THE ROLE OF PRICING IN THE SANTIAGO PUBLIC TRANSPORT SYSTEM

Sergio R. Jara-Díaz*

ABSTRACT

The role of pricing as a short run tool to induce efficiency in the transport system of Santiago is discussed. The spatial distribution of demand shows that both income and car ownership are fairly concentrated in the northeast zone; however, transit trips account for two thirds of the total at a city wide level. Present pricing policy is shown to be quite inefficient from an overall viewpoint: car trips are underpriced, buses seem to charge more than the optimal fare, and underground railroad prices are rational within an internal logic only. It is shown that reaching a socially optimal pricing scheme seems to be politically unfeasible, but refusing to charge for congestion could generate second best prices which are unequitable given the spatial distribution of car ownership. As a compromise, a combination of congestion pricing, ridership maximizing fares (underground railroad) and a flat bus fare, seems to be a reasonable goal.

SINTESIS

Este trabajo analiza el rol de los precios como una herramienta de corto plazo para promover eficiencia en el sistema de transporte en Santiago. La distribución espacial de la demanda muestra que tanto el ingreso como la posesión de un vehículo están bastante concentrados en la zona nororiente; sin embargo, los viajes representan un tercio del total a nivel de la ciudad como un todo. Se muestra que la actual política de precios es bastante ineficiente desde un punto de vista global; los viajes en automóvil están considerados a un precio bajo, los microbuses parecen cobrar más que la tarifa óptima, y los precios del ferrocarril subterráneo resultan racionales sólo dentro de una lógica interna. Se muestra, asimismo, que para alcanzar un esquema de precios socialmente óptimo no parece ser políticamente factible, pero el negarse a cobrar por la congestión generaría precios *second best* que no son equitativos dada la distribución espacial de la posesión de automóviles. A manera de un compromiso, una combinación de cobro por la congestión, tarifas de uso de transporte público maximizadoras (ferrocarril subterráneo) y una tarifa única para microbuses, parece ser una meta razonable.

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1. INTRODUCTION

Finding and setting appropriate prices for the provision of transport services is becoming one of the key issues in both urban and interurban analysis. This has been recently recognized as one of the main results from a recent international Delphy study synthesized in the opening ceremony during the last World Conference on Transport Research; there, all segments in the survey (government, academia and consultancy) identified transport pricing and integrated land use-transport analysis as the main areas that ought to be studied and developed within the next five years (Hensher, 1995). In our view, out of the many tools available to improve the operation of a transport system, pricing is a short term alternative which can be used effectively provided its components are properly introduced. Although prices are usually understood as the result of supply-demand interaction, the fact is that they can be seen as an element to induce behaviour under the presence of a price-sensitive demand, which is the case in most transport systems. However, the fact that these usually serve many markets further complicates the problem; even if we reduce the analysis to passenger flows only, individuals move from many origins to many destinations during many different periods.

The aforementioned picture applies indeed to the urban case, where public and private transport coexist, and where demand can be represented in the short term by a series of origin-destination-period specific willingness to travel functions. As most of these markets are served by more than one transport mode, a choice appears and prices (fares) can play a role.

In general, the urban scene presents a huge number of different users distributed in space, who are served by a transport system which includes private modes (i.e., auto), public modes (e.g., bus, underground railroad) and combinations. In this paper we discuss the role of pricing as a short run tool to induce efficiency in the transport system of Santiago. In the following section we

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describe the urban area and its transport network with particular emphasis on the spatial distribution of demand. Next, prevailing pricing practices are discussed within the framework of modally independent optimal pricing schemes. In section four a general discussion on the possible scenarios is presented. A synthesis and final comments are offered in the last section.

2. THE SANTIAGO TRANSPORT SYSTEM

Around 4.5 million people live in Santiago, where 34 municipalities cover little more than 420 km². Figure 1 shows an aggregate description with six areas. From a socio-economic viewpoint, the North-east area concentrates professionals, entrepreneurs and high income families. Average car ownership in Santiago is 0.39 vehicles/home, but its spatial concentration is very much related with income; accordingly, the North-east area shows little more than one car per house while the remaining areas show less than one third.

According to the 1991 O-D survey (Sectra, 1991), 8.4 million trips are made within Santiago during an average working day. Taking into account that nearly 20 percent are walking trips, only 6.7 million are motorized trips; out of these, aggregate modal split is 20 percent auto, 60 percent bus, 8 percent underground railroad (or combination) and 3.3 percent some form of taxi. In table 1 we show the aggregate modal split by area corresponding to total daily trips starting at a zone. It should be noted that auto trips from the North East area are nearly four times the proportion of auto trips starting North, West or South; this phenomenon is amplified if only the morning peak hour is considered.

TABLE 1

TRIP GENERATION BY AREA AND MODE (DAILY)

	Modal Split (percent area)			
Area	Auto	Subway	Surface Transit	Walking + other
North	8.9	1.3	57.0	32.8
West	8.7	6.9	50.1	34.3
North East	34.9	7.5	39.6	18.0
CBD	13.4	12.7	59.3	14.6
South	8.8	5.9	50.1	35.2
South East	12.1	1.1	52.7	34.1
Santiago	15.8	6.4	50.2	27.3

FIGURE 1
SANTIAGO: ZONES AND CHARACTERISTICS

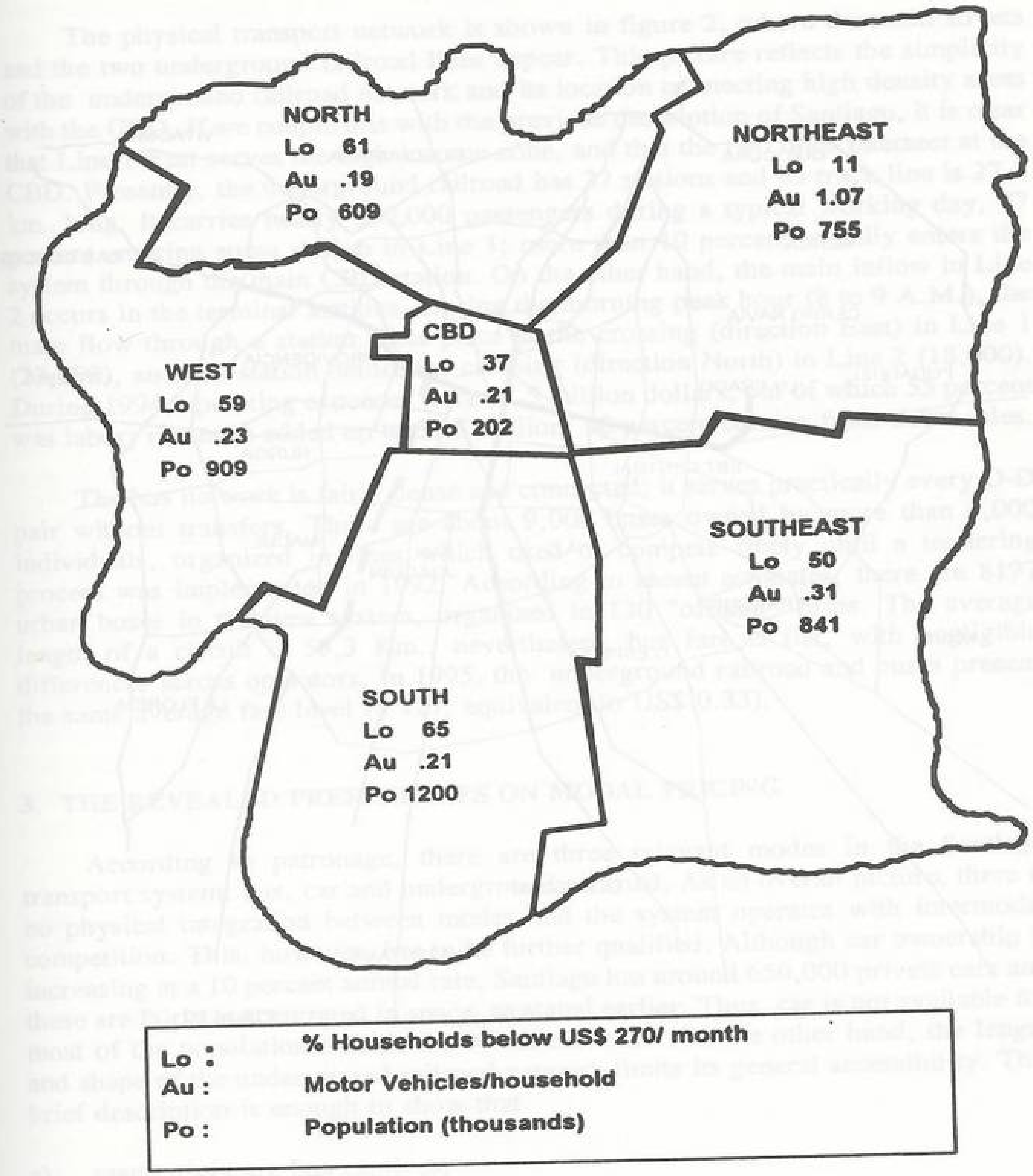
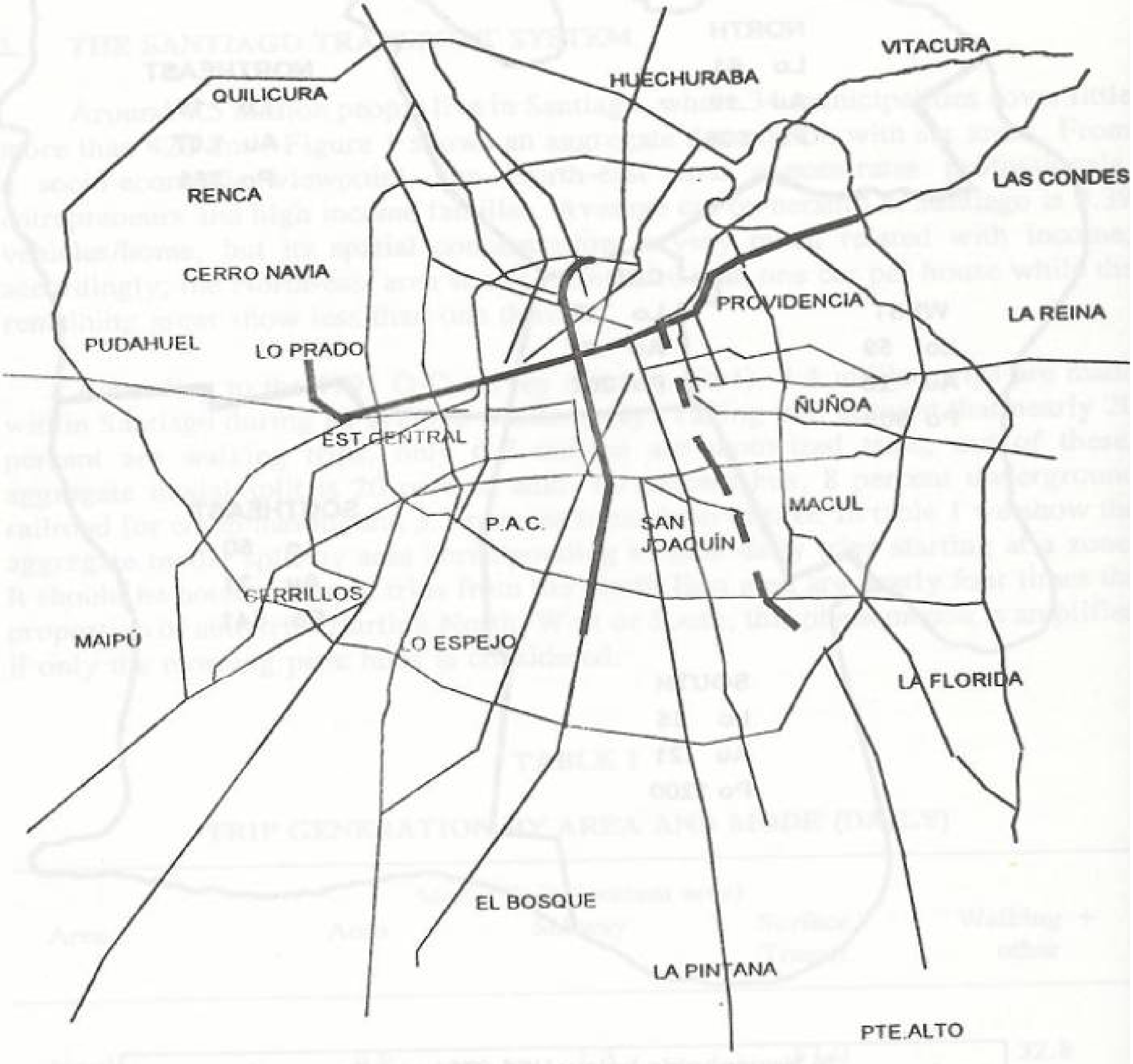


FIGURE 2
SANTIAGO: TRANSPORT NETWORK



Regarding trip purpose, generation is roughly evenly distributed across areas among work, study and others (excepting the Central Business District, CBD) in terms of daily trips.

The physical transport network is shown in figure 2, where the main streets and the two underground railroad lines appear. This picture reflects the simplicity of the underground railroad network and its location connecting high density areas with the CBD. If we couple this with the previous description of Santiago, it is clear that Line 1 East serves the high income zone, and that the two lines intersect at the CBD. Presently, the underground railroad has 37 stations and its track line is 27.3 km. long. It carries nearly 600,000 passengers during a typical working day, 77 percent entering some station in Line 1; more than 10 percent actually enters the system through the main CBD station. On the other hand, the main inflow in Line 2 occurs in the terminal stations. During the morning peak hour (8 to 9 A.M.), the main flow through a station takes place at the crossing (direction East) in Line 1 (27,000), and one station before the crossing (direction North) in Line 2 (18,000). During 1994, operating expenses were 37.5 million dollars, out of which 55 percent was labor; revenues added up to 55.3 million, 90 percent coming from ticket sales.

The bus network is fairly dense and connected; it serves practically every O-D pair without transfers. There are about 9,000 buses owned by more than 4,000 individuals, organized in lines which used to compete freely until a tendering process was implemented in 1992. According to recent estimates, there are 8197 urban buses in the new system, organized in 130 "official" firms. The average length of a circuit is 59,3 Km.; nevertheless, bus fare is flat, with negligible differences across operators. In 1995, the underground railroad and buses present the same average fare level ($ 127, equivalent to US$ 0.33).

3. THE REVEALED PREFERENCES ON MODAL PRICING

According to patronage, there are three relevant modes in the Santiago transport system: bus, car and underground railroad. As an overall picture, there is no physical integration between modes and the system operates with intermodal competition. This, however, has to be further qualified. Although car ownership is increasing at a 10 percent annual rate, Santiago has around 650,000 private cars and these are fairly concentrated in space, as stated earlier. Thus, car is not available for most of the population outside the North-east area. On the other hand, the length and shape of the underground railroad network limits its general accessibility. This brief description is enough to show that

a) many users are bus captives;
b) tri-modal competition is the rule for North-east users;
c) bi-modal competition seems to rank second for all other zones;
d) the underground railroad faces modal competition everywhere.

Viewed in isolation, each of the three relevant urban modes presents a very different picture regarding present pricing practice. Let us begin the discussion with the car case, which can be used also to explain a basic element in optimal transport pricing: the role of users' cost. In the "production" of trips, operators have to incur in costs that are related with the use of resources for which they pay, but a trip also requires a resource which is "paid" by the users: their own time. In the case of car trips (where users and operators coincide), the consumption of time for a given trip increases as the flow increases if congestion is present. This makes the marginal time (which accounts for the additional time induced by one car on the other cars) greater than the average trip time (e.g., the actual individual time); as marginal cost pricing is optimal, car users are "paying" less than what they should. If one can find the money equivalent of the difference and charge the user that amount, then that trip will be in fact priced at marginal cost; the money equivalent is simply the physical time difference (which can be calculated using known flow theory techniques) times the subjective value of travel time of the user (which can be calculated using known demand techniques). This is, in essence, congestion pricing.

Although congestion pricing has been raised as an issue since 1990[1], the fact is that the corresponding law has not been approved by the parliament. There has been a number of misunderstandings around this topic while being discussed in the political arena. Just as an example, congestion pricing is seen only as a device to reduce car trips. As every transport analyst knows, even if demand was absolutely inelastic, congestion pricing means charging for the unpaid external cost (time); this would induce the best use of available capacity resulting in the minimum total travel time for users as a whole. Nevertheless, and in spite of its rationality, car users are not being charged for congestion which means that they pay less than the marginal cost when making a trip by car under congested conditions. As will be seen later, this is particularly relevant during the morning peak hour for car trips starting either in the North-east zone (where most of the cars are) or in the South-east zone (where capacity is low).

If we turn to buses, we find small differences in fares across firms according to route, and even these are slowly vanishing. Thus, it is fair to say that the bus fare is flat and constant across periods and routes. Regarding costs, there are some sources of variation across firms; however, calculations have been made by the MTT for a firm serving the average circuit (59,3 km.), including all operating costs (labour, fuel, spare parts, maintenance, lubricants, administration, parking) and vehicle depreciation, which yields an average cost that nearly replicates the average fare. This figure, however, requires an estimate of daily patronage per bus, which

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[1] Although the Ministry of Transport and Telecommunications (MTT) was created by 1974, it was not until 1990 that it became a proper technical secretariat.

varies according to the source (i.e., the 1991 O-D survey or the bus owners association); we trust more the official information from the demand side which indicates 600 passengers per bus-day.

How far is the bus fare scheme from a socially optimum pricing structure? From the pioneering work of Mohring (1972) including users' inputs (time) in the economic analysis of bus services, it has been accepted that there are increasing returns to scale. Intuitively, it seems clear that a higher bus demand along a corridor induces higher frequencies and lower waiting times; similarly, if demand expands in space, the required bus network becomes more dense, reducing access time. As clearly shown by Jansson (1984) for scheduled transit systems, the optimal price should be equal to the difference between total marginal cost (operator plus users) and the users' average cost; under increasing returns, this optimal price falls below the operator's average cost (note that the reverse happens for cars, which provides the rationale for congestion pricing). In Santiago, the urban bus fleet is one hundred percent privately owned and, according to the explicit policy settled by the MTT during 1990, there is no intention to change the rules of the game; besides, subsidies are out of question. This means that prices can not follow the first-best rule, and a second-best approach could be appropriate.

As real bus ownership is so "atomized" in Santiago, we can not claim that the fleet adapts to demand variation in time, which means that the number of buses serving the different routes along the day does not vary significantly. In other words, most buses operate during as many periods as possible. Thus, it would be inappropriate to apply the results on second-best pricing for buses suggested in the literature, as they rest upon other assumptions. In fact, the simple average cost-like flat fare might be viewed as a type of second-best scheme which could be improved by charging more to long distance travellers due to the higher marginal cost to the operators. Thus, on average, the present fare level plus the average user cost is above total marginal cost, but the reverse might be true for long distance travellers.

Finally, the case of the underground railroad is extremely interesting, because it operates with an explicit policy regarding fares. Until 1990, the underground railroad company was owned by the State and administered as a public entity which was part of the Ministry of Public Works. From then on, it has operated as an autonomous enterprise, Metro S.A., which has two owners: Corfo, the ministry that promotes production, and the State itself. It is organized in practice as a private company administratively, but the declared objective is to maximise ridership providing good service at no operating losses. This means, for instance, to run trains with no more than four minutes headway even in the low demand periods within a working day. But, also, it means an adequate set of fares in space and time.

During 1986, the TOM system, an optimal pricing computer package, was developed in order to assist the pricing policy of the underground railroad. It was the first attempt to induce a rational attitude in that particular dimension. The main idea was to manipulate fares in order to take advantage of the different price elasticities in space and time. It was designed assuming both trip generation and distribution constant at a city-wide level. Thus, changes in modal split were the only effects considered, and fares were the only real policy variables, as level of service was given for the underground railroad and assumed as given for the remaining modes (a fairly realistic asumption due to the competitive tendering process for the buses). TOM was designed to find the optimal set of fares (one for each O-D pair and period) for various possible objectives, i.e., to maximize profits, welfare or ridership. In the second and third objectives, a budget constraint is imposed, which means that revenue should cover operating cost plus depreciation of equipment. The information needed to solve each problem analytically, includes an underground railroad O-D matrix, mode choice models for each O-D pair and period, level of service for all modes (for each O-D pair and period), fares for all other modes, fixed operating costs and depreciation, and estimates of marginal costs. The latter are calculated in an ad-hoc manner for each underground railroad line, assigning all items that can be identified with the operation of each one; only the contribution of energy consumption to expenses comes from an econometric model (Jara-Díaz and Valenzuela, 1985). It should be said that flow information in the underground railroad can be recovered at a very detailed level regarding entry to stations data; also, a well designed O-D survey is performed once a year with an enthusiastic response by underground railroad users. The analitics of the three problems depicted is explained to some detail in Jara-Díaz (1986).

Once the fares which maximize ridership covering costs (with each price greater than marginal cost) are calculated, the set is reduced in order to obtain a reasonable number of prices, including the existence of return and ten-trip tickets. As a result, the underground railroad now presents different fares according to period (low, medium and high demand), line (one and two) and type of ticket. The highest price is for the peak period, single trip ticket for line 1. The lowest price is charged in both lines before 7 A.M. an after 9 P.M. Note that the policy regarding minimum frequency makes the average user cost (passenger travel time) nearly equal to the marginal user cost.

In summary, the three main modes in Santiago are presently priced implicitly according to the rules schematically represented by figures 3, 4 and 5. Car trips are not charged for congestion, which means that the generalized price P_c is below the marginal cost m_c (figure 3). Bus trips are charged something similar to the mean average operating cost, but the generalized price is p_b (on average) above the marginal cost m_b; however, if marginal operating cost increases with distance, long distance trips might be underpriced (figures 4a and 4b). Finally, the underground railroad is roughly priced following a kind of second-best scheme, charging above marginal cost everywhere, with higher prices in the more inelastic markets (peak period, line 1), as shown in figure 5.

FIGURE 3
PRESENT IMPLICIT PRICING POLICY: CARS

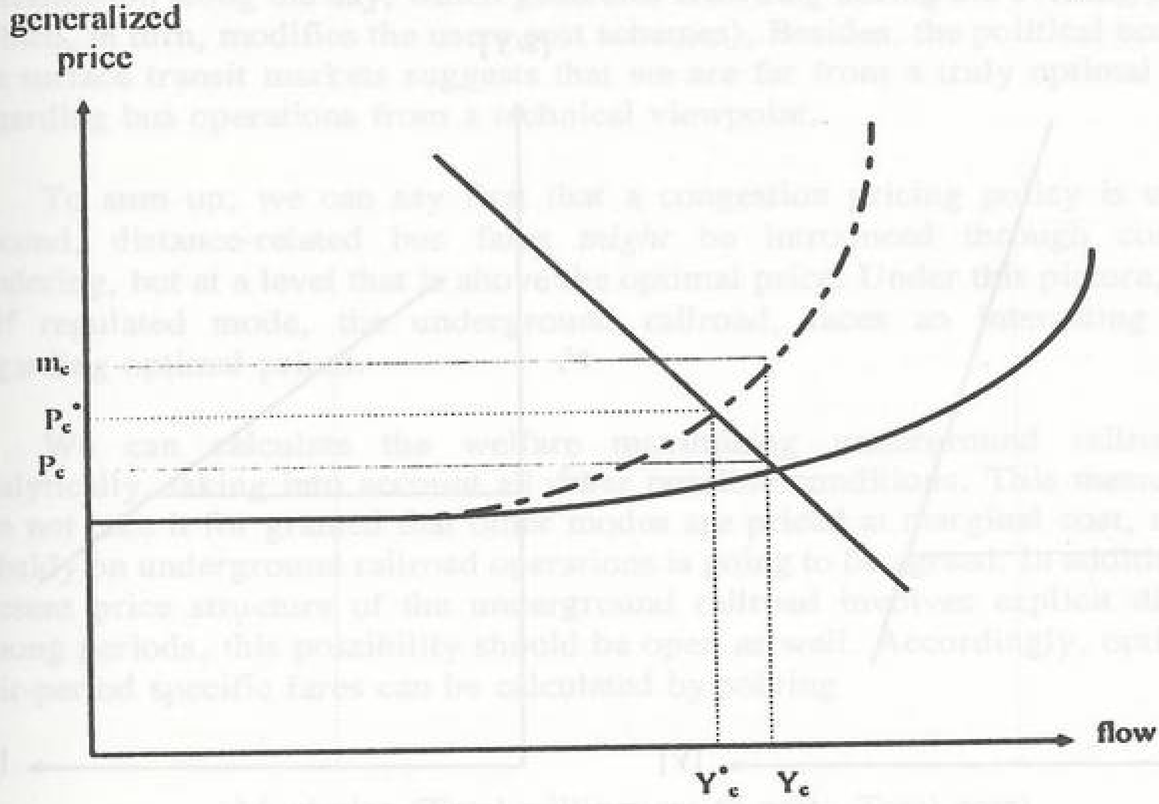
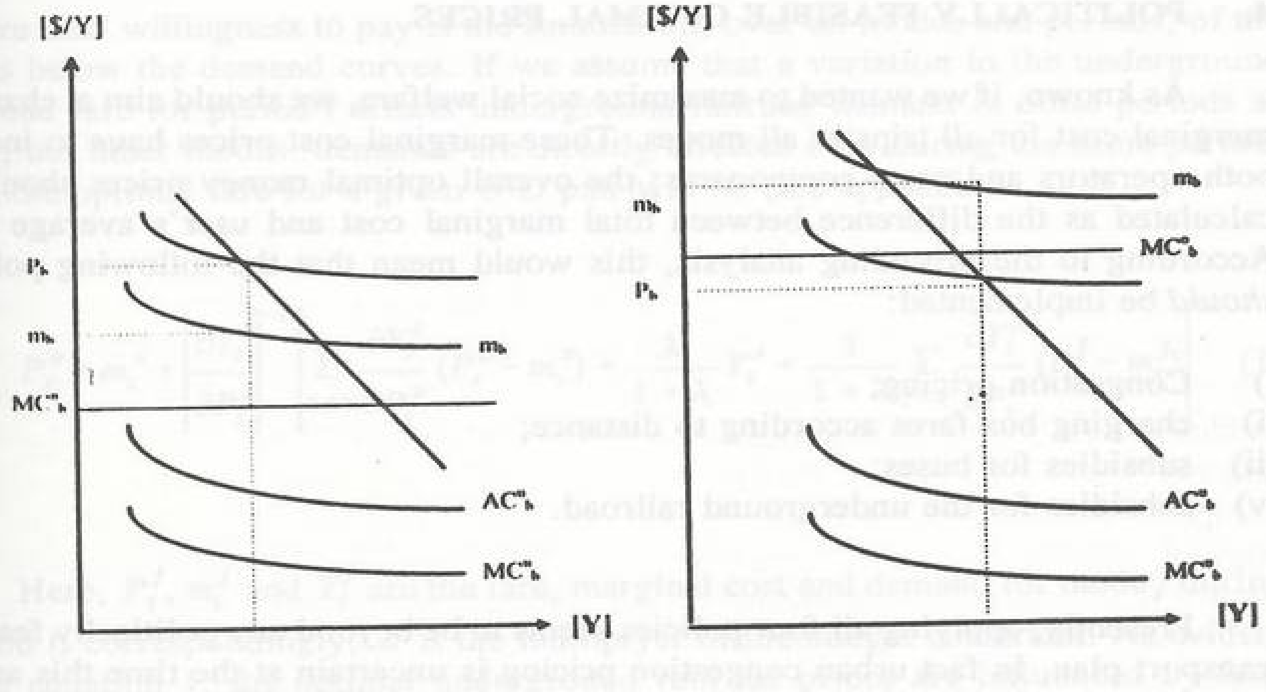
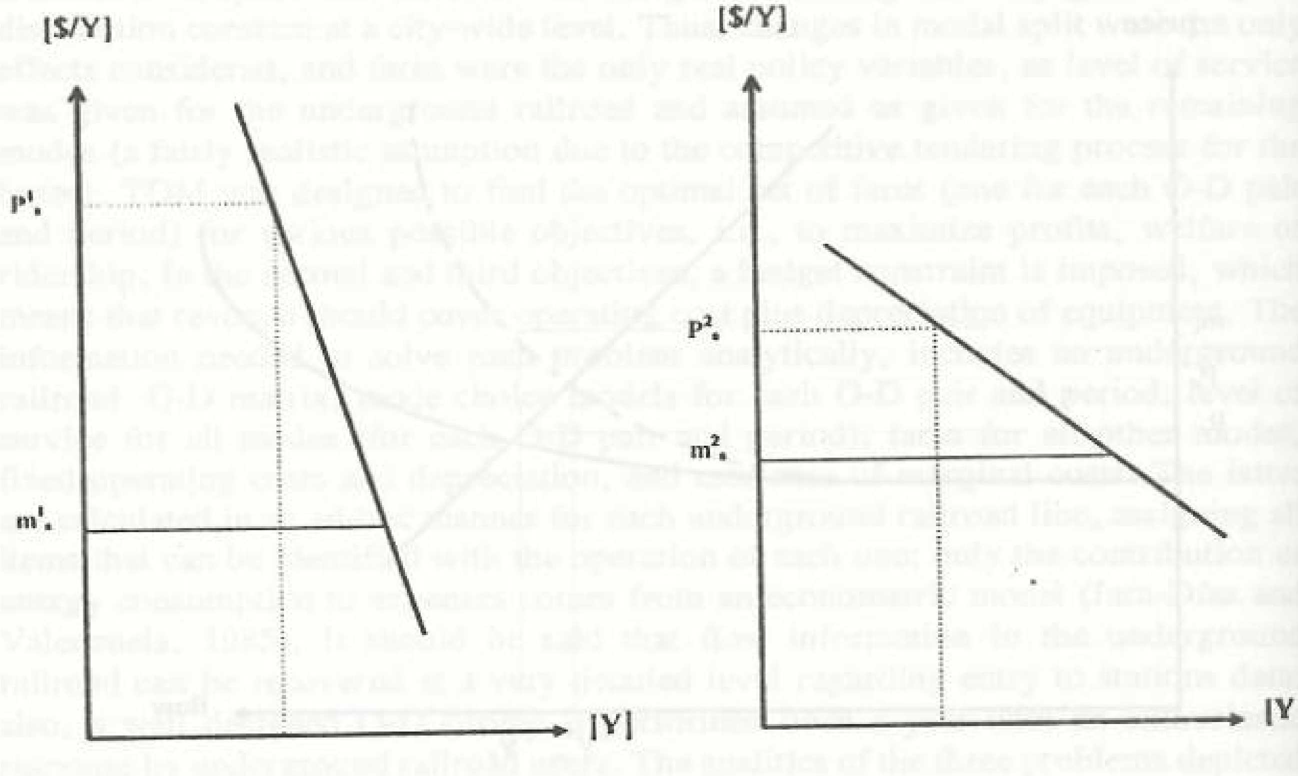


FIGURE 4
PRESENT PRICING POLICY: BUSES



a) Average distance trips   b) Long distance trips

FIGURE 5
PRESENT PRICING POLICY: SUBWAY



a) Line one, peak period.

b) Off-peak

4. POLITICALLY FEASIBLE OPTIMAL PRICES

As known, if we wanted to maximize social welfare, we should aim at charging marginal cost for all trips in all modes. These marginal cost prices have to include both operators and users components; the overall optimal money prices should be calculated as the difference between total marginal cost and user's average cost. According to the preceding analysis, this would mean that the following policies *should* be implemented:

i) Congestion pricing;
ii) charging bus fares according to distance;
iii) subsidies for buses;
iv) subsidies for the underground railroad.

Presently, applying all four policies seems to be beyond any politically feasible transport plan. In fact urban congestion pricing is uncertain at the time this article is being written, but certainly all sort of subsidies can be disregarded under the present political conditions. Regarding policy ii), it is difficult to fully understand the economics of bus operators, particularly because of the constant frequency phenomenon along the day, which generates crowding during the evening peak hour (which, in turn, modifies the users cost schemes). Besides, the political economy of the surface transit markets suggests that we are far from a truly optimal situation regarding bus operations from a technical viewpoint.

To sum up, we can say first that a congestion pricing policy is uncertain. Second, distance-related bus fares *might* be introduced through competitive tendering, but at a level that is above the optimal price. Under this picture, the only self regulated mode, the underground railroad, faces an interesting problem regarding optimal prices.

We can calculate the welfare maximizing underground railroad fares analytically, taking into account all other possible conditions. This means that we can not take it for granted that other modes are priced at marginal cost, nor that a subsidy on underground railroad operations is going to be agreed. In addition, as the present price structure of the underground railroad involves explicit differences among periods, this possibility should be open as well. Accordingly, optimal O-D pair-period specific fares can be calculated by solving

$$\underset{\{P\}}{\text{Maximize}}\ (\text{Total willingness to pay - Total cost})$$

subject to

$$\text{Revenue - Underground railroad cost} \geq 0\ ,$$

where total willingness to pay is the summation, over all modes and periods, of the areas below the demand curves. If we assume that a variation in the underground railroad fare for period *t* affects underground railroad demand in other periods as well, but other modes' demands are directly affected only during the same period, then the optimal fare for a given O-D pair will be (see appendix).

$$P_t^s = m_t^s + \left|\frac{\partial Y_t^s}{\partial P_t^s}\right|^{-1} \left[\sum_{r \neq t} \frac{\partial Y_r^s}{\partial P_t^s} (P_r^s - m_r^s) + \frac{\lambda}{1+\lambda} Y_t^s + \frac{1}{1+\lambda} \sum_{j \neq s} \frac{\partial Y_t^j}{\partial P_t^s} (P_t^j - m_t^j) \right] \qquad (1)$$

Here, P_t^j, m_t^j and Y_t^j are the fare, marginal cost and demand for mode *j* during period *t*, correspondingly; λ is the multiplyer of the budget constraint. As evident from equation 1, the optimal underground railroad prices are influenced by four components: underground railroad marginal cost, the effect on other periods, the need to cover costs, and the effect on other modes. All effects depend upon price elasticities of demand, and also on the corresponding differences between price and marginal cost. The former are zone and period specific and the latter depend upon the pricing policies in other modes as well. It should be noted that if prices were equal to marginal costs for all other modes and periods, then the inverse elasticity rule for period t would be optimal. On the other hand, if revenue was not required to cover costs, equation 1 would yield the general second-best scheme; lastly, overall marginal cost pricing would be the rule if no constraints were imposed.

What are the possible future scenarios regarding prices in the Santiago transport system? First of all, congestion pricing might be applied or not; both possibilities are open. Secondly, competitive tendering for buses is very likely to go on, but nothing has been mentioned in terms of distance related prices. Third, the underground railroad can handle prices within some limits, as the firm is subject to scrutiny by different ministries. In order to explore possible outcomes, we can take a normative viewpoint within the limits of key political decisions. Thus, we can imagine the future either with or without congestion pricing, but we can accept that buses will keep the average cost-like pricing policy. Under these circumstances, underground railroad prices can act as a modest regulator according to the optimal pricing scheme represented by equation 1. Alternatively, we can take the present underground railroad pricing policy as given, assuming that "something" can be imagined for bus fares. In what follows, we will examine only the first alternative.

Let us first analyze the optimistic scenario, i.e., the one with congestion pricing, which means moving from (P_c,Y_c) to (P_c^*,Y_c^*) in figure 3. This has two effects in terms of the optimal underground railroad fares according to equation 1: first, the last term vanishes for j = car, and second, car trips diminish with some impact on underground railroad demand. All other terms influence price in such a way that the underground railroad fares should be high during the peak periods and low during the off-peaks, mainly because of the impact of the own price demand elasticity, marginal cost and patronage. On the other hand, as the difference between bus price and marginal cost decreases with distance, the last term would tend to make the underground railroad prices lower for long distance trips; however, proper calculation of marginal costs for the underground railroad would yield higher figures for such trips, which suggests counterbalancing effects. When looking at trip origins, the three "legs" of the underground railroad can be assigned high, medium and low relative price elasticities when moving from West to South to East; accordingly, prices should increase in the same order.

Let us assume now that congestion pricing is not approved. Then the last term in equation 1 will include a negative figure for all those O-D pairs and periods which present congested conditions for car trips competing with underground railroad services; these include particularly the Northeast-Center corridors during both morning and evening peaks. The intuitive explanation is simple: as car trips are underpriced, more than optimal trips are made by car, which pushes optimal underground railroad prices down in order to counterbalance the undesired effect through modal competition. Thus, the scenario without congestion pricing induces a corrective effect on the optimal underground railroad pricing scheme depicted in the preceding paragraph. Although this result has been shown to be reasonable on efficiency grounds, it does not seem appealing in terms of equity, as it would mean to reduce the underground railroad fare for those users that have car and move on congested corridors (i.e., the wealthy users). However, this could be viewed as an undesired effect provoked by the lack of congestion pricing. Thus, if one wants efficiency and political feasibility, congestion pricing might be the preferred path.

Finally, a sophisticated second-best scheme, as represented by equation 1, might look too discriminatory among users, as it involves both own and cross price elasticities, which vary according to zone (income-related) and period (activity related). The fact is that a first-best scheme in transport does not avoid the problem, it only hides it. The main reason is that the relevant optimal pricing figure is operators plus users marginal costs, and these, in turn, depend upon subjective valuations of trip characteristics, as the value of time.

5. SYNTHESIS, DISCUSSION AND CONCLUSIONS

In Santiago, public transport produces more than three times the number of passenger trips which are made by car. Cars (and income) are unevenly distributed, mainly concentrated in the north-east zone; bus lines cover all the metropolitan area at a flat fare, and the two underground railroad lines serve the main east-west and center-south corridors. This is the synthetic picture of transport supply in Santiago, and is the background for a pricing policy that, until now, seems to be quite inefficient from an overall viewpoint: car trips are underpriced, buses seem to charge more than optimal, and underground railroad prices are rational within an internal logic only. But reaching a social optimum seems to require presently unfeasible political decisions.

The key aspect in the analysis of pricing within any urban transport system, is the concept of marginal social cost, which includes both user and operator components. As the average user cost can be roughly associated with the perceived money equivalent of travel time, the source of the difference with the marginal cost is the individual effect on the travel time of other users. Thus, the increasing interaction among vehicles as patronage increase makes the marginal user cost higher than the average user cost for cars; but the increasing connectivity and frequency of bus as the number of users gets larger, causes the opposite effect on scheduled transport services in general. As these differences have to be multiplied by the subjective valuation of time, and this generally increases with income, the underpricing of car trips can be in fact more relevant in money terms than the overpricing of buses. And this, of course, is more relevant where the cars are. Some figures have been estimated using present data with the Estraus model (Sectu, 1990), which yield optimal congestion prices of about $ 1500 per long distance trip from the north-east zone to the CBD. Estimating similar figures for bus trips is presently difficult, as it requires some idea on how the bus network would evolve according to patronage.

Actually, estimating operators marginal costs is a whole problem in itself. One could think that bus services would have to be extended in order to reach the more distant origins or destinations, increasing both cycle time and frequency (or bus size, or both); accordingly, marginal cost should increase with distance. The fact is that bus lines are fairly stable such that, in practice, they nearly resemble underground railroad lines. In this latter type of service, one can actually calculate the value of the additional resources required to reach one further station for a given track line, but it has not been done. Thus, both the form in which bus operators act ("en route" competition) and the level of service policy of the underground railroad, make proper estimation of O-D related marginal costs a special task; it is, however, a requisite to move on.

For a given infrastructure, there is an optimal transport pricing policy that induces the best use of available resources. Identifying that policy requires information on the political and operating constraints acting on each transport mode. An unrestricted optimum in Santiago would require congestion pricing and a dramatic change in surface transit. Under this scenario, further analytical and empirical work is required to calculate proper marginal costs for scheduled public transport services. On the other hand, if present operating conditions prevail, we will have to rely upon an adequate underground railroad policy, taking into account that a new line serving the south-east zone is going to be in operation soon; thus, the role of "modest regulator" that we have given to the underground railroad can turn into something more relevant than that. Under this second scenario, the research needed is even more demanding. As shown here, though, optimal fares for public transport without congestion pricing would result in an extremely unfair spatial structure, as the correction required would favor the high income areas.

Whatever the future of urban transport regulations is in Santiago, there is a wide open space to help assigning resources in a better way. It should be clear, however, that long run policies (i.e., infrastructure investment) are going to be efficient only if operating policies are efficient and prices are correct.

APPENDIX

OPTIMAL PRICING FOR A SELF-FINANCED SINGLE OPERATOR IN A MULTIPERIOD CONTEXT FACING MODAL COMPETITION

Let $C_l(Y^l)$ be the total cost of mode l as a function of product (flow vector) Y^l, with components Y^l_{kr} denoting flow in O-D pair k, period t.

Then the maximum welfare subject to a budget constraint is obtained solving

$$\underset{P}{Max} \sum_{l} \sum_{k} \sum_{r} \int_{0}^{Y^{l}_{kr}} P^l_{kr}(w)\, dw - \sum_{l} C_l(Y^l)$$

subject to

$$\sum_{k} \sum_{r} P^s_{kr} Y^s_{kr} - C_s(Y^s) \geq 0$$

where s stands for underground railroad. If we assume that

$$\frac{\partial Y^l_{ir}}{\partial P^s_{it}} = 0 \qquad \forall l \neq s, \forall r \quad and \tag{c}$$

$$\frac{\partial Y^l_{jt}}{\partial P^s_{it}} = 0 \qquad \forall j \neq i, \forall l \quad , \tag{d}$$

then the first-order condition for P^s_{it} is

$$\sum_{l} \sum_{r} P^l_{ir} \frac{\partial Y^l_{ir}}{\partial P^s_{it}} - \sum_{l} \frac{\partial C_l}{\partial Y^l_{it}} \frac{\partial Y^l_{it}}{\partial P^s_{it}} - \sum_{r \neq t} \frac{\partial C_s}{\partial Y^s_{ir}} \frac{\partial Y^s_{ir}}{\partial P^s_{it}} +$$

$$+ \lambda \left[Y^s_{it} + P^s_{it} \frac{\partial Y^s_{it}}{\partial P^s_{it}} + \sum_{r \neq t} P^s_{ir} \frac{\partial Y^s_{ir}}{\partial P^s_{it}} - \frac{\partial C_s}{\partial Y^s_{it}} \frac{\partial Y^s_{it}}{\partial P^s_{it}} - \sum_{r \neq t} \frac{\partial C_s}{\partial Y^s_{ir}} \frac{\partial Y^s_{ir}}{\partial P^s_{it}} \right] = 0 \tag{e}$$

which leads to

$$P_{it}^{s} = m_{it}^{s} + \left|\frac{\partial Y_{it}^{s}}{\partial P_{it}^{s}}\right|^{-1}\left[\sum_{r \neq t}\frac{\partial Y_{ir}^{s}}{\partial P_{it}^{s}}(P_{ir}^{s} - m_{ir}^{s}) + \frac{\lambda}{1+\lambda}Y_{it}^{s} + \frac{1}{1+\lambda}\sum_{l \neq s}\frac{\partial Y_{it}^{l}}{\partial P_{it}^{s}}(P_{it}^{l} - m_{it}^{l})\right] \qquad \text{(f)}$$

where m_{ir}^{l} is the marginal cost of mode l, O-D pair i, period t.

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