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Teaching modeling skills using a massively multiplayer online mathematics game

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Abstract One important challenge in mathematics education is teaching modeling skills. We analyze the logs from a game-based learning system used in a massively multiplayer online tournament. Students had to detect an input–output pattern across 20 rounds. For each round, they received an input and had 2 minutes to predict the output by selecting a binary option (2 points if correct, −1 otherwise), or writing a model (4 points if model prediction was correct, −4 otherwise), or refraining (1 point). Thousands of 3rd to 10th grade students from hundreds of schools simultaneously played together on the web. We identified different types of players using cluster analysis. From 5th grade onwards, we found a cluster of students that wrote models with correct predictions. Half of the 7th to 10th grade students that detected patterns were able to express them with models. The analysis also shows diffusion within the teams of modeling strategies for simple patterns.

Keywords online tournaments · data mining · modeling · common core standards · PISA mathematical competencies

1 Introduction

One important challenge in K-12 mathematics and science education is developing students' modeling skills. For example, the new Common Core State Standards [9] in the US establish

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Standards for Mathematical Practice, which mandate that students must be able to do modeling. Students have to be able to "identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas … and analyze those relationships mathematically to draw conclusions." The Common Core State Standards define modeling as, "The process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions." OECD/PISA [21, 22] also establishes modeling as one of its cognitive mathematical competencies. The 2003 PISA Assessment Framework has three defined competence clusters: the reproduction cluster, the connections cluster, and the reflection cluster, all of which include modeling. For the reproduction cluster, modeling involves "recognizing, recollecting, activating, and exploiting well-structured familiar models … and elementary communication about model results." For the connections cluster, modeling involves "structuring the field or situation to be modeled" and "translating reality into mathematical structures in contexts that are not too complex." For the reflection cluster, modeling involves "monitoring the modeling process and validating the resulting model."

One of the problems with teaching modeling skills is that it is a relatively new requirement. Teachers do not have enough experience with teaching modeling skills, as they have neither been educated nor trained to do so. Most teachers do not know what modeling skills are and almost all elementary school teachers have never heard the word "modeling" in the context of mathematics. Furthermore, after being told what modeling is, most teachers are not able to provide examples of interesting real life situations that could be modeled. For example, [18] reviews the experience in eight countries and concludes with four obstacles present in countries with modeling in their curriculum: teachers' perceptions of mathematics, teachers' understanding of modeling, a lack of adequate textbooks and modeling tasks, and a lack of adequate assessment methods. We believe that it would be extremely helpful for teachers if they knew a great variety of examples of models appropriate for different contexts and student interests. However, this goal might be too daunting a task. Instead, it would be much better if teachers were taught a small number of simple classes of models. These classes of models have to be powerful enough to be easily adjusted to different interests and types of student knowledge and to the different mathematics and science content that the curricula require.

In this work, we present a multimedia e-learning based strategy to teach modeling skills to thousands of elementary and middle school teachers and students. During 2 months of preparation, teachers and students can access multimedia resources and synchronous e-learning technologies to learn modeling skills. Then, in a national synchronous tournament, they play a game that requires them to find patterns and build a model. The model belongs to a simple but powerful class of mathematical models. The class of models used can be directly connected to multiple real life contexts and can also easily include different mathematic content as demanded by the curricula. Furthermore, it can be easily turned into an engaging game that can be played in teams, and that promotes learning mathematical concepts and strategies through team cooperation.

This model belongs to a very simple class of models, since it attempts to predict the simplest possible outcome: a binary outcome— or, as we will see later, a black or white outcome. For example, this can represent the challenge of predicting whether a child will behave well or not, or predicting if a bug will turn left or right, or predicting if a team will win or lose a soccer game, etc. These are situations that students frequently encounter in their lives.

The model is powerful enough to account for real world interrelation between variables. In typical situations, there are usually variables that could help predict the outcome. In the first of the previous examples, predicting a child's behavior, it could be important to know what kind of friends the child has, his present mood, and the types of optional activities available to them. In the following example, in order to predict a bug's change of direction it could be important to know the location of other bugs, the location of food, and the location of predators. In the third example, to predict the outcome of a soccer game, it would be helpful to know the ranking of both teams and each team's previous performances, including average number of goals per game, etc. These types of variables that relate to future outcomes are also common aspects of most students' everyday life.

Additionally, a history of data can be progressively included, which is very important to finding patterns and making decisions. Normally, in real life situations, there are previous cases with information on the variables and associated outcomes. This information is critical for finding input–output patterns, trying to match the present conditions to similar conditions in the past, and making good decisions. According to our experience, this poses an interesting challenge for children; one that they can easily relate to multiple personal experiences from their everyday life. In fact, we organized a tournament with a special version of this model adjusted for the 2010 FIFA Soccer World Cup. It was played by thousands of students, teachers, and parents. The specific results of this version are not reported here, but the enthusiasm for building automatic betting models was so great that it attracted the interest of local media outlets. Before each game in the World Cup, the press published the statistics of the bets submitted by our students' models.

This class of model can easily include different mathematic content, from whole numbers, to decimals, fractions, operations, and powers, as well as algebra and statistics. It is very important that fractions and algebra can easily be introduced given that students are known to struggle with these concepts [30].

Furthermore, this class of model can be easily packaged as a game. We converted it into a betting game, where each round the students receive a new input, and after looking for patterns have to decide what the outcome will be for the present input. In order to make it more tangible, the game is presented as betting on what is inside a box. This experience is similar to trying to guess what is inside a Christmas or birthday gift. The characteristics of the box, like its size, give some hints, but the history of previous gifts you have received from your parents does as well. We turned this situation into a betting game establishing that what was inside was simply a white or black cell, and that the color of the cell had been selected by following a rule. A computer program read the length, width, height, and color of the box, and used the rule to compute the color of the associated cell. Boxes are selected randomly for each round. After the students see the box they have to bet on the color of the cell inside.

The game has been used in several massively multiplayer online tournaments. In this article we extend our previous analysis of the data gathered from the fourth tournament [5].

2 Why games?

Learning requires attention and abundant practice [26], but studies of student affective states show that mathematics classes are the least attractive [17]. Students prefer to talk and spend time with friends. Normally, teachers and parents try to encourage students to study by using extrinsic motivation. Examples of long-term extrinsic motivation include suggesting that several years later they will succeed in getting a good job, and short-term motivation includes allowing children to play only after completing the proposed tasks. A great challenge in education is to design intrinsically motivating academic activities.

A natural strategy is to use games, since games are a learning mechanism used not only by humans but also by other mammals, birds, and even reptiles [8]. For example, rough-and-tumble play prepares animals for fighting and hunting, social games like playing with dolls, as well as games with rules like soccer and chess all help prepare for social interaction. At least in some mammals, distinct neural systems have already been identified which contribute to the generation of rough-and-tumble play [23]. Since ancient times, people have realized the great educational value of play. Plato wrote, "Teach your children not in a compulsory way but by game playing, and you will be able to better tell the natural abilities of each one." However, determining exactly how games help in this context has turned out to be a less than straightforward task. One approach in game playing research has been to study its adaptive value. That is, considering that young animals spend much of their time and energy playing, about 10 % of time and energy for humans [6], it is necessary to identify the benefits and determine if they outweigh the costs. There is some consensus that play is an inborn mechanism allowing young animals to prepare for adult life: to hunt, mate, interact with others, lead, work in teams, behave appropriately in social hierarchies, etc. According to Bjorklund [6], games also teach how to learn, how to generate solutions to new problems, and provide a means for discovery. Also, games make learning easier, more enjoyable, and more flexible.

In the case of humans, it is important to distinguish between games without rules and games with rules [25]. The former are very similar to those played by other mammals, like rough-and-tumble play, play fighting, etc. These games are most popular among preschoolers. Games with rules are the most frequently played at school, mainly by older students. These include sports games like soccer and basketball, and board games like checkers and chess. In the 1960's, the New Game Movement arose, proposing the use of multi-player games in schools to promote intra-team collaboration as well as inter-team competition [32]. This movement has published several books, promoted tournaments, and has had a major impact on school playground games and physical education games.

With mathematics and science games, we pursue two goals. On the one hand, there is an affective goal: to motivate students to approach mathematics and science, see them as attractive, and perceive them as something they are capable of doing and enjoying. One of the most popular strategies is to employ multimedia and videogames to induce a state of pleasure called "flow" [11], an enjoyable state of great effortless attention [20, 27]. On the other hand, we have a cognitive goal: to help student players understand concepts that are biologically secondary [15, 16]. These are very different from biologically primary concepts, which are very basic and learned by everybody even without explicit instruction. Examples of biologically primary concepts are speaking, walking, number sense, geometry of shapes, sense of risk, etc. Learning biologically secondary concepts like fractions, algebra [4], natural selection, inertia, laws of motion, etc. is the real educational challenge.

A well-designed game is a powerful metaphor [10]. It is an activity that maps abstract biologically secondary concepts into a biologically primary activity. For instance, chess is a metaphor for battle. In this paper we analyze Magical Surprises, a mathematical game aimed at guessing what is inside given opaque boxes. This game is a metaphor for pattern recognition, modeling, and statistical concepts [1, 2].

3 Why massively multi-player online games?

This class of model could have been implemented as a single-player game. Instead, we designed a multiplayer version where students play simultaneously against others and a ranking is computed and shown for each round. Furthermore, we designed it as a game played by teams. Each member of the team has to bet on the color of a numbered cell assigned to him/her. We use 12 cells, so that teams are formed by 12 students. The boxes are all the same, but for each cell a particular rule is used. Therefore, each student of the team has a different pattern detection problem than the rest of their team. This serves as individualized accountability for each student. Since time is short, the probability of having another team member solve the problem and suggest what to bet is very low. There is little time to analyze efficiently all the data and search for patterns.

Game-based learning systems have the potential to promote collaborative learning [19]. We designed a team version of the game with the goal of promoting collaboration and peer-to-peer instruction of mathematical strategies for how to play better. This is expected and promoted in the months before the tournament. Two synchronous preparation tournaments are held to promote team cooperation and learning of the required mathematical contents and strategies. As is typical with synchronous learning modes, students' queries are quickly answered [19], allowing for a quick diffusion of mathematical strategies which allow for better play. During these preparation events, a game instructor and coordinator broadcasts online streaming video commentaries of the game, suggests strategies, and answers online questions from all the connected players. This is an excellent and powerful opportunity to discuss and analyze the concepts and strategies needed to improve the performance of the game with a large community of students and teachers; the potential impact is enormous. This is particularly interesting for teaching modeling skills, since modeling is new and unknown to most teachers.

Computer-based games in science and mathematics available on the market are primarily single-player or few-player ones. This means that they do not take advantage of online networking opportunities that could provide strong motivation based on the appeal of a massively multiplayer game, like many of the recent, highly-popular, commercial games. Playing single-player or few-player games does not allow for a sense of identity or for a global community to arise. In this work we report on the use of a massively multi-player online game, where thousands of students from different regions of Chile simultaneously and synchronously participated in tournaments. According to game designer Chris Crawford [8], the massively multiplayer game "is one of the most interesting ideas that emerged from the introduction of the computer in game playing."

The most common massively multiplayer games are RPGs (Role playing games). A comprehensive literature review [35] and an Internet search show the absence of this type of game designed especially for science education. There are, however, studies on the educational impact of popular massively multiplayer games designed with exclusively commercial goals. For example [12], analyzes the impact of the games Everquest and Second Life played by 36 university students in the classroom [7]. studied the impact of these games on military training, particularly in decision-making, leadership, and conflict resolution. The Institute for Advanced Research in Human Sciences and Technology (HSTAR) of Stanford has recently begun to investigate the potential of massive multiplayer games commercially available to support the teaching of mathematics for elementary and middle school students [13, 14]. Another exception is NASA [28, 29], which through the NASA Learning Technologies Project has developed Moonbase Alpha, a massively multiplayer online game recently released to the educational system. However, we do not know of any studies of its impact on learning. Today, followers of the New Game Movement [24] propose to extend the movement into the digital age with massive multiplayer online games, drawing lessons from successful physical games.

4 National tournaments of online mathematics and science games

Here we report on the implementation of the National Tournaments of Online Mathematics and Science games in Chile. A similar competition in the country is the Mathematical and Scientific Olympic Games. But, unlike the tournaments, Olympic Games do not use information and communication technologies and are intended only for very talented students. In an internet survey we found only one type of tournament involving online games. This was the DimensionM [36] tournament, an educational game with a 3D engine. Two teams, each of about 15 students selected from one school, compete in a gym. Taking advantage of the highly appealing 3D engine, students drive vehicles or virtual agents and every so often they must answer mathematical questions in order to go on playing. These tournaments are not massively multiplayer games because there are just two small teams playing, all located in the same gym, and most of the students participate only through cheering and supporting the teams. That is, only the viewers and supporters are massive in number, as in a basketball or soccer championship.

This paper reports the results of a 3 year effort implementing tournaments involving massively multiplayer games to support mathematical contents in statistics and algebra with students from 3rd to 10th grade, and now also covers physics and biology. Semester after semester, participation has been strongly increasing, including thousands of students from all regions of the country. In each tournament, students in the same grade simultaneously and synchronously play the same game online. Every 2 min a round closes, and all students must submit their bets during these 2 min lapses. The game schedule is defined through pairs of grade levels: 3rd and 4th graders play at 09:00, 5th and 6th graders play at 10:30, 7th and 8thgraders play at 12:00 and the 9th and 10th graders play at 13:30 h. The game is played in a 12×1 mode, in which one computer is assigned to a team of 12 students, with a team representative entering the personal bet of each member. Games are so designed to optimize computing resources and to let many students play even in schools with small labs, few computers, and very narrow bandwidth (the computer can be connected to the internet even with just minimal bandwidth). To run the game, each team of 12 students uses one computer only, which must have a permanent Internet connection, Windows or Linux operating system, Java 1.6, a 1024×768 resolution video card, and at least 512 MB of RAM. On the other hand, we use a data base server (PosgreSQL 8.3) for user information (teams, ages, schools and passwords), a server for the game logic (Reddwarf), and a file server (Nginx).

Our massively multiplayer online mathematics games contain several multimedia-based strategies to support teacher and student modeling learning. For instance, we have developed a registration system where teachers register their schools and teams online, there is a national ranking system by region and county, an automatic system for generating participation certificates, an empathetic intelligence system that recognizes and acknowledges good moves from each team or student in a team, and gives encouragement to those who might not be winning. Since it is impossible to offer in situ training to teachers and students from all over the country, we have developed a web video and an online training strategy with teams playing synchronously. First, students and teachers get acquainted with the game by reading instructions and watching videos available on the web. Then, on two occasions, approximately 40 and then 20 days before the tournament, an online synchronous game is played. On these occasions, a live web transmission is broadcasted where a trainer comments on the game and the strategies, and makes suggestions to the different teams. At the same time, teams receive via chat online answers to their questions and also appropriate personalized feedback. In addition, the game is open on the internet for teams to practice at any time. Here we analyze the data of the final of the third tournaments that took place on October 23, 2009, where 3,432 students participated out of the nearly 10,000 students registered.

5 Game description

In this paper we report the results of the game Magical Surprises, originally designed and implemented in 1995 [1], but now adapted to a web version. In this version, the game is played online simultaneously by thousands of students. Nearly 10,000 students, coming from 356 schools from all the regions in Chile, registered for the fourth tournament. The children play in teams of 12 students from the same class, and send their personal bets to a central server every 2 min. Each competition lasts about 1 h and takes place within school hours. Students are pre-registered and trained by their school teacher. The teacher is responsible for reviewing curriculum contents useful for improving performance in the game and coaches those students participating in trial tournaments that occur a few weeks before the national tournament. All bets are recorded in a database that enables us to later analyze each student's bet sequences. In this game, each team member must try to find a pattern. For each round, a box appears in a different size and color, containing a card with 12 cells, one for each team member. Each player must bet on the color, white or black, of their corresponding cell. Once the time is over, after about 2 min, the box opens to display its contents. If the student succeeds, they get 2 points, if they do not, then they lose a point. The Magical Surprises version used in the third and fourth tournament has two additional variants: first, each player can bet "gray", meaning "I do not know," and earn one point. Moreover, each player can also bet by writing a model which is expressed as a rule. For example:

```
IF
length + 2.5 x width > 3
THEN
cell color = white
ELSE
cell color = black.
```

If applying the rule to the current box data predicts the right color for his/her cell, then the player earns 4 points. If the rule predicts the wrong color for the slot, then he loses 4 points. So, betting by writing a rule is appealing but also more risky because the loss is higher. The team's accumulated points are displayed immediately in a ranking with several tabs: national ranking, county ranking, level ranking and students ranking within the team.

Each student's decisions over the course of the game and associated team response times are registered to be subsequently analyzed. The game server tracks all the bets: "white", "black", "gray," or an explicit rule. This way we can estimate not only whether the student seems to have discovered a pattern, but also if they can describe this pattern explicitly in algebraic language. This type of betting allows us to discern explicit versus implicit pattern discovery. Our register allows us to investigate cognitive aspects such as student strategy selection [30, 31] in different rounds and its evolution as the game progresses. For example, [3] reports an analysis of strategies used in the second tournament with a previous version of Magic Surprises.

6 Learning

As the game progresses, the students begin to hit the right cell colors. From round 10 onwards, students are hitting significantly above random guessing. The games have different degrees of difficulty depending on the level. For Level 1, intended for 3d and 4th grade students, Magic Surprises was loaded with two variables describing externally the boxes. One is the (three valued) *color* variable and the other is the dimension variable *length*, which is always a whole number. Moreover, patterns are defined by a single variable. For example, in one cell when the external box *color* is red, then the chosen cell is black. On another cell the pattern could be: when the *length* of the box is less than 5 cm then the chosen cell is white. Since there are 12 cells and each student of the team is in charge of one cell, then the type of pattern can be different for different students and the difficulty can vary. However, the difficulty of cell 1 is identical for all teams, just as the difficulty of cell 2 is identical for all teams, etc. In all other levels, 4 variables were used: box *color*, *height, width,* and *length.* This expands the number of possible patterns. Moreover, variables were decimal or fraction valued. Additionally, at higher levels, patterns were introduced that were defined by two variables. For example, when the *length* of the box is at least twice the *width*, the chosen cell is white.

A quick analysis shows how the scores achieved vary in a statistically significant way with respect to some monitored variables. As shown in Table 1, student performance depends on *type of school* (except for the first level corresponding to grades 3 and 4). Moreover, for the two higher levels, *grade*, *age, gender, cell,* and *type of school* impacted the game scores in a statistically significant way. We also noticed that for each level (which comprises two grades), *grade* has an impact on the score of success.

7 Clusters of players

From the stored information that contains the bets of each student in each round, we tried to find types or clusters of players, i.e. to determine whether there are clearly defined ways to play which we could use to classify the students. In order to do so we considered several basic variables that record performance and type of bets made:

- *Number of "gray" bets from round 1 onwards*
- *Number of "gray" bets from round 7 onwards*
- *Number of "gray" bets from round 15 onwards*
- *Number of hits from round 1 onwards*
- *Number of hits from round 7 onwards*
- *Number of hits from round 15 onwards*

Table 1 For each level, p-values of the null hypothesis that the average performance on the game is equal for different grades, ages, genders, and types of school.

Level	Grade	Age	Gender	Cell	Type of school
(3rd & 4th)	0.000	0.000	0.467	0.853	0.148
(5th & 6th)	0.007	0.569	0.201	0.11	0.000
(7th & 8th)	0.000	0.035	0.049	0.016	0.000
(9th & 10th)	0.002	0.033	0.000	0.000	0.000

- *Number of bets using a rule from round 1 onwards*
- *Number of bets using a rule from round 7 onwards*
- *Number of bets using a rule from round 15 onwards*
- *Hits using rule from round 1 onwards*
- *Hits using rule from round 7 onwards*
- *Number of hits using rule from round 15 onwards*
- *Number of hits before the first rule was used*
- *Number of errors before the first rule was used*
- Number of *"gray"* bets before the first rule was used

With these variables we used the *Two-Step algorithm* to search for clusters using SPSS 15.0 software. This proposal is similar to [37], searching for clusters or groupings of individuals in ways that ensure statistically optimal grouping. This analysis is different from that undertaken in [3], which was based on determining the frequency of strategies pre-defined by the researchers [33, 34]. Three main clusters were found:

– "White-Blackists", who bet white or black most of the time.
– "Grayists" who primarily bet "gray."
– "Rulists" that from a given round onwards bet using rules.

For 3rd and 4th graders, "white-blackists" dominate. This cluster decreases for older students, but consistently remains the largest. The "rulist" cluster appears for 5th and 6th graders onwards, which indicates that the students are using algebraic language to express the patterns they have found and input them as a mathematical model. About 20 % of 9th and 10th grade students are already "rulists."

As seen in Figure 1, white-blackists bet mainly white or black, although in the first two rounds we found roughly one third of "gray" bets. Success rate stabilizes to 60 % after round 10, which indicates a level of learning above the random hitting. Later, two sub-clusters are analyzed within this cluster: those who detect the right pattern and those who do not.

The grayist cluster is the conservative cluster, consisting of students that do not take risks. From round 4 onwards they bet gray about 60 % of the time. These students play without showing discovery of a pattern.

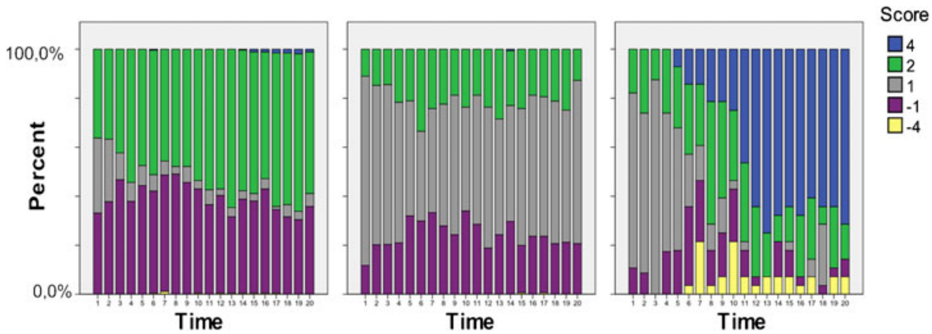

Figure 1 (*Left*) White-blackist cluster for 5th and 6th graders, consisting of students betting black or white. (*Center*) Grayist Cluster for 5th and 6th grade conservative students who take no risks. (*Right*) Cluster of students belonging to grades 7 and 8 who play following a rule that explicitly defines a pattern. Score 4: student bets and hits following a rule; score 2: student bets white or black and hits; score 1: student bets gray; score −1: student bets black or white and does not hit, score −4: student uses a rule but does not hit.

The “rulist” cluster starts by playing gray, but by round 8 they are betting using a rule about 20 % of the time, and from round 13 onwards they bet using a rule over 80 % of the time, making very few mistakes. These students have learned the pattern and know how to make it explicit using algebraic language.

In Figure 2 we can see that “rulists” appear more frequently in paid private schools, then in subsidized private schools and only rarely in municipal schools. There is also a gender bias. There are more “white-blackists” among girls, and there are more “grayists” and “rulists” among boys.

From the 8th round onwards more than 50 % of the “rulists” predict correctly, and from the 11th round, they are right more than 80 % of the time. Moreover, from Figure 1 we can conclude that the “grayists” do not show evidence that they would have detected any pattern. Furthermore, Figure 1 shows that the “white-blackists” hit 60 % of the time from round 12 onwards. A cluster analysis of “white-blackists” shows two sub-clusters: the sub-cluster “white-blackist1” composed by those students who are detecting a pattern and therefore play better than at random, and the sub-cluster “white-blackist2” formed by those “white-blackist” students who are not detecting any pattern and play at random. “White-blackist1” represents 42.3 % of the “white-blackists.” “White-blackist2”, the “random bettors,” form 57.7 % of the “white-blackists”. Interestingly, those “white-blackists” who detect patterns and hit better than random are concentrated in the lowest level: 3^{d} and 4th grade. This can be explained because many of the older students in higher levels who detect a pattern make it explicit by giving a rule, and so they do not appear in the cluster of “white-blackists”.

8 Learning modeling skills

In the tournaments we manipulated the complexity of the different cells in order to help understand the conceptual difficulties, the modeling skills, and the reactions to uncertainty of the students. We are particularly interested in understanding two modeling skills: ability to detect patterns and the capacity to express the patterns using algebraic language. We want to understand how both abilities depend on the complexity of the hidden input–output patterns.

With this objective in mind, we designed different kinds of cells. The simplest one is “Color”. In this case, the cell is black or white according to a rule that only includes the color

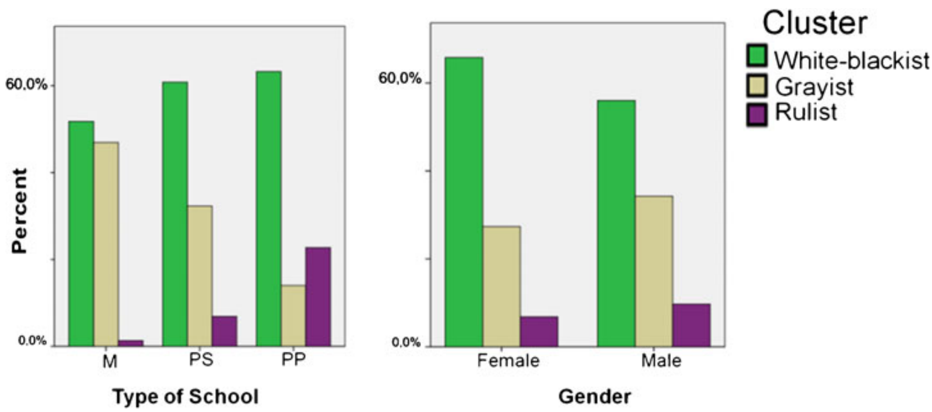

Figure 2 (*Left*) Percentages by type of school (M municipal, PS subsidized private, PP paid private), for students of all levels. (*Right*) Percentage by gender for all levels.

of the box. For example*: if color=red or blue, then cell is white, otherwise it is black*. According to log data this is the simplest kind of pattern for the student. Color is perceptually a very salient feature, and one immediately recognizes a pattern with colors. In any case, in the game students only know that there is a hidden input–output pattern specified by a rule, but they know neither the exact rule nor the kind of cell they have. They only know that cells have been colored according to a rule that includes the three dimensions of the box as well as its color. Therefore, they have to discover whether the color of the box is involved.

Another kind of cell is "Unilateral". This is a cell where the pattern is designed with one inequality and that includes only one of the three numeric variables: *height, length,* and *width* of the box. For example*, if width <2 then cell is white, otherwise it is black*.

Another kind of cell is "Bilateral". This is a cell whose color is defined by a rule involving one of the three numerical variables and two inequalities. For example, *if width >3 and width <5 then cell is white, otherwise it is black.*

Next is the kind "Bivariate", with cells whose color is defined by a rule with two variables, such as if *width <2 and height >5 then cell is white, otherwise it is black.*

Finally, there is the kind "Bivariate2", with cells defined by an inequality with a linear combination of two numerical variables. For example, *if 2width + 4height < 8 then cell is white, otherwise it is black*.

According to these definitions, our assumption regarding complexity is that the color cells are the simplest, followed by unilateral, then bilateral, then bivariate1, with the most complex cells being bivariate2 cells. For this reason, in 3rd and 4th grade we included only color and univariate. As shown in Figure 3, there are more white-blackist1 when the kind of cell is color. In 4th and 6th grade, we included bilateral cells as well. This type of cell turned out to have similar clusters as unilateral cells. In 7th and 8th grade, we also included

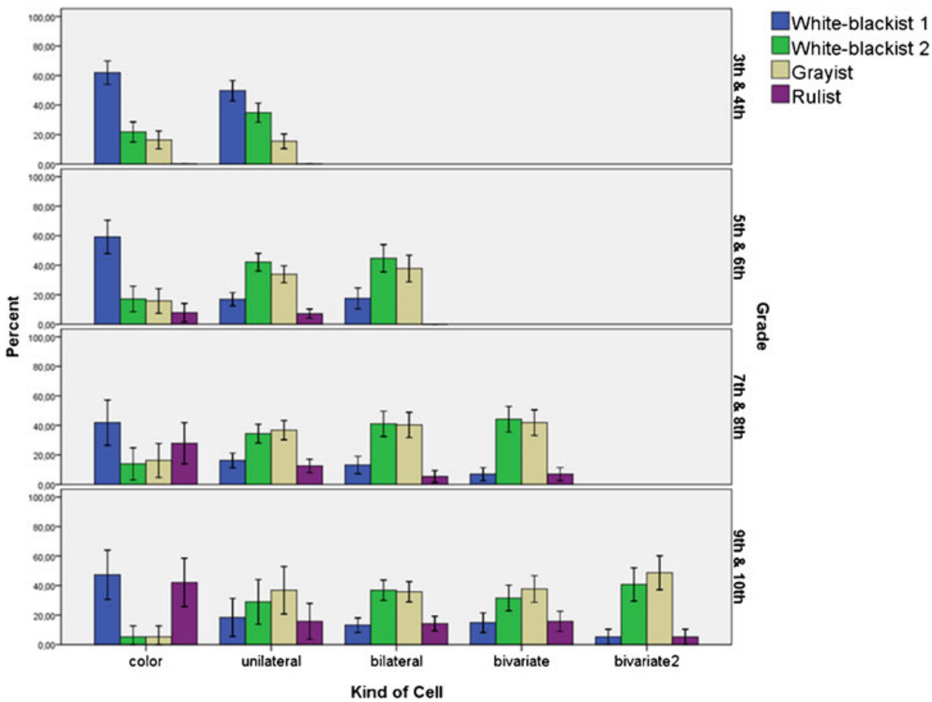

Figure 3 Percentages of the clusters of students for each grade and kind of cell.

bivariate cells. In the 9th and 10th grades, we also included the bivariate2 cells. This kind of cell was more difficult. The proportions of the rulist and white-blackist1 clusters were smaller than on the rest of the cells. It is interesting to note that from 7th grade onwards, half of the students that detected the hidden patterns were able to correctly predict them with a model.

Another of our objectives is to explore the ability to disseminate knowledge and promote team learning of this new kind of web-based instruction framework. Having thousands of students from hundreds of schools simultaneously playing facilitates the game coordinator's ability to disseminate mathematics and science concepts and strategies. By video streaming in the preparation phase and during the synchronous practice tournaments, the coordinator can comment on different possible strategies and their impacts during the game. This setup integrates game-based, internet, and multimedia learning. The instruction provided is much more relevant to the students since they can apply the concepts and strategies immediately and obtain results in the game in a maximum of 2 min. Internet and multimedia resources are provided to the students to help them improve their strategies. Moreover, in the synchronous practice tournaments, the game coordinator also promotes the idea that this is a team tournament and that the team members, along with their teachers, should also analyze strategies and discuss the strengths and weakness of the strategies in order to adequately prepare for the tournament. Anecdotal evidence, videos sent in by teachers, and an increasing number of schools participating in the tournaments all suggest that students are having fun competing against teams from other schools.

We also want to detect if there is peer-to-peer tutoring occurring, in addition to instruction and support from the teachers. It is practically impossible for us to directly observe if such peer-to-peer diffusion of strategies is effectively achieved in-between practice tournaments. However, by analyzing the strategies of team members playing with cells of the same kind, we can obtain an estimate as to whether the students imitate or learn from each other, and as a result play similar strategies. In other words, whether or not there is some kind of diffusion of strategies inside the teams, most probably occurring during the months of preparation leading up to the tournament. The goal is to test if the probability that a team member will use a certain type of strategy changes or not based on whether there is another member of the team using the same strategy with a similar kind of cell. To make this comparison, we need kinds of cells with at least two cells of the kind in a given grade. For example, there was only one color cell from 7th grade onwards and only one unilateral cell for 9th and 10th graders. Therefore it does not make sense to look for diffusion with those cells.

For each kind of cell and grade, we tested if the number of students on the teams belonging to the rulist cluster followed a binomial distribution. For example, if in a given grade there were 2 cells of certain kind, if the proportion of rulist is p, and if the pairs of students of these cells played independently, then we would expect a binomial B(2,p). We found evidence of diffusion from grade 7 onwards, but only for cells that are not too complex. In 7th and 8th grade, the test rejected independence for unilateral cells (p-value=0.000012), and in 9th and 10th grade the test rejected independence for bilateral cells (p-value=0.000078). This means that in the aforementioned grades, there was a diffusion of the rulist strategy. With younger students, there was seemingly no diffusion.

It is important to underline that this does not mean that students copied the results. Each one had a different cell with a different hidden pattern, and there was very limited time for one student to analyze the data of another student. Moreover, they did not know that they had cells that were similar to those of other members of their team. Also, if a teacher had helped their students, then we would have detected dependence for other classes of cells as well. These statistics suggest that during the preparation for the tournament some teams learned how to detect input–output patterns, how to express them with rules, and then internally diffused those strategies. However, they were successful in sharing only the simplest types of hidden input–output patterns.

9 Conclusions

We have presented a multimedia e-learning based strategy to teach modeling skills to thousands of elementary and middle school teachers and students. This is a game-based learning system played synchronously on the web. We have also analyzed the logs of thousands of students playing an online tournament. This tournament has been designed to help students develop modeling skills and other concepts required by school curricula, like the number strand contents such as whole numbers, decimals, fraction, and their location on the number line; operations, powers, and graphs; contents of the algebra strand, such as formulas, regions defined by one inequality with one variable or two variables, and linear inequations; and contents from statistics, like different graphs and data processing. In terms of helping to develop modeling skills, like in real life, students in the game have to search for input–output patterns, progressively build a model, use it to make predictions and decisions, get results, compare with reality and then adjust the model, and repeat the whole cycle multiple times. In the language of the OECD/PISA modeling competencies, players have to recognize, recollect, activate, and exploit a type of model and its results, transform them into a mathematical structure using variables, inequations, and rules, as well as monitor, adapt, and validate their particular models according to the results. The modeling skills promoted in the game are the same as those required for data mining, one of the most important areas in the 21st century given its applications [31]. The game is basically an opportunity to introduce pattern mining, considered as being [31] "amongst the most important and challenging techniques in data mining."

The data recorded on students' bets in the different rounds allow us to estimate whether students are in fact able to detect patterns and write a mathematical model with correct predictions. Since the required data analysis techniques to detect patterns in the game and express them as mathematical models are not directly taught in schools, we hope that the online web material, videos, training sessions, and trial tournaments have made it possible for teachers and students to apply their mathematical knowledge to pattern discovery problems.

Additionally, from the betting information registered, four distinct types of groupings or clusters of players appear. The "rulist" cluster is the most interesting one, because it entails a greater understanding of the detected patterns. These students are characterized by betting with an algebraic rule. We noted that in general when students use rules they guess right, which means not only that they are using algebraic language, but that they are building effective mathematical models. Indeed they are building models and writing mini-computer programs that create bets for them. From 7th grade onwards, about half of the students that detect patterns are able to express them in algebraic language and bet with models. Future studies should investigate situations in which although students respond erroneously, evidence suggests they are indeed learning and that this will become noticeable in their future responses. This evidence would come from successes in the near future

This more advanced strategy is most common in private schools with students of higher socio-economic status and with higher scores on the national standard assessments. Subsidized private school students have the second highest proportion of "rulists". They also score better on the national assessments than the public schools, but perform lower than the full-tuition private schools. Therefore, there is a parallel between the use of more advanced strategies and the performance in national mathematics assessments. There is also a gender difference: a higher proportion of males than females use the more advanced strategy. This data supports an important gender bias present in student mathematical performance in Chile. Through analyzing the logs we have found evidence that there is some diffusion of modeling strategies between students of the same teams. This is true for students from 7th grade onwards.

Teachers have not been formally interviewed in order to record the impact of the tournaments on their students, but there has been increasing interest in the tournaments throughout the years since the first tournament was held, with 600 students participating. Last year, the first Latin-American tournament was run with participants from different countries www.torneoslatinoamericanos.org. The data from this new tournament will allow us to explore differences across countries as well.

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