Solar Wind Thermally Induced Magnetic Fluctuations

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A kinetic description of Alfvén-cyclotron magnetic fluctuations for anisotropic electron-proton quasistable plasmas is studied. An analytical treatment, based on the fluctuation-dissipation theorem, consistently shows that spontaneous fluctuations in plasmas with stable distributions significantly contribute to the observed magnetic fluctuations in the solar wind, as seen, for example, in [S. D. Bale *et al.*, Phys. Rev. Lett. 103, 211101 (2009)], even far below from the instability thresholds. Furthermore, these results, which do not require any adjustable parameters or wave excitations, are consistent with the results provided by hybrid simulations. It is expected that this analysis contributes to our understanding of the nature of magnetic fluctuations in the solar wind.

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Magnetic field fluctuations associated with collisionless dissipation is a fundamental topic in space and astrophysical plasmas. In these environments, the time scale for energy exchange due to Coulomb collisions τ_c can often be much larger than the characteristic time scales of wave-particle interactions and plasma microinstabilities. Consequently, in such cases, collisions are not frequent enough to keep particle distributions near a Maxwellian. In particular, in the solar wind, where the tenuous plasma is mostly not at local thermodynamic equilibrium (LTE), *in situ* measurements of the proton velocity distribution functions (VDF) show that deviations from LTE can be represented by bi-Maxwellian VDFs with different temperatures $T_\parallel$ and $T_\perp$ relative to the mean magnetic field $\boldsymbol{B}_0$ [1]. Indeed, in the past few years, there has been quite a lot of interest about the statistics of solar wind as a function of the plasma beta $\beta_\parallel = 8\pi n k_B T_\parallel / B_0^2$ and the anisotropy $T_\perp/T_\parallel$, and, in particular, about the magnetic fluctuations as shown for example in Ref. [2]. Here, n is the proton density.

In this Letter, we present an analytic kinetic description of Alfvén-cyclotron fluctuations in an anisotropic, but quasistable, electron-proton plasma close and below the linear instability thresholds. Using the fluctuation-dissipation theorem [3–6], we show that spontaneously generated magnetic field fluctuations follow the same ($\beta_\parallel$, $T_\perp/T_\parallel$) pattern as those observed in the solar wind as presented in Fig. 1(b) of Ref. [2], thus representing the final state of the collisionless relaxation of proton anisotropy. As far as we know, this is the first theoretical model that addresses the problem of fluctuations as a function of thermal anisotropy with respect to a background magnetic field in quasiequilibrium plasmas, thus providing an analytic account for such observations.

Although several previous works show that the proton VDFs can sometimes be more complicated than bi-Maxwellians, for example, displaying a clear core and beam [7], in what follows we will assume a bi-Maxwellian VDF since the measurements we analyze below are organized in terms of these two parameters, namely $T_\parallel$ and $T_\perp$. The theory of thermally induced fluctuations for more complicated VDFs will be considered elsewhere.

To characterize the plasma, we note that Coulomb scattering and pressure-anisotropy instabilities are well-known kinetic mechanisms for shaping particle distributions towards more isotropic states ($T_\perp \approx T_\parallel$). However, whenever the ratio of τ_c to the expansion time of the solar wind τ_e is smaller than one, collisional relaxation becomes ineffective in establishing LTE [8]. Indeed, the relative importance of Coulomb collisions in isotropization of protons has been estimated in detail using a large data set of independent observations from the Wind spacecraft at 1 AU [2]. Classifying the data according to the collisional age parameter $\tau = \tau_e/\tau_c$ [9,10], it has been shown in Refs. [2,11] that old parcels of solar wind plasma ($\tau \gg 1$) contain isotropic and Maxwellian proton VDFs. In contrast, young parcels exhibit anisotropic VDFs, such that $T_\perp/T_\parallel \neq 1$, confirming that anisotropic plasmas are relatively collisionless.

It is known from linear Vlasov theory that the anisotropic proton VDFs in the low-collisional plasma parcels can become unstable to microinstabilities and drive electromagnetic fluctuations. In turn, the resulting growing waves scatter particles and the plasma would then be constrained close to marginal states [12,13]. The anisotropy-driven kinetic instabilities, which are probably most important for regulating the proton VDFs, are the Alfvén cyclotron and oblique mirror instabilities when anisotropy $T_\perp/T_\parallel > 1$, and the parallel and oblique firehose instabilities when $T_\perp/T_\parallel < 1$. Measurements, both in the solar wind and laboratory, show that the predicted thresholds from linear theory and computer simulations represent observable bounds on VDFs anisotropies [14]. The corresponding instabilities are characterized by the proton temperature anisotropy $T_\perp/T_\parallel$ and the proton parallel plasma $\beta_\parallel$. In the $(\beta_\parallel,\ T_\perp/T_\parallel)$ plane, it appears that for $\beta_\parallel \geq 1$, the experimental data are bounded by the oblique mirror and firehose instabilities, and that the observed range of $T_\perp/T_\parallel$ values narrows for increasing values of $\beta_\parallel$ [2,7,14–16].

Although linear theory can describe the empirical bounds for proton anisotropies, a significant number of data points are localized far below from the unstable boundaries. Such sites in the $(\beta_\parallel,\ T_\perp/T_\parallel)$ plane correspond to relative stable VDFs so that according to the linear theory, no wave activity from instabilities should be expected. However, power of magnetic fluctuations with small amplitude is perfectly detectable from samples of solar wind plasmas [2], raising, thus, the question of how these fluctuating fields originate. In principle, according to linear theory, these fluctuations should damp exponentially and, therefore, they should not be detectable. But linear theory fails to describe the proper competition between a nonlinear transfer and dissipation which may preserve these fluctuations. An alternative approach is to consider the linear stability as an initial value problem, in which transient growth of small initial perturbations can scatter particles, reaching at the end a state with finite fluctuations far from the linear instability thresholds [17].

Spontaneous fluctuations are a fundamental and ubiquitous feature of many-body systems, and of particular importance to many laboratory and space plasmas (e.g., Refs. [18,19]). The connection of stable VDFs and spontaneous emission of Alfvén-cyclotron fluctuations was already suggested in Refs. [20,21]. However, a quantitative description of the relation between the magnetic fluctuation intensity and the corresponding parameters in the $(\beta_\parallel,\ T_\perp/T_\parallel)$ plane has so far not been available. A variant considers an ensemble of initially unstable states in novel quasilinear simulations with local B field intensity variations [22]. They found that distribution of the saturated states qualitatively reproduces the anisotropy-beta relationship. A similar result can be recovered if an imposed level of magnetic field fluctuations is used in Vlasov simulations [23]. Although complementary to the present Letter, these findings cannot explain the inherent thermal fluctuations, since a large wave amplitude must be excited first.

Hence, we now construct an analytic description of the thermally induced fluctuations. We assume transverse electromagnetic fluctuations propagating along the average magnetic field $\boldsymbol{B}_0$ in a homogeneous plasma of bi-Maxwellian protons and cold electrons. Then, the transverse part of the Fourier-transformed Maxwell equations reduces to

$$\Lambda_\pm^{(0)} E_\pm = \frac{4\pi}{i\omega} J_\pm, \tag{1}$$

where $E_\pm = E_x \pm iE_y$ and $J_\pm$ are the transverse electric field and total transverse current; $\Lambda_\pm^{(0)} = 1 - c^2k_\parallel^2/\omega^2$; and ω and $k_\parallel$ are the usual wave frequency and wave number, respectively.

Following Sinteko's approach [5], we assume that $J_\pm$ is proportional to $E_\pm + \tilde{e}_\pm$, where $\tilde{e}_\pm$ can be understood as a microscopic or secondary electric field. We then obtain

$$E_\pm = \left[\frac{\Lambda_\pm^{(0)}}{\Lambda_\pm} - 1\right]\tilde{e}_\pm, \tag{2}$$

where $\Lambda_\pm$ is given in Ref. [24] as

$$\Lambda_\pm = 1 - \frac{c^2k_\parallel^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\left[A + (\xi + A\xi_\pm)Z(\xi_\pm) \pm \frac{\omega}{\Omega}\right]. \tag{3}$$

Here, $\omega_p^2 = 4\pi e^2 n/m$ is the square of the proton plasma frequency, $\Omega = eB_0/mc$ is the proton cyclotron frequency, m is the proton mass, $\xi = \omega/k_\parallel u_\parallel$, $\xi_\pm = (\omega \pm \Omega)/k_\parallel u_\parallel$, $u_\parallel^2 = 2k_B T_\parallel/m$ is the square of the parallel thermal speed, and $A = T_\perp/T_\parallel - 1$ is a measurement of the proton thermal anisotropy. $Z(\xi)$ is the standard plasma dispersion function [25]. $\Lambda_\pm = 0$ represents the dispersion relation for transverse waves. Following the indications of Ref. [26] p. 254, the average of $E_\pm$ over the phase space of the system near our quasiequilibrium, with different temperatures $T_\perp$ and $T_\parallel$, can be written in terms of the correlation function as

$$E_\pm - E_\pm^* = -\frac{1}{k_B T_\perp}[\langle J_+ E_\pm^*\rangle\tilde{e}_+ + \langle J_- E_\pm^*\rangle\tilde{e}_-]. \tag{4}$$

Combining Eqs. (1), (2), and (4), and using the Maxwell-Faraday equation $|B_\pm| = (ck_\parallel/\omega)|E_\pm|$, then the classical (dimensionless) spectral distribution of transverse magnetic field fluctuations can be expressed as

$$\frac{n\Omega}{B_0^2}\langle|B_\pm|^2\rangle = -\beta_\parallel \frac{T_\perp}{T_\parallel}\frac{c^2k_\parallel^2\Omega}{\omega^3}\mathrm{Im}\left(\frac{1}{\Lambda_\pm}\right). \tag{5}$$

The longitudinal magnetic fluctuations are identically zero for parallel propagation. Notice that for thermally isotropic systems $T_\perp/T_\parallel = 1$, Eq. (5) reduces to those considered in Refs. [5,21,27].

Equation (5) can be evaluated for frequencies and wave numbers where $\Lambda_\pm \neq 0$ and applies as long as wave instabilities are not present. In Fig. 1(a), we plot Eq. (5)

in logarithmic color scale for $\beta_\parallel = 0.6$ and $T_\perp/T_\parallel = 1.4$. We also show the dispersion relation $\Lambda_- = 0$ for the same parameters, from which a weak ion-cyclotron instability develops with a maximum growth rate $\mathrm{Im}(\omega/\Omega) \simeq 2\times10^{-5}$. Besides the normal modes crossing at $\mathrm{Re}(\omega/\Omega) = 0$, Eq. (3) also allows for an infinite number of heavily damped waves under and above the proton cyclotron frequency, denominated as higher-order modes (HOM) [28]. The four least damped HOM are also plotted in Fig. 1(a), whose damping is proportional to $k_\parallel$ and to their slopes. The fluctuation level decreases 100 times between two consecutive HOM, and they concentrate near the normal modes of the system. It is interesting to note, although not plotted here, that the trianglelike zone defined by the HOM—and, thus, the magnetic fluctuations structure—is broadened (narrowed) for higher (lower) values of $\beta_\parallel$, whereas the real part of the normal modes does not change qualitatively, a result that agrees with both proton [21] and electron scales [27] for isotropic plasmas.

In Fig. 1(b), we show the anisotropic case $T_\perp/T_\parallel = 0.1$ for the same $\beta_\parallel = 0.6$, where a weak firehose instability develops, with a maximum growth rate $\mathrm{Im}(\omega/\Omega) \simeq 2.6\times10^{-5}$. Qualitatively, the HOM and fluctuations maintain their structure for fixed $\beta_\parallel$ and different values of $T_\perp/T_\parallel$, but the magnetic fluctuations concentrate mostly along the unstable mode. Although instabilities are present in both Figs. 1(a) and 1(b), their growth time scale is still small compared to one proton gyroperiod.

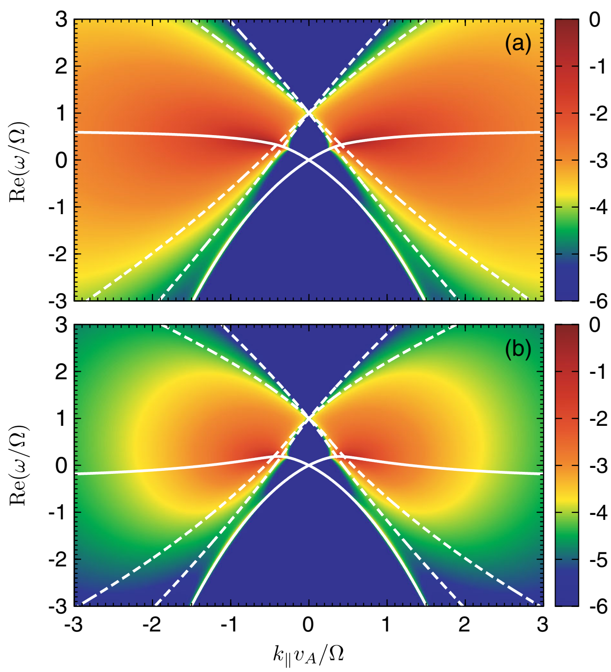

FIG. 1 (color online). Magnetic field fluctuations $n\Omega\langle|B_-|^2\rangle/B_0^2$, normalized to its maximum value, in color logarithmic scale, calculated from Eq. (5) for $v_A/c = 10^{-4}$, $\beta_\parallel = 0.6$, and (a) $T_\perp/T_\parallel = 1.4$, and (b) $T_\perp/T_\parallel = 0.1$. The over plotted curves are normal modes crossing at $\mathrm{Re}(\omega/\Omega) = 0$ (Alfvén-cyclotron and fast modes, solid curves) and four of the least damped HOM crossing at $\mathrm{Re}(\omega/\Omega) = 1$ (dashed curves), as calculated from $\Lambda_- = 0$ in Eq. (3).

Due to the thermal motion of the particles, thermally induced electromagnetic fluctuations can occupy a relevant part of the Fourier spectrum even in absence of free energy for plasma instabilities, as can be seen in Fig. 2(a), where we plot the transverse magnetic spectrum, produced in a hybrid simulation for $\beta_\parallel = 0.6$ and $T_\perp/T_\parallel = 1.4$. The hybrid code [21,29] treats ions as fully kinetic particles and the electrons as a massless charge neutralizing fluid, representing a collisionless, homogeneous, and magnetized plasma. A 1D simulation box with 2048 grid cells is used with 1000 particles per cell, a system length of $502.6 v_A/\Omega$, where $v_A = B_0/\sqrt{4\pi n m}$ is the Alfvén speed, and a time step of $\Omega\Delta t = 0.02$. The boundary conditions are periodic for both particles and fields, and protons are loaded as a bi-Maxwellian distribution at $\Omega t = 0$. In Fig. 2(a), we can clearly see the normal circularly polarized modes in the simulation. A relevant level of thermally induced fluctuations appears for a wide range of frequencies and wave numbers, and in fact, contains a large fraction of the electromagnetic energy. The magnetic fluctuations are enhanced near the Alfvén-cyclotron mode, which quickly lose importance relative to the fluctuations for large values of $k_\parallel$. From linear theory, we expect a high damping on the Alfvén-cyclotron mode for these values of $k_\parallel$, reason why the fluctuations show a greater activity than that of the Alfvén-cyclotron mode. A similar result can be seen in

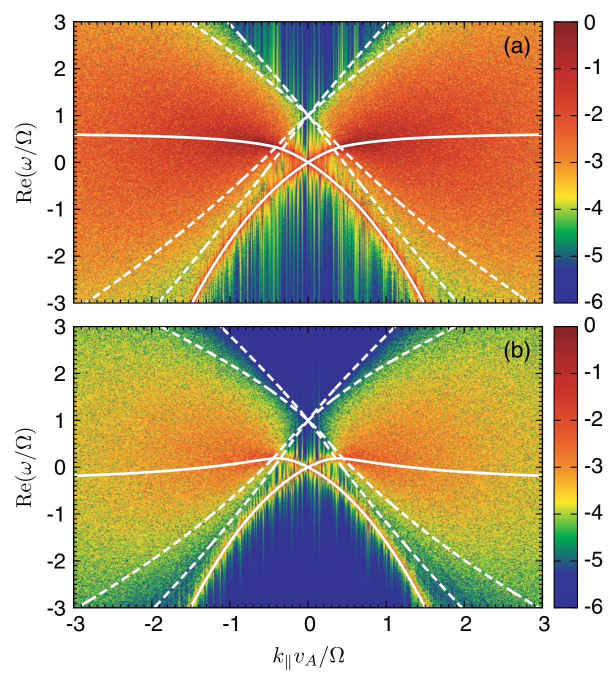

FIG. 2 (color online). Power spectrum (normalized to its maximum value) in logarithmic color scale of the transverse magnetic field fluctuations from a 1.5D hybrid simulation for the same parameters as Fig. 1.

Fig. 2(b) for $\beta_\parallel = 0.6$ and $T_\perp/T_\parallel = 0.1$, in agreement with the theoretical results presented in Fig. 1.

We now show analytically that these transverse magnetic fluctuations can be a relevant contribution to the measured fluctuations in the solar wind, by defining the total fluctuating magnetic energy density as

$$W_\pm = \frac{1}{8\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_\parallel d\omega \langle |B_\pm|^2\rangle. \tag{6}$$

In Fig. 3(a), we plot the dimensionless quantity $nv_A W_-/(\Omega B_0^2)$ as a function of $\beta_\parallel$ and $T_\perp/T_\parallel$ in logarithmic color scale. We have restricted the calculation to regions far from the Alfvén cyclotron or firehose instabilities (which are calculated numerically through the dispersion relation by setting $\mathrm{Im}(\omega_{\max}/\Omega) = 10^{-4}$ as the threshold value for instability). The magnetic energy is clearly enhanced near the instability thresholds, and its features are similar to solar wind observations near 1 AU, as reported in Fig. 1(b) of Ref. [2]. Figure 3(a) shows that the proton VDF can produce considerable amounts of thermally induced fluctuations, even in quasiequilibrium states.

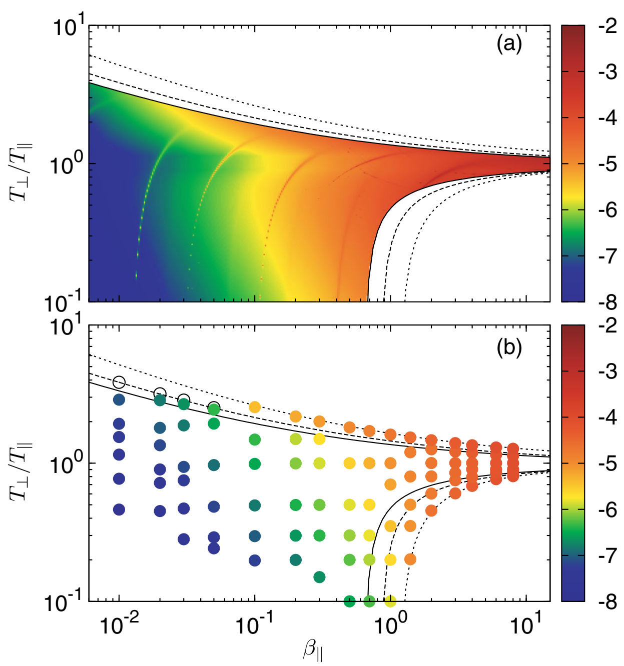

FIG. 3 (color online). (a) Fluctuating magnetic energy $nv_A W_-/(\Omega B_0^2)$, given by Eq. (6) and normalized to its maximum value, that can be related with the measured statistics of $\delta B/B$ given in Fig. 1(b) of Ref. [2]. (b) Final state distribution from hybrid simulations as a function of the initial $\beta_\parallel$ and final $T_\perp/T_\parallel$. Included are also contours of the maximum growth rate $\mathrm{Im}(\omega/\Omega) = 10^{-4}$ (solid curves), 10^{-3} (dashed curves), and 10^{-2} (dotted curves) for parallel propagation. The set of initial states that change significantly from the final states are shown as open circles. Color logarithmic scales are used in both figures.

We have also conducted a systematic study of hybrid simulations in the ($\beta_\parallel$, $T_\perp/T_\parallel$) plane in order to compare the production of thermal fluctuations with our analytic treatment. Figure 3(b) shows the final value of the space-averaged magnetic field intensity $(\delta B_\perp/B_0)^2$, as function of the initial value of $\beta_\parallel$ and final value of $T_\perp/T_\parallel$, obtained from a simulation ensemble of initial states in the ($\beta_\parallel$, $T_\perp/T_\parallel$) plane. In order to isolate the spontaneous fluctuations from collective instabilities, the initial points were chosen below and at the instability threshold borders. As a consequence, most of the final points are not significantly displaced from the initial ones as noted in Refs. [12,13], except for a few cases close to the ion-cyclotron instability threshold at $T_\perp/T_\parallel > 1$ and $\beta_\parallel < 10^{-1}$, whose initial state we represent as open circles in the figure. In that region, these points move downwards below the instability threshold, suggesting that a slight isotropization process occurs.

Since the thermal fluctuation level Eq. (5) essentially scales with $\beta_\parallel$ and $T_\perp/T_\parallel$, we expect an absence of thermal noise for low $\beta_\parallel$, and the anisotropy relaxation process can take place without the effects of thermal scattering. The free energy available for isotropization is nevertheless very low and thus scarce energy exchange between perpendicular and parallel directions can occur. This fact can explain why the experimental solar wind data cannot be fitted with one threshold curve only [20]. It is interesting to note that these results are consistent with those of Ref. [12], in which a sufficiently large initial proton temperature anisotropy can reduce the proton anisotropy to values slightly below the instability threshold through scattering by the enhanced fluctuations. However, it is difficult to use this mechanism to explain the magnetic fluctuations far below from the instability threshold, unless we consider other processes such as streaming for small plasma beta [30,31]. On the other side, for $\beta_\parallel > 0.1$ the noncoherent scattering produced by the spontaneous fluctuations is not negligible and competes with the scattering from normal modes, making the relaxation of the thermal anisotropy a much slower process, contrary to the fast relaxation predicted by quasilinear theory. Furthermore, even though any unstable mode can be observable only if it exceeds the spontaneous fluctuation level, a finite normal mode wave activity is maintained until the end of the simulations, and as a result both the Alfvén-cyclotron and the right-handed or fast modes survive linear damping (see Figs. 1 and 2).

In conclusion, we point out that spontaneous magnetic fluctuations resulting from stable VDFs can significantly contribute to observations in the solar wind. These fluctuations can also play an important role in the isotropization processes in a magnetized ion plasma, thus providing a possible explanation to the limited anisotropies of solar wind data, complementing studies on marginal instabilities [14,31], and quasilinear [22], and Vlasov simulations [23].

We also mention on the high correlation between the magnetic fluctuations structure and the highly damped solutions of the dispersion relation (HOM), even for anisotropic $T_{\perp}/T_{\parallel} \neq 1$ systems, a result which generalizes those suggested in Refs. [21,27].

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