Quantifying the Costs of Investment Limits for Chilean Pension Funds*

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Abstract

Since the creation of the Chilean pension fund industry in 1981, pension fund administrators have not been free to choose their investment portfolios because of stringent regulation of investment limits. The diagnosis implicit with the imposition of limits was that the Chilean capital market was not deep, that there was an important demand for funds to finance the expansion of the productive sector and that, due to principal–agent problems, protection for uninformed account holders was needed. As this regulation entails an inefficient combination of risk and return, this paper quantifies its costs.

I. Introduction

Many countries have conducted reforms of their social security systems in recent years, switching from pay-as-you-go (PAYG) to fully funded (FF) systems with individual accounts. One of the main reasons for doing so is

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that the demographic transition observed around the world implies declining birth rates and declining ratios of workers to retired people.[1]

Usually run by the government, PAYG systems are not generally efficient. Furthermore, due to political pressures, in many cases the funds are used for different purposes. On the other hand, the administration of FF systems has usually been delegated to private firms known as pension fund administrators (PFAs). Competition among them is expected to lead to efficient investment of resources and provision of pensions. However, in response to the principal–agent problem that might arise in this market, regulations impose important restrictions on investments. This market is also characterised by compulsory contributions and government guarantees, aspects that might induce a lack of interest in pension products on the part of customers, weakening market competition.

Regarded as a pioneering example of this transition, Chile started its pension fund reform in 1981.[2] Since then, several regulations have been adopted and changed. This paper focuses on the effects of one of them, namely the regulation that has prevented PFAs from freely choosing their portfolio allocations. We analyse the potential costs of these limits on the accumulated savings of account holders and on the profile of risks and returns that they have faced.[3] This analysis might be relevant for other countries in Latin America and Eastern Europe where similar systems have been implemented with even more stringent investment restrictions in some cases.

Any general equilibrium implications that the implementation of the FF system or its regulation might have had are absent from our analysis. Therefore, in quantifying the costs, we build counterfactual scenarios by imposing restrictions that make our analysis as realistic as possible, but we do not address some possible benefits of the regulations that might arise in general equilibrium. Corbo and Schmidt-Hebbel (2003) quantify the effects of the Chilean pension fund system on the development of the capital market, on resource allocations and on growth.[4]

The paper is organised as follows: Section II briefly describes the regulation of investment limits that PFAs have faced; Section III discusses the methodology used to evaluate the consequences of the regulation; Section IV presents the results; and finally, Section V concludes.

[1]For a detailed analysis of pension fund reforms, see Valdés (1997).

[2]See Cheyre (1991) and Superintendence of Pension Funds Administrators (2003).

[3]Cardinale (2003) attempts to find the optimal portfolio in the absence of limits on investment abroad, but does not consider the specific investment regulation in Chile.

[4]See Vittas (1996).

II. The Chilean pension fund industry

Being a market in which workers are compelled to contribute (i.e. to buy the product) and may not be well informed about the specific characteristics of what they are buying, the pension fund industry is subject to heavy investment regulation.[5] The services that the PFAs provide to their contributors (the pensions they can offer and the information they have to provide) are also regulated. Prices are regulated to some extent as well, in the sense that they have to be a fixed amount per contribution and/or a percentage of income per contribution.[6]

The assumption underlying these regulations is that agency problems mean that mechanisms need to be implemented that ensure the safe and adequate management of funds. When the system was designed, competition was expected to lead to adequate risk–return combinations being offered, low prices and efficient provision of services. Nevertheless, intending to protect uninformed customers, the regulation limited differentiation and competition, which is considered necessary in the case of a compulsory product with government guarantees involved.

1. Investment regulation

With the intention of guaranteeing the safekeeping of pension funds, the regulation states that resources must be invested only in instruments that are authorised by the Law (DL 3.500 of 1980). These instruments are financial assets on public offer; if they are not issued by the government or the Central Bank of Chile or another country, their issuers must be supervised by some government agency such as the Superintendence of Securities and Insurance, the Superintendence of Banks and Financial Institutions or their equivalents in other countries.[7]

Notes and Source to Table 1
[a]The limit is reduced to 30 per cent if the duration is shorter than 1 year.
[b]A 1 per cent limit was established for 1 year, with plans to increase it by 1 percentage point each year up to 5 years. After the fifth year, it was to be increased to 10 per cent.
[c]The distinction between corporations with concentrated property and those with unconcentrated property was eliminated.
[d]This limit is joint with variable income investment abroad.
Note: Each range in the table is the range established by Law within which the Central Bank has to set the maximum investment limit.
Source: Superintendence of Pension Funds Administrators (SAFP).

[5]Even after 24 years of the Chilean pension system, account holders appear not to be well informed about prices, returns and other important variables. According to surveys conducted in 2001 (Barómetro-CERC) and 2002 (EPS Survey), more than 90 per cent of them did not know how much they were charged by PFAs for administration fees. Lack of information on pensions or financial education in general appears to be not just a Chilean but a worldwide characteristic (Bernheim, 1998; D'ambrosio, 2003).

[6]The income considered for contributions has a ceiling of 60UF (around US$2,040) a month.

[7]Walker and Valk (1995) analyse investment regulations and their performance.

TABLE 1

Changes in permitted ranges for investment limits

<table>
<tr><th>Instrument</th><th>1981</th><th>1985</th><th colspan="2">1989</th><th colspan="2">1990</th><th colspan="2">1994</th><th colspan="2">1996</th><th colspan="2">1999</th></tr>
<tr><td>State-issued</td><td>100%</td><td>100%</td><td colspan="2">50%</td><td colspan="2">45%</td><td colspan="2">35–50%</td><td colspan="2">35–50%</td><td colspan="2">35–50%</td></tr>
<tr><td>Mortgage notes</td><td>40–100%</td><td>40–100%</td><td colspan="2">40–100%</td><td colspan="2">40–100%</td><td colspan="2">35–50%</td><td colspan="2">35–50%</td><td colspan="2">35–50%</td></tr>
<tr><td>Fixed income issued by financial institutions</td><td>40[a]–100%</td><td>40[a]–100%</td><td colspan="2">40[a]–100%</td><td colspan="2">40[a]–100%</td><td colspan="2">30–50%</td><td colspan="2">30–50%</td><td colspan="2">30–50%</td></tr>
<tr><td>Bonds from public and private corporations</td><td>60–100%</td><td>30–100%</td><td colspan="2">30–100%</td><td colspan="2">30–100%</td><td colspan="2">30–50%</td><td colspan="2">30–50%</td><td colspan="2">30–50%</td></tr>
<tr><td>Stocks from open corporations</td><td>Not allowed</td><td>10–30%</td><td>–</td><td rowspan="3">10–30%</td><td>10–30%</td><td rowspan="3">20–40%</td><td colspan="2">30–40%[c]</td><td colspan="2">30–40%</td><td colspan="2">30–40%</td></tr>
<tr><td>Real estate corporate stock</td><td>Not allowed</td><td>Not allowed</td><td>–</td><td>10–30%</td><td colspan="2">10–20%</td><td colspan="2">10–20%</td><td colspan="2">10–20%</td></tr>
<tr><td>Stocks from open corporations (concentrated)</td><td>Not allowed</td><td>Not allowed</td><td>10%</td><td>10–30%</td><td colspan="2">–</td><td colspan="2">–</td><td colspan="2">–</td></tr>
<tr><td>National investment funds</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">10–20%</td><td colspan="2">5–10%</td><td colspan="2">5–10%</td><td colspan="2">5–10%</td></tr>
<tr><td>International investment funds</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">3–6%[d]</td><td colspan="2">3–6%[d]</td></tr>
<tr><td>Fixed income foreign instruments</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">1%[b]</td><td>6–12%</td><td rowspan="2">6–12%</td><td>6–12%</td><td rowspan="2">6–12%</td><td>10–20%</td><td rowspan="2">10–20%</td></tr>
<tr><td>Variable income foreign instruments</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">Not allowed</td><td>3–6%</td><td>3–6%</td><td>5–10%</td></tr>
<tr><td>Hedging instruments</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">5–15%</td><td colspan="2">5–15%</td><td colspan="2">10–25%</td></tr>
<tr><td>Others approved by Central Bank</td><td>Not allowed</td><td>Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">Not allowed</td><td colspan="2">1–5%</td><td colspan="2">1–5%</td><td colspan="2">1–5%</td></tr>
</table>

Among the financial instruments that are currently authorised by the Pension Fund Law, we find the following: state-issued titles of the Central Bank of Chile and the General Treasury of the Republic, previsional past-service bonuses, instruments issued by financial institutions (deposits, promissory notes, mortgage notes, bonds and stocks), corporate bonds, stocks, shares of investment funds and foreign instruments. Within this last category, there are instruments issued by states and governments, and corporate bonds and stocks. Pension funds are also allowed to carry out hedging operations by using derivatives in domestic and international markets.

Pension funds are allowed to invest in an extensive list of instruments rated and approved by a Risk Rating Commission (CCR). In the case of fixed income instruments, they are required to have a rating between AAA and BBB or equivalent. In the case of stocks, they have to be explicitly approved by the CCR, or meet some specific requirements with respect to results and assets.

Moreover, the Law specifies a range for the maximum percentage of the fund that can be invested in each instrument, and the Central Bank sets the actual limit within this range (see Table 1). There are limits per instrument, per issuer, per risk and per group of instruments, and some specific limits for issuers that have property relations with the pension fund manager.

The limits per instrument have been slackening significantly over time. In 1981, investment was allowed only in national fixed income instruments. As the local capital market developed, investment in some stocks was allowed, with a limit of 30 per cent of the portfolio set in 1985. In 1990, more equity risk was allowed through a new vehicle: shares of investment funds, with a limit that went up to 20 per cent.

It was not until 1990 that investment abroad was allowed. The limit was set by the Central Bank at 2.5 per cent in January 1992 and has increased steadily since then, reaching limits of 15 per cent and 20 per cent, for variable and fixed income respectively, in 2002.[8]

The limits per issuer are expressed as a percentage of the fund and as a percentage of the assets of the issuer. The former aims to achieve a higher diversification and the latter to avoid the possibility of having a pension fund manager as controller of a specific issuer. Nevertheless, these limits are significantly decreased when the issuer has a property relationship with the pension fund manager. For example, in the case of stocks, the limit determined as a percentage of the issuer’s assets is downsized from 7 per cent to 2 per cent.

[8]Investment limits abroad have continued to increase. By Law, this limit can go up to 30 per cent with no distinction between variable and fixed income instruments. The limit is currently set by the Central Bank at the maximum level of 30 per cent.

Additionally, pension funds are subject to minimum return regulation. During the sample period of this study, the Law established that the managers are responsible for ensuring an average real return over the past 12 months that must exceed the average return of all the funds minus 2 percentage points, or 50 per cent of the average return of all funds, whichever is lower.[9] For this purpose, PFAs must keep 1 per cent of the value of the fund they manage (as a reserve that must have the same portfolio as the pension funds). These resources are used if the returns go below the lower bound.[10] When the difference is not covered by the reserve or the funds of the administrator, the government must cover it. However, in this case or when the cash reserve is not restored after being used, the PFA is liquidated.

The regulations described above have had effects on the way PFAs have chosen their portfolios and, by doing so, on the risk the account holders face, the return on the investments they make and the pensions received once they retire (Arrau and Chumacero, 1998; Valdés and Ramirez, 1999).

During 2002, there was an important amendment to the Law that allowed pension funds to invest in five different portfolios, from which account holders have to choose the one that best suits their risk–return preferences. However, the regulation to which these funds are submitted is still in terms of quantitative restrictions with a similar structure to the one that prevailed in the case of one fund. Additionally, there is a minimum return that is now computed for each fund, with a bandwidth that is larger for riskier funds. These changes in regulation may reduce the costs that are computed here, but analysis of their effects goes beyond the scope of the paper.

2. Evolution of the PFAs' portfolios

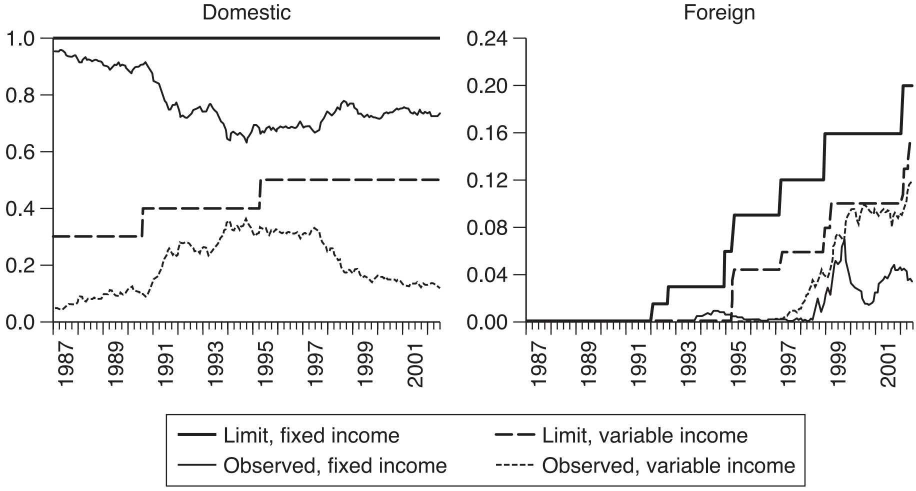
As mentioned above and observed in Figure 1, investment limits have changed significantly over time and PFAs have taken advantage of this increased flexibility. In fact, the share of variable income instruments in Chile increased steadily from 1985, when they were first allowed, until 1991. In that year, the limit was increased from 30 per cent to 40 per cent in the case of stocks, and investment fund shares were incorporated as instruments of investment, raising the limit to 50 per cent for variable income instruments as a whole.

[9] In August 2002, the average rate of return to compute the minimum return was changed from an average over the past 12 months to an average over the past 36 months.

[10] When the rate of return is above the average plus 2 percentage points, or 150 per cent of that average, whichever is higher, the excess returns must be used as a reserve if returns go below the minimum threshold.

FIGURE 1

Investment limits and observed portfolios



Source: Superintendence of Pension Funds Administrators (SAFP).

Investment abroad was permitted in 1990. At the beginning, it was circumscribed to investment in fixed income instruments and there were very few investments abroad. The limit has increased steadily and investments overseas have become more important, especially in variable income instruments, where the limit has been almost binding since 2000.[11]

International diversification of pension funds has been a crucial element in recent reforms, contributing to better portfolio management. As there is evidence of home bias in voluntary savings, and given that the most important asset of workers (their human capital) is also significantly correlated with the domestic situation, international diversification of pension funds would allow diversification of these risks.[12] However, as a developing country may need internal financing, general equilibrium effects should be considered when setting regulation. Additionally, cross-border investment must be carefully evaluated, taking into consideration the probability of an international conflict that might interrupt the international payment system, having an adverse effect on future pensioners.[13]

Many Latin American and Eastern European countries that have reformed their pension systems prohibit investment abroad or have very small limits on it (Uruguay and Dominican Republic, among others, forbid foreign investment). However, as in the cases of Chile and Mexico, this

[11]See Zurita and Jara (1999) for an analysis of the performance of pension funds in Chile.

[12]See French and Poterba (1991) and Lewis (1999).

[13]We thank a referee for pointing this out.

situation has been changing as the systems mature. The Mexican reform started in 1997 and regulation of investment abroad was issued in 2005, with a limit of 20 per cent of pension funds.

III. Characteristics of the exercise

This paper aims to provide a quantitative approximation of the costs of investment limits. This task is not easy, as a plausible counterfactual scenario must be provided. That is, we have to evaluate how the PFAs would have chosen their investment portfolios in the absence of limits.

The basic premiss that we try to follow is to prioritise the construction of realistic scenarios and, when in doubt, we choose to model decisions that lead to underestimation of the costs of these limits, thus most likely providing lower bounds.

To construct the counterfactual scenario, we need to make the following explicit:

- The *instruments* in which PFAs could have invested. PFAs have numerous instruments to choose from when making their portfolio decisions. We assume that they can be grouped in four categories, which can be represented by the following set of instruments: Chilean fixed income, Chilean variable income, foreign fixed income and foreign variable income.[14] Representative prices for these categories are proxied by promissory notes of the Central Bank of Chile with maturity of 8 years (PRC8), the Chilean general index of stock prices (IGPA), 6-month Treasury Bills of the United States and the Dow-Jones index respectively.[15]
- The *returns* of each of these instruments. We assume that the returns are independent of the decisions taken by PFAs. In general equilibrium, the returns of some of these assets (the Chilean ones) may have been affected by the decisions of PFAs and the investment restrictions they faced. If Chilean PFAs had market power, portfolio choices would have to internalise the effects of the actions on asset prices. We do not consider this possibility here but discuss some implications below.
- The way in which the *portfolio* would have been chosen. This point deserves further scrutiny. As what is intended here is to quantify the costs of regulation, we need to compare what did happen with what

[14] Appendix A provides further details of the data.

[15] As pointed out by the referees, one could use a more diversified global portfolio (such as the MSCI world index) instead of the Dow-Jones. However, the contemporaneous correlation of the monthly returns between the indices is 0.82. Furthermore, as discussed below, our results are robust to using the MSCI index.

would have happened in its absence. To construct this comparison, we need to define the way in which PFAs would have chosen their portfolios. We approach this problem by considering several strategies that they may have followed, the most popular being the construction of minimum-variance portfolios. However, we also consider other cases, such as variants of VaR (value-at-risk) efficient portfolios.[16]

- The law of motion of the *assets* that would have been managed. Comparing the performance of different investment strategies depends on at least two dimensions: the returns of a given portfolio and the total amount of assets invested. Denote by W_t^i the total assets available at time t when portfolio strategy i is followed. Its law of motion is given by

  (1) $$W_t^i = W_{t-1}^i r_t^i + A_t^i$$

  where A_t^i is the amount of net inflows received in period t when portfolio strategy i is followed[17] and r_t^i is the gross return of the portfolio chosen in period t when strategy i is followed. This gross return is computed as

  (2) $$r_t^i = x_t' w_{t-1}^i$$

  where x_t is the vector of gross returns of the k assets available for investment (assumed to be independent of i) and w_{t-1}^i is the k-vector of portfolio shares chosen in period t–1. In each period, PFAs have to choose how they will invest the assets that they have available. Given the instruments, expected returns and volatilities, constraints and an objective function to optimise, PFAs are assumed to choose their portfolio.

  The ground rules that we use to obtain these inputs are as follows:

  – The initial total assets, W_0, are fixed independently of i to the value observed in February 1987, which is the earliest period available for computing returns of the assets considered.
  – The assets considered and their returns are the four discussed above.
  – In constructing each portfolio strategy i, we assume that PFAs make their choices based on forecasts of returns and volatilities and not based on the returns observed *ex post*. These expectations are computed using information available at the time. That is, in order to estimate the expected returns of the assets in period t, we compute the

[16]Appendix B defines and briefly describes the approaches considered for modelling the portfolio choice of PFAs.

[17]Operationally, the net inflows are computed from (1) by using the information on total assets and observed returns of the system reported by the Superintendence of Pension Funds Administrators every month. A methodological description of the valuation of assets can be found in its Circular #1216 of July 2002.

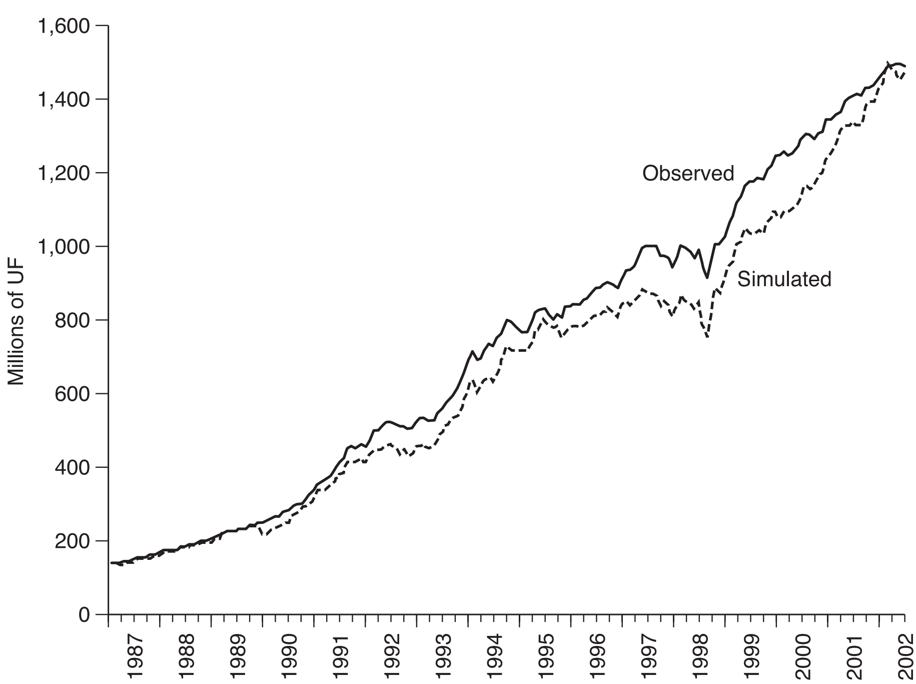
vector average returns and covariance matrix using information between periods t–1 and t–H for $H > 1$. If H is large and the stochastic process followed by the returns is persistent, the estimated first moments will not be good forecasters of the returns, but would arguably be better estimates of the second moments.[18]

– Finally, we consider the sequence of A_t^i as deterministic and independent of i. This assumption will underestimate the cost of regulating the investment limits as net inflows made by account holders would most likely have increased if the returns of the investment had been greater than observed.[19]

Before reporting the results of the exercise, we need to construct a useful benchmark. Given that we assume in our simulations that PFAs choose their portfolio from a subset of the assets considered in reality, we compute what we call our simulated portfolio. It uses portfolio weights actually observed

FIGURE 2

Observed and simulated total assets



[18]We explored several settings for H and fixed it at 36 (three years) for the numerical exercises reported below.

[19]Voluntary contributions to the system were always allowed. Changes in regulation in 2002 made this option even more attractive.

(grouped in the four categories), but constructs r in (2) by using the returns of the four instruments considered in our exercise (not all the instruments that PFAs use). Given a value for W_0, and the sequences of A and x, we construct sequences for r and W.

Figure 2 presents a comparison of the evolution of observed and simulated total assets. The observed and simulated returns behave similarly (the sample correlation of the monthly observed and simulated returns is 0.7), but as the simulated assets are obtained using the observed portfolio with a more restrictive set of assets, the simulated assets are usually smaller.

To level the field when assessing the effects of investment limit regulation, we will use the simulated assets and returns instead of the observed ones as a benchmark, because we are considering the evolution of these variables using a restricted set of assets from which to choose the portfolios.

In summary, when possible, we construct conservative estimates of the costs of investment limits. For example, we consider a very restrictive set of instruments from which PFAs can form their portfolios. The wider the variety of instruments considered, the heavier the costs associated with investment limits might be. In addition, we consider naive models for forecasting expected returns and volatilities. Furthermore, we do not allow for hedging operations, which, once again, restricts the set of instruments. Finally, in our counterfactual scenarios, we consider that the net inflows would not have been influenced by better performance of the funds.

However, there are some aspects that may imply overestimation of the costs. For instance, we do not consider transaction costs. According to our model, in the absence of limits, PFAs would have changed their portfolios more frequently and more abruptly than with limits, thus arguably incurring higher transaction costs. Furthermore, some of the potential benefits of investment limits (particularly in the early stages) may have been their beneficial effect on the development of local capital markets.

Finally, there are some factors associated with the size of pension funds in the local market that are not considered. If pension funds had market power, some strategic behaviour, not considered here, would have been possible. In this sense, prices are assumed to be unchanged, independently of the strategy followed by PFAs. The sign of this effect on our computations of the costs is ambiguous. On the one hand, pension funds could have taken advantage of their market power, but on the other hand, this could have been an additional constraint for their portfolio selection.

IV. Results

As discussed above, to quantify the cost of the regulation, we intend to compute the portfolio that would have been chosen in the absence of investment limits. For that purpose, we need to take a stance regarding the way in which PFAs choose their portfolios. As no explicit mechanism is known, we use several standard models for portfolio selection with their respective optimisation problems. The three strategies for selecting portfolios that we consider are (without short sales): the minimum-variance portfolio (portfolio ***p***), the quadratic preferences portfolio (portfolio ***q***) and the VaR efficient portfolio (portfolio ***v***).[20]

For each of these problems we proceed as follows:

- First, select the portfolio that is consistent with the corresponding model subject to the constraints imposed by the investment limits. For each period, the expected return of the portfolio that is chosen replicates as closely as possible the average return of the simulated portfolio in a window of J periods.[21]
  - In the case of the minimum-variance portfolio, there is no need for an additional constraint to the one that asks our hypothetical investor to match the expected return when there are investment limits.
  - For the case of the quadratic preferences portfolio, we calibrate the parameter B of expression (B.10) in Appendix B to replicate the average return observed in the window.
  - For the VaR efficient portfolio, we set the loss probability, α, to 0.05 and calibrate the parameter $\overline{VaR}$ of (B.14) to replicate the average return of the whole sample.[22]
- Next, consider the optimisation problem without the investment limits constraints. As this problem has fewer constraints, it is expected to perform better. We force the PFAs to choose a portfolio that matches the expected volatility of the portfolio with limits (described above). The reason for doing this is that if risk were volatility, the portfolio without limits would have been exposed to the same risk as the one with limits.[23] This amounts to

[20]See Appendix B for details and definitions.

[21]We set $J = 36$ (three-year average).

[22]Alexander and Baptista (2002) show that when returns are normally distributed, minimum-VaR portfolios converge to minimum-variance portfolios as the confidence level at which VaR is computed increases (α decreases). As we chose to obtain a minimum-VaR portfolio that matches the average return of the whole sample, we calibrated $\overline{VaR}$ only once.

[23]Alternatively, the portfolio without limits could have been chosen by lifting the restrictions and using the values for parameters such as B and $\overline{VaR}$ obtained from the procedures described above. This would have led to 'better' return–volatility combinations for the agent (the PFAs) than the ones obtained

- finding the expected return that is necessary for the unconstrained portfolio to match the expected volatility of the constrained $\boldsymbol{p}$ portfolio;
- finding the value of B that is necessary to match the expected volatility of the constrained $\boldsymbol{q}$ portfolio;
- finding the value of $\overline{VaR}$ that is necessary to match the expected volatility of the constrained $\boldsymbol{v}$ portfolio.

As the strategy followed by PFAs is not necessarily the same as the ones used by us, we perform every exercise by asking our hypothetical PFA to face the optimisation problems with and without limits to determine lower bounds to the cost of the restrictions.

Figure 3 presents a comparison of the evolution of the ratio of total assets obtained without and with limits for each strategy. From it, we gather that the costs in terms of the amounts of total assets managed by the PFAs appear to be substantial. In particular, the exercise suggests that with the minimum-variance portfolio, at least 30 per cent more assets would have been on the system by July 2002 without increasing the volatility of the returns.[24] Thus, not only may investment limits have been costly in terms of not allowing proper risk diversification, but also they could have had a cost in terms of forgone assets. The figures obtained with the other investment strategies follow the same general pattern as the minimum-variance portfolio but imply lower costs. The reason is that the other types of investment strategies tend to be more conservative (particularly in the case of the VaR efficient portfolio). At any rate, according to these exercises, by the end of the sample period, the amount of forgone total assets may have been at least 10 per cent.

In terms of the characteristics of the portfolios that would have been chosen in each case, the models predict a much heavier weight of fixed and variable foreign instruments than the one observed. More importantly, the portfolio allocations in the model with limits were chosen so that the expected returns (and thus total assets) were consistent with the observed trajectory of total assets and the portfolio selection strategy considered.

here. However, this would have made it impossible to evaluate the effects of regulation on the principal (account holders) without knowing their preferences. By matching volatilities, we obtain straightforward (and conservative) estimates of the costs of investment limits.

[24]Interestingly, the investment limits were beneficial during the 'black Monday' episode (and in general at the end of the 1980s and beginning of the 1990s). As investment abroad was not allowed, total assets with limits were greater than the assets that would have been obtained without limits.

FIGURE 3

Ratios of total assets, no limits / limits

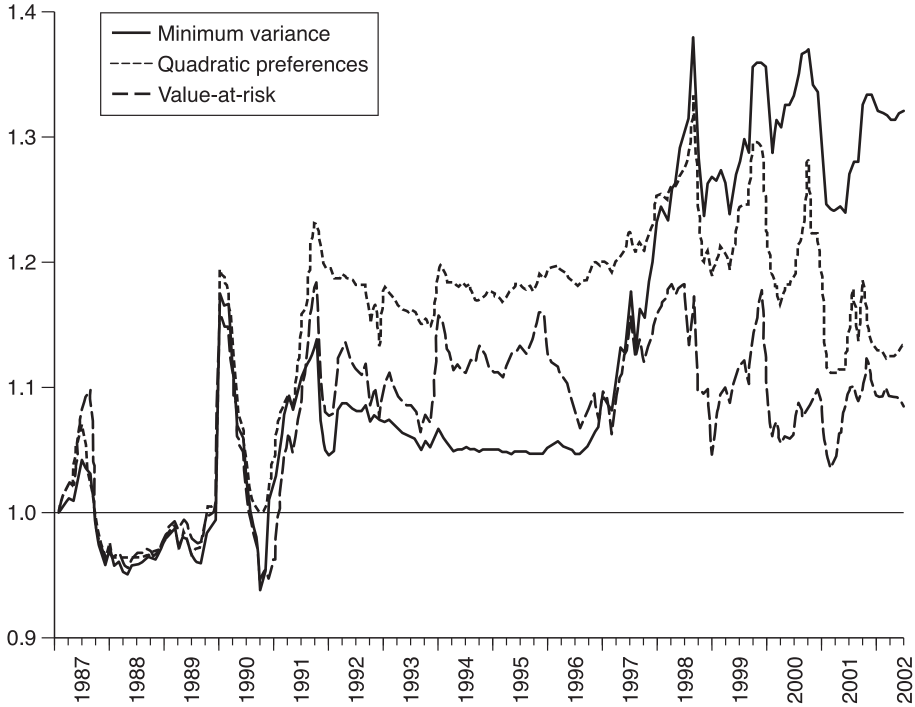


Table 2 presents other summary statistics that describe the nature of the results. They suggest that the costs of investment limits could have been substantial. In particular, the portfolios consistent with limits have returns significantly below the ones obtained in the absence of limits, without increasing their risk (as the expected volatility is made to coincide). This is something that should not be surprising, as the models consider that the average share of investments abroad should have been more than 30 per cent and, independent of the strategy considered, the limits might have been binding 90 per cent of the time. In terms of costs, not only could total assets have been at least 10 per cent higher, but also, depending on the investment strategy, the average account holder may have lost more than UF20 (approximately US$680).[25]

[25]The results are robust to the choice of instrument used for foreign variable income. For example, if the MSCI world index were used instead of the Dow-Jones index and if portfolio strategy ***q*** were selected, the average portfolio composition in fixed income domestic, variable income domestic, fixed income foreign and variable income foreign instruments would have been 54 per cent, 14 per cent, 17 per cent and 15 per cent respectively. The probability that the limit on foreign variable income instruments would have been binding is 86 per cent, and the average gap per account holder would have been UF27 (approximately US$920).

TABLE 2

Costs of investment limits

	*Portfolio **p***	*Portfolio **q***	*Portfolio **v***
Average monthly return without limits	0.84%	0.75%	0.70%
Standard deviation of monthly return with and without limits	2.46%	2.38%	2.57%
Average monthly return with limits	0.66%	0.62%	0.61%
Average shares, 1988–2002			
Fixed national	49.4%	51.9%	44.8%
Variable national	17.9%	15.5%	18.9%
Fixed foreign	18.6%	20.2%	17.3%
Variable foreign	14.1%	12.5%	19.1%
Probability of binding limit			
Fixed national	0.0%	0.0%	0.0%
Variable national	15.1%	14.0%	9.1%
Fixed foreign	62.4%	66.7%	61.3%
Variable foreign	90.3%	91.9%	86.6%
Average gap (UF per account holder)	38	36	20
Standard deviation of gap (UF per account holder)	46	27	15
Maximum gap (UF per account holder)	141	106	58

Despite the fact that we construct the portfolios with limits so that the expected return matches the average return of the simulated portfolio, the composition of the portfolios selected may differ from the ones observed in reality. Figure 4 presents a comparison of the evolution of the portfolio shares in each of the four instruments.[26] For illustrative purposes, the limits and the actual portfolio shares are also included.[27]

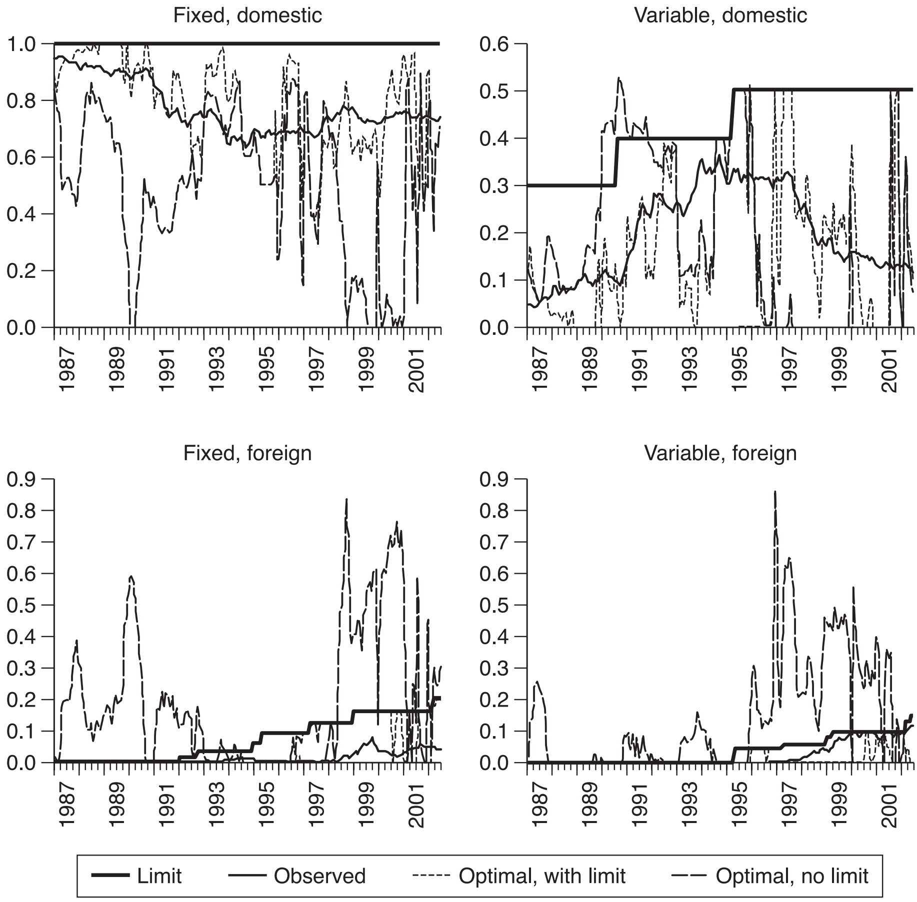
Even though the portfolio shares obtained for the portfolio with limits follow the behaviour of the portfolio that the PFAs actually chose, our weights are much more volatile. There are several candidates for explaining the excessive volatility of our portfolio weights. In our exercise, transaction costs in adjusting portfolios are absent; this makes portfolio changes less

[26]The results for the ***q*** and ***v*** portfolios are qualitatively similar to those for the ***p*** portfolio and are not included.

[27]A referee suggested considering a different counterfactual scenario with more flexible limits. Figure 4 suggests that if (for example) the investment limits prevailing at the end of the sample period were in place from the beginning, the costs would still have been substantial as the limits on investing abroad would have been binding at least 38 per cent of the time.

FIGURE 4

Portfolio shares implied by the **p** *model*



costly. While transaction costs may explain some of the persistence of portfolio allocations, some other feature may be at play. One possibility is the choice of the length of the window with which the vector of expected returns and the conditional covariance matrix are computed (the parameters *H* and *J* discussed above). As we set them to 36 (three years), the information that is used to 'forecast' future returns is not very precise. That is, if returns were relatively persistent, the average returns observed in the past three years may not be good candidates for forecasting next month's return.

As the portfolios actually selected have much smoother trajectories than the ones chosen by our model, we next consider the importance of this feature. For this purpose, we consider a portfolio strategy in which PFAs choose their portfolios as a weighted average of the portfolios that solve the optimisation problem presented in Figure 4 and the portfolio chosen the previous period. Thus, if we denote by w_t^i the optimal portfolio chosen following strategy *i* in period *t*, we now compute the smoothed portfolio as

$$(3) \qquad \tilde{w}_t^i = \gamma \tilde{w}_{t-1}^i + (1-\gamma) w_t^i$$

and set $\tilde{w}_1^i = w_1^i$ (for $0 \leq \gamma \leq 1$). With $\gamma = 1$, the portfolio chosen is constant (and equal to the initial portfolio); with $\gamma = 0$, the portfolio chosen is equal to the optimal portfolio. Thus, the closer γ is to 1, the smoother the portfolio allocation would be.

Table 3 presents the results of this exercise for different values of γ. As expected, the smaller the value of this parameter, the closer the model is to the portfolio reported in Table 2 and the larger are the costs of investment limits. Thus, the more sluggish the portfolio allocations, the lower are the costs of regulation because both the expected returns and the costs of investment limits are decreasing functions of γ. At any rate, even with very

TABLE 3

Costs of investment limits
(with smoothed portfolios and minimum-variance objective function)

	$\gamma = 0.25$	$\gamma = 0.50$	$\gamma = 0.75$
Average monthly return without limits	0.81%	0.76%	0.72%
Standard deviation of monthly return with and without limits	2.45%	2.45%	2.46%
Average monthly return with limits	0.63%	0.59%	0.54%
Average shares, 1988–2002			
Fixed national	49.4%	49.5%	49.8%
Variable national	17.9%	17.9%	17.9%
Fixed foreign	18.6%	18.5%	18.3%
Variable foreign	14.1%	14.1%	14.1%
Probability of binding limit			
Fixed national	0.0%	0.0%	0.0%
Variable national	11.8%	12.4%	11.8%
Fixed foreign	62.4%	64.5%	66.1%
Variable foreign	89.8%	91.4%	91.9%
Average gap (UF per account holder)	37	37	37
Standard deviation of gap (UF per account holder)	43	40	40
Maximum gap (UF per account holder)	135	127	121

sluggish portfolio allocations ($\gamma = 0.75$), the main results of the paper hold and the estimated costs of the limits appear to be substantial.[28]

V. Concluding remarks

Since the creation of the Chilean pension fund industry in 1981, pension fund administrators have not been free to choose their investment portfolios because of stringent regulation of investment limits. The diagnosis implicit with the imposition of limits was that the Chilean capital market was not deep, that there was an important demand for funds to finance the expansion of the productive sector and that, due to principal–agent problems, protection for uninformed account holders was needed.

As this regulation entails an inefficient combination of risk and return, this paper aimed to quantify its costs. For that purpose, we constructed counterfactual scenarios for the evolution of the assets and returns that PFAs would have administered had this regulation been absent. In constructing these counterfactual scenarios, we tried to be as conservative as possible. That is, we consider that the costs computed here are most likely lower bounds, because we have always tried to construct scenarios in which, if a bias were present, it would be towards underestimating the costs.

Our results suggest that the costs may have been substantial and that, in the absence of limits, the total assets managed by PFAs could have been at least 10 per cent larger, that pension fund account holders could have been exposed to less volatility, that the investment limits may have been binding approximately 90 per cent of the time and that on average each account holder lost between US$680 and US$1,300.

However, our analysis abstracted from any possible endogeneity with respect to the role of pension funds in the development of local capital markets, which may have been important in the early stages.

Appendix A. The data

Information on the portfolios of PFAs (by investment instrument) was available from June 1981; however, prices for selected instruments were not all available for years before 1984. Also, the first three years were needed to compute the 36-period average for the variance–covariance matrix. Therefore the sample period for the analysis was February 1987 to July 2002.

[28]In fact, as Table 3 suggests, the differences in portfolio allocations are not substantial, and the trajectory of the ratio of assets with and without limits is in accordance with that reported in Figure 3.

The whole set of instruments that were available for pension fund investments were grouped into four categories: national fixed income, national variable income, foreign fixed income and foreign variable income.

The first category included state-issued instruments, mortgage notes, deposits, bonds and titles guaranteed by financial institutions, and bonds from public and private corporations. The representative instrument in this category was chosen to be promissory notes of the Central Bank of Chile with maturity of 8 years (PRC8). Prices were calculated by using the market interest rate observed in each period. For this category, mortgage notes interest rates were also used as an alternative, with no significant impact on the results. The correlation between the 10–12-year mortgage interest rate and PRC8 for the sample period was 0.7.

The second category incorporated stocks from open corporations, real estate corporate stocks, and national investment funds. The representative price for this category was the Chilean general index for stock prices (IGPA) for the national variable income instruments deflated by a unit of account indexed to past inflation (known as the Unidad de Fomento, UF).[29]

The third category (foreign fixed income instruments) comprised credit titles, securities and negotiable titles issued by foreign states, foreign banks or central banks, and bonds issued by overseas companies. The 6-month Treasury Bills index from Bloomberg was used to represent these instruments. The returns were expressed in terms of UFs by using the observed exchange rate figures of the Central Bank of Chile.

Under variable income foreign investment, we considered international investment funds, stocks issued by foreign companies, and international mutual funds. For this type of instrument, the Dow-Jones was used as a proxy for prices. Even if this is only representative of US equity, the results appear to be robust to other alternatives such as the MSCI world index.[30] The real returns in this case were also computed by using the exchange rates reported by the Central Bank of Chile.

Appendix B. Portfolio selection

This appendix defines the various strategies for portfolio selection used in the paper. We begin by considering the conventional CAPM model and other strategies derived from quadratic objective functions, and then consider the increasingly popular value-at-risk model.

[29]See Shiller (2002) for a discussion about the use of the Chilean UF and indexed units of account around the world.

[30]The correlation between the Dow-Jones and the MSCI world index over the sample period was 0.82.

1. Quadratic loss functions

Following Campbell, Lo and MacKinlay (1997), let there be n risky assets with mean vector m and covariance matrix V. Define w_a as the n-vector of portfolio weights for an arbitrary portfolio a with weights summing to unity. Portfolio a has mean return $\mu_a = w_a' m$ and variance $w_a' V w_a$.

Definition 1. *Portfolio* **p** *is the minimum-variance portfolio of all portfolios with mean return* μ *if its portfolio weight vector is the solution to the following constrained optimisation:*

(B.1) $$\min_w \frac{1}{2} w' V w$$

subject to

(B.2) $$w' m = \mu$$

(B.3) $$w' \iota = 1$$

where ι *is an n-vector of ones.*

To solve this problem, we form the Lagrangian function L_1, differentiate with respect to w, set the resulting equations to zero and then solve for w.

$$L_1 = \frac{1}{2} w' V w + \lambda_1 \left(\mu - w' m\right) + \lambda_2 \left(1 - w' \iota\right)$$

where λ_1 and λ_2 are the Lagrange multipliers of (B.2) and (B.3) respectively.

The first-order conditions for this problem are

(B.4) $$V w_p - \lambda_1 m - \lambda_2 \iota = 0 .$$

Combining (B.2), (B.3) and (B.4), we find the solution

(B.5) $$w_p = G + H \mu$$

where G and H are n-vectors,

$$G = \frac{1}{D}\left[B V^{-1} \iota - A V^{-1} m \right]$$

$$H = \frac{1}{D}\left[C V^{-1} m - A V^{-1} \iota \right]$$

and $A = \iota' V^{-1} m$, $B = m' V^{-1} m$, $C = \iota' V^{-1} \iota$ and $D = BC - A^2$.

The optimal portfolio (B.5) admits short sales (some of the weights may be negative). When short sales are not allowed, the Lagrangian function is

$$L_2 = \frac{1}{2} w'Vw + \lambda_1 \left(\mu - w'm\right) + \lambda_2 \left(1 - w'\iota\right) + w'\delta$$

where δ is an n-vector of Lagrange multipliers that imposes the constraints

(B.6) $\quad w \geq 0$.

The first-order conditions of this problem,

(B.7) $\quad Vw_p - \lambda_1 m - \lambda_2 \iota + \delta = 0$,

along with (B.2), (B.3) and the slackness conditions

$$\delta_i w_i = 0 \quad \text{for} \quad i = 1,\ldots,n,$$

are used to solve for the minimum-variance portfolio when short sales are not allowed (Lai et al., 1992).

If a risk-free asset is introduced, the portfolio weights of the risky assets are not constrained to sum to 1, since $1 - w'\iota$ can be invested in the risk-free asset.

Definition 2. *Given a risk-free asset return f, portfolio* ***c*** *is the minimum-variance portfolio of all portfolios with mean return μ if its portfolio weight vector is the solution to the minimisation of (B.1) subject to*

(B.8) $\quad w'm + \left(1 - w'\iota\right) f = \mu$.

To solve this problem, we form the Lagrangian function L_3, differentiate with respect to w, set the resulting equations to zero and then solve for w.

$$L_3 = \frac{1}{2} w'Vw + \lambda_1 \left(\mu - w'm - \left(1 - w'\iota\right) f\right)$$

where λ_1 is the Lagrange multiplier of (B.8).

The first-order conditions for this problem are

$$Vw_c - \lambda_1 \left(m - f\iota\right) = 0,$$

which, combined with (B.8), yields

(B.9) $\quad w_c = JV^{-1}\left(m - f\iota\right)$

where

$$J = \frac{\mu - f}{(m - f\iota)' V^{-1} (m - f\iota)}.$$

When short sales are not allowed, the constrained optimisation problem must be modified to guarantee that (B.6) and $w'\iota \leq 1$ hold.

If risk is equated with volatility, the minimum-variance portfolio problem is closely related to the optimisation problem in which an agent maximises expected utility with quadratic preferences (see Brandimarte (2002), Huang and Litzenberger (1988) or LeRoy and Werner (2001)).

Definition 3. *Portfolio* ***q*** *is the optimal portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimisation:*

(B.10) $$\max_{w} \left(w'm - \frac{1}{2} B w' V w \right)$$

subject to (B.2) and (B.3).

In this case, *B* is a parameter linked to risk aversion, with higher values indicating more risk aversion. The solution to this problem is

(B.11) $$w_q = \frac{1}{B} V^{-1} (m - E\iota)$$

where

$$E = \frac{A - B}{C}.$$

If short sales are not allowed, we proceed as discussed above.

Definition 4. *Given a risk-free asset return f, portfolio* **s** *is the optimal portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimisation:*

(B.12) $$\max_{w} \left(w'm + (1 - w'\iota) - \frac{1}{2} B w' V w \right)$$

subject to (B.8).

The solution to this problem is

$$w_s = \frac{1}{B} V^{-1} \left(m - f\iota \right).$$

As in the previous case, when short sales are not allowed, we proceed by maximising (B.12) subject to (B.6) and $w'\iota \leq 1$.

2. Value-at-risk

Value-at-risk (VaR) has become a key tool for risk management of financial institutions. Usually defined as the maximum expected loss over a given horizon period at a given level of significance, it is intended to provide quantitative and synthetic measures of risk.[31]

Following Gourieroux, Laurent and Scaillet (2000), if P_t is the conditional distribution of future asset prices given the information at time t and if a loss probability of level α is considered, the value-at-risk, $VaR(w,\alpha)$, is defined as

$$P_t \left[W_{t+1}(w) - W_t(w) + VaR_t(w,\alpha) < 0 \right] = \alpha .$$

In particular, if the VaR is computed under the assumption of normality of the returns, with conditional mean m_t and covariance matrix V_t, then

$$VaR_t(w,\alpha) = -w'm_t + \left(w'V_t w \right)^{1/2} z_{1-\alpha} ,$$

with $z_{1-\alpha}$ being the quantile of level $1-\alpha$ of the normal distribution.

Definition 5. *Portfolio* $\mathbf{v}$ *is the VaR efficient portfolio if its portfolio weight vector is the solution to the following constrained optimisation:*

(B.13) $\max_{w} w'm$

subject to (B.8) and

(B.14) $VaR(w,\alpha) = \overline{VaR}$

where $\overline{VaR}$ *is the bound considered.*

This portfolio is a function of the loss probability, α, and the bound considered, $\overline{VaR}$, and it satisfies the following first-order conditions:

[31] See Dowd (1998) and Johnson (2001) for details.

$$
\text{(B.15)} \quad \begin{aligned} m &= -\lambda_t \frac{\partial VaR_t}{\partial w}(w,\alpha) \\ VaR_t(w,\alpha) &= \overline{VaR} \end{aligned}
$$

where λ_t is the Lagrange multiplier of the constraint (B.14). If the assumption of gaussianity holds,

$$
\frac{\partial VaR_t}{\partial w} = -m_t + \frac{V_t w}{\left(w' V_t w\right)^{1/2}} z_{1-\alpha} .
$$

The conditions stated in (B.15) form a non-linear system of equations that can be solved to obtain the VaR efficient portfolio.

If short sales are not allowed, we solve a constrained optimisation problem in which constraints (B.3) and (B.6) are included.

References

Alexander, G. and Baptista, A. (2002), 'Economic implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis', *Journal of Economic Dynamics and Control*, vol. 26, pp. 1159–93.

Arrau, P. and Chumacero, R. (1998), 'Tamaño de los fondos de pensiones en Chile y su desempeño financiero', *Cuadernos de Economía*, vol. 105, pp. 205–35.

Bernheim, D. (1998), 'Financial illiteracy, education, and retirement savings', in O. Mitchell and S. Schieber (eds), *Living with Defined Contribution Plans*, Philadelphia, PA: University of Pennsylvania Press.

Brandimarte, P. (2002), *Numerical Methods in Finance: A MATLAB-Based Introduction*, New York, NY: John Wiley & Sons.

Campbell, J., Lo, A. and MacKinlay, C. (1997), *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

Cardinale, M. (2003), 'Optimal portfolio allocations and funded pension systems: the case of Chile', Imperial College Management School, Watson Wyatt Technical Paper no. 2003-GP02.

Cheyre, H. (1991), *La previsión en Chile ayer y hoy*, second edition, Santiago: Centro de Estudios Públicos.

Corbo, V. and Schmidt-Hebbel, K. (2003), 'Efectos macroeconómicos de la reforma de pensiones en Chile: resultados y desafíos de las reformas a las pensiones', edited by Corporation for Research, Study and Development of Social Security (CIEDESS), Santiago: Federación Internacional de Administradoras de Fondos de Pensiones.

D'ambrosio, M. (2003), 'Ignorance is not bliss: the importance of financial education', Contractual Savings Conference: Supervisory and Regulatory Issues in Private Pensions and Life Insurance, World Bank, Washington, DC.

D.L. 3.500 (1980), Regulations D.L. no. 3.500 and D.F.L. no. 101 (updated text, February 2002).

Dowd, K. (1998), *Beyond Value at Risk: The New Science of Risk Management*, Chichester, NY: Wiley & Sons.

French, K. and Poterba, J. (1991), 'Investor diversification and international equity markets', *American Economic Review*, vol. 81, pp. 222–6.

Gourieroux, C., Laurent, J. and Scaillet, O. (2000), 'Sensitivity analysis of values at risk', *Journal of Empirical Finance*, vol. 7, pp. 225–45.

Huang, C. and Litzenberger, R. (1988), *Foundations for Financial Economics*, Englewood Cliffs, NJ: Prentice Hall.

Johnson, C. (2001), 'Value-at-risk: teoría y aplicaciones', *Estudios de Economía*, vol. 28, pp. 217–47.

Lai, T., Wang, K., Chan, S. and Lee, D. (1992), 'A note on optimal portfolio selection and diversification benefits with short sale restrictions on real estate assets', *Journal of Real Estate Research*, vol. 7, pp. 493–501.

LeRoy, S. and Werner, J. (2001), *Principles of Financial Economics*, Cambridge: Cambridge University Press.

Lewis, K. (1999), 'Trying to explain home bias in equities and consumption', *Journal of Economic Literature*, vol. 37, pp. 571–608.

Shiller, R. (2002), 'Indexed units of account: theory and assessment of historical experience', in F. Lefort and K. Schmidt-Hebbel (eds), *Indexation, Inflation, and Monetary Policy*, Series on Central Banking, Analysis, and Economic Policies 2, Santiago: Central Bank of Chile.

Superintendence of Pension Funds Administrators (2003), *The Chilean Pension System based on Individual Capitalization*, fourth edition, edited by A. Ferreiro, Santiago.

Valdés, S. (1997), 'Financing a pension reform toward private funded pensions', in S. Valdés-Prieto (ed.), *The Economics of Pensions*, Cambridge: Cambridge University Press.

— and Ramirez, M. C. (1999), 'Herding in DC pension funds: regulation or market forces?', Pontificia Universidad Católica de Chile, Instituto de Economía, mimeo.

Vittas, D. (1996), 'Pension funds and capital markets', World Bank, Financial Sector Development, Note no. 71.

Walker, E. and Valk, E. (1995), 'La inversión de los fondos de pensiones: historia, normativa y resultados', in S. Baeza and F. Margozzini (eds), *Quince años después: una mirada al sistema privado de pensiones*, Santiago: Centro de Estudios Públicos.

Zurita, S. and Jara, C. (1999), 'Desempeño financiero de fondos de pensiones', *Estudios Públicos*, vol. 74, pp. 227–54.