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Package Size Decisions

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We describe a model examining how a firm might choose the package size and price for a product that deteriorates over time. Our model considers four factors: (1) the usable life of the product, (2) the rates at which consumers use the product, (3) the relation between package size and the variable cost of the product, and (4) the minimum quantities consumers seek to consume for each dollar they spend (we call these reservation quantities). We allow heterogeneity in the usage rates and reservation quantities for the consumers. We show that when the cost increases as a linear or convex function of the package size, the firm should make packages of the smallest possible size. Smaller packages reduce waste and allow consumers to more closely match their purchases with desired consumption. This in turn allows the firm to charge a higher unit price and also sell more unit volume. The results imply that in a market with multiple package sizes (produced by the same or competing firms), at least one of the packages must have the smallest possible size, provided the fixed cost of making the product is sufficiently low. For concave cost functions, the firm may find it optimal to make larger than smallest-size packages.

Key words: package size; pricing; product design; product policy
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1. Introduction

There is substantial marketing literature on product policy, but there is little research concerning the choice of package sizes for goods such as foods and drugs, which deteriorate over time. Changes in chemical composition, microbial growth, and varying storage and handling conditions all affect the taste, quality, and/or efficacy of such products. The consequent limits on their usable lives constrain the sizes of the package that can be sold to consumers.

We propose a framework for analyzing how a profit-maximizing firm might choose the package size and price for a product that deteriorates over time. Besides usable life, we consider three other factors that are important when selecting a package size: (1) the costs associated with making packages of different sizes, (2) consumer usage rates, and (3) the utilities obtained by users from consuming—rather than just owning—a product. For example, suppose a product has a short usable life, and consumers have low usage rates. Both of these factors would suggest that the firm should make a small package size. However, if smaller packages have a higher marginal cost per unit than larger packages, there could be an incentive for the firm to sell large packages. But the effects of these three factors—usable life, usage rates, and marginal cost—still are not sufficient for determining the optimal package size. How much users value consumption also matters. If, for example, the marginal value of consumption is high for some consumers, those consumers might be willing to buy a large package and pay a higher price even if they have to waste a substantial amount of the product.[1] These considerations are further complicated by the fact that the usage rates and consumption utilities can vary substantially among users. So the firm must additionally decide which consumers to serve and which to forgo.

A notable feature of the proposed model is that it allows consumers to purchase any (integer) number of packages. Demand is obtained by adding the number of packages purchased by all consumers. How many packages a consumer buys, if any, depends on his reservation quantity and consumption rate, and on the package size and its price.[2] We define the reservation quantity as the ratio of the minimum quantity a

[1] Assuming that they value consumption and do not explicitly incur disutility from wasting the product.

[2] Purchase affects consumption in our model only to the extent that a consumer might choose to consume a quantity of the product that he might have otherwise wasted. However, there is no marginal value for this consumption in our model. For a discussion of the effects of stockpiling on consumption, see, for example, Ailawadi and Neslin (1998), Chandon and Wansink (2002), and Wansink (1996).

person must be able to consume from a package and the price of a package. For simplicity, we assume that (1) the product has a fixed usable life that is common for all consumers, (2) the usage rates of consumers can be represented by a uniform distribution, and (3) the reservation quantities for consumers can be represented by an independent uniform distribution.

Our model shows that when the cost to produce a product's package rises at a linear or increasing rate the firm should package the product in the smallest package possible. Small packages allow consumers to most closely match their actual purchases to the number of units of the product that they can consume. As a result, the firm can charge a higher price and sell a larger quantity of the product. The lower the unit cost of the product, the more salient is the effect of small package size on unit price. However, sufficiently large economies of size can offset the advantage of smaller packages for the firm. We discuss the implications for markets that offer multiple package sizes, possibly by different firms, and show that when the cost function is linear or convex, the market should have at least one package with the smallest size possible.

To the best of our knowledge, there are only two papers in the marketing literature that explicitly consider package sizing and pricing decisions. The first is by Desai et al. (2008), who describe a model for package sizes in emerging markets. Their model considers two firms, one making a high-quality product and the other a low-quality product. There are two consumer segments, one of which has an income constraint limiting its ability to buy the high-quality product. The authors show that the firm making the high-quality product can sometimes reduce both price and package size to serve the income-constrained consumers and that the firm making the low-quality product should then respond by raising its price. The second model, by Gerstner and Hess (1987), emphasizes the effects of transaction costs and shows that a firm can use differences in package size for price discrimination, potentially charging a higher unit price for a larger package if there are customer segments with different inventory costs.[3]

1.1. Organization of the Paper

Section 2 introduces the model, derives the demand function, and characterizes the optimal package size and price when the variable cost of a package linearly increases with its size. Section 3 considers certain aspects of demand, profit, and quantities wasted by consumers; examines the sensitivity of the optimal solution to the model's parameters; and discusses the implications for product lines and competition. Section 4 extends the analysis to consider nonlinear cost functions, identifying a sufficient condition under which the firm may not want to produce the smallest possible package size. Section 5 discusses some implications of the results and concludes the paper.

2. Model

We start with an overview of the model. Consider a firm making a product that deteriorates over time. The product has a fixed usable life, T, after which it becomes unsuitable for consumption. The usable life can depend on the inherent rate of product deterioration, storage and handling conditions, and the time a package remains unsold on a store shelf. The size of a package, s, is measured in units such as grams or milliliters. The cost of making a package, $c(s)$, is an increasing function of its size. The value of $c(s)$ includes the cost of producing the product, its packaging, and holding, breakage, damage, and insurance costs. The firm's objective is to choose the package size, s, and the package price, p, so as to maximize its profit.

The demand for the product depends on the size of the potential market, the fraction of the market that buys the product, and the number of packages purchased by each buyer. At most one package—call it the "last" package—is not fully consumed by a buyer. The decision to buy this last package depends on two quantities, which we call the unit valuation and the reservation quantity. The unit valuation is the ratio of the quantity consumed from the last package and the package price. The reservation quantity is the minimum value this ratio must reach for a consumer to buy the last package. We assume that reservation quantities and usage rates are independently and uniformly distributed across consumers. The total demand for packages of a given size is obtained by adding the number of units purchased by buyers with different usage rates. We use the demand function and a cost function to determine the optimal package size and price for a product.

2.1. Consumer Model

Let p denote the price of a package of size s. A consumer who buys $i \geq 1$ packages completely uses $i-1$ packages and a quantity f from the ith package, where $0 < f \leq s$. We assume that the consumer obtains value from using, not merely owning, the product and that the utility from the ith package is

$$u = \gamma f - p, \quad 0 < f \leq s,$$

where γ denotes the consumer's valuation of a unit of consumption and is measured in dollars per unit.

[3] An empirical study of paper towels by Cohen (2008) found that 34%–46% of the unit price variation among package sizes was consistent with price discrimination; the rest was attributable to differences in costs.

The total utility obtained by a consumer buying $i \geq 1$ packages is

$$U = \gamma\{(i-1)s + f\} - ip, \quad i \geq 1.$$

We assume that the consumer buys the ith package only if $u > 0$. As $s \geq f$, the condition $u > 0$ is sufficient for the consumer to buy each of the $i-1$ fully used packages.

Let

$$r = f/p \quad \text{and} \quad R = 1/\gamma.$$

Then the condition $u = \gamma f - p > 0$ can be rewritten as $r > R$. We interpret this condition as follows. A consumer using a quantity f from a package obtains, for each dollar he spends, the use of r units of the product. This quantity must exceed a minimum threshold, R, for the consumer to buy the ith unit of the product. We call r the valuation (or unit valuation) and R the reservation quantity for a consumer. A lower value of R implies that a consumer requires less use per dollar to buy the ith unit of the product or, equivalently, is willing to pay more per unit of consumption.

The value of r depends on a consumer's usage rate, θ, and the package price, p. The value of R depends on how much a consumer values consumption of the product. We allow heterogeneity in both usage rates and reservation quantities across consumers. Let $\theta \in [A_1, A_2]$ and $R \in [B_1, B_2]$. Then $A_1 > 0$ because a consumer, by definition, must want to use the product, and $A_2 < \infty$ because there is a finite limit to how many units a buyer can consume before the product becomes unusable. Similarly, $B_1 > 0$ because no consumer is willing to pay an infinitely high price for consuming a unit of the product, and $B_2 < \infty$ because every consumer must be willing to pay some positive amount for consuming a unit of the product. For simplicity, we assume that the heterogeneities in usage rate, θ, and reservation quantity, R, are characterized by uniform density functions, $f(\theta) = 1/(A_2 - A_1)$ and $g(R) = 1/(B_2 - B_1)$, respectively.

2.2. Demand Function

We first obtain an expression for the demand for packages by imposing the restriction that no consumer can buy more than n packages. We then allow n to be any arbitrary, positive integer. The primary reason for using this two-step approach is that it simplifies the derivation of various results. A secondary reason is that the effects of certain factors not considered in this paper—primarily the ordering cost for buyers and the order-processing cost for sellers—can, to some extent, be reflected by imposing an upper limit on the value of n. As we will see (and as one might expect), the optimal package size decreases as n increases. However, ordering and order-processing costs generally increase with n. Beyond a point, the firm may not want to make and consumers may not want to buy packages of still smaller sizes.

Recall that only those consumers with sufficiently low reservation quantities and sufficiently high usage rates buy a package of a given size at a given price. Lemma 1 characterizes the necessary condition for a consumer to buy i packages of the product.[4]

Lemma 1. *A consumer with reservation quantity R will buy i packages only if $s \geq pR$ and $\theta \geq \{pR + (i-1)s\}/T$, where $1 \leq i \leq n$.*

Let D_{iR} denote the number of consumers with reservation quantity R who buy at least i packages. Following Lemma 1,

$$D_{iR} = \int_{[pR+s(i-1)]/T}^{A_2} f(\theta)\, d\theta = 1 - F\left(\frac{pR + s(i-1)}{T}\right)$$
$$= 1 - \frac{pR + s(i-1) - A_1 T}{(A_2 - A_1)T} \quad \text{for all } 1 \leq i \leq n.$$

The value of D_{iR} increases with the product's usable life, T, decreases with package size, s, package price, p, and the consumer's reservation quantity, R.

Let d_i denote the demand for exactly i packages and D_i the demand for at least i packages of the product for all $1 \leq i \leq n$. Define $D_{n+1} = 0$. Then

$$d_i = D_i - D_{i+1} \quad \text{for all } 1 \leq i \leq n,$$

and

$$D = 1 \cdot d_1 + \cdots + n \cdot d_n.$$

Substituting for d_i in the preceding expression and simplifying gives Lemma 2.

Lemma 2.

$$D = D_1 + \cdots + D_n.$$

Suppose $s/p < B_2$. Consider a consumer whose reservation quantity, R, is greater than s/p. As $f \leq s$, we have $r = f/p \leq s/p < R$ for such a consumer. But $r > R$ is a necessary condition for a consumer to buy the ith package. It follows that no consumer for whom $s/p < R \leq B_2$ will buy the product. As $\theta \geq A_1$, Lemma 1 implies that only those consumers with reservation quantities $R \geq [A_1 T - s(i-1)]/p$ will buy the product. The demand from consumers who buy at least i packages is

$$D_i = \int_{\max\{B_1, [A_1 T - s(i-1)]/p\}}^{\min\{B_2, s/p\}} D_{iR}\; g(R)\, dR \qquad \text{for all } i = 1, \ldots, n.$$

The value of D_i is the largest when $s = pB_2$ because (1) a package of size $s > pB_2$ does not attract any

[4] Proofs of Lemma 1, Theorems 1 and 2, and Claim 1 are given in the appendix.

more buyers than does a package of size $s = pB_2$ and (2) a package of size $s \leq pB_2$ is only purchased by consumers with reservation quantities $R \in [B_1, s/p]$. We therefore only need to consider the case $s/p \leq B_2$. We also restrict the analysis to the case where $\max\{B_1, [A_1T - s(i-1)]/p\} = B_1$ for all $1 \leq i \leq n$ or, equivalently, $A_1 \leq B_1 p/T$. This condition requires "sufficient" heterogeneity in usage rates and implies that even consumers who have the lowest reservation quantity, B_1, do not purchase the product if they have low consumption rates. The demand function can then be written as

$$D_i = \int_{B_1}^{s/p} \frac{D_{iR}}{B_2 - B_1}\, dR.$$

Substituting for D_{iR} in the preceding expression, summing across D_i, and simplifying gives

$$\begin{aligned} D &= D_1 + \cdots + D_n \\ &= \left(\frac{s/p - B_1}{B_2 - B_1}\right)\left(\frac{A_2}{A_2 - A_1} - \frac{pB_1 + sn}{2(A_2 - A_1)T}\right)n. \end{aligned}$$

The demand for packages increases with the quantity per dollar sold in a package, s/p, the usable life of the product, T, and the maximum usage rate, A_2. Both demand and the quantity purchased decrease as the minimum reservation quantity, B_1, increases. As the ranges of the usage rate, $A_2 - A_1$, and the reservation price, $B_2 - B_1$, increase, fewer consumers are satisfied by a single package size, and demand for the product decreases.

2.3. Optimal Decisions

We consider a firm that chooses the package size and price to maximize its profit:

$$\max_{s,\,p}\ \pi = (p - c(s))D.$$

We assume first that the cost is a linear function of package size: $c(s) = \alpha s$, $\alpha > 0$. We relax this assumption in §4. Following the development in §2.2, we consider a situation in which no consumer can buy more than n packages of the product, where $n \geq 1$ is an integer. We then allow n to take any arbitrary value.

Theorem 1. *Let $c(s) = \alpha s$, $\alpha > 0$ be the cost function for a firm. Let $n \geq 1$ denote the maximum number of packages that can be purchased by any consumer. The firm should make the product only if $\alpha < 1/B_1$, and then choose a package of size $s^* = (1-k)A_2T/n$ and a package price of $p^* = kA_2T/B_1$, where $k = 1/\sqrt{(n+1)(n/(B_1\alpha)+1)}$.*

One implication of Theorem 1, discussed in §3.1, is that it is optimal for a firm to increase unit markup for smaller packages. This happens because smaller packages allow consumers to more closely match the quantity they purchase with the amount they wish to consume over the usable life of the product. Note that the value of k increases with $B_1\alpha$, where $0 < B_1\alpha < 1$. A lower unit cost increases the optimal package size, as does a lower reservation quantity for those buyers who obtain the most value from consuming the product. The minimum package size, $A_2T/(n+1)$, is obtained as $B_1\alpha$ approaches zero, and the maximum package size, A_2T/n, is obtained as $B_1\alpha$ approaches unity. The optimal package size and price increase with the usable life, T, and the highest usage rate, A_2.

Consider a buyer with the highest usage rate, A_2. If the package size is A_2T/n, then such a buyer consumes exactly n packages in time T. Similarly, if the package size is no larger than $A_2T/(n+1)$, the buyer surely will want to purchase more than n packages. When the package size is optimal, a consumer with the highest usage rate prefers not to buy more than n packages or to waste any part of the n packages purchased.

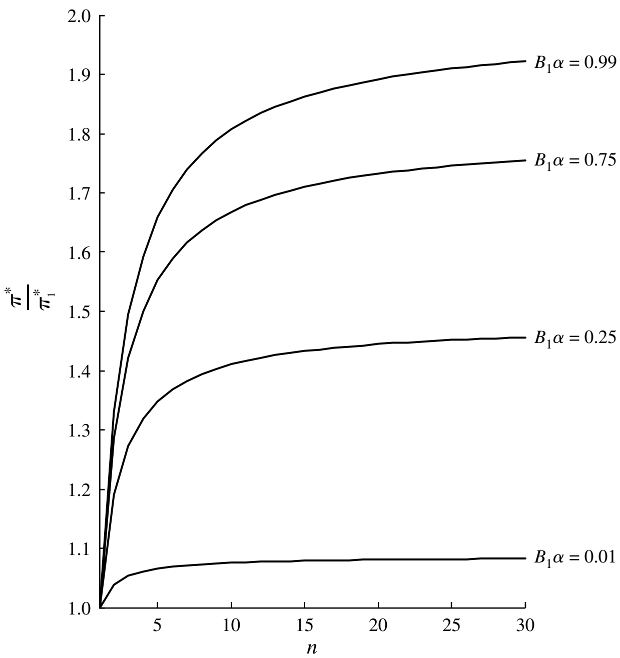
The lower bound on the package size, $A_2T/(n+1)$, appears to be a consequence of two assumptions in our model: (1) the uniform distribution of usage rates across consumers and (2) the linear cost function, which we discuss in §4. Consider the assumption of uniformly distributed usage rates. Buyers with the highest usage rate can buy and use the largest number of units of the product. Reducing package size can draw in some consumers with lower usage rates (and/or higher reservation quantities) but does so at the expense of sales to buyers with higher usage rates (who may then want to buy more but are artificially constrained to purchasing at most n packages). If usage rates are uniformly distributed, there is no disproportionate gain in the number of consumers when the firm makes a package that is smaller than $A_2T/(n+1)$. However, if there were disproportionately more consumers with low usage rates, the loss from restricting sales to the buyers with highest usage rate could potentially be offset by additional sales to consumers with lower usage rates. The firm would then have an incentive to make a still smaller package for a given value of n, accentuating the main result that the optimal decision for a firm is to make the smallest possible package size.

If we substitute the expressions for s^*, p^*, and D^* into the profit equation, we obtain

$$\pi^* = \frac{A_2^2 T}{2(A_2 - A_1)(B_2 - B_1)}\left(1 + \left(1 + \frac{2}{n}\right)B_1\alpha - \frac{2B_1\alpha}{nk}\right).$$

As expected, the firm's profit decreases as the unit cost, α, increases. A market with greater heterogeneity in usage rates and reservation quantities (i.e., larger values of $A_2 - A_1$ and $B_2 - B_1$) forces a firm to forgo sales to some consumers to attract others and this reduces its profitability.

Figure 1 $r = \pi^*/\pi_1^*$ **as a Function of** n **and** $B_1\alpha$



So far, we have only considered optimal decisions given an arbitrary upper limit, n, on the maximum number of units sold to any one consumer. To find the optimal package size across the possible values of n, we observe that

$$\frac{d\pi^*}{dn} = \frac{A_2^2 T B_1 \alpha k(\sqrt{B_1\alpha(n+1)} - \sqrt{n + B_1\alpha})^2}{2(A_2 - A_1)(B_2 - B_1)n^2 B_1\alpha} > 0,$$

which implies that the optimal profit increases with n. That is, the optimal decision for the firm is to choose a value of n that is as large as possible and then select a package size and a price consistent with the result in Theorem 1. As n increases, the optimal package size and price decrease,[5] and the optimal profit approaches the limiting value:

$$\pi^*_{\max} = \lim_{n\to\infty} \pi_n^* = \frac{A_2^2 T(1 - \sqrt{B_1\alpha})^2}{2(A_2 - A_1)(B_2 - B_1)}.$$

Let π_1^* denote the profit π^* when $n = 1$. Figure 1 plots the ratio π^*/π_1^* as a function of n and $B_1\alpha$.

The smaller the value of $B_1\alpha$, the smaller the value of $\pi^*_{\max}/\pi_1^*$: a firm has less reason to make a smaller package size if the unit cost α is small even if the buyers who are least sensitive to price are still quite price sensitive (i.e., have a lower value of B_1). For each value of $B_1\alpha$, the value of the profit ratio initially increases rapidly with n but then levels off quickly to its asymptotic value. For example, when $B_1\alpha = 0.01$, the profit ratio is within 1% of the maximum for $n = 10$; when $B_1\alpha = 0.99$, the profit ratio is about 2% of the maximum for $n = 30$. Thus, in practice, $n = 30$ is large enough for a firm to obtain about the maximum possible profit; in most cases, $n = 10$ is sufficiently large.

The range for the optimal price also decreases with n, because (1) $s^* > A_2T/(n+1)$ and (2) $s^*/B_2 \le p^* < A_2T/(B_1(n+1))$, and these conditions together imply that

$$\frac{A_2 T}{(n+1)B_2} < p^* < \frac{A_2 T}{(n+1)B_1}.$$

The lower bound of the price range depends on the maximum reservation quantity, B_2, and the upper bound of the price range depends on the minimum reservation quantity, B_1.

[5] Note that

$$s^* = A_2 T\left\{\sqrt{(n+1)\left(\frac{n}{B_1\alpha}+1\right)} - 1\right\} \Big/ \left\{n\sqrt{(n+1)\left(\frac{n}{B_1\alpha}+1\right)}\right\},$$

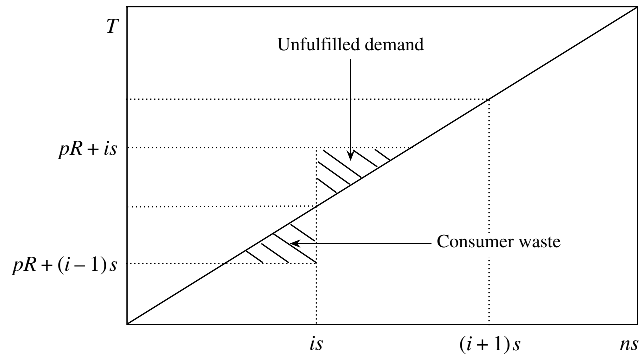
and it is easy to verify that the denominator grows faster with n than the numerator.

3. Characteristics of the Optimal Solution with a Linear Cost Function

3.1. Product Waste and Unfulfilled Demand

From a consumer perspective, the package size problem shares certain aspects of inventory problems. Some consumers, for example, may not be able to buy as much as they can possibly consume. Others may buy in excess and waste a part of a package because the product is no longer usable. This is a type of inefficiency introduced by quantity bundling. Its effect is that consumers may be less willing to pay because they do not value the wasted quantity. We refer to the total quantity that is purchased but not used by buyers as "consumer waste" and to the lost sales to consumers who buy less than they can consume as "unfulfilled demand." Figure 2 illustrates the difference between consumer waste and unfulfilled demand.

Figure 2 **Consumer Waste and Unfulfilled Demand**



Consider a consumer with consumption rate θ and reservation quantity R. If $\theta T \le is$, then the consumer will buy i packages only if $\{p^*R + (i-1)s^*\}/T \le \theta \le is/T$. Such a consumer will waste a quantity $w = is^* - \theta T$. As only those customers for whom $s^*/p^* > R$ buy i units of the product, the total quantity wasted across consumers is given by the expression

$$W = \sum_{i=1}^{n} \int_{B_1}^{s^*/p^*} \int_{(p^*R+(i-1)s^*)/T}^{(is^*)/T} \left(is^* - \theta T\right) f(\theta)\, d\theta g(R)\, dR$$
$$= \frac{(s^* - B_1 p^*)^3 n}{6(A_2 - A_1)(B_2 - B_1)p^* T}.$$

Thus, waste increases with the optimal package size. Substituting for p^* and s^* from Theorem 1 into this expression gives

$$W = \frac{A_2^2 T(\sqrt{B_1\alpha(n+1)(n+B_1\alpha)} - B_1\alpha(n+1))^3}{6(A_2-A_1)(B_2-B_1)B_1\alpha^2 n^2 (n+1)(n+B_1\alpha)}.$$

As n increases, the optimal package size becomes smaller, consumers can better match purchased quantities with consumption, and total waste decreases. It can be verified that the preceding expression for consumer waste approaches zero as n becomes arbitrarily large. A higher unit cost, α, reduces package size and so reduces waste. Recall that consumers fully consume the first $(i-1)$ packages and waste only a part of the ith package. The smaller the package size, the lesser the possible waste.

If $is^*/T \le \theta \le \{p^*R + is^*\}/T$, then the unfulfilled demand for a customer with usage rate θ is $h = \theta T - is^*$. Integrating over R and summing over the units bought by customers gives the total unfulfilled demand:

$$H = \sum_{i=1}^{n} \int_{B_1}^{s^*/p^*} \int_{(is^*)/T}^{(p^*R+is^*)/T} \left(\theta T - is^*\right) f(\theta)\, d\theta g(R)\, dR$$
$$= \frac{s^{*3} - (B_1 p^*)^3}{6(A_2 - A_1)(B_2 - B_1)p^* T}\; n.$$

Substituting the values of s^* and p^* from Theorem 1 into this expression gives

$$H = \frac{A_2^2 T[(\sqrt{B_1\alpha(n+1)(n+B_1\alpha)} - B_1\alpha)^3 - (B_1\alpha n)^3]}{6(A_2-A_1)(B_2-B_1)B_1\alpha^2 n^2 (n+1)(n+B_1\alpha)}.$$

Unfulfilled demand approaches zero as n becomes arbitrarily large. The ratio W/H has the value

$$\frac{W}{H} = \frac{(s^* - B_1 p^*)^3}{s^{*3} - (B_1 p^*)^3} < 1.$$

That is, the amount wasted by customers is always less than the total unfulfilled demand. This is one reason the firm reduces package sizes: it stands to gain more in additional sales (from nonbuyers, and from those consuming less than they can) than it stands to lose because some consumers buy more than they can use of a larger package.

3.2. Unit Price

Theorem 1 implies that the unit price for the product is

$$\frac{p^*}{s^*} = \frac{n}{B_1(1/k - 1)} = \frac{n}{B_1(\sqrt{(n+1)(n/(B_1\alpha)+1)} - 1)}.$$

As $B_1\alpha < 1$, the term under the square root in this expression is greater than $n+1$. It follows that the unit price, p^*/s^*, is an increasing function of n. As s^* decreases with n, the optimal unit price decreases with the optimal package size. However, even the smallest package size cannot have a higher unit price than $1/B_1$, the maximum any consumer is, by definition, willing to pay. Larger package sizes have a lower unit price not because there is a quantity discount (there is only one package size in our model) but because some buyers of smaller packages would incur more waste with a larger package and so refuse to buy the larger package at the same unit price. That is, the willingness to pay for a package is greater when a consumer can use a larger fraction of the package.

3.3. Number of Packages and Units

Theorem 1 implies that, for any given value of n, the total demand for packages is

$$D^* = \frac{1}{2(1 - A_1/A_2)(B_2/B_1 - 1)} \cdot \left(\sqrt{(n+1)\left(\frac{n}{B_1\alpha} + 1\right)} - (n+1)\right).$$

A larger value of n increases the demand for packages because (1) consumers need to buy more smaller-sized packages to meet their need for the product and (2) some consumers buy a larger quantity (more packages) because the smaller package size allows them to waste less. The combined effect of these two factors more than compensates for any potential loss in sales to consumers who no longer buy the product (or buy less) because the unit price, p^*/s^*, is higher for a larger value of n. The total number of units sold is

$$Q^* = s^* D^*$$
$$= \frac{A_2 T}{2(1 - A_1/A_2)(B_2/B_1 - 1)} \cdot \left(1 - \frac{1}{\sqrt{(n+1)(n/(B_1\alpha)+1)}}\right) \cdot \left(\sqrt{\left(1+\frac{1}{n}\right)\left(\frac{1}{B_1\alpha} + \frac{1}{n}\right)} - \left(1 + \frac{1}{n}\right)\right).$$

As n becomes arbitrarily large, Q^* approaches the value

$$Q^*_{\max} = \lim_{n\to\infty} Q^*$$
$$= \frac{A_2 T}{2(1 - A_1/A_2)(B_2/B_1 - 1)}\left(\frac{1}{\sqrt{B_1\alpha}} - 1\right).$$

Note that the condition $A_1 \leq \{B_1 p + s(i-1)\}/T$ implies that $A_1/A_2 \leq k$ and the condition $B_2 \geq s/p$ implies that $B_2/B_1 \geq (1-k)/(nk)$. Thus, D^*, Q^*, and $Q^*_{\max}$ cannot increase without bound.

3.4. Implications for Product Lines and Competition

So far, we have considered a firm making only a single package size. Now let us consider the implications of this analysis for markets in which there are multiple package sizes offered. Some of these packages might be produced by one firm and others by competing firms.

As long as at least one package offered is the smallest size possible, all consumers, regardless of usage rate, can potentially buy the product. Some may not purchase because the unit price is too high, but no one will forgo purchasing because the package is too large. On the other hand, if none of the packages are the minimum size, then there can be consumers who forgo purchasing because even the smallest package size offered is too large for them. Such a market allows for the possibility that a firm might profitably produce a smaller package. The following result claims that a minimum package size will indeed be profitable.

Claim 1. *In a market with free entry, a competitive equilibrium requires provision of at least one product with minimum package size.*

Observe that this claim does not require the derivation of a competitive equilibrium; it only requires showing that the market is not in (pure) equilibrium if there is no minimum package size, because it is then possible for an entrant to profitably introduce such a product.[6] The magnitude of this profit depends on the size of the smallest existing package. The larger that package is, the greater the profit a firm can make by introducing a product with the minimum package size. Whether the smallest package size is produced depends on the fixed cost of entry. The smaller this cost, the greater the profit from making the minimally sized package. Notably, the fixed cost is low for many of the everyday products for which we have seen the emergence of single-serve packages in developing countries—products like salt, ketchup, bread, cookies, and tea (Prahalad 2005). This claim is appropriate only if, as in the forgoing analysis, the marginal cost increases at a constant rate with package size (and, as we discuss later, also if it increases at an increasing rate with package size).

[6] Only a pure equilibrium appears to be meaningful in the context of product launch decisions.

4. Nonlinear Cost Functions

We now examine nonlinear cost functions. Suppose the cost of making a package increases at an increasing rate with its size; that is, $c(s)$ is a convex function, with $dc(s)/ds > 0$ and $d^2c(s)/ds^2 > 0$. Then a firm should still make a package of the smallest possible size, because its marginal cost increases with larger packages and demand does not depend on cost. However, that result need not hold for a concave cost function; i.e., if $dc(s)/ds > 0$ and $d^2c(s)/ds^2 < 0$. This is because the diminishing marginal cost creates an incentive for the firm to make a larger package. The optimal package size, then, depends on two opposing effects. On one hand, a smaller package allows consumers to self-select and buy only as much as they can use, and, because they waste less, they are willing to pay more per unit purchased. On the other hand, the firm can lower its average cost by making a larger package. If the marginal cost declines sufficiently rapidly, the firm can make a greater profit by increasing the package size even though some consumers may not be willing to pay the higher unit price (because they will waste a part of their purchases) and others may buy less or not at all. Although it is difficult to generalize, there can indeed be situations in which package cost is a concave function of the package size. For example, the cost of packaging materials increases at a decreasing rate with the volume of a package (volume is a cubic function and surface area is a quadratic function, of the linear dimensions of a package). Larger packages may also require lesser handling and involve lower transportation costs. The following theorem gives a sufficient condition under which a firm's profit decreases with n and increases with s^*.

Theorem 2. *Let $\tilde{c} \equiv \tilde{c}(s)$ be a concave cost function. Let $n = n^*$ denote the largest value of n for which a linear cost function $\hat{c} = \hat{\alpha} s$ satisfies $\hat{\alpha} < \alpha_{\max}$, where*

$$\begin{aligned}\alpha_{\max} = \frac{1}{B_1 n}\Big[&2 + 2B_1\alpha - 4\sqrt{B_1\alpha} + n(2 + B_1\alpha - 2\sqrt{B_1\alpha})\\ &-2\Big\{(n+1)[(1-4\sqrt{B_1\alpha})\\ &\qquad + B_1\alpha(6 + B_1\alpha - 4\sqrt{B_1\alpha})\\ &\qquad + n(1 + B_1\alpha - 2\sqrt{B_1\alpha})]\Big\}^{1/2}\Big].\end{aligned}$$

Then the firm's profit decreases with n for all $n \geq n^*$.

Theorem 2 says that for a cost function that is "sufficiently" concave there will be some $n = n^*$ such that the firm will make a greater profit by choosing a package size between $A_2T/(n^*+1)$ and A_2T/n^* than by making a package that is smaller than $A_2T/(n^*+1)$. The actual value of the package size in this range will depend on the cost function. The value of n^*, and the optimal package size and price will depend on the rate at which marginal cost decreases with package size. The greater this rate of change, the larger the optimal package size.

5. Implications and Conclusion

This paper proposes a framework for analyzing package size decisions for products that deteriorate over time. It allows consumers to buy multiple packages, considers heterogeneous usage rates and product valuations, and presents some initial results concerning the optimal package size and price for a profit-maximizing firm. One such result is that the size of the package should be as small as possible when cost increases as a linear or convex function with package size. This allows consumers to more closely match their purchases with their requirements. The result is less waste, more buyers, a higher unit price, and a greater profit for the firm. A market that offers multiple packages by the same or competing firms should offer at least one minimum-size package.

Our results may be relevant to the "single-serve revolution" in developing countries, which Prahalad (2005) notes has enabled the poor to buy products that they could not otherwise afford. The present results suggest that single-serve packages may be appropriate even if buyers do not have income constraints. The key requirements are that (1) the variable cost must increase at a linear or increasing rate with package size and (2) the ordering cost for the buyers and the order-processing cost for the sellers must be small (fixed costs, once incurred, are not relevant for transaction costs). The latter is possible in dense urban areas of countries like India, where retail stores are located close to buyers and keep long hours. The product's usable life, T, can be shorter for some products and consumers in developing markets (due, for example, to high temperatures and poor storage conditions). As our model suggests, a smaller value of T implies a smaller package size.[7] Consistent with Prahalad's observation, minimum-size packages can increase the number of consumers who buy a product, increase total consumption, decrease product waste, and lead to greater profits for a firm, in part because of higher unit prices. However, these higher unit prices do not have to be a poverty penalty to buyers who do not have the income needed to buy and store larger packages. Relaxing the assumption of uniform heterogeneity distributions to allow disproportionately more consumers with lower usage rates and higher reservation quantities is likely to strengthen the result that a firm should make packages of minimum size.

[7] We thank a reviewer for bringing this point to our attention.

Our results may also be relevant for pricing and sizing of products sold by warehouse stores like Costco and Sam's Club in the United States. These retailers sell packages that are so large that most noninstitutional consumers buy one package at a time and some of them still waste part of its contents. The exclusive focus on large package sizes suggests a decision by these firms to serve only consumers with high usage rates and low inventory holding costs, which corresponds to the choice of a small value of n (a supply constraint) in our model.

We conclude by noting some possible extensions of the present research. First, the assumptions that buyers have negligible ordering cost and seller have negligible order-processing cost might be less reasonable for developed markets than it is for developing markets.[8] If the assumption is relaxed, it may not be optimal for the firm to make the smallest package size possible, because buying and selling a larger number of packages can add to the costs for the consumers and the firm. Second, it could be useful to introduce dynamics into the model, allowing consumers to make repeat purchases. Heterogeneity in purchase cycles and consumer transaction costs could then be included. One likely effect is that consumers will make decisions concerning how much to buy based on their purchase cycles (which, for some products, could depend on when they make the next trip to a store), and some may prefer to avoid waste by never buying more than they can fully use.[9] Third, the present results only suggest that a minimum package size should exist in a market that offers multiple packages provided that the entry cost is small. A useful avenue for future research would be to fully characterize optimal package sizes for product lines and examine equilibrium package sizes in a competitive market. The recent work by Desai et al. (2008) is a useful step in this direction. Finally, one could extend the approach presented in the present paper to study the design of services, such as calling cards, museum memberships, and railway passes for which there is no inherent deterioration but the seller imposes a limit on the life of the service. The package size in this case would comprise the bundling of an amount of use (or number of uses) of a service and the usable life itself becomes a decision variable for the firm.

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The authors are listed in alphabetical order and contributed equally to this paper. They thank two anonymous

[8] Note that the higher ordering cost to consumers will not be due to their having to make multiple trips to a store—there is no repeat buying in our model—but because it can be inconvenient for them to store many small packages instead of a few larger ones.

[9] We thank a reviewer for bringing this point to our attention.

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Appendix. Proofs

Proof of Lemma 1. As the consumer uses $f \le s$ units of the ith package, we have $s/p \ge f/p \ge R$, which implies $s \ge pR$. Next, a consumer who buys i packages must consume $i-1$ packages, each of size s, and $f \le s$ units from the ith package. Thus, the usage rate for the consumer is $\theta = [f + (i-1)s]/T$. Now $r = f/p \ge R$ implies $f \ge pR$, and so $\theta = [f + (i-1)s]/T \ge [pR + (i-1)s]/T$. $\square$

Proof of Theorem 1. Recall that $R \in [B_1, B_2]$, where $R = B_1$ corresponds to the reservation quantity for the least price sensitive customer, who is willing to pay a unit price $1/B_1$ to buy the product. Thus, if the unit selling price exceeds $1/B_1$, no customer will buy the product. However, if $\alpha \ge 1/B_1$, the firm will make no profit at a unit price below $1/B_1$. It follows that the firm will not make the product if $\alpha \ge 1/B_1$.

Now consider $\alpha < 1/B_1$. The firm's problem is to maximize profit over package size s and price p:

$$\max_{s,p} \pi = (p - \alpha s)\left(\frac{s/p - B_1}{B_2 - B_1}\right) \cdot \left(\frac{1}{1 - A_1/A_2} - \frac{pB_1 + sn}{2(A_2 - A_1)T}\right)n.$$

The first-order conditions are

$$\frac{\partial \pi}{\partial p} = \frac{B_1 n p^2(2B_1 p + (n-1)s - 2A_2 T) - ns(B_1^2 p^2 + s(ns - 2A_2 T))\alpha}{2(A_2 - A_1)(B_2 - B_1)p^2 T} = 0,$$

$$\frac{\partial \pi}{\partial s} = \Big\{ n[-B_1^2 \alpha p^2 + B_1((n-1)p + 2(-ns + s + A_2 T)\alpha)p + 2A_2 T(p - 2s\alpha) + ns(3s\alpha - 2p)] \Big\} \cdot [2(A_2 - A_1)(B_2 - B_1)pT]^{-1} = 0.$$

We obtain four solutions, of which only the following has positive profit:

$$s^* = (1-k)A_2 T/n, \quad p^* = kA_2 T/B_1,$$

where

$$k = 1 \bigg/ \sqrt{(n+1)\left(\frac{n}{B_1 \alpha} + 1\right)}.$$

The expressions for the corresponding demand, D^*, and profit, π^*, are

$$D^* = \frac{A_2\sqrt{(n+1)B_1\alpha(n + B_1\alpha)} - (n+1)B_1\alpha}{2\alpha(A_2 - A_1)(B_2 - B_1)},$$

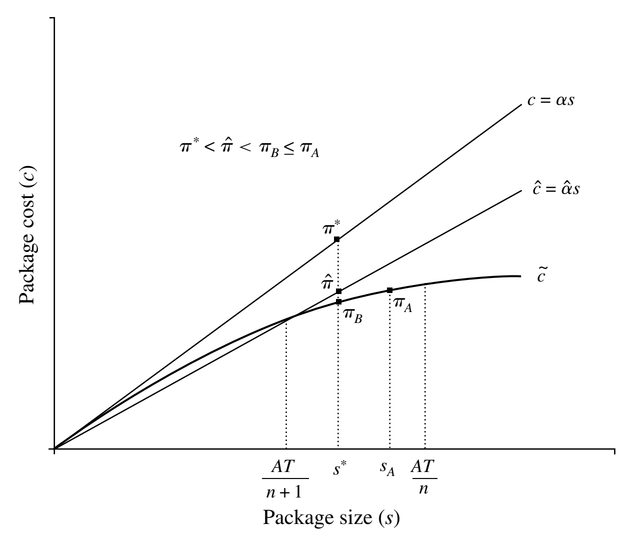
$$\pi^* = \frac{A_2^2 T(\sqrt{n + B_1\alpha} - \sqrt{B_1\alpha(n+1)})^2}{2(A_2 - A_1)(B_2 - B_1)n}. \quad \square$$

Proof of Claim 1. Consider a market with m products. Let s_i denote package size for the ith product and p_i its price, $i = 1, \ldots, m$. Without loss of generality, let $s_1 \le \cdots \le s_m$. Suppose s_1 is not the minimum package size. We claim that these m products cannot comprise an equilibrium set in a market in which the entry cost is zero. That is, if the cost of introducing another package size is zero, then the smallest package size cannot be larger than a minimum package size. Let θ_1 denote the lowest usage rate for any consumer buying the smallest package, which has size s_1. If $\theta_1 > A_1$, where A_1 is the lowest possible usage rate across consumers, then s_1 is not a minimum-size package. There are consumers with usage rates $\theta \in [A_1, \theta_1]$ who do not purchase the product. Thus, there is an opportunity for some firm to introduce a product for these consumers. Following Theorem 1, such a product will have the smallest possible size. However, this contradicts the assumption that the smallest package size that the market can sustain, s_1, is larger than the smallest possible size. $\square$

Proof of Theorem 2. Consider the concave cost function, $\tilde{c} \equiv \tilde{c}(s)$ shown in Figure A.1. The linear cost function, $c = \alpha s$, lies everywhere above $\tilde{c}$ and minimally dominates $\tilde{c}$ in the sense that $\lim_{s\to 0} d\tilde{c}(s)/ds = \alpha$. The point s^* on the horizontal axis corresponds to the optimal package size for the linear cost function, $c = \alpha s$; as noted earlier, it lies between $A_2 T/(n+1)$ and $A_2 T/n$ for $n \ge 1$. Let π^* denote the profit the firm would earn if it had the linear cost $c = \alpha s$. Let s_A and p_A denote the optimal package size and price for the cost function $\tilde{c}(s)$, and let $\pi_A \equiv \pi_A(s_A, p_A)$ denote the associated profit. Consider the linear cost function $\hat{c} = \hat{\alpha} s$, which goes through the origin and intersects the concave cost function $\tilde{c}(s)$ at the point $s = A_2 T/(n+1)$.

The condition $\lim_{s\to 0} d\tilde{c}(s)/ds = \alpha$ implies that $\lim_{n\to\infty} \pi_A = \lim_{n\to\infty} \pi^* = \pi^*_{\max}$. That is, as s approaches zero, π_A, the firm's optimal profit under the concave cost function $\tilde{c}(s)$, approaches $\pi^*_{\max}$, the maximum possible profit

Figure A.1 **Package Size and Package Cost Relationships for Linear and Concave Cost Function**



Notes. This figure illustrates the relation $\pi^* < \hat{\pi} < \pi_B \le \pi_A$, where s_A denotes the optimal package size when the package cost is specified by the concave function $\tilde{c} = \tilde{c}(s)$, and the associated profit is $\pi_A \equiv \pi_A(s_A, p_A)$; s^* denotes the optimal package size when the package cost is $c = \alpha s$, and the associated profits is $\pi^* \equiv \pi^*(s^*, p^*)$; $\hat{\pi} \equiv \hat{\pi}(s^*, p^*)$ is the profit associated with (s^*, p^*) if the cost is $c = \hat{\alpha} s^*$, where $\hat{\alpha}$ satisfies $\hat{c} = \hat{\alpha} s = \tilde{c}$ for $s = AT/(n+1)$; and $\pi_B \equiv \pi_B(s^*, p^*)$ is the profit associated with (s^*, p^*) if the cost is $\tilde{c} = \tilde{c}(s^*)$.

under the linear cost function $c = \alpha s$; see Figure A.1. Let $\pi_B = \pi_B(s^*, p^*)$ denote the profit at (s^*, p^*) if we use the cost $\tilde{c}(s^*)$ instead of the cost $c = \alpha s^*$ when computing the profit at (s^*, p^*). Then $\pi^* < \pi_B \leq \pi_A$, where (1) $\pi^* < \pi_B$ because $\tilde{c}(s^*) < \alpha s^*$ and (2) $\pi_B \leq \pi_A$ because (s_A, p_A) is the optimal solution when the cost is given by the concave function $\tilde{c}(s)$. Next, consider the linear cost function $\hat{c} = \hat{\alpha} s$, which goes through the origin and intersects the concave cost function $\tilde{c}(s)$ at the point $s = A_2 T/(n+1)$. Then $\tilde{c}(s) < \hat{c} < c$ for all $s \geq A_2 T/(n+1)$, because $\hat{c} = \hat{\alpha} s$ (1) lies everywhere below $c = \alpha s$ and (2) lies above $\tilde{c}(s)$ for $s > A_2 T/(n+1)$. Let $\hat{\pi} = \hat{\pi}(s^*, p^*)$ denote the profit if we use the cost $\hat{c} = \hat{\alpha} s^*$ instead of the cost $c = \alpha s$ to compute the profit at (s^*, p^*). Then $\pi^* < \hat{\pi} < \pi_B$, where (1) $\pi^* < \hat{\pi}$ because $\hat{\alpha} s^* < \alpha s^*$ and (2) $\hat{\pi} < \pi_B$ because $\tilde{c}(s^*) < \hat{\alpha} s^*$. The inequalities $\pi^* < \pi_B \leq \pi_A$ and $\pi^* < \hat{\pi} < \pi_B$ together imply that $\pi^* < \hat{\pi} < \pi_B \leq \pi_A$. Subtracting $\pi^*_{\max}$ from each term of the preceding inequality gives $\pi^* - \pi^*_{\max} < \hat{\pi} - \pi^*_{\max} < \pi_B - \pi^*_{\max} \leq \pi_A - \pi^*_{\max}$. We showed in the previous section that $\pi^* - \pi^*_{\max} < 0$. However, if $\hat{\pi} - \pi^*_{\max} > 0$, then the preceding inequality implies that $\pi_A - \pi^*_{\max} > 0$. Thus, a sufficient condition under which the maximum profit for a concave cost function $\tilde{c}(s)$ is obtained for a finite value of $n = n^*$ is given by the condition $\hat{\pi} - \pi^*_{\max} > 0$, which, upon substituting

$$\hat{\pi}^* = \frac{A_2^2 T\left(\sqrt{n^* + B_1\hat{\alpha}} - \sqrt{B_1\hat{\alpha}(n^*+1)}\right)^2}{2(A_2 - A_1)(B_2 - B_1)n^*},$$

$$\pi^*_{\max} = \lim_{n\to\infty} \pi^*_n = \frac{A_2^2(1-\sqrt{B_1\alpha})^2 T}{2(A_2 - A_1)(B_2 - B_1)},$$

yields the desired condition $\hat{\alpha} < \alpha_{\max}$, where

$$\begin{aligned}\alpha_{\max} = \frac{1}{B_1 n^*}\Big[&2+2B_1\alpha - 4\sqrt{B_1\alpha} + n^*(2+B_1\alpha - 2\sqrt{B_1\alpha})\\ &-2\Big\{(n^*+1)[(1-4\sqrt{B_1\alpha}) + B_1\alpha(6+B_1\alpha-4\sqrt{B_1\alpha})\\ &+n^*(1+B_1\alpha-2\sqrt{B_1\alpha})]\Big\}^{1/2}\Big].\end{aligned}$$

Finally, if $\hat{\alpha} < \alpha_{\max}$ for $n = n^*$, then it is also true for all $n \geq n^*$, because $\hat{\alpha}$ increases with n. $\square$

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