HYSTERESIS PROVIDES SELF-ORGANIZATION IN A PLASMA MODEL

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Abstract. The magnetosphere is a multi-scale spatio-temporal complex dynamical system. Self-organization is a possible solution to the seemingly contradicting observation of the repeatable and coherent substorm phenomena with underlying complex behavior in the plasma sheet. Self-organization, through spatio-temporal chaos, emerges naturally in a plasma physics model with sporadic dissipation.

Keywords: self-organization, space plasmas, magnetospheric dynamics

1. Introduction

There is mounting evidence that plasmas can demonstrate very complex behavior, that includes multi-scale dynamics, emergence and self-organization, phase transitions, turbulence, spatio-temporal chaos, etc. (Lu, 1995; Carreras *et al.*, 1996; Biskamp, 2000).

In the magnetosphere, there are two seemingly contradicting observations: (a) the magnetotail plasma sheet appears to be a dynamic and turbulent region (Borovsky *et al.*, 1997; Ohtani *et al.*, 1998), and (b) the substorm cycle seems coherent and repeatable with identifiable distinct phases (Baker *et al.*, 1999) and predictable geomagnetic indices (Vassiliadis *et al.*, 1995; Valdivia *et al.*, 1996, 1999).

We suggest that these seemingly contradicting statements may be reconciled by proposing that the plasma sheet is driven into a non-equilibrium self-organized (SO) "global" state (Chang, 1999), as suggested initially by Chang (1992), that is characterized by critical behavior with scale invariant events, self-similar spatial structure, and multi-fractal topology. This paradigm is in sharp contrast to the standard picture of plasma sheet transport with laminar earthward flow in a well-ordered magnetic field. Instead they are more consistent with the presence of elementary transport events, probably bursty bulk flows (Baumjohan *et al.*, 1990; Angelopoulos *et al.*, 1992), that are accelerated in local reconnection regions (see Figure 1a).

There is mounting evidence that such a SO state occurs in the magnetosphere. Consolini (1997) found a power-law distribution of burst strength in the AL index.

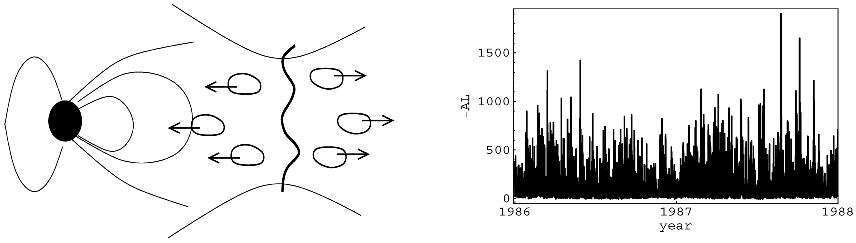

Figure 1. (a) Conceptual view of the complex magnetosphere. (b) AL time series.

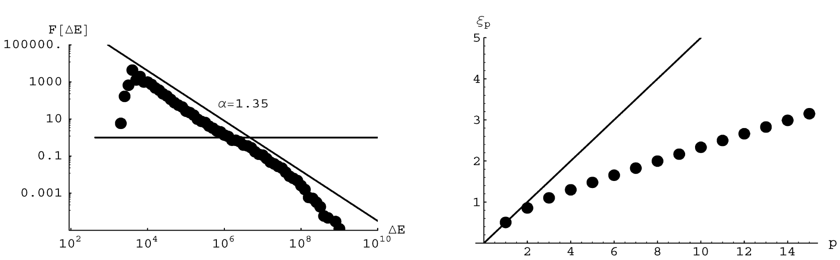

Figure 2. (a) The energy release distribution for the proxy AL^2. (b) ξ_p.

For the 2 years of the AL index shown in Figure 1b, let us use AL^2 as a rough proxy for the energy dissipation rate. Obviously, this is not correct, for we do not have the conductivity nor the effective area of dissipation. Still, we computed the event distribution of the energy dissipated ΔE, when $AL^2 > (50\,nT)^2$ (see Figure 2a). If we assume $P(\Delta E \,|\, \alpha) = \Delta E^{-\alpha}/\zeta(\alpha)$, with $\zeta(\alpha)$ as the Riemann zeta function, and apply a Bayesian argument to the measured sequence, we estimate $\alpha \sim 1.35$ from the maximum of (Goldstein *et al.*, 2004)

$$P(\alpha \,|\, (\Delta E)) \sim \Pi_i P(\Delta E_i \,|\, \alpha) = e^{-\alpha \sum_i^N \ln(\Delta E_i) - N\zeta(\alpha)}$$

independently of the binning process (we assume a smooth prior $P(\alpha)$). A nonlinear exponent ξ_p with p, in the structure function

$$\langle \,|\, AL^2(t+\tau) - AL^2(t) \,|\,^p \rangle \sim \tau^{\xi_p}$$

is also a good indication of the intermittent multi-fractal dissipation in the spatio-temporal system, as suggested by Figure 2b.

Given that we are dealing with a complex spatio-temporal system, the analysis of the single time series representation suggest just the possibility of a SO state. An analysis of the spatio-temporal ionospheric energy dissipation from Polar UVI images also found power-law distributions (Lui *et al.*, 2000; Uritsky *et al.*, 2003). For the case of actual measurements in the tail, we can mention the work of Angelopoulos *et al.* (1999) that studied the nature of the intermittent properties of the BBFs. More detailed arguments in favor of this SO state can be found in Klimas *et al.* (2000) and Valdivia *et al.* (2005).

If the plasma sheet is in a SO state, then understanding self-organization may be the key to understand the substorm evolution. Even though the SO state is a dynamical state in nature with a superimposed unpredictable behavior, its "global" structure is inevitable and repeatable (this is true of sandpile systems as well (Bak *et al.*, 1987)). Thus, we are led to study substorm phenomena as an ensemble of multi-scale dissipation and flow burst events in the turbulent plasma sheet under the assumption that it can reach a global SO state (see Figure 1a).

2. Modeling

We go beyond sandpile analogues (e.g., Takalo *et al.*, 1999) to develop plasma physics models that evolve naturally into a SO state. Take

$$\left(\frac{\partial \mu}{\partial t} + \frac{U_j \partial \mu}{\partial x_j}\right) = -\mu \nabla \times \mathbf{U}$$

$$\mu\left(\frac{\partial \mathbf{U}}{\partial t} + \frac{U_j \partial \mathbf{U}}{\partial x_j}\right) = \mathbf{J} \times \mathbf{B} - \nabla P + \nu \nabla^2 \mathbf{U}$$

$$\left(\frac{\partial P}{\partial t} + \frac{U_j \partial P}{\partial x_j}\right) = -\gamma P \nabla \cdot \mathbf{U} + (\gamma - 1)\mathbf{J} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) - \nabla \mathbf{Q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} + \mathbf{U} \times \mathbf{B} = \eta \mathbf{J} + \alpha_1 \mathbf{J} \times \mathbf{B} + \alpha_2 \frac{\partial \mathbf{J}}{\partial t} + \alpha_3 \nabla \mathbf{P_e} + \cdots \tag{1}$$

For now let $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\nabla \mathbf{Q} = 0$, and $B = \nabla \times A$. Klimas *et al.* (2000) and Valdivia *et al.* (2003) derived from the plasma equations, but using anomalous localized dissipation, a continuous 1D model of magnetic annihilation that displays self-similar event behavior, reminiscent of a SO state. Indeed, simplifying $\mathbf{A} = A_y(z, t)\hat{y}$ we obtain

$$\frac{\partial A_y}{\partial t} = \eta \frac{\partial^2 A_y}{\partial z^2} + S(z, t), \quad J = \frac{-\partial^2 A_y}{\partial z^2} \tag{2}$$

with $S = (\mathbf{U} \times \mathbf{B})_y$, which provides the starting point to simulate a SO state by incorporating the localized dissipation (Lu, 1995)

$$\frac{d\eta}{dt} = \frac{(q(J) - \eta)}{\tau}, \quad q(J) = \begin{bmatrix} \eta_{\max} & |\,J\,| > J_c \\ \eta_{\min} & |\,J\,| < \beta J_c \end{bmatrix} \tag{3}$$

with a hysteretic trigger function q having two states. At a given position, q will transition from $q = \eta_{\min}$ to $q = \eta_{\max}$ when $|\,J\,| > J_c$, but will not transition to the low state $q = \eta_{\min}$ until $|\,J\,| < \beta J_c$ ($\beta < 1$). This 1D model displays self-similar

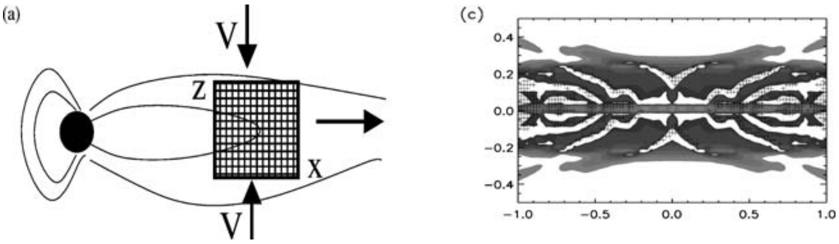

Figure 3. (a) The 2D geometry. (b) The 2D current density J at a given time. $\beta = 0.9$.

behavior through spatio-temporal chaos for a range of conditions (Klimas *et al.*, 2000; Valdivia *et al.*, 2005).

As explained in Valdivia *et al.* (2003, 2005), this model represents the dynamics of magnetic field in the diffusion region of the magnetotail. As we add the plasma evolution in a 2D model, given by Equation (1) but including the dynamical η, we found that the annihilated magnetic energy is transferred to the plasma in an intermittent manner, generating bursts through a $J \times B$ force at the localized dissipation regions (see Valdivia *et al.*, 2005 for details). Figure 3b displays the evolution of Equation (1) in 2D at some particular time t. The geometry of the system, represented in Figure 3a, is similar to that of Ugai and Tsuda (1977) with a symmetric system at both $x = 0$ and $z = 0$, and an imposed constant inflow z-velocity $U_{z,o}$ and magnetic field $B_{x,o}$ at $z = \pm L_z$. We have outgoing conditions at $x = \pm L_x$ (see Valdivia *et al.*, 2003, 2005, for more details). We can already see from Figure 3 that even though we have strong underlying turbulence, there is a well-defined global state that permits the dissipation and transport of energy through the system, but in a bursty fashion (see also Klimas *et al.*, 2004). This picture is very reminiscent of the behavior expected by Antonova *et al.* (1999).

Even though we are treating the plasma sheet as a magnetofluid, the spatially-dependent η brings the necessary intermittent dissipation that is not present in regular MHD. We use this nonlinear resistivity in an attempt to characterize some of the complex microphysics behavior, with q acting like a physical current driven instability (Papadopoulos *et al.*, 1985) with a threshold J_c that is higher than the value required to maintain the instability. Indeed, Chang *et al.* (2004) review a physical microscopic behavior that may produce hysteresis as a spatio-temporal coarse-grained dissipation. Here, we are concerned with the event statistics of the collective effects of many such interacting instability sites, derived from observations and data analysis, in a complementary manner to microphysics. Furthermore, the introduction of the hysteretic loop is crucial in the generation of the loading–unloading mechanism that produces intermittent behavior, and is present in virtually all SO models including sandpiles. If we take $\beta = 1$, then η relaxes very quickly after q is turned on, hence we destroy loading–unloading cycle. If we make $\beta \ll 1$, we give more time for $\eta_{\min}$ to smooth the spatial profile of A during the driving

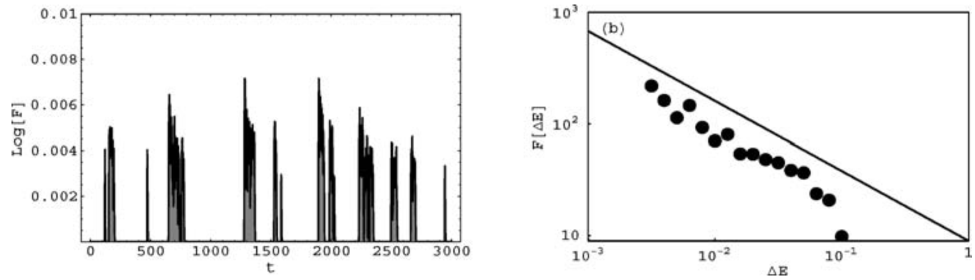

Figure 4. (a) $F(t)$. (b) Event distribution of energy dissipated.

time, favoring a quasi-periodic evolution (Valdivia *et al.*, 2005). It is important to stress that hysteresis is a natural phenomena that appears even in the simplest of systems, e.g., the constantly driven pendulum (Ott, 1993) which can be used as a starting point for a simulation of the charge dynamics in a slowly varying magnetic field. Before tackling this 2D model (to be published elsewhere), we propose to go back to the 1D model, which is more manageable and study its spatio-temporal chaos and multi-scale behavior.

As an illustration, let Us take $-L \leq z \leq L$, with $L = 20$, $\Delta x = 0.1$, $S(z) = S_0 \cos(\pi z/2L)$, $J_c = 0.04$, $\eta_{\max} = 5$ (normalized to $c^2 L V_{\mathrm{a}}/4\pi$), V_{a} a reference Alfven's speed, $\tau = 1$, and $J = 0$ at the boundaries. The dissipation rate $F(t) = \int \eta(x,t) J(x,t)^2 dx$ is shown in Figure 4a for $S_0 = 0.001$, $\beta = 0.9$, and $\eta_{\min} = 0$. In Figure 4b, we computed the event distribution of energy dissipated, in which an event is defined for $F > F_{\min} = 10^{-5}$. Using the technique discussed earlier, we estimated a power-law index of $\alpha \approx 0.6$. Clearly, if $S_0 < J_c \eta_{\min}$ or $S_0 > J_c \eta_{\max}$, we can have a steady-state solution. The bifurcation diagram with S_0 depends strongly on $\eta_{\min}$, $\eta_{\max}$, and β, and is illustrated in Figure 5:

1. For $J_c \eta_{\min} < S_0 < S_p$, we can have a quasi-periodic situation (see Valdivia *et al.*, 2005 for an example using $\beta = 0.5$). This regime depends on the ratio $\eta_{\min}/\eta_{\max}$ and β (Tangri *et al.*, 2003).
2. For $S_p < S_0 < S_c$, we have a SO state, with a well-defined global B_x and intermittent dissipation with self-similar statistics. As $S_0 \to S_c$, the duration of loading and unloading cycles become the same. The time duration distribution follows a power-law with $\alpha \approx 1.4$ and suggests an explanation for the distribution of BBFs observed by Angelopoulos *et al.* (1999).

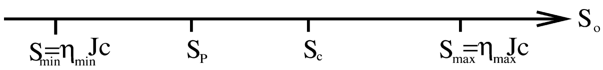

Figure 5. The phase diagram with S_0.

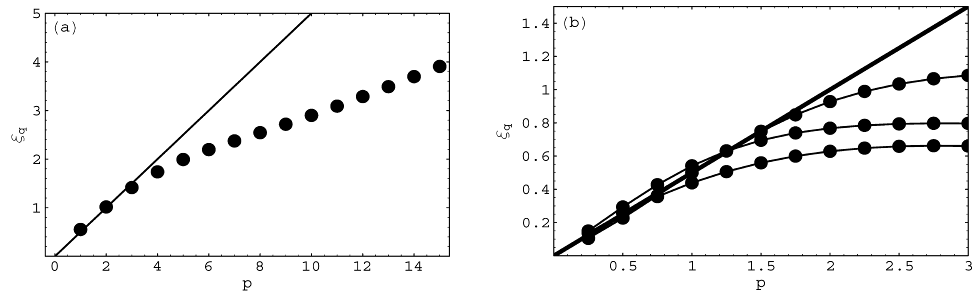

Figure 6. Singularity analysis of (a) $F(t)$ and (b) ηJ^2 in space, at three instances.

3. For $S_c < S_0 < J_c \eta_{\max}$, we have a chaotic behavior but without the loading–unloading cycle, as the separation tends to zero as $S_0 \rightarrow S_c$.
4. For $S_0 > J_c \eta_{\max}$, the system responds directly to the driver. It is important not to over extrapolate, but the transition at $S_0 \sim J_c \eta_{\max}$ seems like a first-order phase transition, and may explain the observation of Sitnov *et al.* (2000) and Uritsky *et al.* (2002).

Whether each of these behaviors is actually displayed by the magnetosphere remains to be determined, but it is suggestive to mention the following: (1) saw-tooth-like oscillations, (2) turbulent self-similar evolution, (3) directly driven state, and (4) steady magnetospheric convection.

A singularity analysis of F is shown in Figure 6a for the time series of Figure 4a. We note that there is a clear multi-fractal behavior, and that it is strikingly similar to Figure 2b. The singularity analysis can also be applied to the spatial dependence of the dissipation ηJ^2, as illustrated in Figure 6b at three different instants during the same dissipation event.

3. Conclusions and Outlook

In the magnetosphere, the robust SO state is a possible solution to the seemingly contradicting observations of the repeatable and coherent substorm phenomena with underlying complex behavior in the plasma sheet. This work suggests that hysteresis may have an important role in the self-similar behavior of the magnetotail. Even though the exact details of the microphysics (ballooning, cross-field current, variant of tearing, etc.) may not be accounted by the simple parameterization, it is expected that the statistical behavior of complex distributed systems is more a property of their SO state than the details of the physical processes that allow such state. This is a general characteristic of systems that are close to criticality where many systems belong to the same universality class, suggesting that it is probable that the statistics of substorms, pseudobreakups, and even the evolution of the growth and expansion phases are unrelated to the details of the dissipation process (Shay *et al.*, 1998) other than the dissipation allowed for the establishment of a SO state. Even though, the 2D model is clearly a more appealing description of the intermittent dissipation in the magnetotail, the 1D model is more manageable and permits a more comprehensive study of the parameter space. Furthermore, some of the parameters of the model may be estimated from actual measurements, e.g., plasma sheet eddy diffusion coefficient (Borovsky *et al.*, 1997).

The intriguing spatio-temporal multi-fractal chaotic behavior of the 1D model needs to be characterized in detail. For now, it is interesting to note that the system described by Equation (3) can be discretized as an $n = L/\Delta x \geq 1$ dimensional system (the local instability size becomes a fourth relevant parameter). For $n = 1$, we have a simple nonlinear oscillator, and as we increase n, the system can become spatio-temporal chaotic through a nonstandard transition (Ott, 1993) that needs to be studied in detail. Finally, the bifurcation diagram will become even more interesting as the driver $S(t)$ is made stochastic.

Acknowledgements

We appreciate financial support from FONDECyT under contract nos. 1030727, 3060029, 1060830 and 1030957, and fruitful discussion with A. Klimas and V. Uritsky.

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