Journal of Business Research 68 (2015) 1574–1580

Contents lists available at ScienceDirect

Journal of Business Research

Aggregation methods to calculate the average price☆

José M. Merigó [a,*], Daniel Palacios-Marqués [b], María del Mar Benavides-Espinosa [c]

[a] Department of Management Control and Information Systems, University of Chile, Av. Diagonal Paraguay 257, 8330015 Santiago, Chile
[b] Department of Business Organization, Universitat Politècnica de València, Camino Vera s/n, 46022 València, Spain
[c] Department of Business Administration, Universitat de València, Av. Los Naranjos s/n, 46022 València, Spain

ARTICLE INFO

Available online 16 March 2015

Keywords:
Average price
Aggregation systems
Unified aggregation operator
OWA operator

ABSTRACT

Average price is a numerical value that represents a set of prices, which may relate to firms, countries, or regions. This study presents new methods of average price aggregation that build on the unified aggregation operator (UAO). The UAO combines a wide range of sub-aggregation processes into a single formulation capable of accounting for the importance of each concept in the analysis. The aggregation system is flexible, can adapt to different environments, and provides a complete representation of relevant information. The UAO can calculate the average price for numerous geographical contexts such as supranational regions and countries. The study illustrates the UAO's utility by presenting an example of how to calculate the world average price of a product while considering a range of opinions and environmental uncertainties.

1. Introduction

Average price is a representative value for a set of varying prices (Chen, 2006; Silver & Ioannidis, 2001). Average prices are useful to analyze prices within sets of firms, countries, or regions and are a fundamental part of pricing research (Johnson & Cui, 2013; Leone, Robinson, Bragge, & Somervuori, 2012). Usually, analysis of the average price uses the simple average or the weighted average. However, many other aggregation techniques exist (Beliakov, Pradera, & Calvo, 2007; Grabisch, Marichal, Mesiar, & Pap, 2011). An increasingly popular aggregation operator is the ordered weighted average (OWA) (Yager, 1988; Yager, Kacprzyk, & Beliakov, 2011). The OWA is an aggregation operator that provides a parameterized family of aggregation operators that range between the minimum and the maximum. The main advantage is its efficiency in representing the decision maker's attitudinal character.

Recent studies employ, several generalizations of the OWA operator and related techniques, which include the integration of weighted averages (Torra, 1997; Xu & Da, 2003) and probabilities with the OWA operator (Engemann, Filev, & Yager, 1996; Yager, Engemann, & Filev, 1995). Merigó (2012) introduces the probabilistic OWA operator and further generalizations that include the weighted average (Merigó, Lobato-Carral, & Carrilero-Castillo, 2012). The main advantage of this approach is the flexibility to be able to consider different information sources in the same formulation. An additional practical development is the unified aggregation operator (UAO) (Merigó, 2011). The UAO has a more general structure and is capable of additional aggregation that can capture numerous sub-aggregation processes within the specific problem while accounting for each concept's importance.

This study sets forth new methods for calculating the average price by using the UAO and related techniques. This approach provides a more general representation of the information by considering different information sources such as the experts' opinion, probabilities, weighted averages, and the decision maker's attitudinal character. In addition, the UAO draws on information from firms, countries, and regions. The UAO's main advantage is its flexibility in adapting to the specific needs of the environment.

This study addresses key examples including the average price in the European Union, North America, and Asia. This approach lets decision makers analyze different information types and integrate these information types into a representative result that fits the decision maker's interests. The study presents an example for the world average price to illustrate the method numerically. To demonstrate the UAO's capacity to include sub-structures in average price calculation, the study also analyzes the US average price and the average price of the individual US states.

Section 2 briefly reviews some key aggregation systems. Section 3 introduces new methods for addressing the average prices. Section 4 presents an illustrative example. Finally, Section 5 summarizes the study's main findings.

☆ The authors would like to thank Professor Domingo Ribeiro, University of Valencia, Professor Jian-Bo Yang, University of Manchester, Professor Pedro Soto, University of Murcia, Professor Shouzhen Zeng, Zhejiang Wanli University, the guest editors of this special issue, the anonymous reviewers and the participants of the GIKA 2014 conference for their valuable comments, which have improved the quality of this research. The authors also gratefully acknowledge the support from the European Commission (PIEF-GA-2011-300062) and the Spanish Ministry of Economy and Competitiveness (DER2012-39223-C02-02).

* Corresponding author.
E-mail addresses: jmerigo@fen.uchile.cl (J.M. Merigó), dapamar@doe.upv.es (D. Palacios-Marqués), maria.m.benavides@uv.es (M. del Mar Benavides-Espinosa).

http://dx.doi.org/10.1016/j.jbusres.2015.01.054
0148-2963/

2. Aggregation systems

Aggregation systems are very common in the literature (Beliakov et al., 2007; Grabisch et al., 2011). They capture initial information, summarize results, and give decision makers a better representation of the available information. The simple average, the weighted average and the ordered weighted average (OWA) (Yager, 1988) are the most popular aggregation systems. Assessing complex information, however, requires methods that are more general. A useful tool for assessing complex information is the unified aggregation operator (UAO) (Merigó, 2011). The definition of the UAO is as follows.

Definition 1. A unified aggregation operator of dimension m is a mapping UAO: $R^m \times R^n \rightarrow R$, with an associated weighting vector C of dimension m representing concepts with a degree of importance C_h, such that

$$UAO(a_1, \ldots, a_n) = \sum_{h=1}^{m} \sum_{i=1}^{n} C_h w_i^h a_i, \tag{1}$$

where C_h is the degree of importance of each concept in the aggregation such that $C_h \in [0, 1]$ and $\sum_{h=1}^{m} C_h = 1$; and w_i^h is the ith weight of the hth weighting vector W such that $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$.

The UAO includes a wide range of aggregation operators including the weighted average, the OWA operator and the POWAWA operator (Merigó et al., 2012). Note that if some of the aggregation methods do not appear in the order according to i, the reordering of these sub-aggregation operators to the ordering of i is necessary.

The UAO contains the POWAWA operator when using probability, the weighted average and the OWA operator. Thus,

$$f(a_1, \ldots, a_n) = C_1 \sum_{j=1}^{n} w_j b_j + C_2 \sum_{i=1}^{n} v_i a_i + C_3 \sum_{i=1}^{n} p_i a_i. \tag{2}$$

According to Eq. (1), conversion of w_j and b_j is possible using x_i and a_i, where x_i is the ith weight w_j ordered according to the initial positions i. This formulation, automatically yields other basic cases as follows:

- If $C_1 = 1$, the formulation yields the OWA operator.
- If $C_2 = 1$, the formulation yields the weighted average.
- If $C_3 = 1$, the formulation yields the probabilistic aggregation.
- If $C_1 = 0$, the formulation yields the PWA operator.
- If $C_2 = 0$, the formulation yields the POWA operator.
- If $C_3 = 0$, the formulation yields the OWAWA operator.

Note that the UAO includes many other types of aggregation operators. The idea is to use the specific cases pertinent to the problem at hand. Other popular aggregation operators are the Choquet integrals (Belles, Merigó, Guillen, & Santolino, 2014), distance measures (Zeng, Su, & Le, 2012), generalized aggregation operators (Zhou, Chen, & Liu, 2013) and fuzzy systems (Zhao, Lin, & Wei, 2013).

3. Analysis of average prices

Many methods analyze and calculate average prices (Silver & Ioannidis, 2001). This research presents new methods of average price calculation that can represent the information more completely. These approaches must be flexible to adapt to the specific needs of the problem. The UAO offers this flexibility. The UAO generalizes a range of aggregation operators. Using the UAO in the simplest representation of the average price gives the following expression:

$$P = \sum_{h=1}^{m} \sum_{i=1}^{n} C_h w_i^h p_i, \tag{3}$$

where p_i is the price of the ith firm or region; w_i^h is the ith weight of the hth weighting vector W such that $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$; and C_h is the firm's or region's importance in the aggregation such that $C_h \in [0, 1]$ and $\sum_{h=1}^{m} C_h = 1$.

This formulation considers only one set of elements with specific prices. In the real world, however, more sets are often available, and problems must sometimes take into account firms, regions, and countries. Such cases require a more general structure. Manipulating Eq. (3) yields the following:

$$P = \sum_{h=1}^{m} \sum_{g=1}^{o} \sum_{i=1}^{n} C_h v_{hg} w_i^{hg} p_{ihg}, \tag{4}$$

where p_{ihg} is the price for the gth country in the ith region (or firm) and hth opinion; w_i^{hg} is the weight (or market share) of the ith region in the gth country such that $w_{ig} \in [0, 1]$ and $\sum_{i=1}^{n} w_{ig} = 1$.

Observe that many examples could follow this direction. For example, under the assumption that three weighting vectors represent the objective and subjective information and the attitudinal character, the POWAWA operator would be suitable for analysis. Here, Eq. (4) could take the following form:

$$P = C_1 P_{OWA} + C_2 P_{WA} + C_3 P_{PA} \tag{5}$$

or:

$$P = C_1 \sum_{g=1}^{o} \sum_{i=1}^{n} v_{1g} w_i^{1g} p_{i1g} + C_2 \sum_{g=1}^{o} \sum_{i=1}^{n} v_{2g} w_i^{2g} p_{i2g} + + C_3 \sum_{g=1}^{o} \sum_{i=1}^{n} v_{3g} w_i^{3g} p_{i3g}. \tag{6}$$

Scholars could study many other variations of this approach following the UAO approach (Merigó, 2011).

The next part of the discussion explores some interesting real world examples. To develop these examples, the first step is to present the formulas for calculating the average price in some representative supranational regions. Table 1 displays the results.

In this example, each country comprises several sub-aggregations (i.e., states, provinces, and firms). Table 2 displays such a structure for the USA.

The order of each formula runs from the highest economy to the lowest economy. Standard definitions from well-known sources define the regions. Some small differences may appear, however, because of the specific conditions in each country. Note that the real world is more complex because of the need to consider firms, scenarios, and differences between cities and towns.

A major advantage of using this approach is that the analysis does not lose information because the minimum and the maximum bound the results:

$$\text{Min}\{p_i\} \leq \text{AP} \leq \text{Max}\{p_i\}, \tag{7}$$

where AP is the average price that an aggregation system (e.g., simple average of the UAO) yields.

Observe that the UAO admits numerous partial bounds that account for the minimum and the maximum when considering additional information. A representation of this idea is as follows:

$$\text{Min}\{p_i\} \leq \ldots \leq \text{Min-UAO}\{p_i\} \leq \text{AP} \leq \text{Max-UAO}\{p_i\} \leq \ldots \leq \text{Max}\{p_i\}, \tag{8}$$

where Min-UAO and Max-UAO indicate an aggregation process that combines the minimum and the maximum with the UAO, and Min-UAO and Max-UAO are the minimum and maximum of the additional aggregation integrated within the UAO.

Finally, this methodology closely resembles box-plot analysis (Tukey, 1977) and other statistical methods. The difference in the UAO approach is that the weighting vectors represent a specific attitude against the uncertainty of the environment. Moreover, the sub-

Table 1
Building the average price using the unified aggregation operator.

Average price	Formulation
General	$P = \sum_{h=1}^{m}\sum_{g=1}^{o}\sum_{i=1}^{n} C_h v_{hg} w_i^{hg} P_{ihg} = \sum_{h=1}^{m} C_h \left(\sum_{g=1}^{o}\sum_{i=1}^{n} v_{hg} w_{ihg} P_{ihg}\right)$
World	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{USA}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{CHN}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{200h-\mathrm{TUV}}\left(\sum_{i=1}^{n} w_{ih200}P_{ih200}\right)\right]$
European Union	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{GER}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{FRA}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{28h-\mathrm{MAL}}\left(\sum_{i=1}^{n} w_{ih28}P_{ih28}\right)\right]$
North America	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{USA}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{MEX}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + v_{3h-\mathrm{CAN}}\left(\sum_{i=1}^{n} w_{ih3}P_{ih3}\right)\right]$
South America	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{BRA}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{ARG}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{13h-\mathrm{GUY}}\left(\sum_{i=1}^{n} w_{ih3}P_{ih3}\right)\right]$
Africa	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{S.A.}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{NIG}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{52h-\mathrm{S.T.}}\left(\sum_{i=1}^{n} w_{ih52}P_{ih52}\right)\right]$
Asia	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{CHN}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{JAP}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{41h-\mathrm{MLD}}\left(\sum_{i=1}^{n} w_{ih41}P_{ih41}\right)\right]$
Oceania	$P = \sum_{h=1}^{m} C_h \left[v_{1h-\mathrm{AUS}}\left(\sum_{i=1}^{n} w_{ih1}P_{ih1}\right) + v_{2h-\mathrm{N.Z.}}\left(\sum_{i=1}^{n} w_{ih2}P_{ih2}\right) + \ldots + v_{14h-\mathrm{TUV}}\left(\sum_{i=1}^{n} w_{ih14}P_{ih14}\right)\right]$

Abbreviations: CHN = China; TUV = Tuvalu; GER = Germany; FRA = France; MAL = Malta; MEX = Mexico; CAN = Canada; BRA = Brazil; ARG = Argentina; GUY = Guyana; S.A. = South Africa; NIG = Nigeria; S.T. = Sao Tome and Principe; JAP = Japan; MLD = Maldives; AUS = Australia; N.Z. = New Zealand.

Table 2
The structure of the average prices in the USA.

		Weights 1			…		Weights L		
US state	AP_1	C_{11}	…	C_{m1}	…	AP_L	C_{1L}	…	C_{mL}
Alabama	P_{ALB1}	W_{ALB1}	…	W_{ALBm}	…	P_{ALBL}	X_{ALB1}	…	X_{ALBm}
Alaska	P_{ALS1}	W_{ALS1}	…	W_{ALSm}	…	P_{ALSL}	X_{ALS1}	…	X_{ALSm}
Arizona	P_{ARI1}	W_{ARI1}	…	W_{ARIm}	…	P_{ARIL}	X_{ARI1}	…	X_{ARIm}
…	…	…	…	…	…	…	…	…	…
Wisconsin	P_{WIS1}	W_{WIS1}	…	W_{WISm}	…	P_{WISL}	X_{WIS1}	…	X_{WISm}
Wyoming	P_{WYO1}	W_{WYO1}	…	W_{WYOm}	…	P_{WYOL}	X_{WYO1}	…	X_{WYOm}

The states appear in alphabetical order.
Abbreviations: AP = average price for the scenario/opinion 1; C = degree of importance of each weighting vector; P = price for a specific US state; W = weights for each type of weighting vector for the scenario/opinion 1; X = weights for each type of weighting vector for the scenario/opinion L.

Table 3
Sub-aggregations of the unified aggregation operator by state.

State	W_1	W_2	W_3	W_4	UAO
Alabama	0.012	0.011	0.013	0.011	0.0117
Alaska	0.001	0.002	0.002	0.003	0.0019
Arizona	0.018	0.016	0.014	0.015	0.0160
Arkansas	0.008	0.007	0.009	0.008	0.0079
California	0.125	0.126	0.121	0.124	0.1243
Colorado	0.018	0.016	0.015	0.017	0.0166
Connecticut	0.014	0.012	0.013	0.012	0.0128
Delaware	0.003	0.004	0.004	0.005	0.0039
D. C.	0.002	0.003	0.004	0.003	0.0029
Florida	0.064	0.066	0.062	0.065	0.0644
Georgia	0.029	0.025	0.026	0.027	0.0268
Hawaii	0.003	0.004	0.003	0.004	0.0035
Idaho	0.005	0.006	0.005	0.006	0.0055
Illinois	0.046	0.048	0.042	0.045	0.0456
Indiana	0.022	0.021	0.023	0.024	0.0223
Iowa	0.008	0.009	0.010	0.011	0.0093
Kansas	0.009	0.007	0.008	0.007	0.0078
Kentucky	0.012	0.009	0.011	0.012	0.0109
Louisiana	0.015	0.013	0.012	0.014	0.0136
Maine	0.004	0.005	0.004	0.005	0.0045
Maryland	0.018	0.019	0.018	0.017	0.0181
Massachusetts	0.023	0.024	0.022	0.023	0.0231
Michigan	0.028	0.031	0.032	0.029	0.0299
Minnesota	0.016	0.017	0.015	0.016	0.0161
Mississippi	0.008	0.009	0.008	0.009	0.0085
Missouri	0.018	0.020	0.021	0.022	0.0200
Montana	0.003	0.003	0.004	0.003	0.0032
Nebraska	0.006	0.007	0.006	0.007	0.0065
Nevada	0.008	0.009	0.007	0.008	0.0081
New Hampshire	0.004	0.003	0.004	0.003	0.0035
New Jersey	0.029	0.025	0.026	0.027	0.0268
New Mexico	0.006	0.007	0.008	0.007	0.0069
New York	0.075	0.071	0.073	0.072	0.0728
North Carolina	0.032	0.031	0.032	0.033	0.0319
North Dakota	0.002	0.003	0.004	0.003	0.0029
Ohio	0.034	0.031	0.032	0.033	0.0325
Oklahoma	0.012	0.011	0.013	0.011	0.0117
Oregon	0.011	0.012	0.013	0.011	0.0117
Pennsylvania	0.047	0.045	0.046	0.045	0.0458
Rhode Island	0.003	0.002	0.003	0.002	0.0025
South Carolina	0.016	0.019	0.018	0.017	0.0175
South Dakota	0.002	0.003	0.004	0.003	0.0029
Tennessee	0.021	0.022	0.020	0.021	0.0211
Texas	0.082	0.081	0.082	0.081	0.0815
Utah	0.009	0.007	0.008	0.007	0.0078
Vermont	0.002	0.003	0.004	0.003	0.0029
Virginia	0.023	0.026	0.025	0.024	0.0245
Washington	0.021	0.022	0.023	0.021	0.0217
West Virginia	0.006	0.007	0.008	0.007	0.0069
Wisconsin	0.015	0.017	0.016	0.014	0.0156
Wyoming	0.002	0.003	0.004	0.003	0.0029

aggregation processes aim to address the key sub-structures to ensure that information analysis is correct.

4. Illustrative example

The previous section demonstrates how to analyze average prices using a range of aggregation operators to address the complexities and granularities of uncertain environments. This section presents a numerical example to reflect the real world. This example demonstrates how to calculate the average price of a real product. To make the demonstration more realistic, an additional example shows how to calculate the internal sub-aggregations of the USA, a highly representative case. The example considers only one additional aggregation level. In the real world, however, considering more levels is important to account for differences between firms, towns, and cities within states or provinces.

Assume that a group of experts wants to calculate the world average price of a specific product *A*. To do so, the experts have access to data for all the states and provinces in the world. Here, the assumption is that the data already contain each province's average price, which consists of the average of firms, cities, and towns within the province. Thus, the experts must construct the prices, accounting for the importance of each state. Because the experts want to forecast the average price for the next period, however, they do not yet know each state's exact importance.

Several scenarios may occur, so expected results are unclear. The assumption under which the analysis takes place is that each future scenario falls into one of two general groups of scenarios. The first group of scenarios is a situation whereby the economy flourishes and prices tend to increase more than the average trend from the previous years. The second group of scenarios is pessimistic: the economy experiences a downturn and prices tend to decrease. Many other scenarios and monetary policies are possible, but this simple example works with the assumption of these two general scenarios. In this example the experts have already reached a consensus, so all experts provide the same result. In a more general study, a previous step would appear

Table 4
US average prices by state.

State	Scenario 1				Scenario 2				Collective results			
	AP	MS	SB	UAO	AP	MS	SB	UAO	AP	MS	SB	UAO
Alabama	63	0.011	0.012	0.0117	54	0.012	0.013	0.0114	59.4	0.0114	0.0124	0.01158
Alaska	61	0.002	0.001	0.0019	52	0.003	0.002	0.0017	57.4	0.0024	0.0014	0.00182
Arizona	57	0.018	0.017	0.0160	49	0.016	0.018	0.0156	53.8	0.0172	0.0174	0.01584
Arkansas	62	0.009	0.011	0.0079	50	0.008	0.012	0.0072	57.2	0.0086	0.0114	0.00762
California	67	0.129	0.125	0.1243	61	0.131	0.123	0.1237	64.6	0.1298	0.1242	0.12406
Colorado	63	0.019	0.018	0.0166	55	0.018	0.017	0.0159	59.8	0.0186	0.0176	0.01632
Connecticut	66	0.013	0.012	0.0128	58	0.012	0.014	0.0133	62.8	0.0126	0.0128	0.01300
Delaware	63	0.004	0.003	0.0039	54	0.003	0.003	0.0032	59.4	0.0036	0.0030	0.00362
D. C.	72	0.004	0.005	0.0029	64	0.006	0.003	0.0035	68.8	0.0048	0.0042	0.00314
Florida	53	0.062	0.061	0.0644	47	0.068	0.064	0.0641	50.6	0.0644	0.0622	0.06428
Georgia	57	0.028	0.027	0.0268	53	0.025	0.026	0.0271	55.4	0.0268	0.0266	0.02692
Hawaii	54	0.004	0.005	0.0035	52	0.003	0.004	0.0037	53.2	0.0036	0.0046	0.00358
Idaho	60	0.005	0.004	0.0055	53	0.004	0.003	0.0058	57.2	0.0046	0.0036	0.00562
Illinois	73	0.045	0.043	0.0456	67	0.042	0.044	0.0459	70.6	0.0438	0.0434	0.04572
Indiana	67	0.021	0.023	0.0223	60	0.022	0.024	0.0217	64.2	0.0214	0.0234	0.02206
Iowa	64	0.007	0.006	0.0093	57	0.006	0.005	0.0096	61.2	0.0066	0.0056	0.00942
Kansas	65	0.008	0.007	0.0078	59	0.007	0.006	0.0081	62.6	0.0076	0.0066	0.00792
Kentucky	67	0.011	0.012	0.0109	60	0.013	0.011	0.0114	64.2	0.0118	0.0116	0.01110
Louisiana	54	0.014	0.013	0.0136	49	0.012	0.012	0.0134	52.0	0.0132	0.0126	0.01352
Maine	62	0.004	0.005	0.0045	56	0.003	0.004	0.0042	59.6	0.0036	0.0046	0.00438
Maryland	65	0.019	0.017	0.0181	62	0.018	0.016	0.0176	63.8	0.0186	0.0166	0.01790
Massachusetts	72	0.025	0.024	0.0231	70	0.023	0.023	0.0227	71.2	0.0242	0.0236	0.02294
Michigan	70	0.029	0.028	0.0299	65	0.027	0.029	0.0301	68.0	0.0282	0.0284	0.02998
Minnesota	71	0.015	0.017	0.0161	64	0.014	0.018	0.0157	68.2	0.0146	0.0174	0.01594
Mississippi	61	0.007	0.008	0.0085	53	0.006	0.009	0.0087	57.8	0.0066	0.0084	0.00858
Missouri	59	0.017	0.016	0.0200	52	0.016	0.018	0.0173	56.2	0.0166	0.0168	0.01892
Montana	58	0.003	0.004	0.0032	54	0.004	0.003	0.0029	56.4	0.0034	0.0036	0.00308
Nebraska	56	0.005	0.005	0.0065	50	0.006	0.006	0.0067	53.6	0.0054	0.0054	0.00658
Nevada	60	0.007	0.008	0.0081	53	0.007	0.009	0.0078	57.2	0.0070	0.0084	0.00798
New Hampshire	64	0.005	0.006	0.0035	58	0.004	0.007	0.0029	61.6	0.0046	0.0064	0.00326
New Jersey	72	0.028	0.027	0.0268	66	0.029	0.027	0.0274	69.6	0.0284	0.0270	0.02704
New Mexico	59	0.005	0.004	0.0069	52	0.007	0.005	0.0077	56.2	0.0058	0.0044	0.00722
New York	78	0.075	0.078	0.0728	73	0.079	0.074	0.0734	76.0	0.0766	0.0764	0.07304
North Carolina	64	0.031	0.032	0.0319	59	0.032	0.035	0.0323	62.0	0.0314	0.0332	0.03206
North Dakota	62	0.003	0.002	0.0029	56	0.002	0.003	0.0031	59.6	0.0026	0.0024	0.00298
Ohio	65	0.034	0.036	0.0325	59	0.033	0.035	0.0328	62.6	0.0336	0.0356	0.03262
Oklahoma	61	0.011	0.012	0.0117	53	0.013	0.013	0.0121	57.8	0.0118	0.0124	0.01186
Oregon	63	0.012	0.013	0.0117	56	0.011	0.014	0.0123	60.2	0.0116	0.0134	0.01194
Pennsylvania	73	0.046	0.049	0.0458	69	0.046	0.041	0.0463	71.4	0.0460	0.0458	0.04600
Rhode Island	69	0.003	0.002	0.0025	63	0.002	0.003	0.0027	66.6	0.0026	0.0024	0.00258
South Carolina	62	0.015	0.016	0.0175	53	0.018	0.017	0.0169	58.4	0.0162	0.0164	0.01726
South Dakota	60	0.002	0.003	0.0029	51	0.003	0.004	0.0032	56.4	0.0024	0.0034	0.00302
Tennessee	59	0.022	0.021	0.0211	50	0.021	0.022	0.0207	55.4	0.0216	0.0214	0.02094
Texas	67	0.085	0.083	0.0815	61	0.086	0.081	0.0819	64.6	0.0854	0.0822	0.08166
Utah	63	0.008	0.009	0.0078	58	0.007	0.008	0.0083	61.0	0.0076	0.0086	0.00800
Vermont	65	0.003	0.002	0.0029	59	0.004	0.003	0.0031	62.6	0.0034	0.0024	0.00298
Virginia	66	0.022	0.021	0.0245	57	0.023	0.022	0.0247	62.4	0.0224	0.0214	0.02458
Washington	64	0.022	0.023	0.0217	55	0.021	0.024	0.0219	60.4	0.0216	0.0234	0.02178
West Virginia	65	0.007	0.006	0.0069	56	0.006	0.007	0.0074	61.4	0.0066	0.0064	0.00710
Wisconsin	64	0.014	0.015	0.0156	53	0.015	0.014	0.0157	59.6	0.0144	0.0146	0.01564
Wyoming	62	0.002	0.003	0.0029	54	0.003	0.002	0.0032	58.8	0.0024	0.0026	0.00302
US average	65.8				59.5				63.3			
US Min	53.0				47.0				50.6			
UAO	65.7				59.4				63.2			
US Max	78.0				73.0				76.0			

Abbreviations: AP = average price; MS = initial market share; SB = subjective belief (weighted average); UAO = unified aggregation operator (the opinion of four experts).

in which individual expert opinions would feed into the model through an additional aggregation process.

To form the world average price, the first step is to combine information from different states and provinces to find the average price for each country. For simplicity, the study presents only the case of the USA. In the real world, however, all countries should follow this previous step to calculate their average prices.

The importance of the states and countries depends on their importance regarding the product in question on their future progress. Currently, the experts have information from the last period only. By using this information, they can forecast the future price. Three weighting vectors summarize the degree of importance. These vectors represent initial market share, subjective belief, and the UAO, which is the result of integrating several weighting vectors. Table 3 presents the weighting vectors that form the UAO. In Table 3, the assumption is that this operator collects the information from four experts who provide different opinions regarding each state's importance in determining the USA's average price for this specific product. Note that $C = (0.3, 0.3, 0.2, 0.2)$.

Next, consider the average price for the USA. Forecasting for the average price in the next period takes place under two scenarios. For each scenario, the table considers the initial market share, a general subjective belief regarding each state's degree of importance and the UAO that represents four experts' opinions. Table 3 presents the process for

Table 5
World average prices.

	Scenario 1				Scenario 2				Collective results			
Country	AP	MS	SB	UAO	AP	MS	SB	UAO	AP	MS	SB	UAO
USA	66	0.055	0.072	0.074	59	0.059	0.083	0.089	63.3	0.0566	0.0764	0.0800
Canada	64	0.007	0.011	0.010	60	0.009	0.012	0.013	62.4	0.0078	0.0114	0.0112
Mexico	59	0.016	0.014	0.015	53	0.016	0.014	0.014	56.6	0.0160	0.0140	0.0146
Central America	57	0.006	0.005	0.005	52	0.006	0.005	0.004	55.0	0.0060	0.0050	0.0046
Caribbean	58	0.006	0.005	0.006	53	0.006	0.004	0.005	56.0	0.0060	0.0046	0.0056
Brazil	61	0.029	0.026	0.027	56	0.029	0.025	0.026	59.0	0.0290	0.0256	0.0266
Argentina	60	0.006	0.005	0.005	54	0.006	0.004	0.005	57.6	0.0060	0.0046	0.0050
Colombia	53	0.006	0.004	0.005	49	0.006	0.004	0.004	51.4	0.0060	0.0040	0.0046
Venezuela	54	0.004	0.003	0.003	47	0.004	0.002	0.003	51.2	0.0040	0.0026	0.0030
Peru	48	0.004	0.003	0.004	42	0.004	0.002	0.003	45.6	0.0040	0.0026	0.0036
Chile	57	0.002	0.002	0.002	51	0.002	0.002	0.003	54.6	0.0020	0.0020	0.0024
South America	50	0.006	0.004	0.005	45	0.005	0.003	0.004	48.0	0.0056	0.0036	0.0046
Germany	67	0.014	0.021	0.020	60	0.018	0.026	0.029	64.2	0.0156	0.0230	0.0236
France	64	0.010	0.015	0.015	59	0.013	0.019	0.021	62.0	0.0112	0.0166	0.0174
UK	65	0.010	0.015	0.014	60	0.013	0.018	0.021	63.0	0.0112	0.0162	0.0168
Italy	62	0.009	0.013	0.012	56	0.011	0.016	0.019	59.6	0.0098	0.0142	0.0148
Spain	60	0.007	0.010	0.009	55	0.009	0.011	0.012	58.0	0.0078	0.0104	0.0102
Poland	56	0.006	0.006	0.006	51	0.006	0.006	0.006	54.0	0.0060	0.0060	0.0060
Netherlands	69	0.003	0.006	0.006	63	0.004	0.008	0.008	66.6	0.0034	0.0068	0.0068
Belgium	68	0.002	0.005	0.004	62	0.003	0.006	0.006	65.6	0.0024	0.0054	0.0048
Sweden	70	0.002	0.005	0.004	63	0.003	0.005	0.006	67.2	0.0024	0.0050	0.0048
Austria	67	0.002	0.004	0.004	61	0.003	0.004	0.005	64.6	0.0024	0.0040	0.0044
Romania	49	0.001	0.001	0.001	43	0.001	0.001	0.001	46.6	0.0010	0.0010	0.0010
Other EU	56	0.009	0.010	0.009	50	0.008	0.009	0.008	53.6	0.0086	0.0096	0.0086
Ukraine	46	0.005	0.005	0.004	40	0.005	0.004	0.003	43.6	0.0050	0.0046	0.0036
Norway	78	0.002	0.004	0.004	74	0.002	0.005	0.005	76.4	0.0020	0.0044	0.0044
Switzerland	79	0.002	0.004	0.004	73	0.002	0.005	0.005	76.6	0.0020	0.0044	0.0044
Other Europe	66	0.012	0.012	0.011	60	0.011	0.011	0.010	63.6	0.0116	0.0116	0.0106
Russia	60	0.022	0.022	0.021	55	0.021	0.021	0.022	58.0	0.0216	0.0216	0.0214
Central Asia	56	0.008	0.007	0.007	51	0.007	0.006	0.006	54.0	0.0076	0.0066	0.0066
Turkey	57	0.011	0.010	0.009	52	0.010	0.009	0.008	55.0	0.0106	0.0096	0.0086
Saudi Arabia	62	0.003	0.003	0.004	59	0.003	0.004	0.004	60.8	0.0030	0.0034	0.0040
Middle East	54	0.014	0.011	0.013	50	0.012	0.009	0.010	52.4	0.0132	0.0102	0.0118
Iran	57	0.009	0.007	0.008	51	0.008	0.006	0.007	54.6	0.0086	0.0066	0.0076
India	41	0.171	0.152	0.155	36	0.167	0.144	0.139	39.0	0.1694	0.1488	0.1486
Pakistan	42	0.027	0.023	0.022	36	0.025	0.021	0.019	39.6	0.0262	0.0222	0.0208
Bangladesh	40	0.023	0.020	0.021	35	0.022	0.018	0.017	38.0	0.0226	0.0192	0.0194
South Asia	42	0.014	0.012	0.012	36	0.013	0.010	0.011	39.6	0.0136	0.0112	0.0116
Indonesia	44	0.035	0.031	0.032	39	0.034	0.030	0.031	42.0	0.0346	0.0306	0.3160
Singapore	69	0.001	0.003	0.003	64	0.002	0.004	0.004	67.0	0.0014	0.0034	0.0034
Vietnam	49	0.011	0.009	0.010	43	0.010	0.007	0.008	46.6	0.0106	0.0082	0.0092
Philippines	47	0.013	0.010	0.010	45	0.012	0.008	0.009	46.2	0.0126	0.0092	0.0096
Thailand	49	0.008	0.007	0.007	44	0.007	0.006	0.007	47.0	0.0076	0.0066	0.0070
Southeast Asia	48	0.007	0.006	0.006	42	0.006	0.005	0.006	45.6	0.0066	0.0056	0.0060
China	56	0.193	0.191	0.192	51	0.195	0.194	0.193	54.0	0.1938	0.1922	0.1924
Japan	67	0.021	0.031	0.029	60	0.025	0.038	0.036	64.2	0.0226	0.0338	0.0318
South Korea	63	0.008	0.015	0.011	57	0.010	0.018	0.015	60.6	0.0088	0.0162	0.0126
Other East Asia	60	0.007	0.007	0.007	54	0.007	0.008	0.008	57.6	0.0070	0.0074	0.0074
Australia	65	0.006	0.011	0.010	61	0.008	0.015	0.014	63.4	0.0068	0.0126	0.0116
Other Oceania	64	0.004	0.008	0.007	59	0.005	0.011	0.009	62.0	0.0044	0.0092	0.0078
South Africa	58	0.007	0.008	0.007	50	0.008	0.009	0.007	54.8	0.0074	0.0084	0.0070
Nigeria	45	0.023	0.018	0.019	39	0.021	0.014	0.013	42.6	0.0222	0.0164	0.0166
Egypt	48	0.012	0.009	0.011	42	0.011	0.007	0.008	45.6	0.0116	0.0082	0.0098
Algeria	49	0.005	0.004	0.004	41	0.005	0.003	0.003	45.8	0.0050	0.0036	0.0036
Sudan	39	0.006	0.004	0.004	34	0.004	0.004	0.003	37.0	0.0520	0.0040	0.0036
Ethiopia	41	0.013	0.009	0.010	36	0.010	0.006	0.005	39.0	0.0118	0.0078	0.0080
Other Africa	43	0.075	0.062	0.061	39	0.068	0.051	0.040	41.4	0.0722	0.0576	0.0526
World average	52.1				47.4				50.2			
World Min	39.0				34.0				37.0			
UAO	53.1				49.1				51.5			
World Max	79.0				74.0				76.6			

Abbreviations: AP = average price; MS = initial market share; SB = subjective belief (weighted average); UAO = unified aggregation operator.

obtaining the UAO for the first scenario. Although the calculation does not appear in the study, calculating the UAO in the second scenario would follow the same method. The calculations to obtain the average price of the USA appear in Table 4. According to Table 4, the importance of initial market share and the importance of subjective belief are 0.3, whereas the importance of the UAO is 0.4.

The study of each state's average prices takes place separately. The process yields the average price then combines these individual prices using a range of weighting vectors that represent the expectations regarding each state's degree of importance in this specific product's price.

A similar process would yield the average price for each of the other countries. Table 5 presents the world average price for this product considering the two main scenarios that may arise in this problem. Thus, the table presents forecasts considering the different attributes available in the analysis. To simplify the analysis, only the major countries receive individual attention. All other countries appear in groups according to geographical regions. Note the assumption that the importance of initial market share is 0.3, the importance of subjective belief is 0.2, and the importance of UAO is 0.5. The first scenario's importance is 0.6, and the second scenario's possibility of occurrence is 0.4.

Results differ depending on the country under consideration. To build the world average price, the last rows indicate the average result with the initial market share and with the integration of the three weighting vectors. Table 5 also indicates the minimum and maximum to show how the price can fluctuate without losing information. This issue is also useful for designing business and economic strategies because varying prices between countries may lead to different policies in different countries. Table 5 displays the results for each scenario and provides an integrated result that represents the main expectations regarding outcomes.

5. Conclusions

The study proposes new methods to calculate average prices, focusing on the use of the UAO in the average price. Thus, the study considers a range of sub-aggregations in the problem according to the available information and the decision maker's needs. Key examples of the average price include the average price of the European Union and North America. The main advantage of using the UAO is that the UAO enables a more complete assessment of the problem and thereby avoids any potential loss of information. Thus, the UAO improves on previous methods by offering a flexible aggregation process that can adapt to a range of situations.

The study presents an illustrative example to demonstrate how these methods can apply to real-world problems. The calculation of the world average price of a specific product has been studied. First, general calculations indicate how to aggregate the available information from a range of countries. A previous step indicates that calculating the average price for each country requires an additional aggregation process because prices can vary across regions, cities, or towns. An example of these sub-aggregation processes is the calculation of the average price for the USA. This average price calculation consists of integrating the average price of all the states.

This approach creates new opportunities in average price analysis by providing flexible techniques that can adapt to the complexities of uncertain environments. This study therefore improves the analysis of price information by addressing relevant sub-structures. Nevertheless, future research should address many other issues to provide a complete approach that is adaptable to all real-world uncertainties. For example, many other sub-structures warrant attention, but the average is sometimes insufficient to assess these sub-structures. Therefore, the use of interval numbers or other such techniques may be useful to avoid the loss of information. Furthermore, information is dynamic and may change over time, which complicates price forecasting. By considering related issues, scholars could develop a complete method capable of correctly representing and forecasting price information.

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