Anderson localization in a periodic photonic lattice with a disordered boundary

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We investigate experimentally the light evolution inside a two-dimensional finite periodic array of weakly-coupled optical waveguides with a disordered boundary. For a completely localized initial condition away from the surface, we find that the disordered boundary induces an asymptotic localization in the bulk, centered around the initial position of the input beam.

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The original concept of Anderson localization assumes a periodic structure where disorder is introduced via a random change of the local properties at each site of the lattice, leading to wave localization due to interference between multiple scattering paths [1]. Although it was first described in the context of condensed matter [1,2], there are now examples in many other fields: Acoustics [3], microwaves [4], Bose-Einstein condensates [5] and optics [6–10], to name a few. Most importantly, in the optical domain it was proven in experiment and theory that a random displacement of the lattice sites yields the same localization results as changing the local properties of the individual sites [11,12].

In rough or corrugated channels, the phenomenon of Anderson localization is connected to the transmission of electrons or optical pulses. In these systems, the channel surface is disordered along the propagation direction. Theoretical studies showed transitions between diffusive and localized regimes [13–15]. Transport behavior was also considered in [16–18], where multiple scattering from longitudinal surface roughness caused localization of waves, although regimes of coexistence between ballistic, diffusive and localized transport, depending on the symmetry of the corrugation profile, have been predicted [19].

Recently, a different kind of "corrugated waveguide" was considered theoretically [20]. Here the disorder is only in the transverse direction, and does not change along the direction of propagation. Also, the optical medium in the bulk, away from the corrugated surface, possesses a periodic index of refraction along the two transversal directions, thus forming an array of weakly coupled waveguides [21]. The disorder is imposed on the boundary by random displacement. Despite the weak disorder, a light beam still tends to localize in the center of such a system far from the boundary. In our work, we experimentally prove this prediction.

Let us consider a 2D rectangular $N \times M$ waveguide array (Fig.1). In the coupled-mode approach, the electric field $E(\mathbf{r}, z)$ propagating along the waveguides can be written as a superposition of the waveguide modes, $u(\mathbf{r}, z) = \sum_{\mathbf{n}} u_{\mathbf{n}}(z)\phi(\mathbf{r}-\mathbf{n})$, where $\mathbf{r} = (x, y)$, $u_{\mathbf{n}}$ is the amplitude of the single modes $\phi_{\mathbf{n}}$ centered around site $\mathbf{n} = (p, q)$. The evolution equations for the mode amplitudes $u_{\mathbf{n}}$ are

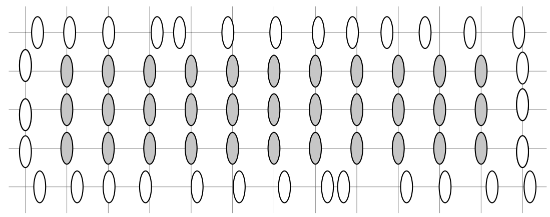

Fig. 1. Scheme of a finite two-dimensional coupled array of elliptical waveguides with disordered boundary.

$$\sum_{j=p\pm1} C_{v_j} u_{j,q} + \sum_{j=q\pm1} C_{h_j} u_{p,j} = -i\frac{du_{pq}}{dz}\ , \qquad (1)$$

where $\mathbf{n} = (p, q)$ denotes the position of the guide center, z is the longitudinal distance, and the $C_{v,h}$ are the coupling coefficients between nearest-neighbors guides. They decay exponentially with the mutual distance between the guides. To keep our approach general, we assume anisotropic coupling in the horizontal and vertical direction. At the boundary of the array, randomness is introduced by displacing the guides from their (ordered) positions, along the boundary surface. This creates random couplings among the boundary guides, for the horizontal coupling $C_h \to C_h e^{-\omega_h \Delta}$ as well as for the vertical coupling $C_v \to C_v e^{-\omega_v \Delta}$. The quantity Δ is a random number in $[-1, 1]$ and the randomness strengths ω_h and ω_v along the horizontal and vertical boundaries respectively are different due to the ellipticity of the guides. The bulk, that represents the ordered part of the array, is connected to the disordered boundary layer with coupling values computed from the mean pythagorean distance. These conditions are close to the possibilities of experimental realization [12]. The entire system could be viewed as a single “corrugated” photonic crystal waveguide, with fixed transversal corrugation.

First, in order to illustrate the localization mechanisms in the system under consideration, we analyze a waveguide array with 13×5 waveguides and disordered boundary and perform a direct numerical integration of Eq. (1) with a single-site excitation at the array center $(n_c, m_c) = (7, 3)$. The parameters chosen for the simulation of the disordered case (left column of Fig. 2) are: $C_h = 2$, $C_v = 1.7$, $\omega_h = 11/28$, $\omega_v = 4/17$ [22]. For comparison, we compute the evolution for the ordered case and its pure discrete diffraction (right column of Fig. 2). For small propagation distances ($z \simeq 5$), both systems show discrete diffraction with maxima at the outer lobes. The amplitude of the initially excited site decreases steadily with z. However, after a transition distance of $z_t \simeq 10$, that is necessary for a complete backscattering cycle, the influence of the boundary layer becomes apparent. In the case of the disordered boundary, for $z > 10$, the amplitude of the center site is non-vanishing for all propagation lengths. It continues to oscillate with z, but finally saturates to a nearly stationary mode, showing the persistence and stability of localization due to the disordered boundary. The main difference to the case of a completely disordered system (bulk + surface) is that the initial oscillations due to discrete diffraction are stronger.

For our experiments, we fabricated various waveguide arrays in polished bulk fused silica glass, using the laser direct-writing technology [23]. Each guide has dimensions of $4 \times 12\ \mu\text{m}^2$ and exhibits a refractive index change of $\approx 5 \times 10^{-4}$ [22]. We prepared 60 disordered-boundary waveguide arrays with a length of 101 mm, with either $N \times M = 5 \times 5$ or with $N \times M = 13 \times 5$ waveguides each. The inter-guide separation in the ordered bulk, which is also the mean separation at the (disordered) boundary, was of 14 μm and 17 μm, along the horizontal and vertical directions, respectively. Disorder in the waveguide spacing was induced by varying the inter-guide distance: $d_h = 14 \pm 5.5\Delta\ \mu$m and $d_v = 17 \pm 4\Delta\ \mu$m, with $\Delta \in [-1, 1]$ randomly equally distributed.

In each array the individual central waveguide was excited using a Ti:Sapphire laser system at low input power to ensure linear propagation. At the end facet, the intensity patterns were recorded with a CCD camera. For the observation of localization in the 5×5 arrays, a wavelength of 800 nm was used, corresponding to coupling coefficients [22] of $C_h(C_v) \simeq 2.0(1.7)\ \text{cm}^{-1}$. To enhance localization in the arrays of 13×5 waveguides, the wavelength was increased to 840 nm, leading to higher coupling coefficients and therefore decreased coupling lengths. This allows the observation of localization effects at effectively shorter evolution distances.

The experimental results, shown in Figs. 3 and 4, demonstrate a clear localization tendency around the position of the input beam, for both, 5×5 (Fig. 3) and 13×5 (Fig. 4) arrays. Figure 3(a) shows the output intensity at the end facet, averaged over 30 different boundary-disorder realizations. The exponential localization behavior of the light intensities in the horizontal direction is shown and clearly seen in Fig. 3(b). Since the global coupling in the vertical direction is weaker, the decay here is remarkably stronger (Fig. 3(c)).

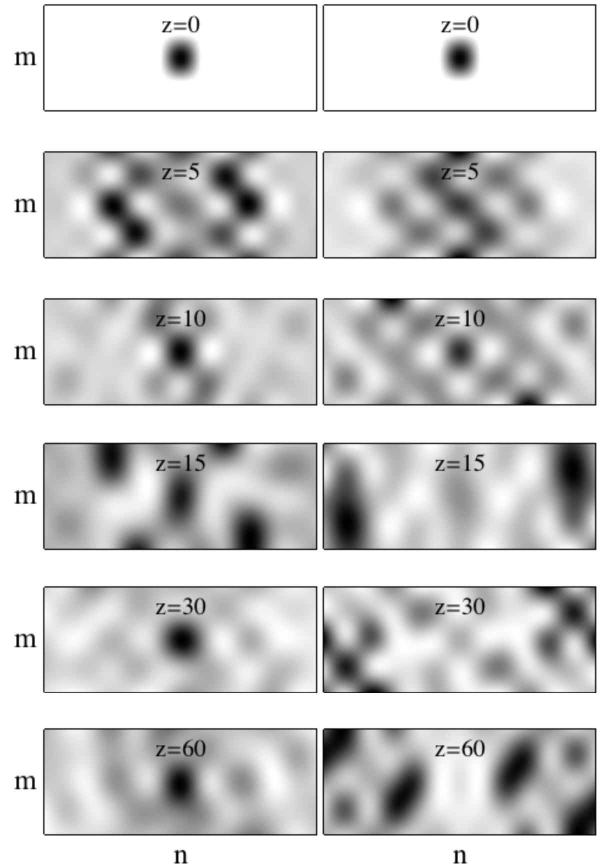

Fig. 2. Comparison between simulations of an ordered 13×5 array (right column), and the disordered boundary waveguide array)of eq. (1) (left column) with $C_\alpha = 2$, $C_\beta = 1.7$, $\omega_\alpha = 11/28$, $\omega_\beta = 4/17$, averaged over 100 realizations.

A microscopic image of a single realization of the 13×5 disordered boundary array is shown in Fig. 4(a), where the ordered center as well as the boundary layer with disordered spacing can be seen. For comparison we also show a microscopic image of the completely ordered 13×5 waveguide array. In Fig. 4(c), the mean output intensity of 30 realizations is shown, where localization on the central input site is clearly observed. When comparing with the intensity distribution in an ordered array (Fig. 4(d)) we find that the propagation distance is sufficiently long for complete delocalization in the ordered case. Thus, the observed localization effects can be attributed to the disordered boundary layer. The stronger

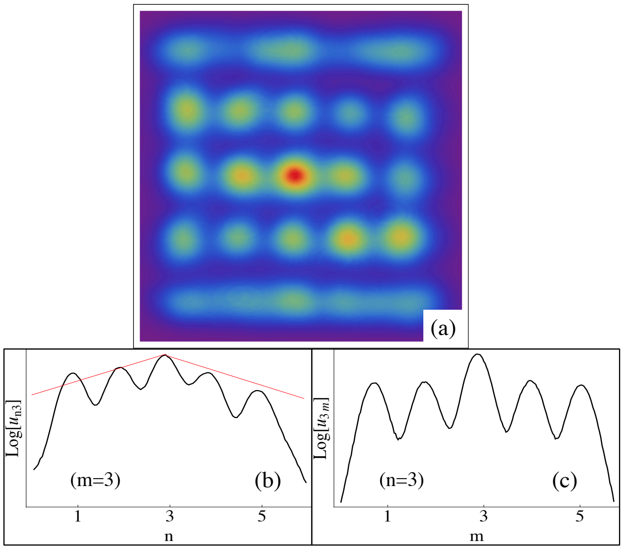

Fig. 3. Experimental results for the 5×5 arrays: (a) mean output of 30 realizations, $\lambda = 800\ nm$, (b), (c) logarithmic plots of $|u_{n,m_c}|$, $|u_{n_c,m}|$ respectively.

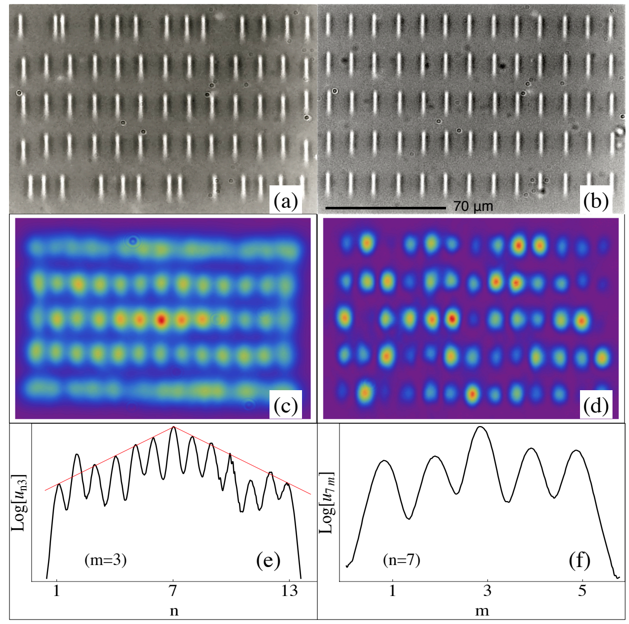

Fig. 4. Experimental results for the 13×5 arrays: (a) Microscope image of one configuration of the disordered boundary, (b) ordered array, (c) mean output of 30 realizations of the disordered boundary waveguide array, $\lambda = 840\ nm$ (d) output of the ordered array, (e), (f) logarithmic plots of $|u_{n,m_c}|$, $|u_{n_c,m}|$.

coupling obtained in the 13×5 array due to the longer wavelength leads to better exponential localization in both coupling directions, shown in Figs. 4(e) and 4(f).

In summary, we have investigated experimentally the influence of a disordered boundary in a finite 2D coupled waveguide array and found asymptotic partial localization of the wave packet in the center of the bulk region far away from the boundary. We conclude that the presence of a disordered boundary can give rise to Anderson localization in regions away from the boundary. We conjecture that this result could be extrapolated to larger finite lattices, if one allows for a sufficiently long propagation distance.

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