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Contract market power and its impact on the efficiency of the electricity sector



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HIGHLIGHTS

- The paper analyzes the pro-competitive impact of contracts for difference.
- The reference price of contracts is the average spot price.
- Installed capacity increases with total quantity of energy contracted.
- Social welfare is maximized when energy contracted equals the efficient capacity.
- An aggregation of all consumers would choose to auction the efficient quantity.

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ABSTRACT

This paper analyzes the pro-competitive effects of financial long-term contracts in oligopolistic electricity markets. This is done in a model that incorporates the main features of the industry: non-storable production, time-varying price-elastic demand, and sequential investment and production decisions. The paper considers contracts for difference that have as reference price the average spot price. Assuming that the spot market coordinator sets competitive prices, the paper shows that installed capacity increases with the quantity of energy contracted, reaching the welfare-maximizing capacity when energy contracted equals this same level. Next, the paper studies the case where the quantity of energy contracted is endogenous and contracts are traded before capacity decisions are taken. Regarding purchasers of contracts, two polar cases are considered: either they are price-taker speculators or they are an aggregation of consumers that auctions a long (buy) contract for a given energy quantity. In the former case the strike price equals the reference price, i.e., arbitrage is perfect, and the quantity of energy contracted falls short of the efficient level. In turn, in the latter case, the strike price equals the average efficient spot price. Moreover, an aggregation of all consumers would choose to auction the social optimum quantity.

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1. Introduction

This paper discusses the impact of introducing a contract market in the efficiency of the electricity sector using a model that considers the main features of the industry: a non-storable product, a time-varying and price-elastic demand, and the sequential nature of investment and production decisions.^{1,2} These characteristics of electricity markets, particularly the first two, impose the need to balance demand and supply in real-time. Indeed, even if almost all consumption were purchased in forward markets, a mechanism to handle supply and demand short-run

deviations from contracts would still be required. This paper assumes that this mechanism is a spot market, which is the choice of most countries that have liberalized their electricity sectors.

Given the concentrated nature of electricity markets, the spot market is susceptible to non-competitive pricing by generators when demand is at or near its peak. In fact, given that, in the short run, capacity is fixed and no inventories are available, producers have incentives to withhold capacity.³ In those markets where the regulation mandates holding day-ahead auctions to receive supply offers, generators tend to bid above marginal cost.⁴ These behaviors

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E-mail addresses: pserra@econ.uchile.cl, serra.banfi@gmail.com¹ Since the paper focuses on power generation, transmission and distribution activities are omitted.² A fourth feature, which is in part a consequence of the other three, is that the installed capacity typically includes units with different technologies.³ Kwoka and Sabodash (2011) found that strategic withholding of production occurred in the New York system during the summer of 2001 and resulted in unusually high prices.⁴ Von der Fehr and Harbord (1993), assuming that the coordinator of the spot market holds day ahead supply auctions, found that in high-demand realizations (all suppliers needed to cover demand) prices exceed the cost of even the most inefficient supplier.

are fostered by the lack of demand participation in the spot market. Although the idiosyncratic characteristics of the electricity sector combine to produce significant fluctuations in spot prices, retail prices remain constant in the short and even in the medium term in most electricity markets.

The literature broadly concurs that the policy response to market power in the electricity sector should combine the introduction of contract markets and demand participation in the spot market.⁵ The intuition is that producers have less incentive to raise the spot price if part of their production is sold prior to the spot market clearing.⁶ The empirical evidence supports the view that forward contracting has a pro-competitive impact on those markets (Fabra and Toro, 2005; Bushnell et al., 2008; Petrella and Sapio, 2011). In addition, a greater demand participation in the spot market would result in higher price elasticity, the critical factor in determining market power.

Although demand participation in the spot market still confronts some practical difficulties, recent technological developments known as smart grid – which allow for bi-directional flows of information in the grid – could aid consumers to manage their electricity needs more efficiently. For instance, smart switches could turn on/off high-consuming appliances depending on real-time prices. There is thus a real possibility of increasing demand participation in the spot wholesale market.⁷

We assume demand participation in the spot market and concentrate our analysis on the impact that contracts have on the industry's efficiency. This effect depends on the degree of market power exercised by generators on the contract market. We thus focus the attention on the effect that market power on contract trading has on the industry. To do so, we assume competitive pricing in the spot market because it simplifies the analysis without detracting from the main objective. Moreover, this condition holds in a number of real markets. Indeed, legislation in a number of countries, especially in Latin America, entrusts the spot market coordinator to set competitive prices.⁸

Our analysis is formalized first in a two-stage game with exogenous contracts. In the first stage, each generator decides on its capacity by taking its rivals' capacity as given and considering standing contracts. In the second stage, the spot market coordinator sets the competitive spot price for each time segment. Thus, the model considers the variability of demand and the non-storability of the product (instantaneous clearing of the spot market), and the sequential nature of investment and production decisions (the two-stage game nature of the model).

The paper further assumes that parties trade two-way contracts for difference (*CfDs*), where the reference price is set equal to the average spot price. Thus, generators pay (or are paid by) their counterparties the difference between the reference price and the strike price times the quantity contracted. Accordingly, generators that sold supply contracts have incentives to lower the reference price by installing more capacity. In addition, all firms are identical, all parties have perfect foresight, and contracts are observable and enforceable. For simplicity, discount factors are ignored.

Within this framework, the paper shows that the Nash equilibrium aggregate capacity is increasing in total quantity of energy

contracted and that the welfare-maximizing level is reached when the quantity contracted equals the welfare-maximizing aggregate capacity. Consequently, social welfare is increasing in contracted energy as long as it does not exceed the welfare maximizing capacity.

The analysis then turns to the case where the quantity of energy contracted is endogenous and contracts are traded before capacity is committed. Two polar market structures are examined; first, the counterparty of generators is a competitive fringe of speculators; second, the counterparty is an aggregation of consumers that auctions a long (buy) contract for a given energy quantity. For the former case, this paper shows that the emergence of a contract market curbs but does not eliminate the exercise of market power by generators. In fact, introducing a *CfD* market increases the industry's capacity, but not by enough to reach the social welfare maximizing level. Accordingly, the emergence of a contract market lowers spot prices (in those time segments where capacity is binding), but not to their efficient levels. Moreover, arbitrage is perfect, i.e., the strike price equals the reference price.

The paper then addresses the case where an aggregation of consumers awards a long (buy) *CfD* contract in a sealed first-price auction. In this situation, the strike price equals the average efficient spot price, reflecting the fact that generators are price-setters in the contract market. Hence, those consumers who participate in the auction benefit both from a reduction in the spot prices, as do all other consumers, and from the auction of the contract. Furthermore, the paper proves that an aggregate of all consumers would auction a contract for the quantity that ensures welfare maximization.⁹

Thus, the effect of contracts on the efficiency of electricity markets hinges both on the structure of the contract market and on the sequencing of investment and contracting decisions. In fact, in this paper, regulated spot prices depend solely on capacity and time demand, neither of which can be modified by contracts traded after capacity decisions are taken. To achieve the efficient solution, contracts have to be settled before investments are committed and generators must be price-takers in the contract market.

Long-term contracts, i.e., contracts that are awarded before capacity is committed, have been implemented in a number of countries. For instance, regulations in Brazil and Chile require distribution companies to auction contracts to supply energy at least 3 years ahead of the delivery date (Moreno et al., 2010). Demand requirements are auctioned with supposedly enough lead-time to allow for the entry of new firms and for existing ones to expand their capacity. Distribution companies auction on behalf of their consumers given that there is a pass-through of contract prices to end-consumers. There is also evidence that large energy consumers auction their energy supply with enough anticipation to let bidders build new capacity if necessary.

This paper builds on the pioneering work of Allaz and Vila (1993), who modeled the interactions between a contract market and a spot market of an oligopolistic industry in a two-stage game. In the first stage, firms and competitive speculators trade contracts that close in the second stage; in stage 2, given standing contracts, firms compete à la Cournot in the spot market. Firms have constant marginal costs and do not face capacity constraints. Within this context, they find that forward markets have a pro-competitive impact on the spot market.¹⁰

⁵ See, for instance, Borenstein (2005) and Joskow (2008).

⁶ Firms end up worse when they all trade in the forward market. However each generator has incentives to trade forward because, by moving first, it gains a strategic advantage in the spot market (Allaz and Vila, 1993).

⁷ Moreover, halfway solutions such as time-of-use pricing have been put into practice by quite a few countries.

⁸ As shown by Castro-Rodríguez et al. (2009), among others, in an oligopolistic industry with a regulated spot market, producers can still exercise market power by investing below the social optimum level.

⁹ We have derived similar results in a working paper that considers a single-period and uncertain supply.

¹⁰ Bushnell (2007) extends the work of Allaz and Vila (1993) by introducing increasing marginal costs. Calibrating the model with parameters of existing electricity markets, he concludes that, when forward contracts are present, the

Recalling Kreps and Scheinkman (1983), the second stage in Allaz and Vila (1993) might be seen as the synthesis of two phases; while in the first phase firms decide on their capacities, in the second they compete à la Bertrand. Thus, the results on the Allaz and Vila paper seem appropriate for studying forward markets that take place before the investment decisions are made such as those addressed by this paper. Mahenc and Salanie (2004) in turn, assuming differentiated Bertrand competition on the spot market, find that producers may benefit from softening competition by taking long positions in the contract market. The Bertrand conjectures in the second stage better reflect the impact of forward markets that occur after investments are committed.

In many countries legislation confers the responsibility of holding next-day supply offer auctions on the coordinator of the spot market. Two competing approaches have been used to characterize these auctions: the supply function model by Klemperer and Meyer (1989) and the discrete-bid auction model by von der Fehr and Harbord (1993). Newbery (1998) and Green (1999) use the former approach. Newbery, assuming that generators behave as Cournot players in the contract market, finds that contracting makes the spot market more competitive. Green (1999) in turn, using linear marginal costs and assuming competition à la Bertrand in the contract market, shows that generators will set prices equal to marginal costs. de Frutos and Fabra (2012) relying on the latter approach, find that forward contracts are generally pro-competitive, but might have anti-competitive effects if awarded to firms that have little market power.

Murphy and Smeers (2010) introduce a three-stage game that separates investment decisions from production decisions. In the first stage, firms decide their capacities anticipating their impact on the forward and spot equilibriums. In the second stage, the forward-market equilibrium is found, given the capacities and taking into account the ensuing spot equilibrium. Finally, the spot-market equilibrium is derived given the capacities and forward positions. Firms behave like Cournot players in the three markets (capacity, contracts, and spot). Within this framework, these authors show that with demand certainty a forward market does not affect market power, while with uncertain demand a forward market can enhance or mitigate generators' market power.¹¹

Arellano and Serra (2010) also model a three-stage game, but they assume that investments are committed after contracts are awarded and that consumers, not firms, are the quantity setters in the contract market. Further, they model a two-technology industry (baseload and peak technologies) and a price inelastic load duration curve. In the first-stage, a fraction of the load curve is auctioned, while, in the second stage, firms decide on their investment. In the third stage, peak-load pricing is used to set energy and capacity spot prices. Within this context, they show that the auction leads firms to invest more in baseload capacity, reducing the average marginal cost of energy and, consequently, the spot prices of energy.

Murphy and Smeers (2012) construct a model where two firms with differing technologies invest in a first stage, contract part of their production in the second stage and sell the rest in the spot market that takes place in the third stage. Firms behave as Cournot players in all stages. A price elastic demand function for each time segment of the load curve is considered. They find that anything

can happen in terms of investment in their model, a result that should also hold in more complex models. They conclude that regulators or competition authorities cannot rely on contracts to induce sufficient capacity expansion by reducing market power and, consequently, other approaches to mitigating market power need to be developed. Their results reinforce the idea that short-term contracts do not necessarily reduce market power in oligopolistic industries.

The rest of this paper is organized as follows: Section 2 presents the model, Section 3 considers the impact of vested *CfDs* on the spot market, Section 4 studies the emergence of a long-term contract market, Section 5 analyses the effects of auctioning a long *CfD* by an aggregation of consumers, and the final section concludes and addresses future work.

2. The basic model

The supply side of the model considers a single, linear cost technology, where investment and production decisions are made sequentially. The unit operating cost is denoted c and the unit capacity cost r . Plants can adjust their production instantaneously and without costs. By normalization one unit of capacity produces one unit of electricity. In what follows K denotes the total installed capacity of the industry.

Demand is price elastic and time varying. In order to keep the model manageable, demand is assumed to be linear and cross-time price elasticities are ignored. Thus, demand at time segment t is given by the expression $q(t) = a(t) - p(t)$, where $p(t)$ denotes the price and $a(t)$ the highest price consumers would pay for electricity,¹² both for time t . Time demand is rearranged in decreasing order and total time is normalized to 1. Thus, t corresponds to the time segment with the t -th highest demand, and consequently $a(t)$ is a decreasing function that attains its maximum at time 0 and its minimum at time 1. In order to simplify mathematical proofs we assume that function $a(t)$ is continuously differentiable.

Dispatch is mandatory and the spot market coordinator is entrusted to set the competitive price for each time segment. Hence, capacity is fully employed as long as $a(t) - K \geq c$, i.e., as long as the spot price computed as $a(t) - K$ exceeds the unit operating cost c . On the contrary, if $a(t) - K < c$, then there is idle capacity at time t , the price is set equal to the unit operating cost c and, consequently, consumption equals $a(t) - c$.¹³ In what follows we assume that $a(1) > c$, ensuring that consumption is always strictly positive.

Let τ denote the inverse function of $a(t)$, then τ is continuously differentiable and decreases over its domain $[a(1), a(0)]$. Moreover, $t \leq \tau(K+c)$ if and only if $a(t) \geq K+c$. Hence, capacity binds if and only if $t \geq \tau(K+c)$ and, consequently, $\tau(K+c)$ corresponds to the length of time that capacity is fully employed. Thus, for installed capacity K , the spot price at time t is

$$p(t, K) = \begin{cases} a(t) - K & \text{if } t \leq \tau(K+c) \\ c & \text{otherwise} \end{cases} \quad (1)$$

In addition, consumption (and production) at time t is given by

$$q(t, K) = \begin{cases} K & \text{if } t \leq \tau(K+c) \\ a(t) - c & \text{otherwise} \end{cases} \quad (2)$$

(footnote continued)

importance of supplier concentration is greatly magnified relative to other determinants of market power such as demand elasticity.

¹¹ Using a similar framework, Adilov (2012) shows that forward trading improves social welfare when demand uncertainty is high. He argues that firms underutilize their capacities during low-demand stages and find it more difficult to eliminate the price-reducing effect of a forward market by restricting their first stage investments.

¹² Thus, $a(t)$ corresponds to the Y-axis intercept of the demand curve at time t .

¹³ The simplicity of expressions derives from the following normalizations and assumptions: one unit of capacity produces one unit of electricity when dispatched; the slope of the demand curve is minus one and; the spot market coordinator sets the competitive price for each time segment.

2.1. Social welfare

In what follows, the installed capacity that maximizes social welfare is derived. Given installed capacity K , the consumer surplus at time t – $s(t, K)$ – is given by the following expression:

$$s(t, K) = \int_0^{q(t, K)} (a(t) - q - p(t, K)) dq \quad (3)$$

Recalling Eqs. (1) and (2), Eq. (3) can be rewritten as

$$s(t, K) = \begin{cases} \int_0^K (K - q) dq = \frac{1}{2} K^2 & \text{if } t \leq \tau(K + c) \\ \int_0^{a(t) - c} (a(t) - q - c) dq = \frac{1}{2} (a(t) - c)^2 & \text{otherwise} \end{cases} \quad (4)$$

when $K < a(1) - c$, capacity binds in all time segments, so it seems proper to define $\tau(K + c) \equiv 1$. On the contrary, if $K > a(0) - c$, then capacity never binds and defining $\tau(K + c) \equiv 0$ is suitable. Then, given a total installed capacity K , the aggregate overtime consumer surplus $-S(K)$ – is given by the following expression:

$$S(K) = \int_0^1 s(t, K) dt = \frac{K^2}{2} \tau(K + c) + \frac{1}{2} \int_{\tau(K + c)}^1 (a(t) - c)^2 dt. \quad (5)$$

If $K < a(1) - c$, the second term in the right-hand side of Eq. (5) equals zero. In contrast, if $K > a(0) - c$, then the first term in the right-hand side of Eq. (5) is zero. Differentiating Eq. (5) results in $dS(K)/dK = K\tau(K + c)$. Hence, consumer surplus is increasing in installed capital within the interval $[0, a(0) - c]$, and is constant beyond this interval.

In turn, producer profits are given by the expression

$$\Pi(K) = \int_0^1 q(t, K)(p(t, K) - c) dt - rK. \quad (6)$$

Eqs. (1) and (2) imply that

$$\Pi(K) = K \int_0^{\tau(K + c)} (a(t) - K - c) dt - rK. \quad (7)$$

Capacity has to bind for a positive length of time for investment to be recovered given competitive pricing in the spot market. Hence, $\tau(K + c)$ must be greater than zero and, consequently, $K < a(0) - c$. In the rest of the paper we assume that the latter condition holds.

Assuming that social welfare is measured by consumer surplus plus producer profits, the social welfare function is given by¹⁴

$$W(K) = K \int_0^{\tau(K + c)} (a(t) - K/2 - c) dt + \frac{1}{2} \int_{\tau(K + c)}^1 (a(t) - c)^2 dt - rK \quad (8)$$

Differentiating this function with respect to K results in

$$\frac{dW(K)}{dK} = \int_0^{\tau(K + c)} (a(t) - K - c) dt - r. \quad (9)$$

The social welfare function is concave given that its second derivative is $-\tau(K + c)$. Thus, the efficient capacity, i.e., the capacity that maximizes social welfare, which will be denoted K^* , is unique and determined by the conditions $dW/dK \leq 0$ and $KdW/dK = 0$. From Eq. (8) follows that $W(K)$ goes to minus infinity when K tends to infinity. Thus, a sufficient condition to ensure the existence of a strictly positive solution is that the derivative of the welfare function be strictly positive at $K = 0$.¹⁵

Assumption 1. The derivative of the social welfare function $W(K)$ is strictly positive at $K = 0$, i.e., $\int_0^{\tau(c)} (a(t) - c) dt > r$.

Given Assumption 1, the efficient capacity K^* is implicitly defined by the equation:

$$\int_0^{\tau(K^* + c)} (a(t) - K^* - c) dt = r. \quad (10)$$

The concavity of the social welfare function implies that welfare is a positive function of capacity within the interval $[0, K^*]$ and a negative function of capacity for values above K^* .

Differentiating Eq. (7) results in

$$\frac{d\Pi(K)}{dK} = \int_0^{\tau(K + c)} (a(t) - 2K - c) dt - r. \quad (11)$$

Hence, the second derivative of the industry's profit function is

$$\frac{d^2\Pi(K)}{dK^2} = -2\tau(K + c) - K\tau'(K + c). \quad (12)$$

In what follows we assume that the industry's profit function $\Pi(K)$ is concave in the interval $[0, K^*]$, i.e., that the following condition holds:

Assumption 2. The expression $2\tau(K + c) + K\tau'(K + c)$ is positive for $K \in [0, K^*]$.

Regarding the intuition of Assumption 2, it is useful to rewrite it as

$$-\frac{ta'(t)}{2a(t) - c} > 1 \quad t \in [\tau(K^* + c), T]. \quad (13)$$

Hence the condition is satisfied when function $a(t)$ is highly time-elastic over the interval $[\tau(K^* + c), T]$. Since K^* is decreasing in both r and c , an increase in either of those two parameters makes the fulfillment of the condition more likely.¹⁶

It follows from Eqs. (7) and (10) that producer profits at K^* are zero, i.e., $\Pi(K^*) = 0$. Thus, if there is free entry to the industry, then optimal regulation of the spot market leads to the welfare maximizing capacity. From Eqs. (7) and (9) it follows that $\Pi(K) = KdW(K)/dK$. Hence, the industry's profit is non-negative in the interval $[0, K^*]$ and non-positive for $K > K^*$.

2.2. Oligopolistic equilibrium

Next we find the equilibrium of the industry assuming the existence of $n \geq 2$ identical firms. The model considers two stages: in stage 1, firms decide their capacities and in stage 2, the spot market coordinator that has power to mandate dispatch sets the competitive price for each time segment.

Next we derive the first-stage Nash equilibrium solution, assuming that firms behave as Cournot players. Let k_i denote the installed capacity of firm i and k the n -tuple of capacities, i.e., $k = (k_1, \dots, k_n)$; hence $K = \sum_i k_i$. In those time segments where the industry's capacity is not binding, i.e., when $t \geq \tau(K + c)$, it is inconsequential who produces what because the spot price equals the unit operating cost. Thus, firm i 's payoff function is given by the following expression:

$$\pi_i(k) = k_i \int_0^{\tau(K + c)} (p(t, K) - c) dt - rk_i = k_i \int_0^{\tau(K + c)} (a(t) - K - c) dt - rk_i, \quad (14)$$

and from Eqs. (7) and 14 it follows that

$$\pi_i(k) = \frac{k_i}{K} \Pi(K). \quad (15)$$

Let K_{-i} denote the capacity installed by firms other than firm i , i.e., $K_{-i} = \sum_{j \neq i} k_j$. Then $k_i > (K^* - K_{-i})$ implies that total installed

¹⁴ Hence, consumers and producers weigh equally in social preferences.

¹⁵ This is not a restrictive assumption because otherwise there would be no consumption of electricity.

¹⁶ For instance, if $a(t) = a_0 - t$ where a_0 is a constant, then $K^* = a_0 - c - (2r)^{1/2}$ and Assumption 2 holds when $r \geq (a_0 - c)^2/8$.

capacity is greater than K^* and consequently $\pi_i(k) < 0$. Thus a firm never chooses a capacity that exceeds $K^* - K_{-i}$. Differentiating Eq. (15) with respect to k_i results in

$$\frac{\partial \pi_i(k)}{\partial k_i} = \int_0^{\tau(K+c)} (a(t) - K - k_i - c) dt - r. \quad (16)$$

Since firms decide on their capacities by assuming that the capacities of the other players are given, the best-response function of firm i , $i = 1, \dots, n$, is the capacity k_i that maximizes $\pi_i(k)$ for each value K_{-i} . Let $k^c = (k_1^c, \dots, k_n^c)$ denote the Nash equilibrium of the Cournot game. We assume that the Nash equilibrium is the (k_1^c, \dots, k_n^c) that solves the system of equations $\partial \pi_i / \partial k_i = 0$, $i = 1, \dots, n$, a condition that is verified in Appendix A (Lemma 2). Let K^c denote the industry's Nash equilibrium capacity, i.e., $K^c = \sum_i k_i^c$. Hence, the Nash equilibrium is determined by the following system of equations:

$$\int_0^{\tau(K^c+c)} (a(t) - K^c - k_i^c - c) dt - r = 0, \quad i = 1, \dots, n. \quad (17)$$

It follows from Eq. (17) that the industry's Nash equilibrium capacity is implicitly defined by

$$\int_0^{\tau(K^c+c)} (a(t) - (1 + 1/n)K^c - c) dt = r. \quad (18)$$

We then show that when firms behave as Cournot players, the industry's installed capacity is below the social optimum level.

Proposition 1. *If firms compete à la Cournot in capacity, the industry's installed capacity is below the social optimum level, i.e., $K^c < K^*$.*

Proof. Consider the following function:

$$g(\beta, K) = \int_0^{\tau(K+c)} (a(t) - \beta K - c) dt - r. \quad (19)$$

Hence, if $g(\beta, K) = 0$ and the conditions of the Implicit Function Theorem hold, then

$$\frac{dK}{d\beta} = - \frac{\partial g(\beta, K) / \partial \beta}{\partial g(\beta, K) / \partial K} = - \frac{K \tau(K+c)}{\beta \tau(K+c) + (\beta-1)K \tau'(K+c)}. \quad (20)$$

Given that function τ is continuously differentiable, both $\partial g / \partial \beta$ and $\partial g / \partial K$ are continuous and consequently function g is also continuously differentiable. From Eq. (10) it follows that $g(1, K^*) = 0$ and $\partial g(1, K^*) / \partial K = \tau(K^* + c)$; therefore the conditions of the Implicit Function Theorem hold at $\beta = 1$. Accordingly, $dK(1) / d\beta = -K^*$ and consequently $dK/d\beta$ is negative in a vicinity of $\beta = 1$. Thus, for values of β marginally above 1, the K that solve $g(\beta, K) = 0$ are below K^* . In addition, $\beta / (\beta - 1) > 2$ when β is contained in the interval $[1, 2]$; hence from Assumption 2 it follows that $\partial g / \partial K > 0$, which in turn implies that $dK/d\beta < 0$. Accordingly, successive marginal increases in β imply that the K that solve equations $g(\beta, K) = 0$ decrease with β in the interval $[1, 2]$. In turn, from Eq. (18) it follows that $g((1 + 1/n), K^c) = 0$. Since $(1 + 1/n) > 1$, we conclude that $K^c < K^*$. □

Thus, in oligopolistic electricity markets where entry barriers impede the arrival of new competitors, even when the spot market is regulated, incumbent firms attain positive profits by investing less than in a competitive environment.

3. Vested contracts

This section derives the equilibrium of the industry given the existence of vested CfDs that are observable by all parties and

enforceable. Again we consider a two-stage game. In the first stage, firms decide their capacities by taking their rivals' capacities as given and considering standing contracts, while in the second stage the spot market coordinator sets real-time competitive prices.

The reference price of contracts $-p^r$ is the average spot price, i.e.:

$$p^r = \int_0^1 p(t, K) dt = \int_0^{\tau(K+c)} (a(t) - K - c) dt + c. \quad (21)$$

Thus, generators pay (or are paid by) their counterparties the difference between the average spot price and the strike price times the quantity specified in the contract. For simplicity, the discount rate is assumed to be zero. Let p^s denote the strike price of CfDs, x_i the quantity of energy contracted by firm i , and x the n -tuple of contracts, i.e., $x = (x_1, \dots, x_n)$. Then, given the standing CfDs, firm i 's payoff function is

$$\psi_i(k, x, p^s) = \pi_i(k) + \left(p^s - \int_0^1 p(t, K) dt \right) x_i, \quad (22)$$

where the first term in the right-hand side of Eq. (22) corresponds to profits in the spot market and the second term to profits in the contract market. Differentiating Eq. (22) with respect to k_i results in

$$\frac{\partial \psi_i}{\partial k_i} = \frac{\partial \pi_i}{\partial k_i} + x_i \tau(K+c) = \int_0^{\tau(K+c)} (a(t) + x_i - K - k_i - c) dt - r, \quad (23)$$

Since firms base their capacity decisions on the assumption that their competitors are committed to a certain capacity, the best response function is given by k_i that maximizes $\psi_i(k, x, p^s)$ for each value of K_{-i} . We assume that the Nash equilibrium solution of the Cournot game is determined by the system of equations $\partial \psi_i / \partial k_i = 0$, $i = 1, \dots, n$, a condition whose compliance we confirm in Appendix A (Lemma 3). Hence, the Nash equilibrium capacities of firms, which will be denoted $\kappa_i(x)$, $i = 1, \dots, n$, are implicitly defined by the subsequent system of equations:

$$\int_0^{\tau(K(x)+c)} (a(t) + x_i - \kappa_i(x) - K(x) - c) dt = r, \quad i = 1, \dots, n, \quad (24)$$

where $K(x)$ denotes Nash equilibrium aggregate capacity, i.e., $K(x) = \sum_i \kappa_i(x)$. In Appendix B it is shown that functions $\kappa_i(x)$ are locally continuously differentiable. Adding over i equations in (24) results in

$$\int_0^{\tau(K(x)+c)} (a(t) + X/n - (1 + 1/n)K(x) - c) dt = r. \quad (25)$$

Notice that $K(x)$ depends on the firms' energy contracts through their aggregation. Consequently, in what follows (abusing notation) we write $K(X)$.

Next we show that the Nash equilibrium aggregate capacity is at its social optimum level when total energy contracted equals the efficient capacity.

Proposition 2. *First, if total energy contracted equals the efficient capacity, then firms install the efficient capacity. Second, the Nash equilibrium aggregate capacity increases with total quantity of energy contracted, but at a rate less than or equal to $1/(n-1)$.*

Proof. We define the function

$$f(X, K) = \int_0^{\tau(K+c)} (a(t) + X/n - (1 + 1/n)K - c) dt - r. \quad (26)$$

If $f(X, K) = 0$ and the conditions of the Implicit Function Theorem hold, then

$$\frac{dK(X)}{dX} = \frac{\tau(K(X) + c)}{(n + 1)\tau(K(X) + c) + (K(X) - X)\tau'(K(X) + c)}. \quad (27)$$

Function $f(X, K)$ is continuously differentiable. From Eq. (17) it follows that $f(0, K^c) = 0$. Given that $K^c < K^*$, Assumption 2 implies that the denominator in Eq. 27 ($\partial f(0, K^c)/\partial K$) is contained in $[(n-1)\tau(K^c+c), (n+1)\tau(K^c+c)]$. Hence, the Theorem applies and, therefore, $dK(0)/dX$ is contained in the interval $[1/(n+1), 1/(n-1)]$. Subsequent marginal increases in X lead to higher values of $K(X)$ that solve the equation $f(X, K(X)) = 0$. In fact, as long as the K that solves the equation $f(X, K(X)) = 0$ satisfies the condition $K \leq K^*$, Assumption 2 implies that $\partial f(X, K(X))/\partial K > 0$ and, accordingly, that the derivative $dK(X)/dX$ is contained in the interval $[(n-1)\tau(K+c), (n+1)\tau(K+c)]$.

From Eqs. (10) and (25) it follows that (i) $f(K^*, K^*) = 0$; (ii) $\partial f(K^*, K^*)/\partial K = (n+1)\tau(K^*+c)$; and (iii) $dK(K^*)/dX = 1/(1+n)$. Hence, for values of X below K^* , the derivative $dK(X)/dX$ is contained in the interval $[(n-1)\tau(K(X)+c), (n+1)\tau(K(X)+c)]$. In turn, for values of X marginally above K^* , $K(X)$ is greater than X , and given that function τ is decreasing, the numerator in Eq. (27) is greater than $(n+1)\tau(K(X)+c)$, and as a result the value of $dK(X)/dX$ is the interval $(0, 1/(n+1))$. Continuous marginal increases in the values of X leads to the conclusion that the derivative $dK(X)/dX$ is restricted to the interval $(0, 1/(n+1))$ when X is greater than X^* .¹⁷ □

Corollary 1. *If total energy contracted is below [above] the efficient capacity, then the actual capacity installed by firms is below [above] the efficient level but above [below] the total energy contracted.*

Proof. The corollary follows from the facts that (i) firms install the efficient capacity when the energy contracted equals the efficient capacity and (ii) the value of the derivative $dK(X)/dX$ is contained in the interval $[0, 1/(n-1)]$. □

Eqs. (24) and 25 imply that the Nash equilibrium capacity of firm i is characterized by the expression:

$$\kappa_i(x) = \text{Max} \left\{ \frac{K(X)}{n} + x_i - \frac{X}{n}, 0 \right\} \quad (28)$$

From Corollary 1 it follows that if the total quantity of energy contracted is less than K^* , then the actual capacity installed is also smaller but by less than the energy contracted, and consequently $\kappa_i(x) \geq x_i$. In turn, if X is greater than K^* , then $K(X) \geq X$, and consequently $\kappa_i(x) \leq x_i$. In this case, firms that have no contracts, or that contracted a relatively small quantity of energy, do not invest as it can be inferred from Eq. (28). This happens when $x_i < (X - K(X))/n$ and consequently $\kappa_i(x)$ is zero. Thus the determination of the stage 1 Nash equilibrium capacities considers that these firms do not invest.

In turn from Eqs. (10) and (14) and Corollary 1 it follows that spot market profits π_i of those firms that do invest are negative when $X > K^*$. Hence those firms that do install capacity do not

recover their investment with their spot market sales. However, they do invest to lower the reference price specified in their financial contracts, given that the spot market average price declines with installed capacity.

4. Contract market

In this section the model is expanded to incorporate a contract market that takes place before investments are committed. Firms offer to sell energy contracts that are purchased by competitive speculators. All parties are assumed to have perfect foresight. The interaction between the spot, the capacity and the contract market is modeled as a three stage game. In the first stage, the contract market quantities and strike prices are determined, in the second, generators simultaneously set their capacities given the standing *CfDs* and, in the third stage, the spot market coordinator sets the competitive spot price for each time segment and the *CfDs* are settled. In stages 1 and 2, firms behave as Cournot players. The game is solved by backwards induction.

Stage 3: The spot market coordinator sets the price $p(t, K)$ for each $t \in [0, 1]$.

Stage 2: Generators decide their capacities, which are given by Eq. 28.

Stage 1: In this stage firms simultaneously choose the quantity of energy they want to sell in the contract market.

The payoff function of firm i in stage 1 – that will be denoted $\mu_i(x, p^s)$ – is obtained by plugging into Eq. (22) the capacity functions $\kappa_i(x)$ derived in stage 2. Hence, recalling Eq. (15), it follows that the payoff function of firm i is

$$\mu_i(x, p^s) = \frac{\kappa_i(x)}{K(X)} \Pi(K(X)) + \left(p^s - \int_0^1 p(t, K(X)) dt \right) x_i \quad (29)$$

The first term in the right-hand-side of Eq. (29) corresponds to profits in the spot market, while the second term refers to profits in the contract market. Perfect foresight by all parties combined with the assumption that speculators are price-takers implies that the strike price equals the reference price (the average spot price). Thus, the payoff function simplifies to

$$\mu_i(x, p^s) = \frac{\Pi(K(X))}{K(X)} \kappa_i(x). \quad (30)$$

Differentiating the payoff function of firm i with respect to x_i results in

$$\frac{\partial \mu_i}{\partial x_i} = \left[\int_0^{\tau(K(X)+c)} (a(t) - K(X) - c) dt - r \right] \frac{\partial \kappa_i(x)}{\partial x_i} - \tau(K(X) + c) \kappa_i(x) \frac{dK(X)}{dx_i}. \quad (31)$$

Because of Eqs. (24) and 25, Eq. (31) can be rewritten:

$$\frac{\partial \mu_i}{\partial x_i} = \tau(K(X) + c) \left[(\kappa_i(x) - x_i) \frac{\partial \kappa_i(x)}{\partial x_i} - \kappa_i(x) \frac{dK(X)}{dx_i} \right]. \quad (32)$$

In what follows, the paper assumes that the payoff function of firm $i - \mu_i(x, p^s)$ – is concave in x_i , $i = 1, \dots, n$.¹⁸ Thus, given that firms behave as Cournot competitors in the contract market, the best response of firm i is the solution to equations $\partial \mu_i / \partial x_i \leq 0$ and $x_i \partial \mu_i / \partial x_i \leq 0$. Since all firms contract energy (as it will be seen later on), the Nash equilibrium solution is determined by the system of equations $\partial \mu_i / \partial x_i = 0$, $i = 1, \dots, n$. Therefore, the Nash equilibrium quantities of *CfDs* traded are the x_i^c , $i = 1, 2, \dots, n$, that solve the

¹⁷ The Implicit Function Theorem states that for a pair (X, K) that solves the system $f(X, K) = 0$ there is a continuous differentiable function $K(X)$ defined in a vicinity of X such that $f(X, K(X)) = 0$. Given that for each X in the compact $[0, a(0) - c]$ there is a locally continuous function, it can be proven that $K(X)$ is continuous in the interval. Analogously, functions $\kappa_i(x)$ are continuous.

¹⁸ Appendix C shows that a sufficient condition for the concavity of the payoff function is the concavity of function τ .

system of equations:

$$(\kappa_i(x^c) - x_i^c) \frac{\partial \kappa_i(x^c)}{\partial x_i} = \kappa_i(x^c) \frac{dK(X^c)}{dx_i}, \quad (33)$$

where $x^c = (x_1^c, \dots, x_n^c)$ and X^c denotes the aggregate quantity of contracts, i.e., $X^c = \sum_i x_i^c$.

Next we show that the emergence of a contract market mitigates but does not eliminate generators' market power.

Proposition 3. *The emergence of a contract market increases the industry's capacity but not by enough to reach the social optimum level.*

Proof. Recalling Eq. (28), Eq. 33 can be rearranged as follows:

$$x_i^c = \frac{(n-1)(1 - dK(X^c)/dx_i)}{n-1 + dK(X^c)/dx_i} \kappa_i(x^c) \quad (34)$$

Thus, from Eq. (34) and the fact that the value of $dK(X)/dX$ is contained in the interval $(0, 1/(n-1)]$ (Proposition 2), we can infer that $0 < x_i^c < \kappa_i(x^c)$ for each firm i , and consequently that $0 < X^c < K(X^c)$. From Corollary 1 it follows that $X^c < K(X^c) < K^*$. Moreover, recalling Eqs. (18) and (25) it follows that $K(0) = K^c$, and given that the installed capacity is increasing in the standing CfDs we conclude that $K(X^c) > K^c$. □

5. Auctions

This section assumes that an aggregation of consumers awards a CfD in a sealed first-price auction before investment decisions are taken. If two or more firms submit the same price bid, they draw lots to choose the winner. Thus, a three-stage game is played. In the first stage a contract is auctioned, in the second, producers decide their capacities, and in the third stage, the spot market coordinator sets real-time competitive spot prices and the CfDs are settled. The game is solved by backwards induction.

First, we analyze the case where the quantity of energy auctioned, which will be denoted X^a , is less than the efficient capacity K^* .

Stage 3: In this stage, for each time segment t the spot market coordinator sets the price $p(t, K) = \text{Max}\{c, a(t) - K\}$.

Stage 2: Firms simultaneously choose their capacities given the auction outcome and assuming that their rivals' capacities are given. Let firm 1 be the one that won the contract in stage 1. Defining $x^a = (X^a, 0, \dots, 0)$, then Eq. (28) implies that firm 1 invests

$$\kappa_1(x^a) = \frac{K(X^a)}{n} + \frac{n-1}{n} X^a, \quad (35)$$

while the other firms invest

$$\kappa_i(x^a) = \text{Max}\left\{\frac{K(X^a)}{n} - \frac{X^a}{n}, 0\right\} \quad i = 2, \dots, n. \quad (36)$$

Given that we are analyzing the case $X^a < K^*$, Corollary 1 implies that the $\kappa_i(x^a)$ are strictly positive for $i = 1, \dots, n$.

Stage 1: In Stage 1, an aggregation of consumers awards a long-term CfD for a quantity X^a in a sealed first-price auction. Let p^a denote the auction winning price. Given that firm 1 is the contract-winning firm, from Eqs. (29), (35), and (36) it can be inferred for $i = 2, \dots, n$ that

$$\mu_1(x^a, p^a) = \mu_i(x^a, p^a) + \left(p^a - \int_0^1 p(t, K(X^a)) dt\right) X^a + \frac{\Pi(K(X^a))}{K(X^a)} X^a. \quad (37)$$

Profits of firms that do not win the contract do not depend on the strike price p^a , while profits of the contract-winning firm is an affine function of this price (Eq. (29)). There is thus only one strike price $- p^a -$ that equalizes profits of all firms. From Eq. (37) it follows that this price is given by the expression

$$p^a = \int_0^1 p(t, K(X^a)) dt - \frac{\Pi(K(X^a))}{K(X^a)} \quad (38)$$

From Eqs. (7) and 21 it follows that

$$p^a = \int_0^{\tau(K(X^a)+c)} (a(t) - K(X^a) - c) dt + c - \int_0^{\tau(K(X^a)+c)} (a(t) - K(X^a) - c) dt + r = c + r \quad (39)$$

The Nash equilibria of the second stage of the game are those solutions where at least two firms (including the contract winning firm) bid the price p^a and the rest bid prices above p^a . In fact, no firm would have incentives to change its bid and tender a price above the strike price as its profits would not change. Neither do firms have reasons to undercut the price p^a . In fact, if a firm did bid a price below p^a then it would be awarded the contract, but as profits are an increasing function of the strike price, this firm would see its profits fall.

Next we show that the strike price equals the average efficient spot price, i.e., the average spot price when capacity is at its social optimum.

Lemma 1. *The strike price equals the average efficient spot price.*

Proof. From Eq. (1), it follows that the average efficient spot price is given by

$$\int_0^1 p(K^*, t) dt = \int_0^{\tau(K^*+c)} (a(t) - K^* - c) dt + c \quad (40)$$

From Eq. (10) it follows that

$$\int_0^1 p(K^*, t) dt = r + c. \quad (41)$$

□

Thus, the auction of a contract has two effects. First, as a result of contract trading, installed capacity rises and, accordingly, spot prices fall. Second, it equalizes the strike price to the average efficient spot price. The latter effect is a consequence of an aggregation of consumers instead of producers exerting market power in the contract market. Thus, the aggregation of consumers benefits both from a lower spot price and the profit derived from the contract. The latter benefit disappears when spot prices are at their efficient level.

In what follows the paper tackles the case where the aggregation of consumers auctions a contract for a quantity of energy greater than or equal to the efficient capacity K^* . From Corollary 1 and Eq. (36) it follows that only the bid-winning firm invests in capacity. Hence, this firm becomes a monopolist in the spot market facing a monopsony (the aggregate of consumers). However, this situation causes no difficulties given that the spot market is regulated. Moreover, the monopolist still invests to prevent the reference price escalating too high. Recalling Eq. (24), the investment of firm 1 $-\kappa_1(x^a)$ - is the solution to equation:

$$\int_0^{\tau(\kappa_1(x^a)+c)} (a(t) + X^a - 2\kappa_1(x^a) - c) dt - r = 0. \quad (42)$$

Moreover, Corollary 1 implies that firm 1 invests more than the socially efficient capacity K^* and, consequently, net revenues from the spot market are insufficient to recover the investment

disbursements. Therefore, the auction's winning bid has to exceed the reference price to compensate for losses in the spot market.

The Nash-equilibrium strike price of the auction is the price that makes that the total profits of the bid-winning firm, deriving both from the spot market and the contract market, add up to zero. A higher price would incentivize other firms to undercut this price. On the other hand, no firm would bid a price below it because it would win the contract but lose money. From Eq. (29) it follows that the strike price of the Nash equilibrium solution is given by

$$p^a = \int_0^1 p(t, k_1(x^a)) dt - \frac{1}{X^a} \Pi(k_1(x^a)) = c + r + \left(1 - \frac{k_1(x^a)}{X^a}\right) \frac{\Pi(k_1(x^a))}{k_1(x^a)}. \quad (43)$$

Given that $\Pi(k_1(x^a))$ has a negative value (Corollary 1 implies that the firm loses money in the spot market), the strike price p^a is greater than the reference price p^r . Moreover, the condition $k_1(x^a) < X^a$ implies that the strike price is below the average efficient spot price.

Finally, the paper shows that an aggregate that includes all consumers auctions a contract for the quantity that ensures the efficient solution.

Proposition 4. *An aggregate of all consumers will auction a contract for a quantity of energy that equals the efficient capacity.*

Proof. The auction of the efficient capacity K^* maximizes social welfare and reduces industry's profits to zero. In turn, if the consumer aggregate auctioned a supply contract for a quantity below [above] K^* , then social welfare would not be maximized and the industry would have positive [zero] profits. Thus, a fortiori, consumer surplus is maximized when the quantity auctioned is K^* . □

6. Conclusions and future work

We analyze the impact of financial contracts in oligopolistic electricity markets. In a model where the spot market is characterized by price regulation and demand participation, it is shown that the implementation of a long-term contract market reduces market power in the industry. The long-term nature of contracts is crucial for the result. In fact, given regulation, spot prices solely depend on capacity and instant demand and are not affected by contracts traded after investment decisions are taken. In addition we show that if an aggregate of consumers auctions a long (buy) contract, then the strike price of the contract is the average of efficient spot prices. The reason is that an auction of a contract by consumers takes the market power in the contract market away from firms.

This paper analyzes financial contracts, but physical contracts could also be implemented. In principle, given that demand is time-varying, the long-term physical contracts would have to specify a quantity of energy for each time segment, something that seems cumbersome at least. Though contracts to supply energy for specific time hours are traded in power exchanges, these contracts differ from the long-term contracts analyzed in this paper. The latter are normally used to buy the bulk of forecasted consumption, while short-term contracts are used to make adjustments as the period gets closer and the available capacity and demand projections become more precise.

An option would be implementing a market for capacity contracts where the reference price would be the net income per unit of capacity contracted. Physical capacity contracts are easier to implement than physical energy contracts as there is no need to specify time varying quantities. And the firms that sold physical capacity contracts simply would have to make the

capacity available to purchasers of the contracts, and the latter would produce the energy they desire.¹⁹

Although the model captures most stylized facts of electricity markets such as a bounded short-term capacity, endogenous investment decisions, and a time-varying demand that must be supplied instantaneously, other traits such as stochastic supply and demand are not considered. Moreover, this paper models a one-technology industry, but the installed generating capacity usually includes units with low operating costs and high capacity costs and others that have high operating costs and low capacity costs. Future research should extend the model to consider these features.

Another simplification in this paper is the assumption of no intertemporal substitutions in consumption. A benefit of real-time pricing is diminishing demand peaks by incentivizing users to shift demand from periods in which the market is tight and therefore spot prices are high, to those in which the market is lax and accordingly spot prices are low. This peak-load shaving lowers the investment requirements, which in turn should result in lower prices. This paper, however, does not address this effect given that cross time price elasticities are omitted. Forthcoming work will incorporate intertemporal substitution in consumption.

Finally, future research should consider the fundamentals that justify modeling an oligopolistic electricity market. Although the empirical evidence is overwhelming regarding the concentrated nature of these markets, there are contending explanations that have distinct policy implications. Concentration could be caused by entry barriers, i.e., costs that apply to entrants but not to firms already operating. A conspicuous entry barrier to power generation is the cornering of some crucial resources in the hands of incumbent firms. These firms tend to concentrate the best locations for new coal fired thermoelectric power plants (near ports) and the non-consumptive water rights to develop the most profitable hydroelectric projects, all of which were acquired at little or no cost in the past.

In turn, a weak institutional framework constitutes an additional risk for potential entrants given the centralized character of dispatch that requires the coordination of all firms, especially if incumbent firms have close links to the system operator. For instance, Michaels (2008) holds that the responses of the Market Monitoring Institutions (MMI) –the institutions in charge of overseeing competition in U.S. energy markets– to the practice of “virtual bidding” differed with the relative strengths of different interests in the governance of those organizations.²⁰ In addition, approving environmental permits, an inescapable and usually murky stage in the installation of new power plants, is also likely to be more difficult for new entrants that lack ties with authorities and, consequently, the lobbying capacities of incumbent firms

An alternative (and complementary) explanation for the high concentration observed in electricity markets is the existence of strong scale economies. These economies tend to occur at the central organization level rather than at the plant operational level. Understanding the complex sectoral and environmental regulations and participating in the spot and forward markets demand resources that are independent of firm sizes. The scale economies and sunk costs characteristic of the sector raise entry barriers.²¹ Thus, understanding the cause of concentration is of

¹⁹ An alternative would be that firms who sold contracts produce the energy requested by purchasers of contracts bound by the capacity contracted and, in turn, receive a pre-established price.

²⁰ Regional Transmission Organizations (RTO) and Market Monitoring Institutions (MMI), that oversee the markets that RTOs operate, are new institutions that came along with the restructuring of wholesale electricity markets in the U.S.

²¹ McAfee et al. (2004) argue that while scale economies are not in themselves a barrier to entry, they can aggravate other barriers to entry within the system.

foremost importance for policy-making. This paper provides the framework in which all these issues will be analyzed in the future.

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Appendix A

First we show that the Nash equilibrium of the Cournot game described in Section 2 is the n -tuple $k^c=(k_1^c, \dots, k_n^c)$ that solves the system of equations $\partial\pi_i/\partial k_i=0, i=1, \dots, n$. Let K_{-i}^c denote the capacity of firms other than firm i , i.e., $K_{-i}^c=\sum_{j \neq i} k_j^c$, k_{-i}^c the n -tuple that is obtained replacing the i -th component of k^c with zero, i.e., $k_{-i}^c=(k_1^c, \dots, 0, \dots, k_n^c)$, and k_m^c the n -tuple that it is obtained substituting the i -th component of k^c with K^*-K_{-i} , i.e., $k_m^c=(k_1^c, \dots, K^*-K_{-i}, \dots, k_n^c)$.

Lemma 2. *The Nash equilibrium of the Cournot game described in Section 2 is the n -tuple $k^c=(k_1^c, \dots, k_n^c)$ that solves the system of equations $\partial\pi_i/\partial k_i=0, i=1, \dots, n$.*

Proof. Recalling Eq. (16), the second derivative of the payoff function π_i with respect to k_i is

$$\frac{\partial^2 \pi_i(k)}{\partial k_i^2} = -2\tau(K+c) - k_i \tau'(K+c). \tag{a.1}$$

Assumption 2 ensures the concavity of the payoff function π_i with respect to k_i within the range $[0, K^*-K_{-i}]$ for $i=1, \dots, n$. Therefore, we need to show that $\partial\pi_i/\partial k_i$ evaluated at k_{-i}^c is non-negative and evaluated at k_m^c is non-positive. From Eq. (16) it follows that

$$\frac{\partial\pi_i(k_{-i}^c)}{\partial k_i} = \int_0^{\tau((1-1/n)K^c+c)} (a(t) - (1-1/n)K^c - c) dt - r \tag{a.2}$$

Recalling Eq. (9), it follows that $\partial\pi_i/\partial k_i$ evaluated at k_{-i}^c equals dW/dK evaluated at $(1-1/n)K^c$. Since $K^c \leq K^*$, the concavity of the welfare function implies that $\partial\pi_i(k_{-i}^c)/\partial k_i \geq 0$. In turn

$$\frac{\partial\pi_i(k_m^c)}{\partial k_i} = \int_0^{\tau(K^c+c)} (a(t) - 2K^* + (1-1/n)K^c - c) dt - r \tag{a.3}$$

Recalling Eq. (10), Eq. (a.3) reduces to

$$\frac{\partial\pi_i(k_m^c)}{\partial k_i} = -(K^* - (1-1/n)K^c) \tau(K^* + c), \tag{a.4}$$

and the result follows from the fact that $K^c \leq K^*$. \square

Next we show that the Nash equilibrium of the Cournot game in Section 3 is the $\kappa(x)=(\kappa_1(x), \dots, \kappa_n(x))$ that solves the system of equations $\partial\psi_i/\partial k_i=0, i=1, \dots, n$. Let $K_{-i}(x)$ denote the capacity of firms other than firm i , i.e., $K_{-i}(x)=\sum_{j \neq i} k_j(x)$, $\kappa_{-i}(x)$ the n -tuple obtained replacing the i -th component of $\kappa(x)$ with a zero, i.e., $\kappa_{-i}(x)=(\kappa_1(x), \dots, 0, \dots, \kappa_n(x))$, and $\kappa_m(x)$ the n -tuple obtained replacing the i -th component of $\kappa(x)$ with a $K^*-K_{-i}(x)$, i.e., $\kappa_m(x)=(\kappa_1(x), \dots, K^*-K_{-i}(x), \dots, \kappa_n(x))$.

Lemma 3. *The Nash equilibrium of the Cournot game in Section 3 is the n -tuple $\kappa(x)=(\kappa_1(x), \dots, \kappa_n(x))$ that solves the system of equations $\partial\psi_i/\partial k_i=0, i=1, \dots, n$.*

Proof. Differentiating Eq. (24) results in

$$\frac{\partial^2 \psi_i}{\partial k_i^2} = -2\tau(K+c) - (k_i - x_i) \tau'(K+c). \tag{a.5}$$

Hence, the payoff function ψ_i is concave in $k_i, i=1, \dots, n$ when $k_i \leq x_i$. Moreover, Assumption 2 ensures the concavity when $k_i \leq (K^* - K_{-i})$. Thus we need to show that $\partial\psi_i/\partial k_i$ evaluated at $\kappa_{-i}(x)$ is non-negative and evaluated at $\kappa_m(x)$ or x_i is non-positive. First we assume that $X \leq K^*$. From Eq. (24), it follows that

$$\frac{\partial\psi_i(\kappa_{-i}(x))}{\partial k_i} = \int_0^{\tau(K_{-i}(x)+c)} (a(t) + x_i - K_{-i}(x) - c) dt - r \tag{a.6}$$

Recalling Eq. (9), it follows that

$$\frac{\partial\psi_i(\kappa_{-i}(x))}{\partial k_i} = \frac{dW(K_{-i}(x))}{dK} + x_i \tau(K_{-i}(x) + c). \tag{a.7}$$

Corollary 1 implies that $K(X) \leq K^*$. Consequently dW/dK evaluated at $K_{-i}(x)$ takes a non-negative value, and accordingly $\partial\psi_i/\partial k_i$ evaluated at $\kappa_{-i}(x)$ is non-negative. In turn

$$\frac{\partial\psi_i(\kappa_m(x))}{\partial k_i} = \int_0^{\tau(K^c+c)} (a(t) + x_i - 2K^* + K_{-i}(x) - c) dt - r \tag{a.8}$$

Eq. (10) implies that

$$\frac{\partial\psi_i(\kappa_m(x))}{\partial k_i} = (x_i - K^* + K_{-i}(x)) \tau(K^* + c) \tag{a.9}$$

Since $X \leq K^*$, from Eq. (28) it follows that all $\kappa_i(x)$ are non-negative and that

$$\sum_{j \neq i} \kappa_j(x) = \frac{n-1}{n} K(X) - x_i + \frac{X}{n} \tag{a.10}$$

Thus

$$x_i - K^* + K_{-i}(x) = \frac{n-1}{n} K(X) + \frac{X}{n} - K^* \tag{a.11}$$

Since $K^* \geq K(X) \geq X$, $\partial\psi_i/\partial k_i$ evaluated at $\kappa_m(x)$ is non-positive.

Next we consider the case $X > K^*$, but assume that $x_i \geq (X - K(X))/n$ for all i . Then from Eq. (28) it follows that $0 \leq \kappa_i(x) \leq x_i$. Thus Eq. (a.5) implies that the payoff functions are concave. By definition $\partial\psi_i(\kappa_i(x))/\partial k_i=0$, hence the concavity of function ψ_i with respect to the i -th component implies that $\partial\psi_i(0)/\partial k_i \geq 0$ and the condition holds. If $x_i < (X - K(X))/n$, then firm i does not invest, and the Nash equilibrium solution is determined excluding this firm from the game. \square

Appendix B

Let us define the system of n equations:

$$f_i(k_1, \dots, k_n, x_1, \dots, x_n) = \int_0^{\tau(\sum_{j \neq i} k_j + c)} (a(t) + x_i - k_i - \sum_{j \neq i} k_j - c) dt - r, i = 1, \dots, n. \tag{b.1}$$

Then

$$\frac{\partial f_i}{\partial k_j} = -(1 + \delta_{ij}) \tau(\sum_{l \neq i} k_l + c) + (x_i - k_i) \tau'(\sum_{l \neq i} k_l + c) \tag{b.2}$$

$$\frac{\partial f_i}{\partial x_j} = \delta_{ij} \tau(\sum_{l \neq i} k_l + c) \tag{b.3}$$

where δ_{ij} denotes the Kronecker delta. Given that by assumption function τ is continuously differentiable, then functions f_i are continuously differentiable. Moreover the determinant of the Jacobian of the system of Eq. (b.1) with respect to k is given by the expression

$$\Delta = (-1)^n (\tau(K+c))^{n-1} ((n+1)\tau(K+c) + (K-X)\tau'(K+c)) \quad (\text{b.4})$$

In order to calculate the determinant we transform the Jacobian into a triangular matrix. First, we subtract the column n from the other columns. Then we subtract from column n the other columns multiplied by the corresponding expression to eliminate all but the last component in column n .

Notice that the determinant Δ is different from zero if either $K \leq K^*$ (Assumption 2) or $K \leq X$ given that τ' takes negative values. Hence, if either of the two inequalities is satisfied, then the conditions of the Implicit Function Theorem hold and the system b.1 can be locally solved at a ball around a solution by implicitly defined functions that are continuously differentiable. Thus for any n -tuple $(x_1, \dots, x_n, k_1, \dots, k_n)$ such that $f_i(x_1, \dots, x_n, k_1, \dots, k_n) = 0$, $i = 1, \dots, n$, there are functions $\kappa_i(x)$ that satisfy the equations:

$$f_i(x_1, \dots, x_n, \kappa_1(x), \dots, \kappa_n(x)) = \int_0^{\tau(\sum_j \kappa_j(x) + c)} (a(t) + x_i - \kappa_i(x) - \sum_j \kappa_j(x) - c) dt - r = 0 \quad (\text{b.5})$$

Moreover

$$\frac{\partial \kappa_i(x)}{\partial x_i} = \frac{(-1)^n (\tau(K+c))^{n-1} (n\tau(K(X)+c) + (n-1)(K(X)-c)\tau'(K(X)+c)/n)}{\Delta} \quad (\text{b.6})$$

Rearranging terms

$$\frac{\partial \kappa_i(x)}{\partial x_i} = \frac{\tau(K+c)}{n[(n+1)\tau(K(X)+c) + (K(X)-X)\tau'(K(X)+c)]} + \frac{n-1}{n} \quad (\text{b.7})$$

Moreover

$$\frac{\partial \kappa_i(x)}{\partial x_j} = \frac{\tau(K+c)}{n[(n+1)\tau(K(X)+c) + (K(X)-X)\tau'(K(X)+c)]} - \frac{1}{n}, \quad (\text{b.8})$$

and

$$\frac{dK(X)}{dX} = \frac{\tau(K(X)+c)}{(n+1)\tau(K(X)+c) + (K(X)-X)\tau'(K(X)+c)}. \quad (\text{b.9})$$

Appendix C

In order to simplify notation, arguments of functions are omitted and $dK(X)/dX$ is denoted K' . From Eqs. (28) and (32) it follows that

$$\frac{\partial \mu_i}{\partial x_i} = [(\kappa_i - x_i)(n-1) - ((n-1)\kappa_i + x_i)K'] \frac{\tau}{n} \quad (\text{c.1})$$

Differentiation of Eq. (c.1) leads to

$$\frac{\partial^2 \mu_i}{\partial x_i^2} = - \left[(n-1) + (n^2 - 2n + 2)K' + (n-1)(K')^2 + ((n-1)\kappa_i + x_i)K'' \right] \frac{\tau}{n^2} - \frac{\tau'}{\tau} \frac{\partial \mu_i}{\partial x_i} \quad (\text{c.2})$$

Resorting to Eqs. (28) and (32) we rewrite equation c.2 as

$$\frac{\partial^2 \mu_i}{\partial x_i^2} = - \frac{1}{n^2} \left[2((n-1) + K')K'\tau + n\kappa_i(K')^2\tau' + ((n-1)\kappa_i + x_i)\tau K'' \right] \quad (\text{c.3})$$

In turn, differentiating Eq. (27) results in

$$\tau K'' = (K')^2\tau' - [(n+1)\tau'K' + (K'-1)\tau' + (K-X)K'\tau''] (K')^2 \quad (\text{c.4})$$

Reordering terms:

$$\tau K'' = [2 - (n+2)K'] (K')^2\tau' - (K-X)(K')^3\tau'' \quad (\text{c.5})$$

From Eqs. (c.3) and (c.5) it follows that

$$\frac{\partial^2 \mu_i}{\partial x_i^2} = - \frac{(K')^2}{n} \left[2((n-1)/K' + 1)\tau + A\tau' - ((n-1)\kappa_i + x_i)(K-X)K'\tau'' \right] \quad (\text{c.6})$$

where

$$A = [(3n-2)\kappa_i + 2x_i - ((n-1)\kappa_i + x_i)(n+2)K'] \quad (\text{c.7})$$

We assume that $X < K^*$; consequently, Corollary 1 implies that $X < K < K^*$. Thus, function μ_i is concave when the following condition is satisfied:

$$\tau'' < \frac{2((n-1)/K' + 1)\tau + A\tau'}{((n-1)\kappa_i + x_i)(K-X)K'} \quad (\text{c.8})$$

Proposition 2 states that $1/(n+1) > K' \leq 1/(n-1)$, and this condition ensures that the numerator in c.8 is positive.

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