Optim Lett (2012) 6:421–430
DOI 10.1007/s11590-010-0267-x

ORIGINAL PAPER

Bargaining and negative externalities

Sigifredo Laengle · Gino Loyola

Received: 21 May 2010 / Accepted: 17 November 2010 / Published online: 5 December 2010

Abstract Two important issues in distributive bargaining theory are, first, the conditions under which a negotiation breakdown occurs, and second, what and how source of parties' bargaining powers influences the properties of a possible agreement. Research based on classic Nash's demand game has explored both questions by sophisticating the original game a lot. As an attempt to deal with both issues under a simpler framework, we propose a modification of the Nash demand game in which bargainers suffer negative externalities proportional to the share of the surplus captured by their rival. It is shown that the negotiator experiencing a relatively high externality level has greater bargaining power and thus, appropriates a larger proportion of the surplus at stake. However, if externality levels are sufficiently high, bargaining powers become incompatible and a negotiation breakdown emerges from the bargaining process. We compare our results with the previous literature, and argue that they can be especially relevant in negotiations held under highly polarized environments.

Keywords Bargaining · Externalities · Nash demand game · High polarized negotiation

1 Introduction

Since the pioneering work of Nash [10], two important questions remain open in distributive bargaining theory. First, what are the conditions under which a negotiation

S. Laengle (✉) · G. Loyola
Department of Management Control, Universidad de Chile, Diagonal Paraguay 257, 8330015 Santiago, Chile
e-mail: slaengle@fen.uchile.cl

G. Loyola
e-mail: gloyola@fen.uchile.cl

breakdown can emerge as an outcome. Second, in case of reaching an agreement, what and how source of bargaining power of parties shape the properties of the deal. Indeed, in the original game proposed by Nash (the so-called Nash demand game), whereas disagreement is not possible, there is a multiplicity of equilibria that implies a continuum of agreements. Subsequent literature on negotiation games has explored both issues, but in general through the high sophistication of the original game or the proposition of a new game at all. In fact, this literature has offered answers to these questions by either introducing perturbations into the original game, adopting an incomplete information environment, or using a dynamic approach to model a distributive bargaining situation.

In an attempt to address both issues under an even simpler approach, this paper examines the impact of negative externalities on equilibrium properties of the classic Nash demand game. Specifically, we analyze how externalities between players can explain negotiation breakdowns, or, if an agreement exists, provide a more precise characterization of its properties. To this end, we construct a model of distributive negotiation in which bargainers suffer a negative externality proportional to the surplus captured by their rival.

The remainder of this paper is structured as follows. In Sect. 2, we compare the literature related to our work. In Sect. 3, a model of bargaining with externalities is setup, and the main properties of its equilibrium are examined. In Sect. 4, we discuss possible applications of our model to polarized conflicts. Finally, Sect. 5 concludes and outlines some extensions. The proof of our main result is contained in the Appendix.

2 Comparison to the previous literature

Previous articles also describe Nash demand games with a unique equilibrium, but achieve this either by introducing uncertainty into the game's information structure [10], extending the game to a multi-stage framework with a schedule of offers and counteroffers [11] or formulating an evolutionary version of the game [13].

Within this literature, the work that perhaps comes closest to ours is Muthoo [9], in which a two-stage negotiation game is proposed. At the first stage, both bargainers simultaneously adopt partial commitment tactics that consist of a share of the unit size cake that they would like to obtain in a subsequent distributive bargaining game. At the second stage, bargainers play the distributive negotiation game so that they can costly revoke their previous commitments. When the distributive negotiation stage is modeled as the Nash demand game, the paper establishes that commitments allow to select a unique equilibrium in this classic game under certain assumptions on revoking cost functions. Interestingly, its main insight on the equilibrium's properties has a similar flavor to ours in the sense that a first glance negotiator's weakness becomes eventually a stronger bargaining position. In particular, it is shown that a party's equilibrium share is increasing in his marginal cost of revoking a commitment and decreasing in that of his counterpart. Despite this similarity, our model exhibits important differences with the framework constructed by Muthoo [9]. These differences mainly rest on the presence of externalities in our bargaining game, which introduces an additional source of cross-player strategic behavior absent in the Muthoo's model. As a result, whereas disagreement is not possible in the Muthoo's work, we are able to characterize a negotiation breakdown under mild (and realistic) conditions. In fact, as our approach can be applied to highly polarized conflicts, a disagreement appears as a natural outcome in bargaining process conducted under this environment.

Although the present paper has much in common with all the research cited above, an important difference is that in our model, the presence of externalities is sufficient to attain a unique equilibrium *without* the need to adopt a dynamic approach. Furthermore, and in contrast to this literature, our framework is able to rationalize a disagreement as a bargaining outcome under realistic conditions.

In addition, the present work is directly related to the literature examining the impact of externalities on the equilibrium of negotiation games under complete information. All this literature uses, however, a setting that differs from our model in two crucial aspects. First, it considers a game with more than two players in which the seller wants to allocate a single indivisible good among several buyers. Second, the complete game is a dynamic one in which some player enjoys a first-mover advantage. Within this approach, the article closest to ours is Chowdhury [2], who develops a two-stage bargaining framework in which two potential buyers make simultaneous offers to a seller. Both buyers exhibit positive and symmetric externalities if the other reaches an agreement. In a couple of results reminiscent of our conclusions, he finds that disagreement is a *probable* outcome if externalities are large enough, and a unique equilibrium (a subgame perfect equilibrium) emerges if externalities are relatively small. In terms of welfare properties of the outcome, the main results of this model are as follows. On the one hand, when externalities are low, the payoff of each potential buyer is the same (the level of externality), irrespective of he is who reach the agreement or not. Although the seller's payoff is decreasing with the level of externality, it is positive. On the other hand, when externalities are high, the first-mover advantage is decisive for the eventual buyer to extract all the bargaining surplus from the seller. The buyer not getting the object obtains, however, a higher payoff due to the externality. Nevertheless, and contrary to our work, the symmetric-buyers assumption in Chowdhury's model prevents to predict who specific buyer will obtain a higher payoff or identify any source of bargaining power between them.

Other contributions [4,5] also study the role played by externalities among several buyers in negotiations held with a seller, but in a multiperiod context. Other differences with respect to Chowdhury [2] are the facts that in this class of models the first-mover player is the seller and asymmetric externalities are possible. Under this set-up, it is shown that large enough negative externalities can be a source of bargaining *delays* irrespective of the existence of a deadline. The framework of this literature is not able, however, to yield a complete bargaining disagreement, or alternatively, an indefinite delay.

Although more indirectly related to our work, there is another theoretical research that analyzes possible disagreements in the presence of externalities, but in a context somehow different from a bargaining game. This is the case of Jehiel et al. [6,7], who characterize the revenue-maximizing selling procedure in a one period version of the Jehiel and Moldovanu's set-up. Among other results, their model predicts that a sort of disagreement is possible as it may be optimal for the seller to keep the object whenever total externalities exceed all buyer's valuations.

Finally, the current paper has also connections to the literature on games with envy between players. For instance, Rusinowska [12] characterizes the Nash equilibria of a family of bargaining models in which parties can exhibit a jealous or friendly behavior. This paper shows that by considering players' attitudes towards their opponents' payoffs, the problem of multiplicity of equilibria existing in this family of models is, in general, eliminated or mitigated. Thus, her main result shares the same idea with our findings that envy (or externalities in our terminology) can play a role as a refinement process of Nash equilibria. Nevertheless, the framework of Rusinowska differs from ours in at least two relevant aspects. First, whereas her family of bargaining models is based upon the so-called *exact potential game*, the Nash demand game is not examined. Second, she models jealous and friendly behavior in a different way from that adopted by our approach. Under the Rusinowska's framework when a player has to compare two outcomes, his preferences are firstly driven by his own payoff. Then, he cares about his rival's payoffs only if his own payoff is the same in both outcomes. On the contrary, in our model a player prefers an outcome based upon a utility function that depends *simultaneously* on both his own and his opponent's payoff.

3 A model of negotiation with externalities

The previous section showed how the extant literature has addressed questions about conditions of a disagreement, or how bargaining powers influence the outcome of an agreement. We argue that the presence of externalities is sufficient to deal with both issues without complicating too much the Nash demand game. In the following two sections, we pose our model and discuss some applications to illustrate the power of the approach.

3.1 The model

Consider a modification of the Nash demand game [10] in which two bargainers, Emile and Frances, want to divide a surplus represented by a pie of size 1. Each player simultaneously chooses a demand on the surplus at stake, given by x (for Emile) and y (for Frances) such that $0 \leq x, y \leq 1$.

Both bargainers experience a negative externality proportional to the share of the surplus captured by their counterpart. As a consequence, the utility functions of Emile and Frances are given by $U_E(x, y) \doteq x - \gamma_E y$ and $U_F(x, y) \doteq y - \gamma_F x$, respectively. The constants γ_E and γ_F are non-negative ($\gamma_E, \gamma_F \geq 0$) and represent, therefore, the level of marginal externalities suffered by both players. The problems the bargainers face are defined as follows:[1]

Definition 1 Let $(x^*, y^*) \in \mathbb{R}^2$ be a *Nash equilibrium strategy pair* of the game. The strategies therefore satisfy

$$U_E(x^*, y^*) = \max\{x - \gamma_E y^* : x \geq 0, x + y^* \leq 1 \text{ and } x - \gamma_E y^* \geq 0\}$$

[1] Alternatively, the equilibrium problem presented here can also be generalized and formulated using variational inequality models (see for instance Chinchuluun et al. [1], Giannessi et al. [3] and Konnov [8]).

and

$$U_F(x^*, y^*) = \max\{y - \gamma_F x^* : y \geq 0, x^* + y \leq 1 \quad \text{and} \quad y - \gamma_F x^* \geq 0\}.$$

Each player's problem must satisfy three classes of constraints: a *feasibility* constraint ($x, y \geq 0$), an *agreement* constraint ($x + y \leq 1$), and a *participation* constraint ($x - \gamma_E y \geq 0$ and $y - \gamma_F x \geq 0$). Observe that given these constraints, the strategies must belong to the interval [0, 1], i.e., $x, y \in [0, 1]$. For instance, since Frances' strategy is $y \geq 0$, a feasible strategy for Emile satisfying the constraint $x + y \leq 1$ necessarily implies that $x \leq 1$.

The following proposition characterizes the Nash equilibrium strategy pair (x^*, y^*) computed on the basis of Definition 1.

Proposition 1 *The Nash equilibrium strategy pair* (x^*, y^*) *is given by:*

(a) *If* $\gamma_E\gamma_F > 1$*, the strategy pair satisfying the Nash equilibrium conditions does not exist.*
(b) *If* $\gamma_E\gamma_F < 1$*, then* $\gamma_E(1 + \gamma_E)^{-1} \leq x^* \leq (1 + \gamma_F)^{-1}$, $\gamma_F(1 + \gamma_F)^{-1} \leq y^* \leq (1 + \gamma_E)^{-1}$ *and* $x^* + y^* = 1$.
(c) *If* $\gamma_E\gamma_F = 1$*, the Nash equilibrium pair is unique and given by* $x^* = (1 + \gamma_F)^{-1}$ *and* $y^* = (1+\gamma_E)^{-1}$*, or equivalently,* $x^* = \gamma_E(1+\gamma_E)^{-1}$ *and* $y^* = \gamma_F(1+\gamma_F)^{-1}$.

Proof See Appendix.

3.2 Properties of the equilibrium

The Nash demand game is a special case of the distributive negotiation game presented here. This result derives from Proposition 1, part (*b*); other results are explained below with the help of Figs. 1 and 2.

The players' best-response functions are a rich source of analysis. In our model these functions are readily obtained from the proof of Proposition 1. Emile's best-response function is given by

$$R_E(y) = \begin{cases} 1 - y & \text{if } y \leq (1 + \gamma_E)^{-1} \\ \varnothing & \text{otherwise.} \end{cases}$$

Similarly, Frances' best-response function is

$$R_F(x) = \begin{cases} 1 - x & \text{if } x \leq (1 + \gamma_F)^{-1} \\ \varnothing & \text{otherwise.} \end{cases}$$

All strategy pairs (x^*, y^*), such that $x^* = R_E(y^*)$ and $y^* = R_F(x^*)$, belong to the set of Nash equilibria, as shown in Fig. 2.

On the one hand, when both negotiators experience sufficiently high externality levels, their bargaining powers become *incompatible* and negotiation breaks down. According to Proposition 1, part (*a*), this occurs when $\gamma_E\gamma_F > 1$. On the other hand,

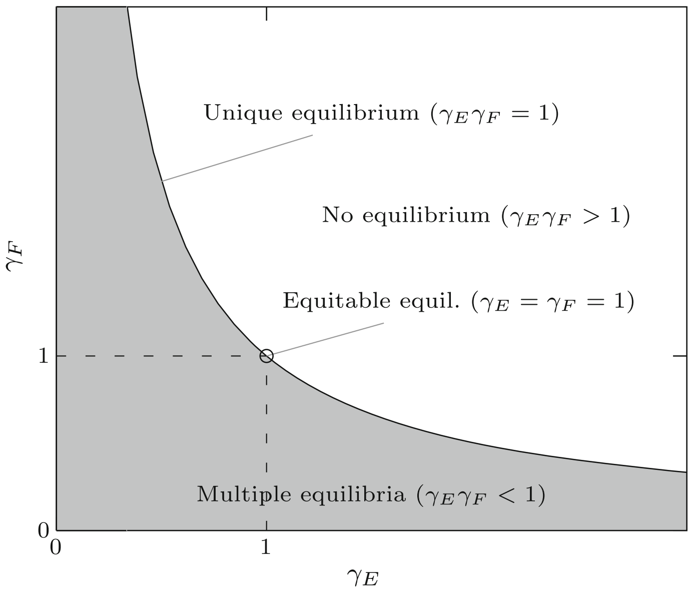

Fig. 1 Equilibrium and compatibility of bargaining powers based on externalities. The *shaded region* represents a compatible bargaining power relationship ($\gamma_F \gamma_E \leq 1$). On the *curve* ($\gamma_F \gamma_E = 1$) a unique equilibrium is attained. A particular perfectly equitable solution exists under symmetry ($\gamma_F = \gamma_E = 1$). A negotiation breakdown occurs if bargaining powers are incompatible because the externalities for both players are sufficiently high ($\gamma_F \gamma_E > 1$)

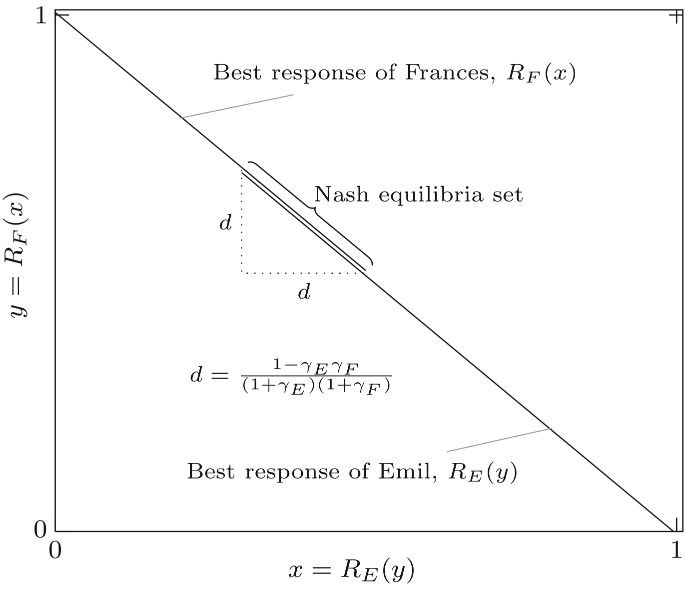

Fig. 2 Nash equilibrium in a bargaining model with externalities. When bargaining powers are compatible, ($\gamma_F \gamma_E \leq 1$), $d \geq 0$ and the Nash equilibria set is therefore not empty. If, on the other hand, bargaining powers are incompatible, negotiation breaks down

a *compatible* bargaining power relationship does exist as long as $\gamma_E \gamma_F \leq 1$, which in turn guarantees that an equilibrium exists and the players reach an agreement. Figure 1 clearly shows both results while Fig. 2 depicts the Nash equilibria set, which is empty if $d < 0$ or equivalently if $\gamma_E \gamma_F > 1$.

When bargaining powers are compatible, the set of equilibria is *smaller* than that of the original Nash demand game, as is established by Proposition 1, part (*b*). The set still lies on the line $x + y = 1$, however, as can be seen in Fig. 2.

When $\gamma_F \gamma_E = 1$ the agreement region is reduced to a single point and the equilibrium is therefore *unique*, as is demonstrated by Proposition 1, part (*c*). A particular case of a unique equilibrium is the *inequitable equilibrium*. The present game setting allows for an inequitable equilibrium if we assume an *asymmetric* structure of externalities, i.e., $\gamma_E \neq \gamma_F$. In this environment the most favored player is the one who suffers a larger marginal externality, that is, the one with the larger γ. Finally, another unique equilibrium case is the *perfectly equitable equilibrium* , which comes from assuming a *symmetric* externality structure, i.e., $\gamma_E = \gamma_F = 1$ with the Nash strategy pair taking the values $x^* = \frac{1}{2}$ and $y^* = \frac{1}{2}$.

4 Discussion on applications

The results derived here can be applied to real-world bargaining situations characterized by two features: (i) a highly polarized environment, and (ii) a delegated negotiation process.

To do that, one can think about conflicts in which high polarization—because ideological, religious, political, or racial causes—leads each party to exhibit some sort of *envy*, and thus, to experience a negative externality when its counterpart obtains some surplus. As in that situations bargainers frequently act on behalf of their respective parties, the parameters $\gamma' s$ can be re-interpreted in our model as follows. We can say that γ represents the penalty that one party imposes on its representative negotiator when some surplus is *given up* to her or his counterpart. This could occur if parties perceive any concession made by its delegated negotiator as the *treason* of a fundamental moral or political principle.

Examples of political delegated negotiations conducted under high polarization include, among others, peacemaking processes, disarming agreements, and political transitions (from dictatorship to democracy). In economics, conflicts of this nature can emerge from wage negotiations between management and labor unions, or international trade disputes between national governments.[2]

As a more concrete example where our framework may be applicable, consider the case of a government engaged in a disarmament process with a terrorist group. In this situation, some of the government's supporters may consider any concession to be an act of treason that violates a sacred moral principle. Hence, the government could suffer political costs from its supporters by means of electoral punishment, mass protest,

[2] Consider, for instance, the so-called *sugar war case*, a highly polarized trade dispute between Australian and Japanese companies that climbed up until their respective national governments (see Welch and Wilkinson [14]).

or lower popularity in opinion polls. Similar attitudes might be found among the more radical members of the terrorist group, who may impose a political cost—or even a human one—to the group's representative negotiator in response to any agreement arrived at.

It is important to stress that in highly polarized conflicts, a negotiation breakdown turns out to be a natural outcome of the bargaining process. Our model contributes, therefore, to provide the formal conditions under which this class of conflicts can face severe failures of negotiations. In the context of a delegated bargaining process, these conditions are also realistic as they establish that a negotiation breakdown can emerge if penalties that represented parties impose on their representative bargainers are too large.

5 Conclusions

The parsimonious extension of the classic Nash demand game developed in this paper allows us to obtain the following conclusions. First, as opposed to the classical distribution game without externalities, a negotiation breakdown is possible. This occurs if the level of externalities experienced by both parties is high enough so that relative bargaining powers become *incompatible*. Second, if bargaining powers are *compatible*, the agreement region is smaller than that attained without externalities. This reduces the degree of indeterminacy of the equilibrium in the classical Nash demand game. Third, we characterize the circumstances under which a unique equilibrium is fully equitable and identify the conditions that determine both the winning and the losing party in an asymmetric solution. In particular, it is shown that the party experiencing a relatively high externality level has greater bargaining power and, therefore, appropriates a larger proportion of the surplus at stake. Lastly, we further argue that our results can be especially applicable to highly polarized conflicts.

As for future work, an interesting avenue for further research would be to make the externality parameter endogenous in a stage previous to the distributive game itself. In this context, it would be worthy to model how parties strategically form its externality levels so that (dis)agreement conditions characterized in this article are met. In a related matter, it would also be interesting to extend our model to a dynamic game approach, which would allow us to account for the resolution path (or recurrent negotiation failures) observed in several real-world polarized conflicts.

6 Appendix

Proof of Proposition 1

Proof If (x, y) is a Nash equilibrium strategy pair, then x is an optimum of Emile's optimization problem and must therefore satisfy the KKT conditions. Moreover, since the problem is concave, x is a global optimum. In other words, there exist $\lambda_{1E}, \lambda_{2E}, \lambda_{3E} \in \mathbb{R}$ that satisfy the following properties: (1) stationarity:

$$\frac{d}{dx}(U_E(x, y) + \lambda_{1E}x - \lambda_{2E}(x + y - 1) + \lambda_{3E}(x - \gamma_E y))$$
$$= 1 + \lambda_{1E} - \lambda_{2E} + \lambda_{3E} = 0;$$

(2) primal feasibility: (2a) $x \geq 0$, (2b) $x + y \leq 1$ and (2c) $x - \gamma_E y \geq 0$; (3) dual feasibility: $\lambda_{1E}, \lambda_{2E}, \lambda_{3E} \geq 0$; and (4) complementary slackness: (4a) $\lambda_{1E}x = 0$, (4b) $\lambda_{2E}(x + y - 1) = 0$ and (4c) $\lambda_{3E}(x - \gamma_E y) = 0$. Analogously, y is an optimum for Frances' optimization problem. There then exist multipliers that satisfy equations (1) through (4) above in which the corresponding substitutions have previously been made (subscript F for E, y for x and x for y). We now compute the strategies x, y that satisfy the Nash equilibrium. *First*, let us assume that $\lambda_{2E} = 0$. Then $\lambda_{1E} + \lambda_{3E} = -1$ by (1), which contradicts (3). Thus, λ_{2E} must be not equal to 0, therefore $x + y = 1$ by (4b). *Second*, the equality $x + y = 1$ and the condition (2c) $x - \gamma_E y \geq 0$ prove that x and y must satisfy the conditions $x \geq \gamma_E(1 + \gamma_E)^{-1}$ and $y \leq (1 + \gamma_E)^{-1}$ respectively. Likewise in the Frances' case, since $\lambda_{2F} \neq 0$ implies $x + y = 1$, thus $x \leq (1+\gamma_F)^{-1}$ and $y \geq \gamma_F(1+\gamma_F)^{-1}$. In other words, x must be in the closed interval $[\gamma_E(1+\gamma_E)^{-1}, (1+\gamma_F)^{-1}]$ and y in the closed interval $[\gamma_F(1+\gamma_F)^{-1}, (1+\gamma_E)^{-1}]$. Given these results, it is not difficult to prove that the Nash equilibrium strategies exist if only if $\gamma_E\gamma_F \leq 1$. Indeed, if $\gamma_E\gamma_F \leq 1$ then $\gamma_E(1 + \gamma_F) \leq (1 + \gamma_F)$ and $\gamma_E(1 + \gamma_E)^{-1} \leq (1 + \gamma_F)^{-1}$, implying that Emile's interval strategy is not empty. Likewise, if we suppose that $\gamma_E\gamma_F \leq 1$, the interval solution for Frances is not empty either. *Third*, if $\gamma_E\gamma_F = 1$ and λ_{3E} and λ_{3F} are any values satisfying (3), then Emile's strategy is given by $x = \gamma_E(1 + \gamma_E)^{-1} = (1 + \gamma_F)^{-1}$ and that of Frances by $y = \gamma_F(1 + \gamma_F)^{-1} = (1 + \gamma_E)^{-1}$, i.e., the interval strategies are reduced to numbers x, y such that $x+y = 1$. *Fourth*, as we showed in the second step, the strategy intervals are empty if $\gamma_E\gamma_F > 1$ for any value of $\lambda_{3E} \geq 0$ and $\lambda_{3F} \geq 0$.

References

1. Chinchuluun, A., Pardalos, P., Migdalas, A., Pitsoulis, L. (eds.): Pareto Optimality, Game Theory and Equilibria. Springer, Berlin (2008)
2. Chowdhury, P.R.: Externalities and bargaining disagreement. Econ. Lett. **61**, 61–65 (1998)
3. Giannessi, F., Maugeri, A., Pardalos, P. (eds.): Equilibrium Problems: Nonsmooth Optimization and Variational Inequality Models. Springer, Berlin (2002)
4. Jehiel, P., Moldovanu, B.: Negative externalities may cause delay in negotiation. Econometrica **63**(6), 1321–1335 (1995)
5. Jehiel, P., Moldovanu, B.: Cyclical delay in bargaining with externalities. Rev. Econ. Stud. **62**(4), 619–637 (1995)
6. Jehiel, P., Moldovanu, B., Stacchetti, E.: How (not) to sell nuclear weapons. Am. Econ. Rev. **86**(4), 814–829 (1996)
7. Jehiel, P., Moldovanu, B., Stacchetti, E.: Multidimensional mechanism design for auctions with externalities. J. Econ. Theory **85**, 258–293 (1999)
8. Konnov, I.: On variational inequalities for auction market problems. Optim. Lett. **1**(2), 155–162 (2007)
9. Muthoo, A.: A bargaining model based on the commitment tactic. J. Econ. Theory **69**, 134–152 (1996)
10. Nash, J.F.: Two-person cooperative games. Econometrica **21**(1), 128–140 (1953)
11. Rubinstein, A.: Perfect equilibrium in a bargaining model. Econometrica **50**(1), 97–110 (1982)
12. Rusinowska, A.: Refinements of Nash equilibria in view of jealous or friendly behaviour of players. Int. Game Theory Rev. **4**(3), 281–299 (2002)

13. Young, H.P.: An evolutionary model of bargaining. J. Econ. Theory **59**(1), 145–168 (1993)
14. Welch, C., Wilkinson, I.: Network perspectives on interfirm conflict: reassessing a critical case in international business. J. Bus. Res. **58**(2), 205–213 (2005)