TECHNICAL NOTE:

A SIMPLE SEARCH MODEL APPLIED TO R&D*

Jaime E. Vatter**

I. INTRODUCTION

There are several papers that use the simple search paradigm to analyze R&D investment.[1] Here the firm invests a fixed amount each period, at the end of which, the value of the innovation is determined. Having this information available, the firm decides whether to stop the R&D project or not, i.e., to accept or reject what that period offers.

The purpose of this note is to describe the simple search model in the R&D cases, as well as to elaborate on some of the implications of the model. It is of particular interest to compare two frameworks for subsidies; one based on the cost of R&D and the other on the benefits obtained from it.

II. THE MODEL

In this section a simple model of R&D based on sequential search is elaborated. As explained earlier, R&D is viewed as a searching process in which the firm engaged in R&D will continue its program until an innovation of some value is found.[2]

The searching process can be described as follows. There is a single and infinitely lived firm that invests c dollars in an R&D project at the beginning of each period. R&D is the effort made by a firm to discover a new product or production process, or to improve an existing product or production process. Since the firm can analyze more than one project at a time, c is the total cost of

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[1] See Reinganum (1989) for a good review. Also see Reinganum (1982) and Telser (1982).

[2] I will focus on innovations or inventions only, the diffusion process is not considered here.

the specific project, which includes wages and other variables, and fixed costs. For simplicity sake, it is assumed that c is constant over time. At the end of each period the firm knows with certainty the "value" of the new (improved) product or process, which is called p_t.[3]

The "value" of the new (improved) product or process comes from a known distribution function. Note that the distribution function is not affected by the firm, however one can think that a greater effort in R&D might improve the distribution offer. This line of reasoning is not pursued here and is left for later research. Furthermore there is no learning involved in the process, however when there is, the model becomes more cumbersome, but in general the results do not change.[4]

The "value" p_t is defined as the net present value of all future cash flow associated with the technology at its current stage.[5] The firm follows the current p_t offer at any and all periods.

It is assumed that the firm can adopt any innovation at zero cost. Then the firm will adopt all innovations that have a value higher than the current technology. This means that recall is allowed. Then p_t^* is defined as the best offer after t periods or, similarly, the value of the actual technology. Note that p_t^* is non decreasing.[6] If $p_t \geq p_{t-1}^*$ the firm adopts the improved technology at its current stage. However, in order to decide whether to continue the project, the firm compares the best value of the innovation so far (p_t^*) with the expected net value after investing (searching) for one more period.

Therefore, the firm compares p_t^* with a point of reference, called the reservation value, here denoted by α. If the best value of the innovation at period t is higher or equal than the reservation value, then the firm will stop this R&D project. At any given moment the firm has to decide whether to adopt or stop. The rule is to adopt if $p_t \geq p_{t-1}^*$ and to stop if $p_t^* \geq \alpha$. To stop means to discontinue the R&D project on this specific product or process.

The simple model has the following assumptions:

A.1. One single firm, no entry threats, no patents.
A.2. Infinite horizon.

[3] In a different approach the firm does not know with certainty the value of the project. See Lippman and McCardle (1990).

[4] In Vatter (1992, Chapter 1) it is proved that under mild assumptions the results of an adaptive search model are the same than the ones obtained with the simple model.

[5] The NPV is net with respect to the production costs of the new (improved) good or the variable costs of the new (improved) process, not net with respect to the R&D costs.

[6] If $p_{t+1} > p_t^*$ then $p_{t+1}^* = p_{t+1}$, and if $p_{t+1} \leq p_t^*$, then $p_{t+1}^* = p_t^*$, therefore $p_{t+1}^* \geq p_t^*$.

A.3. The value of innovation in period t, P_t, is a non-negative random variable with known and stationary cumulative density function F.

A.4. The firm receives one "value offer" (hereafter "offer") p_t, which is a realization of P_t, per period.

A.5. $p_t \leq M < \infty$, for all t. The maximum value an innovation can take, M, is exogenous and constant during the R&D project.

A.6. The cost per period of the R&D program is c. It is paid at the beginning of each period and is constant over time.

A.7. Recall is allowed.

A.8. No adoption cost.

A.9. β, the discount factor is strictly less than one.[7]

Define V(x) as the maximum discounted expected value when the searcher has an offer x and follows an optimum policy thereafter. Then the functional equation that defines the problem can be written as follows:

$$V(p_t^*) = \max\{p_t^*, -c + \beta \int_0^{p_t^*} V(p_{t+1}) f(p_{t+1}) dp_{t+1} + \beta \int_{p_t^*}^{M} V(p_{t+1}) f(p_{t+1}) dp_{t+1}\} \qquad (1)$$

The intuition from the equation (1) is that the searcher has to compare its best offer so far with the net expected value of the innovation after one extra searching period. Zero adoption cost implies that recall is allowed, thus any offer below the value of the current technology is rejected. That is, if $p_{t+1} \leq p_t^*$ then $V(p_{t+1}) = V(p_t^*)$. Then (1) is rewritten as

$$V(p_t^*) = \max\{p_t^*, -c + \beta V(p_t^*) F(p_t^*) + \beta \int_{p_t^*}^{M} V(p_{t+1}) f(p_{t+1}) dp_{t+1}\} \qquad (2)$$

The reservation value is obtained from the equalization of the best offer and the expected value. Calling α the best offer that satisfies the equalization and realizing that $V(p_t) = p_t$ for all $p_t \geq \alpha$ and $V(p_t) = \alpha$ for all $p_t < \alpha$,[8] it results then that,

$$\alpha = -\frac{1}{1-\beta} c + \frac{\beta}{1-\beta} H(\alpha) \qquad (3)$$

where $H(\alpha)$ is a non-negative, decreasing and convex function[9] defined as follows

[7] The discount factor may be risk-adjusted. If this is the case there is no need to assume that the firm is risk neutral.

[8] This comes from the guessed solution, see Sargent (1987) chapter 2.

[9] $H'(z) = -(1 - F(z))$, therefore $H''(z) > 0$.

$$H(\alpha) = \int_{\alpha}^{M} (p_{t+1} - \alpha) f(p_{t+1}) dp_{t+1} \quad (4)$$

The intuition of (3), which is the fundamental equation of this problem, is that the total searching cost for one extra period is, at the optimum equal to the present value of the expected gain. Furthermore, it is worth to note that there are two types of cost for searching one extra period: c and the cost represented by the opportunity cost of the reservation value.

The searcher looks at the current offer p_t, then computes the best offer so far p_t^* and compares it with the reservation value in order to decide whether to stop or not. Then the optimum policy is to stop searching if and only if $p_t^* \geq \alpha$, for any $\alpha \geq 0$.[10] The firm invests in R&D as long as $p_t^* < \alpha$. Every period in which the value of the innovation is "large enough" the new (or improved) product or process is adopted. However, only when p_t^* is larger than the reservation value the firm stops the R&D project. As noted elsewhere, this searching policy exhibits the reservation value and myopic properties.[11] The reservation value property means that in order to decide whether to stop or not, the searcher compares the best value with this reference value. The myopic property, on the other hand, means that the best value is compared with the expected value of the innovation in the following period, net of the cost of searching. That is, the searcher carries on as if only one period were left.

These two properties in the optimum policy, reservation and myopic properties, can hold true independently of each other, that is there can be a policy which is myopic without the reservation property and vice versa.[12] For the myopic property is sufficient to have a stationary distribution, though not necessary.

The relevant variable to analyze is the reservation value. Understanding the determinant variables, makes the analysis of R&D possible. A higher reservation value implies that the firm invests more in R&D, since more periods are required before an innovation is adopted in expected value.[13] Also a higher reservation value implies a higher expected value for the innovation.

[10] $\alpha \geq 0$ is the necessary condition to initiate any R&D project.

[11] Lippman and McCall (1976) and Reinganum (1982), among others, show that when recall is not allowed these properties hold. It is well known that in the stationary case the recall assumption is innocuous, therefore in the recall case the properties mentioned in the text hold as well.

[12] See McMillan and Rothschild (1989).

[13] If N is the number of periods in which the firm invests in R&D, then $E[N] = 1/[1-F(\alpha)]$. E[N] is increasing in α.

Some comparative statistics can be designed from this simple setting. The effect of a change in the discount factor and in the cost of R&D are analyzed here. By differentiating (3):

$$\frac{d\alpha}{dr} \leq 0 \tag{5}$$

and

$$\frac{d\alpha}{dc} \leq 0 \tag{6}$$

Thus, an increase in the relevant interest rate implies a reduction in the reservation value; search will stop earlier and less R&D is undertaken. Note that a change in the interest rate does not modify the offer distribution and, therefore, the expected value of the offer is not affected. However, since the reservation value is lower, the probability of accepting an innovation of lower value is higher; consequently the expected value of the innovation is lower. The expected value of an innovation is $E[\mathbb{P}_t | p_t^* \geq \alpha]$, clearly decreasing in α. Hence, an increase in the interest rate reduces the expected length and value of the project. Also an increase in the cost of R&D will affect these variables negatively.

Therefore, in the simple model, R&D can be induced by subsidizing the current cost. What if there is a subsidy on the "prize"? That is, the government decides to subsidize every innovation. In this case, the new value of the innovation is a random variable $\mathbb{Q}_t = \mathbb{P}_t + s$, where s ($>0$) represents the subsidy per unit.[14] Consequently, at each achievement p_t increases by a fixed dollar amount. The subsidy affects the distribution of offers and induces, on the one hand, a higher reservation value, and thus more R&D, but on the other hand, a shorter expected length for the R&D project. To observe these effects, define G(q) as the distribution function of the best offers with the subsidy, then

$$G(q) = F[q - s] \tag{7}$$

Therefore $\mathbb{Q}_t$ dominates $\mathbb{P}_t$ stochastically in first order, since g(q), the density function, is to the right of f(p).[15] Both density functions are exactly equal, but with different location parameters. It is also known that for any non-decreasing function μ the following holds true

$$E[\mu(q)] \geq E[\mu(p)] \tag{8}$$

[14] The result also holds for a percentage subsidy.

[15] Proof: $\mathbb{Q} \sim G()$, $\mathbb{P} \sim F()$, then $\mathbb{Q}\ D_1\ \mathbb{P}$ iff $F(t)-G(t) \geq 0$, for all t, but $G(t)=F(t-s)$, then $F(t)-F(t-s) \geq 0$, which holds for all t, $s \geq 0$, since F( ) is increasing.

Define $\mu(x) = x - \alpha$, for all $x \geq \alpha$, and 0 otherwise, then $\mu(x)$ is non-decreasing. Then

$$E[\mu(q)] = \int_{\alpha}^{M}(q - \alpha)dG(q) = H_G(\alpha_G) \tag{9}$$

$$E[\mu(p)] = \int_{\alpha}^{M}(p - \alpha)dF(p) = H_F(\alpha_F) \tag{10}$$

(8), (9) and (10) imply that

$$H_G(\alpha_G) \geq H_F(\alpha_F) \tag{11}$$

and finally

$$\alpha_G \geq \alpha_F \tag{12}$$

That is, the reservation value is higher under the existence of the subsidy. But, as stated above, the expected length of the project is also affected by the change in the offer's distribution. This second effect is a result of the stochastic dominance as well. Since $G(x) \leq F(x)$, for all x, then for a given α

$$E_G(N) = \frac{1}{1 - G(\alpha)} \leq \frac{1}{1 - F(\alpha)} = E_F(N) \tag{13}$$

where N is the length of the R&D project. The intuition behind this result is in view that the offer's distribution is improved, the chance of obtaining an offer higher than α increases. Then this second effect implies that, for a given reservation value, the expected length of the R&D project decreases.

Therefore, there are two counteracting forces. For a given α the expected length of the R&D project goes down. However, α itself goes up implying an increase in the expected length of the R&D.

Lippman and McCall (1986) show that in this case, and when β is less than 1, the expected length of search decreases. That is, when the prize of the innovation receives a subsidy per unit, the investment in R&D decreases. However, the expected value of the innovation (before subsidy) increases, due to the rise in the reservation value. On the other hand, if the subsidy is a percentage of the innovation's value, the opposite result holds true. That is, the expected length of R&D increases.[16]

The comparison of these three frameworks is not trivial. In all cases the reservation value goes up and, therefore, the innovation's expected value goes up. However, it is not clear where among these three, the reservation value increases more. On the other hand, if α increases the same under the cost-subsidy case as in the prize-subsidy case, it is not clear which program will cost more. However, in the cases of lump-sum subsidies on returns, the expected length of R&D goes down, if urgency is required for the innovation, this alternative might be better than the subsidy on the cost of R&D.

To sum up, the simple model has straightforward results: (i) R&D can be analyzed as a problem in searching, (ii) there exists a stopping rule, and (iii) it is myopic. The firm establishes a reservation value and invests in R&D until the net present value of the innovation is at least equal to the reservation value, i.e., until the reward today is at least equal to the present value of the expected reward, net of search costs, of tomorrow's innovation. Similar to other searching problems, if there is an increase in the interest rate or in the cost of searching, then R&D investment is negatively affected. The analysis of different subsidy frameworks to induce more R&D yields an interesting result. If the time of the discovery is important, a lump-sum subsidy on the positive outcome may be more appropriate than a subsidy to the cost of R&D or a subsidy on the outcome based on percentage.

[16] Lippman and McCall (1986) state this result and Vatter (1992, Chspter 2) gives a formal proof.

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