Space-Time Network Coding With Transmit Antenna Selection and Maximal-Ratio Combining

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***Abstract*—In this paper, we investigate space-time network coding (STNC) in cooperative multiple-input multiple-output networks, where U users communicate with a common destination D with the aid of R decode-and-forward relays. The transmit antenna selection with maximal-ratio combining (TAS/MRC) is adopted in user-destination and relay-destination links where a single transmit antenna that maximizes the instantaneous received signal-to-noise ratio is selected and fed back to transmitter by receiver and all the receive antennas are combined with MRC. In the presence of perfect feedback, we derive new exact and asymptotic closed-form expressions for the outage probability (OP) and the symbol error rate (SER) of STNC with TAS/MRC in independent but not necessarily identically distributed Rayleigh fading channels. We demonstrate that STNC with TAS/MRC guarantees full diversity order. To quantify the impact of delayed feedback, we further derive new exact and asymptotic OP and SER expressions in closed form. We prove that the delayed feedback degrades the full diversity order to $(R+1)N_{\mathrm{D}}$, where N_{D} is the antenna number of the destination D. Numerical and Monte Carlo simulation results are provided to demonstrate the accuracy of our theoretical analysis and evaluate the impact of network parameters on the performance of STNC with TAS/MRC.**

***Index Terms*—Cooperative communications, space-time network coding (STNC), multiple-input multiple-output (MIMO), delayed feedback.**

I. Introduction

COOPERATIVE communications have recently attracted considerable attention in emerging wireless applications due to their advantages of network coverage extension and capacity expansion [1]–[3]. The inherent concept of cooperative communications is to provide spatial diversity by employing relay nodes to forward signals from the source to the destination. Against this background, various cooperative diversity schemes have been proposed and analyzed [4], [5]. We note that most research contributions in cooperative communications assumed perfect synchronization. Such an assumption may be difficult or impossible in practical multi-node wireless networks as it is very challenging to align all the signals from multiple sources at multiple destinations [6], [7]. High-accuracy synchronization requires complicated control mechanisms and extra control messages, which leads to high system complexities and overheads [7]. When synchronization is imperfect, the performance of cooperative communications can be severely degraded.

To overcome the problem caused by imperfect synchronization, the space-time network coding (STNC) scheme was proposed in [8], which uses time-division multiple access (TDMA) to deal with the imperfect synchronization issues. Importantly, this scheme achieves the full diversity order. STNC combines information from different sources at each relay node and transmits the combined signal in dedicated time slots, which jointly exploit the benefits of both network coding and space-time coding. To quantify the benefits offered by STNC, the symbol error rates (SERs) of STNC over Rayleigh and Nakagami-m fading channels were investigated in [8] and [9], respectively, and the outage probability (OP) of STNC in Rayleigh fading channel was analyzed in [10]. In [8], [9], it was assumed that the channel state information (CSI) is known at the receivers. To avoid the requirement of channel estimation, the differential space-time network coding (DSTNC) and distributed differential space-time-frequency network coding (DSTFNC) schemes were designed for narrowband and broadband cooperative communication systems, respectively, in [11]. Similarly to STNC, both the DSTNC and DSTFNC schemes provide the full diversity. By allowing each relay to exploit the overheard signals transmitted from not only the sources but also the previous relays, a new STNC scheme with overhearing relays was proposed in [12]. When there is no dedicated relay in the systems, users are required to help each other to exploit the cooperative diversity gain [13], [14]. In [13], a novel clustering based STNC scheme was proposed to achieve a better tradeoff between diversity gain and bandwidth efficiency. The core idea of this scheme is to divide the whole network into several small clusters and allow different clusters to help each other to relay signals. In [14], the STNC with optimal node selection scheme was presented to allow multiple users to exchange their data simultaneously. We note that in [8]–[14], each node in the network is equipped with a single antenna. Since multiple-input multiple-output (MIMO) technology, in which communication nodes are equipped with multiple transmit and/or receive antennas, can significantly increase communication reliability through the use of spatial diversity [15], we focus on multi-antenna STNC in this work.

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In this paper, we examine STNC in the cooperative MIMO network, where U users communicate with a common destination D with the assistance of R relays and all the nodes are equipped with multiple antennas. We consider the independent but not necessarily identically distributed (i.n.i.d.) Rayleigh fading channels and include the direct links between the users and destination. Particularly, we focus on decode-and-forward (DF) relaying protocol due to its application in the Third-Generation Partnership Project Long-Term Evolution and IEEE 802.16m [16], [17] such that the relays need to decode the received signal, reencode it, and forward it to the destination. For user-destination $(u-\mathrm{D})$ and relay-destination $(r-\mathrm{D})$ links, we adopt transmit antenna selection with maximal-ratio combining (TAS/MRC) [18], [19], where a single transmit antenna that maximizes the output signal-to-noise ratio (SNR) is selected and all the receive antennas are combined using MRC. As such, the transmitter can be easily implemented with a single front-end and an analog switch, and the receiver only needs to feed back the index of the selected transmit antenna.

We first consider perfect feedback and derive new closed-form expressions for the exact and asymptotic OP and SER. As TAS could be performed by using outdated CSI due to feedback delays, we then quantify the effect of delayed feedback in $u-\mathrm{D}$ and $r-\mathrm{D}$ links on the OP and SER. In doing so, we derive new closed-form expressions for the exact and asymptotic OP and SER by taking the delayed feedback into account. The primary analytical contributions of this paper are summarized as follows.

1) We integrate cascaded TAS/MRC into STNC as a solution to preserve full transmit and receive diversity with low computational complexity and reduced feedback overhead;
2) We derive new closed-form exact and asymptotic expressions for the OP and SER. These results are valid for general operating scenarios with arbitrary number of antennas and arbitrary number of relays;
3) We derive new closed-form exact and asymptotic expressions for the OP and SER with feedback delays to examine the impact of outdated CSI on the performance. Based on our results, it is demonstrated that the transmit diversity vanishes and that the diversity order is entirely independent of the number of transmit antennas.

The rest of the paper is organized as follows. The system model is presented in Section II. In Section III, the exact and asymptotic performance of STNC with TAS/MRC in cooperative MIMO network in the presence of perfect feedback is analyzed. In Section IV, the impact of delayed feedback on the performance of STNC with TAS/MRC is quantified. Numerical and simulation results are presented in Section V. The conclusions are drawn in Section VI.

II. System Model

We consider a cooperative MIMO network where U users transmit their own information to a common destination D with the aid of R relays. In this MIMO network, user u, $1 \leq u \leq U$, relay r, $1 \leq r \leq R$, and destination D are equipped with N_u, N_r, and N_{D} antennas, respectively. The channel coefficient between the jth antenna of transmitter μ and the ith antenna of receiver ν is defined as $h_{ij}^{\nu\mu} \sim \mathcal{CN}(0, d_{\mu\nu}^{-\alpha})$, $\mu \in \{u, r\}$, $\nu \in \{r, D\}$, $\mu \neq \nu$, where $d_{\mu\nu}$ and α denote the distance between μ and ν and the path loss exponent, respectively.

The transmission of STNC takes place over $U+R$ time slots, which are divided into two consecutive phases [8]. In the first phase, the U users take turns to broadcast their symbols to the relays and destination in the first U time slots, i.e., user u broadcasts its symbol in time slot u while other users staying in silence. In the second phase, R relays combine the overheard symbols from multiple users during the first phase to a single symbol and then take turns to transmit it to destination in the last R time slots, i.e., relay r transmits its symbol in time slot $U+r$. We outline the STNC with TAS/MRC in the cooperative MIMO network as follows.

In the first phase, TAS/MRC is applied between each user and the destination D. The optimal antenna amongst the N_u antennas at user u is selected to maximize the instantaneous SNR of the signal from user u to the destination D. Therefore, the index of the optimal transmit antenna, n_u^*, is determined as $n_u^* = \arg\max_{1 \leq n_u \leq N_u} \|h_{n_u\mathrm{D}}\|_F$, where $h_{n_u\mathrm{D}}$ is the channel vector between the n_uth antenna at user u and the multiple antennas at the destination D, and $\|\cdot\|_F$ denotes the Frobenius norm. Since the multiple antennas at one node do not appear colocated and the MIMO links are close to independent and identically distributed (i.i.d.) Rayleigh fading in the rich-scattering scenario [20], we assume that $h_{n_u\mathrm{D}}$ has i.i.d. Rayleigh entries. The signals received at the destination D and relay r from user u in time slot u are

$$y_{u\mathrm{D}}(t) = h_{n_u^*\mathrm{D}} \sqrt{P_{0u}} x_u s_u(t) + n_{u\mathrm{D}}(t) \tag{1}$$

and

$$y_{ur}(t) = h_{n_u^* r} \sqrt{P_{0u}} x_u s_u(t) + n_{ur}(t) \tag{2}$$

respectively, where $h_{n_u^*\mathrm{D}}$ and $h_{n_u^* r}$ denote the Rayleigh channel vectors between the n_u^*th antenna at user u and multiple antennas at the destination D and between the n_u^*th antenna at user u and multiple antennas at relay r respectively, $n_{u\mathrm{D}}(t)$ and $n_{ur}(t)$ are the additive white Gaussian noise (AWGN) vectors with zero mean and variances of $N_0\mathbf{I}_{N_{\mathrm{D}}}$ and $N_0\mathbf{I}_{N_r}$ respectively, $\mathbf{I}_n$, $n \in \{N_{\mathrm{D}}, N_r\}$, denotes the identity matrix of size n, P_{0u} is the transmit power of x_u, and x_u and $s_u(t)$ denote the symbol with unit energy transmitted by user u and the related spreading code, respectively. The cross correlation between $s_u(t)$ and $s_{u'}(t)$ is defined as $\rho_{uu'} = \langle s_u(t), s_{u'}(t)\rangle$, where $\langle f(t), g(t)\rangle \triangleq \frac{1}{T}\int_0^T f(t) g^*(t) dt$ is the inner product between $f(t)$ and $g(t)$ with the symbol interval T, and $g^*(t)$ is the complex conjugate of $g(t)$. Moreover, we assume that $\rho_{uu} = \|s_u(t)\|_F^2 = 1$.

After combining the received signal replicas from different antennas using MRC and applying matched-filtering, the received signals at the destination D and relay r are given by

$$y_{u\mathrm{D}}^u = \langle w_{u\mathrm{D}} y_{u\mathrm{D}}(t), s_u(t)\rangle = \|h_{n_u^*\mathrm{D}}\|_F \sqrt{P_{0u}} x_u + n_{u\mathrm{D}}^u \tag{3}$$

and

$$y_{ur}^{u} = \langle w_{ur} y_{ur}(t), s_u(t) \rangle = \|h_{n_u^* r}\|_F \sqrt{P_{0u}} x_u + n_{ur}^{u} \tag{4}$$

respectively, where weight vectors $w_{u\mathrm{D}} = h_{n_u^*\mathrm{D}}^{\dagger} / \|h_{n_u^*\mathrm{D}}\|_F$ and $w_{ur} = h_{n_u^* r}^{\dagger} / \|h_{n_u^* r}\|_F$, $(\cdot)^{\dagger}$ denotes conjugate transpose, and $n_{u\mathrm{D}}^{u}$ and n_{ur}^{u} are AWGN with zero mean and variance of N_0.

In the second phase, TAS/MRC is applied between each relay and the destination D. The index of the optimal transmit antenna at relay r, n_r^*, is determined as $n_r^* = \arg\max_{1 \le n_r \le N_r} \|h_{n_r\mathrm{D}}\|_F$, where $h_{n_r\mathrm{D}}$ is the channel vector between the n_rth antenna at relay r and the multiple antennas at the destination D with i.i.d. Rayleigh entries. Each relay linearly combines the overheard symbols during the first phase to a single encoded signal and then forwards it to the destination D in the second phase. The signal received at the destination D from relay r in time slot $U + r$ is

$$y_{r\mathrm{D}}(t) = h_{n_r^*\mathrm{D}} \sum_{u=1}^{U} \beta_{ru} \sqrt{P_{ru}} x_u s_u(t) + n_{r\mathrm{D}}(t) \tag{5}$$

where $h_{n_r^*\mathrm{D}}$ denotes the Rayleigh channel vector between the n_r^*th antenna at relay r and multiple antennas at the destination D, $n_{r\mathrm{D}}(t)$ is the AWGN vector with zero mean and variance of $N_0 \mathbf{I}_{N_\mathrm{D}}$, P_{ru} is the transmit power of x_u at relay r, and β_{ru} denotes the detection state of relay r on x_u. If relay r correctly decodes x_u, $\beta_{ru} = 1$; otherwise, $\beta_{ru} = 0$. The relay detection state can possibly be done by examining the included cyclic-redundancy-check digits or the received SNR levels. Here, we assume that the destination D knows the detection states of the relays, which can be obtained through the indicators sent by the relays.

By combining the received signal replicas from multiple antennas with MRC and applying matched-filtering, the received signal at the destination D from user u' through relay r is

$$\begin{aligned} y_{r\mathrm{D}}^{u'} &= \langle w_{r\mathrm{D}} y_{r\mathrm{D}}(t), s_{u'}(t) \rangle \\ &= \|h_{n_r^*\mathrm{D}}\|_F \sum_{u=1}^{U} \beta_{ru} \sqrt{P_{ru}} x_u \rho_{uu'} + n_{r\mathrm{D}}^{u'} \end{aligned} \tag{6}$$

where weight vector $w_{r\mathrm{D}} = h_{n_r^*\mathrm{D}}^{\dagger} / \|h_{n_r^*\mathrm{D}}\|_F$, and $n_{r\mathrm{D}}^{u'}$ is AWGN with zero mean and variance of N_0. Rewriting (6) into matrix form yields

$$\tilde{y}_{r\mathrm{D}} = \|h_{n_r^*\mathrm{D}}\|_F \mathbf{R} \mathbf{P}_r x + \tilde{n}_{r\mathrm{D}} \tag{7}$$

where

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{21} & \dots & \rho_{U1} \\ \rho_{12} & 1 & \dots & \rho_{U2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1U} & \rho_{2U} & \dots & 1 \end{bmatrix}$$

with $\tilde{y}_{r\mathrm{D}} = [y_{r\mathrm{D}}^1, y_{r\mathrm{D}}^2, \dots, y_{r\mathrm{D}}^U]^T$, $\mathbf{P}_r = \mathrm{diag}\{\beta_{r1}\sqrt{P_{r1}}, \beta_{r2}\sqrt{P_{r2}}, \dots, \beta_{rU}\sqrt{P_{rU}}\}$, $x = [x_1, x_2, \dots, x_U]^T$, $\tilde{n}_{r\mathrm{D}} = [n_{r\mathrm{D}}^1, n_{r\mathrm{D}}^2, \dots, n_{r\mathrm{D}}^U]^T$ is the AWGN vector with zero mean and variance matrix of $N_0 \mathbf{R}$, and $[\cdot]^T$ denotes transpose. Here, the correlation amongst the entries of $\tilde{n}_{r\mathrm{D}}$ is from the matched-filtering operation. Assuming that $\mathbf{R}$ is invertible with the inverse matrix $\mathbf{R}^{-1}$ [21], the soft symbol of x_u from relay r is derived by multiplying the both sides of (7) with $\mathbf{R}^{-1}$, which yields

$$\tilde{y}_{r\mathrm{D}}^{u} = \|h_{n_r^*\mathrm{D}}\|_F \beta_{ru} \sqrt{P_{ru}} x_u + \tilde{n}_{r\mathrm{D}}^{u} \tag{8}$$

where $\tilde{n}_{r\mathrm{D}}^{u}$ is the AWGN with zero mean and variance of $N_0 \theta_u$ with θ_u being the uth diagonal element of matrix $\mathbf{R}^{-1}$ associated with symbol x_u.

By combining the information on x_u from user u and R relays with MRC, the instantaneous end-to-end SNR of x_u at the destination D is written as

$$\Upsilon_u = \Upsilon_{u\mathrm{D}} + \sum_{r=1}^{R} \beta_{ru} \Upsilon_{r\mathrm{D}} \tag{9}$$

where $\Upsilon_{u\mathrm{D}} = \|h_{n_u^*\mathrm{D}}\|_F^2 P_{0u} / N_0$ and $\Upsilon_{r\mathrm{D}} = \|h_{n_r^*\mathrm{D}}\|_F^2 P_{ru} / (N_0 \theta_u)$. Similarly, the instantaneous SNR of x_u at relay r can be given by $\Upsilon_{ur} = \|h_{n_u^* r}\|_F^2 P_{0u} / N_0$. By defining the active relay set for user u as $\mathcal{D}_u = \{r : \beta_{ru} = 1, r = 1, 2, \dots, R\}$, (9) is rewritten as

$$\Upsilon_u = \Upsilon_{u\mathrm{D}} + \sum_{r \in \mathcal{D}_u} \Upsilon_{r\mathrm{D}}. \tag{10}$$

When all the relays fail to decode the symbol x_u, i.e., $\mathcal{D}_u = \phi$, it is obvious that the STNC with TAS/MRC reduces to the conventional single hop TAS/MRC MIMO scheme.

We note that the optimal transmit antenna for the destination D corresponds to a random transmit antenna for relay r, which is due to the fact that the optimal antenna at user u is entirely determined by the channel state between user u and the destination D and that the channels from user u to the destination D and to relay r are mutually independent. Therefore, the output SNR at r after MRC combing, Υ_{ur}, is the sum of the SNR at each antenna of r. As such, the probability density function (PDF) of Υ_{ur} is the convolution of exponential PDFs, and is given by [22]

$$f_{\Upsilon_{ur}}(\gamma) = \frac{\gamma^{N_r - 1} e^{-\frac{\gamma}{\bar{\gamma}_{ur}}}}{\Gamma(N_r) \bar{\gamma}_{ur}^{N_r}} \tag{11}$$

where $\bar{\gamma}_{ur} = E[\|h_{ij}^{ru}\|_F^2] P_{0u} / N_0$ is the average per-antenna SNR between user u and relay r, and $E[x]$ denotes the expectation of x. Since $h_{n_u^*\mathrm{D}}$ is the channel vector between the optimal transmit antenna n_u^* at user u and N_D receive antennas at the destination D, the PDF of $\Upsilon_{u\mathrm{D}}$ is

$$\begin{aligned} f_{\Upsilon_{u\mathrm{D}}}(\gamma) = &\frac{N_u}{\Gamma(N_\mathrm{D})} \sum_{g_0=0}^{N_u - 1} \binom{N_u - 1}{g_0} \frac{\Phi_g e^{-\xi_0 \gamma}}{(-1)^{g_0} \bar{\gamma}_{u\mathrm{D}}} \\ &\times \left(\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}\right)^{\phi_g + N_{\mathrm{D}-1}} \end{aligned} \tag{12}$$

where $\bar{\gamma}_{u\mathrm{D}} = E[\|h_{ij}^{\mathrm{D}u}\|_F^2] P_{0u} / N_0$ is the average per-antenna SNR between user u and the destination D, and Φ_g, ϕ_g, and ξ_0 are defined in Appendix A.

Proof: The proof is presented in Appendix A. ■

Following the similar procedures of deriving the PDF of $\Upsilon_{u\mathrm{D}}$, we can derive the the PDF of $\Upsilon_{r\mathrm{D}}$ as

$$f_{\Upsilon_{r\mathrm{D}}}(\gamma) = \frac{N_r}{\Gamma(N_\mathrm{D})} \sum_{h_{r,0}=0}^{N_r-1} \binom{N_r-1}{h_{r,0}} \frac{\Phi_r e^{-\xi_{r,0}\gamma}}{(-1)^{h_{r,0}}\bar{\gamma}_{r\mathrm{D}}} \times \left(\frac{\gamma}{\bar{\gamma}_{r\mathrm{D}}}\right)^{\phi_r+N_\mathrm{D}-1} \tag{13}$$

where $\Phi_r = \prod_{i=1}^{N_\mathrm{D}-1} (\sum_{h_{r,i}=0}^{h_{r,i-1}} \binom{h_{r,i-1}}{h_{r,i}} (\frac{1}{i!})^{h_{r,i}-h_{r,i+1}})$, $\phi_r = \sum_{i=1}^{N_\mathrm{D}-1} h_{r,i}$, $h_{r,N_\mathrm{D}} = 0$, $\xi_{r,0} = (h_{r,0}+1)/\bar{\gamma}_{r\mathrm{D}}$, and $\bar{\gamma}_{r\mathrm{D}} = E[\|h_{ij}^{\mathrm{D}r}\|_F^2] P_{ru}/(N_0\theta_u)$ is the average per-antenna SNR between relay r and the destination D.

In (12) and (13), the MIMO links between the same transmitter/receiver pair are assumed i.i.d. Rayleigh distributed. If they are i.n.i.d. Rayleigh distributed, we have

$$f_{\Upsilon_{\mu\mathrm{D}}}(\gamma) = \sum_{j=1}^{N_\mu} \alpha_j \sum_{i=1}^{N_\mathrm{D}} \beta_{ij} e^{-\frac{\gamma}{\bar{\gamma}_{ij}^{\mathrm{D}\mu}}} \sum_{\Omega_m} 1^{m_0} \times \prod_{j'=1, j'\neq j}^{N_\mu} \left(-\alpha_{j'} \sum_{i'=1}^{N_\mathrm{D}} \beta_{i'j'} \bar{\gamma}_{i'j'}^{\mathrm{D}\mu} e^{-\frac{\gamma}{\bar{\gamma}_{i'j'}^{\mathrm{D}\mu}}}\right)^{m_{j'}} \tag{14}$$

where $\mu \in \{u, r\}$, Ω_m is the set of nonnegative integers $\{m_0, m_1, \ldots, m_{j-1}, m_{j+1}, \ldots, m_{N_\mu}\}$ such that $\sum_{j'=0, j'\neq j}^{N_\mu} m_{j'} = N_\mu - 1$ with $0 \le m_0 \le N_\mu - 1$ and $m_{j'} \in \{0, 1\}$, $1 \le j' \le N_\mu$, and α_j, β_{ij}, and $\bar{\gamma}_{ij}^{\mathrm{D}\mu}$ are defined in Appendix B.

Proof: The proof is presented in Appendix B. ■

The PDF of $\Upsilon_{\mu\mathrm{D}}$ given by (14) is the summation of exponential functions. Therefore, (14) is in the same form as (12) and (13). As such, it is easy to generate our theoretical analysis in the following sections to the i.n.i.d. Rayleigh fading case.

III. Performance With Perfect Feedback

In this section, we analyze the performance of STNC in the cooperative network without feedback delays. We first derive the new closed-form expressions for the exact OP and SER. We then present the asymptotic expressions for the OP and SER to provide the useful insights into the behavior of STNC with TAS/MRC in the high SNR regime.

A. Exact Performance

1) OP: The outage probability is an important quality-of-service measure as it characterizes the probability that the instantaneous end-to-end SNR falls below a predetermined threshold Υ_th. Here, we define $\Upsilon_\mathrm{th} = 2^{\mathcal{R}} - 1$, where $\mathcal{R}$ is the target transmission rate of the users.

From (10), the OP of STNC associated with x_u can be given by

$$P_{out,u} = \sum_{|\mathcal{D}_u|=0}^{R} \Pr(\Upsilon_u < \Upsilon_\mathrm{th} | \mathcal{D}_u) \Pr(\mathcal{D}_u) \tag{15}$$

where $|\mathcal{D}_u|$ is the size of $\mathcal{D}_u$. As there are $\vartheta = \binom{R}{|\mathcal{D}_u|}$ possible choices of the active relay set $\mathcal{D}_u$ for a given $|\mathcal{D}_u|$, (15) is rewritten as

$$P_{out,u} = \sum_{|\mathcal{D}_u|=0}^{R} \sum_{v=1}^{\vartheta} \Pr(\Upsilon_u < \Upsilon_\mathrm{th} | \mathcal{D}_{u,v}) \Pr(\mathcal{D}_{u,v}) \tag{16}$$

where $\mathcal{D}_{u,v}$ is the vth possible choice of $|\mathcal{D}_u|$ relays from the R relays, and the probability $\Pr(\mathcal{D}_{u,v})$ is

$$\Pr(\mathcal{D}_{u,v}) = \prod_{r\in\mathcal{D}_{u,v}} (1 - P_{s,ur}) \prod_{r\notin\mathcal{D}_{u,v}} P_{s,ur} \tag{17}$$

where $P_{s,ur}$ is the SER of detecting x_u at relay r. We will derive $P_{s,ur}$ later in this subsection. In the following, we proceed to evaluate the conditional probability $\Pr(\Upsilon_u < \Upsilon_\mathrm{th} | \mathcal{D}_{u,v})$ and the probability $\Pr(\mathcal{D}_{u,v})$.

Without loss of generality, we now re-number the indices of the active relays belonging to $\mathcal{D}_{u,v}$ as $\{1, 2, \ldots, \rho\}$, where $\rho = |\mathcal{D}_u|$. As such, the end-to-end SNR of x_u conditioned on active relay set $\mathcal{D}_{u,v}$, $\Upsilon_{u|\mathcal{D}_{u,v}}$, is

$$\Upsilon_{u|\mathcal{D}_{u,v}} = \Upsilon_{u\mathrm{D}} + \sum_{r=1}^{\rho} \Upsilon_{r\mathrm{D}}. \tag{18}$$

Given that $\Upsilon_{u|\mathcal{D}_{u,v}}$ is the summation of $\rho + 1$ independent largest order statistics of Gamma distributed variables, we present the PDF of $\Upsilon_{u|\mathcal{D}_{u,v}}$ in the following lemma.

Lemma 1: The PDF of $\Upsilon_{u|\mathcal{D}_{u,v}}$ is derived as

$$f_{\Upsilon_{u|\mathcal{D}_{u,v}}}(\gamma) = \sum_{g_0=0}^{N_u-1} \sum_{h_{1,0}=0}^{N_1-1} \cdots \sum_{h_{\rho,0}=0}^{N_\rho-1} \Psi \left(\sum_{p=1}^{\phi_g+N_\mathrm{D}} \frac{c_u \gamma^{p-1} e^{-\xi_0\gamma}}{\Gamma(p)} + \sum_{r=1}^{\rho} \sum_{p=1}^{\phi_r+N_\mathrm{D}} \frac{c_r \gamma^{p-1} e^{-\xi_{r,0}\gamma}}{\Gamma(p)} \right) \tag{19}$$

where Ψ, c_u, and c_r are defined in Appendix C.

Proof: The proof is presented in Appendix C. ■

Integrating (19) from 0 to γ with the aid of the definite integral of the exponential function, which is given by [23, Eq. (3.351.1)], we obtain the corresponding cumulative distribution function (CDF) of $\Upsilon_{u|\mathcal{D}_{u,v}}$, as shown in (20), shown at the bottom of the page.

$$\begin{aligned} F_{\Upsilon_{u|\mathcal{D}_{u,v}}}(\gamma) &= \sum_{g_0=0}^{N_u-1} \sum_{h_{1,0}=0}^{N_1-1} \cdots \sum_{h_{\rho,0}=0}^{N_\rho-1} \Psi \left(\sum_{p=1}^{\phi_g+N_\mathrm{D}} \frac{c_u \int_0^\gamma x^{p-1} e^{-\xi_0 x} dx}{\Gamma(p)} + \sum_{r=1}^{\rho} \sum_{p=1}^{\phi_r+N_\mathrm{D}} \frac{c_r \int_0^\gamma x^{p-1} e^{-\xi_{r,0} x} dx}{\Gamma(p)} \right) \\ &= 1 - \sum_{g_0=0}^{N_u-1} \sum_{h_{1,0}=0}^{N_1-1} \cdots \sum_{h_{\rho,0}=0}^{N_\rho-1} \Psi \left(\sum_{p=1}^{\phi_g+N_\mathrm{D}} \sum_{k=0}^{p-1} \frac{c_u \gamma^k e^{-\xi_0\gamma}}{k!\,\xi_0^{p-k}} + \sum_{r=1}^{\rho} \sum_{p=1}^{\phi_r+N_\mathrm{D}} \sum_{k=0}^{p-1} \frac{c_r \gamma^k e^{-\xi_{r,0}\gamma}}{k!\,\xi_{r,0}^{p-k}} \right) \end{aligned} \tag{20}$$

Based on (20), it is easy to derive the conditional probability $\Pr(\Upsilon_u < \Upsilon_{\rm th}|\mathcal{D}_{u,v})$ as

$$\Pr(\Upsilon_u < \Upsilon_{\rm th}|\mathcal{D}_{u,v}) = F_{\Upsilon_{u|\mathcal{D}_{u,v}}}(\Upsilon_{\rm th}) \tag{21}$$

where $F_{\Upsilon_{u|\mathcal{D}_{u,v}}}(\Upsilon_{\rm th})$ is the CDF of $\Upsilon_{u|\mathcal{D}_{u,v}}$ evaluated at $\gamma = \Upsilon_{\rm th}$.

We now derive the SER $P_{s,ur}$ to calculate the $\Pr(\mathcal{D}_{u,v})$ according to (17). The closed-form expression for SER could be given directly in terms of the CDF of the received SNR, $F(\gamma)$, as [24]

$$P_s = \frac{a}{2}\sqrt{\frac{b}{\pi}}\int_0^\infty \frac{F(\gamma)}{\sqrt{\gamma}}e^{-b\gamma}d\gamma \tag{22}$$

where the parameters a and b are up to a specific used modulation scheme, which encompasses a variety of modulations such as binary phase-shift keying (BPSK) ($a = b = 1$), M-ary phase-shift keying ($a = 2$, $b = \sin^2(\pi/M)$), and M-ary quadrature amplitude modulation ($a = 4(\sqrt{M}-1)/\sqrt{M}$, $b = 3/(2(M-1))$) [22]. Integrating (11) with the aid of [23, Eq. (3.351.1)] and then substituting the outcome into (22) with the aid of identity [23, Eq. (3.371)] yield

$$P_{s,ur} = \frac{a}{2} - \frac{a\sqrt{b}}{2}\sum_{i=0}^{N_r-1}\frac{(2i-1)!!}{i!2^i\bar{\gamma}_{ur}^i}\left(\frac{1}{\bar{\gamma}_{ur}}+b\right)^{-i-\frac{1}{2}}. \tag{23}$$

Substituting (23) into (17), the probability of the active relay set $\mathcal{D}_{u,v}$ is obtained. Therefore, we can derive the exact closed-form expression for the OP of STNC with TAS/MRC by substituting (17) and (21) into (16).

2) SER: Similar to (16), the SER of STNC associated with x_u is given by

$$P_{s,u} = \sum_{|\mathcal{D}_u|=0}^{R}\sum_{v=1}^{\vartheta} P_{s,u|\mathcal{D}_{u,v}}\Pr(\mathcal{D}_{u,v}) \tag{24}$$

where $P_{s,u|\mathcal{D}_{u,v}}$ denotes the SER of detecting x_u at the destination D conditioned on active relay set $\mathcal{D}_{u,v}$.

Substituting the CDF of end-to-end SNR associated with x_u, (20), into (22) with the aid of identity [23, Eq. (3.371)], we derive the exact SER of x_u conditioned on $\mathcal{D}_{u,v}$, as shown in (25), shown at the bottom of the page. Substituting (17) and (25) into (24) yields the exact closed-form SER expression for STNC with TAS/MRC. The SER expression in (24) is valid for a variety of modulations and applies to arbitrary numbers of users and relays and arbitrary antenna numbers at the nodes. It is noticed that our result encompasses the SER expression for single-antenna STNC in [8] as a special case.

B. Asymptotic Performance

In this subsection, we derive the asymptotic OP and SER expressions in the high SNR regime to characterize the behavior of STNC with TAS/MRC.

1) OP: In the high SNR regime, the probability that relay r decodes the symbol x_u correctly approaches one, i.e., $\lim_{\bar{\gamma}_{ur}\to\infty}\beta_{ru} = 1$. In such a scenario,

$$\Upsilon_u^\infty = \Upsilon_{u\rm D}^\infty + \sum_{r=1}^{R}\Upsilon_{r\rm D}^\infty. \tag{26}$$

To obtain the CDF of Υ_u^∞, the first order expansions of the PDFs of $\Upsilon_{u\rm D}$ and $\Upsilon_{r\rm D}$ are needed. Applying the Taylor series expansion of the exponential function in (12) and (13) and retaining the first order term, we obtain $f_{\Upsilon_{u\rm D}^\infty}(\gamma)$ and $f_{\Upsilon_{r\rm D}^\infty}(\gamma)$ as

$$f_{\Upsilon_{u\rm D}^\infty}(\gamma) \approx \frac{N_u\gamma^{N_{\rm D}N_u-1}}{\Gamma(N_{\rm D})\left(\Gamma(N_{\rm D}+1)\right)^{N_u-1}\bar{\gamma}_{u\rm D}^{N_{\rm D}N_u}} \tag{27}$$

and

$$f_{\Upsilon_{r\rm D}^\infty}(\gamma) \approx \frac{N_r\gamma^{N_{\rm D}N_r-1}}{\Gamma(N_{\rm D})\left(\Gamma(N_{\rm D}+1)\right)^{N_r-1}\bar{\gamma}_{r\rm D}^{N_{\rm D}N_r}} \tag{28}$$

respectively. Performing the Laplace transforms of (27) and (28) with the aid of [23, Eq. (3.351.3)] to calculate the moment generating functions (MGFs) of $\Upsilon_{u\rm D}^\infty$ and $\Upsilon_{r\rm D}^\infty$ and multiplying the MGFs together, we have the MGF of Υ_u^∞ as

$$\mathcal{M}_{\Upsilon_u^\infty}(s) \approx \Theta(s\bar{\gamma}_{u\rm D})^{-G_d} \tag{29}$$

where

$$\Theta = \frac{N_u\Gamma(N_{\rm D}N_u)}{\Gamma(N_{\rm D})\left(\Gamma(N_{\rm D}+1)\right)^{N_u-1}} \times\prod_{r=1}^{R}\frac{N_r\Gamma(N_{\rm D}N_r)\kappa_r^{N_{\rm D}N_r}}{\Gamma(N_{\rm D})\left(\Gamma(N_{\rm D}+1)\right)^{N_r-1}}$$

with $\kappa_r = \bar{\gamma}_{u\rm D}/\bar{\gamma}_{r\rm D}$ and $G_d = (N_u + \sum_{r=1}^{R}N_r)N_{\rm D}$. Performing the inverse Laplace transform of (29) and integrating the outcome produce the CDF of Υ_u^∞ as

$$F_{\Upsilon_u^\infty}(\gamma) \approx \frac{\Theta}{\Gamma(G_d+1)\bar{\gamma}_{u\rm D}^{G_d}}\gamma^{G_d}. \tag{30}$$

By evaluating $F_{\Upsilon_u^\infty}(\gamma)$ at $\gamma = \Upsilon_{\rm th}$, the OP of the end-to-end SNR associated with x_u in the high SNR regime is given by

$$P_{out,u}^\infty = F_{\Upsilon_u^\infty}(\Upsilon_{\rm th}). \tag{31}$$

$$P_{s,u|\mathcal{D}_{u,v}} = \frac{a}{2} - \frac{a\sqrt{b}}{2}\sum_{g_0=0}^{N_u-1}\sum_{h_{1,0}=0}^{N_1-1}\cdots\sum_{h_{\rho,0}=0}^{N_\rho-1}\Psi\left(\sum_{p=1}^{\phi_g+N_{\rm D}}\sum_{k=0}^{p-1}\frac{c_u(2k-1)!!\xi_0^{k-p}}{k!2^k(\xi_0+b)^{k+\frac{1}{2}}} + \sum_{r=1}^{\rho}\sum_{p=1}^{\phi_r+N_{\rm D}}\sum_{k=0}^{p-1}\frac{c_r(2k-1)!!\xi_{r,0}^{k-p}}{k!2^k(\xi_{r,0}+b)^{k+\frac{1}{2}}}\right) \tag{25}$$

2) SER: We now derive the asymptotic SER based on $F_{\Upsilon_u^\infty}(\gamma)$. Substituting (30) into (22) and calculating the resultant integral, we derive the asymptotic SER as

$$P_{s,u}^{\infty} \approx (G_a \bar{\gamma}_{u\mathrm{D}})^{-G_d} \tag{32}$$

where the array gain $G_a = b\left(\frac{a\Theta(2G_d-1)!!}{\Gamma(G_d+1)2^{G_d+1}}\right)^{-\frac{1}{G_d}}$. Based on (32), we demonstrate that STNC with TAS/MRC achieves the full diversity order of $G_d = (N_u + \sum_{r=1}^{R} N_r)N_\mathrm{D}$ in the cooperative MIMO network and the system performance is significantly improved through integrating multiple antennas in the nodes. The contributions of the $u-\mathrm{D}$ and $r-\mathrm{D}$ links to the diversity order are $N_u N_\mathrm{D}$ and $N_r N_\mathrm{D}$ respectively, and their contributions to the array gain are $\frac{N_u\Gamma(N_\mathrm{D}N_u)}{\Gamma(N_\mathrm{D})(\Gamma(N_\mathrm{D}+1))^{N_u-1}}$ and $\frac{N_r\Gamma(N_\mathrm{D}N_r)\kappa_r^{N_\mathrm{D}N_r}}{\Gamma(N_\mathrm{D})(\Gamma(N_\mathrm{D}+1))^{N_r-1}}$ respectively. In the special case where all the nodes are equipped with a single antenna with $N_u = N_r = N_\mathrm{D} = 1$, the diversity order reduces to $R+1$, which agrees with the result given in [8], [10].

IV. Performance With Delayed Feedback

In the presence of perfect feedback, the optimal transmit antenna is selected based on accurate CSI. However, the feedback processes usually lead to delay, which results in that the TAS is performed based on outdated CSI. In this section, we examine the detrimental impact of delayed feedback on the performance of STNC with TAS/MRC.

A. Exact Performance

Due to the delay between the instants of channel estimation and data transmission, we assume that the transmit antennas of user u and relay r are selected based on the outdated CSI with τ_u and τ_r time delays, respectively.

To model the relationship between $h_{ij}^{\mathrm{D}\mu}(t)$ and $h_{ij}^{\mathrm{D}\mu}(t-\tau_\mu)$, $\mu \in \{u, r\}$, we employ the time-varying channel feedback error model to express the channel coefficient as $h_{ij}^{\mathrm{D}\mu}(t) = \rho_\mu h_{ij}^{\mathrm{D}\mu}(t-\tau_\mu) + \sqrt{1-|\rho_\mu|^2}e_\mu(t)$[25], [26], where $e_\mu(t) \sim \mathcal{CN}(0, d_{\mu\mathrm{D}}^{-\alpha})$ and ρ_μ is the normalized correlation coefficient between $h_{ij}^{\mathrm{D}\mu}(t)$ and $h_{ij}^{\mathrm{D}\mu}(t-\tau_\mu)$. For Clarke's fading spectrum, $\rho_\mu = \mathcal{J}_0(2\pi f_\mu \tau_\mu)$, where f_μ is the Doppler frequency and $\mathcal{J}_0(\cdot)$ is the zeroth-order Bessel function of the first kind [23, Eq. (8.402)]. Define $\tilde{\Upsilon}_{\mu\mathrm{D}}$, $\mu \in \{u, r\}$, as the outdated SNR, and the following lemma gives the PDF of $\tilde{\Upsilon}_{\mu\mathrm{D}}$.

Lemma 2: The PDFs of $\tilde{\Upsilon}_{u\mathrm{D}}$ and $\tilde{\Upsilon}_{r\mathrm{D}}$ are

$$f_{\tilde{\Upsilon}_{u\mathrm{D}}}(\tilde{\gamma}) = \sum_{g_0=0}^{N_u-1}\sum_{k=0}^{\phi_g} \frac{\tilde{\Psi}_{0,k}\tilde{\gamma}^{N_\mathrm{D}+k-1}}{\Gamma(N_\mathrm{D}+k)} e^{-\tilde{\xi}_0\tilde{\gamma}} \tag{33}$$

and

$$f_{\tilde{\Upsilon}_{r\mathrm{D}}}(\tilde{\gamma}) = \sum_{h_{r,0}=0}^{N_r-1}\sum_{k_r=0}^{\phi_r} \frac{\tilde{\Psi}_{r_0,k_r}\tilde{\gamma}^{N_\mathrm{D}+k_r-1}}{\Gamma(N_\mathrm{D}+k_r)} e^{-\tilde{\xi}_{r,0}\tilde{\gamma}} \tag{34}$$

respectively, where $\tilde{\Psi}_{0,k}$, $\tilde{\Psi}_{r_0,k_r}$, $\tilde{\xi}_0$, and $\tilde{\xi}_{r,0}$ are defined in Appendix D.

Proof: The proof is presented in Appendix D. ∎

In the presence of perfect feedback, i.e., $\rho_\mu = 1$, $\tilde{\Upsilon}_{\mu\mathrm{D}} = \Upsilon_{\mu\mathrm{D}}$. In such a case, it is easy to demonstrate that (33) and (34) are equal to (12) and (13), respectively, by setting $k = \phi_g$ in (33) and $k_r = \phi_r$ in (34). For the case of fully delayed feedback, i.e., $\rho_\mu = 0$, $\tilde{\Upsilon}_{\mu\mathrm{D}}$ reduces to a gamma distributed variable with shape parameter N_D and scale parameter $\bar{\gamma}_{\mu\mathrm{D}}$, and we demonstrate that (33) and (34) reduce to the same form as that in (11) by setting $k=0$ and $k_r = 0$ in (33) and (34), respectively.

1) OP: Following the procedure outlined in Section III-A1, the end-to-end SNR of x_u conditioned on active relay set $\mathcal{D}_{u,v}$ with delayed feedback is

$$\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}} = \tilde{\Upsilon}_{u\mathrm{D}} + \sum_{r=1}^{\rho}\tilde{\Upsilon}_{r\mathrm{D}}. \tag{35}$$

Based on the PDFs of $\tilde{\Upsilon}_{u\mathrm{D}}$ and $\tilde{\Upsilon}_{r\mathrm{D}}$, the PDF of $\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}$ is presented in the following lemma.

Lemma 3: The PDF of $\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}$ is derived as

$$f_{\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}}(\tilde{\gamma}) = \sum_{g_0=0}^{N_u-1}\sum_{k=0}^{\phi_g}\sum_{h_{1,0}=0}^{N_1-1}\sum_{k_1=0}^{\phi_1}\cdots\sum_{h_{\rho,0}=0}^{N_\rho-1}\sum_{k_\rho=0}^{\phi_\rho}\tilde{\Psi} \times\left(\sum_{p=1}^{N_\mathrm{D}+k}\frac{\tilde{c}_u\tilde{\gamma}^{p-1}e^{-\tilde{\xi}_0\tilde{\gamma}}}{\Gamma(p)} + \sum_{r=1}^{\rho}\sum_{p=1}^{N_\mathrm{D}+k_r}\frac{\tilde{c}_r\tilde{\gamma}^{p-1}e^{-\tilde{\xi}_{r,0}\tilde{\gamma}}}{\Gamma(p)}\right) \tag{36}$$

where

$$\tilde{c}_u = (-1)^{N_\mathrm{D}+k-p}\sum_{\tilde{\Omega}_u}\prod_{r=1}^{\rho}\frac{\binom{N_\mathrm{D}+k_r+i_r-1}{i_r}}{(\tilde{\xi}_{r,0}-\tilde{\xi}_0)^{N_\mathrm{D}+k_r+i_r}}$$

$$\tilde{c}_r = (-1)^{N_\mathrm{D}+k_r-p}\sum_{\tilde{\Omega}_r}\frac{\binom{N_\mathrm{D}+k+i_0-1}{i_0}}{(\tilde{\xi}_0-\tilde{\xi}_{r,0})^{N_\mathrm{D}+k+i_0}} \times\prod_{r'=1,r'\neq r}^{\rho}\frac{\binom{N_\mathrm{D}+k_{r'}+i_{r'}-1}{i_{r'}}}{(\tilde{\xi}_{r',0}-\tilde{\xi}_{r,0})^{N_\mathrm{D}+k_{r'}+i_{r'}}}$$

with $\tilde{\Omega}_u$ is the set of nonnegative integers $\{i_1, i_2, \ldots, i_\rho\}$, such that $\sum_{j=1}^{\rho} i_j = N_\mathrm{D}+k-p$, $\tilde{\Omega}_r$ is the set of nonnegative integers $\{i_0, i_1, \ldots, i_{r-1}, i_{r+1}, \ldots, i_\rho\}$, such that $\sum_{j=0,j\neq r}^{\rho} i_j = N_\mathrm{D}+k_r-p$, and $\tilde{\Psi} = \tilde{\Psi}_{0,k}\prod_{r=1}^{\rho}\tilde{\Psi}_{r_0,k_r}$.

Proof: Following the similar procedures outlined in Appendix C, we derive the MGF of $\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}$ by multiplying the MGFs of $\tilde{\Upsilon}_{u\mathrm{D}}$ and $\tilde{\Upsilon}_{r\mathrm{D}}$. Here, the MGFs of $\tilde{\Upsilon}_{u\mathrm{D}}$ and $\tilde{\Upsilon}_{r\mathrm{D}}$ can be obtained based on the PDFs of $\tilde{\Upsilon}_{u\mathrm{D}}$ and $\tilde{\Upsilon}_{r\mathrm{D}}$. Then performing the inverse Laplace transform with some algebraic manipulations, we derive the PDF of $\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}$ as (36). ∎

Integrating (36) with the aid of [23, Eq. (3.351.1)], we obtain the CDF of $\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}$ as

$$F_{\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}}(\tilde{\gamma}) = 1 - \sum_{g_0=0}^{N_u-1}\sum_{k=0}^{\phi_g}\sum_{h_{1,0}=0}^{N_1-1}\sum_{k_1=0}^{\phi_1}\cdots\sum_{h_{\rho,0}=0}^{N_\rho-1}\sum_{k_\rho=0}^{\phi_\rho}\tilde{\Psi} \times\left(\sum_{p=1}^{N_{\rm D}+k}\sum_{q=0}^{p-1}\frac{\tilde{c}_u\tilde{\gamma}^q e^{-\tilde{\xi}_0\tilde{\gamma}}}{q!\tilde{\xi}_0^{p-q}} + \sum_{r=1}^{\rho}\sum_{p=1}^{N_{\rm D}+k_r}\sum_{q=0}^{p-1}\frac{\tilde{c}_r\tilde{\gamma}^q e^{-\tilde{\xi}_{r,0}\tilde{\gamma}}}{q!\tilde{\xi}_{r,0}^{p-q}}\right). \tag{37}$$

In the presence of perfect feedback, i.e., $\rho_u = \rho_r = 1$ and $\tilde{\Upsilon}_u = \Upsilon_u$, it is easy to demonstrate that (37) is equal to (20) by setting $k = \phi_g$ and $k_r = \phi_r$ in (37).

As we mentioned previously that the optimal transmit antenna of user u for the destination D corresponds to a random transmit antenna for relay r in the first phase, the delayed feedback has no impact on the SNR of x_u at relay r. Hence, substituting the conditional CDF $\Pr(\tilde{\Upsilon}_u < \Upsilon_{\rm th}|\mathcal{D}_{u,v}) = F_{\tilde{\Upsilon}_{u|\mathcal{D}_{u,v}}}(\Upsilon_{\rm th})$ and (17) into (16), we can derive the exact closed-form expression for the OP of STNC with TAS/MRC in the presence of delayed feedback.

2) SER: In the presence of delayed feedback, the SER of STNC associated with x_u is given by

$$\tilde{P}_{s,u} = \sum_{|\mathcal{D}_u|=0}^{R}\sum_{v=1}^{\vartheta}\tilde{P}_{s,u|\mathcal{D}_{u,v}}\Pr(\mathcal{D}_{u,v}) \tag{38}$$

where $\tilde{P}_{s,u|\mathcal{D}_{u,v}}$ denotes the SER of detecting x_u at the destination D conditioned on active relay set $\mathcal{D}_{u,v}$ in the presence of delayed feedback. Substituting (37) into (22) and applying the identity [23, Eq. (3.371)] to solve the resultant integral, we derive the exact closed-form expression for $\tilde{P}_{s,u|\mathcal{D}_{u,v}}$ as

$$\tilde{P}_{s,u|\mathcal{D}_{u,v}} = \frac{a}{2} - \frac{a\sqrt{b}}{2}\sum_{g_0=0}^{N_u-1}\sum_{k=0}^{\phi_g}\sum_{h_{1,0}=0}^{N_1-1}\sum_{k_1=0}^{\phi_1}\cdots\sum_{h_{\rho,0}=0}^{N_\rho-1}\sum_{k_\rho=0}^{\phi_\rho}\tilde{\Psi} \times\left(\sum_{p=1}^{N_{\rm D}+k}\sum_{q=0}^{p-1}\frac{\tilde{c}_u(2q-1)!!\tilde{\xi}_0^{q-p}}{q!2^q(\tilde{\xi}_0+b)^{q+\frac{1}{2}}} + \sum_{r=1}^{\rho}\sum_{p=1}^{N_{\rm D}+k_r}\sum_{q=0}^{p-1}\frac{\tilde{c}_r(2q-1)!!\tilde{\xi}_{r,0}^{q-p}}{q!2^q(\tilde{\xi}_{r,0}+b)^{q+\frac{1}{2}}}\right). \tag{39}$$

For the extreme case of perfect feedback with $\rho_u = \rho_r = 1$, it is easy to verify that (39) is equivalent to (25) by setting $k = \phi_g$ and $k_r = \phi_r$ in (39). Substituting (17) and (39) into (38) yields the exact closed-form SER expression $\tilde{P}_{s,u}$ with delayed feedback.

B. Asymptotic Performance

In this subsection, we derive the asymptotic OP and SER expressions in the high SNR regime to quantify the detrimental impact of delayed feedback and offer useful insights into the behavior of STNC with TAS/MRC.

1) OP: In the presence of delayed feedback, the end-to-end SNR of x_u in the high SNR regime is given by

$$\tilde{\Upsilon}_u^\infty = \tilde{\Upsilon}_{u{\rm D}}^\infty + \sum_{r=1}^{R}\tilde{\Upsilon}_{r{\rm D}}^\infty \tag{40}$$

where $\tilde{\Upsilon}_{u{\rm D}}^\infty$ and $\tilde{\Upsilon}_{r{\rm D}}^\infty$ are the received SNRs of x_u in the high SNR regime through the $u-{\rm D}$ and $r-{\rm D}$ links respectively, whose asymptotic PDFs can be derived by applying the Taylor series expansion of the exponential function in (33) and (34) and discarding the high order items as follows

$$f_{\tilde{\Upsilon}_{u{\rm D}}^\infty}(\tilde{\gamma}) \approx \sum_{g_0=0}^{N_u-1}\binom{N_u-1}{g_0} \times\frac{N_u\Phi_g(-1)^{g_0}\Gamma(N_{\rm D}+\phi_g)\left(1-\rho_u^2\right)^{\phi_g}}{(\Gamma(N_{\rm D}))^2\left(1+(1-\rho_u^2)g_0\right)^{N_{\rm D}+\phi_g}\bar{\gamma}_{u{\rm D}}^{N_{\rm D}}}\tilde{\gamma}^{N_{\rm D}-1} \tag{41}$$

$$f_{\tilde{\Upsilon}_{r{\rm D}}^\infty}(\tilde{\gamma}) \approx \sum_{h_{r,0}=0}^{N_r-1}\binom{N_r-1}{h_{r,0}} \times\frac{N_r\Phi_r(-1)^{h_{r,0}}\Gamma(N_{\rm D}+\phi_r)\left(1-\rho_r^2\right)^{\phi_r}}{(\Gamma(N_{\rm D}))^2\left(1+(1-\rho_r^2)h_{r,0}\right)^{N_{\rm D}+\phi_r}\bar{\gamma}_{r{\rm D}}^{N_{\rm D}}}\tilde{\gamma}^{N_{\rm D}-1}. \tag{42}$$

Performing the Laplace transforms of (41) and (42) with the aid of [23, Eq. (3.351.3)] and multiplying the resulted expressions together, we have the MGF of $\tilde{\Upsilon}_u^\infty$ as

$$\mathcal{M}_{\tilde{\Upsilon}_u^\infty}(s) \approx \tilde{\Theta}(s\bar{\gamma}_{u{\rm D}})^{-(R+1)N_{\rm D}} \tag{43}$$

where

$$\tilde{\Theta} = \sum_{g_0=0}^{N_u-1}\binom{N_u-1}{g_0}\frac{N_u\Phi_g(-1)^{g_0}\Gamma(N_{\rm D}+\phi_g)\left(1-\rho_u^2\right)^{\phi_g}}{\Gamma(N_{\rm D})1+(1-\rho_u^2)g_0)^{N_{\rm D}+\phi_g}} \times\prod_{r=1}^{R}\sum_{h_{r,0}=0}^{N_r-1}\binom{N_r-1}{h_{r,0}} \times\frac{N_r\Phi_r(-1)^{h_{r,0}}\Gamma(N_{\rm D}+\phi_r)\left(1-\rho_r^2\right)^{\phi_r}\kappa_r^{N_{\rm D}}}{\Gamma(N_{\rm D})\left(1+(1-\rho_r^2)h_{r,0}\right)^{N_{\rm D}+\phi_r}}.$$

Performing the inverse Laplace transform of (43) yields the PDF of $\tilde{\Upsilon}_u^\infty$, from which we derive the asymptotic CDF of $\tilde{\Upsilon}_u^\infty$ in the high SNR regime as

$$F_{\tilde{\Upsilon}_u^\infty}(\tilde{\gamma}) \approx \frac{\tilde{\Theta}}{\Gamma((R+1)N_{\rm D}+1)}\frac{\tilde{\gamma}^{(R+1)N_{\rm D}}}{\bar{\gamma}_{u{\rm D}}^{(R+1)N_{\rm D}}}. \tag{44}$$

Through evaluating $F_{\tilde{\Upsilon}_u^\infty}(\tilde{\gamma})$ at $\tilde{\gamma} = \Upsilon_{\rm th}$, the asymptotic OP of the end-to-end SNR associated with x_u under delayed feedback conditions is given by

$$\tilde{P}_{out,u}^\infty = F_{\tilde{\Upsilon}_u^\infty}(\Upsilon_{\rm th}). \tag{45}$$

2) SER: Based on $F_{\tilde{\Upsilon}_u^\infty}(\tilde{\gamma})$, we now derive the asymptotic SER of STNC with TAS/MRC in the high SNR regime. Substituting (44) into (22) and calculating the resultant integral, we derive the asymptotic SER as

$$\tilde{P}_{s,u}^\infty \approx (\tilde{G}_a\bar{\gamma}_{u{\rm D}})^{-\tilde{G}_d} \tag{46}$$

where the diversity order $\tilde{G}_d = (R+1)N_{\rm D}$ and array gain $\tilde{G}_a = b\left(\frac{a\tilde{\Theta}(2\tilde{G}_d-1)!!}{\Gamma(\tilde{G}_d+1)2^{\tilde{G}_d+1}}\right)^{-\frac{1}{\tilde{G}_d}}$.

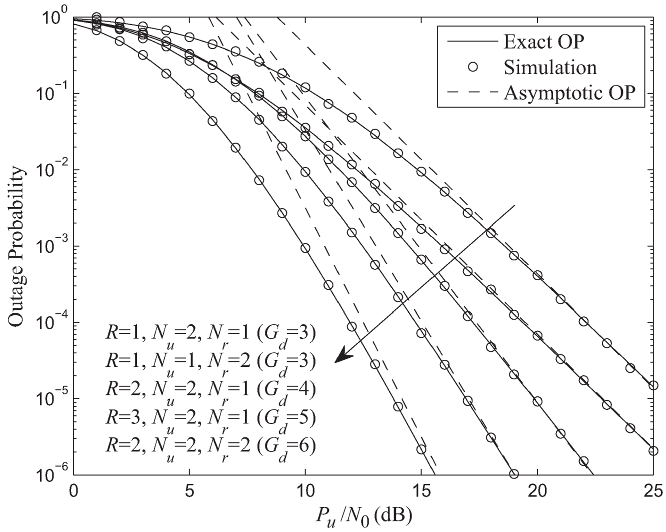

Fig. 1. Outage probability of STNC for a threshold $\Upsilon_{\text{th}} = 10$ dB in the presence of perfect feedback ($N_{\text{D}} = 1$).

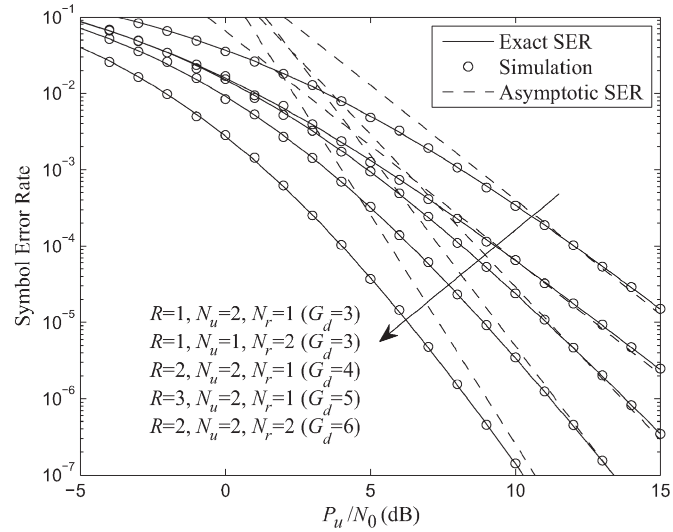

Fig. 2. Symbol error rate of BPSK modulation of STNC in the presence of perfect feedback ($N_{\text{D}} = 1$).

Comparing (46) with (32), it is evident that delayed feedback has a severely detrimental effect on the SER and degrades the diversity order from $(N_u + \sum_{r=1}^{R} N_r)N_{\text{D}}$ to $(R+1)N_{\text{D}}$, which indicates that the contribution of the multiple antennas at the transmit end to the diversity vanishes due to the delayed feedback. In the extreme case of fully delayed feedback, the optimal transmit antenna selected based on the fully outdated CSI actually corresponds to a random transmit antenna. The significant impact of delayed feedback implies that a high feedback rate may be required in practice to attain the full benefits of transmit antenna selection.

V. Numerical Results

In this section, numerical and simulation results are presented to show the validity of our analysis and the impacts of the network parameters on the OP and SER of STNC with TAS/MRC. We assume that the coordinations of the destination D, relay r, and user u are (0,0), $((d + r\Delta d)\cos(r\psi), (d + r\Delta d)\sin(r\psi))$, and $(\cos(u\psi'), \sin(u\psi'))$ respectively, which implies that the distances from the destination D to relay r and to user u are $d + r\Delta d$ and 1 respectively. In the simulations, $d = 0.4$, $\Delta d = 0.1$, and $\psi = \psi' = \pi/18$. The cross correlations between different spread codes are set to be zero, and the value of the path-loss exponent α is 3.5 [27]. We also assume equal transmit power at each node.

Figs. 1 and 2 plot the OP and SER of STNC versus transmit SNR P_u/N_0 for different antenna configurations in the presence of perfect feedback, respectively, where $P_u = P_{0u} + \sum_{r=1}^{R} P_{ru}$. The arrows in the figures are used to link each curve with corresponding network parameters. It is observed that the simulation results perfectly match with exact theoretical results and exact curves approach asymptotic curves in the high SNR regime, which validate the accuracy of our theoretical analysis in Section III. In the presence of perfect feedback, STNC with TAS/MRC provides full diversity order of $\left(N_u + \sum_{r=1}^{R} N_r\right) N_{\text{D}}$, as indicated by (32). Notably, the diversity order increases when the antenna number and relay number increase, which in turns leads to a pronounced SNR gain. For the single relay case in Figs. 1 and 2, it is shown that the antenna configuration of $N_u = 1$ and $N_r = 2$ exhibits a better performance than that of $N_u = 2$ and $N_r = 1$, although both of them have the same diversity order. This is due to the fact that the relay is involved in both transmission phases.

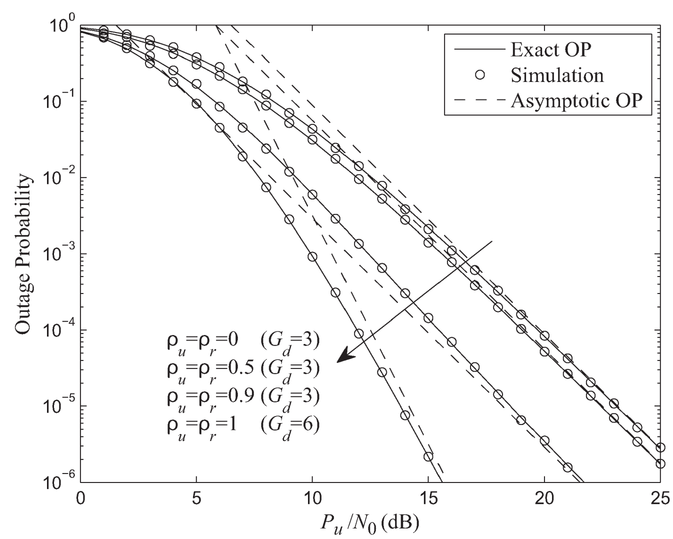

Fig. 3. Outage probability of STNC for a threshold $\Upsilon_{\text{th}} = 10$ dB in the presence of delayed feedback ($R = 2$, $N_u = N_r = 2$, $N_{\text{D}} = 1$).

Fig. 3 plots the OP of STNC with different delay correlation coefficients versus transmit SNR P_u/N_0. Figs. 4 and 5 plot the SERs of STNC with different modulations and different delay correlation coefficients versus transmit SNR P_u/N_0. The simulation results are marked by "•" in Fig. 5. The precise agreement between the theoretical and the simulation results in Figs. 3 and 4 verifies our theoretical analysis in Section IV. In Fig. 5, the theoretical and simulation curves are consistent in the medium and high SNR regime; whereas there exists a very minor gap between the theoretical and the simulation curves in the low SNR regime, especially for 16QAM. This is due to the fact that the closed-form SER expression (22) is an

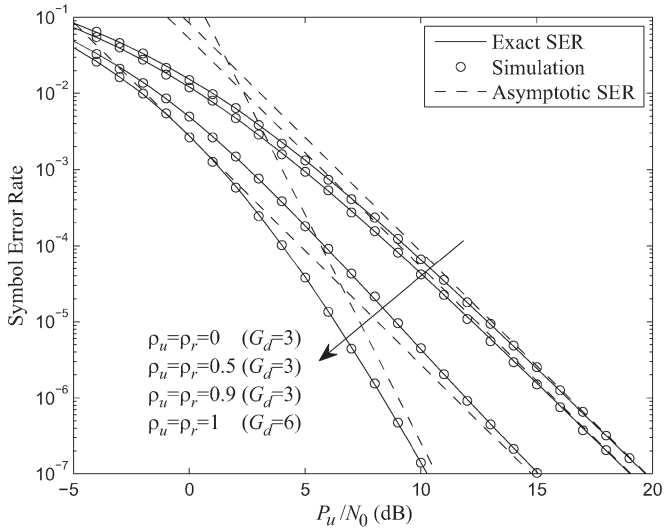

Fig. 4. Symbol error rate of BPSK modulation of STNC in the presence of delayed feedback ($R = 2$, $N_u = N_r = 2$, $N_\mathrm{D} = 1$).

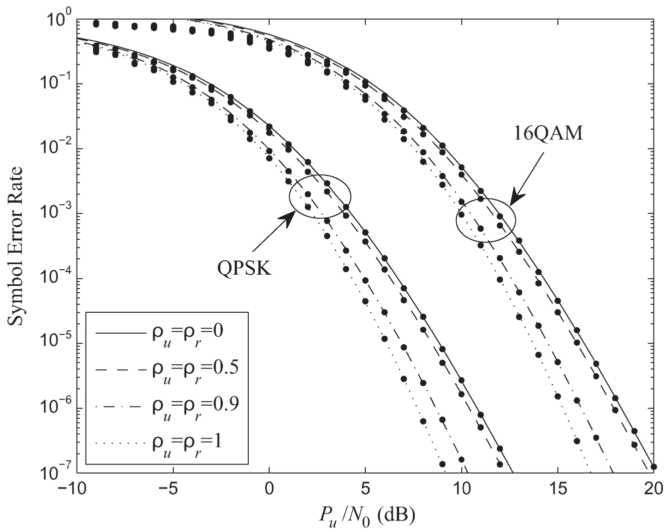

Fig. 5. Symbol error rates of QPSK and 16QAM modulations of STNC in the presence of delayed feedback ($R = 2$, $N_u = N_r = N_\mathrm{D} = 2$).

approximation for higher degree of modulation, e.g., QPSK and 16QAM. We find that the performance of the STNC improves as the delay correlation coefficients ρ_u and ρ_r increase. It is shown that, in the presence of delayed feedback, the asymptotic curves in Figs. 3 and 4 keep the same slope of -3, which means that the delayed feedback reduces the full diversity order to $(R+1)N_\mathrm{D}$ regardless of the values of ρ_u and ρ_r. This indicates that the diversity advantage from transmit end vanishes and only the one from receive end remains due to the delayed feedback. In Fig. 5, it is shown that fully delayed feedback incurs an SNR loss of approximately 3 dB compared to perfect feedback at an SER of 10^{-6}.

VI. Conclusion

In this paper, we have analyzed the performance of STNC in cooperative MIMO network in terms of OP and SER, where multi-antenna diversity is guaranteed via TAS/MRC. We have derived new closed-form expressions of the exact and asymptotic OP and SER for both perfect and delayed feedback. In the presence of perfect feedback, we have confirmed that STNC with TAS/MRC preserves the full diversity order of $\left(N_u + \sum_{r=1}^{R} N_r\right) N_\mathrm{D}$. In the presence of delayed feedback, we have quantified the detrimental effect of delayed feedback on the OP and SER of STNC. It is shown that the diversity advantage from the transmit end vanishes and the diversity order is degraded to $(R+1)N_\mathrm{D}$ due to delayed feedback.

Appendix A
Proof of Equation (12)

We denote γ_u as the instantaneous SNR of the channel from a random transmit antenna at user u to the N_D antennas at the destination D. The PDF and CDF of γ_u, $f_{\gamma_u}(\gamma)$ and $F_{\gamma_u}(\gamma)$, are given by [22]

$$f_{\gamma_u}(\gamma) = \frac{\gamma^{N_\mathrm{D}-1} e^{-\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}}}{\Gamma(N_\mathrm{D})\bar{\gamma}_{u\mathrm{D}}^{N_\mathrm{D}}} \tag{47}$$

and

$$F_{\gamma_u}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}} \sum_{i=0}^{N_\mathrm{D}-1} \frac{\gamma^i}{i!\bar{\gamma}_{u\mathrm{D}}^i} \tag{48}$$

respectively. Based on (47) and (48), we have

$$\begin{aligned} f_{\Upsilon_{u\mathrm{D}}}(\gamma) &= \frac{N_u \gamma^{N_\mathrm{D}-1} e^{-\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}}}{\Gamma(N_\mathrm{D})\bar{\gamma}_{u\mathrm{D}}^{N_\mathrm{D}}} \left(1 - e^{-\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}} \sum_{i=0}^{N_\mathrm{D}-1} \frac{\gamma^i}{i!\bar{\gamma}_{u\mathrm{D}}^i}\right)^{N_u-1} \\ &= \frac{N_u \gamma^{N_\mathrm{D}-1} e^{-\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}}}{\Gamma(N_\mathrm{D})\bar{\gamma}_{u\mathrm{D}}^{N_\mathrm{D}}} \sum_{g_0=0}^{N_u-1} \binom{N_u-1}{g_0} (-1)^{g_0} e^{-\frac{g_0\gamma}{\bar{\gamma}_{u\mathrm{D}}}} \\ &\quad \times \left(\sum_{i=0}^{N_\mathrm{D}-1} \frac{\gamma^i}{i!\bar{\gamma}_{u\mathrm{D}}^i}\right)^{g_0}. \end{aligned} \tag{49}$$

According to [28, Eq. (9)], we have

$$\left(\sum_{i=0}^{N_\mathrm{D}-1} \frac{\gamma^i}{i!\bar{\gamma}_{u\mathrm{D}}^i}\right)^{g_0} = \Phi_g \left(\frac{\gamma}{\bar{\gamma}_{u\mathrm{D}}}\right)^{\phi_g} \tag{50}$$

where $\Phi_g = \prod_{i=1}^{N_\mathrm{D}-1} \left(\sum_{g_i=0}^{g_{i-1}} \binom{g_{i-1}}{g_i} \left(\frac{1}{i!}\right)^{g_i - g_{i+1}}\right)$, $\phi_g = \sum_{i=1}^{N_\mathrm{D}-1} g_i$, and $g_{N_\mathrm{D}} = 0$. Substituting (50) into (49) yields (12), where $\xi_0 = (g_0+1)/\bar{\gamma}_{u\mathrm{D}}$.

Appendix B
Proof of Equation (14)

Using $\gamma_{ij}^{\mathrm{D}\mu}$, $\mu \in \{u, r\}$, $1 \le i \le N_\mathrm{D}$, and $1 \le j \le N_\mu$, to denote the i.n.i.d. distributed SNR between the jth antenna of transmitter μ and the ith antenna of destination D, we have

$$f_{\gamma_{ij}^{\mathrm{D}\mu}}(\gamma) = \frac{1}{\bar{\gamma}_{ij}^{\mathrm{D}\mu}} e^{-\frac{\gamma}{\bar{\gamma}_{ij}^{\mathrm{D}\mu}}} \tag{51}$$

where $\bar{\gamma}_{ij}^{D\mu} = E[\gamma_{ij}^{D\mu}]$. Based on (51), the MGF of $\gamma_{ij}^{D\mu}$ is $\mathcal{M}_{\gamma_{ij}^{D\mu}}(s) = 1/\left(s\bar{\gamma}_{ij}^{D\mu}+1\right)$. If we MRC combine the N_D signal replicas from the jth antenna of transmitter μ at the destination D, the total SNR, γ_μ, is the summation of $\gamma_{ij}^{D\mu}$, $1 \le i \le N_D$. As such, the MGF of γ_μ is

$$\mathcal{M}_{\gamma_\mu}(s) = \prod_{i=1}^{N_D} \frac{1}{s\bar{\gamma}_{ij}^{D\mu}+1} = \alpha_j \prod_{i=1}^{N_D}\left(s+\frac{1}{\bar{\gamma}_{ij}^{D\mu}}\right)^{-1} \tag{52}$$

where $\alpha_j = \prod_{i=1}^{N_D}\frac{1}{\bar{\gamma}_{ij}^{D\mu}}$. Expanding $\prod_{i=1}^{N_D}\left(s+\frac{1}{\bar{\gamma}_{ij}^{D\mu}}\right)^{-1}$ in poles and residuals with the aid of partial fraction decomposition [23, Eq. (2.102)] and performing inverse Laplace transform on (52), we obtain

$$f_{\gamma_\mu}(\gamma) = \alpha_j \sum_{i=1}^{N_D} \prod_{i'=1,i'\neq i}^{N_D}\left(\frac{1}{\bar{\gamma}_{i'j}^{D\mu}}-\frac{1}{\bar{\gamma}_{ij}^{D\mu}}\right)^{-1} e^{-\frac{\gamma}{\bar{\gamma}_{ij}^{D\mu}}}. \tag{53}$$

Defining $\beta_{ij} = \prod_{i'=1,i'\neq i}^{N_D}\left(\frac{1}{\bar{\gamma}_{i'j}^{D\mu}}-\frac{1}{\bar{\gamma}_{ij}^{D\mu}}\right)^{-1}$, the CDF of γ_μ is given by $F_{\gamma_\mu}(\gamma) = 1-\alpha_j\sum_{i=1}^{N_D}\beta_{ij}\bar{\gamma}_{ij}^{D\mu}e^{-\frac{\gamma}{\bar{\gamma}_{ij}^{D\mu}}}$. Hence, PDF of $\Upsilon_{\mu D}$ is given by

$$f_{\Upsilon_{\mu D}}(\gamma) = \sum_{j=1}^{N_\mu}\alpha_j\sum_{i=1}^{N_D}\beta_{ij}e^{-\frac{\gamma}{\bar{\gamma}_{ij}^{D\mu}}} \times \prod_{j'=1,j'\neq j}^{N_\mu}\left(1-\alpha_{j'}\sum_{i'=1}^{N_D}\beta_{i'j'}\bar{\gamma}_{i'j'}^{D\mu}e^{-\frac{\gamma}{\bar{\gamma}_{i'j'}^{D\mu}}}\right). \tag{54}$$

Based on (54), (14) can be derived directly by expanding the product term.

Appendix C
Proof of Lemma 1

Here, we first derive the MGF of $\Upsilon_{u|\mathcal{D}_{u,v}}$, and then obtain the PDF of $\Upsilon_{u|\mathcal{D}_{u,v}}$ based on the MGF.

From (12) and (13), the MGFs of Υ_{uD} and Υ_{rD} are obtained by performing the Laplace transform with the aid of the definite integral of the exponential function [23, Eq. (3.351.3)] as

$$\mathcal{M}_{\Upsilon_{uD}}(s) = \sum_{g_0=0}^{N_u-1}\Psi_0(\xi_0+s)^{-(\phi_g+N_D)} \tag{55}$$

and

$$\mathcal{M}_{\Upsilon_{rD}}(s) = \sum_{h_{r,0}=0}^{N_r-1}\Psi_{r,0}(\xi_{r,0}+s)^{-(\phi_r+N_D)} \tag{56}$$

respectively, where $\Psi_0 = \binom{N_u-1}{g_0}\frac{N_u\Gamma(\phi_g+N_D)\Phi_g}{\Gamma(N_D)(-1)^{g_0}\bar{\gamma}_{uD}^{\phi_g+N_D}}$, and $\Psi_{r,0} = \binom{N_r-1}{h_{r,0}}\frac{N_r\Gamma(\phi_r+N_D)\Phi_r}{\Gamma(N_D)(-1)^{h_{r,0}}\bar{\gamma}_{rD}^{\phi_r+N_D}}$.

Since the MGF of the sum of multiple independent random variables is equal to the product of the MGFs of the random variables [22], the MGF of $\Upsilon_{u|\mathcal{D}_{u,v}}$ is

$$\mathcal{M}_{\Upsilon_{u|\mathcal{D}_{u,v}}}(s) = \sum_{g_0=0}^{N_u-1}\Psi_0(\xi_0+s)^{-(\phi_g+N_D)} \times \prod_{r=1}^{\rho}\left(\sum_{h_{r,0}=0}^{N_r-1}\Psi_{r,0}(\xi_{r,0}+s)^{-(\phi_r+N_D)}\right). \tag{57}$$

Expanding (57) in poles and residuals with the aid of partial fraction decomposition [23, Eq. (2.102)] and Faa di Bruno Formula and performing inverse Laplace transform yield (19), where

$$c_u = (-1)^{\phi_g+N_D-p}\sum_{\Omega_u}\prod_{r=1}^{\rho}\frac{\binom{\phi_r+N_D+i_r-1}{i_r}}{(\xi_{r,0}-\xi_0)^{\phi_r+N_D+i_r}}$$

$$c_r = (-1)^{\phi_r+N_D-p}\sum_{\Omega_r}\frac{\binom{\phi_g+N_D+i_0-1}{i_0}}{(\xi_0-\xi_{r,0})^{\phi_g+N_D+i_0}} \times \prod_{r'=1,r'\neq r}^{\rho}\frac{\binom{\phi_{r'}+N_D+i_{r'}-1}{i_{r'}}}{(\xi_{r',0}-\xi_{r,0})^{\phi_{r'}+N_D+i_{r'}}}$$

with $\Psi = \Psi_0\prod_{r=1}^{\rho}\Psi_{r,0}$, Ω_u is the set of nonnegative integers $\{i_1, i_2, \ldots, i_\rho\}$, such that $\sum_{j=1}^{\rho} i_j = \phi_g+N_D-p$, and Ω_r is the set of nonnegative integers $\{i_0, i_1, \ldots, i_{r-1}, i_{r+1}, \ldots, i_\rho\}$, such that $\sum_{j=0,j\neq r}^{\rho} i_j = \phi_r+N_D-p$.

Appendix D
Proof of Lemma 2

Denoting $\tilde{\gamma}_u$ as the outdated SNR relative to the original SNR γ_u, the joint PDF of $\tilde{\gamma}_u$ and γ_u is [29]

$$f_{\tilde{\gamma}_u,\gamma_u}(\tilde{\gamma},\gamma) = \left(\frac{1}{\bar{\gamma}_{uD}}\right)^{N_D+1}\left(\frac{\tilde{\gamma}\gamma}{\rho_u^2}\right)^{\frac{N_D-1}{2}}\frac{e^{-\frac{\tilde{\gamma}+\gamma}{(1-\rho_u^2)\bar{\gamma}_{uD}}}}{\Gamma(N_D)\left(1-\rho_u^2\right)} \times I_{N_D-1}\left(\frac{2\sqrt{\rho_u^2\tilde{\gamma}\gamma}}{\left(1-\rho_u^2\right)\bar{\gamma}_{uD}}\right) \tag{58}$$

where $I_n(\cdot)$ stands for the nth-order modified Bessel function of the first kind [23, Eq. (8.406.1)]. According to the order statistics, the time-delayed SNR after TAS, $\tilde{\Upsilon}_{uD}$, is the induced order statistics of the original ordered SNR after TAS, Υ_{uD} [30]. Therefore, the PDF of $\tilde{\Upsilon}_{uD}$ is derived as

$$f_{\tilde{\Upsilon}_{uD}}(\tilde{\gamma}) = \int_0^\infty \frac{f_{\tilde{\gamma}_u,\gamma_u}(\tilde{\gamma},\gamma)}{f_{\gamma_u}(\gamma)} f_{\Upsilon_{uD}}(\gamma)d\gamma = N_u\int_0^\infty f_{\tilde{\gamma}_u,\gamma_u}(\tilde{\gamma},\gamma)\left(F_{\gamma_u}(\gamma)\right)^{N_u-1}d\gamma. \tag{59}$$

Substituting (58) and the CDF of γ_u, (48), into (59) and performing some algebraic manipulations with the aid of [23, Eq. (6.643.2)] yield

$$f_{\tilde{\Upsilon}_{u\mathrm{D}}}(\tilde{\gamma}) = \sum_{g_0=0}^{N_u-1} \binom{N_u-1}{g_0} \frac{N_u \Phi_g \Gamma(N_\mathrm{D}+\phi_g)\tilde{\gamma}^{\frac{N_\mathrm{D}}{2}-1}}{(\Gamma(N_\mathrm{D}))^2 (-1)^{g_0} \left(\rho_u^2 \bar{\gamma}_{u\mathrm{D}}\right)^{\frac{N_\mathrm{D}}{2}}}$$
$$\times \left(\frac{1}{1-\rho_u^2} + g_0\right)^{-\left(\frac{N_\mathrm{D}}{2}+\phi_g\right)}$$
$$\times e^{-\frac{2\left(1+\left(1-\rho_u^2\right)g_0\right)\tilde{\gamma}-\rho_u^2\tilde{\gamma}}{2\left(1-\rho_u^2\right)\bar{\gamma}_{u\mathrm{D}}\left(1+(1-\rho_u^2)g_0\right)}}$$
$$\times M_{-\left(\frac{N_\mathrm{D}}{2}+\phi_g\right),\frac{N_\mathrm{D}-1}{2}} \left(\frac{\left(1-\rho_u^2\right)^{-1}\rho_u^2\tilde{\gamma}}{\bar{\gamma}_{u\mathrm{D}}\left(1+\left(1-\rho_u^2\right)g_0\right)}\right) \quad (60)$$

where $M_{a,b}(\cdot)$ is the Whittaker function [23, Eq. (9.220.2)]. With the aid of the finite series expansion of Whittaker function [26], we derive the PDF of $\tilde{\Upsilon}_{u\mathrm{D}}$ given in (33), where

$$\tilde{\Psi}_{0,k} = \binom{N_u-1}{g_0}\binom{\phi_g}{k}$$
$$\times \frac{N_u\Phi_g(-1)^{g_0}\Gamma(N_\mathrm{D}+\phi_g)\rho_u^{2k}\left(1-\rho_u^2\right)^{\phi_g-k}}{\Gamma(N_\mathrm{D})\bar{\gamma}_{u\mathrm{D}}^{N_\mathrm{D}+k}\left(1+\left(1-\rho_u^2\right)g_0\right)^{N_\mathrm{D}+\phi_g+k}}$$

and $\tilde{\xi}_0 = \frac{g_0+1}{\left(1+\left(1-\rho_u^2\right)g_0\right)\bar{\gamma}_{u\mathrm{D}}}$. Following the similar procedures of deriving the PDF of $\tilde{\Upsilon}_{u\mathrm{D}}$, we derive the the PDF of $\tilde{\Upsilon}_{r\mathrm{D}}$, as shown in (34), where

$$\tilde{\Psi}_{r_0,k_r} = \binom{N_r-1}{h_{r,0}}\binom{\phi_r}{k_r}$$
$$\times \frac{N_r\Phi_r(-1)^{h_{r,0}}\Gamma(N_\mathrm{D}+\phi_r)\rho_r^{2k_r}\left(1-\rho_r^2\right)^{\phi_r-k_r}}{\Gamma(N_\mathrm{D})\bar{\gamma}_{r\mathrm{D}}^{N_\mathrm{D}+k_r}\left(1+\left(1-\rho_r^2\right)h_{r,0}\right)^{N_\mathrm{D}+\phi_r+k_r}}$$

and $\tilde{\xi}_{r,0} = \frac{1+h_{r,0}}{\left(1+\left(1-\rho_r^2\right)h_{r,0}\right)\bar{\gamma}_{r\mathrm{D}}}$.

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