SEQUENTIAL PEAK-LOAD PRICING: THE CASE OF AIRPORTS AND AIRLINES

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Abstract: This paper investigates airport peak-load pricing using a vertical structure of airport and airlines. We consider a private, unregulated airport and a public airport that maximizes social welfare. We find that compared to the public airport which may or may not be budget-constrained, a profit-maximizing airport would charge higher peak and off-peak runway prices, as well as a higher peak/off-peak price differential. Consequently, airport privatisation would lead to both fewer total air passengers and fewer passengers in the peak period. Although peak-traveling passengers benefit from fewer delays, overall it is not efficient to have such a low level of peak congestion, suggesting that airport privatization cannot be judged based on its effect on congestion delays alone. We also examine pricing behaviour of a private airport strategically collaborating with the airlines, and of a public airport that is constrained to charge a time independent price.

Keywords: Peak-load pricing, Vertical airport-airline structure, Runway charges, Airport privatization, Market power

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1. INTRODUCTION

During the last several years airlines and passengers have been suffering from runway congestion and delays at busy airports, and airport delays have become a major public policy issue. Since the early work of Levine (1969), Carlin and Park (1970) and Borins (1978), economists have approached runway congestion by calling for the use of price mechanism, under which landing fees are based on a flight's contribution to congestion. While congestion pricing is economically desirable, it has not really been practiced. The existing landing fees depend on aircraft weight, and the fee rates are based on the accountancy principle of cost recovery required usually for a public enterprise.[1] Airports have traditionally been owned by governments, national or local. This is changing, however. Starting with the privatization of seven airports in the UK to BAA plc. in 1987, many airports around the world have been, or are in the process of being, privatized.[2] One of the leading arguments for airport privatization is that privatised airports might well shift toward peak-load congestion pricing of runway services they provide to airlines, thus reducing delays in peak travel times (Poole, 1990; Gillen, 1994; Vasigh and Haririan, 1996). For example, Gillen (1994) argues that privatization does a better job of producing efficient runway pricing mechanisms compared to public ownership.

Taken together, today's shortage of airport capacity has revived much of the recent discussions about peak-load congestion pricing and airport privatization. In this paper we carry out an analysis of peak-load congestion pricing for a private, profit-maximizing airport, for a public airport that maximizes social welfare, and for a public, welfare-maximizing airport that is subject to a financial break-even constraint. The comparison then allows us to shed some light on their pricing policies and traffic allocations to the peak and off-peak periods. We find that compared to the two types of public airport, a profit-maximizing airport would charge higher peak and off-peak runway prices, as well as a higher peak/off-peak price differential. As a consequence, privatisation would lead to both fewer total air

[1] Airport charges include landing fees, aircraft parking and hangar fees, passenger terminal fees and air traffic control charges (if the service is provided by the airport authority), with landing fees being most dominant. The revenues derived from these charges are referred to as aeronautical revenues. In addition, busy airports derive significant revenues from non-aeronautical business, such as concessions and other commercial activities. As Daniel (2001) pointed out, landing fees in the U.S. traditionally recovered the "residual" costs –those remaining after all other revenue sources are fully exploited– with the fee rate equalling the annual residual costs divided by the weight of all aircraft landing during the year.

[2] A number of major airports in Europe, Australia, New Zealand and Asia were recently privatized, or are in the process of being privatized. In the U.S., on the other hand, the airports that are used by scheduled airlines are virtually all publicly owned facilities run by the local (city) government or by an agency on behalf of the local government. Canada may represent a middle-of-the-road case in which airports recently devolved from direct Federal control to become autonomous entities and major airports, though still government-owned, are now managed by private not-for-profit (but subject to cost recovery) corporations.

passengers and fewer passengers using the peak hours of the day for their travel. Hence, the alleged benefits of privatization would appear to have been achieved, as those passengers who still use the peak period indeed face less congestion delays; but overall it is not economically efficient to have such a lower level of peak congestion. This suggests that airport privatization cannot be judged based on its effect on congestion delays alone. Our analysis also shows that whilst a private airport will always use peak-load pricing, somewhat surprisingly, a public airport may actually charge a peak price that is *lower* than the off-peak price. Here the public airport, on the surface, is not practicing the peak-load pricing, but such pricing structure is nevertheless socially optimal.

We further investigate the case of a public airport that is constrained to charge a time independent landing fee. Somewhat surprisingly, such airport would not choose a fee that is in between the peak and off-peak prices –as the *a priori* intuition may have suggested– but would charge the off-peak price throughout the day. Thus if by external reasons a public airport cannot use PLP but a private airport can, privatization may indeed do a better job at solving the congestion problem. Further, if the problem of congestion is sufficiently important, the gain from reduced congestion might even outweigh the welfare loss from a privatized airport's exploitation of market power, suggesting that moving public airports toward the use of PLP would be worthwhile. Also, we examine .a case where a private airport strategically collaborates with the airlines so that it maximizes the joint airport-airline profits, since it has been often argued that greater airlines' *countervailing power* or more strategic collaboration between airports and airlines may improve efficiency of privatized airports by allowing a better alignment of incentives. The analysis shows that while the airport's pricing practices would induce a collusive outcome in the airline market, they would also, owing to the elimination of "double marginalization," induce greater total traffic and greater peak traffic than a pure (no-collaboration) private airport. Nevertheless, assuming no differences in technical efficiency, both figures will still be smaller than those for a pure, or budget-constrained, public airport.

As indicated above, the present paper investigates airport peak-load pricing (PLP) and analyzes both the price level and price structure (peak vs. off-peak). This is in contrast to the majority of airport pricing studies which did not address inter-temporal pricing across different travel periods. In these *congestion pricing* studies, there is only one demand function (i.e., a single-period model) for the airport, which is obtained by aggregating the demands of many agents –in this case, the airlines. Since runways are congestible, when an airline decides to schedule a new flight, it induces extra-delays on every other flight. The airline however would only internalize the delays it imposes on its own

flights and not others. Congestion pricing then looks at the price the airport, or a regulatory authority, should charge to the airline for the new flight, in order for the airline to internalize all the congestion it produces (e.g., Morrison, 1987; Zhang and Zhang, 2003, 2006; Pels and Verhoef, 2004; Basso, 2005). Notice that under congestion pricing, since time-varying congestion is absent, there is only one way for either the airport or the airlines to internalize congestion: raising prices to suppress the demands. In a PLP framework, on the other hand, excess demand problems arise because of the variability of demands during the reference times of the day. If the same price was charged throughout the day, there would be peak periods at which the demand would be much higher than at off-peak periods. PLP looks at the optimal time-schedule of prices so as to flatten the demand curve and make better use of existing capacity. As discussed below, both airports and airlines may engage in such demand spreading by using PLP. Note that in this PLP framework, the airport is still a congestible facility, which implies that in the resulting optimal price-schedule, prices at peak periods would still have to correct for un-internalized congestion: peak-load prices will have a congestion pricing component. Moreover, as demonstrated in the text, the PLP/congestion pricing distinction is also important in that a single-period congestion toll is not optimal unless it is charged on top of the optimal charge in the off-peak period, *which may not be the marginal cost*. In other words, restricting the analysis to the toll that should be charged during the peak hours offers only a partial view of the problem.

Another major feature of our analysis lies in the basic model structure used, which has strong implications for peak-load pricing. Here an airport, as an input provider, makes its price decisions prior to the airlines' output decisions. This vertical structure gives rise to *sequential* PLP: The PLP schemes implemented by the downstream airlines induce a different periodic demand for the upstream airport, with the shape of that demand depending, among other things, on the number of downstream carriers and the type of competition they exert. The airport then would have an incentive to use PLP as well, which in turn affects the way the downstream firms use PLP. Although several very useful models of airport peak-load congestion pricing have been developed (e.g., Morrison, 1983; Morrison and Winston, 1989; Oum and Zhang, 1990; Arnott, De Palma and Lindsey, 1993; Daniel, 1995, 2001), these studies considered PLP primarily at the airport level. Brueckner (2002, 2005), on the other hand, investigated PLP primarily at the airline level. Most of

these studies considered only a public airport that maximizes social welfare, making no assessments about the effects of privatization on airport price structures.[3]

There is an extensive body of literature on peak-load pricing. The classical papers (Boiteaux 1949; Steiner 1957; Hirschleifer 1958; Williamson 1966) focused on normative rules for pricing a public utility's non-storable service subject to periodic demands. Some of the usual assumptions were: (i) demand is constant within each pricing period; (ii) demand in one period is independent of demand in other periods; (iii) constant marginal costs; (iv) the length of pricing periods is fixed and exogenous; (v) the number of pricing periods is exogenous; and (vi) peak time is known. Many authors have since contributed to the generalization of PLP results by relaxing one or a group of these assumptions, including Pressman (1970), Panzar (1976), Dansby (1978), Craven (1971, 1985), Crew and Kleindorfer (1986, 1991), Gersten (1986), De Palma and Lindsey (1998), Dana (1999), Laffont and Tirole (2000), Shy (2001) and Calzada (2003).[4] However, the case of sequential peak-load pricing, be it for public or private utilities, has yet been analyzed. In the telecommunications research, for instance, Laffont and Tirole look at PLP only at the upstream level (the network access charge) whilst Calzada considers PLP only at the downstream level. Because of this, we think our paper could be a contribution to the general peak-load pricing literature as well.

The paper is organized as follows: Section 2 sets up the model. Section 3 analyzes the output-market equilibrium, paying particular attention to the peak and off-peak derived demands for airport services. Section 4 examines the airport's pricing behaviour and discusses how the airport ownership influences the peak and off-peak prices, traffic volumes, delays and welfare. Section 5 contains concluding remarks.

2. THE MODEL

We consider a two-stage model of airport and airline behavior, in which N air carriers service a congestible airport. In the first stage the airport decides on its runway charges on airlines, and in the second stage each carrier chooses its

[3] As discussed in detail in the model section, we follow Brueckner (2002, 2005) in assuming a "single crossing property" when specifying consumers' travel benefit functions (from where we derive consumer demands). Unlike Brueckner, who further imposes an *interior crossing* condition so that the peak and off-peak periods are not vertically differentiated for all consumers, however, we will consider both Brueckner's interior-crossing case and the vertical-differentiated case (i.e., peak travel is preferred to off-peak travel by all consumers if airfares and travel delays are equal). As shown in the paper, our results hold for both cases.

[4] See Crew, Fernando and Kleindorfer (1995) for an excellent survey of the peak-load pricing literature.

output in terms of the number of flights. We shall consider a discrete choice model in which the consumer chooses between three mutually exclusive alternatives, namely: $h=p$, travel during peak hours of a day; $h=o$, off-peak period travel; and $h=n$, not traveling. There is a continuum of consumers labelled by θ. We denote $B_h(\theta)$ the gross benefit for consumer θ from traveling in period h and D_h the flight delay associated with travel in period h. We assume that consumers' utility functions are quasilinear, so that the direct utility functions are $U = x + B_h(\theta) - \alpha D_h$, where x is consumption expenditure and α is a consumer's value of time, making αD_h the monetary costs of delays to passengers. Consumers maximize utility by choosing x and $h \in \{p, o, n\}$ subject to the budget constraint $x + t_h \le I(\theta)$, where t_h is the ticket price (airfare) of traveling in period h, and $I(\theta)$ is consumer θ's income. We can then focus on the part of the utility function that determines the discrete choice. This *conditional indirect utility function* is given by:

$$V_h(\theta) = B_h(\theta) - \alpha D_h - t_h \tag{1}$$

The flight delay at period h, for $h=p,o$, may be given by $D_h = D(Q_h; L_h, K)$, where Q_h is the total number of flights in the period, L_h is the length (duration) of the pricing period, and K is the airport's runway capacity (measured in terms of the maximum number of flights that the airport's runways can handle per hour). In this paper we consider that K and L_h are exogenously given.[5] We further assume L_o is sufficiently long so that $D(Q_o; L_o, K) = 0$ throughout the relevant range of our analysis. In other words, whilst the narrow peak period is congestible, congestion never arises in the broader off-peak period.[6] For the peak delay function, we make the standard assumption that $D_p = D(Q_p)$ is differentiable in Q_p and

$$D_p' = \frac{dD}{dQ_p} > 0, \qquad D_p'' = \frac{d^2 D}{dQ_p^2} \ge 0 \tag{2}$$

This assumption is quite general, requiring only that for given airport capacity, increasing peak traffic will increase congestion of the peak period and the effect is more pronounced when there is more congestion; that is, for given

[5] The case of variable and endogenous capacity is examined in Basso (2005) and Zhang and Zhang (2006) in a congestion-pricing framework.

[6] This is similar to the two-period (peak/off-peak) formulation developed in Brueckner (2002). If the off-peak period is also congestible (but not too serious to cause a "peak reversal"), the analysis will become more complicated although our main insights will continue to hold. We discuss the issue further in the concluding remarks.

capacity, the peak delay is convex in traffic volume. The assumption is certainly satisfied under a linear delay function, $D(Q_p; L_p, K) = \delta \cdot Q_p / (L_p K)$ –which has been used by, e.g., Pels and Verhoef (2004)– or under the functional form suggested by Lave and de Salvo (1968), that is, $D(Q_p; L_p, K) = Q_p \left[L_p K (K - (Q_p / L_p)) \right]^{-1}$.[7]

To obtain the consumer demands for peak and off-peak travel, we first follow Brueckner (2002, 2005) in assuming that consumers' benefits functions fulfill $B_p^{'}(\theta) > B_o^{'}(\theta) > 0$. These three conditions say that no two passengers have the same peak or off-peak benefits, that consumers are ordered (according to θ) in increasing order of benefits, and that the peak benefit function is steeper than the off-peak benefit function everywhere. The latter is a *single crossing property* which holds if, for example, θ is seen as an index of the passenger's tendency to travel in business (Brueckner, 2002). From (1), these conditions directly imply that $V_p^{'}(\theta) > V_o^{'}(\theta) > 0$; thus, setting $B_n = 0$, the following characteristics about the allocation of consumers can be easily shown to hold : (i) if consumer θ_1 flies, then consumers $\theta \geq \theta_1$ fly; (ii) if consumer θ_1 does not fly, then consumers $\theta < \theta_1$ do not fly; and (iii) if θ^* denotes the consumer who is indifferent between traveling in the peak and off-peak periods, then passengers $\theta \geq \theta^*$ choose peak travel whereas passengers $\theta < \theta^*$ choose off-peak travel or non-travel.[8] Hence, if we denote θ^f the consumer who is indifferent between flying and not flying, (i), (ii) and (iii) above imply, in the case of an interior solution, that $\underline{\theta} < \theta^f < \theta^* < \overline{\theta}$. We assume for now the allocation is interior, but later shall find conditions on the parameters for this to hold.

[7] This functional form was previously estimated from steady-state queuing theory and is further discussed in U.S. Federal Aviation Administration (1969) and Horonjeff and McKelvey (1983). It has been used by, e.g., Morrison (1987), Zhang and Zhang (2003), and Basso (2005).

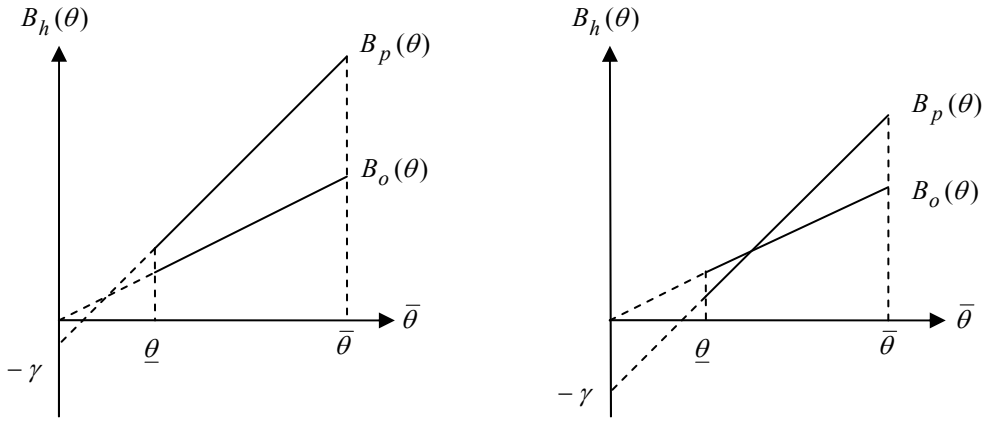
[8] *Proof:* (i) θ_1 flies if $V_h(\theta_1) \geq 0$ for *h=p,o*. If $\theta \geq \theta_1$, $V_h(\theta) \geq V_h(\theta_1) \geq 0$ and so θ flies. (ii) is analogous. (iii) Let $\Delta V(\theta) \equiv V_p(\theta) - V_o(\theta)$, and suppose θ flies. Then if $\Delta V(\theta) \geq 0$, θ chooses to fly in the peak period. If $\Delta V(\theta) < 0$, θ chooses to fly in the off-peak period. Now, suppose that there exists θ^* such that $\Delta V(\theta^*) = 0$ (interior solution). Then it follows, since $\Delta V'(\theta) > 0$, that if $\theta \geq \theta^*$, θ chooses peak travel and if $\theta < \theta^*$, θ chooses off-peak travel or non-travel. ■

For simplicity, we assume θ (>0) is distributed uniformly on $\left[\underline{\theta}, \overline{\theta}\right]$ and normalize the number of total consumers to $\overline{\theta} - \underline{\theta}$, so the number of passengers with type belonging to $[\theta_1, \theta_2]$ is directly given by $\theta_2 - \theta_1$. We further assume that the benefit functions follow simple linear forms:

$$B_o(\theta) = B_o \cdot \theta, \qquad B_p(\theta) = B_p \cdot \theta - \gamma \tag{3}$$

with $B_p > B_o > 0$ and $\gamma \geq 0$. These obviously fulfill the assumed conditions for the benefit functions and it is also easy to see that, if $\gamma < \gamma_1 \equiv (B_p - B_o)\underline{\theta}$, then the peak benefit function is above the off-peak benefit function $\forall \theta \in [\underline{\theta}, \overline{\theta}]$. In this case, with identical airfares and delays, consumers always prefer traveling in the peak period to off-peak traveling. Thus, peak travel and off-peak travel would be *vertically differentiated*: Controlling for fares and delay costs, passengers regard a peak flight as a better product than an off-peak flight. This vertical-differentiation feature of air travel can arise if the peak period represents the day's more desirable travel times. Since people want to travel in those "popular" hours, the (unfettered) demand approaches or exceeds the capacity of the existing infrastructure, thereby resulting in (potential) congestion during the peak hours. If, on the other hand, $\gamma_1 \leq \gamma < \gamma_2 \equiv (B_p - B_o)\overline{\theta}$, the benefit functions intersect at an intermediate value of θ, thus indicating that $B_p(\theta) > B_o(\theta)$ for large values of θ but $B_p(\theta) < B_o(\theta)$ for small values of θ. In this case, periods are not vertically differentiated for all consumers. This *interior crossing* case was the one considered by Brueckner (2002, 2005), who imposed it to avoid a degenerate (corner) equilibrium in his analysis. Both cases are graphically represented in Figure 1. By keeping γ as a parameter we nest both the vertically differentiated case and Brueckner's interior-crossing case in our model. As is to be seen below though, $\gamma < \gamma_1$ does not necessarily lead to a degenerate equilibrium in our model and, in fact, the value of γ does not play an essential role in any of our results.[9]

[9]Note that the case of $\gamma \geq \gamma_2$ is not relevant as it implies that with identical airfares and delays, consumers always prefer traveling in the off-peak period to peak traveling, violating our peak/off-peak formulation. We also note that our demand problem is identical to the one that will result if we fix θ but allow the value of time α to have a distribution among consumers (in simple models with endogenous hours of work, the consumers' "opportunity cost" of time lost in delays is proportional to their wages). One could also argue that θ and α are related (Yuen and Zhang, 2005), but we do not do this here.



(a) Vertically differentiated periods case $0 \le \gamma < \gamma_1$

(b) Brueckner (2002)'s *interior crossing* case $\gamma_1 \le \gamma < \gamma_2$

Figure 1: Cases for the benefit functions

Using q_h to denote the total number of passengers in period h, then $q_p = \bar{\theta} - \theta^*$ and $q_o = \theta^* - \theta^f$. Since runway charges are imposed on aircraft (flights), we need to transform the passenger-based demands q_p and q_o into per-flight demand functions. As in Brueckner (2002), Pels and Verhoef (2004), Basso (2005) and Zhang and Zhang (2006), we make a "fixed proportions" assumption, i.e., $S \equiv$ Aircraft Size × Load Factor, is constant and the same across carriers.[10] It then follows immediately that $q_p = Q_p S = \bar{\theta} - \theta^*$ and $q_o = Q_o S = \theta^* - \theta^f$, or equivalently,

$$\theta^* = \bar{\theta} - Q_p S, \qquad \theta^f = \theta^* - Q_o S \tag{4}$$

From (1) and (3), the indifferent flyer θ^* is determined by $\theta^*(B_p - B_o) - \gamma = (t_p - t_o) + \alpha D_p$ (recall that $D_o = 0$), i.e., a passenger's gain of shifting from the off-peak to the peak period, is balanced by the fare differential and the congestion cost. The final flyer θ^f is determined by $\theta^f B_o = t_o$. Replacing θ^* and θ^f in (4) we obtain:

$$t_o(Q_o, Q_p) = B_o\bar{\theta} - B_o S Q_o - B_o S Q_p \tag{5}$$

$$t_p(Q_o, Q_p) = (B_p\bar{\theta} - \gamma) - B_o S Q_o - B_p S Q_p - \alpha D(Q_p) \tag{6}$$

[10] That is, the number of passengers in each flight is constant. This assumption also allows us to abstract away from the issue of weight-based pricing as aircraft here have the same weight, and thereby focus on the main issue of peak-load congestion pricing.

Equation (5) is the (inverse) consumer demand function faced by the airlines for the off-peak period, whereas (6) is the consumer demand function for the peak period. Note that this demand system is not linear if D is not. Further, the peak and off-peak flights are substitutes for the final passengers, which gives the room for airlines to "spread the demand" across the peak and off-peak periods by using peak-load airfares.

We now turn to the airlines. They have identical cost functions, given by:

$$c_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \sum_{h=p,o}[c + P_h + \beta D(Q_h)]Q_h^i \tag{7}$$

where Q_h^i is the number of airline i's flights in period h, $\mathbf{Q}_h^{-i}$ denotes the vector of flights of airlines other than i, c is the airline's operating cost per flight, and P_h is the airport landing fee in period h.[11] Further, parameter β (>0) measures the delay costs to an airline per flight, which may include wasted fuel burned while taxiing in line or holding/circling in the air, extra wear and tear on the aircraft, and salaries of flight crews. Airlines' profit functions can then be written as:

$$\phi^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \sum_{h=p,o}[t_h(Q_o, Q_p)Q_h^i S - c_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h)] \tag{8}$$

Using these functions, we shall investigate the subgame perfect equilibrium of our two-stage airport-airlines game.

3. ANALYSIS OF OUTPUT-MARKET EQUILIBRIUM

To solve for the subgame perfect equilibrium we start with the analysis of the second-stage airline competition. Given the airport's runway charges P_p and P_o, the N carriers choose their quantities to maximize profits, and the Cournot equilibrium is characterized by the first-order conditions, $\partial\phi^i / \partial Q_h^i = 0$, $h=p,o$ (note the second-order conditions are satisfied).[12] Imposing symmetry, $Q_h^i = Q_h / N$, and re-arranging, the first-order conditions can be expressed as:

$$\Omega^o(Q_o, Q_p, P_o, N) \equiv (B_o\bar{\theta}S - c - P_o) - Q_o\frac{B_o S^2(N+1)}{N} - Q_p\frac{B_o S^2(N+1)}{N} = 0 \tag{9}$$

[11] As indicated earlier, airport charges usually include landing and terminal charges (charges for aircraft parking are minor). While landing fees are based on aircraft movements, terminal charges are typically per-passenger based. Since the present paper is concerned with runway congestion, we shall focus on landing fees.

[12] We have assumed a Cournot game in the output-market competition. Brander and Zhang (1990, 1993), for example, find some empirical evidence that rivalry between duopoly airlines is consistent with Cournot behaviour.

$$\Omega^p(Q_o,Q_p,P_p,N)\equiv(B_p\bar{\theta}S-\gamma S-c-P_p)-Q_o\frac{B_oS^2(N+1)}{N}-Q_p\frac{B_pS^2(N+1)}{N}$$
$$-(\alpha S+\beta)\left(D(Q_p)+\frac{Q_p}{N}D'(Q_p)\right)=0 \tag{10}$$

A useful equation obtained from (9) and (10) is:

$$-\Omega^p+\Omega^o=Q_p\frac{(B_p-B_o)S^2(N+1)}{N}+(\alpha S+\beta)\left(D(Q_p)+\frac{Q_p}{N}D'(Q_p)\right)$$
$$+(P_p-P_o)-\bar{\theta}S(B_p-B_o)+\gamma S=0 \tag{11}$$

Since equation (11) depends on Q_p but not on Q_o, it implicitly defines Q_p as a function of P_o, P_p and N. Substituting this function into (9), we obtain Q_o as a function of P_o, P_p and N, leading to:

$$Q_p=Q_p(P_o,P_p;N),\qquad Q_o=Q_o(P_o,P_p;N) \tag{12}$$

Equations (12) are the *airport*'s demands for the use of its peak and off-peak periods, respectively. Here it is worth stressing that, while $t_o(Q_o,Q_p)$ and $t_p(Q_o,Q_p)$ –defined by equations (5) and (6)– capture the *final* consumer demands for air travel, $Q_o(P_o,P_p;N)$ and $Q_p(P_o,P_p;N)$ are the *derived* demands faced by the airport. As demonstrated in the Appendix (Proposition A.1), there exist conditions on the parameters that guarantee interior solutions, that is, $\underline{\theta}<\theta^f<\theta^*<\bar{\theta}$ or equivalently, $Q_p,Q_o,Q_n>0$. For example, the peak period is used if the per-passenger airport peak/off-peak price differential is smaller than the incremental gross benefit, for the highest consumer type $\bar{\theta}$, of shifting from off-peak travel to peak travel. In particular, when the airport does not practice peak-load pricing (so $P_p=P_o$), the peak period is always used as long as $\gamma<\gamma_2$, that is, the benefit functions cross each other before $\bar{\theta}$. The proof also reveals that a smaller airport peak/off-peak price differential increases the likelihood of both the peak and off-peak periods being used, and that the off-peak period is always used if $\bar{\theta}$ is large enough.[13] In the remainder of the paper we shall restrict our attention to interior allocations.

[13] These results suggest that Brueckner (2002, 2005)'s condition of the benefit functions crossing at some point within $(\underline{\theta},\bar{\theta})$, which was introduced to guarantee the existence of a non-empty peak/off-peak interior solution, may not be

We now characterize the airport's demands $Q_p(P_o, P_p; N)$ and $Q_o(P_o, P_p; N)$. For example, totally differentiating (11) with respect to P_p yields:

$$\frac{\partial Q_p}{\partial P_p} = -\frac{N}{(B_p - B_o)S^2(N+1) + (\alpha S + \beta)\left((N+1)D'(Q_p) + Q_p D''(Q_p)\right)} < 0 \tag{13}$$

where the inequality follows from (2) and (3). So the airport's demand for the peak period is, as expected, downward-sloping in the peak charge. Similarly, we can obtain:

$$\begin{aligned}
&\frac{\partial Q_p}{\partial P_p} < 0, && \frac{\partial Q_p}{\partial P_o} = -\frac{\partial Q_p}{\partial P_p} > 0, \\
&\frac{\partial Q_o}{\partial P_p} = -\frac{\partial Q_p}{\partial P_p} = \frac{\partial Q_p}{\partial P_o} > 0, && \frac{\partial Q_o}{\partial P_o} = -\frac{\partial Q_p}{\partial P_o} - \frac{N}{B_o S^2(N+1)} < 0, \\
&\frac{\partial (Q_o + Q_p)}{\partial P_p} = 0, && \frac{\partial (Q_o + Q_p)}{\partial P_o} = -\frac{N}{B_o S^2(N+1)} < 0
\end{aligned} \tag{14}$$

We can see that, *ceteris paribus*, the airport peak charge does not influence total traffic but only the allocation of traffic to the peak and off-peak periods. Furthermore, from (11) and (13) we get:

$$\frac{\partial Q_p}{\partial \Delta P_{p-o}} = \frac{\partial Q_p}{\partial P_p} < 0 \tag{15}$$

where $\Delta P_{p-o} \equiv P_p - P_o$. Both the above results and straightforward comparative statics with respect to *N* lead to:

Remark 1: The airport's demands $Q_p(P_o, P_p; N)$ and $Q_o(P_o, P_p; N)$ have the following properties:

(i) They are downward-sloping in own prices;

(ii) The peak and off-peak periods are gross substitutes;

(iii) The off-peak runway charge (P_o) determines the amount of total traffic, while the difference between the peak and off peak charges (ΔP_{p-o}) determines the partition of that traffic into the two periods, with peak traffic declining with the charge differential.

(iv) $0 < \partial Q_p / \partial N < Q_p / \left(N(N+1)\right)$, so the number of peak-period passengers increases with *N*;

needed here. The peak benefit function may be above the off-peak benefit function everywhere, yet an interior equilibrium may still exist (see Figure 1).

(v) $\partial(Q_o + Q_p)/\partial N = (Q_o + Q_p)/\big(N(N+1)\big) > 0$, so the number of total passengers increases with *N*;

(vi) $\partial Q_o / \partial N > Q_o /\big(N(N+1)\big)$, so if the off-peak period is used ($Q_o > 0$) then the number of passengers traveling in the off-peak period increase with *N*.

Notice that Part (ii) of Remark 1 shows that the airport has the room to "spread the flights" across the peak and off-peak periods by using peak-load landing fees. Together with the discussion following equations (5) and (6), therefore, our vertical airport-airline structure gives rise to a possible *sequential PLP*: the PLP schemes implemented by the downstream airlines (higher peak airfare) induce a different periodic demand for the upstream airport. On the other hand, parts (iv)-(vi) show that the shape of the demands depend on the number of downstream carriers. Given that we consider interior solutions, conditional on runway fees P_p and P_o, both the peak and off-peak traffic volumes increase with the number of firms in the output market.

The final ingredient to characterize the Cournot equilibrium in the output market is related to the important issue of airfares: For given airport charges, how do the peak and off-peak airfares compare with each other? From (5) and (6) it follows that

$$\Delta t_{p-o} \equiv t_p - t_o = \overline{\theta}(B_p - B_o) - \gamma - Q_p S(B_p - B_o) - \alpha D(Q_p) \tag{16}$$

From the equilibrium condition (11) we obtain an expression for $\overline{\theta}(B_p - B_o) - \gamma$. Replacing that expression in (16) gives rise to the following airfare-differential formula, evaluated at the Cournot equilibrium:

$$\begin{aligned} \Delta t_{p-o}\big|_{\text{Cournot eq}} = {} & \frac{P_p - P_o}{S} + \frac{\beta}{S} D(Q_p) \\ & + \frac{\beta}{S}\frac{Q_p}{N} D'(Q_p) + \alpha \frac{Q_p}{N} D'(Q_p) + Q_p \frac{(B_p - B_o)S}{N} \end{aligned} \tag{17}$$

It is clear from (17) that if $P_p \geq P_o$, then $\Delta t_{p-o} > 0$, that is, if the airport uses peak-load pricing (in the sense that it charges a higher landing fee in the peak than in the off-peak), airlines will also use PLP (i.e., higher peak airfares) in equilibrium. More interesting perhaps is the fact that, even if the airport prices the periods *backwards*, i.e., $P_p < P_o$, the airlines may still use peak-load pricing in equilibrium, because the remaining four terms in (17) are all positive.

To further interpret (17), first note that holding P_p and P_o constant, $\partial \Delta t_{p-o} / \partial N$ is negative, which can be seen by differentiating (16) and recalling, from Remark 1, that sub-game equilibrium Q_p and Q_o increase in N. This implies that a monopoly airline would have the largest airfare differential. Since, from (5) $\partial t_o / \partial N$ is also negative, the lower the N, the larger the off-peak fare. These two observations are consistent with what we already have in Remark 1 with respect to total peak and off-peak traffic. Next, it can be seen that for very large N, the airfare differential approaches to the difference between an airline's peak and off-peak per-passenger average costs, i.e., the first and second terms on the right-hand side (RHS) of (17). When there is an oligopoly, however, three extra terms are added. Specifically, the third term on the RHS of (17) is the cost of extra congestion on an airline's own flights and caused by an additional passenger flying in the peak period. Thus, the first three terms on the RHS of (17) represent the difference between an airline's peak and off-peak marginal costs. The fourth term represents the money value of extra congestion to an airline's passengers when a new passenger chooses to fly in the peak period, whereas the fifth term is the mark-up term that arises from the carriers' exploitation of market power. Hence, as it is now known, oligopoly airlines only internalize (charge for) the congestion they impose on their own flights, which has two cost components: extra operating costs for the airline, and extra delay costs for its passengers (Brueckner, 2002). When there is a monopoly airline, congestion is perfectly internalized but exploitation of market power is at its highest degree. When N is large, exploitation of the market power is small but un-internalized congestion is large.

These points can be made more clearly if the Cournot case is compared to the case in which a social planner maximizes total surplus in the second-stage game. To do this we first need a measure of consumer surplus (*CS*). Given the linearity of our conditional indirect utility function in (1), *CS* is given by:

$$CS = \int_{\theta^*}^{\bar{\theta}} \left[\theta B_p - \alpha D(Q_p) - t_p(Q_p, Q_o)\right] f(\theta) d\theta + \int_{\theta^f}^{\theta^*} \left[\theta B_o - t_o(Q_p, Q_o)\right] f(\theta) d\theta \qquad (18)$$

where $f(\theta)$ is the density function. Using (5) and (6) for t_o and t_p, solving the integrals and replacing θ^* and θ^f with (4), we finally obtain:

$$CS = \frac{S^2}{2}\left(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2\right) \qquad (19)$$

The planner maximizes, for given airport charges, the sum of consumer surplus, given by (19), and airline profits:

$$CS + \Phi \equiv CS + \sum_{i=1}^{N} \phi^i \tag{20}$$

where Φ denotes the aggregate airline (equilibrium) profits. The first-order conditions of (20) with respect to airline quantities together with the imposition of symmetry lead to two equations, analogous to (9) and (10), which characterize the optimum. Subtracting the two equations from each other yields:

$$Q_p(B_p - B_o)S^2 + (\alpha S + \beta)\big(D(Q_p) + Q_p D'(Q_p)\big) + (P_p - P_o) - \bar{\theta} S(B_p - B_o) + \gamma S = 0 \tag{21}$$

Using (21) to obtain a new expression for $\bar{\theta}(B_p - B_o) - \gamma S$ and replacing the term in (16), we get:

$$\Delta t_{p-o}\Big|_{\substack{\text{efficient}\\ \text{output}}} = \frac{P_p - P_o}{S} + \frac{\beta}{S} D(Q_p) + \frac{\alpha S + \beta}{S} Q_p D'(Q_p) \tag{22}$$

Conditional on the airport charges and the airline market structure, (22) gives the socially efficient difference between the peak and off-peak airfares. This fare differential is equal to the difference between an airline's peak and off-peak average costs (the first and second terms on the RHS of (22)), plus all the external costs associated with a new flyer in the peak period, with the latter being the extra congestion cost of *all* the airlines and passengers, not just that of the airline that carries the new peak passenger. Obviously, the last two terms represent the portion of the optimal airfare differential that is not directly affected by the airport's pricing practices.

4. AIRPORT PRICING, TRAFFIC, DELAY AND WELFARE COMPARISONS

We have shown that the airport decisions, namely, P_p and P_o, can influence the subsequent output-market competition among airlines. When deciding its runway charges in the first stage, therefore, the airport will take the second-stage equilibrium output into account. These decisions may in reality be set by a public airport or a privatized airport. Consequently, the objective of an airport may be to maximize welfare or to maximize profit. In this section, we first compare airport charges and consequent airfares for these two airport types. We then discuss three extensions, namely, the case of a budget-constrained public airport,, the case of a public airport that is constrained to charge a time independent fee, and airport-airlines collaboration in a private setting.

4.1 Maximization of social welfare

Consider first a public airport that chooses P_p and P_o to maximize welfare. With three agents –namely, airport, airlines, and passengers– social welfare (*SW*) is the sum of their payoffs:

$$SW(P_o, P_p; N) = \pi(P_o, P_p; N) + CS + \Phi \tag{23}$$

where the airport's profit, π, is given by

$$\pi(P_o, P_p; N) = P_o Q_o + P_p Q_p - C \cdot (Q_o + Q_p) \tag{24}$$

In (23), $Q_o = Q_o(P_o, P_p; N)$ and $Q_p = Q_p(P_o, P_p; N)$ are the airport's peak and off-peak demands respectively (given by (12)) while *C* is the unit runway operating cost of the airport (recall we are considering a fixed capacity case, so the capacity cost is omitted here). Note that we have assumed that the marginal operating cost is constant, since the estimation of cost functions has shown that airport runways have relatively constant return to scale (e.g., Morrison, 1983; Pels, Nijkamp and Rietveld, 2003). Consumer surplus, *CS*, is given by (19), whereas each airline's equilibrium profit is $\phi^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h) = \phi^1\left(Q_o(P_o, P_p; N), Q_p(P_o, P_p; N), P_h\right)$. Noting the downstream equilibrium is symmetric, the aggregate airline profit is given by $\Phi(P_o, P_p; N) = N \cdot \phi^1(P_o, P_p; N)$, that is,

$$\begin{aligned}\Phi(P_o, P_p; N) = {} & \overline{\theta} S(B_p Q_p + B_o Q_o) - S^2\left(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2\right) - (\alpha S + \beta) Q_p D(Q_p) \\ & - (c + P_o) Q_o - (c + P_p) Q_p - \gamma S Q_p\end{aligned} \tag{25}$$

Substituting Φ, π, and *CS* into (23) we obtain:

$$\begin{aligned}SW = {} & \overline{\theta} S(B_p Q_p + B_o Q_o) - c \cdot (Q_p + Q_o) - C \cdot (Q_p + Q_o) - \gamma S Q_p \\ & - S^2 (B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2)/2 - (\alpha S + \beta) Q_p D(Q_p)\end{aligned} \tag{26}$$

Derivation of the pricing formulas then follows from the first-order conditions (details are in the Appendix; P_o^W, P_p^W denote the welfare-maximizing runway charges):

$$P_o^W = C - \frac{Q_o S^2 B_o}{N} - \frac{Q_p S^2 B_o}{N} \tag{27}$$

$$\Delta P_{p-o}^W = \frac{N-1}{N}(\alpha S + \beta) Q_p D'(Q_p) - \frac{Q_p S^2 (B_p - B_o)}{N} \tag{28}$$

The above welfare-maximizing airport pricing may be seen as if the fees were determined in two phases. First, choice of an off-peak price P_o^W induces the (socially) right amount of total traffic; as can be seen from (27), P_o^W is below the airport's marginal cost. This is needed because exploitation of market power in the airline market would induce

allocative inefficiencies by producing too little output. A welfare-maximizing airport fixes this inefficiency by providing a "subsidy" to the airlines and hence lowering their marginal costs in the off-peak period. The exact amount of the subsidy depends in part on the extent of market power, which here is captured by *N*. Once the total traffic is set to its optimal level, the next phase is concerned with the optimal allocation of this traffic to the peak and off-peak periods, which is, as indicated earlier, determined by ΔP_{p-o}. In particular, the public airport sets the peak/off-peak price differential to ΔP_{p-o}^{W} that will induce the optimal airfare differential downstream. This is apparent from substituting (28) into (17), which yields

$$\Delta t_{p-o}^{W}\Big|_{\text{Cournot eq}} = \frac{\beta}{S} D(Q_p) + \frac{\alpha S + \beta}{S} Q_p D'(Q_p) > 0 \tag{29}$$

The RHS of (29) is equal to the optimal airfare differential that is not directly affected by the airport's pricing practices, as discussed in (22). Hence, the outcome is the same as if the airport were to set $P_o = P_p$, which is optimal because there are no differences in costs, and then social welfare is maximized in the airline market.

Brueckner (2002) identified the first term in (28) as the per-flight toll that should be charged by the airport authorities to address the problem of un-internalized congestion (note that when *N*=1, this toll is equal to zero). Pels and Verhoef (2004), Basso (2005) and Zhang and Zhang (2006) pointed out that the optimal toll should also include the second term, the market-power effect;[14] they did this, however, using models of congestion pricing (one period), while Brueckner (2002) and the present paper use models of peak-load pricing. This distinction is important because a toll equal to the two terms, thereby capturing both the congestion and market power effects, will not be optimal unless it is charged on top of the optimal charge in the off-peak period, *which is not the marginal cost*. In other words, restricting the analysis to the toll that should be charged during the peak hours offers only a partial view of the problem.

Notice further that the charge differential, ΔP_{p-o}^{W}, given in (28), is not signed *a priori*. Hence, *it may happen that the airport charge is smaller in the peak period than in the off-peak period*. More specifically, the airport charge differential will be negative for small *N*. This is so because a "tight" airline oligopoly has an airfare differential that is too large due to strong market power, while congestion is largely internalized. As a consequence, the airport price

[14] To be fair, although Brueckner did not formally consider the second term in the toll to be charged, he did point out that, depending on the size of the market-power term, a pure congestion toll could be detrimental for social welfare.

differential is driven predominantly by the market-power effect. When N is large, on the other hand, the airport price differential will be positive. This is so because a "loose" oligopoly would have an airfare differential that is too small due to un-internalized congestion, whereas market power is relatively weak. The airport charge differential is then driven by the congestion effect. Note from (29) that although P_p^W (the welfare-maximizing peak charge) may be less than P_o^W, final passengers will, nevertheless, *always* pay higher peak airfare than off-peak airfare. The above discussion may be summarised in the following proposition:

Proposition 1: For a public, welfare-maximizing airport, (i) the off-peak runway charge is below its marginal cost; (ii) for small N, the off-peak runway charge may be *greater* than its peak runway charge; in this sense, it appears that the airport does not use peak-load pricing; (iii) although the airport's peak charge may be less than its off-peak charge, final passengers will nevertheless always pay higher peak airfare than off-peak airfare.

4.2 Maximization of airport profit

Next, consider a private, unregulated airport. The airport's profit is given by (24). The airport will choose P_p and P_o to maximize its profit, and the first-order conditions lead to (P_o^π, P_p^π denoting the profit-maximizing airport charges):

$$P_o^\pi - C = \frac{P_o^\pi}{\varepsilon_{oo}} + \frac{(P_p^\pi - C)Q_p\varepsilon_{po}}{Q_o\varepsilon_{oo}}, \qquad P_p^\pi - C = \frac{P_p^\pi}{\varepsilon_{pp}} + \frac{(P_o^\pi - C)Q_o\varepsilon_{op}}{Q_p\varepsilon_{pp}}$$

where $\varepsilon_{oo} \equiv -(\partial Q_o / \partial P_o)(P_o / Q_o)$ is the (positive) price elasticity of off-peak airport demand, $\varepsilon_{po} \equiv (\partial Q_p / \partial P_o)(P_o / Q_p)$ is a cross-price elasticity, and ε_{pp} and ε_{op} are defined analogously. Since $\partial Q_p / \partial P_o > 0$ and $\partial Q_o / \partial P_p > 0$ –see (14) or Remark 1– both ε_{op} and ε_{po} are positive, implying that the airport charges are higher than would be if the peak and off-peak charges were chosen independently (in which case the mark-ups would be proportional to the inverse of own-price demand elasticities only). This is a well-known result for multi-product monopolies that produce substitutes.

We can simplify the pricing equations: replacing the elasticities' definitions and using the fact that $\partial Q_o / \partial P_p = -\partial Q_p / \partial P_p$ in (14) and then equation (13) yield:

$$P_o^{\pi} = C + \frac{Q_o S^2 B_o (N+1)}{N} + \frac{Q_p S^2 B_o (N+1)}{N} \tag{30}$$

$$\Delta P_{p-o}^{\pi} = \frac{\alpha S + \beta}{N} Q_p \left[(N+1) D'(Q_p) + Q_p D''(Q_p) \right] + \frac{Q_p S^2 (B_p - B_o)(N+1)}{N} \tag{31}$$

The RHS of (31) is, by (2) and (3), positive and hence $P_p^{\pi} > P_o^{\pi}$: The private airport charges higher runway fees in the peak period than in the off-peak period, and this is true for any *N*. Thus, a profit-maximizing airport has an incentive to use peak-load pricing. Furthermore, since $(\alpha S + \beta) Q_p (N-1)/N$ is the extra cost each airline induces by not fully internalizing congestion, the first term on the RHS of (31) shows that the private airport will overcharge for congestion. Moreover, notice from (30) that the off-peak charge, which determines the amount of total traffic, is above marginal cost. This is a result of monopoly power on the part of the airport. There is, therefore, a "double marginalization" problem, which is typical of an uncoordinated vertical structure. The discussion leads to the following proposition:

Proposition 2: A private, profit-maximizing airport would use peak-load pricing but would charge more than the cost of un-internalized congestion. Further, it would charge an off-peak runway fee that is above its marginal cost.

4.3 Performance comparisons between the private outcome and first-best

Having derived and characterized the pricing structures for both the public and private airports, we now want to compare them. To have a clearer picture about their performance differences, we shall compare not only the off-peak runway fees and the peak/off-peak fee differentials, but also the induced traffic levels, delays and total surplus. Moreover, we want to assess how these differences (if any) change with the number of airlines, *N*, which is exogenously given and can be considered as a proxy for airline market structure. We summarize our findings in the following proposition (the proof is provided in the Appendix):

Proposition 3: Comparisons of airport pricing, traffic, delay and welfare between the private and public airports are as follows:

(i) $P_o^W < P_o^{\pi}$ and $\frac{dP_o^W}{dN} > \frac{dP_o^{\pi}}{dN} = 0$;

(ii) $\Delta P_{p-o}^W < \Delta P_{p-o}^{\pi}$ and $\frac{d\Delta P_{p-o}^W}{dN} > 0$. If the delay function is linear, then $\frac{d\Delta P_{p-o}^{\pi}}{dN} = 0$;

(iii) $Q_p^W > Q_p^\pi$ and $\frac{dQ_p^\pi}{dN} > \frac{dQ_p^W}{dN} = 0$;

(iv) $Q_t^W > Q_t^\pi$ and $\frac{dQ_t^\pi}{dN} > \frac{dQ_t^W}{dN} = 0$, where $Q_t \equiv Q_p + Q_o$ is total traffic volume;

(v) $D_p^W > D_p^\pi$ and $\frac{dD_p^\pi}{dN} > \frac{dD_p^W}{dN} = 0$;

(vi) $SW^W > SW^\pi$ and $\frac{dSW^\pi}{dN} > \frac{dSW^W}{dN} = 0$.

From Proposition 3 we see that a private, profit-maximizing airport would induce too small total traffic as compared to the first-best outcome, thereby resulting in allocative inefficiencies. Additionally, a private airport has a greater peak/off-peak runway charge differential than a public airport. Hence, with a private airport, the peak period would be underused not only because the airport has smaller total traffic, but also because its charge differential is too large. Although those passengers who still use the peak period benefit from less delays as part (v) enounces, overall it is not economically efficient. Note that part of the welfare loss arises if the consumers (or some of them, with the number depending on γ) denied from peak travel view traveling in the peak times as a higher quality product than traveling in the off-peak times. To help better understand this proposition, we also offer a schematic representation of the findings in Figure 2.

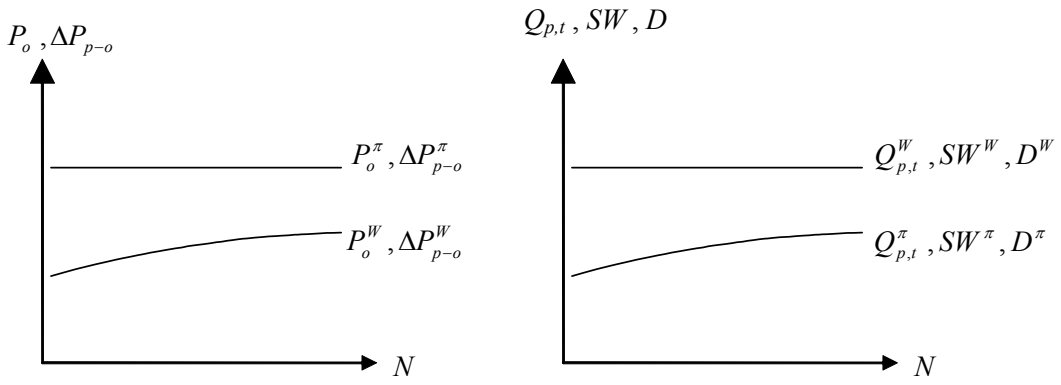


Figure 2: Schematic representation of the results in Proposition 3

This discussion highlights an important issue: one of the main ideas behind airport privatization has been that it would allow airports to use peak-load pricing and thus help solve the congestion problems. But if privatization is measured solely by its effect on congestion delays, it may be seen as a better idea than it actually is and important deadweight losses may be overlooked. This result, which holds here for a fixed capacity–peak-load pricing model, was also found by Basso (2005) in a congestion pricing model with variable (endogenous) capacity.

We have seen that the public airport is indifferent between values of N –although Basso (2005) showed that this may not be the case if airlines are not homogenous or if passengers are affected by schedule delay cost. Given that the (welfare) performance of a private airport improves as the number of airlines rises (see Figure 2), it seems important to know what would be the preferred N of a private airport itself. From (24) we have:

$$\left|\frac{d\pi}{dN}\right|_{P_o^\pi, P_p^\pi} = \left|\frac{\partial\pi}{\partial N}\right|_{P_o^\pi, P_p^\pi} = \left(P_o^\pi - C\right)\frac{\partial Q_o}{\partial N} + \left(P_p^\pi - C\right)\frac{\partial Q_p}{\partial N} > 0$$

where the first equality follows from the envelope theorem and the inequality follows from Remark 1 and the fact that prices are above marginal costs. Thus, the private airport prefers a large N, which is a desirable property, given the findings of Proposition 3.

4.4 Constrained public airport: the financial break-even case and the uniform pricing case

In the above analysis of public airport's pricing behaviour, we have not included a financial break-even constraint on the airport, which may represent a more realistic case nowadays. The unconstrained first-best solution may lead to budget inadequacy. One possible solution for this would be to use two-part tariff, that is, to charge a fixed fee in addition to the marginal (per flight) charge. However, the use of a fixed-fee may not be feasible for a number of reasons. One of them is the absence of symmetry at the airline level; as a consequence, the airport would need to charge differentiated fixed fees (depending on the airlines) in order to achieve the first-best, something that may encounter strong opposition and may even be unlawful. If lump-sum transfers are not possible, then Ramsey-Boiteaux prices should be considered. In general, as demonstrated analytically in Basso (2005) and Zhang and Zhang (2006), the pricing formula of a budget-constrained public airport varies between the formula for an unconstrained welfare-maximizing airport and the formula for a profit-maximizing airport. Likewise, in terms of social welfare this *second-best* situation would fall in between the first-best and the private outcome, and would move towards the latter when

the severity of the budget constraint rises. Analytically, there is little more to say regarding this issue. A rudimentary numerical simulation shows that for N=4 (for example) whilst the private outcome would attain 71% of the first-best welfare level, the budget-constrained public airport would attain 99%, very close to the first-best outcome.[15]

Another major concern of our public-private comparison in Section 4.3 is that such comparison may not be the most relevant one. As indicated in the introduction, public airports have not really practised peak-load pricing. For whatever reason, most public airports currently charge landing fees that are undifferentiated by time of day. We now consider pricing behaviour of a public airport that is constrained to use such a uniform pricing scheme. Formally, the airport's problem is the maximization of social welfare (26), subject to the constraint $P_p = P_o$. Derivation of the pricing rules follows from maximization of the corresponding Lagrangean function, yielding:[16]

$$P_o = C - \frac{Q_o S^2 B_o}{N} - \frac{Q_p S^2 B_o}{N} = P_o^W$$

$$P_p = P_o$$

Hence, if the public airport is constrained to charge a flat landing fee, it would not choose a fee that is in between the peak and off-peak prices –as the *a priori* intuition may have suggested– but would charge the off-peak price P_o^W throughout the day! This implies that the airport would be doing nothing regarding congestion, irrespective of how acute the problem is. In particular, when N is large and hence un-internalized congestion is severe (while airlines' market power is weak), the airport would still charge below marginal cost throughout the day, rather than charging for congestion. In other words, if the airport charges a flat fee, it would not be the case that it distorts the charging and meets "in the middle" of prices P_o^W and P_p^W; it is actually not taking congestion into account whatsoever. As a result, congestion would certainly worsen as the number of airlines increases.

Now, because P_o is below the marginal cost, in this case the airport would always run a deficit, even for large N, unless two-part tariff is feasible. However, adding a budget constraint to the problem does not change things much. From what we have just found, if the airport needs to break even but charges a flat fee, it would obviously raise the

[15] Details of this and others illustrations (see below) are available upon request.

[16] The derivation of these pricing rules is quite similar to the derivation of equations (27) and (28), which is provided in the Appendix. It is sufficient to add $\mu(P_p - P_o)$ to SW in (26) in order to form the Lagrangean (where μ is the multiplier) and then proceed in a similar fashion.

(uniform) price until it can cover its costs, something that happens when that the price equals marginal cost: $P_o = C = P_p$ (this is indeed easy to show formally). So, again, the airport would not really be doing anything to deal with the congestion problem.

These observations are important: on one hand, if by external reasons a public airport cannot use PLP but a private airport can, then privatization may indeed do a better job at solving the congestion problem. In terms of social welfare, the comparison of the two cases would obviously depend on several parameter values. For example, for N=50 in our numerical example, the private outcome attains 80% of the first-best welfare level, while a budget-constrained public airport charging a uniform price attains 88%, a quite smaller gap than before. More generally, if the problem of congestion is sufficiently important and if N is large, the gain from reduced congestion could analytically outweigh the welfare loss from a privatized airport's exploitation of market power. On the other hand, if public airports have no institutional reasons to avoid peak-load pricing, moving public airports toward the use of PLP is something that is worthwhile and urgent to do.

4.5 Airport-airlines strategic collaboration

Consider next an airport that has some sort of strategic agreements with the airlines using it. The reasons why it is interesting to look at this case are two-fold: on one hand, a simple pricing mechanism, two-part tariff, may be enough for the outcome of joint profit maximization to arise. On the other hand, it has been often argued that greater airlines' *countervailing power* or more strategic collaboration between airlines and airports may improve efficiency of privatized airports by allowing a better alignment of incentives, and even may make price regulation unnecessary (see, e.g., Beesley, 1999; Condie 2000; Forsyth, 1997; Starkie, 2001; Productivity Commission, 2002; Civil Aviation Authority UK, 2004). The analysis of joint profit maximization may then serve as another benchmark case for our comparison. Formally, the objective faced by this airport is to maximize the sum of the airport's profit and airlines' profits, that is, maximize $\pi + \Phi$ with respect to P_o and P_p. The pricing formula then follow from the first-order conditions of the problem (P_o^{JP} and P_p^{JP} denoting the joint profit-maximizing airport charges):

$$P_o^{JP} = C + \frac{Q_o S^2 B_o (N-1)}{N} + \frac{Q_p S^2 B_o (N-1)}{N} \tag{32}$$

$$\Delta P_{p-o}^{JP} = \frac{N-1}{N}(\alpha S + \beta)Q_p D'(Q_p) + \frac{Q_p S^2 (B_p - B_o)(N-1)}{N} \tag{33}$$

The interpretation of the JP airport's pricing rules (32) and (32) is as follows. As before, this airport may be seen as deciding its runway fees in two phases; first, it induces a contraction of total traffic by choosing an off-peak price P_o^{JP} above its marginal cost. It does so because the failure of coordination among the airlines results in their producing too much with respect to what would be best for them as a whole (referred to as the "business stealing effect"). The amount of excess production depends on how tight the oligopoly is, which is why the off-peak mark-up decreases with *N*. In particular, when the airline market is monopolized, (32) shows that $P_o^{JP} = C$ so the airport does not need to charge the mark-up at all. Comparison of (32) and (30), however, shows that the total traffic contraction in the JP case is smaller than that in the pure private case; indeed, coordination of the vertical airport-airlines chain resolves the double-marginalization problem. In the second phase, the airport chooses the (non-negative) price differential ΔP_{p-o}^{JP} that will induce the airlines' cartel outcome, destroying airline competition downstream. Hence, the outcome is the same as if the airport were to set $P_o = P_p$ and a cartel were running the airline market.

This result, which was obtained by Basso (2005) in a congestion pricing, two-part tariff setting, has different intuitions depending on why the maximization of joint profits is the relevant case. With two-part tariffs, the private airport uses the variable prices, peak and off-peak, to destroy competition downstream in order to maximize the airline profits, which are later captured by the airport through the fixed fee. When the joint profit maximization arises because of collaboration between airlines and the airport, what happens is that the airlines would like to collude in order to increase profits, but are unable to do so themselves. What they manage to do, however, is to "capture" an input provider (airport) to run the cartel for them. By altering the prices of the inputs (runway services) and hence the downstream marginal costs in both the peak and off-peak periods, the input provider induces both the collusive total output and the "right" (to the airlines) allocation of passengers to the peak and off-peak periods. The upstream firm is then rewarded with part of the collusive profits, which is where bargaining power enters the picture. Note that the airport pricing rules (32) and (33) take into account the congestion externality and the business-stealing effect at both

pricing phases: the airport's price differential has two parts.[17] When N=1, there is no business-stealing effect and congestion is perfectly internalized by the monopolist. Consequently, both terms vanish: *with a monopoly airline, this type of airport would not use peak-load pricing*.

Now, despite the fact that the result is as if airlines were colluding, this case is not worse, in terms of social welfare, than a private airport charging linear prices as in Section 4.2. This is because, here, the two other harmful externalities, namely, the vertical double marginalization and the congestion externality, have been dealt with. In effect, we can show that the JP case represents a middle-of-the-road case between the profit maximization and first-best: in Proposition 3, the runway fees, traffic volumes, delays and social welfare will be in between those of the private and public airport cases. And in Figure 2, the curves pertaining to the JP case would be parallel displacements of the public airport curves, lying in between the two existing public and private curves.[18] Strategic collaboration between the airport and the airlines smoothes the airport-charge problem. But recall that the downstream airfares would be as if the airlines were colluding, so we cannot expect that the JP ends up being very close to the first-best. In effect, our numerical simulation has shown that the JP would correspond to 81% of the maximum social welfare attainable, which is better than the pure private outcome but is worse than the performance of a budget-constrained public airport.

5. CONCLUDING REMARKS

Our main objective in writing this paper is to analyze the sequential peak-load pricing (PLP) problem that arises when airports are recognized as input providers for final consumer markets facing periodic demands, and to contribute to the understanding of the effect of airport privatization on congestion delay and social welfare. We have analyzed this PLP problem for a private airport and a private airport that strategically collaborates with the airlines, as well as for various types of public airport. We found that privatization would not induce efficient PLP schemes as it has been argued in some studies. While a private airport always has an incentive to use PLP –higher runway fees in the peak than off-

[17] This idea of an upstream firm running the cartel for the downstream firms has been discussed in the vertical control literature and, particularly, in the input joint-venture case. For example, Chen and Ross (2003) formalized the conjecture that input joint-ventures can facilitate collusion and push a market toward the monopoly outcome. If airport provision was seen as an input joint-venture by the airlines, our results show three things in addition to what Chen and Ross have found. First, the results hold even in a peak-load pricing setting, i.e., when demand is periodic. Second, if there are externalities, the input prices are, additionally, used to force their internalization by downstream competitors. Third, when marginal costs downstream are not constant, the outcome is not as in a monopoly or a downstream merger, but as in a cartel.

[18] The proofs of the results discussed in this paragraph are available upon request.

peak periods– irrespective of the number of airlines servicing the airport and even when the airlines have used PLP themselves, it would overcharge for congestion and its pricing structure would induce insufficient total traffic and peak traffic as compared to the socially optimal levels or the levels associated with a budget-constrained public airport. Somewhat surprisingly, depending on the degree of carriers' market power, a public airport may choose a peak charge that is lower than the off-peak charge, so as to offset the market power downstream at the airline level. Here, the public airport, on the surface, is not practicing the peak-load pricing, but such pricing structure is nevertheless socially optimal. Another surprising new result is related to the case of a public airport that is constrained to charge a time independent landing fee. Such airport would not choose a fee that is in between the peak and off-peak prices –as the *a priori* intuition may have suggested– but would charge the off-peak price. Thus if by external reasons a public airport cannot use PLP but a private airport can, privatization may indeed do a better job at solving the congestion problem. Further, if the problem of congestion is sufficiently important, the gain from reduced congestion might even outweigh the welfare loss from a privatized airport's exploitation of market power. This suggests that if public airports have no institutional reasons to avoid peak-load pricing, moving public airports toward the use of PLP would be worthwhile. Finally, a private airport that strategically collaborates with the airlines would induce greater total traffic and peak traffic than a pure private airport, but both figures will still be smaller than those for a pure public airport or a budget-constrained public airport. If the airport collaborates with a monopoly airline, it would not use peak-load pricing.

The paper has raised several other issues and avenues for future research. First, while the model covers the monopoly and perfect competitive market structures (at the airline level), it limits the oligopoly analysis to a symmetric Cournot-Nash equilibrium. As pointed out by an anonymous referee, assuming a symmetric airline equilibrium is most reasonable for airports like (in the U.S.) Los Angeles, LaGuardia, JFK, Boston, Washington National, and perhaps Chicago O'Hare and Dallas/Ft. Worth. A number of other hub airports however, typically have one airline with over half of the departure traffic. Here, the large airline is not a monopoly, and the many airlines with small market shares may be modelled as competitive fringe or as Stackelberg followers. In these cases, if the large airline moves a flight from the peak to off-peak periods, it may induce some fringe carriers' flights to move from the off-peak to peak periods, especially if the off-peak period is also congestible. As demonstrated by Daniel and Harback (2007), when a Stackelberg dominant airline faces an equilibrium in which relative congestion levels are determined by the equilibrium behaviour of other airlines, its ability to internalize congestion is limited. Therefore, our assumptions

(symmetric Cournot-Nash equilibrium) lead to a role of congestion which is less important than would be under Daniel and Harback's assumptions. Further investigating how the dominant carrier interacts with the small carriers and the associated impacts on airport capacity, peak-load pricing and congestion would make the analysis and policy implications applicable to a larger number of hub airports. Second, we have assumed a monopoly airport. This assumption is quite common in the airport literature and is understandable given the local monopoly nature of an airport. The situation is changing, however, as a result of the continuing rapid growth in air transport demand and the dramatic growth of low cost carriers, which has enabled some secondary airports to cut into the catchment areas of large airports. Many of the major markets in the U.S. and elsewhere now have at least two commercial airports, e.g., New York, Los Angeles, Chicago, Washington DC, San Francisco, and London. Moreover, privatization would probably increase competition from regional airports. A recent analysis on pricing and capacity competition between two congestible airports was Basso and Zhang (2007), which casts the problem in a one-period congestion model. We see analysis of the two-period peak-load pricing with competing airports as a natural and important extension.

Although the airline industry is chosen for analysis, our basic model structure, in which airports, as input providers, make their pricing decisions prior to airlines' strategic interactions in the final output market, is highly relevant to several other industries including electricity, telecommunications, and transport terminals (e.g., the vertical chain of ports-shipping liners-shippers). In telecommunications, for example, at the upstream level there are the network owners, while downstream there are carriers who must use the network in order to produce the final good (telephone calls). Like airports, these industries are undergoing privatization in a number of countries. We note that the sequential PLP method used in the present paper may be useful in examining similar issues in those sectors as well, particularly when these industries are prone to have congestion problems.

REFERENCES

Arnott, R., A. De Palma and R. Lindsey (1993) "A structural model of peak-load congestion: A traffic bottleneck with elastic demand," *American Economic Review,* 83, 161-179.

Basso, L. J. (2005) "Airport Ownership: Effects on Pricing and Capacity". Working Paper, The University of British Columbia (http://ssrn.com/abstract=849584).

Basso, L. J. and A. Zhang (2007) "Congestible Facility Rivalry in Vertical Structures," *Journal of Urban Economics*, 61(2), 218-237.

Beesley, M.E. (1999) "Airport regulation," in: M.E. Beesley (ed.), *Regulating Utilities: a New Era?*, Institute of Economic Affairs, London.

Boiteux, M. (1949) "La Tarification des demandes en point: Application de la theorie de la vente au cout marginal," *Revue Generale de L'Electricite*, 58, 321-340; translated as Boiteux, M. (1960) "Peak load pricing," *Journal of Business*, 33, 157–179.

Borins, S.F. (1978) "Pricing and investment in a transportation network: The case of Toronto airport," *Canadian Journal of Economics*, 11, 680-700.

Brander, J.A. and A. Zhang (1990) "Market conduct in the airline industry," *Rand Journal of Economics*, 21, 567-583.

Brander, J.A. and A. Zhang (1993) "Dynamic behaviour in the airline industry," *International Journal of Industrial Organization*, 11, 407-435.

Brueckner, J.K. (2002) "Airport congestion when carriers have market power," *American Economic Review*, 92: 1357-1375.

Brueckner, J.K. (2005) "Internalization of airport congestion: A network analysis," *International Journal of Industrial Organization*, 23, 599-614.

Calzada, J. (2003) "Capacity charge and peak-load pricing," Working paper, Universitat de Barcelona.

Carlin, A. and R.E. Park (1970) "Marginal cost pricing of airport runway capacity," *American Economic Review*, 60, 310-319.

Chen, Z. Q. and T.W. Ross (2003) "Cooperating upstream while competing downstream: a theory of input joint ventures". *International Journal of Industrial Organization*, 21(3), 381-397

Civil Aviation Authority (2004) *Airport Regulation: Looking to the Future, Learning from the Past*, Civil Aviation Authority, London.

Condie, S. (2000) "Whither airport regulation?" in: W. Bradshaw and H.L. Smith (eds.), *Privatization and Deregulation of Transport*, St. Martin's Press, Inc, New York.

Craven, J. (1971) "On the optimal time periods for a surplus-maximising utility subject to fluctuating demand," *Bell Journal of Economics*, 2, 495-502.

Craven, J. (1985) "Peak-load pricing and short-run marginal cost," *Economic Journal*, 95, 778-780.

Crew, M. and P. Kleindorfer (1986) *The Economics of Public Utility Regulation*, Cambridge, Mass., MIT Press.

Crew, M. and P. Kleindorfer (1991) "Peak-load pricing of postal service and competition," in M. Crew and P. Kleidorfer (eds.) *Competition and Innovation in Postal Sciences*, Kluwer Academic Publisher, pp. 37–56.

Crew, M., C.S. Fernando and P. Kleindorfer (1995) "The theory of peak-load pricing: A survey," *Journal of Regulatory Economics*, 8, 215-248.

Dana, J.D. (1999) "Using yield management to shift demand when the peak time is unknown," *RAND Journal of Economics*, 30, 456-474.

Daniel, J.I. (1995) "Congestion pricing and capacity of large hub airports: A bottleneck model with stochastic queues," *Econometrica*, 63, 327-370.

Daniel, J.I. (2001) "Distributional consequences of airport congestion pricing," *Journal of Urban Economics*, 50, 230-258.

Daniel, J.I. and K.T. Harback (2007) "(When) do hub airlines internalize their self-imposed congestion delays?" *Journal of Urban Economics*, forthcoming.

Dansby, R. (1978) "Capacity constrained peak-load pricing," *Quarterly Journal of Economics*, 92, 387-98.

De Palma, A. and R. Lindsey (1998) "Information, welfare and usage of congestible facilities under different pricing regimes," *Canadian Journal of Economics,* 31, 666-692.

Forsyth, P. (1997) "Price regulation of airports: Principles with Australian applications," *Transportation Research E*, 33, 297-309.

Gersten, J. (1986) "Peak-load pricing in competitive markets," *Economic Inquiry*, 24, 349-361.

Gillen, D.W. (1994) "The market and aviation infrastructure: Pricing, productivity and privatization," *Transportation Research Circular*, No. 425 (App): 82-86.

Hirschleifer, J. (1958) "Peak-loads and efficient pricing: A comment," *Quarterly journal of Economics*, 72, 451-462.

Horonjeff, R. and McKelvey, F.X. (1983) *Planning and Design of Airports*, third edition, McGraw-Hill.

Laffont, J.J. and J. Tirole (2000) *Competition in Telecommunications*, Massachusetts, MIT Press.

Lave, L.B. and J.S. DeSalvo (1968) "Congestion, tolls, and the economic capacity of a waterway," Journal of Political Economy, 76, 375-391.

Levine, M.E. (1969) "Landing fees and the airport congestion problem," *Journal of Law and Economics*, 12, 79-108.

Morrison, S.A. (1983) "Estimation of long-run prices and investment levels for airport runways," *Research in Transportation Economics,* 1, 103-130.

Morrison, S.A. (1987) "The equity and efficiency of runway pricing," *Journal of Public Economics*, 34, 45-60.

Morrison, S.A. and C. Winston (1989) "Enhancing the performance of the deregulated air transportation system," *Brookings Papers on Economic Activity, Microeconomics* (1989) Special Issue, 61-123.

Oum, T.H. and Y. Zhang (1990) "Congestion tolls, lumpy investment and cost recovery," *Journal of Public Economics*, 43, 353-374.

Panzar, J. (1976) "A neoclassical approach to peak-load pricing," *Bell Journal of Economics*, 7, 521-530.

Pels, E. and E.T. Verhoef (2004) "The economics of airport congestion pricing," *Journal of Urban Economics*, 55, 257-277.

Pels, E., P. Nijkamp and P. Rietveld (2003) "Inefficiencies and scale economies of European airport operations," *Transportation Research E*, 39, 341-361.

Poole, R.W. (1990) *Airport Privatization: What the Record Shows*, The Reason Foundation Policy Study No. 124, The Reason Foundation, Los Angeles, CA.

Pressman, I. (1970) "A mathematical formulation of the peak-load pricing," *Bell Journal of Economics*, 1, 304–326.

Productivity Commission (2002), *Price Regulation of Airport Services*, Report No. 19, AusInfo Canberra, Australia.

Shy, O. (2001) "Dynamic peak-load pricing," Working paper, University of Haifa and Stockholm School of Economics.

Starkie, D. (2001) "Reforming UK airport regulation," *Journal of Transport Economics and Policy*, 35, 119-135.

Steiner, P. (1957) "Peak-loads and Efficient Pricing," *Quarterly journal of Economics*, 71, 585–610.

U.S. Federal Aviation Administration (1969) *Airport Capacity Handbook*, 2nd edition, Washington, D.C.

Vasigh, B. and M. Haririan (1996) "Airport privatization: A simple welfare analysis," *Journal of Economics and Finance*, 20: 89-95.

Williamson, O. (1966) "Peak-load pricing and optimal capacity constraints," *American Economic Review*, 56, 810–827.

Yuen, A. and A. Zhang (2005) "Airport congestion and market structure: The case of variable passenger time costs," Working paper, University of British Columbia.

Zhang, A. and Y. Zhang (2003) "Airport charges and capacity expansion: Effects of concessions and privatization," *Journal of Urban Economics*, 53, 54-75.

Zhang, A. and Y. Zhang (2006) "Airport capacity and congestion when carriers have market power," *Journal of urban Economics*, 60, 229-247.

APPENDIX

- **Proposition A.1**: Conditions for an interior allocation of consumers

(i) If $(P_p - P_o)/S < \bar{\theta}(B_p - B_o) - \gamma S$, then the peak period is used, that is $\theta^* < \bar{\theta}$.

(ii) If $\underline{\theta} B_o < (c + P_o)/S$, then some consumers will not fly, that is $\theta^f > \underline{\theta}$.

(iii) If $\bar{\theta}$ is large enough, then the off-peak period is used, that is $\theta^* > \theta^f$.

Proof:

First, equivalent conditions for interior allocations, but in terms of Q_p and Q_o are:

The peak is used: $\theta^* < \bar{\theta} \Leftrightarrow (\bar{\theta} - \theta^*)/S > 0 \Leftrightarrow Q_o > 0$

Some consumers do not fly: $\theta^f > \underline{\theta} \Leftrightarrow (\bar{\theta} - \theta^f)/S < (\bar{\theta} - \underline{\theta})/S \Leftrightarrow Q_o + Q_p < (\bar{\theta} - \underline{\theta})/S$

The off-peak is used: $\theta^* > \theta^f \Leftrightarrow (\theta^* - \theta^f)/S > 0 \Leftrightarrow Q_o > 0$

With this, the proofs of each part are:

(i) Note that $(-\Omega^p + \Omega^o)$ in (11) is strictly increasing in Q_p, and $\left(-\Omega^p + \Omega^o\right)_{Q_p \to \infty} > 0$. Also, $\left(-\Omega^p + \Omega^o\right)_{Q_p = o} = (P_p - P_o) - \bar{\theta} S(B_p - B_o) + \gamma S$. Hence, if $(P_p - P_o) < \bar{\theta} S(B_p - B_o) - \gamma S$, then $\left(-\Omega^p + \Omega^o\right)_{Q_p = o} < 0$ and $Q_p > 0$. ■

(ii) From Ω^o=0 in (9) we get that $(Q_o + Q_p)B_o S^2(N+1)/N = B_o\bar{\theta}S - c - P_o$, This imply that $Q_o + Q_p < (B_o\bar{\theta}S - c - P_o)/(B_o S^2)$. Hence, a sufficient condition for $Q_o + Q_p < \frac{\bar{\theta} - \underline{\theta}}{S}$ is: $(B_o\bar{\theta}S - c - P_o)/(B_o S^2) < \bar{\theta} - \underline{\theta}/S$, which leads to $\underline{\theta}B_o < (c + P_o)/S$. ■

(iii) From Ω^o=0 we know that $Q_o + Q_p = \frac{(B_o\bar{\theta}S - c - P_o)N}{B_o S^2(N+1)}$. Hence, $Q_o > 0$ is equivalent to $Q_p < \frac{(B_o\bar{\theta}S - c - P_o)N}{B_o S^2(N+1)} \equiv \tilde{Q}_p$. In order to ensure $Q_p < \tilde{Q}_p$, we need that $\left(-\Omega^p + \Omega^o\right)_{Q_p = \tilde{Q}_p} > 0$ (see proof of part i). Straightforward algebra gives us $(-\Omega^p + \Omega^o)(\tilde{Q}_p) = (\alpha S + \beta)\left[D(\tilde{Q}_p) + \frac{\tilde{Q}_p}{N}D'(\tilde{Q}_p) + \frac{(P_p - P_o) + \gamma S}{(\alpha S + \beta)} - \frac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)}\right]$, so that a sufficient condition for $\left(-\Omega^p + \Omega^o\right)_{\tilde{Q}_p} > 0$ is $D(\tilde{Q}_p) > \frac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)} - \frac{(P_p - P_o) + \gamma S}{(\alpha S + \beta)}$. And since $\partial\tilde{Q}_p/\partial\bar{\theta} > 0$, the condition is always fulfilled for $\bar{\theta}$ large enough, even when $\gamma < \gamma_1$ – i.e. when the peak benefit function is above the off peak benefit function everywhere in $(\underline{\theta}, \bar{\theta})$. ■

Part (i) says that the peak period is used if the airport price differential between peak and off-peak is not too large. Specifically, the per-passenger airport price differential has to be smaller than the incremental benefit, for the highest consumer type, of changing from the off-peak to the peak. If the airport does not practice PLP, the peak is used as long as $\gamma < \gamma_2$. Part (ii) says that if $\underline{\theta}$ is low enough, then some consumers will not fly. In particular, the lowest consumer type must have a willingness to pay for off-peak travel that is smaller than the airlines' per-passenger marginal cost for an off-peak flight. Finally, part (iii) implies that Brueckner (2002, 2005)'s *interior crossing property*, which imposes that $B_p(\theta) < B_o(\theta)$ for small θ values, may not be needed to have a non-empty off-peak, and that a smaller airport price differential between peak and off-peak increases the likelihood of the off-peak been used. The lower bound for $\bar{\theta}$ cannot be made explicit because of the non-linearity of the delay function. For a linear delay function $D(Q_p, K) = \delta Q_p / K$, the lower bound on $\bar{\theta}$ is given by $\tilde{\theta} = \frac{SK(N+1)}{\delta B_o(\alpha S + \beta)N}\left((B_p - B_o)(c + P_o) - B_o(P_p - P_o) - B_o\gamma S\right) + c + P_o$, while a lower bound not depending on *N*, would be $2\tilde{\theta}N/(N+1)$.

- **Derivation of equations (27) and (28)**

Consider first the derivative of (26) with respect to P_p:

$$\frac{\partial SW}{\partial P_p} = \left\{\bar{\theta}SB_p - \gamma S - c - C - Q_o B_o S^2 - Q_p B_p S^2 - (\alpha S + \beta)\left(D(Q_p) + Q_p D_Q(Q_p)\right)\right\}\frac{\partial Q_p}{\partial P_p}$$

$$- \left\{\bar{\theta}SB_o - c - C - Q_o B_o S^2 - Q_p B_o S^2\right\}\frac{\partial Q_o}{\partial P_p}$$

But, from (14), $\partial Q_o / \partial P_p = -\partial Q_p / \partial P_p$. Replacing and re-arranging, we get the following:

$$\frac{\partial SW}{\partial P_p} = \left\{\bar{\theta}S(B_p - B_o) - Q_p(B_p - B_o)S^2 - (\alpha S + \beta)\left(D(Q_p) + Q_p D_Q(Q_p)\right)\right\}\frac{\partial Q_p}{\partial P_p} = 0 \qquad \text{(A.2)}$$

Next, consider the derivative of (26) with respect to P_o:

$$\frac{\partial SW}{\partial P_o} = \left\{\bar{\theta}SB_p - \gamma S - c - C - Q_o B_o S^2 - Q_p B_p S^2 - (\alpha S + \beta)\left(D(Q_p) + Q_p D_Q(Q_p)\right)\right\}\frac{\partial Q_p}{\partial P_o}$$

$$- \left\{\bar{\theta}SB_o - c - C - Q_o B_o S^2 - Q_p B_o S^2\right\}\frac{\partial Q_o}{\partial P_o}$$

From (14), we have that $\frac{\partial Q_o}{\partial P_o} = -\frac{\partial Q_p}{\partial P_o} - \frac{N}{B_o S^2 (N+1)}$. Replacing and re-arranging, we get the following:

$$\frac{\partial SW}{\partial P_o} = \left\{\bar{\theta}S(B_p - B_o) - \gamma S - Q_p(B_p - B_o)S^2 - (\alpha S + \beta)\left(D(Q_p) + Q_p D_Q(Q_p)\right)\right\}\frac{\partial Q_p}{\partial P_o}$$

$$- \left\{\bar{\theta}SB_o - c - C - Q_o B_o S^2 - Q_p B_o S^2\right\}\frac{N}{B_o S^2 (N+1)} = 0$$

The first term in brackets on the RHS is 0 by $\partial SW / \partial P_p$ in (A.2). Using Ω^o in (9), we also get that $\bar{\theta}SB_o - c = P_o + \frac{Q_o B_o S^2 (N+1)}{N} + \frac{Q_p B_p S^2 (N+1)}{N}$. Thus we get

$$\frac{\partial SW}{\partial P_o} = -\left\{P_o + \frac{Q_o B_o S^2}{N} + \frac{Q_p B_p S^2}{N} - C\right\}\frac{N}{B_o S^2 (N+1)} = 0$$

From where equation (27) is obtained. Next, $(-\Omega^p + \Omega^o)$ in (11), we get

$$\bar{\theta}S(B_p - B_o) - \gamma S = P_p - P_o + \frac{Q_p S^2 (B_p - B_o)(N+1)}{N} + (\alpha S + \beta)\left(D(Q_p) + \frac{Q_p}{N} D_Q(Q_p)\right)$$

which, when replaced in (A.2) leads to equation (28).

- **Proof of Proposition 3**

To prove parts (i) and (ii), it is useful to first state the following Lemma:

Lemma A.2: If two prices P_1 and P_2 are given by the fixed points $P_1 = f(Q(P_1))$ and $P_2 = g(Q(P_2))$ respectively, where f is continuously differentiable in $(Q(P_1); Q(P_2))$, Q is continuously differentiable in $(P_1; P_2)$, $f(Q(P_2)) > g(Q(P_2))$, and either $Q(\cdot)$ is non-increasing and $f(\cdot)$ is non-decreasing, or $Q(\cdot)$ is non-decreasing and $f(\cdot)$ is non-increasing, then $P_1 > P_2$.

Proof: We prove this by contradiction. Suppose that $P_1 \le P_2$. Denote $\widetilde{P}_2 = f(Q(P_2))$. Applying the mean-value theorem to $P_1 = f(Q(P_1))$ and $\widetilde{P}_2 = f(Q(P_2))$ yields

$$P_1 - \widetilde{P}_2 = f'(\overline{Q})(Q(P_1) - Q(P_2))$$

where $\overline{Q}$ is some point between $Q(P_1)$ and $Q(P_2)$. Further applying the mean-value theorem to $Q(P_1)$ and $Q(P_2)$, the above equation becomes:

$$P_1 - \widetilde{P}_2 = f'(\overline{Q})(Q(P_1) - Q(P_2)) = f'(\overline{Q})Q'(\overline{P})(P_1 - P_2) \ge 0$$

where the inequality arises because $f'(\overline{Q})Q'(\overline{P}) \le 0$ and the assumption that $P_1 \le P_2$. Thus, $P_1 \ge \widetilde{P}_2$. But since by assumption $f(Q(P_2)) > g(Q(P_2))$ or, equivalently, $\widetilde{P}_2 > P_2$, we obtain $P_1 > P_2$, thus resulting in a contradiction. ■

Now, we can prove Proposition 3.

(i) That $P_o^W < P_o^\pi$ follows from writing the pricing rules (27) and (30) as $P_o^\pi = f_o(Q_o + Q_p)$ and $P_o^W = g_o(Q_o + Q_p)$. Since total traffic $Q_0 + Q_p$ is, by (14), downward sloping in P_0, $f_o(\cdot)$ is increasing and $f_o(\cdot) > g_o(\cdot)$, then $P_o^W < P_o^\pi$ by Lemma A.2.

Next, taking derivative of (27) and (30) with respect to N gives us:

$$\frac{dP_o^W}{dN} = S^2 B_o \left[\frac{Q_o + Q_p}{N^2} - \frac{\partial(Q_o + Q_p)}{\partial N} \frac{1}{N} \right] \text{ and } \frac{dP_o^\pi}{dN} = S^2 B_o \left[\frac{\partial(Q_o + Q_p)}{\partial N} \left(\frac{N+1}{N} \right) - \frac{Q_o + Q_p}{N^2} \right].$$

But, from Remark 1. we know that $\partial(Q_o + Q_p)/\partial N = (Q_o + Q_p)/\big(N(N+1)\big)$. Therefore

$$\frac{dP_o^W}{dN} = \frac{S^2 B_o (Q_o + Q_p)}{N(N+1)} > 0 \text{ and } dP_o^\pi / dN = 0.$$ ■

(ii) That $\Delta P_{p-o}^W < \Delta P_{p-o}^\pi$ follows from writing the pricing rules (28) and (31) as $\Delta P_{p-o}^\pi = f_{p-o}(Q_p)$ and $\Delta P_{p-o}^W = g_{p-o}(Q_p)$. Since peak traffic Q_p is, by (15), downward sloping in ΔP_{p-o}, $f_{p-o}(\cdot)$ is non-

decreasing as long as the (unsigned) term $D'''(\cdot)$ is non-negative (or if is negative, its magnitude is not too large) and $f_{p-o}(\cdot) > g_{g-o}(\cdot)$, then $\Delta P^{W}_{p-o} < \Delta P^{\pi}_{p-o}$ by Lemma A.2.

$\frac{d\Delta P^{W}_{p-o}}{dN} > 0$ follows from differentiation of (28) with respect to *N*. We get:

$$\frac{d\Delta P^{W}_{p-o}}{dN} = (\alpha S + \beta)\left[\frac{Q_p D'(Q_p)}{N^2} + \frac{\partial Q_p}{\partial N}\frac{(N-1)}{N}D'(Q_p) + Q_p D''(Q_p)\frac{(N-1)}{N}\right] + S^2(B_p - B_o)\left[\frac{Q_p}{N^2} - \frac{\partial Q_p}{\partial N}\frac{1}{N}\right]$$

Since $\partial Q_p / \partial N > 0$ by Remark 1.4, the first term in the RHS is positive. Since $\partial Q_p / \partial N < Q_p / \left(N(N+1)\right)$ by Remark 1.4, $\frac{Q_p}{N^2} > \frac{\partial Q_p}{\partial N}\frac{1}{N}$ making the second term in the RHS positive as well. Therefore, $d\Delta P^{W}_{p-o} / dN > 0$.

$\frac{d\Delta P^{\pi}_{p-o}}{dN} = 0$ if the delay function is linear, follows from differentiating (31) and then imposing $D''(Q_p) = 0$, $D'''(Q_p) = 0$. We get:

$\frac{d\Delta P^{\pi}_{p-o}}{dN} = \left((\alpha S + \beta)D'(Q_p) + (B_p - B_o)S^2\right)\left[-\frac{Q_p}{N^2} + \frac{\partial Q_p}{\partial N}\frac{(N+1)}{N}\right]$, where $D'(Q_p)$ is a constant. With a linear delay function Remark 1.4 changes to $\partial Q_p / \partial N = Q_p / \left(N(N+1)\right)$, making $d\Delta P^{\pi}_{p-o} / dN = 0$. If we consider $D''(Q_p) > 0$, the sign is then undetermined and depends on the values of, for example, B_p, B_o and $D'''(Q_p)$. ■

(iii) $Q^{W}_{p} > Q^{\pi}_{p}$ flows from part (ii), and the comparative statics in (15) or Remark 1.3. $\frac{dQ^{\pi}_{p}}{dN} > \frac{dQ^{W}_{p}}{dN} = 0$ follows from replacing ΔP^{π}_{p-o} in the sub-game equilibrium equation (11); we get:

$$\left(-\Omega^{p} + \Omega^{o}\right)(\Delta P^{\pi}_{p-o}) = \frac{2Q^{\pi}_{p}(B_p - B_o)S^2(N+1)}{N} - \bar{\theta}S(B_p - B_o) - \gamma S + (\alpha S + \beta)\left(D(Q^{\pi}_{p}) + \frac{N+2}{N}Q^{\pi}_{p}D'(Q^{\pi}_{p}) + \left(Q^{\pi}_{p}\right)^2 D''(Q^{\pi}_{p})\right) = 0$$

Use this to calculate $\frac{dQ_p^{\pi}}{dN} = -\frac{\partial(-\Omega^p + \Omega^o)(\Delta P_{p-o}^{\pi})/\partial N}{\partial(-\Omega^p + \Omega^o)(\Delta P_{p-o}^{\pi})/\partial Q_p}$ and to prove that $0 < dQ_p^{\pi}/dN < Q_p^{\pi}/\big(N(N+1)\big)$, which shows that peak traffic increases with *N*.

Similarly, replacing ΔP_{p-o}^{W} in the sub-game equilibrium equation (11), we get:

$$\big(-\Omega^p + \Omega^o\big)(\Delta P_{p-o}^{W}) = Q_p^W (B_p - B_o)S^2 - \bar{\theta}S(B_p - B_o) - \gamma S + (\alpha S + \beta)\big(D(Q_p^W) - Q_p^W D'(Q_p^W)\big) = 0$$

Which does not depend on *N*, hence $dQ_p^W/dN = 0$. ■

(iv) $Q_t^W > Q_t^{\pi}$ flows from part (i) and the comparative statics in (14) or Remark 1.1. To prove $\frac{dQ_t^{\pi}}{dN} > \frac{dQ_t^W}{dN} = 0$, use the sub-game equilibrium equation (9) to prove $\frac{dQ_o^{\pi}}{dN} > \frac{dQ_o^W}{dN} = 0$ in an analogous way as in part (iii). The result then follows directly from this and part (iii).

(v) Direct from part (iii) and $D'(Q_p) > 0$ (equation 2). ■

(vi) Consider the SW function in (26). We can rewrite it in terms of total traffic, Q_t, and peak-traffic, by replacing $Q_o = Q_t - Q_p$. This gives us:

$$SW(Q_t, Q_p) = \bar{\theta}S(B_p Q_p + B_o(Q_t - Q_p)) - c(Q_t) - C(Q_t) - \gamma S Q_p - \frac{S^2}{2}(B_o(Q_t - Q_p)^2 + 2B_o(Q_t - Q_p)Q_p + B_p Q_p^2) - (\alpha S + \beta)Q_p D(Q_p)$$

Now, *SW* is globally concave in (Q_t, Q_p) because

$$\frac{\partial^2 SW}{\partial Q_t^{\,2}} = -B_o S^2 < 0,\ \frac{\partial^2 SW}{\partial Q_p^{\,2}} = -(B_p - B_o)S^2 - (\alpha S + \beta)\big(2D'(Q_p) + Q_p D''(Q_p)\big) < 0 \text{ and}$$

$$\frac{\partial^2 SW}{\partial Q_t^{\,2}} \cdot \frac{\partial^2 SW}{\partial Q_p^{\,2}} - \left(\frac{\partial^2 SW}{\partial Q_t \partial Q_p}\right)^2 = B_o S^2\big[(B_p - B_o)S^2 + (\alpha S + \beta)\big(2D'(Q_p) + Q_p D''(Q_p)\big)\big] > 0.$$

Since (Q_t^W, Q_p^W) maximizes *SW*, and from parts (iii) and (iv), $(Q_t^W, Q_p^W) > (Q_t^{\pi}, Q_p^{\pi})$, then $SW^W > SW^{\pi}$. Finally, since (Q_t^{π}, Q_p^{π}) increases with *N*, SW^{π} increases with *N*, while SW^W does not change with *N* because (Q_t^W, Q_p^W) does not. ■