A tale of two elasticities: A general theoretical framework for the environmental Kuznets curve analysis

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ABSTRACT

We show that a variety of seemingly diverse concepts used to theoretically explaining the EKC have a common origin in two key preference and production elasticities. We also prove that they jointly correspond to a unique, underlying preference-technology theoretical framework.

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1. Introduction

According to the environmental Kuznets curve (EKC), the emissions or the concentrations of a particular contaminant in the environment initially rise as the income per capita of a country or a city increases over time, reach a maximum and then finally decline even though income per capita keeps growing.[1]

There is a significant number of theoretical models that have attempted to explain the EKC. The topic is popular because understanding and untangling the underlying mechanisms that explain the empirical occurrence of an EKC could allow assessment of the extent to which an EKC is automatic and/or policy induced. It might also reveal which regulatory or other public policy measures can speed up economic growth and/or bring down the external costs of growth.

However, the literature on the EKC is fragmented and lacks a common structure. The most widely accepted theoretical models that explain the EKC disguise their key explanatory parameters — the income-elasticity of marginal utility and the elasticity of substitution between factors — behind a variety of seemingly different concepts. We discuss these key parameters and, on the one hand, we unveil their common origin and, on the other, we prove

that they correspond jointly to a unique, underlying preference-technology theoretical framework. The theoretical generalization we present here is relevant and useful because it allows a better understanding of the interconnections between the different approaches and shows that EKC models based on preferences require weaker restrictions than models based on abatement technologies.

In Section 2, a general framework based on Lopez (1994) and Copeland and Taylor (2003) is presented. Section 3 shows that EKC models that impose restrictions on abatement technologies are particular cases of the general framework proposed. Section 4 concludes.

2. A general theoretical framework for the EKC

We present here a unified general framework for analyzing the theoretical literature on the EKC. Let $f = f(K, L)$ denote a standard production function or an "aggregator of the total conventional factors of production" (Lopez, 1994) where K is capital and L is labor. Let x be the level of pollution. In a static general equilibrium setting, revenue R is devoted exclusively to consumption C which is assumed to be a function of $f(\cdot)$ and x, i.e. $C = R(f, x)$. Thus, pollution is a factor of production. The social welfare function $\mu = \mu(C, x)$ is a conventional utility function that depends on both C and x. The optimal level of pollution x^* is given by the following first-order condition:

$$Z = \mu_1(C, x) R_2(f, x) + \mu_2(C, x) = 0$$

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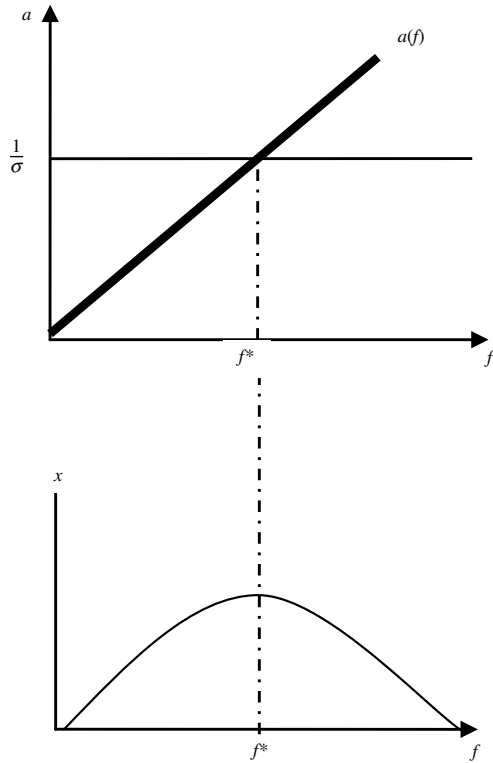[1] For empirical illustrations, see Dinda (2004).

Fig. 1. The EKC.

where μ_1 and μ_2 represent the marginal utility of consumption and pollution, respectively, and $R_2 = \partial R/\partial x$.

Under additive preferences we have that (see Lopez, 1994)[2]:

$$\frac{dx}{df} \geq (\leq) \, 0 \quad \text{if and only if } a \leq (\geq)\frac{1}{\sigma}, \tag{1}$$

where $a = -\mu_{11}C/\mu_1$ is the income-elasticity of marginal utility and $\sigma = (R_1R_2/R_{12}R)$ is the elasticity of substitution in production between factors f and x. According to (1), the sign of the income-pollution path depends on whether the income-elasticity of marginal utility a is above or below the threshold value given by the inverse of σ.

Lopez (1994) shows the particular case of a increasing in f and $1/\sigma$ constant such that there will be a unique point where a crosses from below the threshold value of $1/\sigma$ (see Figs. 1 here and in (Lopez, 1994, p. 172)).

Note however that it is also possible to obtain the same result if a is constant and $1/\sigma$ is decreasing in f. We now show that this is precisely the case in models with restrictions on abatement technology.

3. Models with abatement technologies

Unlike models based on preferences as described in Section 2, all models that generate the EKC based on abatement technology — with the exception of Andreoni and Levinson (2001) — rest on the assumption that a constraint is binding at high income levels. In these threshold models, at low levels of income, abatement is zero and thus pollution increases monotonically with income. At high income levels, abatement is positive and pollution decreases with income if some additional restrictions on preferences are imposed. In these models, unlike the model presented in Section 2,

pollution is a by-product of consumption. Moreover, total income $f(K.L)$ is used for consumption C and abatement A, i.e., the budget constraint is $C + A = f(K, L)$.

The pollution function is $x = x(C, A)$. Pollution increases with consumption C and decreases with abatement A; moreover, $x_1(C, A) > 0, x_2(C, A) < 0, x_{11}(C, A) > 0, x_{22}(C, A) > 0$ and — for simplicity — $x_{12}(C, A) = 0$.

Even though in this specification pollution is a by-product of consumption and not a factor of production, under some regularity conditions, it is possible to invert the pollution function $x = x(C, A)$, and convert it into a production function $C = R(f(\cdot), x)$ with pollution as a factor of production.

McConnell (1997) proposes a general model of abatement technology. In his model utility is given by $\mu = \mu(C, x)$, the pollution function is given by $x = x(C, A)$, and the representative consumer maximizes $\mu(C, x)$ subject to $x = x(C, A)$, the budget constraint $C + A = f(K, L)$ and $0 \leq C \leq f(K, L)$. Solving the consumer problem, the following first-order condition is obtained:

$$\mu_1 \geq \mu_2(x_2(C, A) - x_1(C, A)). \tag{2}$$

Income threshold f^* is obtained by solving (2) with equality and assuming $A = 0$

$$\mu_1(f^*, x(f^*, 0)) = -\mu_2(f^*, x(f^*, 0))(x_1(f^*, 0) - x_2(f^*, 0)).$$

For income lower than the income threshold, $f < f^*$, abatement is zero, consumption is equal to income, $C = f(K, L)$, and pollution grows monotonically with income according to the function $x = x(C, A) = x(f(\cdot), 0)$. Thus, the transformed function $C = R(f, x)$ is given by $C = R(f, x(f, 0))$. This is a fixed-proportions production function, and therefore the elasticity of substitution between pollution x and total conventional factors f is zero, $\sigma = 0$.

For an interior solution, the first-order condition in (2) can be written as:

$$-\frac{\mu_2}{\mu_1} = \frac{1}{x_1(C, A) - x_2(C, A)}. \tag{3}$$

Differentiation of (3) in x and f allows McConnell to express the slope of the income-pollution path as:

$$\frac{dx}{df} = -\frac{1}{\triangle}[x_1(-\mu_2 x_{22}) + x_2(-\mu_{11} - \mu_2 x_{11})] \tag{4}$$

where $\triangle$ is the negative determinant of the comparative statics matrix. Thus, pollution decreases with income, i.e., $dx/df < 0$, if the negative second term on the right-hand side outweighs the positive first term. We demonstrate below that within this condition there is implicit another condition which is sufficient in obtaining the decreasing part of the EKC shown in the theoretical framework proposed in Section 2.

If $\frac{dx}{df} < 0$, rearranging (4) it is possible to obtain:

$$-\frac{\mu_{11}}{\mu_1} > \frac{\mu_2}{\mu_1}\frac{x_1 x_{22} + x_2 x_{11}}{x_2}. \tag{5}$$

Using the first-order condition in (3), multiplying both sides of (5) by C, and recalling that the left-hand side of (5) times C defines the income elasticity of marginal utility a, it is possible to write:

$$a > -\frac{R}{x_1 - x_2}\frac{x_1 x_{22} + x_2 x_{11}}{x_2}. \tag{6}$$

Inverting the pollution function $x = x(C, A) = x(C, f - C)$ to obtain a revenue production function of the type $C = R(f, x)$ and calculating the elasticity of substitution between factors from this production function, it is possible to show that the condition in (6) collapses to[3]:

$$a > \frac{1}{\overline{\sigma}} \quad \text{with } \overline{\sigma} = -\frac{x_2(x_1 - x_2)}{(x_1 x_{22} + x_2 x_{11})R}.$$

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[2] Also see Copeland and Taylor (2003) for the case of $\sigma = 1$.

[3] $R_2 = \frac{1}{x_1 - x_2}, R_1 = -\frac{x_2}{x_1 - x_2}, R_{12} = \frac{x_1 x_{22} + x_2 x_{11}}{(x_1 - x_2)^3}$ and $\sigma = \frac{R_1 R_2}{R_{12} R}$.
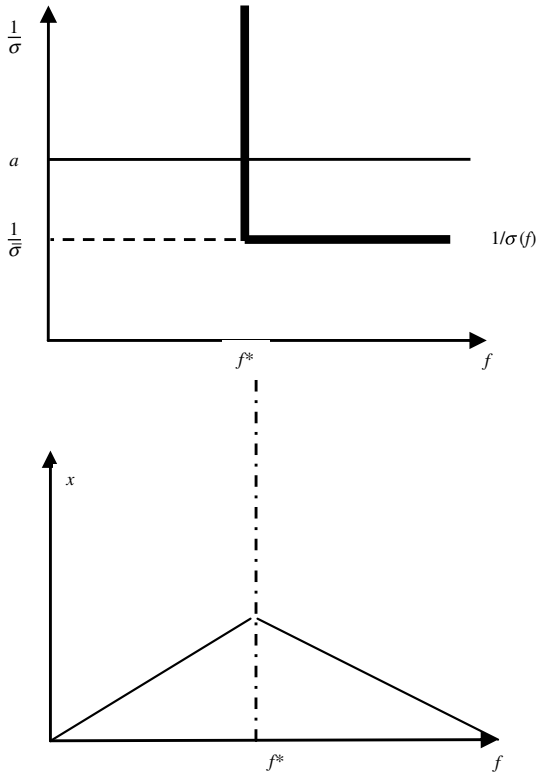
Fig. 2. McConnell (1997).

Fig. 3. Stokey (1998).

In summary, McConnell (1997) implicitly assumes a shift in the elasticity of substitution between conventional inputs and pollution σ such that

$$\sigma(f) = \begin{cases} 0 & f < f^* \\ \overline{\sigma} & f > f^*. \end{cases}$$

The pattern of $1/\sigma$ is illustrated in Fig. 2. From the figure it seems straightforward that a sufficient condition for the EKC to occur is $a > 1/\overline{\sigma}$ which is implicit in Eq. (4); thus McConnell (1997) is a particular case of the framework presented in Section 2.

Another example of a threshold model is Stokey (1998). In her model, similar to McConnell's model, pollution is a by-product of consumption and total income $f(K, L)$ is used for consumption C and abatement A, i.e., $C + A = f(K, L)$.

In Stokey's model, consumption is produced according to the function $C = zf(K, L)$, where $z \in [0, 1]$ and $f(K, L)$ is potential income. The choice of z translates into a choice of abatement expenditures via $A = (1 - z)f(K, L)$ and Stokey's pollution function is given by:

$$x = z^\delta f(K, L) = C^\delta f(\cdot)^{1-\delta} \tag{7}$$

with $\delta > 1$; thus, a higher z leads to more consumption and more pollution. Stokey assumes the following utility function:

$$\mu(C, x) = \frac{1}{1-a}\left(C^{1-a} - 1\right)$$
$$- \frac{B}{\gamma}(x)^\gamma \quad B > 0, a > 0, \gamma > 1. \tag{8}$$

If $z = 1$ then $A = 0$ and $C = x = f(\cdot)$; thus, the production function is given by $C = R(f, x) = \min(f, x)$ which is a fixed-proportions production function with elasticity of substitution between factors $\sigma = 0$. For $z < 1, A > 0$ and it is possible to invert the pollution function in (7) to obtain the production function
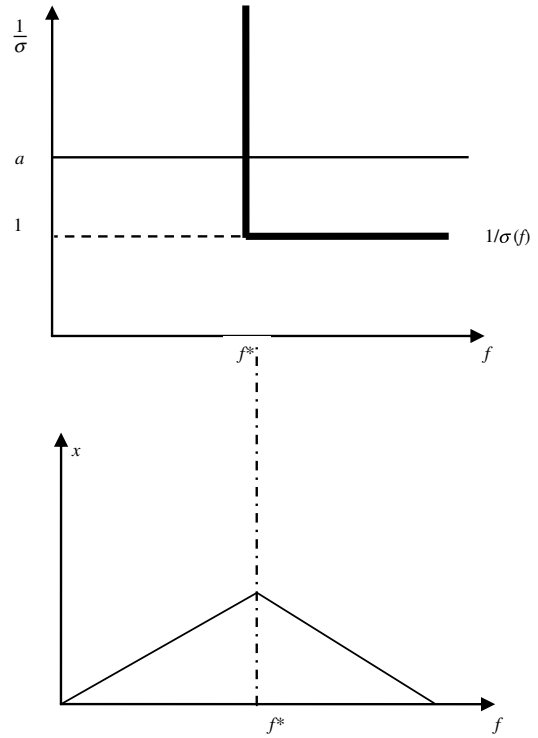
$C = x^\alpha f(\cdot)^{1-\alpha}$ with $\alpha = 1/\delta < 1$. This is a Cobb–Douglas production function with elasticity of substitution between factors $\overline{\sigma} = 1$, i.e. σ is given by

$$\sigma(f) = \begin{cases} 0 & f < f^* \\ 1 & f > f^*. \end{cases}$$

Looking at Fig. 3, it is straightforward to show that $a > 1$ is a sufficient condition to display the EKC. This is precisely the sufficient condition at which Stokey (1998) arrived.

Another interesting threshold model is Bousquet and Favart (2000). In their model utility is $\mu(C, x) = C^\alpha - x$ with $0 < \alpha < 1$, abatement technology is $x(C, A) = \gamma C - \delta A$ with γ and δ being positive parameters and the budget constraint is $C + A = f(K, L)$. The first-order condition is:

$$\alpha C^{\alpha-1} - (\gamma + \delta) \geq 0.$$

It is possible to show that a corner solution with $A = 0$ and $C = f(K, L)$ holds for any $f < f^* = ((\gamma + \delta)/\alpha)^{\frac{1}{\alpha-1}}$, thus, according to the pollution function $x(C, 0) = \gamma f$, i.e. pollution x grows monotonically with f. On the other hand, for income levels $f > f^* = ((\gamma + \delta)/\alpha)^{\frac{1}{\alpha-1}}$ the first-order condition holds with equality and pollution is given by $x = (\gamma + \delta)((\gamma + \delta)/\alpha)^{\frac{1}{\alpha-1}} - \delta f$ which is a decreasing function of f. Therefore, pollution describes the EKC (in fact it is an inverted V rather than an inverted U). For income lower than f^*, the production function is $C = R(f, x) = \min(f, (1/\gamma)x)$. This is a fixed-proportions production function with $\sigma = 0$. For income higher than f^*, the production function is given by: $C = R(f, x) = ((\delta f + x)/(\gamma + \delta))$, which is linear in f and x, and therefore it displays an infinite elasticity of substitution between factors, i.e.,

$$\sigma(f) = \begin{cases} 0 & f < f^* \\ \infty & f > f^*. \end{cases}$$

Note that the income-elasticity of marginal utility is $a = 1 - \alpha$, and — with reference to Fig. 4 — it seems evident that the sufficient
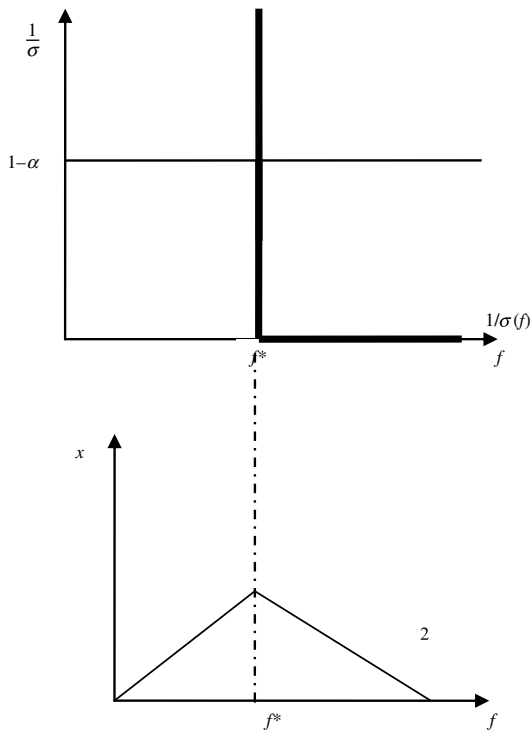
Fig. 4. Bousquet and Favart (2000).

condition to obtain the EKC is $1 - \alpha > 0$ which is satisfied by the assumption $\alpha \in (0, 1)$.

4. Conclusion

In this paper we analyze the effect of economic growth on pollution and show that despite the seemingly different concepts and assumptions used to explain the EKC, they all have a common origin and amount to a unique underlying theoretical framework. We show that every model imposes different restrictions on the trajectory — as income grows — of two well known parameters: the income elasticity of marginal utility a, and the elasticity of substitution between pollution and conventional factors of production σ.

We show that models based on preferences require restriction on a, while models based on abatement technologies require restrictions on both σ and a. Thus, we conclude that in order to explain the empirical occurrence of the EKC, theoretical models based on preferences require weaker restrictions than models based on abatement technologies.

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