

An optimization algorithm to assess different reserves models for open pit short term planning

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Abstract

This paper summarizes a research to develop an optimization algorithm used to support short term planning decisions in an open pit operation. In particular, it is sought to use this algorithm to evaluate block models that have been computed using different geostatistical techniques. This leads to several production schedules, each representing the best production option for a given block model. Then, these schedules are compared against a most likely optimal schedule that uses blast hole grade information as a main input. This comparison provides sufficient information to decide upon the geostatistical method that better forecasts a given production schedule.

The optimization algorithm is oriented to minimize production deviations with respect to the medium term plan (yearly budget) subject to uniform grade fed to the plant over a period of time (e.g. a week), minimum tonnage fed, shovels digability, among others. The algorithm will report the path that needs to be followed at different mining faces in order to reach optimality. The method proposed is of direct optimization, in which all the possible combinations in short time steps are evaluated explicitly hence allowing decision making regarding the path to be followed. Finally, the geostatistical method that better predicts the actual mining outcome is determined. This path minimizes deviations regarding the medium term production targets.

The method is implemented and validated with actual data: a reserves model is built from drillhole data using several geostatistical models, namely, ordinary kriging and Gaussian simulation. The models are compared with blasthole data that is effectively used to make the decision of sending the blocks to the processing plant or to the waste dump. The responses from different geostatistical models are compared with the optimized schedule determined with the optimization methodology described.

Keywords: block model; geostatistical simulation; grade control; short term plan

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1. Introduction

Estimating geological resources is one of the key aspects in any feasibility study. These resources must be converted to reserves once a mine design and plan have been defined and all considerations regarding metallurgical behaviour and selectivity, among others, have been taken into account.

At an early stage, a long term plan is designed in order to define the sequence and expected grade and tonnage over large production periods, usually yearly. However, during production, the variability of the grade and tonnages of ore, waste and low-grade ore to be stocked has a significant impact in the project's performance. Therefore, forecasting this short term variability at an early stage of the study and with limited information is required. Understanding the variability in ore and waste grade and tonnage is extremely relevant for defining the appropriate fleet of equipment (size and quantity), and to design the mineral processing facilities. Large variability in the grade and tonnage or in the metallurgical characteristics of the ore will hinder the processing plant performance and should be forecasted prior to further investments.

Geostatistical estimation techniques are routinely applied in the mining industry to predict the resources and reserves. However, estimated block models are smooth and should not be used to assess the performance of different processes that involve several blocks at a time. Typical examples of these processes are the selection of blocks above a cutoff during a time period such as a shift or a day. The total tonnage and grade over that time period should be assessed with geostatistical simulation techniques, that allow the construction of numerical models of the deposit that honour the spatial variability of the true block grades. Furthermore, several such models can be built, allowing for the quantification of the uncertainty on any performance measure (Journel and Kyriakidis, 2004).

In this article, we discuss a methodology to assess the performance of different estimation and simulation techniques to predict the expected grade and tonnage variability of the ore fed to the processing plant over a time period. In order to provide a comparable and repeatable extraction sequence, an optimization approach has been devised to define the extraction order and classification of blocks as ore or waste, to facilitate the evaluation of the grade and tonnage variability at the primary crusher.

Due to the size of the block model, a stepwise optimization over allowable paths of the shovels at several working faces is proposed and developed, in order to avoid the huge dimensionality of a full optimization.

The proposed approach is based on a local optimization for the best shovel paths considering a given number of blocks. With these local optimum paths, a decision is made at each face to extract a given block. From these new starting points, a new local optimization is performed to decide upon the next blocks to be extracted. This allows defining a path to extract the blocks for each shovel, without solving the full optimization problem, but providing a result consisting of local short-term optimums. The optimization is done considering several criteria: constant average grade and tonnage to

the processing plant, maximum profit, geometrical constraints for the shovel displacement, etc.

The paper is presented as follows. Section 2 describes the short term planning process. Section 3 presents the techniques currently used to calculate the resources and reserves for mine planning. Section 4 describes the optimization algorithm and its implementation. Section 5 shows a case study. Finally, some comments, recommendations and conclusions are provided in Section 6.

2. Short term planning

In most open pits on hard rock, short term planning is made based on the result of sampling blast holes or performing advanced drilling for sampling purposes. This information, usually in a dense pseudo-regular grid, is used for grade control (Kim, 1987). The typical procedure is:

- Map the blast hole or advanced drilling grades in plan view on a bench by bench basis.
- Assign a block grade to a selective mining unit, usually through an estimation technique such as polygonal or ordinary kriging. The selective mining unit size is related to the mining equipment that will extract the ore and waste.
- Define a practical dig limit based on these block grades, considering the operational constraints due to the size of the equipment.

Based on this procedure, a variable ore grade and tonnage is delivered to the primary crusher and fed to the processing plant. These variations may affect significantly the plant's performance, thus at early evaluation stages of the mining project a reliable prediction of this variability is required. However, prior to production, only long term drilling is available for forecasting. The challenge is to predict the variability accounting for the *information effect*, that is, considering that at the time of the decision, the information available will be much denser.

3. Resources and reserves calculation for mine planning

Mine planning addresses the estimation of the mineral inventory for different time spans. Depending on the stage, different information will be available and different goals sought after. Long-term mine planning requires a resources and reserves model for mine design and planning (Osanloo et al., 2007), usually calculated from drillhole data at a relatively wide spacing, with some denser (in-fill) drilling in the areas of forthcoming production.

Medium-term planning uses the production data (usually from blast hole samples) to adjust the long term plan and fit a budget and target production for a quarter or a year. Short term planning deals mainly with the day-to-day scheduling to satisfy the requirements set in the long and medium-term plans.

Short-term performance is well understood once abundant information is available during production, but is difficult to forecast during the early stages of the project, when the advance exploration drilling grid is the only information available. At this stage, geostatistical simulation is used to create plausible representations of the grade distribution in the deposit, honoring the global histogram, the available data, and the spatial continuity of the grades, but without local accuracy. This means that the stochastic realizations should not be used to find the best estimate at every location, but rather, they can be used to evaluate the performance when faced to a particular process. In our application, the process is the definition of an extraction path and the classification of the blocks as ore or waste. This procedure is evaluated at the processing plant, by considering the tonnage of ore fed and its grade.

Defining the extraction paths of a number of shovels at different faces of the mine should account for several considerations:

- The extracted blocks should feed the plant with a regular tonnage and grade, over each shift. This will ensure the reasonable performance of the processing plant by avoiding large variations in head grade and tonnage.
- Additionally, the metal quantity fed to the plant should be even in time, to ensure the budget is satisfied.
- The total tonnage (ore + waste) should not change significantly. Abrupt changes would require availability of additional mining equipment.
- The shovel must extract all the ore and waste from an area before moving to an adjacent area. This means that the extraction path must consider the operational constraints of the mining process.

Any other consideration deemed necessary could be used when defining the extraction paths.

In the next section, we describe an approach to define the extraction path of several shovels, considering multiple constraints.

4. Optimizing the extraction paths subject to constraints

Consider the case where s shovels are working on independent faces, and where each face consists of a model of N blocks. Each block is characterized by its grade and tonnage. If each shovel could extract n blocks freely following a path that can move to any of the four adjacent blocks to the current location of the shovel, then the possible number of combinations would be $(4)^{n \cdot s}$. This number becomes quite large rapidly. For example, considering 3 shovels ($s = 3$) and trying to find the jointly optimum path of 10 blocks ($n = 10$) for each of them would require the evaluation of over 10^{18} combinations.

Because this number is so large, we propose the following approach for defining the optimum path of several shovels operating at independent faces:

- Define a limited number of blocks for the path to optimize $n' \ll n$
- Compute all the possible paths within that limited combinatorial scenario: $(4)^{n' \cdot s}$

- Evaluate an objective function that accounts for departures from the target values for tonnage, grade, quantity of metal, revenue, etc., and also considers geometrical constraints about the order in which the blocks are visited along the path.
- Decide upon the optimum paths for each shovel in order to minimize the variability of the ore fed to the processing plant
- Move each shovel to the first block in its optimum path
- Optimize the following n' blocks from the new starting position and repeat the entire procedure

5. Application to evaluate different resources models for short term mine planning

The construction of numerical models of the deposit for planning purposes is subject to many modeling decisions that may significantly affect the forecasting capacity of these models. As mentioned in the introduction, depending on the goals of the study, interpolated models should not be used, but it is still a fairly common practice in industry to see kriged models used for mine planning. In this section, we present a case study where using real data from a porphyry copper mine in Chile, we compare the performance of three models constructed with different techniques:

1. Ordinary kriging
2. Sequential Gaussian simulation of point values representing blast hole samples used for grade control and short term mine planning
3. Sequential Gaussian simulation incorporating multiple-point statistics by updating conditional probabilities (Ortiz and Emery, 2005).

These models are compared considering their capacity to predict the grade and tonnage variability at the primary crusher of the processing plant. For comparison purposes, production (blast holes) data are available. These data represent the truth and are used along with the optimizer of the extraction path to simulate the variability of the material sent to the plant.

The extraction path optimizer was implemented considering the following parameters:

- Number of faces: $s = 2$
- Number of blocks in optimized path: $n' = 6$
- Number of blocks in each model: $n = 625$

The optimization of the paths was done considering the following restrictions:

- Revenue of the path should be maximized. This allows optimizing the path in order to obtain the best possible product.
- Difference with respect to the target head grade at the processing plant should be minimized. This constraint accounts for systematic bias in the grade of the ore fed to the plant.
- Variability of the head grade to plant should be minimized. This constraint accounts for the variability of the material fed to the plant. High variability will have a negative impact in plant performance.

All of these constraints may be modified to account for variable rock types, recoveries, detrimental elements, and could also be implemented in a multivariate framework.

In addition to these constraints, a condition was defined to impose the extraction of all blocks within a region before moving to an adjacent area, by defining a maximum allowable distance of a block in the path to a reference block extracted a number of steps back in the path. This ensures the shovels extract all blocks in an area before continuing. As an example of the effect of this constraint, Figure 1 shows the optimized path for the unconstrained case and for the operative case.

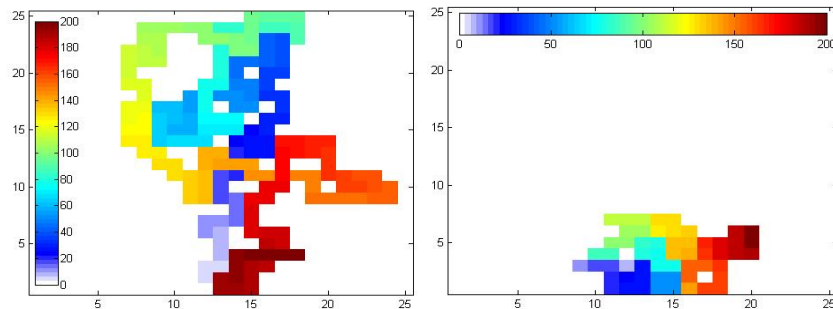


Figure 1: Optimized path without the geometrical constraint (left) and with the constraint (right). The colour scale indicates the step in the path. It is clear that this restriction imposes a more realistic operative extraction of the material.

Let us define the following notation: the grade of each block is Z_j , its associated profit is P_j , which depends on the mining and metallurgical recoveries, metal price, density, block grade, mine, plant and smelter costs.

For a 6 blocks path (i), the total profit is noted as $P_{total}^{(i)} = \sum_{j=1}^6 P_j^{(i)}$ (the superscript notes the

particular path), the average grade is $Z_{avg}^{(i)} = \frac{1}{6} \sum_{j=1}^6 Z_j^{(i)}$, and the variance of block grades

within the path is $Var_Z^{(i)} = \frac{1}{6-1} \sum_{j=1}^6 (Z_j^{(i)} - Z_{avg}^{(i)})^2$.

The algorithm minimizes an objective function that combines all three constraints:

$$\min\{OF\} = \min_{(i)} \left\{ \lambda_1 \cdot \left(\frac{P_{max} - P_{total}^{(i)}}{P_{max}} \right)^2 + \lambda_2 \cdot \sum_{j=1}^6 \left(\frac{Z_{target} - Z_j^{(i)}}{Z_{target}} \right)^2 + \lambda_3 \cdot Var_Z^{(i)} \right\}$$

where P_{\max} is the maximum profit found in any of the paths and Z_{target} is the target grade expected by the processing plant.

Three free parameters $\lambda_1, \lambda_2, \lambda_3$ allow modifying the relative importance of these three competing objectives. Additionally, only paths that satisfy a geometric constraint are allowed by imposing the extraction of a number of blocks before letting the shovel move to location that are farther from the starting point.

The objective function is minimized simultaneously in two faces providing the optimum solution for the combined case. It must be emphasized that once the paths of length 6 blocks have been optimized, shovels are moved only one block in the model, and the optimum paths are calculated again from the new starting points, in order to include a new block in the evaluation horizon.

The algorithm was implemented to assess the forecasting capacity of three geological models.

1. Ordinary kriging using long term data from the drillhole advanced exploration campaign (Isaaks and Srivastava, 1989). At every location, the block grade is estimated as:

$$Z^{OK}(\mathbf{u}) = \sum_{i=1}^n \lambda_i \cdot Z(\mathbf{u}_i)$$

where the $Z(\mathbf{u}_i)$ are the grades at sample locations and the optimum weights λ_i are calculated by solving the following system of $(n+1)$ linear equations:

$$\sum_{i=1}^n C(\mathbf{u}_j, \mathbf{u}_i) \cdot \lambda_i - \mu = \bar{C}(\mathbf{u}_j, \mathbf{u}) \quad \forall j = 1, \dots, n$$

$$\sum_{i=1}^n \lambda_i = 1$$

In these equations $C(\mathbf{u}_j, \mathbf{u}_i)$ stands for the spatial covariance function evaluated for the vector that separates locations \mathbf{u}_j and \mathbf{u}_i , $\bar{C}(\mathbf{u}_j, \mathbf{u})$ is the average covariance between location \mathbf{u}_j and the volume of the block located at \mathbf{u} , and μ is a Lagrange parameter to impose the unbiasedness constraint on the weights.

2. Sequential Gaussian simulation models built considering long term data from the drillhole advanced exploration campaign (Deutsch and Journel, 1997). Several realizations are built that reproduce the spatial continuity of the variable, its histogram and return the data values at sample locations. The simulation algorithm works by considering the normal scores of the data, that is, the original distribution must be transformed to a standard Gaussian distribution. The domain is discretized by a grid of nodes that are visited randomly. At every node, a simulated value is drawn from the conditional distribution, which is computed as a Gaussian distribution with mean and variance defined by the simple kriging estimate and variance of the available information in a search neighbourhood. This information can be sample data (transformed to normal scores) or previously

- simulated values. Once all nodes in the grid have been visited, the values are back transformed to original grade units. The grid values represent blast hole samples that are then used for grade control. Several such models are built by changing the visiting order of the nodes and the drawn values in the conditional distributions.
3. Sequential Gaussian simulation models accounting for multiple-point statistics are built using data from the drillhole advanced exploration campaign for conditioning and blast hole data from previously extracted benches to infer pattern (multiple-point) statistics (Ortiz and Deutsch, 2004; Ortiz and Emery, 2005). The algorithm proceeds similarly to the previous one, but at every node, the conditional distribution is updated using the conditional probability extracted from the blast hole data base, and depending on the information available at the four adjacent nodes in the simulation grid, as shown in Figure 2. From the blast hole information, the following conditional probability can be obtained for $Z(\mathbf{u})$ to be less than or equal to a cutoff: $P(\mathbf{A} | \mathbf{C}) = [\text{Prob}\{Z(\mathbf{u}) \leq z_k | (m)\}]_{MP}^*$. From the Gaussian simulation, the conditional probability can be calculated as: $P(\mathbf{A} | \mathbf{B}) = [\text{Prob}\{Z(\mathbf{u}) \leq z_k | (n)\}]_{SK}^* = [\text{Prob}\{Y(\mathbf{u}) \leq y_k | (n)\}]_{SK}^* \sim N(m_Y(\mathbf{u}), \sigma_Y^2(\mathbf{u}))$. These two probabilities are combined using a conditional independence assumption:

$$P(\mathbf{A} | \mathbf{B}, \mathbf{C}) = \frac{\frac{1 - P(\mathbf{A})}{P(\mathbf{A})}}{\frac{1 - P(\mathbf{A})}{P(\mathbf{A})} + \frac{1 - P(\mathbf{A} | \mathbf{B})}{P(\mathbf{A} | \mathbf{B})} \cdot \frac{1 - P(\mathbf{A} | \mathbf{C})}{P(\mathbf{A} | \mathbf{C})}}$$

Figure 3 shows plan views of the same bench. The first model (Figure 3a) represents the truth, since it is the block model that was finally mined out and it was constructed by performing ordinary kriging of block grades with the blast hole sample data of the bench. Figure 3b shows the kriged models using only the drillhole sample data, that is, only the long term information. Figure 3c shows one particular realization built with sequential Gaussian simulation. Figure 3d displays one realization that accounts for multiple point statistics.

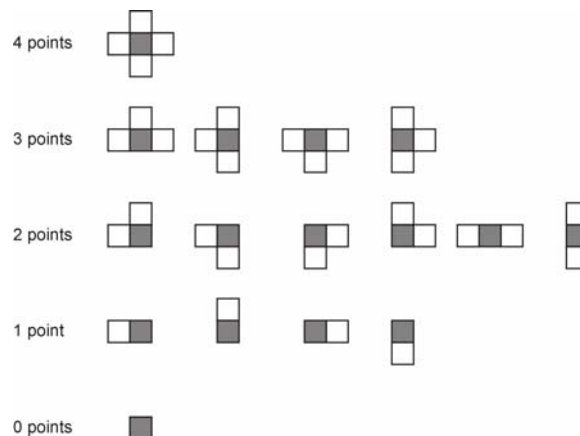


Figure 2: Patterns used for updating the conditional probabilities at every grid node

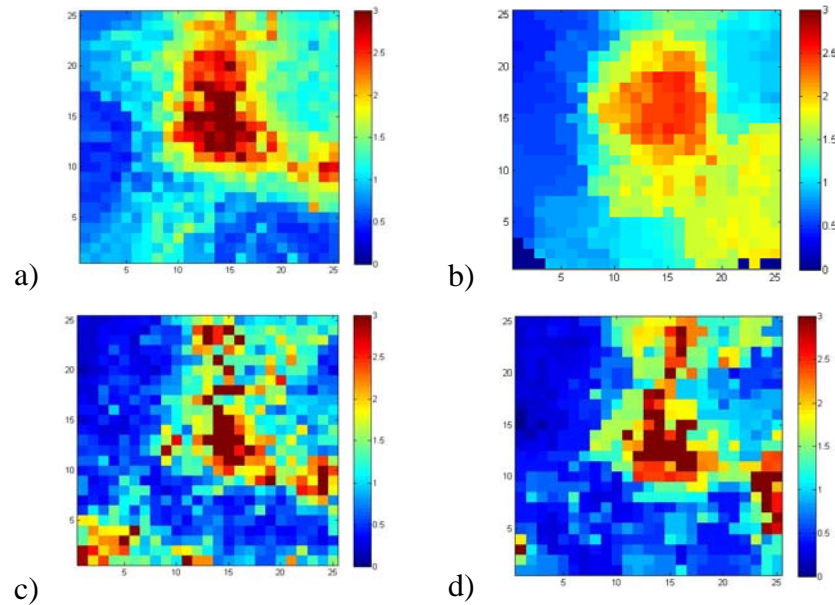


Figure 3: Models used for the analysis. a) True block grades; b) Ordinary kriging; c) Sequential Gaussian simulation; d) Sequential Gaussian simulation accounting for multiple-point statistics.

Figure 4 shows some of the output charts obtained by processing each model. Figure 4a presents the grades of blocks extracted at each one of the two faces, their average and the target grade required by the processing plant. Figure 4b shows the tonnage of ore sent to the plant.

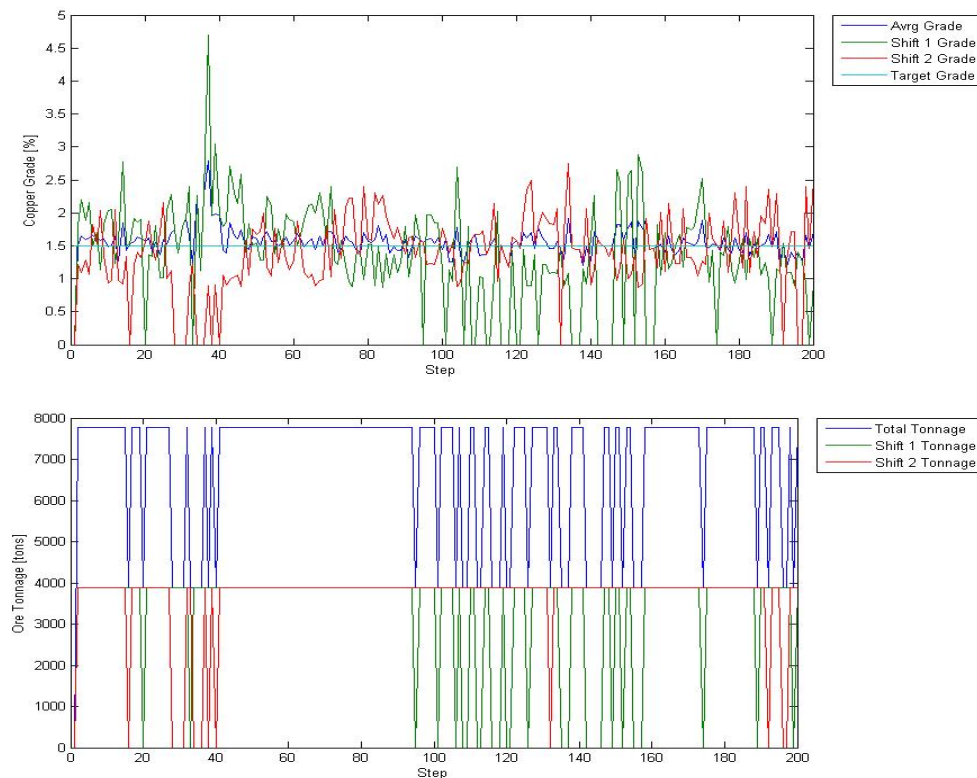


Figure 4: Output charts after the definition of the optimum paths.

Model performance was compared by statistical means for different time steps. Comparisons on a block by block basis, shift basis and weekly basis were performed. Table 1 shows the results for a block by block comparison. It can be seen that the block model estimated with ordinary kriging performed poorly, while the simulated models improved significantly the forecasting of the grades and tonnages sent to the plant. Similar results (not shown) were obtained for shift and weekly averages, although, the differences in performance diminish due to the effect of averaging over those time horizons.

	OK	SGS	SGS-MP
<i>Grade to the plant</i>			
Mean Error	0.25	-0.05	-0.03
Error Variance	0.47	0.11	0.1
MAE	0.42	0.2	0.18
MSE	0.53	0.11	0.1
<i>Tonnage to the plant</i>			
Mean Error	719	-541	-551
Error Variance	5,481,077	2,488,128	2,646,070
MAE	1,419	1,047	1,057
MSE	5,971,035	2,768,595	2,936,578

Table 1: Statistical performance comparison for the block by block output.

6. Conclusions

In this article we have shown a practical tool to define the optimum extraction path when multiple constraints must be met and shovels operating at different faces must be simultaneously taken into account when programming the short term operation. The algorithm provides a fairly simple framework for incorporating additional constraints and could be calibrated to the practice of each mining operation. It could be easily extended to account for multiple variables.

The proposed optimization algorithm was used to provide a repeatable and comparable definition of the short term planning procedures to assess the forecasting capacity of three reserves block models. Results showed that using an estimated model will underestimate the true variability of grades fed to the processing plant, impeding the proper actions to handle these variations in grade, tonnage and ore types. By using simulated models, the variability in these parameters can be anticipated and proper actions taken to avoid significant difficulties at the plant, with the consequent reductions in recovery, increases in costs, and operational complications. The model that incorporates multiple-point statistics performs marginally better in some cases than the standard simulation approach, which can be due to the small pattern size used. A larger pattern might improve its performance.

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Authors' Biographies

Dr. Ortiz is a Mining Engineer from University of Chile and holds a Ph.D. degree in Mining Engineering (Geostatistics) from University of Alberta. He currently teaches and conducts research into ore reserve estimation and geostatistics at the Department of Mining Engineering at University of Chile. He also collaborates as an Assistant Adjunct Professor at the Civil and Environmental Engineering Department – University of Alberta. He has several publications in journals and international conferences and has worked as a consultant for the mining industry. His main areas of interest are geostatistics, ore reserve estimation and sampling.

Dr. Rubio is a Mining Engineer from University of Chile, MASc in Mining Engineering and PhD in Mining Engineering from The University of British Columbia. His research area is related to the development of reliability models applied to underground production planning systems. Mr. Rubio has written more than 12 articles for international conferences and mining journals. His main areas of interest are mining technology, mine planning, numerical modeling and information systems in mining. During his career he has worked as a mining consultant for companies in Chile, Canada, UK, Indonesia, South Africa, and USA.

Mr. Yarmuch is a Mining Engineer from University of Chile. He currently works at Codelco Chile, Division Codelco Norte in the Trainee Program. His main areas of interest are rock mechanics, mine planning and geostatistics. During his studies he has worked as teacher assistant in rock mechanics at University of Chile.

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