1	A2	1	2
	30%	1.261016%	0.896729%	0.583403%	Baa2	Baa2	Baa1	0	1
Senior 2	**0%**	0.000070%	0.000013%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.181293%	0.084769%	0.027786%	A1	Aa3	Aa2	1	1
	10%	0.979267%	0.585355%	0.293509%	Baa2	Baa1	A2	1	2
	15%	2.160162%	1.450450%	0.868129%	Baa3	Baa3	Baa2	0	1
	20%	3.450444%	2.483862%	1.642482%	Ba1	Ba1	Baa3	0	1
	25%	4.711950%	3.554903%	2.507953%	Ba2	Ba1	Ba1	1	0
	30%	5.887007%	4.595017%	3.393459%	Ba3	Ba2	Ba1	1	1
Mezzanine	**0%**	0.471832%	0.158810%	0.024759%	A3	A1	Aa1	2	3
	5%	4.846613%	2.888830%	1.423118%	Ba2	Ba1	Baa3	1	1
	10%	8.753595%	6.109013%	3.837333%	B1	Ba3	Ba2	1	1
	15%	11.499090%	8.656679%	6.059463%	B2	B1	Ba3	1	1
	20%	13.433396%	10.579610%	7.879355%	B3	B2	B1	1	1
	25%	14.821411%	12.031010%	9.331037%	B3	B2	B1	1	1
	30%	15.827754%	13.131366%	10.481583%	B3	B2	B2	1	0
Equity	**0%**	63.753841%	56.993851%	50.030081%	Caa3	Caa3	Caa3	0	0
	5%	61.385037%	55.544025%	49.303107%	Caa3	Caa3	Caa3	0	0
	10%	58.616633%	53.425450%	47.827691%	Caa3	Caa3	Caa3	0	0
	15%	55.958117%	51.226423%	46.113954%	Caa3	Caa3	Caa3	0	0
	20%	53.439715%	49.061982%	44.333502%	Caa3	Caa3	Caa3	0	0
	25%	51.040828%	46.952940%	42.542969%	Caa3	Caa3	Caa3	0	0
	30%	48.736973%	44.896917%	40.761067%	Caa3	Caa3	Caa3	0	0

MIDGARD		Recovery Rate Following a Beta Distribution							
Tranche Name	Default Correlation	95% Confidence Interval							
		Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.000359%	0.000024%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.142519%	0.017981%	0.000000%	A1	Aa1	Aaa	3	1
	10%	0.661303%	0.120841%	0.000000%	Baa1	A1	Aaa	3	4
	15%	1.382707%	0.313766%	0.000000%	Baa3	A2	Aaa	4	5
	20%	2.036863%	0.545848%	0.000000%	Baa3	Baa1	Aaa	2	7
	25%	2.690088%	0.808114%	0.000000%	Ba1	Baa2	Aaa	2	8
	30%	3.267598%	1.073854%	0.000000%	Ba1	Baa2	Aaa	2	8
Class III	0%	0.007075%	0.000670%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.710550%	0.097302%	0.000000%	Aa3	Aa3	Aaa	4	3
	10%	2.064699%	0.402881%	0.000000%	A3	A3	Aaa	3	6
	15%	3.371427%	0.818373%	0.000000%	Baa2	Baa2	Aaa	2	8
	20%	4.500868%	1.240864%	0.000000%	Baa2	Baa2	Aaa	3	8
	25%	5.463193%	1.673561%	0.000000%	Baa3	Baa3	Aaa	2	9
	30%	6.165759%	2.076893%	0.000000%	Baa3	Baa3	Aaa	3	9
Equity	0%	0.241095%	0.025677%	0.000000%	A2	Aa1	Aaa	4	1
	5%	2.025706%	0.327425%	0.000000%	Baa3	A2	Aaa	4	5
	10%	4.034473%	0.858067%	0.000000%	Ba2	Baa2	Aaa	3	8
	15%	5.607443%	1.430560%	0.000000%	Ba3	Baa3	Aaa	3	9
	20%	6.808133%	1.961423%	0.000000%	Ba3	Baa3	Aaa	3	9
	25%	7.632246%	2.450423%	0.000000%	Ba3	Ba1	Aaa	2	10
	30%	8.150596%	2.874937%	0.000000%	B1	Ba1	Aaa	3	10

MIDGARD		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
		95% Confidence Interval							
Tranche Name	**Default Correlation**	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.007901%	0.003262%	0.001117%	Aaa	Aaa	Aaa	0	0
	15%	0.048454%	0.019336%	0.007002%	Aa2	Aa1	Aaa	1	1
	20%	0.157850%	0.074961%	0.029098%	A1	Aa3	Aa2	1	1
	25%	0.323581%	0.174884%	0.080764%	A2	A1	Aa3	1	1
	30%	0.546924%	0.318742%	0.159245%	Baa1	A2	A1	2	1
Class III	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.027873%	0.009072%	0.002240%	Aa2	Aaa	Aaa	2	0
	15%	0.154259%	0.055039%	0.019738%	A1	Aa2	Aa1	2	1
	20%	0.381211%	0.194851%	0.071292%	A3	A1	Aa3	2	1
	25%	0.712314%	0.386194%	0.189445%	Baa1	A3	A1	1	2
	30%	1.105902%	0.638814%	0.333572%	Baa2	Baa1	A3	1	1
Equity	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.003963%	0.000369%	0.000000%	Aaa	Aaa	Aaa	0	0
	10%	0.079510%	0.026208%	0.007881%	Aa3	Aa1	Aaa	2	1
	15%	0.318339%	0.144268%	0.044780%	A2	A1	Aa2	1	2
	20%	0.682041%	0.354710%	0.151032%	Baa1	A3	A1	1	2
	25%	1.155987%	0.641977%	0.313774%	Baa2	Baa1	A2	1	2
	30%	1.639664%	0.993311%	0.520289%	Baa3	Baa2	Baa1	1	1

T-CDX.NA.IG Overlap = 15%		Recovery Rate Following a Beta Distribution							
		95% Confidence Interval							
Tranche	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.027024%	0.002203%	0.000000%	Aa1	Aaa	Aaa	1	0
	5%	1.642464%	0.274960%	0.000000%	Baa3	A2	Aaa	4	5
	10%	3.887151%	0.913200%	0.000000%	Ba2	Baa2	Aaa	3	8
	15%	5.605142%	1.608116%	0.000000%	Ba3	Baa3	Aaa	3	9
	20%	6.809556%	2.237859%	0.000000%	Ba3	Baa3	Aaa	3	9
	25%	7.626360%	2.783571%	0.000000%	Ba3	Ba1	Aaa	2	10
	30%	8.155263%	3.239709%	0.000000%	B1	Ba1	Aaa	3	10
Senior 1	0%	1.068403%	0.101871%	0.000000%	Baa2	Aa3	Aaa	5	3
	5%	9.035142%	1.777416%	0.000000%	B1	Baa3	Aaa	4	9
	10%	12.797468%	3.415784%	0.000000%	B2	Ba1	Aaa	4	10
	15%	14.375370%	4.594593%	0.000000%	B3	Ba2	Aaa	4	11
	20%	14.978112%	5.416676%	0.000000%	B3	Ba2	Aaa	4	11
	25%	15.095483%	5.990401%	0.000000%	B3	Ba3	Aaa	3	12
	30%	14.956371%	6.387542%	0.000000%	B3	Ba3	Aaa	3	12
Senior 2	0%	5.206642%	0.548648%	0.000000%	Ba2	Baa1	Aaa	4	7
	5%	16.076121%	3.447177%	0.000000%	B3	Ba1	Aaa	5	10
	10%	18.884447%	5.354719%	0.000000%	Caa1	Ba2	Aaa	5	11
	15%	19.592840%	6.534451%	0.000000%	Caa1	Ba3	Aaa	4	12
	20%	19.463171%	7.263989%	0.000000%	Caa1	Ba3	Aaa	4	12
	25%	18.995706%	7.721397%	0.000000%	Caa1	B1	Aaa	3	13
	30%	18.341710%	7.992136%	0.000000%	Caa1	B1	Aaa	3	13
Mezzanine 1	0%	11.394720%	1.353511%	0.000000%	B2	Baa3	Aaa	5	9
	5%	21.399961%	4.893894%	0.000000%	Caa1	Ba2	Aaa	5	11
	10%	22.943966%	6.803695%	0.000000%	Caa1	Ba3	Aaa	4	12
	15%	22.852486%	7.889667%	0.000000%	Caa1	B1	Aaa	3	13
	20%	22.259781%	8.516296%	0.000000%	Caa1	B1	Aaa	3	13
	25%	21.380532%	8.865924%	0.000000%	Caa1	B1	Aaa	3	13
	30%	20.360323%	9.032550%	0.000000%	Caa1	B1	Aaa	3	13
Mezzanine 2	0%	22.725228%	3.493395%	0.000000%	Caa1	Ba1	Aaa	6	10
	5%	28.465653%	7.302783%	0.000000%	Caa2	Ba3	Aaa	5	12
	10%	28.095817%	8.993519%	0.000000%	Caa2	B1	Aaa	4	13
	15%	27.053290%	9.863077%	0.000000%	Caa2	B1	Aaa	4	13
	20%	25.709773%	10.291226%	0.000000%	Caa2	B2	Aaa	3	14
	25%	24.305498%	10.455733%	0.000000%	Caa2	B2	Aaa	3	14
	30%	22.915767%	10.459085%	0.000000%	Caa1	B2	Aaa	2	14
Equity	0%	48.000992%	9.587291%	0.000000%	Caa3	B1	Aaa	5	13
	5%	42.544079%	12.423675%	0.000000%	Caa3	B2	Aaa	4	14
	10%	38.342252%	13.334949%	0.000000%	Caa3	B3	Aaa	3	15
	15%	35.186348%	13.643940%	0.000000%	Caa3	B3	Aaa	3	15
	20%	32.512138%	13.625975%	0.000000%	Caa2	B3	Aaa	2	15
	25%	30.139237%	13.414820%	0.000000%	Caa2	B3	Aaa	2	15
	30%	27.873922%	13.086515%	0.000694%	Caa2	B2	Aaa	3	14

T-CDX.NA.IG Overlap = 15%		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
		95% Confidence Interval							
Tranche	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.011251%	0.002227%	0.000027%	Aa1	Aaa	Aaa	1	0
	10%	0.191312%	0.070206%	0.020452%	A1	Aa3	Aa1	1	2
	15%	0.623475%	0.300986%	0.116133%	Baa1	A2	A1	2	1
	20%	1.197194%	0.666179%	0.309977%	Baa2	Baa1	A2	1	2
	25%	1.804745%	1.097090%	0.577742%	Baa3	Baa2	Baa1	1	1
	30%	2.392149%	1.550018%	0.878839%	Ba1	Baa3	Baa2	1	1
Senior 1	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.134424%	0.032451%	0.004755%	A1	Aa2	Aaa	2	2
	10%	1.055325%	0.443678%	0.135929%	Baa2	A3	A1	2	2
	15%	2.275176%	1.212722%	0.522800%	Baa3	Baa2	Baa1	1	1
	20%	3.431958%	2.064496%	1.052844%	Ba1	Baa3	Baa2	1	1
	25%	4.362662%	2.859450%	1.610647%	Ba2	Ba1	Baa3	1	1
	30%	5.054675%	3.514993%	2.172276%	Ba2	Ba1	Baa3	1	1
Senior 2	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.370510%	0.094029%	0.019932%	A3	Aa3	Aa1	3	2
	10%	1.906253%	0.842188%	0.292969%	Baa3	Baa2	A2	1	3
	15%	3.519584%	1.932545%	0.883437%	Ba1	Baa3	Baa2	1	1
	20%	4.791753%	2.985050%	1.592498%	Ba2	Ba1	Baa3	1	1
	25%	5.768975%	3.868801%	2.278376%	Ba3	Ba2	Baa3	1	2
	30%	6.501377%	4.537687%	2.888958%	Ba3	Ba2	Ba1	1	1
Mezzanine 1	0%	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	0.654923%	0.184349%	0.032675%	Baa1	A1	Aa2	3	2
	10%	2.602532%	1.206720%	0.435397%	Ba1	Baa2	A3	2	2
	15%	4.411740%	2.519736%	1.172740%	Ba2	Ba1	Baa2	1	2
	20%	5.764947%	3.641828%	1.986936%	Ba3	Ba1	Baa3	2	1
	25%	6.738409%	4.524399%	2.719985%	Ba3	Ba2	Ba1	1	1
	30%	7.415654%	5.202814%	3.347699%	Ba3	Ba2	Ba1	1	1
Mezzanine 2	0%	0.003190%	0.000066%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	1.294393%	0.394259%	0.086668%	Baa3	A3	Aa3	3	3
	10%	3.812102%	1.856113%	0.704604%	Ba2	Baa3	Baa1	2	2
	15%	5.821654%	3.380212%	1.662951%	Ba3	Ba1	Baa3	2	1
	20%	7.173463%	4.628235%	2.604780%	Ba3	Ba2	Ba1	1	1
	25%	8.114726%	5.513690%	3.385017%	B1	Ba2	Ba1	2	1
	30%	8.698559%	6.180231%	4.024002%	B1	Ba3	Ba2	1	1
Equity	0%	0.149221%	0.011426%	0.000878%	A1	Aa1	Aaa	3	1
	5%	3.573227%	1.310222%	0.360762%	Ba1	Baa3	A3	1	3
	10%	6.959689%	3.574067%	1.565295%	Ba3	Ba1	Baa3	2	1
	15%	9.056215%	5.419339%	2.878645%	B1	Ba2	Ba1	2	1
	20%	10.261601%	6.736136%	3.978403%	B2	Ba3	Ba2	2	1
	25%	10.952092%	7.576994%	4.830256%	B2	Ba3	Ba2	2	1
	30%	11.312673%	8.137817%	5.452659%	B2	B1	Ba2	1	2

T-CDX.NA.HY Overlap = 15%		Recovery Rate Following a Beta Distribution							
		95% Confidence Interval							
Tranche	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	0%	0.969604%	0.065144%	0.000000%	Baa2	Aa3	Aaa	5	3
	5%	14.515791%	2.611196%	0.000000%	B3	Ba1	Aaa	5	10
	10%	19.149683%	4.688493%	0.000000%	Caa1	Ba2	Aaa	5	11
	15%	21.316058%	6.240957%	0.000000%	Caa1	Ba3	Aaa	4	12
	20%	22.421762%	7.431312%	0.000000%	Caa1	Ba3	Aaa	4	12
	25%	22.986881%	8.380203%	0.000000%	Caa1	B1	Aaa	3	13
	30%	23.217650%	9.142488%	0.000000%	Caa1	B1	Aaa	3	13
Senior 1	0%	18.156613%	1.364228%	0.000000%	Caa1	Baa3	Aaa	7	9
	5%	34.576961%	7.214274%	0.000000%	Caa3	Ba3	Aaa	6	12
	10%	35.145525%	9.635254%	0.000000%	Caa3	B1	Aaa	5	13
	15%	34.665900%	11.116741%	0.000000%	Caa3	B2	Aaa	4	14
	20%	33.840671%	12.112002%	0.000000%	Caa3	B2	Aaa	4	14
	25%	32.936953%	12.823432%	0.000000%	Caa3	B2	Aaa	4	14
	30%	31.923290%	13.340288%	0.000000%	Caa2	B3	Aaa	2	15
Senior 2	0%	45.041408%	4.239222%	0.000000%	Caa3	Ba2	Aaa	7	11
	5%	43.475625%	9.806184%	0.000000%	Caa3	B1	Aaa	5	13
	10%	41.325146%	11.932866%	0.000000%	Caa3	B2	Aaa	4	14
	15%	39.528500%	13.185343%	0.000000%	Caa3	B3	Aaa	3	15
	20%	37.932680%	13.997972%	0.000000%	Caa3	B3	Aaa	3	15
	25%	36.436688%	14.550161%	0.000000%	Caa3	B3	Aaa	3	15
	30%	35.016822%	14.923642%	0.000000%	Caa3	B3	Aaa	3	15
Mezzanine	0%	70.763477%	8.654814%	0.000000%	Caa3	B1	Aaa	5	13
	5%	52.562539%	12.771883%	0.000000%	Caa3	B2	Aaa	4	14
	10%	47.710293%	14.435209%	0.000000%	Caa3	B3	Aaa	3	15
	15%	44.689177%	15.389409%	0.000000%	Caa3	B3	Aaa	3	15
	20%	42.184795%	15.976891%	0.000000%	Caa3	B3	Aaa	3	15
	25%	40.080667%	16.335181%	0.000000%	Caa3	B3	Aaa	3	15
	30%	38.183519%	16.553490%	0.000000%	Caa3	B3	Aaa	3	15
Equity	0%	90.407560%	17.271014%	0.000000%	Caa3	B3	Aaa	3	15
	5%	65.047558%	18.391343%	0.000000%	Caa3	Caa1	Aaa	2	16
	10%	57.199359%	19.062757%	0.000000%	Caa3	Caa1	Aaa	2	16
	15%	52.364850%	19.401941%	0.000000%	Caa3	Caa1	Aaa	2	16
	20%	48.722373%	19.532034%	0.000000%	Caa3	Caa1	Aaa	2	16
	25%	45.808229%	19.528907%	0.000000%	Caa3	Caa1	Aaa	2	16
	30%	43.208851%	19.441972%	0.000000%	Caa3	Caa1	Aaa	2	16

T-CDX.NA.HY Overlap = 15%		Recovery Rate Following Depending on Default Probability (Equation 29 from Chapter 5)							
		95% Confidence Interval							
Tranche	Default Correlation	Expected Loss			Rating			Notches	
		Upper Bound	Average	Lower Bound	Upper Bound	Average	Lower Bound	+	-
Super Senior	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	1.365100%	0.632672%	0.250781%	Baa3	Baa1	A2	2	2
	10%	4.553025%	2.882483%	1.673916%	Ba2	Ba1	Baa3	1	1
	15%	7.343726%	5.214469%	3.503536%	Ba3	Ba2	Ba1	1	1
	20%	9.538171%	7.202821%	5.203344%	B1	Ba3	Ba2	1	1
	25%	11.225422%	8.834524%	6.683691%	B2	B1	Ba3	1	1
	30%	12.531969%	10.159278%	7.948646%	B2	B2	B1	0	1
Senior 1	**0%**	0.000000%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	5.063332%	2.620842%	1.171207%	Ba2	Ba1	Baa2	1	2
	10%	10.667224%	7.109207%	4.488533%	B2	Ba3	Ba2	2	1
	15%	14.047012%	10.404674%	7.281291%	B3	B2	Ba3	1	2
	20%	16.161663%	12.674614%	9.470888%	B3	B2	B1	1	1
	25%	17.485736%	14.230865%	11.110665%	Caa1	B3	B2	1	1
	30%	18.368816%	15.329583%	12.342348%	Caa1	B3	B2	1	1
Senior 2	**0%**	0.002404%	0.000000%	0.000000%	Aaa	Aaa	Aaa	0	0
	5%	7.948381%	4.368967%	2.086286%	B1	Ba2	Baa3	2	2
	10%	13.906382%	9.584067%	6.132381%	B3	B1	Ba3	2	1
	15%	17.039459%	12.902616%	9.220293%	B3	B2	B1	1	1
	20%	18.798975%	14.996304%	11.432225%	Caa1	B3	B2	1	1
	25%	19.891676%	16.363501%	12.971436%	Caa1	B3	B2	1	1
	30%	20.571068%	17.247324%	14.084871%	Caa1	B3	B3	1	0
Mezzanine	**0%**	0.093461%	0.007560%	0.000416%	Aa3	Aaa	Aaa	3	0
	5%	11.804842%	6.742978%	3.476081%	B2	Ba3	Ba1	2	2
	10%	17.556942%	12.449709%	8.190960%	Caa1	B2	B1	2	1
	15%	20.199224%	15.585058%	11.410920%	Caa1	B3	B2	1	1
	20%	21.672094%	17.399221%	13.488882%	Caa1	B3	B3	1	0
	25%	22.460265%	18.543509%	14.937662%	Caa1	Caa1	B3	0	1
	30%	22.881158%	19.249903%	15.863439%	Caa1	Caa1	B3	0	1
Equity	**0%**	4.854923%	1.357657%	0.290676%	Ba2	Baa3	A2	2	4
	5%	20.855676%	13.435316%	7.807576%	Caa1	B3	B1	1	2
	10%	24.852140%	18.558978%	13.050985%	Caa2	Caa1	B2	1	2
	15%	26.334527%	20.913712%	15.920215%	Caa2	Caa1	B3	1	1
	20%	26.920490%	22.104997%	17.597605%	Caa2	Caa1	Caa1	1	0
	25%	27.063706%	22.706620%	18.585520%	Caa2	Caa1	Caa1	1	0
	30%	26.927483%	22.985234%	19.166473%	Caa2	Caa1	Caa1	1	0