A diversified multiobjective GA for optimizing reservoir rule curves

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Abstract

The paper develops an efficient macro-evolutionary multiobjective genetic algorithm (MMGA) for optimizing the rule curves of a multi-purpose reservoir system in Taiwan. Macro-evolution is a new kind of high-level species evolution that can avoid premature convergence that may arise during the selection process of conventional GAs. MMGA enriches the capabilities of GA to handle multiobjective problems by diversifying the solution set. Simulation results using a benchmark test problem indicate that the proposed MMGA yields better-spread solutions and converges closer to the true Pareto frontier than the nondominated sorting genetic algorithm-II (NSGA-II). When applied to a real case study, MMGA is able to generate uniformly spread solutions for a two-objective problem involving water supply and hydropower generation. Results of this work indicate that the proposed MMGA is highly competitive and provides a viable alternative to solve multiobjective optimization problems for water resources planning and management.

Keywords: Multi-purpose reservoir; Rule curves; Hedging rules; Multiobjective genetic algorithms; Pareto frontier

1. Introduction

The operation of a reservoir system involves a complex decision-making process, integrating many variables and objectives as well as considerable risk and uncertainty [1]. Although detailed release policies for each reservoir in a system can be prescribed with help from simulation models and optimization tools, a more desirable approach for real-time operation is to have operational rules that minimize the effect of impending supply shortages. Fixed rules governing the operation of a reservoir system commonly are presented in the form of graphs or tables [2]. For multiple-purpose reservoirs, operating policies and the associated rule curves commonly define the desired storage volumes and reservoir releases at any time of the year as a function of existing storage volumes, the time of year, water demands, hydropower demands, and expected inflows [3]. In Taiwan, most reservoirs are operated by rule curves, and management objectives usually include maximizing water supply and hydropower generation. This paper addresses the problem of minimizing water shortages while maximizing hydropower generation through a multiobjective optimization problem.

Optimizing water management strategies is complex, as some impact relations are nonlinear and interdependent [4]. A basic problem of multiobjective optimization is that the various objectives may be conflicting and incommensurable, or may affect different groups of people or interests. The resolution of this problem is often difficult, particularly when system managers cannot easily perceive the tradeoffs among the several purposes, given the existing conditions relevant to system operation [5]. In multiobjective optimization there is no single optimal solution. Instead, the interaction of multiple objectives yields a set of efficient or non-dominated solutions, known as Pareto-optimal solutions, which give a decision maker more flexibility in the selection of a suitable alternative. Traditionally, multiobjective optimization problems have been solved using the

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weighting method or the ε-constraint method [6]. In the weighting method, a single objective function is obtained from the weighted sum of the original multiple objectives. The Pareto-optimal set is obtained by varying the weight associated with each objective and solving the problem sequentially. In the constrained method, all but one of the objectives is incorporated into the constraint set. The objectives included in the constraint set are varied parametrically from the lower bound to the upper bound in order to trace out the Pareto frontier [7]. Multiobjective GAs can alleviate some of the shortcomings attributable to the mathematical programming methods described above. In particular, GAs are able to generate large portions along the Pareto frontier in one iteration, which may render them more efficient than the ε-constraint method. Additionally, GAs can identify convex and non-convex points on the Pareto frontier [8]. The weighting method, on the other hand, has been reported to present problems identifying non-convex sections of the non-dominated frontier. Using diversity-preserving mechanisms, GA can find widely different Pareto-optimal solutions [9]. Multiobjective evolutionary algorithms (MOEAs) have been applied to diverse problems in the field of water resource management in the past two decades. Reed et al. [10] present a design algorithm for the nondominated sorting genetic algorithm-II (NSGA-II) parameters when solving a long-term groundwater quality-monitoring problem. In [10] the objectives are minimizing the cost of sampling as well as the error in estimating contaminant plume characteristics. Kapelan et al. [11] and Cheung et al. [12] provide examples of MOEAs applied to water distribution system design and rehabilitation. Objectives in these cases relate to sustaining minimum pressure head requirements at various nodes in the network, while maintaining rehabilitation costs at a minimum. The work presented by Kapelan et al. [11] is noteworthy because it introduces a stochastic component to NSGA-II, and implements it in studying uncertainty related to demand forecasts as well as to various physical characteristics of the pipes used in network rehabilitation. In the area of watershed management, Yandamuri et al. [13] and Dorn and Ranjithan [14], among others, couple MOGAs with water quality simulation models in order to find trade-offs between required contamination levels in river networks, treatment costs and land use alternatives. The preferred algorithm in most of the aforementioned studies is NSGA-II. According to Dorn and Ranjithan [14], NSGA-II outperforms the Pareto-Archived Evolution Strategy (PAES) and the Strength Pareto Evolutionary Algorithm (SPEA) in terms of computational efficiency and achieving a better spread along the final Pareto frontier. Nevertheless, NSGA-II failed to approximate a "true" Pareto frontier obtained with a single-objective GA and the ε-constraint method [14]. Finally, Kim et al. [15] present an example of MOEA application to multi-reservoir systems. They apply NSGA-II to four interconnected reservoirs in the Han River Basin. The objectives include maximizing reservoir releases and storage levels subject to continuity constraints and end-of-period storage constraints. To the best of our knowledge, there exists no literature that deals with applications of MOEAs to optimize reservoir rule curves.

Macro-evolution algorithm (MA) is a relatively recent heuristic inspired by the dynamics of species extinction and diversification over large time scales. MA has been found to be successful for a wide variety of optimization tasks [16]. In this paper, we present a methodology, called the "macro-evolutionary multiobjective genetic algorithm" (MMGA), which allows the MA to deal with multiobjective optimization problems because of the capability of diversity preservation.

In the next section, we begin by introducing several genetic-based multiobjective optimization techniques and a discussion of their strengths and weaknesses. Then, we present the macro-evolutionary multiobjective genetic algorithm (MMGA) and perform an experimental multiobjective function optimization to test the MMGA capabilities. After that, we use MMGA for obtaining the Pareto-optimal rule curves for a multiobjective reservoir system management problem that considers water supply and hydropower generation in the Tan-Shui River Basin in northern Taiwan. The last section presents a discussion about the findings of this work, as well as future research directions.

2. Multiobjective optimization and genetic algorithms

Multiobjective optimization can be defined as the problem of finding a vector of decision variables that satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions result from the mathematical description of performance criteria, and in most cases are in conflict with each other. A formal notation of Pareto optimality is provided by Fonseca and Fleming [17]. Consider, without loss of generality, the minimization of the m components f_k, $k = 1, \ldots, m$, of a vector function $\mathbf{f}$ of an n-dimensional decision variable $\mathbf{x}$ in a universe U, where $\mathbf{f}(x) = [f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})]$. A decision variable $\mathbf{x}_u \in U$ is said to be Pareto optimal if and only if there is no $\mathbf{x}_v \in U$ for which $\mathbf{v} = \mathbf{f}(\mathbf{x}_v) = [v_1, \ldots, v_m]$ dominates, that is, there is no $\mathbf{u} = \mathbf{f}(\mathbf{x}_u) = [u_1, \ldots, u_m]$ such that

$$\forall i \in \{1, \ldots, m\}, \ v_i \leqslant u_i \quad \text{and} \quad \exists i \in \{1, \ldots, m\}, \ v_i < u_i. \tag{1}$$

In practical applications, searching for all Pareto-optimal solutions is a difficult and time-consuming process. In most cases, the decision maker has no prior knowledge about the shape of the search space in a real-world setting. Usually, the objective functions are inter-dependent, incommensurable, nonlinear and discontinuous, while the Pareto frontier is non-convex, discrete and non-uniform. In general, it is impossible to find an analytical expression of the curve or surface that contains the non-dominated solutions.

More recently genetic algorithms (GAs) have been applied to the search for multi-criteria optima [18,19]. GA has the distinct advantage of being able to handle multiobjective problems that other gradient-based optimizers have failed to solve [20]. GAs seem particularly suitable for solving multiobjective optimization problems because they deal simultaneously with a set of possible solutions. This allows the identification of several members of the Pareto-optimal set in a single run, in contrast to traditional mathematical programming techniques. Additionally, GAs are less susceptible to the shape or continuity properties of the Pareto frontier, whereas these are real concerns for mathematical programming techniques. Schaffer [18] proposed a Vector Evaluated GA (VEGA) for finding multiple solutions to multiobjective problems in a single run. This was achieved by selecting appropriate fractions of parents according to each of the objectives, separately. However, the population tends to split into species particularly strong in each of the objectives if the Pareto trade-off surface is concave and VEGA lacks an explicit mechanism to maintain diversity. Fourman [21] also addressed multiple objectives in a non-aggregating manner. The selection was performed by comparing pairs of individuals, each pair according to one of the objectives. The objective was selected randomly in each comparison. Non-dominated sorting GA (NSGA), first proposed by Goldberg [19], is another Pareto-based fitness assignment method. Goldberg [19] also suggested that using a niching mechanism, such as sharing, would allow the GA to maintain individuals all along the trade-off surface. Several sharing mechanisms were tested to this effect by Sareni and Krahenbuhl [22], who show that clearing achieves good results at maintaining population diversity. Over the years there have been criticisms of NSGA, however. For example, the high complexity of non-dominated sorting makes it computationally expensive for large population sizes because of the non-dominated sorting procedure in every generation [23]. Additionally, NSGA lacks elitism, a procedure that can be used to accelerate the performance of GA. The NSGA-II is more efficient than the original NSGA because it uses elitism and a crowd comparison operator that keeps diversity [24]. On the other hand, Fonseca and Fleming [25] propose a slightly different scheme, whereby an individual's rank is determined by the number of individuals that dominate the individual under consideration in the current population.

3. Macro-evolutionary multiobjective genetic algorithm (MMGA)

All the GA search methods tend to converge to a single Pareto-optimal solution after a large number of generations [8]. Using sharing and niching, GAs as described by Deb and Goldberg [26], Oei et al. [27] and Goldberg [19] bypass this problem. Under the single objective sharing scheme, the fitness values of strings are penalized when the strings are too similar in the decision space to other strings in the population (the more strings that are similar to a particular string, the more the fitness function is penalized). This penalty forces dispersion of solutions in the decision space so that several different optimal and near-optimal solutions can be found.

In this paper, we adopt another improved method, macro-evolution (MA) [16,28], which uses a connectivity matrix W to compare the fitness values and similarities of all the strings (called species) in one generation dynamically. This method replaces the traditional reproduction (selection) operator of GA to produce better and more diversified offspring. With MA, large extinctions can generate coherent population responses that are very different from the slow Darwinian dynamics of a classical GA. Besides, the population of candidate solutions/species might be understood in terms of an ecological system with connections among different species, instead of just a number of independent entities with a given assigned fitness value.

3.1. Macro-evolutionary algorithm

The biological model of macroevolution simulates the dynamics of species extinction and diversification for large time scales. The dynamics are based on the relation between species, and links between species are constructed at each generation to determine whether a species could be alive or extinct. Let N be the number of species, held constant. The relationship between species is represented by a connectivity matrix W, where each item $W_{i,j}(t)$ ($i,j \in \{1,\ldots,N\}$) measures the influence of species j on species i at generation t with a continuous value. At the end of each generation, all extinct species are replaced by the existing species. Now define the n-dimensional fitness function f as a multidimensional function we want to maximize. As with a GA, MA uses a constant population size of N individuals evolving in time through successive updates of the given operators. Each individual in the MA is described by an n-input vector with fitness f. The algorithm is described below.

(1) Connection matrix:

Each individual gathers information about the rest of the population through the strength and sign of its couplings $W_{i,j}$ as

$$W_{i,j} = \frac{f(N_i) - f(N_j)}{\mathrm{dis}(N_i, N_j)}, \tag{2}$$

where N_i are the input parameters of the ith individual, $f(N_i)$ are the objective values of N_i, and $\mathrm{dis}(N_i, N_j)$ is the Euclidean distance between N_i and N_j.

The relation of each species to the rest of the population determines its survival coefficient h as

$$h_i(t) = \sum_{j=1}^{N} W_{i,j}(t), \tag{3}$$

where t is the generation number. Individuals with higher inputs h_i will be favored.

(2) Selection operator:

The selection operator allows for calculating the surviving individuals through their relations, i.e., as a sum of penalties and benefits. The state of a given individual S_i will be given by

$$S_i(t+1) = \begin{cases} 1 & \text{if } h_i(t) \geqslant 0, \quad \text{alive} \\ 0 & \text{otherwise}, \quad \text{extinct} \end{cases} \tag{4}$$

(3) Colonization operator:

The colonization operator allows for filling vacant sites that are freed by extinct individuals (such that $S_i = 0$). This operator is applied to each extinct individual in two ways. With a probability τ, a totally new solution P_{new} will be generated. Otherwise, exploitation of surviving solutions takes place through colonization. For a given extinct solution P_i we choose one of the surviving solutions, say P_b. Now the extinct solution will be "attracted" toward P_b.

Mathematically, a possible, but not unique, choice for this colonization of extinct solutions is

$$P_i(t+1) = \begin{cases} P_b(t) + \rho\lambda[P_b(t) - P_i(t)] & \text{if } \xi > \tau, \\ P_{\text{new}} & \text{if } \xi \leqslant \tau, \end{cases} \tag{5}$$

where $\xi \in [0,1]$ and $\lambda \in [-1,+1]$ are uniformly distributed random numbers, and ρ and τ are given constants in the algorithm. Therefore we can see that ρ describes a maximum radius around surviving solutions and τ acts as a temperature, described by the linear relation

$$\tau(t;G) = 1 - \frac{t}{G}, \tag{6}$$

where G is the total number of generations. In practice, the results of using this linear annealing procedure do not strongly differ from other choices of $\tau(t)$ [16].

In the MA approach, the survival of species/solutions is linked with their relative fitness in relation to all the other species. If the total sum of input connections to a given species is positive, it survives. If the total sum is negative, it disappears from the system. In this sense, the number of removed solutions is not fixed but strongly dynamic. Sometimes, a large extinction event takes place when a very good solution is found. The replacement process guarantees both the exploitation of the high-fit solutions as well as further, random exploration of other domains of the landscape. Because of the connection matrix, the entire population is able to obtain a rather accurate map of the relative importance of the solutions being explored in the landscape. MA is applied successfully to optimization problems that can be formulated in terms of an optimization function even if the function is highly multimodal or highly multidimensional. According to Marin and Sole [16], MA has many advantages when compared with GA using a traditional selection operator. First, MA can reach higher fitness values than traditional GAs for equal population sizes. Second, the probability of success in reaching a good fitness value in a typical run is higher in MA than in GA. Finally, the time needed to reach the optimum using the same population size is lower in MA.

3.2. Using MA for multiobjective optimization

Eq. (2) constitutes the cornerstone of the MA method. It calculates the relationships between each pair of individuals based on the difference between their respective objective values and their distance in the parameter (decision variable) space. If $f(p_i) \geqslant f(p_j)$, the value of $W_{i,j}$ always will be non-negative. It means that p_i tends to survive in the next generation. The purpose of using the distance between two individuals is to avoid survival of too many individuals similar to each other, which would be the case if fitness were the only selection criterion. If two individuals are very close, the one with lower fitness will obtain a negative value of $W_{i,j}$ and eventually will be eliminated from the population. By doing this, MA can preserve the diversity of individuals in any population automatically. In the formulation described above we assume that high diversity in the decision (parameter) space translates into high diversity in the objective space. This condition is problem dependent, and furthermore the relative priority given to diversity in the decision or objective spaces depends on the particular situation being studied. For the sake of simplicity of the formulation, in this research we have given priority to maintain diversity in the decision space. In several test cases conducted (not discussed) we found that gradual changes in the decision space translated into gradual variations in the objective space. This is a topic of further study and practitioners implementing the proposed algorithm should be aware of the implications of these assumptions.

Eq. (4) is used to determine whether an individual is alive or extinct. The threshold value is zero; in other words $\sum_{i=1}^{n}\sum_{j=1}^{n} W_{i,j} = 0$ (where $W_{ii} = 0$). In the case of uniform distribution of $W_{i,j}$, about half the individuals will be extinct in the next generation. MA involves a time scale of order $O(N^2)$, where N is the population size.

Due to the diversity maintenance property of MA described above, it is clear that MA has the potential to solve multiobjective optimization problems. We now combine MA with GA to form the macro-evolutionary multiobjective genetic algorithm (MMGA). It should be noted that MMGA cannot guarantee the convergence of Pareto-optimal solutions during the optimization procedure because it chooses randomly only one objective to optimize at each generation. We present a simple method to discard the inferior solutions based on Eq. (1) at the end of each generation. Because many of the solutions are diversified and non-inferior only a few solutions need to be deleted at a time. This method takes $O(mN)$ computing time, where m is the number of objectives. Therefore, the overall computational requirement of MMGA is $O(N^2)$, which is governed by the MA part of this method when $m < N$. The low computational time required by this approach makes it very promising for engineering optimization problems in which evaluating the objective function value is a time-consuming process.

3.3. Performance measures

In order to allow a quantitative assessment of the performance of a multiobjective optimization algorithm, two issues normally are taken into consideration. First, we seek to maximize the spread of solutions found, so that the solution vectors are distributed as smoothly and uniformly as possible. Second, it is desirable to maximize the number of elements of the Pareto-optimal set. Based on this notion, we adopt a metric to evaluate both of these aspects. The proposed diversity metric (DM) [22] measures the extent of spread achieved among the obtained solutions. Here we are interested in obtaining a set of solutions that spans the entire Pareto-optimal region. We calculate the Euclidean distance d_i between adjacent solutions in the obtained non-dominated set of solutions and the average d of these distances. The following metric is used to calculate the non-uniformity in the distribution:

$$\mathrm{DM} = \frac{d_b + d_e + \sum_{i=1}^{n-1} |d_i - \bar{d}|}{d_b + d_e + (n-1)\bar{d}}. \tag{7}$$

Here, the parameters d_b and d_e are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained non-dominated set, as depicted in Fig. 1, where n is the number of solutions and $\bar{d}$ is the average distance in Eq. (7). A good distribution would make all distances d_i equal to $\bar{d}$ and $d_b = d_e = 0$ (with the existence of extreme solutions in the non-dominated set). Thus, for the most widely and uniformly spread-out set of non-dominated solutions, the numerator of DM would be zero.

3.4. A test problem

We use a test problem to evaluate two different multiobjective GAs: one with macroevolution selection and one without. Veldhuizen [29] cited a number of test problems that have been used in the past to test multiobjective algorithms. Of these, we choose the following [30]:

$$\begin{array}{ll} \text{minimize} & Z_1(x) = x^2, \\ \text{minimize} & Z_2(x) = (x-2)^2, \end{array} \quad -10^3 \leqslant x \leqslant 10^3.$$

Set of non-inferior solutions:

$x \in [0, 2]$.

The decision space is shown in Fig. 2. For this convex problem, a mathematical programming solution for the Pareto frontier was obtained using the ε-constraint method with quadratic programming. Fig. 3 compares the MMGA solutions with the mathematical programming solution. Fig. 4 presents the solutions obtained with a ranking-based GA, together with the mathematical programming solution. The ranking-based GA adopted for this comparison corresponds to the nondominated sorting genetic algorithm (NSGA-II) presented by Deb et al. [31]. NSGA-II is more efficient than the original NSGA, because it uses elitism and a crowd comparison operator that maintains diversity. At each generation, NSGA-II combines the parent (P_t) and child (Q_t) populations to form a single population $R_t = P_t \cup Q_t$ of size $2N$. This allows for elitism to be maintained in successive generations. Then, the population R_t is sorted according to nondomination. The sorting procedure classifies the combined population into several frontiers. The complexity of simple sorting for non-domination in the worst case (when there exists only one solution in each frontier) is $O(m(2N)^3)$. Here, m is the number of objectives and N is the population size. To reduce this computational requirement to $O(m(2N)^2)$, a special procedure is used in NSGA-II. The crowding distance is an estimation of the density of solutions surrounding a particular solution in the population. A solution with a small value of this distance measure is crowded by other solutions and will tend to be discarded. The crowding procedure has

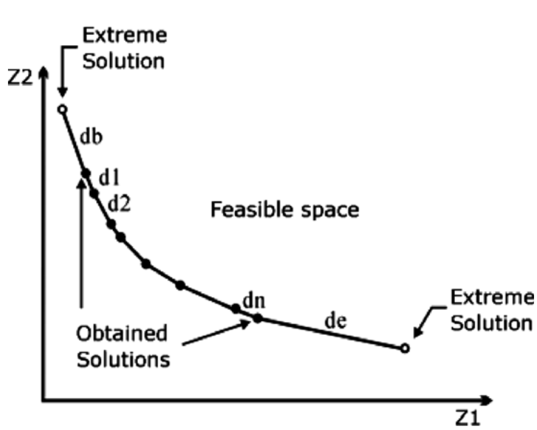

Fig. 1. Diversity metric (DM).

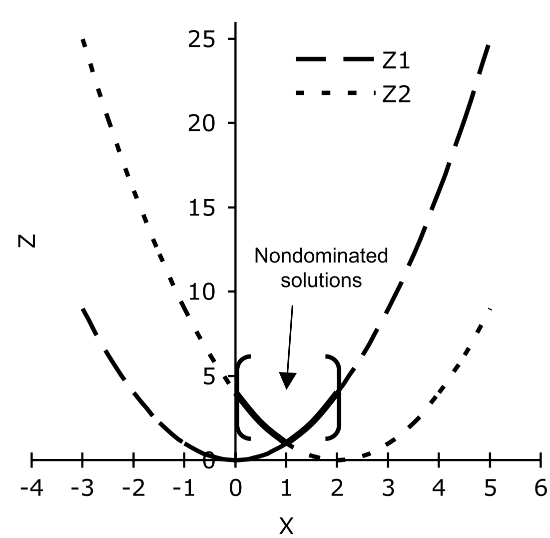

Fig. 2. Decision space for the two-objective function test problem.

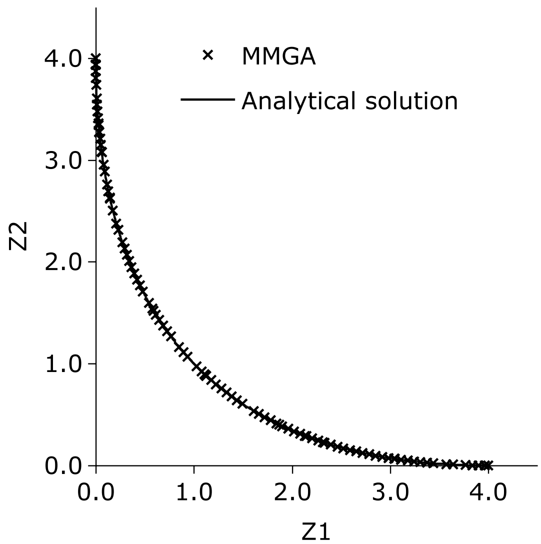

Fig. 3. The Pareto frontier of MMGA (DM = 0.251).

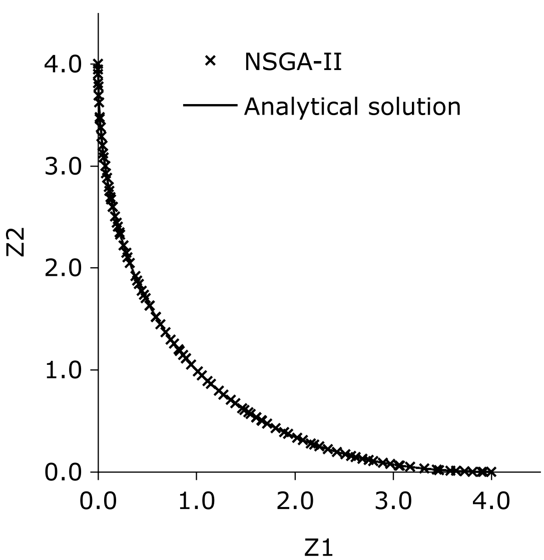

Fig. 4. The Pareto frontier of NSGA-II (DM = 0.412).

$O(m(2N)\log(2N))$ computational complexity. Therefore, the overall complexity of NSGA-II is $O(m(2N)^2)$, which is governed by the nondominated sorting part of the algorithm.

3.5. Results and discussion

NSGA-II and MMGA are compared by running NSGA-II with population size = 100 and 250 generations, whereas MMGA is run with population size = 100 and 1000 generations. The difference in the number of generations was decided in order to maintain a similar level of complexity for both algorithms and therefore to avoid bias the final comparison of the diversity metric. The values of the DM obtained with NSGA-II and MMGA after equivalent runs are 0.412 and 0.251 respectively, which means that for this particular example, after runs with equivalent level of complexity MMGA outperforms NSGA-II significantly. Figs. 3 and 4 show the best Pareto frontiers obtained with MMGA and NSGA-II, together with a reference frontier obtained with the ε-constraint method. For this simple problem, there is no appreciable difference in the visual characterization of the Pareto frontier by each method.

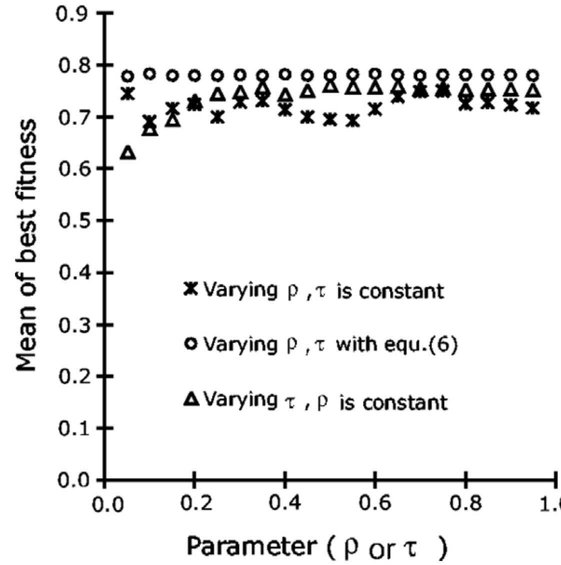

Fig. 5. Parameter behavior of the MMGA.

To understand how the parameters ρ and τ influence the MA behavior, we conducted several numerical experiments using equal population sizes over the same generations. The basic results of these experiments are summarized as follows: There is no well-defined optimum for either ρ or τ. Typically a wide range of parameter values yields similar results. Actually, in most cases Eq. (6) seems to be the best choice, and the range of the specific ρ value used seems to be less important. The results of these experiments also are summarized in Fig. 5. For this reason, MA with $\rho = 0.5$ will be used in our study.

4. Application to multi-purpose reservoir management and operation

We apply MMGA to the management and operation of a complex real-world reservoir system with multiple purposes. Water uses in the reservoir system include hydropower and water supply, and reservoir operation is simulated according to rule curves. These rule curves are established at the planning stage to provide guidelines for operating the reservoir. Although rule curves usually remain unchanged from year to year, they can, if necessary, be updated [32]. We apply MMGA to optimize the reservoir rule curves in order to maximize the benefits derived from hydropower generation while at the same time minimizing water supply shortages. Chen [33] studied this problem as a single-purpose optimization. This paper extends the formulation to a multi-purpose setting.

4.1. System description

The Fei-Tsui reservoir, completed in 1985 with an effective storage capacity of 359 million m^3, is one of the major storage reservoirs in northern Taiwan. Located in the upper stream of the Tan-Shui river basin as shown in Fig. 6, the hydropower plant at Fei-Tsui has a generating capacity of 70 MW. This reservoir is a multi-purpose reservoir for hydroelectric power generation and water supply. The primary water use in the basin is for potable water demand for the city of Taipei. Fig. 7 shows a schematic of the water supply network. The network consists of four inflow nodes, one reservoir (with power plant), three water treatment plants for public water supplies, one diversion node and four junction nodes.

4.2. Input data

The required input data for system simulation includes historic inflows, reservoir and power plant properties, the operating rules, evaporation data, and water demand tar-

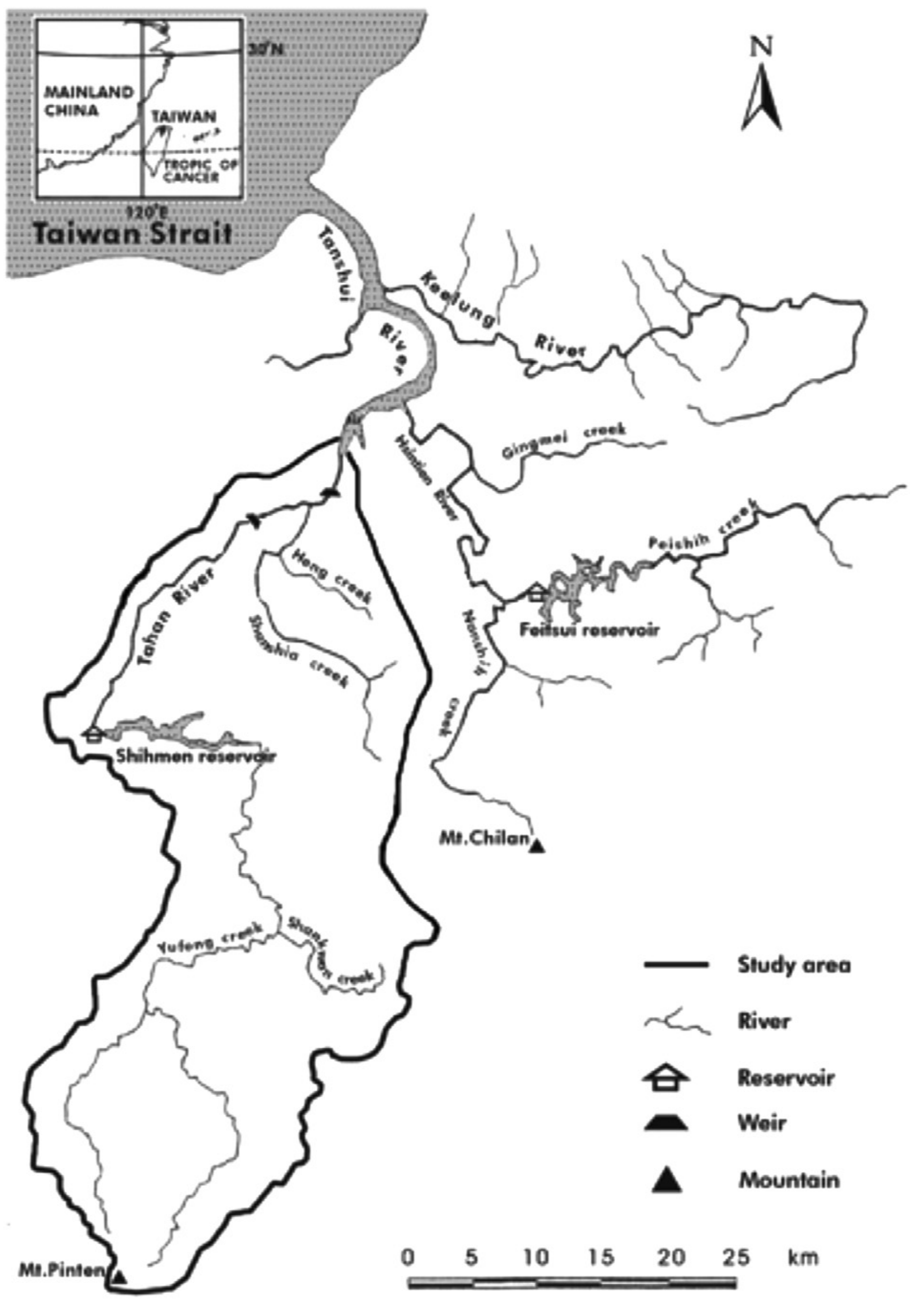

Fig. 6. The Fei-Tsui reservoir and the Tan-Shui river basin in Taiwan.

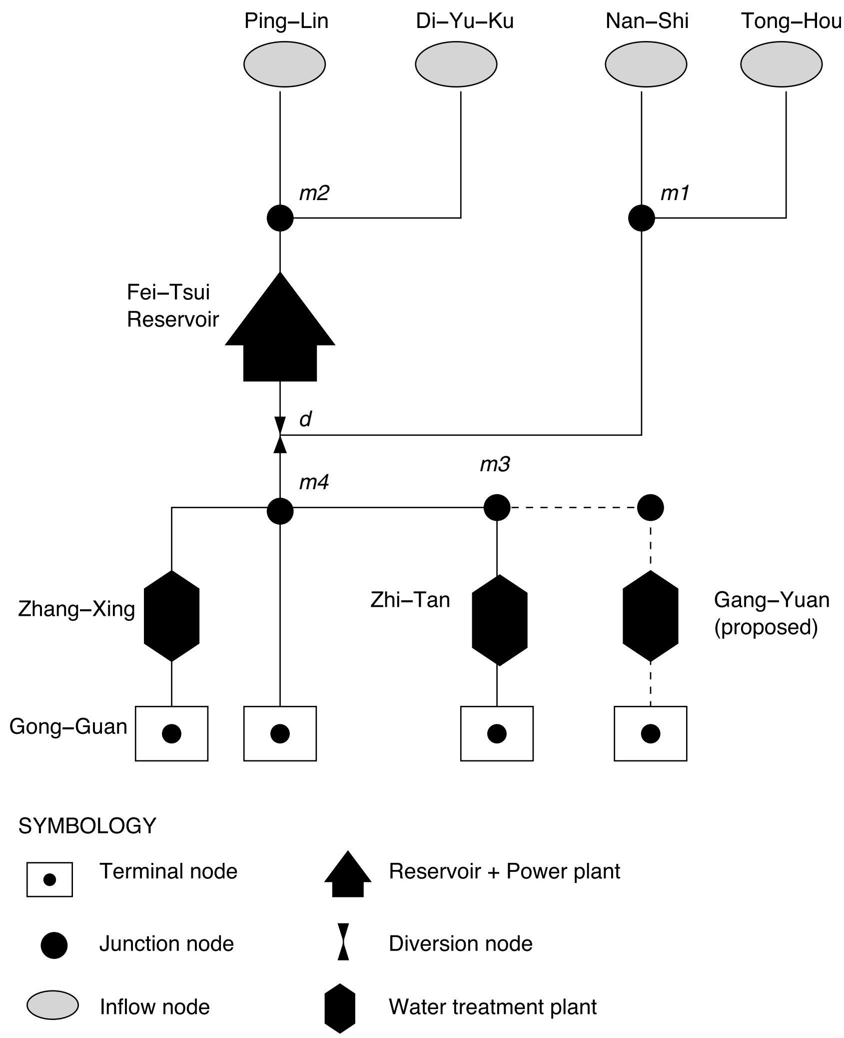

Fig. 7. System description of the Fei-Tsui reservoir.

gets of the system. A total of 41 years of historic inflow data is available. A 10-day time period (the traditional reservoir analysis time unit in Taiwan) was used in the simulation, resulting in a total of 1476 10-day time periods for the planning horizon. The planned water demand was set at the target value for the year 2021.

4.3. Simulation model

The purpose of the simulation model is to recreate the 10-day operations of the Fei-Tsui Reservoir by following its rule curves. The operating rules define the release for each year as a function of the current storage level and overall release target values. The rule curves of the Fei-Tsui reservoir consist of three curves as shown in Fig. 8. The 10-day operations are described as follows:

(1) When the water level is above the upper limit, hydropower generation should be increased to keep the water level at the upper limit.

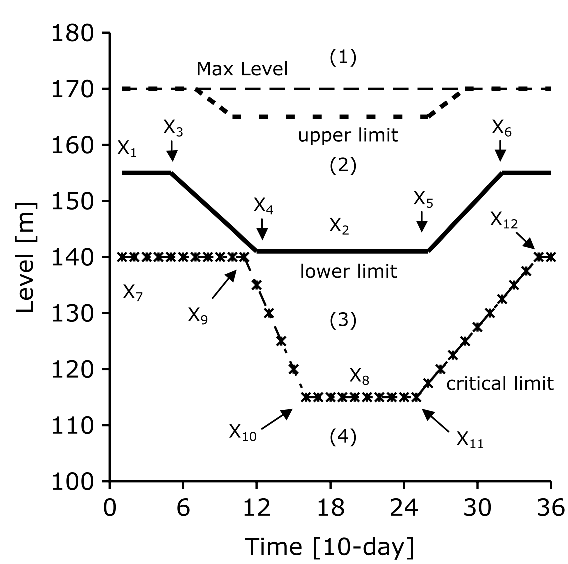

Fig. 8. Decision variable definitions of the original rule curves of the Fei-Tsui reservoir (see Table 2).

(2) When water level is between the upper and lower limits, all operations, including public water supply and hydropower generation are under normal operating condition, but hydropower should be generated at least 6 h per day.
(3) When the water level is between the lower and critical limits, public water can be supplied as usual, but hydropower generation must be halted.
(4) When the water level is below the critical limit, public water supply must be reduced by 30%.

Using the hedging rules along with the rule curves, a balance is achieved between the water supply shortage and the storage objectives [32]. In the following section, we describe how to optimize the rule curves using MMGA.

4.4. Parameter coding

Coding of parameters for solution by GA may vary according to the nature of the problem itself. In this application, the decision variables should describe fully the rule curves so that a change in the shape and location of a particular curve influences the performance of the system. As stated in the previous section, there are three curves (upper limit, lower limit and critical limit), and each is described by six decision variables that are coded in a real string. For each curve, two variables describe a high and low storage level zone (m), while four variables describe the initial and ending times of the linear transition zones between the aforementioned high and low storage levels. In order to maintain the function of flood control, the upper limit curve will not be changed, so the total number of decision variables is 12. The details of these variables are described in Table 1 and Fig. 8. The values of X_1, X_2, X_7 and X_8 are real numbers, which implies that each chromosome is a real-valued vector $\bar{X} = (X_1, X_2, X_7, X_8) \in R^4$, as opposed to binary-coded GA, where chromosomes are 0-1 vectors. However, the values of the other eight variables are integers, namely $X_3, X_4, X_5, X_6, X_9, X_{10}, X_{11}$, and X_{12}. A real-coded GA was used to optimize all of the continuous and integer variables, in order to take advantage of the higher efficiency of real-coded GA. Post-optimization real-coded values of the integer variables were simply rounded-off to obtain the final solution.

The decision variables are subject to the following constraints [33]:

- MAX level $> X_1 > X_2$,
- $X_7 > X_8 >$ MIN level,
- MAX level $> X_1 > X_7$,
- $X_2 > X_8 >$ MIN level,
- $1 < X_3 < X_4 < X_5 < X_6 < 36$,
- $1 < X_9 < X_{10} < X_{11} < X_{12} < 36$,

where MAX level = 170 [m] and MIN level = 110 [m] (Website: http://www.feitsui.gov.tw/).

4.5. Management model

4.5.1. Objective function

Water shortage (Z_1): Water supply is a priority for the Fei-Tsui reservoir system. Minimization of the 10-day period shortage for a total of 41 years (1476 10-day) was adopted as an objective function in order to meet the goal of a stable water supply:

$$\text{Minimize } Z_1 = \sum_{t=1}^{1476} (\text{demand}_t - \text{release}_t) \ [\text{cms}(86,400 \text{ m}^3)], \quad (8)$$

where demand$_t$ equals d_t calculated by using Eq. (16) and release$_t$ equals O_t calculated by Eqs. (13)–(15). We wish to note that the release always is constrained to be less than or equal to the planned demand.

Power generation (Z_2): Power generation is another priority. Power output is proportional to the product of available head and flow rate through the turbines in the power plant. The proportionality factor (power-to-discharge

Table 1
Decision variable definitions for the Fei-Tsui reservoir rule curves

Variable	Represents	Example of Fig. 6 (original value)	Value for Min Deficit	Value for Max Power
X_1	Elevation of upper horizontal segment on the lower limit	155	158.0	163.8
X_2	Elevation of lower horizontal segment on the lower limit	141	141.5	158.6
X_3	Starting time of first inclined line on the lower limit	5	3.0	8.0
X_4	Ending time of first inclined line on the lower limit	12	9.0	11.0
X_5	Starting time of second inclined line on the lower limit	26	11.0	12.0
X_6	Ending time of second inclined line on the lower limit	33	16.0	18.0
X_7	Elevation of upper horizontal segment on the critical limit	140	120.3	140.0
X_8	Elevation of lower horizontal segment on the critical limit	117.5	110.0	137.9
X_9	Starting time of first inclined line on the critical limit	11	25.0	12.0
X_{10}	Ending time of first inclined line on the critical limit	16	28.0	13.0
X_{11}	Starting time of second inclined line on the critical limit	25	31.0	30.0
X_{12}	Ending time of second inclined line on the critical limit	34	34.0	31.0
Z_1	Value of water deficit objective		4319	10,331
Z_2	Value of hydropower objective		83,883	132,941

Table 2
Hydro-power plant characteristics of the Fei-Tsui reservoir

Water level [m]	170	165	155	148.3	140.8	130	125	120	117.1
Hydropower constants [kW/cms]	985.4	948.5	849.8	780.9	697.9	597.6	545.4	489.0	453.7

ratio) [kW/cm] changes with water level, and the values adopted in this study are shown in Table 2. Although power generation is the ultimate objective, it can be replaced by a surrogate objective consisting of the total number of hours of hydropower generation. Selection of the surrogate objective is justified based on the hedging rules currently used by the system's management, which express hydropower operation in terms of the number of hours of generation per day. The second objective is to maximize the total time of power generation (hrs):

$$\text{Maximize } Z_2 = \sum_{t=1}^{1476} (\text{hours of hydropower generation})_t. \tag{9}$$

Direct inclusion of total power output in the objective vector can be achieved easily, although the nonlinearity of the resulting expression could influence the convergence behavior of the MMGA.

4.5.2. System constraints

The water balance of a reservoir system is considered the system constraint. In terms of other constraints, water levels at any period must be higher than the minimum level (intake elevations) and below the flood control level or other limitations. All diversion facility and power-plant equipment capacity limitations in the system must be satisfied. The details follow (see Fig. 7):

The continuity equation for the Fei-Tsui Reservoir can be written as the following:

$$S_t + m_{1,t} - O_t - E_t = S_{t+1}, \tag{10}$$

$$m_{1,t} = I_{\text{Ping-Lin},t} + I_{\text{Di-Yu-Ku},t}, \tag{11}$$

$$S_{\min,t} \leqslant S_t \leqslant S_{\max,t}, \tag{12}$$

where S_t = beginning storage; S_{t+1} = ending storage; I_t = inflow during time period t; O_t = release during time period t; E_t = evaporation loss during time period t; $S_{\min,t}$ = minimum storage; and $S_{\max,t}$ = maximum storage. The locations of m_1, $I_{\text{Ping-Lin}}$ and $I_{\text{Di-Yu-Ku}}$ are shown in Fig. 7.

During each time period t, the relationship between the rule curves and the hedging rules determines the upper bound of a supply link in the system. The following equations represent the relationship:

$$\text{If } S_{\min,t} \leqslant S_t < S_{\text{critical\_limit}}, \quad \text{then } O_t \leqslant 0.7 \cdot d_t \tag{13}$$

$$\text{If } S_{\text{critical\_limit},t} \leqslant S_t < S_{\text{lower\_limit},t}, \quad \text{then } O_t \leqslant d_t \tag{14}$$

$$\text{If } S_{\text{lower\_limit},t} \leqslant S_t < S_{\text{upper\_limit},t}, \quad \text{then } O_t \leqslant \max(d_t, P_t) \tag{15}$$

$$d_t = m_{4,t} - m_{2,t}, \quad \text{If } d_t < 0 \quad \text{then } d_t = 0 \tag{16}$$

where $S_{\text{critical\_limit},t}$ = critical limit storage; $S_{\text{lower\_limit},t}$ = lower limit storage; $S_{\text{upper\_limit},t}$ = upper limit storage; d_t = planned demand of water supply and P_t = planned demand of hydropower generation. The locations of d, m_2, m_4, $I_{\text{Nan-Shi}}$ and $I_{\text{Tong-Hou}}$ are shown in Fig. 7.

5. Results and discussion

Fig. 9 presents the trade-off curve between the water supply shortage (Z_1) and hydropower generation (Z_2) objectives obtained with MMGA. The Pareto frontier was obtained after 500 generations with a population size of 100 and the resulting diversity metric (DM) of the MMGA is 0.477. The minimum water shortage value is approximately 4400 cms when hydropower generation is no more than 85,000 h. On the other hand, the maximum hydropower output is approximately 135,000 h, but at the cost of increasing water shortage more than two-fold, to approximately 10,500 cms.

In general, water shortage and hydropower generation increase when the lower and critical limits are higher. In contrast, both water shortage and hydropower generation decrease when the lower and critical limits are lower. In the feasible space, the lower limit defines the hydropower generation; when it is high there is high water head to generate more power. Furthermore, the critical limit determines the rationing of water supply; when it is high it could cause a greater amount of water shortage.

Figs. 10 and 11 show the ranges (lower bound, upper bound and average value) of the optimal solutions for the lower and critical limit on the Pareto frontier. They

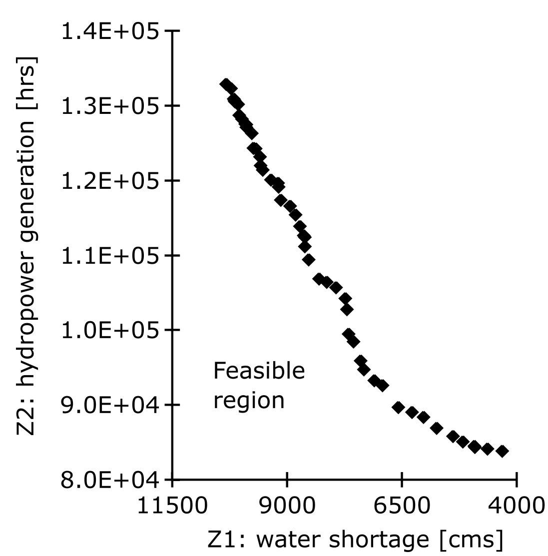

Fig. 9. The Pareto frontier of MMGA (with DM = 0.4774).

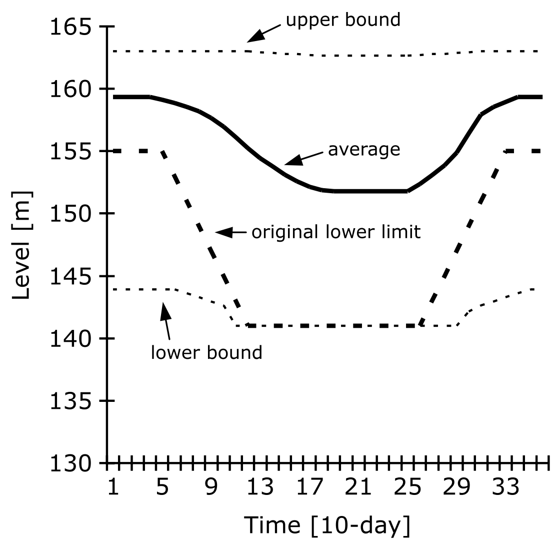

Fig. 10. The Pareto-optimal solution range of the lower limit.

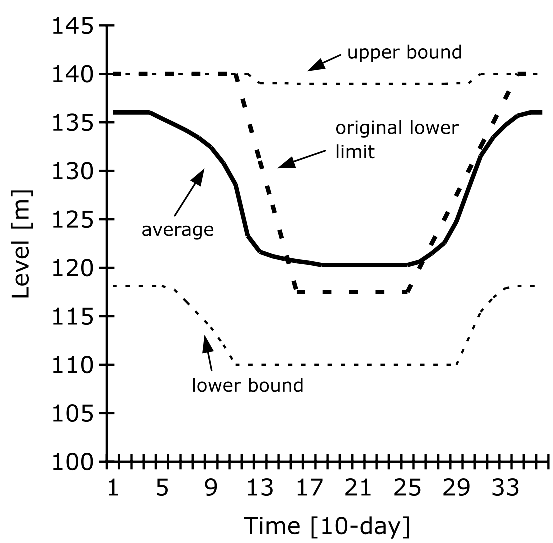

Fig. 11. The Pareto-optimal solution range of the critical lower limit.

indicate that the original lower limit is on the lower bound from May to September (the flood control period) and the original critical limit is on the upper bound from December to February (dry period) of the optimal ranges. In other words, it is obvious that there are plenty of spaces for raising the original lower limit and dropping the original critical lower limit to achieve more hydropower generation and less water shortage.

6. Conclusions

Due to the limitations of conventional multiobjective optimization methods, this paper develops a macro-evolutionary multi-objective genetic algorithm (MMGA). The proposed MMGA is relatively easy to implement and was applied to a test problem taken from the literature, yielding a better spread of solutions than NSGA-II. Trade-off information can be extremely relevant in complex management scenarios, and may aid decision-makers in selecting management policies according to societal, political or other considerations that are difficult to model.

In the case study, we applied MMGA to rule-curve optimization for a multipurpose reservoir system. The problem is highly nonlinear with mixed integer variables and a complex non-convex Pareto frontier. The operating rule curves define long-term target storage levels and target releases and can be used to help system operators make decisions regarding releases for water supply and hydropower generation. The results show that MMGA finds an acceptable spread of Pareto-optimal solutions with a relatively low diversity metric (DM). The results also indicate that MMGA can generate a smooth and well-spread Pareto frontier, representing the trade-off between water shortage and hydropower generation associated with a multi-purpose reservoir system. Additionally, the overall computation time is proportional to the square of the population size (N^2) using the MMGA, which is lower than that of well-established multiobjective algorithms such as NSGA-II, which has a computing complexity of $m(2N)^2$, where m is the number of objective functions. Therefore, the exceptionally low computational time requirement by the MMGA makes it a very promising approach to water resources optimization problems in which the computational cost is a vital issue.

Future work includes extending the presented methodology to problems involving more than two objectives. For reservoir operations, a more complete study should take into account the benefits of flood control, ecological preservation and other purposes of a multi-purpose reservoir.

References

[1] Oliveira R, Loucks DP. Operating rules for multireservoir systems. Water Resour Res 1997;33(4):839–52.
[2] Wurbs RA. Modeling and analysis of reservoir system operations. Englewood Cliffs, NJ: Prentice Hall; 1996.
[3] Loucks DP, Sigvaldason OT. Multiple-reservoir operation in North America. In: Kaczmarek Z, Kindler J, editors. The operation of multiple reservoir systems. International Institute for Applied Systems Analysis, Laxenburg, Austria; 1982.
[4] Vink K, Schot P. Multiple-objective optimization of drinking water production strategies using a genetic algorithm. Water Resour Res 2002;38(9):20-1–20-15.
[5] Yeh W W-G, Becker L. Multiobjective analysis of multireservoir operations. Water Resour Res 1982;18(5):1326–36.
[6] Cohon JL, Marks DM. A review and evaluation of multiobjective programming techniques. Water Resour Res 1975;11(2):208–20.
[7] Louie PWF, Yeh W W-G, Hsu N-S. Multiobjective water resources management planning. J Water Resour Plann Manage-ASCE 1984;110(1):39–56.
[8] Cieniawski SE, Eheart JW, Ranjithan S. Using genetic algorithms to solve a multiobjective groundwater monitoring problem. Water Resour Res 1995;31(2):399–409.
[9] Costa L, Oliveira P. An elitist genetic algorithm for multiobjective optimization. MIC'2001 – 4th Metaheuristics International Conference; 2001. p. 205–9.

[10] Reed P, Minsker BS, Goldberg DE. Simplifying multiobjective optimization: an automated design methodology for the nondominated sorted genetic algorithm-II. Water Resour Res 2003;39(7):1196. doi:10.1029/2002WR00148.
[11] Kapelan ZS, Savic DA, Walters GA. Multiobjective design of water distribution systems under uncertainty. Water Resour Res 2005;41:W11407. doi:10.1029/2004WR00378.
[12] Cheung PB, Reis LFR, Formiga KTM, Chaudhry FH, Ticona WGC. Multiobjective evolutionary algorithms applied to the rehabilitation of a water distribution system: a comparative study. Lecture Notes in Computer Science 2003;2632(January):662–76.
[13] Yandamuri SRM, Srinivasan K, Bhallamudi SM. Multiobjective optimal waste load allocation models for rivers using nondominated sorting genetic algorithm-II. J Water Resour Plan Manage-ASCE 2006;132(3):133–43.
[14] Dorn J, Ranjithan S. Evolutionary multiobjective optimization in watershed quality management. In: Fonseca C, Fleming P, Zitzler E, Deb K, Thiele L, editors. Evolutionary multi-criterion optimization (EMO-2003).
[15] Kim T, Heo J-H, Jeong C-S. Multireservoir system optimization in the Han River basin using multi-objective genetic algorithms. Hydrol Process 2006;20(9):2057–75. doi:10.1002/hyp.604.
[16] Marin J, Sole R. Macroevolutionary algorithms: a new optimization method on fitness landscapes. IEEE Trans Evolut Comput 1999;3(4):272–86.
[17] Fonseca CM, Fleming PJ. An overview of evolutionay algorithms in multiobjective optimization. Evolut Comput 1995;3:1–16.
[18] Schaffer JD. Multiple objective optimization with vector evaluated genetic algorithms. In: Proc 1st int conf genetic algorithm; 1985. p. 93–100.
[19] Goldberg DE. Genetic algorithms in search, optimization and machine learning. Addison-Wesley; 1989.
[20] Man KF, Tang KS, Kwong S. Genetic algorithms. Springer-Verlag London Limited; 1999.
[21] Fourman MP. Compaction of symbolic layout using genetic algorithm. Proc. 1st int conf genetic algorithms; 1985. p. 141–53.
[22] Sareni B, Krahenbuhl L. Fitness sharing and niching methods revisited. IEEE Trans Evolut Comput 1998;2(3):97–106.
[23] Deb K, Pratop A, Agarwal S, Meyarivan T. A fast and elitist multiobjective genetic algorithm, NSGA-II. IEEE Trans Evolut Comput 2002;6(2):182–97.
[24] Prasad TD, Waters GA, Savic DA. Booster disinfection of water supply networks: multiobjective approach. J Water Resour Plan Manage, ASCE 2004;130(5):367–76.
[25] Fonseca CM, Fleming PJ. Genetic algorithms for multiobjective optimization: formulation. Discussion and generalization. In: Forrest S, editor. In: Proc. 5th int. conf. genetic algorithms; 1993. p. 416–23.
[26] Deb K, Goldberg DE. An investigation of niche and species formation in genetic function optimization. Proceedings of the third international conference of genetic algorithms. San Mateo, CA: Morgan Kaufmann; 1989. p. 2–50.
[27] Oei CK, Goldberg DE, Chang SJ. Tournament selection, niching, and the preservation of diversity. IlliGAL Rep. 91011, Ill. Genet. Algorithms Lab., Univ. of Ill. At Urbana-Champaign, Urbana; 1991.
[28] Zhang JH, Xu JQ. A hybrid macro evolutionary algorithm. Advances in natural computation, Pt 3, proceedings lecture notes in computer science 2005;3612:299–308.
[29] Veldhuizen DV. Multiobjective evolutionary algorithms: classifications, analyzes, and new innovations, Air Force Inst. Technol., Dayton, OH, Tech. Rep. AFIT/DS/ENG/99-01; 1999.
[30] Schaffer JD. Multiple objective optimization with vector evaluated genetic algorithms. In: Grefensttete JJ, editor. Proceedings of the first international conference on genetic algorithms. Hillsdale, NJ: Lawrence Erlbaum; 1987. p. 93–100.
[31] Deb K, Pratap A, Agarwal S, Meyarivan T. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 2002;6(2):182–97.
[32] Tu M-Y, Hsu N-S, Yeh W W-G. Optimization of reservoir management and operation with hedging rules. J Water Resour Plan Manage, ASCE 2003;129(2):86–97.
[33] Chen L. Real-coded genetic algorithm optimization of long-term reservoir operation. J Am Water Resour Assoc 2003;39(5):1157–65.