Scattering of a surface plasmon polariton by a localized dielectric surface defect

Rodrigo E. Arias[1*] **and Alexei A. Maradudin**[2]

[1]*Departamento de Física*
Universidad de Chile, FCFM, Santiago, Chile
[2]*Department of Physics and Astronomy*
University of California, Irvine CA 92697, U.S.A.

[*]*rarias@dfi.uchile.cl*

Abstract: On the basis of a rigorous, nonperturbative, purely numerical solution of the corresponding reduced Rayleigh equation for the scattering amplitudes we have studied the scattering of a surface plasmon polariton by a two dimensional dielectric defect on a planar metal surface. The profile of the defect is assumed to be an arbitrary single-valued function of the coordinates in the plane of the metal surface, and to be differentiable with respect to those coordinates. When the defect is circularly symmetric and the dependence of the scattering amplitudes on the azimuthal angle is expressed by a rotational expansion, the reduced Rayleigh equation is transformed into a pair of one-dimensional integral equations for each value of the rotational quantum number. This approach is applied to a defect in the form of an isotropic Gaussian function. The differential cross sections for the scattering of the incident surface plasmon polariton into volume electromagnetic waves in the vacuum above the surface and into other surface plasmon polaritons are calculated, as well as the intensity of the field near the surface. These results differ significantly from the corresponding results for a metallic defect on a metallic substrate.

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A surface plasmon polariton incident on a surface defect is partly scattered into other surface plasmon polaritons and is partly converted into volume electromagnetic waves in the vacuum above the surface. The scattering out of the beam caused by the presence of surface defects decreases the propagation length of the surface plasmon polariton, and it is important for applications of these surface electromagnetic waves to be able to calculate the cross section for such scattering. At the same time surface defects of particular forms and sizes can scatter surface plasmon polaritons in desirable ways, e.g. they can act as mirrors for surface plasmon polaritons or as flashlights, and can focus them as well [1–10]. To exploit the possibilities this offers it is also necessary to be able to calculate the scattering of surface plasmon polaritons by surface defects.

Until now the great majority of such calculations have been carried out for scattering by one-dimensional defects, viz. grooves and ridges. In contrast the scattering of a surface plasmon polariton by a localized two-dimensional surface defect has been little studied. The scattering from indentations (dimples) or protuberances formed from the same metal as the substrate has been studied by a rigorous approach [11]. The scattering from a dielectric rectangular parallelepiped on the planar surface of metallic film in the Kretschmann attenuated total reflection geometry [12] has been studied by a rigorous approach [13]. The scattering from a dielectric defect in the shape of an anisotropic Gaussian and of an anisotropic hemiellipsoid on the planar surface of a semi-infinite metal has been calculated with the use of an effective boundary condition [14]. The interaction of a surface plasmon polariton with a spatially localized dielectric surface defect is of experimental interest [14], and deserves additional study. Such a study is presented in this paper.

Our treatment is based on the reduced Rayleigh equation for the electric field in the vacuum

region above the metal surface and the dielectric defect on it. It is applicable to defects defined by single-valued differentiable profile functions, is rigorous for defects with small slopes, and is computationally tractable. It has worked well in a recent theoretical study of this scattering of light from a planar metal surface coated with a dielectric film whose interface with the vacuum above it is a two-dimensional randomly rough interface [15]. We expect that it will work equally well when the incident volume wave is replaced by a surface electromagnetic wave, and the rough dielectric film is replaced by a localized dielectric defect.

The physical system we consider consists of vacuum (ε_1) in the region $x_3 > \zeta(\mathbf{x}_\parallel)$, where $\mathbf{x}_\parallel = (x_1, x_2, 0)$, a dielectric medium whose dielectric constant is ε_2 in the region $0 < x_3 < \zeta(\mathbf{x}_\parallel)$, and a metal whose dielectric function is ε_3, in the region $x_3 < 0$. The surface profile function $\zeta(\mathbf{x}_\parallel)$ is a non-negative, single-valued function of $\mathbf{x}_\parallel$, that is differentiable with respect to x_1 and x_2, and is sensibly nonzero within a region of radius R on the metal surface $x_3 = 0$. It therefore defines a dielectric defect with a finite footprint on a planar metal surface. We assume that the dielectric function ε_3 is real, because the mean free path of a surface plasmon polariton on the planar portion of the vacuum-metal interface is significantly longer than the linear dimensions of the surface defect.

A surface plasmon polariton of frequency ω is incident on the defect from the region $x_1 < -R$ of the plane $x_3 = 0$, where we have a planar interface between media 1 and 3 at $x_3 = 0$. The total electric field in the region $x_3 > \zeta(\mathbf{x}_\parallel)$ is the sum of the incident field and the scattered field,

$$
\begin{aligned}
\mathbf{E}(\mathbf{x}|\omega) = {} & \frac{c}{\omega}[-i\beta_1(k_\parallel)\hat{\mathbf{k}}_\parallel + k_\parallel\hat{\mathbf{x}}_3]E_{op}(\mathbf{k}_\parallel)\exp[i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel - \beta_1(k_\parallel)x_3] \\
& + \int \frac{d^2q_\parallel}{(2\pi)^2}\left\{\frac{c}{\omega}[i\beta_1(q_\parallel)\hat{\mathbf{q}}_\parallel - q_\parallel\hat{\mathbf{x}}_3]\frac{a_p(\mathbf{q}_\parallel)}{\varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel)} \right. \\
& \left. + (\hat{\mathbf{x}}_3 \times \hat{\mathbf{q}}_\parallel)\frac{a_s(\mathbf{q}_\parallel)}{\beta_1(q_\parallel) + \beta_3(q_\parallel)}\right\}\exp[i\mathbf{q}_\parallel \cdot x_\parallel - \beta_1(q_\parallel)x_3].
\end{aligned}
\tag{1}
$$

In obtaining this equation a time dependence of the field of the form $\exp(-i\omega t)$ has been assumed but not indicated explicitly. The two-dimensional wave vector $\mathbf{k}_\parallel$ is given by

$$
\mathbf{k}_\parallel = k_\parallel(\cos\phi_0, \sin\phi_0, 0),
\tag{2a}
$$

where

$$
k_\parallel = \frac{\omega}{c}\left(\frac{\varepsilon_1\varepsilon_3}{\varepsilon_1 + \varepsilon_3}\right)^{\frac{1}{2}}
\tag{2b}
$$

is the wavenumber of the surface plasmon polariton of frequency ω at the planar interface between vacuum (ε_1) and a metal (ε_3). It is the solution of the equation $\varepsilon_3\beta_1(k_\parallel) + \varepsilon_1\beta_3(k_\parallel) = 0$. The angle ϕ_0 is the azimuthal angle of incidence of the surface plasmon polariton, measured counterclockwise from the positive x_1 axis. The functions $\beta_j(q_\parallel)$ ($j = 1, 2, 3$) are defined by

$$
\beta_j(q_\parallel) = [q_\parallel^2 - \varepsilon_j(\omega/c)^2]^{\frac{1}{2}}, \, Re\beta_j(q_\parallel) > 0, Im\beta_j(\mathbf{q}_\parallel) < 0.
\tag{3}
$$

A caret over a vector indicates that it is a unit vector. Finally, $a_{p,s}(\mathbf{q}_\parallel)$ are the amplitudes of the p- and s-polarized components of the scattered field with respect to the local scattering plane defined by the vectors $\hat{\mathbf{x}}_3$ and $\hat{\mathbf{q}}_\parallel$. The amplitudes $a_p(\mathbf{q}_\parallel)$ and $a_s(\mathbf{q}_\parallel)$ satisfy the pair of coupled

reduced Rayleigh equations [17]:

$$
a_p(\mathbf{p}_\parallel) - \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2\beta_2(p_\parallel)} \int \frac{d^2q_\parallel}{(2\pi)^2} \Bigg\{ (\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_2(p_\parallel))
$$

$$
\times [p_\parallel q_\parallel - \beta_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\beta_1(q_\parallel)] \frac{J^{(+)}(\beta_2(p_\parallel) - \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)}
$$

$$
+ (\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel))[p_\parallel q_\parallel + \beta_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\beta_1(q_\parallel)]
$$

$$
\times \frac{J^{(-)}(\beta_2(p_\parallel) + \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) + \beta_1(q_\parallel)} \Bigg\} \frac{a_p(\mathbf{q}_\parallel)}{\varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel)}
$$

$$
- \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2\beta_2(p_\parallel)} \int \frac{d^2q_\parallel}{(2\pi)^2} \Bigg\{ -i\frac{\omega}{c}\beta_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)_3
$$

$$
\times [(\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_2(p_\parallel))\frac{J^{(+)}(\beta_2(p_\parallel) - \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)}
$$

$$
- (\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel))\frac{J^{(-)}(\beta_2(p_\parallel) + \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) + \beta_1(q_\parallel)}]
$$

$$
\times \frac{a_s(\mathbf{q}_\parallel)}{\beta_3(q_\parallel) + \beta_1(q_\parallel)}
$$

$$
= - \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2\beta_2(p_\parallel)} \Bigg\{ (\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_2(p_\parallel))[p_\parallel k_\parallel - \beta_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\beta_1(k_\parallel)]
$$

$$
\times \frac{J^{(+)}(\beta_2(p_\parallel) - \beta_1(k_\parallel)|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\beta_2(p_\parallel) - \beta_1(k_\parallel)}
$$

$$
+ (\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel))[p_\parallel k_\parallel + \beta_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\beta_1(k_\parallel)]
$$

$$
\times \frac{J^{(-)}(\beta_2(p_\parallel) + \beta_1(k_\parallel)|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\beta_2(p_\parallel) + \beta_1(k_\parallel)} \Bigg\} E_{op}(\mathbf{k}_\parallel) \tag{4a}
$$

$$
a_s(\mathbf{p}_\parallel) - \left(\frac{\omega}{c}\right)^2 \frac{\varepsilon_2 - \varepsilon_1}{2\beta_2(p_\parallel)} \int \frac{d^2q_\parallel}{(2\pi)^2} i\frac{c}{\omega}(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)\beta_1(q_\parallel)
$$

$$
\times \Bigg[(\beta_3(p_\parallel) + \beta_2(p_\parallel))\frac{J^{(+)}(\beta_2(p_\parallel) - \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)}
$$

$$
+ (\beta_3(p_\parallel) - \beta_2(p_\parallel))\frac{J^{(-)}(\beta_2(p_\parallel) + \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) + \beta_1(q_\parallel)}]
$$

$$
\times \frac{a_p(\mathbf{q}_\parallel)}{\varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel)}
$$

$$
- \left(\frac{\omega}{c}\right)^2 \frac{\varepsilon_2 - \varepsilon_1}{2\beta_2(p_\parallel)} \int \frac{d^2q_\parallel}{(2\pi)^2}(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)
$$

$$
\times [(\beta_3(p_\parallel) + \beta_2(p_\parallel))\frac{J^{(+)}(\beta_2(p_\parallel) - \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)}
$$

$$+(\beta_3(p_\parallel)-\beta_2(p_\parallel))\frac{J^{(-)}(\beta_2(p_\parallel)+\beta_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\beta_2(p_\parallel)+\beta_1(q_\parallel)}]$$

$$\times\frac{a_s(\mathbf{q}_\parallel)}{\beta_3(q_\parallel)+\beta_1(q_\parallel)}$$

$$=-\left(\frac{\omega}{c}\right)^2\frac{\varepsilon_2-\varepsilon_1}{2\beta_2(p_\parallel)}i\left(\frac{c}{\omega}\right)(\hat{\mathbf{p}}_\parallel\times\hat{\mathbf{k}}_\parallel)_3\beta_1(k_\parallel)$$

$$\times[(\beta_3(p_\parallel)+\beta_2(p_\parallel))\frac{J^{(+)}(\beta_2(p_\parallel)-\beta_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\beta_2(p_\parallel)-\beta_1(k_\parallel)}$$

$$+(\beta_3(p_\parallel)-\beta_2(p_\parallel))\frac{J^{(-)}(\beta_2(p_\parallel)+\beta_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\beta_2(p_\parallel)+\beta_1(k_\parallel)}]E_{op}(\mathbf{k}_\parallel), \tag{4b}$$

where

$$J^{(\pm)}(\beta_2(p_\parallel)\mp\beta_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)$$
$$=\int d^2x_\parallel\exp[-i(\mathbf{p}_\parallel-\mathbf{q}_\parallel)\cdot\mathbf{x}_\parallel]\{\exp[(\pm\beta_2(p_\parallel)-\beta_1(q_\parallel))\zeta(\mathbf{x}_\parallel)]-1\}. \tag{5}$$

A derivation of these equations is outlined in the Appendix.

Equations (4a) and (4b) are valid for any localized surface defect whose profile function $\zeta(\mathbf{x}_\parallel)$ satisfies the assumptions about it stated above. However, they simplify significantly when $\zeta(\mathbf{x}_\parallel)$ is a function of $\mathbf{x}_\parallel$ only through its magnitude $x_\parallel$. In this case we introduce the expansions

$$a_{p,s}(\mathbf{p}_\parallel)=\sum_{k=-\infty}^{\infty}a_k^{(p,s)}(p_\parallel)\exp(ik\phi_p), \tag{6}$$

where ϕ_p is the azimuthal angle of the vector $\mathbf{p}_\parallel$, measured counterclockwise from the positive x_1 axis. We also have the expansions

$$J^{(+)}(\beta_2(p_\parallel)-\beta_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)=\sum_{k=-\infty}^{\infty}c_k^{(+)}(p_\parallel|q_\parallel)\exp[ik(\phi_p-\phi_q)] \tag{7a}$$

$$J^{(-)}(\beta_2(p_\parallel)+\beta_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)=\sum_{k=-\infty}^{\infty}c_k^{(-)}(p_\parallel|q_\parallel)\exp[ik(\phi_p-\phi_q)] \tag{7b}$$

where

$$c_k^{(+)}(p_\parallel|q_\parallel)=2\pi\sum_{n=1}^{\infty}\frac{(\beta_2(p_\parallel)-\beta_1(q_\parallel))^n}{n!}$$
$$\times\int_0^{\infty}dx_\parallel x_\parallel\zeta^n(x_\parallel)J_k(p_\parallel x_\parallel)J_k(q_\parallel x_\parallel) \tag{8a}$$

$$c_k^{(-)}(p_\parallel|q_\parallel)=2\pi\sum_{n=1}^{\infty}(-1)^n\frac{(\beta_2(p_\parallel)+\beta_1(q_\parallel))^n}{n!}$$
$$\times\int_0^{\infty}dx_\parallel x_\parallel\zeta^n(x_\parallel)J_k(p_\parallel x_\parallel)J_k(q_\parallel x_\parallel), \tag{8b}$$

and $J_k(z)$ is a Bessel function of the first kind and order k. The equations satisfied by the

amplitudes $\{a_k^{(p,s)}(p_\parallel)\}$ are

$$
a_k^{(p)}(p_\parallel) - \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2\beta_2(p_\parallel)} \int_0^\infty \frac{dq_\parallel}{2\pi} q_\parallel \left\{ \frac{\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_2(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)} \right.
$$

$$
\times [p_\parallel q_\parallel c_k^{(+)}(p_\parallel|q_\parallel) - \frac{1}{2}\beta_2(p_\parallel)\beta_1(q_\parallel)(c_{k-1}^{(+)}(p_\parallel|q_\parallel)
$$

$$
+ c_{k+1}^{(+)}(p_\parallel|q_\parallel)) + \frac{\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel)}{\beta_2(p_\parallel) + \beta_1(q_\parallel)}
$$

$$
\times [p_\parallel q_\parallel c_k^{(-)}(p_\parallel|q_\parallel) + \frac{1}{2}\beta_2(p_\parallel)\beta_1(q_\parallel)(c_{k-1}^{(-)}(p_\parallel|q_\parallel)
$$

$$
\left. + c_{k+1}^{(-)}(p_\parallel|q_\parallel))] \right\} \frac{a_k^{(p)}(q_\parallel)}{\varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel)}
$$

$$
- \frac{\varepsilon_2 - \varepsilon_1}{4\varepsilon_2} \frac{\omega}{c} \int_0^\infty \frac{dq_\parallel}{2\pi} q_\parallel \left\{ \frac{\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_3(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)} \right.
$$

$$
\times [c_{k-1}^{(+)}(p_\parallel|q_\parallel) - c_{k+1}^{(+)}(p_\parallel|q_\parallel)]
$$

$$
\left. - \frac{\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel))}{\beta_2(p_\parallel) + \beta_1(q_\parallel)} [c_{k-1}^{(-)}(p_\parallel|q_\parallel) - c_{k+1}^{(-)}(p_\parallel|q_\parallel)] \right\} \frac{a_k^{(s)}(q_\parallel)}{\beta_3(q_\parallel) + \beta_1(q_\parallel)}
$$

$$
= - \frac{(\varepsilon_2 - \varepsilon_1)}{2\varepsilon_2\beta_2(p_\parallel)} \left\{ \frac{\varepsilon_2\beta_3(p_\parallel) + \varepsilon_3\beta_2(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(k_\parallel)} [p_\parallel k_\parallel c_k^{(+)}(p_\parallel|k_\parallel) \right.
$$

$$
- \frac{1}{2}\beta_2(p_\parallel)\beta_1(k_\parallel)(c_{k-1}^{(+)}(p_\parallel|k_\parallel) + c_{k+1}^{(+)}(p_\parallel|k_\parallel))]
$$

$$
+ \frac{\varepsilon_2\beta_3(p_\parallel) - \varepsilon_3\beta_2(p_\parallel)}{\beta_2(p_\parallel) + \beta_1(k_\parallel)} [p_\parallel k_\parallel c_k^{(-)}(p_\parallel|k_\parallel)
$$

$$
\left. + \frac{1}{2}\beta_2(p_\parallel)\beta_1(k_\parallel)(c_{k-1}^{(-)}(p_\parallel|k_\parallel) + c_{k+1}^{(-)}(p_\parallel|k_\parallel))] \right\}
$$

$$
\times \exp(-ik\phi_0) E_{op}(\mathbf{k}_\parallel). \tag{9a}
$$

$$
a_k^{(s)}(p_\parallel) + \frac{\omega}{c} \frac{(\varepsilon_2 - \varepsilon_1)}{4\beta_2(p_\parallel)} \int_0^\infty \frac{dq_\parallel}{2\pi} q_\parallel \left\{ \frac{\beta_3(p_\parallel) + \beta_2(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)} \right.
$$

$$
\times [c_{k-1}^{(+)}(p_\parallel|q_\parallel) - c_{k+1}^{(+)}(p_\parallel|q_\parallel)] + \frac{\beta_3(p_\parallel) - \beta_2(p_\parallel)}{\beta_2(p_\parallel) + \beta_1(q_\parallel)}
$$

$$
\times [c_{k-1}^{(-)}(p_\parallel|q_\parallel) - c_{k+1}^{(-)}(p_\parallel|q_\parallel)] \left. \right\} \frac{\beta_1(q_\parallel) a_k^{(p)}(q_\parallel)}{\varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel)}
$$

$$
- \frac{\omega^2}{c^2} \frac{\varepsilon_2 - \varepsilon_1}{4\beta_2(p_\parallel)} \int_0^\infty \frac{dq_\parallel}{(2\pi)} q_\parallel \left\{ \frac{\beta_3(p_\parallel) + \beta_2(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(q_\parallel)} \right.
$$

$$
\times [c_{k-1}^{(+)}(p_\parallel|q_\parallel) + c_{k+1}^{(+)}(p_\parallel|q_\parallel)] + \frac{\beta_3(p_\parallel) - \beta_2(p_\parallel)}{\beta_2(p_\parallel) + \beta1(q_\parallel)}
$$

$$
\times [c_{k-1}^{(-)}(p_\parallel|q_\parallel) + c_{k+1}^{(-)}(p_\parallel|q_\parallel)] \left. \right\} \frac{a_k^{(s)}(q_\parallel)}{\beta_3(q_\parallel) + \beta_1(q_\parallel)}
$$

$$= \frac{\omega}{c} \frac{\varepsilon_2 - c_1}{4\beta_2(p_\parallel)} \left\{ \frac{\beta_3(p_\parallel) + \beta_2(p_\parallel)}{\beta_2(p_\parallel) - \beta_1(k_\parallel)} [c_{k-1}^{(+)}(p_\parallel|k_\parallel) - c_{k+1}^{(+)}(p_\parallel|k_\parallel)] \right.$$

$$\left. + \frac{\beta_3(p_\parallel) - \beta_2(p_\parallel)}{\beta_2(p_\parallel) + \beta_1(k_\parallel)} [c_{k-1}^{(-)}(p_\parallel|k_\parallel) - c_{k+1}^{(-)}(p_\parallel|k_\parallel)] \right\}$$

$$\times \beta_1(k_\parallel) \exp[-ik\phi_0] E_{op}(\mathbf{k}_\parallel). \tag{9b}$$

Thus, for a circularly symmetric defect the equations for different k values decouple, reducing the problem to one of solving one-dimensional integral equations.

Equations (9) for each value of the azimuthal integer k were solved by transforming them into linear matrix equations following the spirit of the method used in Ref. [18]. The integration over the wavenumber $q_\parallel$ was carried out over a dimensionless variable y defined by $q_\parallel = 2\sqrt{y}/R$, where R is the effective radius of the defect, and a dimensionless frequency Ω was introduced through $(\omega/c) = 2\sqrt{\Omega}/R$. The integration over the wavenumber variable y was split into six intervals: $(0, \Omega)$, $(\Omega, y_p - \delta)$, $(y_p + \delta, 2y_p - \Omega)$, $(2y_p - \Omega, \varepsilon_2\Omega)$, $(\varepsilon_2\Omega, K\Omega)$, $(K\Omega, \infty)$, where K is an integer (chosen to be 7) and ε_2 is the dielectric constant of the defect, plus the contribution from the pole corresponding to the wavenumber of the surface plasmon polariton at $y = y_p$. This separation of the region of integration was based on the behavior of $\beta_1(q_\parallel)$ and $\beta_3(q_\parallel)$ as functions of the wave numbers $q_\parallel$, namely on the values of the latter variable at which they change from purely imaginary to purely real, on the location of the surface plasmon polariton pole, as well as on how the integration to infinite wave number was to be carried out. The integral over the range $(K\Omega, \infty)$ was calculated by Gauss-Laguerre integration, while the integrals over the preceding five intervals were calculated by Gauss-Legendre integration. The number of discretization points in each interval was determined by evaluating the integrals over $q_\parallel$ on the left-hand sides of Eqs. (9a) and (9b), with the expressions for $a_k^{p,s}(q_\parallel)$ (with $k = 1$) appearing in the integrands given by the right-hand sides of these equations, by two different methods. In one method the integral over each interval was evaluated by an integration software package (Mathematica, that uses a method with iteration until it attains a specified accuracy). In the second method a Gaussian integration scheme was used, with the number of abscissas being increased until the same accuracy was achieved. In both of these methods the integrand is known analytically, but the discretization used by them is different. The total number of abscissas used in carrying out the $q_\parallel$ integrations in the region $(0, \infty)$ produced by this approach was 180. The contribution from the surface plasmon polariton pole, i.e. from the interval $(y_p - \delta, y_p + \delta)$, was calculated by the use of the prescription presented in Ref. [11], namely

$$\frac{1}{f_p(q_\parallel)} = P\left(\frac{1}{f_p(q_\parallel)}\right) + i\pi \frac{\varepsilon_1\beta_3(k_\parallel)}{(\varepsilon_1^2 - \varepsilon_3^2)k_\parallel} \delta(q_\parallel - k_\parallel), \tag{10a}$$

where

$$f_p(q_\parallel) \equiv \varepsilon_3\beta_1(q_\parallel) + \varepsilon_1\beta_3(q_\parallel), \tag{10b}$$

P denotes the Cauchy principal part, $\varepsilon(\omega) = \varepsilon_3$ is the dielectric function of the metal, and $k_\parallel$ is the zero of $f_p(q_\parallel)$. Finally, in carrying out these calculations we set the amplitude $E_{op}(\mathbf{k}_\parallel) = 1/(2\pi R)$.

When $a_{p,s}(\mathbf{q}_\parallel)$ have been determined we can use the results to calculate the scattered field in the vacuum region in the far zone by the use of the method described in Ref. [19]. The result

has the form of an outgoing spherical wave

$$
\begin{aligned}
E_{vac}^{(sc)}(\mathbf{x}|\omega) \;=\; & -i\sqrt{\varepsilon_1}\frac{\omega\cos\theta_x}{2\pi c}\frac{e^{i\sqrt{\varepsilon_1}(\omega/c)x}}{x} \\
& \times[\sqrt{\varepsilon_1}\hat{\mathbf{e}}_p A_p(\sqrt{\varepsilon_1}\hat{\mathbf{x}}_\parallel\left(\frac{\omega}{c}\right)\sin\theta_x) \\
& +\hat{\mathbf{e}}_s A_s(\sqrt{e_1}\hat{\mathbf{x}}_\parallel\left(\frac{\omega}{c}\right)\sin\theta_x)], \qquad \sqrt{\varepsilon_1}(\omega/c)x \gg 1.
\end{aligned}
\tag{11}
$$

In this expression $\hat{\mathbf{e}}_p$ and $\hat{\mathbf{e}}_s$ are unit vectors defined by

$$
\hat{\mathbf{e}}_p \;=\; (\cos\theta_x\cos\phi_x, \cos\theta_x\sin\phi_x, -\sin\theta_x) \tag{12a}
$$

$$
\hat{\mathbf{e}}_s \;=\; (-\sin\phi_x, \cos\phi_x, 0), \tag{12b}
$$

with

$$
\cos\theta_x \;=\; \frac{x_3}{x}, \quad \sin\theta_x = \frac{x_\parallel}{x} \tag{13a}
$$

$$
\cos\phi_x \;=\; \frac{x_1}{x_\parallel}, \quad \sin\phi_x = \frac{x_2}{x_\parallel}, \tag{13b}
$$

where (θ_x, ϕ_x) are the polar and azimuthal angles of the vector $\hat{\mathbf{x}}$, while

$$
A_p(\mathbf{q}_\parallel) \;=\; \frac{a_p(\mathbf{q}_\parallel)}{\varepsilon_3\beta_1(q_\parallel)+\varepsilon_1\beta_3(q_\parallel)} \tag{14a}
$$

$$
A_s(\mathbf{q}_\parallel) \;=\; \frac{a_s(\mathbf{q}_\parallel)}{\beta_1(q_\parallel)+\beta_3(q_\parallel)}. \tag{14b}
$$

The surface plasmon polariton contribution to the scattered field is given by the residue at the pole of the integrand in Eq. (1) at $q_\parallel = k_\parallel$. It is given by an outgoing cylindrical wave

$$
\begin{aligned}
E_{spp}^{(sc)}(\mathbf{x}|\omega) \;=\; & -i\frac{e^{ik_\parallel x_\parallel-\beta_1(k_\parallel)x_3-i\frac{\pi}{4}}}{(2\pi k_\parallel x_\parallel)^{\frac{1}{2}}} \\
& \times\frac{c\varepsilon_1\beta_3(k_\parallel)}{\omega}\frac{i\hat{\mathbf{x}}_\parallel\beta_1(k_\parallel)-\hat{\mathbf{x}}_3 k_\parallel}{\varepsilon_3^2-\varepsilon_1^2}a_p(\hat{\mathbf{x}}_\parallel k_\parallel), \qquad k_\parallel x_\parallel \gg 1.
\end{aligned}
\tag{15}
$$

We introduce the differential cross sections, measured in units of length, for scattering into the vacuum and into other surface waves by

$$
\sigma_{vac}(\theta_x, \phi_x) \;=\; \frac{P_{vac}(\theta_x, \phi_x)}{P_{inc}} \tag{16}
$$

$$
\sigma_{spp}(\phi_x) \;=\; \frac{P_{spp}(\phi_x)}{P_{inc}}, \tag{17}
$$

where $P_{vac}(\theta_x, \phi_x)$ is the power scattered into the vacuum away from the surface in the direction $\hat{\mathbf{x}}, P_{spp}(\phi_x)$ is the power scattered into other surface waves in the direction $\hat{\mathbf{x}}_\parallel$, and P_{inc} is the incident power per unit length in the x_2 direction. These powers are given by

$$
\begin{aligned}
P_{vac}(\theta_x, \phi_x) \;=\; & \frac{c}{8\pi}\varepsilon_1\left(\frac{\omega}{2\pi c}\right)^2\cos^2\theta_x \\
& \times\{\varepsilon_1|A_p(\sqrt{\varepsilon_1}\hat{\mathbf{x}}_\parallel\left(\frac{\omega}{c}\right)\sin\theta_x)|^2 \\
& +|A_s(\sqrt{\varepsilon_1}\hat{\mathbf{x}}_\parallel\left(\frac{\omega}{c}\right)\sin\theta_x)|^2\}
\end{aligned}
\tag{18}
$$

$$P_{spp}(\phi_x) = \frac{1}{2}\varepsilon_1^3\left(\frac{c}{4\pi}\right)^2 \frac{\beta_3^2(k_\parallel)}{\omega\beta_1(k_\parallel)} \frac{|a_p(\hat{\mathbf{x}}_\parallel k_\parallel)|^2}{(\varepsilon_3^2-\varepsilon_1^2)^2} \tag{19}$$

$$P_{inc} = \varepsilon_1\frac{c^2}{8\pi\omega}\frac{k_\parallel}{2\beta_1(k_\parallel)}\left(1-\frac{1}{\varepsilon_3^2(\omega)}\right)|E_{op}(\mathbf{k}_\parallel)|^2. \tag{20}$$

In what follows we will neglect the second term on the right-hand side of Eq. (20) due to its smallness relative to the first term.

We present numerical results for a dielectric defect of Gaussian form defined by the surface profile function

$$\zeta(x_\parallel) = A\exp(-x_\parallel^2/R^2), \tag{21}$$

In this case the coefficients $c_k^{(\pm)}(p_\parallel|q_\parallel)$ are given by

$$\begin{aligned}
c_k^{(\pm)}(p_\parallel|q_\parallel) &= \pi R^2 \sum_{n=1}^{\infty} \frac{[\pm A(\beta_2(p_\parallel)\mp\beta_1(q_\parallel))]^n}{n\cdot n!} \\
&\times I_k\left(\frac{R^2 p_\parallel q_\parallel}{2n}\right)\exp\left[\frac{-R^2(p_\parallel^2+q_\parallel^2)}{4n}\right],
\end{aligned} \tag{22}$$

where $I_k(z)$ is a modified Bessel function of the first kind of order k. The Gaussian defect is characterized by a $1/e$ half width $R = 0.25\mu$m. It is assumed to be deposited on a planar aluminum surface. It is illuminated by a surface plasmon polariton propagating in the positive x_1 direction ($\phi_0 = 0°$). The wavelength of the incident surface plasmon polariton is $\lambda =632.8$ nm, and the dielectric function of aluminum at this wavelength is $\varepsilon_3 = \varepsilon_3^r(\omega) + i\varepsilon_3^i(\omega) = -57.19 +i11.19$ [20] of which we used only the real part. The energy mean free path of the surface plasmon polariton at a planar vacuum-aluminum interface at this wavelength is $\ell_{spp}(\omega) = (\lambda/2\pi)|\varepsilon_3^r(\omega)|^{\frac{1}{2}}(|\varepsilon_3^r(\omega)| - \varepsilon_1)^{3/2}/[\varepsilon_1^{3/2}\varepsilon_3^i(\omega)] = 28.67\mu$m. This is more than two orders of magnitude longer than the 0.25μm radius of the surface defect, and justifies our treating ε_3 as real. In these calculations the rotational quantum k ranged in integer setps from -30 to $+30$.

In Fig. 1 we present a contour plot of $\sigma_{vac}(\theta_x, \phi_x)$ for a value of the dielectric constant of the defect $\varepsilon_2 = 2.69$ and (a) $A/R = 0.1$, (b) $A/R = 0.2$, (c) $A/R = 0.3$. For each of these values of A/R the maximum of the intensity of the field radiated into the vacuum occurs at $\theta_x = 70°$, $\phi_x = 0°$. More radiation into smaller values of θ_x with increasing values of A/R is seen, especially in Fig. 1(c). The total cross sections for the waves radiated into the vacuum are (a) 0.0021μm, (b) 0.0057μm, (c) 0.0099 μm respectively. Thus they increase as the amplitude of the defect increases, with the values of the other parameters kept fixed.

When the dielectric constant of the defect is increased to $\varepsilon_2 = 5.0$, with the remaining parameters maintaining the values they have in Fig. 1 (Fig. 2), the maximum of the intensity of the field scattered into the vacuum remains at $\theta_x = 70°, \phi_x = 0°$, but the strength of the scattering is nearly doubled in comparison with the corresponding results presented in connection with Fig. 1. The total cross sections for the waves radiated into the vacuum are (a) 0.0040μm, (b) 0.0125μm, (c) 0.0290μm. Some radiation into the backward directions is seen in Fig. 2(c).

The angular dependence of $\sigma_{spp}(\phi_x)$ is presented in Fig. 3. The dielectric constant of the defect is $\varepsilon_2 = 2.69$, and (a) $A/R = 0.1$, (b) $A/R = 0.2$, (c) $A/R = 0.3$. We see that there is no scattering of the surface plasmon polariton into the backward direction for all three values of A/R: all of the scattering is into the forward direction. These results are due presumably to the
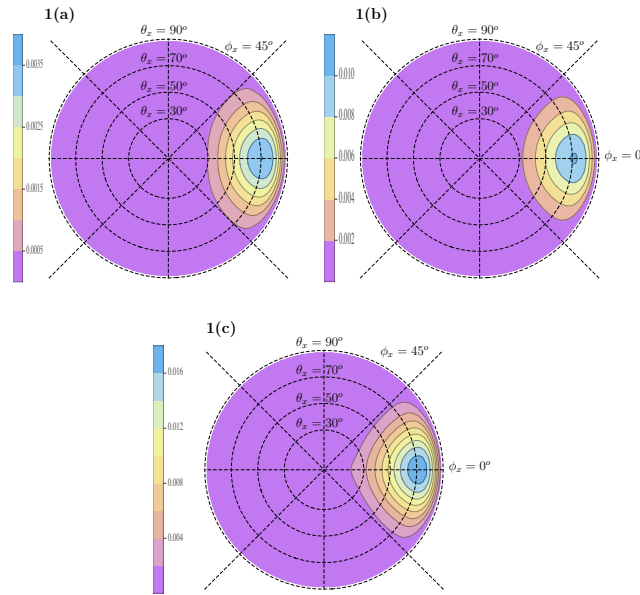
Fig. 1. A contour plot of $\sigma_{vac}(\theta_x, \phi_x)$. The concentric dashed circles are the lines of constant θ_x, with $\theta_x = 0°$ at the center, and $\theta_x = 90°$ at the boundary. The azimuthal angle ϕ_x varies from $0°$ to $360°$. $\varepsilon_2 = 2.69; A/R = 0.1$ (a), 0.2 (b), 0.3 (c).

transparency of the dielectric defect. The shapes of these scattering patterns do not change with increasing values of A/R, but the strength of the scattering does. The total cross sections for the scattering of the incident surface plasmon polariton into other surface plasmon polaritons are (a) 0.0019 μm, (b) 0.0053 μm, (c) 0.0087 μm. These values are nearly the same as the values of $\sigma_{vac}(\theta_x, \phi_x)$ for the corresponding parameter values in Fig. 3.

The same qualitative behavior of $\sigma_{spp}(\phi_x)$ is observed when ε_2 is increased to $\varepsilon_2 = 5.0$. This is seen from the results presented in Fig. 4. The values of A/R for which these results are calculated are (a) $A/R = 0.1$, (b) $A/R = 0.2$, (c) $A/R = 0.3$. All of the scattering is in the forward direction. The shapes of the scattering patterns do not change as the value of A/R increases, while the strength of the scattering increases with increasing values of A/R. The total cross sections for the scattering of the incident surface plasmon polariton into other surface plasmon polaritons are (a) 0.0035μm, (b) 0.0100μm, (c) 0.0176μm. For each value of A/R increasing ε_2 from 2.69 to 5.0 nearly doubles the total scattering cross section from the value it has for the data used in obtaining the corresponding results in connection with Fig. 3.

The forward scattering of the surface plasmon polariton is clearly seen in the results presented in Figs. 5 and 6 which show the field intensity $|\mathbf{E}(\mathbf{x}|\omega)|^2$ as a function of $\mathbf{x}_\parallel$ at 5 nm above the surface profile ($x_3 = \zeta(\mathbf{x}_\parallel) + 5$nm), for $\varepsilon_2 = 2.69$ and 5.0, respectively, while (a) $A/R = 0.1$,
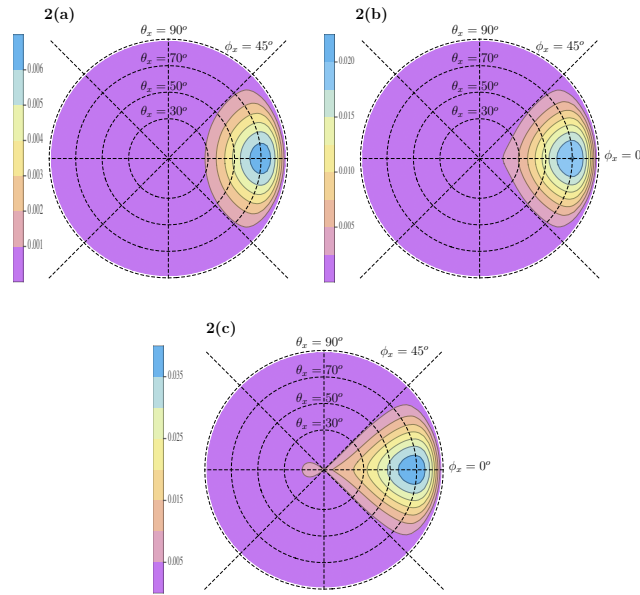
Fig. 2. The same as Fig. 1, but with $\varepsilon_2 = 5.0$.

(b) $A/R = 0.2$, (c) $A/R = 0.3$. Both figures show a weak diffractive spreading of the intensity of the field after its interaction with the defect. There is very little indication of the scattering of the surface plasmon polariton in backward directions, in agreement with the results presented in Figs. 1-2 and 3-4.

The present results qualitatively resemble analogous results obtained in Ref. [14], bearing in mind that in that reference the surface profile functions of the dielectric surface defects were an anisotropic Gaussian and an anisotropic hemiellipsoid.

The preceding results can be understood if in Eqs. (18) and (19) we substitutee for $a_p(\mathbf{q}_\parallel)$ and $a_s(\mathbf{q}_\parallel)$ the expressions for them obtained in the small roughness limit of the first Born approximation, namely

$$a_p(\mathbf{q}_\parallel) \;=\; -\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2}\hat{\zeta}(\mathbf{q}_\parallel - \mathbf{k}_\parallel)[\varepsilon_3 q_\parallel k_\parallel - \varepsilon_2 \beta_3(q_\parallel)(\hat{\mathbf{q}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\beta_1(k_\parallel)]E_{0p}(\mathbf{k}_\parallel) \qquad (23a)$$

$$a_s(\mathbf{q}_\parallel) \;=\; -i(\varepsilon_2 - \varepsilon_1)\hat{\zeta}(\mathbf{q}_\parallel - \mathbf{k}_\parallel)\frac{\omega}{c}(\hat{\mathbf{q}}_\parallel \times \hat{\mathbf{k}}_\parallel)_3 \beta_1(k_\parallel)E_0 p(\mathbf{k}_\parallel), \qquad (23b)$$

where

$$\begin{aligned}
\hat{\zeta}(\mathbf{Q}_\parallel) &= \int d^2 x_\parallel \, \zeta(\mathbf{x}_\parallel)\exp(-i\mathbf{Q}_\parallel \cdot \mathbf{x}_\parallel) \\
&= \pi A R^2 \exp(-R^2 Q_\parallel^2/4). \qquad (24)
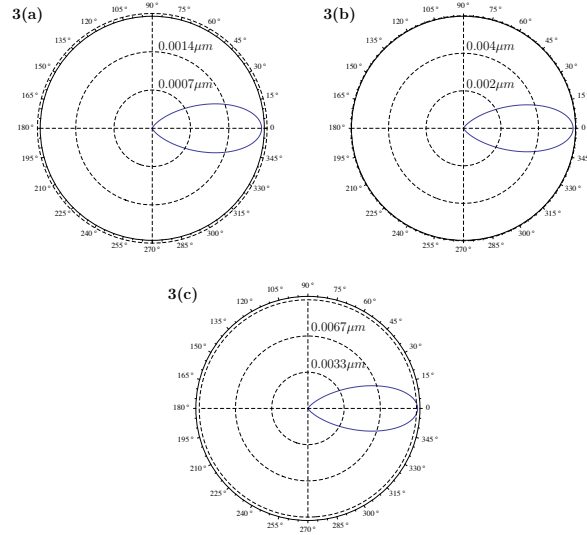\end{aligned}$$

Fig. 3. Plots of $\sigma_{spp}(\phi_x)$. $\varepsilon_2 = 2.69; A/R = 0.1$ (a), 0.2 (b), 0.3 (c).

The second of Eqs. (24) gives the expression for $\hat{\zeta}(\mathbf{Q}_\parallel)$ corresponding to the Gaussian surface profile function (21).

On substituting Eqs. (23) into Eq. (18) and making use of Eqs. (16) and (20) we obtain $\sigma_{vac}(\theta_x, \phi_x)$ in the form

$$
\begin{aligned}
\sigma_{vac}(\theta_x, \phi_x) \;=\;& \frac{1}{2} R \left(\frac{A}{R}\right)^2 \left(\frac{\omega R}{c}\right)^5 \frac{\varepsilon_1^{5/2}}{\varepsilon_2^2 |\varepsilon_3|^{\frac{1}{2}}} \frac{(\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_3^2 - \varepsilon_1^2)} \cos^2 \theta_x \\
&\times \exp[-\frac{R^2}{2}(k_\parallel^2 - 2\sqrt{\varepsilon_1}\frac{\omega}{c} k_\parallel \sin\theta_x \cos\phi_x + \varepsilon_1 \frac{\omega^2}{c^2}\sin^2\theta_x)] \\
&\times \frac{a_0(\theta_x) + a_1(\theta_x)\cos\phi_x + a_2(\theta_x)\cos^2\phi_x}{[|\varepsilon_3| - (|\varepsilon_3| - \varepsilon_1)\sin^2\theta_x]},
\end{aligned}
\tag{25}
$$

where

$$
a_0(\theta_x) \;=\; \varepsilon_2^2 |\varepsilon_3| \cos^2\theta_x + (|\varepsilon_3|^3 + \varepsilon_2^2 \varepsilon_1)\sin^2\theta_x \tag{26a}
$$

$$
a_1(\theta_x) \;=\; 2\varepsilon_2 |\varepsilon_3|^{3/2} \sin\theta_x (|\varepsilon_3| + \varepsilon_1 \sin^2\theta_x)^{\frac{1}{2}} \tag{26b}
$$

$$
a_2(\theta_x) \;=\; \varepsilon_2^2 |\varepsilon_3| \sin^2\theta_x. \tag{26c}
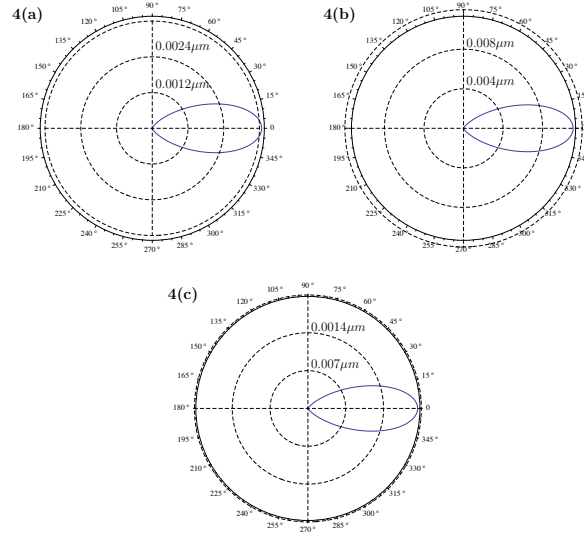$$

Fig. 4. The same as Fig. 3, but with $\varepsilon_2 = 5.0$.

This cross section is proportional to the square of the aspect ratio (A/R) of the defect. It is an increasing function of ε_2, and has a maximum at $(\theta_x, \phi_x) = (70.75°, 0°)$ for the values of the parameters assumed in obtaining Fig. 1, and at $(70.5°, 0°)$ for the values assumed in obtaining Fig. 2. The insensitivity of these angles to changes of ε_2, and their closeness to the angular position of the maximum in these figures, namely $(70°, 0°)$, is gratifying, given the simplicity of this approximate calculation.

When Eq. (23a) is substituted in Eq. (19), and Eqs. (17) and (20) are used, we obtain $\sigma_{spp}(\phi_x)$ in the form

$$
\begin{aligned}
\sigma_{spp}(\theta_x) &= \frac{\pi}{2} R \left(\frac{A}{R} \right) \left(\frac{\omega R}{c} \right)^5 \frac{|\varepsilon_3|^{7/2} \varepsilon_1^{7/2}}{\varepsilon_2^2} \frac{(\varepsilon_2 - \varepsilon_1)^2}{(|\varepsilon_2| - \varepsilon_1)^{9/2}(|\varepsilon_2| + \varepsilon_1)^2} \\
&\quad \times \exp(-2R^2 k_\parallel^2 \sin^2 \tfrac{1}{2} \phi_x)(|\varepsilon_3| + \varepsilon_2 \cos \phi_x)^2.
\end{aligned} \tag{27}
$$

Although the factor $(|\varepsilon_3| + \varepsilon_2 \cos \phi_x)^2$ favors scattering into the forward direction by a small amount, because of the large value of $|\varepsilon_3|$, the suppression of scattering into backward directions is due entirely to the factor $\exp(-2R^2 k_\parallel^2 \sin^2 \tfrac{1}{2} \phi_x)$, which is a consequence of the Gaussian form of $\hat{\zeta}(\mathbf{Q}_\parallel)$, Eq. (24). The cross section (27) is proportional to the square of the defects aspect ratio, and is an increasing function of ε_2.

5(a)

5(b)

5(c)

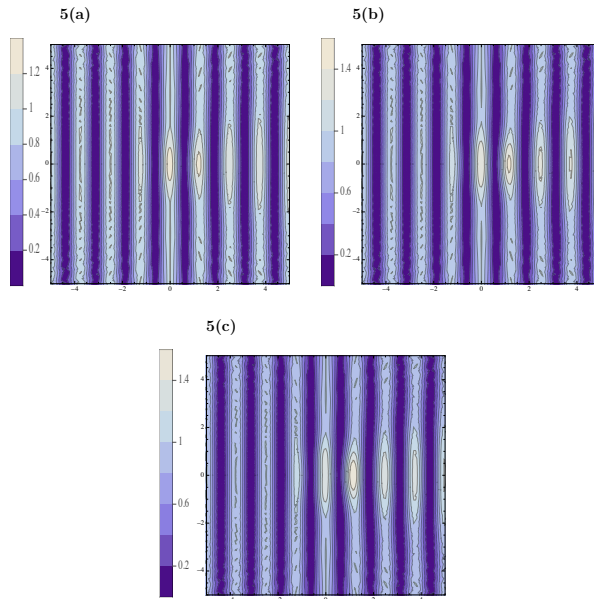Fig. 5. The field intensity $|\mathbf{E}(\mathbf{x}|\omega)|^2$ as a function of $\mathbf{x}_\parallel$ at $x_3 = \zeta(\mathbf{x}_\parallel) + 5$ nm. $\varepsilon_2 = 2.69; A/R$ = 0.1 (a), 0.2 (b), 0.3 (c).

The proportionality of the cross sections (25) and (27) to ω^5 can be understood in the following way. The cross section for the Rayleigh scattering of a volume wave from a d-dimensional object is proportional to ω^{d+1}. Since the scatterer in the present case is defined by $x_3 - \zeta(\mathbf{x}_\parallel) = 0$, it is a three-dimensional scatterer as far as Rayleigh scattering is concerned. The additional factor of ω arises because the decay length of the surface plasmon polariton into the vacuum is proportional to its wavelength parallel to the surface, which reduces the volume within which its interaction with the defect occurs [21].

In this paper we have derived a pair of coupled two-dimensional integral equations – reduced Rayleigh equations – for the amplitudes of the p- and s-polarized components of the scattered elsctric field produced when a surface plasmon polariton is incident on a localized dielectric surface defect on a planar metal surface. When the defect is circularly symmetric with respect to the normal to the surface a rotational expansion of these scattering amplitudes transforms the reduced Rayleigh equations into an infinite set of uncoupled one-dimensional integral equations that have been solved numerically. We have applied this approach to calculate the near-field and far-field angular distributions of the intensity of the field scattered by a dielectric defect in the form of an isotropic Gaussian on a metal surface for two values of its dielectric constant, and three values of its aspect ratio. It is found that the intensity of the volume electromagnetic field scattered into the vacuum above the surface and defect is a maximum for in-plane scattering at a
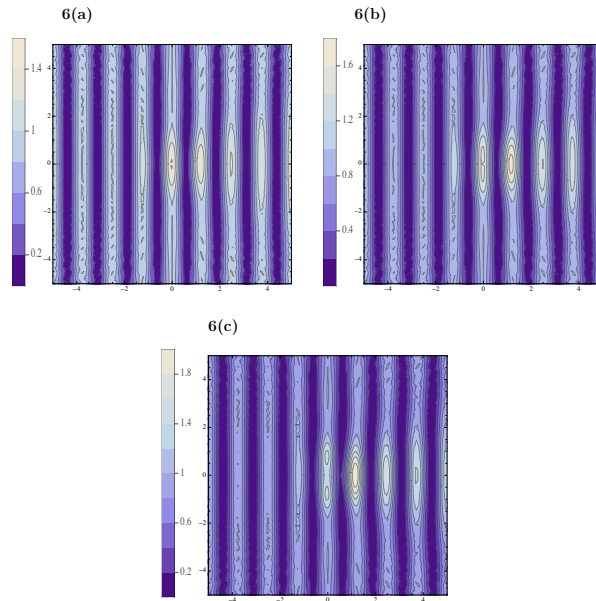
6(a) **6(b)**

6(c)

Fig. 6. The same as Fig. 5, but with $\varepsilon_2 = 5.0$.

polar scattering angle of $\theta_x = 70°$. This value is independent of the aspect ratios and dielectric constants of the defect assumed in our work. It is a significantly larger angle than the angle $\theta_x = 28°$ at which the corresponding maxima occurs in the scattering of a surface plasmon polariton from a circularly symmetric Gaussian indentation (dimple) in a planar metal surface [11]. This result suggests that the excitation of a surface plasma polariton by illuminating this dielectric defect by a p-polarized volume electromagnetic wave will be most efficient for a polar angle of incidence $\theta_0 = 70°$ [22]. It is also found that the surface plasmon polariton is scattered into other surface plasmon polaritons primarily in the forward direction. There is no shadow behind the defect as in the case of the scattering of a surface plasmon polariton from a circularly symmetric indentation in a metal surface [11]. This is due presumably to the transparency of the dielectric defect. Thus there are significant differences between the cross section for the scattering of a surface plasmon polariton from a metallic defect on a metallic surface and for its scattering from a dielectric defect on a metallic surface.

The approach developed here can also be applied to dielectric surface defects that are not circularly symmetric. In this case a rotational expansion of the scattering amplitudes produces a set of one-dimensional integral equations that, unlike Eqs. (9), are coupled for different values of the rotational quantum number k [14].

APPENDIX

In this Appendix we outline the derivation of Eqs. (4). We begin by considering the scattering of a volume electromagnetic wave incident from a dielectric medium in the region $x_3 > d + \zeta(\mathbf{x}_\parallel)$, where $\mathbf{x}_\parallel = (x_1, x_2, 0)$, whose dielectric constant is ε_1, on a dielectric film in the region $0 < x_3 < d + \zeta(\mathbf{x}_\parallel)$ whose dielectric constant is ε_2, that is deposited on a semi-infinite metal whose dielectric constant is ε_3 in the region $x_3 < 0$. The electric fields in these regions are given by

$$\mathbf{E}^>(\mathbf{x}|\omega) = \mathbf{E}_0(\mathbf{k}_\parallel)\exp(i\mathbf{Q}_0 \cdot \mathbf{x}) + \int \frac{d^2 q_\parallel}{(2\pi)^2}\mathbf{A}(\mathbf{q}_\parallel)\exp(i\mathbf{Q}_1 \cdot \mathbf{x}) \tag{A.1}$$

for $x_3 > d + \zeta(\mathbf{x}_\parallel)$,

$$\mathbf{E}^{(f)}(\mathbf{x}|\omega) = \int \frac{d^2 q_\parallel}{(2\pi)^2}[\mathbf{F}^+(\mathbf{q}_\parallel)\exp(i\mathbf{Q}^+ \cdot \mathbf{x}) + \mathbf{F}^-(\mathbf{q}_\parallel)\exp(i\mathbf{Q}^- \cdot \mathbf{x})] \tag{A.2}$$

for $0 < x_3 < d + \zeta(\mathbf{x}_\parallel)$, and

$$\mathbf{E}^<(\mathbf{x}|\omega) = \int \frac{d^2 q_\parallel}{(2\pi)^2}\mathbf{B}(\mathbf{q}_\parallel)\exp(i\mathbf{Q}_2 \cdot \mathbf{x}) \tag{A.3}$$

for $x_3 < 0$. In these equations we have introduced the wave vectors

$$\mathbf{Q}_0(\mathbf{k}_\parallel) = \mathbf{k}_\parallel - \alpha_1(k_\parallel)\hat{\mathbf{x}}_3 \tag{A.4a}$$

$$\mathbf{Q}_1(\mathbf{q}_\parallel) = \mathbf{q}_\parallel + \alpha_1(q_\parallel)\hat{\mathbf{x}}_3 \tag{A.4b}$$

$$\mathbf{Q}^\pm(\mathbf{q}_\parallel) = \mathbf{q}_\parallel \pm \alpha_2(q_\parallel)\hat{\mathbf{x}}_3 \tag{A.4c}$$

$$\mathbf{Q}_2(\mathbf{q}_\parallel) = \mathbf{q}_\parallel - \alpha_3(q_\parallel)\hat{\mathbf{x}}_3, \tag{A.4d}$$

where $(j = 1, 2, 3)$

$$\alpha_j(q_\parallel) = [\varepsilon_j(\omega/c)^2 - q_\parallel^2]^{\frac{1}{2}} \qquad Re\,\alpha_j(q_\parallel) > 0,\, Im\,\alpha_j(q_\parallel) > 0. \tag{A.5}$$

The boundary conditions on the fields at the interface $x_3 = d + \zeta(\mathbf{x}_\parallel)$ can be written

$$\mathbf{n} \times \mathbf{E}^>(\mathbf{x}|\omega) = \mathbf{n} \times \mathbf{E}^{(f)}(\mathbf{x}|\omega) \tag{A.6}$$

$$\mathbf{n} \times (\nabla \times \mathbf{E}^>(\mathbf{x}|\omega)) = \mathbf{n} \times (\nabla \times \mathbf{E}^{(f)}(\mathbf{x}|\omega))) \tag{A.7}$$

$$\varepsilon_1 \mathbf{n} \cdot \mathbf{E}^>(\mathbf{x}|\omega) = \varepsilon_2 \mathbf{n} \cdot \mathbf{E}^{(f)}(\mathbf{x}|\omega), \tag{A.8}$$

where

$$\mathbf{n} = \left(-\frac{\partial \zeta(\mathbf{x}_\parallel)}{\partial x_1}, -\frac{\partial \zeta(\mathbf{x}_\parallel)}{\partial x_2}, 1\right). \tag{A.9}$$

We now take the vector cross product of Eq. (A.6) with $\varepsilon_2 \mathbf{P}^+(\mathbf{p}_\parallel)\exp[-i\mathbf{P}^+(\mathbf{p}_\parallel) \cdot \mathbf{x}_\zeta]$; we then take the product of Eq. (A.7) with $-i\varepsilon_2\exp[-i\mathbf{P}^+(\mathbf{p}_\parallel) \cdot \mathbf{x}_\zeta]$; and we finally multiply Eq. (A.7) by $-\mathbf{P}^+(\mathbf{p}_\parallel)\exp[-i\mathbf{P}^+(\mathbf{p}_\parallel) \cdot \mathbf{x}_\zeta]$. In these expressions we have introduced the vectors $\mathbf{P}^+(\mathbf{p}_\parallel) = \mathbf{p}_\parallel + \alpha_2(p_\parallel)\hat{\mathbf{x}}_3$, where $\mathbf{p}_\parallel$ is an arbitrary two-dimensional wave vector, and $\mathbf{x}_\zeta = \mathbf{x}_\parallel + \zeta(\mathbf{x}_\parallel)\hat{\mathbf{x}}_3$. We add the three equations obtained in this manner and integrate the sum over $\mathbf{x}_\parallel$. In this way we obtain the equation

$$(\varepsilon_2 - \varepsilon_1)e^{-i\alpha_1(k_\parallel)d}\frac{I(\alpha_2(p_\parallel) + \alpha_1(k_\parallel)|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(k_\parallel)}[\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}_\parallel^+(\mathbf{p}_\parallel) \times \mathbf{E}_0(\mathbf{k}_\parallel))]$$

$$+(\varepsilon_2 - \varepsilon_1)\int \frac{d^2 q_\parallel}{(2\pi)^2}e^{i\alpha_1(q_\parallel)d}\frac{I(\alpha_2(p_\parallel) - \alpha_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(q_\parallel)}[\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}^+(\mathbf{p}_\parallel) \times \mathbf{A}(\mathbf{q}_\parallel))]$$

$$= -2\varepsilon_2\alpha_2(p_\parallel)e^{i\alpha_2(p_\parallel)d}\mathbf{F}^+(\mathbf{p}_\parallel), \tag{A.10}$$

where

$$I(\gamma|\mathbf{Q}_\parallel) = \int d^2 x_\parallel e^{-i\mathbf{Q}_\parallel \cdot \mathbf{x}_\parallel} e^{-i\gamma\zeta(\mathbf{x}_\parallel)}. \tag{A.11}$$

We now return to Eqs. (A.6)-(A.8). We take the vector cross product of Eq. (A.6) with $\varepsilon_2 \mathbf{P}^-(\mathbf{p}_\parallel)\exp[-i\mathbf{P}^-(\mathbf{p}_\parallel)\cdot\mathbf{x}_\zeta]$; we next multiply Eq. (A.7) by $-i\varepsilon_2\exp[-i\mathbf{P}^-(\mathbf{p}_\parallel)\cdot\mathbf{x}_\zeta]$; and we finally multiply Eq. (A.8) by $-\mathbf{P}^-(\mathbf{p}_\parallel)\exp[-i\mathbf{P}^-(\mathbf{p}_\parallel)\cdot\mathbf{x}_\zeta]$. Here, the vector $\mathbf{P}^-(\mathbf{p}_\parallel)$ is defined by $\mathbf{P}^-(\mathbf{p}_\parallel) = \mathbf{p}_\parallel - \alpha_2(p_\parallel)\hat{\mathbf{x}}_3$. We add the resulting three equations and integrate the sum over $\mathbf{x}_\parallel$ to obtain

$$-(\varepsilon_2-\varepsilon_1)e^{-i\alpha_1(k_\parallel)d}\frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel|)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)}[\mathbf{P}^-(\mathbf{p}_\parallel)\times(\mathbf{P}^-(\mathbf{p}_\parallel)\times\mathbf{E}_0(\mathbf{k}_\parallel))]$$

$$-(\varepsilon_2-\varepsilon_1)\int\frac{d^2q_\parallel}{(2\pi)^2}e^{i\alpha_1(q_\parallel)d}\frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel|)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)}[\mathbf{P}^-(\mathbf{p}_\parallel)\times(\mathbf{P}^-(\mathbf{p}_\parallel)\times\mathbf{A}(\mathbf{q}_\parallel))]$$

$$= 2\varepsilon_2\alpha_2(p_\parallel)e^{-i\alpha_2(p_\parallel)d}\mathbf{F}^-(\mathbf{p}_\parallel). \tag{A.12}$$

We now turn to the boundary conditions at $x_3 = 0$, which can be written

$$\hat{\mathbf{x}}_3 \times \mathbf{E}^{(f)}(\mathbf{x}|\omega) = \hat{\mathbf{x}}_3 \times \mathbf{E}^<(\mathbf{x}|\omega)) \tag{A.13}$$

$$\hat{\mathbf{x}}_3 \times (\nabla \times \mathbf{E}^{(f)}(\mathbf{x}|\omega)) = \hat{\mathbf{x}}_3 \times (\nabla \times \mathbf{E}^<(\mathbf{x}|\omega)) \tag{A.14}$$

$$\varepsilon_2\hat{\mathbf{x}}_3 \cdot \mathbf{E}^{(f)}(\mathbf{x}|\omega) = \varepsilon_3\hat{\mathbf{x}}_3 \cdot \mathbf{E}^<(\mathbf{x}|\omega). \tag{A.15}$$

These three equations yield three equations for the Fourier amplitudes $\mathbf{F}^+(\mathbf{q}_\parallel), \mathbf{F}^-(\mathbf{q}_\parallel), \mathbf{B}(\mathbf{q}_\parallel)$, namely

$$\hat{\mathbf{x}}_3 \times \mathbf{F}^+(\mathbf{q}_\parallel) + \hat{\mathbf{x}}_3 \times \mathbf{F}^-(\mathbf{q}_\parallel) = \hat{\mathbf{x}}_3 \times \mathbf{B}(\mathbf{q}_\parallel) \tag{A.16}$$

$$i\hat{\mathbf{x}}_3 \times [\mathbf{Q}^+(\mathbf{q}_\parallel) \times \mathbf{F}^+(\mathbf{q}_\parallel)] + i\hat{\mathbf{x}}_3 \times [\mathbf{Q}^-(\mathbf{q}_\parallel) \times \mathbf{F}^-(\mathbf{q}_\parallel)] = i\hat{\mathbf{x}}_3 \times [\mathbf{Q}_2(\mathbf{q}_\parallel) \times \mathbf{B}(\mathbf{q}_\parallel)] \tag{A.17}$$

$$\varepsilon_2\hat{\mathbf{x}}_3 \cdot [\mathbf{F}^+(\mathbf{q}_\parallel) + \mathbf{F}^-(\mathbf{q}_\parallel)] = \varepsilon_3\hat{\mathbf{x}}_3 \cdot \mathbf{B}(\mathbf{q}_\parallel). \tag{A.18}$$

We eliminate $\mathbf{B}(\mathbf{q}_\parallel)$ from these equations and obtain the pair of equations

$$d_p(q_\parallel)[\mathbf{q}_\parallel \cdot \mathbf{F}^+(\mathbf{q}_\parallel)] - \Delta_p(q_\parallel)[\mathbf{q}_\parallel \cdot F^-(\mathbf{q}_\parallel)] = 0 \tag{A.19}$$

$$d_s(q_\parallel)[\hat{\mathbf{x}}_3 \cdot (\mathbf{q}_\parallel \times \mathbf{F}^+(\mathbf{q}_\parallel))] + \Delta_s(q_\parallel)[\hat{\mathbf{x}}_3 \cdot (\mathbf{q}_\parallel \times \mathbf{F}^-(\mathbf{q}_\parallel))] = 0, \tag{A.20}$$

where

$$d_p(q_\parallel) = \varepsilon_2\alpha_3(q_\parallel) + \varepsilon_3\alpha_2(q_\parallel) \tag{A.21a}$$

$$\Delta_p(q_\parallel) = \varepsilon_2\alpha_3(q_\parallel) - \varepsilon_3\alpha_2(q_\parallel) \tag{A.21b}$$

$$d_s(q_\parallel) = \alpha_3(q_\parallel) + \alpha_2(q_\parallel) \tag{A.21c}$$

$$\Delta_s(q_\parallel) = \alpha_3(q_\parallel) - \alpha_2(q_\parallel). \tag{A.21d}$$

When we substitute the expressions for $\mathbf{F}^+(\mathbf{q}_\parallel)$ and $\mathbf{F}^-(\mathbf{q}_\parallel)$ obtained from Eqs. (A.10) and (A.12), respectively, into Eqs. (A.19) and (A.20) we obtain the pair of equations for $\mathbf{A}(\mathbf{q}_\parallel)$

$$
\mathbf{p}_\parallel \cdot \left\{ d_p(p_\parallel) e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d} [\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}^+(\mathbf{p}_\parallel) \times \mathbf{E}_0(\mathbf{k}_\parallel))] \right.
$$

$$
\times \frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)} - \Delta_p(p_\parallel) e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d}
$$

$$
\times [\mathbf{P}^-(\mathbf{p}_\parallel) \times (\mathbf{P}^-(\mathbf{p}_\parallel) \times \mathbf{E}_0(\mathbf{k}_\parallel))] \frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)} \left. \right\}
$$

$$
+\mathbf{p}_\parallel \cdot \int \frac{d^2q_\parallel}{(2\pi)^2} \left\{ d_p(p_\parallel) e^{-i[\alpha_2(p_\parallel)-\alpha_1(q_\parallel)]d} [\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}^+(\mathbf{p}_\parallel) \times \mathbf{A}(\mathbf{q}_\parallel))] \right.
$$

$$
\times \frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)} - \Delta_p(p_\parallel) e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d}
$$

$$
\times [\mathbf{P}^-(\mathbf{p}_\parallel) \times (\mathbf{P}^-(\mathbf{p}_\parallel) \times \mathbf{A}(\mathbf{q}_\parallel))] \frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)} \left. \right\} = 0. \qquad \text{(A.22)}
$$

$$
\hat{\mathbf{x}}_3 \cdot \mathbf{p}_\parallel \times \left\{ d_s(p_\parallel) e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d} [\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}^+(\mathbf{p}_\parallel) \times \mathbf{E}_0(\mathbf{k}_\parallel))] \right.
$$

$$
\times \frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)}] + \Delta_s(p_\parallel) e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d}
$$

$$
\times [\mathbf{P}^-(\mathbf{p}_\parallel) \times (\mathbf{P}^-(\mathbf{p}_\parallel) \times \mathbf{E}_0(\mathbf{k}_\parallel))] \frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)} \left. \right\}
$$

$$
+\hat{\mathbf{x}}_3 \cdot \mathbf{p}_\parallel \times \int \frac{d^2q_\parallel}{(2\pi)^2} \left\{ d_s(p_\parallel) e^{-i[\alpha_2(p_\parallel)-\alpha_1(q_\parallel)]d} \right.
$$

$$
\times [\mathbf{P}^+(\mathbf{p}_\parallel) \times (\mathbf{P}^+(\mathbf{p}_\parallel) \times \mathbf{A}(\mathbf{q}_\parallel))] \frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)}
$$

$$
+\Delta_s(p_\parallel) e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d} [\mathbf{P}^-(\mathbf{p}_\parallel) \times (\mathbf{P}^-(\mathbf{p}_\parallel) \times \mathbf{A}(\mathbf{q}_\parallel))]
$$

$$
\times \frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)} \left. \right\} = 0. \qquad \text{(A.23)}
$$

We now write the amplitude vectors $\mathbf{E}_0(\mathbf{k}_\parallel)$ and $\mathbf{A}(\mathbf{q}_\parallel)$ in the forms

$$
\mathbf{E_0}(\mathbf{k}_\parallel) = \frac{c}{\omega}[\alpha_1(k_\parallel)\hat{\mathbf{k}}_\parallel + k_\parallel \hat{\mathbf{x}}_3] E_{op}(\mathbf{k}_\parallel) + (\hat{\mathbf{x}}_3 \times \hat{\mathbf{k}}_\parallel) E_{0s}(\mathbf{k}_\parallel) \qquad \text{(A.24)}
$$

$$
\mathbf{A}(\mathbf{q}_\parallel) = \frac{c}{\omega}[\alpha_1(q_\parallel)\hat{\mathbf{q}}_\parallel - q_\parallel \hat{\mathbf{x}}_3] A_p(\mathbf{q}_\parallel) + (\hat{\mathbf{x}}_3 \times \hat{\mathbf{q}}_\parallel) A_s(\mathbf{q}_\parallel). \qquad \text{(A.25)}
$$

The coefficients $E_{0p,s}(\mathbf{k}_\parallel)$ and $A_{p,s}(\mathbf{q}_\parallel)$ are the amplitudes of the p- and s-polarized components of the incident and scattered fields with respect to the planes of incidence and scattering, respectively. The substitution of these representations into Eqs. (A.22) and (A.23) yields a pair of equations for $A_p(q_\parallel)$ and $A_s(\mathbf{q}_\parallel)$, which we write as

$$
\int \frac{d^2q_\parallel}{(2\pi)^2} [M_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)A_p(\mathbf{q}_\parallel) + M_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)A_s(\mathbf{q}_\parallel)]
$$

$$
= -[N_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)E_{0p}(\mathbf{k}_\parallel) + N_{ps}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)E_{0s}(\mathbf{k}_\parallel)] \qquad \text{(A.26)}
$$

$$\int \frac{d^2q_\parallel}{(2\pi)^2} [M_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)A_p(\mathbf{q}_\parallel) + M_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)A_s(\mathbf{q}_\parallel)]$$
$$= -[N_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)E_{0p}(\mathbf{k}_\parallel) + N_{ss}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)E_{0s}(\mathbf{k}_\parallel)], \tag{A.27}$$

where

$$
\begin{aligned}
M_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \; & d_p(p_\parallel)e^{-i[\alpha_2(p_\parallel)-\alpha_1(q_\parallel)]d} \\
& \times [\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\alpha_1(q_\parallel) + p_\parallel q_\parallel] \frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)} \\
& -\Delta_p(p_\parallel)e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d} \\
& \times [\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\alpha_1(q_\parallel) - p_\parallel q_\parallel] \frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)}
\end{aligned}
\tag{A.28a}
$$

$$
\begin{aligned}
M_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \; & -\frac{\omega}{c}\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)_3 \\
& \times \Bigg\{ d_p(p_\parallel)e^{-i[\alpha_2(p_\parallel)-\alpha_2(q_\parallel)]d}\frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)} \\
& -\Delta_p(p_\parallel)e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)} \Bigg\}
\end{aligned}
\tag{A.28b}
$$

$$
\begin{aligned}
M_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \; & \frac{c}{\omega}\alpha_1(q_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)_3 \\
& \times \Bigg\{ d_s(p_\parallel)e^{-i[\alpha_2(p_\parallel)-\alpha_1(q_\parallel)]d}\frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)} \\
& +\Delta_s(p_\parallel)e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)} \Bigg\}
\end{aligned}
\tag{A.28c}
$$

$$
\begin{aligned}
M_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \; & (\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel) \\
& \times \Bigg\{ d_s(p_\parallel)e^{-i[\alpha_2(p_\parallel)-\alpha_1(q_\parallel)]d}\frac{I(\alpha_2(p_\parallel)-\alpha_1(q_\parallel)|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(q_\parallel)} \\
& +\Delta_s(p_\parallel)e^{i[\alpha_2(p_\parallel)+\alpha_1(q_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)+\alpha_1(q_\parallel))|\mathbf{p}_\parallel-\mathbf{q}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(q_\parallel)} \Bigg\}
\end{aligned}
\tag{A.28d}
$$

$$
\begin{aligned}
N_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = \; & d_p(p_\parallel)e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d} \\
& \times [\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\alpha_1(k_\parallel) - p_\parallel k_\parallel] \frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)} \\
& -\Delta_p(p_\parallel)e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d} \\
& \times [\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\alpha_1(k_\parallel) + p_\parallel k_\parallel] \frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)}
\end{aligned}
\tag{A.29a}
$$

$$N_{ps}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = -\frac{\omega}{c}\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{k}}_\parallel)_3$$

$$\times \left\{ d_p(p_\parallel)e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d}\frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)} \right.$$

$$\left. -\Delta_p(p_\parallel)e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)} \right\} \quad \text{(A.29b)}$$

$$N_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = \frac{c}{\omega}\alpha_1(k_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{k}}_\parallel)_3$$

$$\times \left\{ d_s(p_\parallel)e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d}\frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)} \right.$$

$$\left. +\Delta_s(p_\parallel)e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)} \right\} \quad \text{(A.29c)}$$

$$N_{ss}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = (\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\left\{ d_s(p_\parallel)e^{-i[\alpha_2(p_\parallel)+\alpha_1(k_\parallel)]d}\frac{I(\alpha_2(p_\parallel)+\alpha_1(k_\parallel)|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)+\alpha_1(k_\parallel)} \right.$$

$$\left. +\Delta_s(p_\parallel)e^{i[\alpha_2(p_\parallel)-\alpha_1(k_\parallel)]d}\frac{I(-(\alpha_2(p_\parallel)-\alpha_1(k_\parallel))|\mathbf{p}_\parallel-\mathbf{k}_\parallel)}{\alpha_2(p_\parallel)-\alpha_1(k_\parallel)} \right\}. \quad \text{(A.29d)}$$

These equations are the basis for the calculations reported in Ref. [16].

To transform them into equations describing the scattering of a surface plasmon polariton from a localized dielectric defect on a planar metallic surface we first set $d = 0$ and restrict $\zeta(\mathbf{x}_\parallel)$ to be a non-negative function of $\mathbf{x}_\parallel$. We also set $E_{0s}(\mathbf{k}_\parallel) \equiv 0$ because the incident field is p polarized, and then rewrite $I(\gamma|\mathbf{Q}_\parallel)$ as

$$I(\gamma|\mathbf{Q}_\parallel) = (2\pi)^2\delta(\mathbf{Q}_\parallel) + J(\gamma|\mathbf{Q}_\parallel), \quad \text{(A.30a)}$$

where

$$J(\gamma|\mathbf{Q}_\parallel) = \int d^2x_\parallel e^{-i\mathbf{Q}_\parallel \cdot \mathbf{x}_\parallel}(e^{-i\gamma\zeta(\mathbf{x}_\parallel)} - 1). \quad \text{(A.30b)}$$

The relevant elements of the matrices $\mathbf{M}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)$ and $\mathbf{N}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)$ then become

$$M_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = (2\pi)^2\delta(\mathbf{p}_\parallel-\mathbf{q}_\parallel)\frac{2\varepsilon_2\alpha_2(p_\parallel)}{\varepsilon_2-\varepsilon_1}[\varepsilon_1\alpha_3(p_\parallel)+\varepsilon_3\alpha_1(p_\parallel)]+\tilde{M}_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel)$$
$$\text{(A.31a)}$$

$$M_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \tilde{M}_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) \quad \text{(A.31b)}$$

$$M_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \tilde{M}_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) \quad \text{(A.31c)}$$

$$M_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = (2\pi)^2\delta(\mathbf{p}_\parallel-\mathbf{q}_\parallel)\frac{c^2}{\omega^2}\frac{2\alpha_2(p_\parallel)}{\varepsilon_2-\varepsilon_1}[\alpha_3(p_\parallel)+\alpha_1(p_\parallel)]+\tilde{M}_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) \quad \text{(A.31d)}$$

$$N_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = (2\pi)^2\delta(\mathbf{p}_\parallel-\mathbf{k}_\parallel)\frac{2\varepsilon_2\alpha_2(k_\parallel)}{\varepsilon_2-\varepsilon_1}[\varepsilon_3\alpha_1(k_\parallel)-\varepsilon_1\alpha_3(k_\parallel)]+\tilde{N}_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel)$$
$$\text{(A.32a)}$$

$$N_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = \tilde{N}_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel), \quad \text{(A.32b)}$$

where

$$\tilde{M}_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = d_p(p_\parallel)[\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\alpha_1(q_\parallel) + p_\parallel q_\parallel]$$
$$\times \frac{J(\alpha_2(p_\parallel) - \alpha_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(q_\parallel)}$$
$$-\Delta_p(p_\parallel)[\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\alpha_1(q_\parallel) - p_\parallel q_\parallel]$$
$$\times \frac{J(-(\alpha_2(p_\parallel) + \alpha_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(q_\parallel)} \tag{A.33a}$$

$$\tilde{M}_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = -\frac{\omega}{c}\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)_3$$
$$\times \left\{ d_p(p_\parallel)\frac{J(\alpha_2(p_\parallel) - \alpha_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(q_\parallel)} \right.$$
$$\left. -\Delta_p(p_\parallel)\frac{J(-(\alpha_2(p_\parallel) + \alpha_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(q_\parallel)} \right\} \tag{A.33b}$$

$$\tilde{M}_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = \frac{c}{\omega}(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{q}}_\parallel)_3\alpha_1(q_\parallel)$$
$$\times \left\{ d_s(p_\parallel)\frac{J(\alpha_2(p_\parallel) - \alpha_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(q_\parallel)} \right.$$
$$\left. +\Delta_s(p_\parallel)\frac{J(-(\alpha_2(p_\parallel) + \alpha_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(q_\parallel)} \right\} \tag{A.33c}$$

$$\tilde{M}_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) = (\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{q}}_\parallel)\left\{ d_s(p_\parallel)\frac{J(\alpha_2(p_\parallel) - \alpha_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(q_\parallel)} \right.$$
$$\left. +\Delta_s(p_\parallel)\frac{J(-(\alpha_2(p_\parallel) + \alpha_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{q}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(q_\parallel)} \right\} \tag{A.33d}$$

$$\tilde{N}_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = d_p(p_\parallel)[\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\alpha_1(k_\parallel) - p_\parallel k_\parallel]$$
$$\times \frac{J(\alpha_2(p_\parallel) + \alpha_1(k_\parallel)|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(k_\parallel)}$$
$$-\Delta_p(p_\parallel)[\alpha_2(p_\parallel)(\hat{\mathbf{p}}_\parallel \cdot \hat{\mathbf{k}}_\parallel)\alpha_1(k_\parallel) + p_\parallel k_\parallel]$$
$$\times \frac{J(-(\alpha_2(p_\parallel) - \alpha_1(k_\parallel))|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(k_\parallel)} \tag{A.34a}$$

$$\tilde{N}_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) = \frac{c}{\omega}(\hat{\mathbf{p}}_\parallel \times \hat{\mathbf{k}}_\parallel)_3\alpha_1(k_\parallel)$$
$$\times \left\{ d_s(p_\parallel)\frac{J(\alpha_2(p_\parallel) + \alpha_1(k_\parallel)|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) + \alpha_1(k_\parallel)} \right.$$
$$\left. +\Delta_s(p_\parallel)\frac{J(-(\alpha_2(p_\parallel) - \alpha_1(k_\parallel))|\mathbf{p}_\parallel - \mathbf{k}_\parallel)}{\alpha_2(p_\parallel) - \alpha_1(k_\parallel)} \right\}. \tag{A.34b}$$

Equations (A.26) and (A.27) now become

$$[\varepsilon_1 \alpha_3(p_\parallel) + \varepsilon_3 \alpha_1(p_\parallel)] A_p(\mathbf{p}_\parallel)$$

$$+ \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2 \alpha_2(p_\parallel)} \int \frac{d^2 q_\parallel}{(2\pi)^2} [\tilde{M}_{pp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) A_p(\mathbf{q}_\parallel) + \tilde{M}_{ps}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) A_s(\mathbf{q}_\parallel)]$$

$$= -\left\{ (2\pi)^2 \delta(\mathbf{p}_\parallel - \mathbf{k}_\parallel)[\varepsilon_3 \alpha_1(k_\parallel) - \varepsilon_1 \alpha_3(k_\parallel)] + \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_2 \alpha_2(p_\parallel)} \tilde{N}_{pp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) \right\} E_{0p}(\mathbf{k}_\parallel) \tag{A.35}$$

$$[\alpha_3(p_\parallel) + \alpha_1(p_\parallel)] A_s(p_\parallel)$$

$$+ \frac{\omega^2}{c^2} \frac{\varepsilon_2 - \varepsilon_1}{2\alpha_2(p_\parallel)} \int \frac{d^2 q_\parallel}{(2\pi)^2} [\tilde{M}_{sp}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) A_p(\mathbf{q}_\parallel) + \tilde{M}_{ss}(\mathbf{p}_\parallel|\mathbf{q}_\parallel) A_s(\mathbf{q}_\parallel)]$$

$$= -\frac{\omega^2}{c^2} \frac{\varepsilon_2 - \varepsilon_1}{2\alpha_2(p_\parallel)} \tilde{N}_{sp}(\mathbf{p}_\parallel|\mathbf{k}_\parallel) E_{0p}(\mathbf{k}_\parallel). \tag{A.36}$$

We now make use of the analytic continuations

$$\alpha_1(k_\parallel) = -i\beta_1(k_\parallel) \tag{A.37a}$$
$$\alpha_1(q_\parallel) = i\beta_1(q_\parallel) \tag{A.37b}$$
$$\alpha_2(q_\parallel) = i\beta_2(q_\parallel) \tag{A.37c}$$
$$\alpha_3(q_\parallel) = i\beta_3(q_\parallel), \tag{A.37d}$$

where

$$\beta_j(q_\parallel) = [q_\parallel^2 - \varepsilon_j(\omega/c)^2]^{\frac{1}{2}}, \quad Re\beta_j(q_\parallel) > 0, \quad Im\beta_j(q_\parallel) < 0. \tag{A.38}$$

With the definitions

$$J(i(\beta_2(p_\parallel) - \beta_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{q}_\parallel)$$
$$= \int d^2 x_\parallel e^{-i(\mathbf{p}_\parallel - \mathbf{q}_\parallel) \cdot \mathbf{x}_\parallel} (e^{(\beta_2(p_\parallel) - \beta_1(q_\parallel))\zeta(\mathbf{x}_\parallel)} - 1)$$
$$= J^{(+)}(\beta_2(p_\parallel) - \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel) \tag{A.39a}$$

$$J(-i(\beta_2(p_\parallel) + \beta_1(q_\parallel))|\mathbf{p}_\parallel - \mathbf{q}_\parallel)$$
$$= \int d^2 x_\parallel e^{-i(\mathbf{p}_\parallel - \mathbf{q}_\parallel) \cdot \mathbf{x}_\parallel} (e^{-(\beta_2(p_\parallel) + \beta_1(q_\parallel))\zeta(\mathbf{x}_\parallel)} - 1)$$
$$= J^{(-)}(\beta_2(p_\parallel) + \beta_1(q_\parallel)|\mathbf{p}_\parallel - \mathbf{q}_\parallel) \tag{A.39b}$$

and

$$A_p(\mathbf{q}_\parallel) = \frac{a_p(\mathbf{q}_\parallel)}{\varepsilon_3 \beta_1(q_\parallel) + \varepsilon_1 \beta_3(q_\parallel)} \tag{A.40a}$$

$$A_s(\mathbf{q}_\parallel) = \frac{a_s(\mathbf{q}_\parallel)}{\beta_1(q_\parallel) + \beta_3(q_\parallel)}, \tag{A.40b}$$

the substitution of Eqs. (A.37)-(A.40) into Eqs. (A.35) and (A.36) yields Eqs. (4). We have used the result that $\varepsilon_3 \beta_1(k_\parallel) + \varepsilon_1 \beta_3(k_\parallel) = 0$. This is the dispersion relation for surface plasmon polaritons at a planar vacuum (ε_1)-metal (ε_3) interface.

Acknowledgments