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PLANIFICACIÓN ÓPTIMA DE GENERACIÓN ELÉCTRICA CONSIDERANDO  
POLÍTICAS DE ENERGÍAS RENOVABLES

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERO CIVIL INDUSTRIAL

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## **Resumen**

### **PLANIFICACIÓN ÓPTIMA DE GENERACIÓN ELÉCTRICA CONSIDERANDO POLÍTICAS DE ENERGÍAS RENOVABLES**

Los sistemas de electricidad en el mundo se enfrentan a retos de proporciones sin precedentes. En respuesta a la crisis del cambio climático, los gobiernos de algunos países desarrollados ya están comprometidos con invertir en tecnologías de generación renovable. En este contexto, se argumenta que tal compromiso con las energías renovables podría causar un aumento en el costo de generación. Sin embargo, este argumento no toma en cuenta los beneficios adicionales asociados a las energías renovables en términos de otras medidas de rendimiento económico, por ejemplo, el riesgo. En este trabajo se propone extender un modelo de optimización (basado en la investigación anterior de Bernales, Moreno, Rudnick e Inzunza, 2014) que determina portafolios de tecnologías de generación, incluyendo las renovables, minimizando el costo medio de inversión y operación, pero al mismo tiempo, limitando la exposición al riesgo asociado a los precios volátiles del combustible y escenarios hidrológicos de incertidumbre.

El modelo es implementado para el Sistema Interconectado Central de Chile (SIC). A partir del análisis aplicado al caso chileno, se evidenció que la generación renovable puede cubrir los riesgos asociados a las variaciones de precios de combustibles y condiciones climáticas. Cuando el objetivo es la minimización de riesgo, se alcanza un 31,8% de generación renovable de manera económicamente óptima, sin la necesidad de aplicar una ley que imponga el cumplimiento de la meta de generación renovable de un 20% para el año 2025. Este resultado es importante porque indica que una alta penetración de tecnologías renovables puede ser justificada económicamente desde la perspectiva de reducción de riesgo. En caso opuesto, cuando el objetivo es exclusivamente la minimización de costos (i.e. planificador neutro al riesgo), la cuota de generación renovable respecto a la electricidad total producida es menor, alcanzando un 18.9%.

Para incentivar el desarrollo de fuentes renovables también se puede aplicar un impuesto a las emisiones de CO<sub>2</sub>. Por esta razón se incluyó en los costos de operación una penalización a las emisiones y se realizó un análisis de sensibilidad con diferentes niveles de impuestos para analizar el efecto sobre la composición de los portafolios óptimos. Se demostró que un impuesto de US\$10 por cada tonelada de CO<sub>2</sub> emitida sería suficiente para inducir un 20% de generación renovable para todos los portafolios, independiente del nivel de riesgo (y bajo los supuestos de costos de este trabajo).

Los portafolios obtenidos al aplicar un impuesto al CO<sub>2</sub> difieren de los obtenidos al imponer un 20% de generación renovable como cota mínima (i.e. implementado como una restricción en el problema de optimización). Por ejemplo, en relación a la tecnología de concentración solar, su instalación se facilita al penalizar las emisiones de CO<sub>2</sub>. Finalmente, se concluye que bajo una planificación óptima no hay una fuente renovable ideal, ya que se complementan entre ellas formando un portafolio óptimo. La distribución óptima depende de los costos de generación, cobertura de riesgos y si las metas en base a políticas energéticas se alcanzan por leyes gubernamentales o mediante incentivos económicos, tales como los impuestos al carbono.

SUMMARY OF THE THESIS TO OBTAIN THE TITLE: Civil Industrial Engineer  
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## **Summary**

### **OPTIMAL ELECTRICITY GENERATION PLANNING UNDER RENEWABLE POLICY TARGETS**

Electricity systems worldwide face challenges of unprecedented proportions. In response to the global climate change crisis, governments of a number of developed countries are already committed to the support of renewable sources of energy. In this context, it is argued that such commitment to renewables could cause a cost increase in generation. However, this argument does not take in account further benefits associated with renewables in terms of other economic performance measures, for example, risk. In this work we propose to extend an optimization model (based on a previous research; Bernales, Moreno, Rudnick and Inzunza, 2014) that determines portfolios of generation technologies, including renewables, by minimizing the mean cost of investment and operation while constraining risk exposure through traditional risk management measures.

We implement the model for the Chilean Central Interconnected System (CIS) to present a potential analyses that can be generated by using our new approach. We find evidence that renewable generation is a valuable hedging tool against risks associated with volatile fuel prices and uncertain hydrological scenarios. We can reach 'economically optimal' generation planning with 31.8% of non-conventional renewable technologies, without any law to reach the Chilean renewable policy target of 20% by 2025, if the objective function is associated with risk decline. This is an important result, because high levels of renewables can be economically justified from the perspective of system risk reduction. In counterpart, when the main objective is exclusively the minimization of costs of generation (i.e. risk-neutral planning), the percentage of the total electricity production attributable to non-conventional renewables is less, reaching a 18.9% and falling short of the Chilean renewable policy target by 2025.

Since renewable policy targets can be reached by applying economic incentives such as a carbon tax, we include a carbon tax in the operational costs of fossil-fuels. We perform a sensitivity analysis with different levels of carbon tax to study the effects on the composition of portfolios. We show that a level of carbon tax of US\$10 per ton of CO<sub>2</sub> could induce 20% of non-conventional renewable generation for all planning portfolios, independent of the level of risk (under the cost assumptions used in the model).

Interestingly, the portfolios obtained with a carbon tax are different to the ones obtained by imposing a 20% lower-bound level of non-conventional renewable generation (i.e. included as a constraint in the optimization model). For example, we show that concentrating solar power (CSP) is installed with a carbon tax, which does not happen if the tax is not imposed.

We conclude that there is not an optimal single-renewable technology in optimal planning. Optimal allocations depend on generation costs, renewable hedge features (regarding fossil-fuel price volatility, hydrological uncertainty and changes in demand), and whether policy targets are reached by strong governmental laws or by economic incentives such as carbon taxes.

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# 1. Introduction

Energy production is critical for the evolution and progress of modern economies. Insufficient electricity generation can (directly and/or indirectly) affect countries in terms of industrial production, consumption, telecommunication, exchanges of goods, financial systems, health services and security structures, to name but a few examples. In fact, inadequate planning for electricity generation may limit economic growth and human development. However, many forms of electricity production –in particular, electricity generation based on fossil-fuels– can also contribute to serious environmental problems such as climate change, rising sea levels, pollution and acid rain. Thus, a number of countries are already committed to the support of renewables and other low carbon-emission generation technologies. For instance, the Renewables 2014 Global Status Report, from REN 21, shows that by early 2014, at least 144 countries had renewable policy targets and 138 countries had renewable energy support policies in place.<sup>1</sup> However, renewable energy production is not without its detractors; for example ExxonMobil (in its 2012 Outlook for Energy to 2040) said, “*advances in technology will be necessary to make [renewable] fuels more practical and economic ... geothermal and solar will remain relatively expensive*”. As such, there is currently a debate about renewable electricity production, with its detractors saying 'renewable energy is too expensive', and its supporters in favor of 'environmental benefits'. Nevertheless, this is still not the complete picture of the problem. On the one hand, the use of different technologies for electricity generation can also be analyzed from a portfolio perspective, in which costs are not only taken into account, but also risks (see, e.g., Awerbuch, 2006; Jansen et al., 2006; and Delarue et al., 2011). Hence, renewable generation may generate benefits in terms of diversification, which may reduce potential risks regarding future fluctuations in fossil-fuel prices and other sources of uncertainty. On the other hand, it is important to consider the security of electricity supply, since the system must be capable of absorbing changes from the intermittent electricity production of renewable technologies (see, e.g., De Jonghe et al. 2010; and Pérez-Arriaga and Battle, 2012; and Inzunza et al. 2014).

The objective and contribution of this thesis is to extend an optimization model (Inzunza et al., 2014) for optimal electricity generation planning, under renewable policy targets. The model is based on a portfolio analysis approach that incorporates different types of electricity generation technologies, which takes into account costs, risks and security supply. The model is solved by a two-stage stochastic linear programming setup, whereby we determine the optimum portfolio of generation technologies in a future power system. We optimize the vector of generation capacities in a first stage, and the operation of the proposed generation infrastructure in a second stage, which are coordinated by a Benders-based method. To quantify the level of risk of the generation portfolio, we use the Conditional Value-at-Risk (CVaR) for two main reasons: firstly it is a coherent measure of risk (see Artzner, 1999), and secondly, optimization problems with CVaR can be reduced to linear programming problems (see Rockafellar and Uryasev, 2002). We also consider operational issues related to maintaining system security levels through scheduling various types of reserves from generation and demand. The model can balance the optimal allocation planning of electricity generation, since the optimization is performed under a large number of stochastic scenarios given by unexpected fossil-fuel prices and/or hydrological conditions. Moreover, our two-stage stochastic programming approach can tackle problems of

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<sup>1</sup> REN21 is an international non-profit association made up of members of international organizations (e.g., European commission, UNIDO and the World Bank) and governments (e.g., United Kingdom, Norway and Brazil).

large dimensions (country level electricity planning, and considering a supply hour by hour in a representative day of the year) given the use of the Benders' method.

Currently, policy makers are starting to give more importance to potential environmental damage caused by electricity generation based on fossil-fuels. Thus, as mentioned above, several governments have imposed policy targets for generation of electricity from renewable sources. For instance, Europe's target is 20% of the total final energy generated from renewables. Renewable policy targets can be reached through different mechanisms such as regulations and standards by law (which impose strict minimum levels of renewable electricity generation), or by creating economic incentives –e.g. carbon tax, subsidies, and carbon tradable permissions for emission– to induce change in the technologies for electricity production (see, e.g., Stern, 2008; Harstad, 2012a,b; Marron and Toder, 2014). Despite the importance of the design of optimal mechanism to reach a specific renewable policy target (see, e.g., McLure, 2014; and Murray et al. 2014; and Acemoglu, 2014; and Golosov, 2014), we focus on a different (but still important) objective. We want to understand the optimal allocations of technologies, given that there is a policy target of  $X\%$  of renewable generation in a country. We would like to answer the following questions: What is the optimal distribution of renewable electricity generation in a country that has a policy target? What are the optimal allocations of traditional fossil-fuel generation to maintain the security of electricity supply? What are the levels of costs and risk exposure for different generation portfolios? These questions are fundamental to optimal planning for future 'clean' electricity production, and thus to sustaining the development of countries. However, surprisingly, there has been limited effort in the economic literature in relation to the analysis of an optimal distribution of technologies for electricity production under renewable policy targets. The goal of our study is to fill this gap.

We implement the model in a country level electricity generation system. The objective is to provide an example of potential planning analyses that can be generated by using our approach in different regions. We use the Chilean Central Interconnected System (CIS) to generate optimal generation planning for the year 2025. The Chilean government has a policy target by 2025 of 20% of electric generation with non-conventional renewable technologies (which encompasses all renewable technologies, except large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts)<sup>2</sup>. The Chilean Ministry of Energy reported that the energy generated in the Central Interconnected System (CIS) in 2014 reached 4,493 GWh, of which just 8.6% came from non-conventional renewable generation.

The implementation of our model in the Chilean generation system is useful in understanding the dynamics between the different generation technologies. We find evidence that renewable generation is a valuable hedging tool against risks associated with volatile fuel prices and uncertain hydrological scenarios. We can reach 'economically optimal' generation planning with 31.8% of non-conventional renewable technologies, without any law to reach the Chilean renewable policy target of 20% by 2025, if the objective function is associated with risk decline. This is an important result, because high levels of renewables can be economically justified from the perspective of system risk reduction.

Nevertheless, the allocation of renewable technologies is not homogeneous for the 2025 Chilean generation planning. The distribution of renewable technologies depends on generation costs, but

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<sup>2</sup> Chilean Non-Conventional Renewable Energy Law (Law 20.257).



also on renewable hedge features in relation to potential fossil-fuel price volatility, uncertain hydrological scenarios and changes in demand. This implementation exercise is useful in understanding that there is not an optimal, single technology to be used in efficient planning but a portfolio of them. For instance, we show that on a typical Chilean day, on average the highest demand requirement is around 1:00pm, which is exactly the time when photovoltaic generation has the maximum availability for electricity production; hence renewable technologies can also be used to provide energy during peak demand conditions and thus potentially displace the need for further generation capacity. In addition, we show that hydro reservoir plants are reduced as we consider portfolios with low levels of risk due to the risk induced in uncertain hydrological scenarios. As the portfolios have lower risk, less hydro-electric capacity is installed and coal can become more attractive to some extent. Coal generation is attractive for maintaining the security supply due to intermittent non-conventional renewable generation. However, the increase in the level of coal electricity production reaches a maximum, since coal generation is also risky in terms of fossil-fuel price volatility. Thus, technologies based on wind are triggered to reduce the levels of risk due to the increase in levels of coal (due to fossil-fuel price volatility), and to protect the system against uncertain hydrological scenarios (especially when portfolios of low risk are annualized).

Following this, we perform an exercise in which we do not impose any constraint in relation to the Chilean renewable policy target of 20%. In this exercise, where the main objective is exclusively the minimization of costs of generation, the percentage of the total electricity production attributable to non-conventional renewables is circa 18.9%, falling short of the Chilean renewable policy target by 2025. Thus policy target laws or economic incentives are required to reach the 2025 goal when cost reduction is the priority, rather than risk diminution. We show that when the policy target is imposed, in general it is biomass generation that is the preferred technology to reach the renewable goal, because it is economically cheaper than the other renewables and it is more stable –i.e. biomass can also be used to maintain the security supply.

In the model, we include variables for optimal electricity planning associated with investment and operational costs, but also variables related to risk exposure and security supply due to several reasons. Historically, electricity generation planning has been designed through optimization models that minimize costs (see, e.g., Neuhoff et al. 2008; Steffen and Weber, 2013; and Eide et al., 2014). Pure costs analysis only reflects the first moment of a stochastic process; which does not consider the possibility of changes in the system variables (e.g., costs volatility and the possibility of diverse hydrological scenarios). Therefore, adequate electricity generation planning should also consider potential risk exposure –the second moment of the distribution. In fact, if we break down electricity production into the different technologies used for generation, each technology can be seen as part of a large portfolio with different assets. Hence, the use of multiple technologies, renewables and fossil-fuels with diverse characteristics and different underlying stochastic processes for their costs and generation capacity, can induce benefits of diversification and thus reduce levels of risk exposure.<sup>3</sup>

In optimal planning for electricity generation, it is also important to take into account operational aspects in relation to the security supply of energy. For instance, Huang and Wu (2008), Gotham et al. (2009), and Vithayasrichareon and MacGill (2014) develop a cost-risk portfolio analysis for

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<sup>3</sup> Portfolio analysis in the energy sector was introduced by Bar-Lev and Katz (1976) and recently by Awerbuch and Berger (2003), Awerbuch (2006), Jansen et al. (2006), Delarue et al. (2011) and Inzunza et al. (2014).

electricity generation planning, in which they incorporate operational constraints. In these studies, the objective is to include in the generation design the potential different levels of renewable generation across periods. Delarue et al. (2011) include additional operational constraints to allow an efficient absorption of the generation system in cases of intermittent renewable outputs. In fact, currently, there is significant concern about the security of supply given the growth of renewable generation. In this context, De Jonghe et al. (2010), Pérez-Arriaga and Battle (2012) and Inzunza et al. (2014) suggest that additional flexibility measures have to be considered, such as higher volumes of generation reserves. Those contributions are incorporated in the model and presented in the Appendix 1 section.

Finally, we present an example of policy exercise that policy makers might implement by using our approach. Since renewable policy targets can be reached by applying economic incentives such as a carbon tax, we include a carbon tax in the operational costs of fossil-fuels. We perform a sensitivity analysis with different levels of carbon tax to study the effects on the composition of portfolios. In this part we want to answer: What is optimal allocation for the different technologies when a carbon tax is imposed? What is the level of carbon tax that would induce 20% of renewable generation in the following 10 years? How different are generation portfolios with and without a carbon tax in reaching a renewable policy target?

In our implementation to the Chilean generation system, the non-conventional renewable policy target of 20% by 2025 is not reached, if we do not impose a policy target constraint and pure cost reduction is the main objective of the planning. Thus, we perform an exercise in which we do not enforce a policy target of a minimum level in renewable generation, but we induce economic incentives in terms of different levels of carbon tax. We show that a level of carbon tax of US\$10 per ton of CO<sub>2</sub> can induce 20% of non-conventional renewable generation for all planning portfolios (independent of the level of risk). A more expensive tax only increases operating costs, without any contribution to developing renewable energy capacity. Interestingly, the portfolios obtained with a carbon tax are different to the ones obtained by imposing a 20% lower-bound level of non-conventional renewable generation. For example, we show that concentrating solar power (CSP) is triggered with a carbon tax, which does not happen if this tax is not imposed. This is due to the fact that when there is a large carbon tax, fossil-fuel energies have to be replaced with other renewable technologies, which are relatively stable, in order to maintain the supply security of the system. We also perform a policy exercise to evaluate the effect of an unexpected carbon tax (e.g., due to a change in renewable policy in subsequent governments in a country concerned about global warming) which is imposed after planning and building the plants for the different technology allocations. We show that the portfolios more affected are the ones with a high level of coal generation, since coal technology has the highest CO<sub>2</sub> emission factor.

The scenario for the coming years in electricity generation presents major challenges. The world energy outlook 2014 report from the international energy agency (IAE), forecast an increase of almost 80% in electricity demand over the period 2012-2040. We think that this challenge is also an opportunity in the electricity sector to expand the matrix with clean energy. However, optimal planning of the technology-type distribution of renewable generation is fundamental. The rest of the thesis is organized as follows. Section 2 presents the model to determine optimum portfolios of electricity generation. Section 3 describes the input data such as simulated scenarios, costs associated with each technology and carbon costs. Section 4 presents the main results. Concluding remarks appear in section 5.

## 2. Model

In this thesis the goal is to extend a cost-risk optimization model based on a previous research (Bernales, Moreno, Rudnick and Inzunza; 2014) which is focused on technical matters related to security of supply. Their main contribution is to add realism to the model in terms of the provision of various generation and demand based frequency control services (to manage the supply-demand balance on a real-time basis) such as: the use of demand side services (customers responding to a signal to change the amount of energy they consume from the power system at a particular time); inclusion of the preservation of system inertia levels (the amount of kinetic energy stored in all spinning turbines and rotors in the system) and spinning and standing reserves (i.e., an optimal reserves allocation of diverse technologies to give stability to the system). The model implementation (scenarios of uncertainty) and solution methodology (Benders decomposition algorithm) used in this thesis remain the same as in the previously cited research.

This section contains the modifications and contributions made to the model developed by Inzunza et al., 2014. Subsection 2.1 shows the importance of adding more renewable technologies to be considered in portfolio. Subsection 2.2 describes the cost related to carbon emissions which was added to the objective function of the optimization problem and subsection 2.3 presents a new constraint that impose a non-conventional renewable policy target. The equations of the model that were not modified are available in Appendix 1.

### 2.1 Inclusion of renewable technologies

The model developed by Andrés Inzunza and his associates determines optimal portfolios of generation technologies by minimizing the mean cost of investment and operation while constraining risk exposure through traditional risk management measures. In that research the portfolio is composed by 7 generation technologies, with wind and solar photovoltaic technologies as the only non conventional renewables<sup>4</sup>.

The portfolio theory tells us that diversification is an effective way to reduce risk. Hence, if we incorporate to the model additional renewable generation, it may generate benefits in terms of diversification, which may protect the entire portfolio from future fluctuations in fossil-fuel prices and other sources of uncertainty. In this study, we consider that the portfolio can be diversified further with the inclusion of more non conventional renewable technologies such as biomass, geothermal, small hydro and concentrated solar power<sup>5</sup>.

Extending the scope of the model will also add realism to the results because the future development of electricity in Chile does not consider wind and solar technologies as the only

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<sup>4</sup> Non-conventional renewable technologies for the Chilean regulation consider all renewable technologies excluding large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts. Large hydropower plants (hydro-reservoirs and run-of-river) are considered as conventional renewable energy sources. This type of energy is the most used in Chile, with a participation of approximately 40% in the current energy matrix.

<sup>5</sup> Concentrated Solar Power (CSP) uses tracking mirrors to reflect and concentrate sunlight onto a central point (receiver) to generate heat which is absorbed by a fluid (e.g., molten salt). The fluid can be stored in a thermal tank for later use. When electricity is required, water is piped into a steam generator where it encounters the high temperature fluid. The steam is used to drive a steam turbine connected to an electrical power generator.

renewables (as it was done in Inzunza et al., 2014). Furthermore, the composition of the portfolio is now consistent with the projection of the future energy matrix made by the Chilean Ministry of Energy (see "Escenarios Energéticos- Chile 2030").

## 2.2 Carbon emission cost

Currently, policy makers are giving more importance to potential environmental damages caused by electricity generation based on fossil fuels. Thus, several countries are promoting clean energy policies to encourage renewable energy. In this thesis we want to contribute with an example of policy exercise that policy makers might implement by using our approach, but, in order to create this example, it is necessary to add additional variables to the optimization problem.

Investment in renewable technologies may be justified by taking account of environmental costs (e.g., emission costs that penalize production from fossil-fuel generation). Since renewable policy targets can be reached by applying economic incentives such as a carbon tax, a carbon tax is included among the operational costs of fossil-fuels. This modification of the model allowed us to analyze the effect of different levels of a carbon tax (0, 10, 20, 30 and 40 U.S dollars per tones of  $CO_2e$  emitted)<sup>6</sup> in the optimal composition of portfolios.

In the next paragraphs we will introduce the optimization model. This will lead to a better understanding of how the carbon emission cost was added to the objective function of the problem:

Consider a future electricity generation system for the year  $YY$ , which incorporates multiple technologies to produce electricity. A generation technology  $i \in \{1, 2, \dots, I\}$  can potentially generate electricity in each hour  $j \in \{1, 2, \dots, 24\}$  of a representative day of the year  $YY$ , depending on the features of the technology (i.e., solar generation can only be obtained when there is sunlight). Suppose that there are set of potential future scenarios  $s \in \{1, 2, \dots, S\}$  that will describe the average day considered in this analysis. A scenario  $s$  is characterized by a group of random variables –i.e., economic and climate conditions– where  $p_s$  reflects its probability of occurrence.

The installed capacity of generation in megawatts [MW] for the technology  $i$  is denoted by  $c_i$ ; while  $g_{i,j,s}$  is the generation in megawatts per hour [MWh] for the technology  $i$ , the hour  $j$  and in the state  $s$  (both are decision variables). The cost of the system,  $C_s$ , facing the scenario  $s$  is given by four components: the annualized investment costs, the operation and maintenance costs, the carbon tax and potential additional costs due to demand shifting. Cost due to demand shifting reflect additional resources required by a change in the demand between two consecutive hours (e.g., it is not possible to generate immediately more energy with wind generation when this technology is in full capacity; hence some energy has to be injected to the system for example

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<sup>6</sup> It is important to emphasize that each energy source can be converted with different types of technology, which have different levels of performance, efficiency, and discharge more/less polluting emissions. Therefore, they should be treated with different emission factors. In this work we do not differentiate between technologies that convert the same energy source. The generation technologies were carefully chosen according to their usage in Chile and development potential over the next decade.

through fossil fuel generation reserves or battery storage). Hence, the cost for the generation system is:

$$C_s = \sum_{i \in I} INV_i \cdot c_i + \sum_{i \in I} \sum_{j \in J} (VOM_{i,s} + CT_i) \cdot g_{i,j,s} + \sum_{j \in J} D_{j,s}^- \cdot dc^- + \sum_{j \in J} D_{j,s}^+ \cdot dc^+ \quad (1)$$

where  $INV_i$  is the annuitized investment cost defined per each technology, which includes yearly fixed maintenance costs,  $VOM_{i,s}$  represents operation and maintenance costs per each technology and scenario, and  $CT_i$  is the cost related to carbon emissions per each technology. The cost of demand decrease (increase) due to demand shifting is  $dc^-$  ( $dc^+$ ), in which the amount of demand change is  $D_{j,s}^-$  ( $D_{j,s}^+$ ).

Regarding the carbon tax variable,  $CT_i$ , Table 1 presents the carbon dioxide equivalent emission factors that are assumed (measured as tones of  $CO_2e$  divided by generation in GWh).<sup>7</sup> These factors are multiplied by the corresponding potential carbon tax [ $\$/TCO_2e$ ] to get a proper 'tax-cost' measure which can then be added to the  $VOM$  costs [ $\$/MWh$ ] in the model in equation (1).

Energy Source	Technology	CO <sub>2</sub> e Emission factors [TCO <sub>2</sub> e/GWh]
Coal	Pulverized Combustion	949
Oil	Diesel fuel	779
LNG	Combined Cycle Gas Turbine	436
Solar PV	Photovoltaic	48
Geothermal	Hydrothermal (steam turbine)	28
Biomass	Integrated gasification combined cycle	24
Solar CSP	Concentrated Solar Power	20
Wind	Onshore wind turbine	11
Hydro	Conventional (dams)	7
Run-of-river	Run of the river	4
Small hydro	Hydroelectric power < 40MW	4

**Table 1. Carbon dioxide equivalent emission factors.**

The table reports the carbon dioxide equivalent emission factors. "Carbon dioxide equivalent" is a term for describing different greenhouse gases in a common unit. For any quantity and type of greenhouse gas,  $CO_2e$  signifies the amount of  $CO_2$  which would have the equivalent global warming impact. Emission factors are related to the technology used to generate electricity from each energy source indicated in the table. These technologies were carefully chosen according to their usage in Chile and development potential over the next decade. The factors shown in the Table are multiplied by the corresponding potential carbon tax [ $\$/TCO_2e$ ], resulting in extra operating costs which are added to the Variable Operation and Maintenance Costs. Data was taken from research named "Energy Scenarios Chile 2030".

In the model, we also include a cost of lost load,  $voll$ , which is the social cost of non-supply electricity for a given demand, which is charged in each hour  $j$  and for a given scenario  $s$ , in which the demand is not fully covered. Thus the total cost of the system is:

$$C_s^{Total} = C_s + voll \cdot \sum_{j \in J} LL_{j,s}, \quad (2)$$

<sup>7</sup> "Carbon dioxide equivalent" or " $CO_2e$ " is a term for describing different greenhouse gases in a common unit. For any quantity and type of greenhouse gas,  $CO_2e$  signifies the amount of  $CO_2$  which would have the equivalent global warming impact (Brander, 2012).

where  $LL_{j,s}$  is the lost load.

The risk exposure of the system is calculated using the conditional value-at-risk,  $CVaR$ , in which bad events (i.e., high costs) could occur with a probability  $\alpha$ . The optimal allocation of electricity generation from a cost-risk perspective is performed by the minimization of costs given a level of risk  $CVaR^*$  as:

$$\min_{c_i, g_{i,j,s}, R_{i,j,s}^{GR}} \sum_{s \in S} p_s \cdot C_s^{Total} \quad (3)$$

s.t.

$$\frac{1}{1-\alpha} \int_{C_s^{Total} \geq VaR_\alpha} C_s^{Total} p_s ds = CVaR^* \quad (4)$$

where  $R_{i,j,s}^{GR}$  are reserves to keep the security of electricity supply.

However, the constraint in equation (4) is not linear, which makes large dimension optimization problems difficult to track. Nevertheless, Rockafellar and Uryasev (2002) and Krokhmal et al. (2002) show that a CVaR constraint can be also written as a linear programming problem by adding a set of linear constraints. Thus, we can re-write equations (3)-(4) as:

$$\min_{c_i, g_{i,j,s}, z} \sum_{s \in S} p_s \cdot C_s^{Total} \quad (5)$$

s.t.

$$z + \frac{1}{1-\alpha} \sum_{s \in S} d_s \cdot p_s \leq CV \quad (6)$$

$$C_s^{Total} - z \leq d_s \quad \forall s \in \{1, 2, \dots, S\} \quad (7)$$

where  $z$  is an auxiliary variable that now is part of the optimization problem; while  $d_s$  is other auxiliary variable that reflects right deviation of the cost with respect to  $z$ . The risk tolerance level in the CVaR is given by  $CV$ , which represents the  $\alpha$ -CVaR's upper bound of generation portfolio costs.

### 2.3 Non-conventional renewable policy target

Another contribution of this work is to analyze the optimal allocations of technologies, given that there is a policy target of  $X\%$  of renewable generation in a country. In the case of Chile, the government has imposed a policy target<sup>8</sup> by 2025 of 20% of electric generation with non-conventional renewable technologies (which encompasses all renewable technologies, except large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts). To represent this requirement in the model, a new constraint should be added to the problem so as to ensure that on average there is enough renewable capacity installed to supply at least  $X\%$  of total electricity generation.

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<sup>8</sup> Chilean Non-Conventional Renewable Energy Law (Law 20.257).

Let impose Equation (8), where  $X$  is a renewable policy target,  $D$  is the total demand for energy in the targeted horizon, while the  $I^{NCR}$  is a subset of the electricity generation technologies ( $I^{NCR} \subseteq I$ ) called 'renewables'.

$$X \leq 100 \cdot \left( \sum_{i \in I^{NCR}} \sum_{j \in J} g_{i,j,s} \right) / D \quad \forall s \in S \quad (8)$$

Equation (8) ensures that the percentage of renewable generation meets the goal of the renewable policy target.

### **3. Model Implementation**

We implement the model for the Chilean Central Interconnected System (CIS) to provide an example of potential analyses that can be generated by using our new approach in other regions. The objective of this implementation is to generate an optimal planning for electricity generation under renewable policy targets. As mentioned above, the Chilean government has a target of 20% by 2025 of electricity generation through non-conventional renewable technologies. However, despite Chile's favorable geographical and environmental conditions and potential to install more renewable energy plants, the Chilean Ministry of Energy reported in 2014 that only 8.6% of the electricity came from non-conventional renewable technologies.

Currently, in Chilean CIS there is already an installed capacity of fossil-fuel (coal, oil and liquefied natural gas), conventional renewable (large hydro reservoirs and run-of-river) and non-conventional renewable (wind, solar photovoltaic, and biomass) technologies. Nevertheless there are initiatives to build plants with new non-conventional renewable technologies such as geothermal, small hydro, and concentrated solar power (CSP).

The model is solved for the 2015 Chilean Central Interconnected System planning by a two-stage stochastic linear programming setup, in which we determine the optimum portfolio of generation technologies of a future power system. We optimize the vector of generation capacities in a first stage, and the operation of the proposed generation infrastructure in a second stage, both of which are coordinated by a Benders-based method. The Benders' Decomposition-based algorithm used for solving the large-scale model is described in Appendix 3.

#### **3.1 Fossil-fuel technologies**

Coal, oil and liquefied natural gas (LNG) are fossil-fuels. They were formed over millions of years by the action of heat from the Earth's core and pressure from rock and soil on the remains of dead plants and animals (i.e. fossils). Hence, fossil-fuels are non renewable. The most common method of generating electricity with fossil-fuels is by steam-electric power. Water is boiled by burning fossil-fuels and the steam is piped directly into a turbine, which generates rotary motion and drives an electrical generator. In hot gas power (gas turbine), turbines are moved directly by gases produced by the combustion of natural gas or oil. The additional heat generated from a gas turbine can be used to raise steam, in a combined cycle plant that improves overall efficiency.

Fossil-fuel generation has benefits and drawbacks. On the one hand, they are in general (at present) cheaper than other non-conventional renewable options. In addition, they are very effective as reserves (headroom) to regulate generation in case of potential contingencies; thus they are fundamental to maintaining security of supply. On the other hand, fossil-fuel generation also contributes to serious environmental problems such as climate change, rising sea levels, pollution and acid rain; and they have a financial risk associated to exposure to the volatility of their prices.

#### **3.2 Renewable technologies**

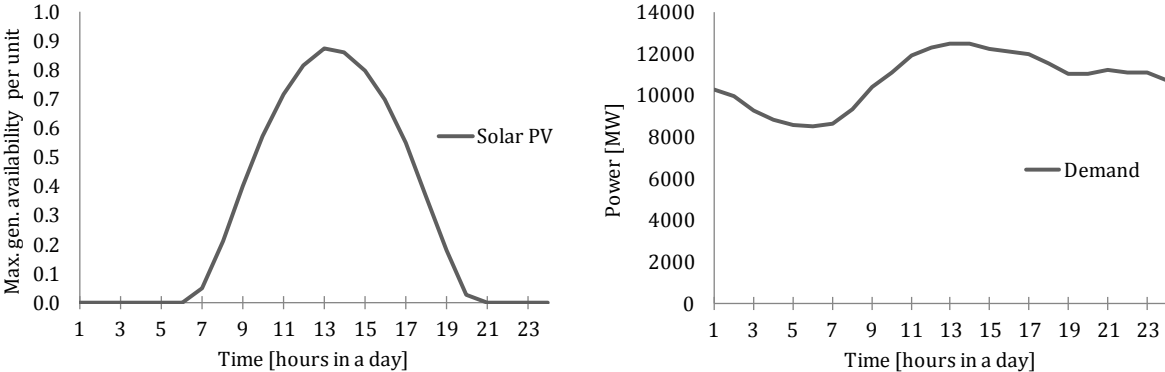
Renewable technologies for electricity generation are the ones that come from resources that are naturally renewed in a human timescale, unlike fossil-fuels that need millions of years to be replenished. Renewable technologies can generate electricity through three main methods: kinetic



energy transformation (with steam), kinetic energy transformation (without steam) and without kinetic energy. For example, concentrating solar power, biomass and geothermal generate electricity by the use of steam, which induces a rotary motion that is transformed to electricity in a turbine, in a similar way to fossil-fuel generation. Wind generation and hydropower plants (by the use of reservoirs and run-of-river ) also transform kinetic energy to electricity through the use of turbines, but kinetic energy is driven by wind, and falling (or flowing) water. In terms of electricity generation without the transformation of kinetic energy to electric power, there is solar photovoltaic generation in which panels with solar cells convert the energy of light directly into electricity.

Renewable generation also has advantages and disadvantages. In terms of advantages, renewable technologies present important environmental benefits given their reduced carbon emissions. In addition, renewable generation can be used as a diversification tool for sources of uncertainty regarding other technologies. For instance, Awerbuch (2006) shows how electricity-generating mixes can be beneficial in terms of diversification from additional renewable electricity production, since some generation systems are highly exposed to fossil-fuel price volatility.

Moreover, renewable technologies can be used to hedge changes in demand (i.e. contribution to peak demand periods). For example, Figure 1 presents the average profile of the maximum availability of solar photovoltaic generation (left hand side), and the average electricity demand (right hand side) per hour in a representative day in 2025 for the Chilean generation system (demand profile was taken from the 2012 average hourly demand and scaled to meet the projections made by the regulator for the objective year). Figure 1 shows that the highest demand requirement is around 1:00pm, which is exactly the time when photovoltaic generation has maximum availability for electricity production. Therefore, diversification benefits can be generated from fossil-fuel prices but also from the interaction between different generation profiles and demands.



**Figure 1. Solar PV generation average profile and demand of electricity per hour in a day.** The left hand side of the figure shows the average profile of the maximum availability of solar photovoltaic generation per hour in a day. This profile was obtained from data provided by the Chilean Ministry of Energy and University of Chile. The right hand side of the figure shows the average electricity demand per hour in a representative day in 2025 for the Chilean generation system. The demand profile used was taken from the 2012 average hourly demand (reported by the National Commission of Energy which is Associated Chilean Ministry of Energy) and scaled to meet the projections made by the regulator for the objective year (2025).

### 3.3 Scenarios of uncertainty

As it was done in the small scale study of Inzunza et al., 2014, we generate 1000 full year-scenarios to reflect potential conditions of a representative day in the year 2025, which represent 24,000 hours of potential generation conditions that we consider for optimal electricity generation planning. We built diverse hydrological scenarios for the Chilean generation system, which reflect the historical situation observed over the last 50 years. These 50 hydrological series were extracted from the National Commission of Energy which is associated to the Chilean Ministry of Energy. The data consists of a series of monthly averages of water inflows for each run-of-river generator, and weekly averages for each generator with a reservoir, between 1960 to March 2010.

Run-of-river inflow data was normalized and averaged using the inflow-to-power rate of each unit and its nominal capacity. In this way, a representative weekly capacity factor profile was generated for each of the 50 series. These profiles were modulated using 2012's run-of-river generation in order to obtain 50 hourly capacity factor profiles. Throughout this procedure, the hour-to-hour variability of this resource was also included in the model. Reservoir generation inflow profiles on the other hand, were summed up in order to obtain the weekly total water inflow to the system's basins. Weekly averages of total inflow were assumed to be constant throughout the hours of each week.<sup>9</sup>

We classify the 50 years of historical hydrological conditions in 10 groups, which represent potential hydrological scenarios. The groups are built according to annual averages of inflows. Table 2 shows summary statistics of the ten scenarios in terms of probabilities of occurrence, average capacity factor of run-of-river profiles, and the average capacity factor equivalent to the amount of water inflow in the reservoir profiles.

# of Scenario	1	2	3	4	5	6	7	8	9	10
Probability [p.u.]	0,06	0,06	0,13	0,17	0,13	0,11	0,17	0,08	0,04	0,06
Reservoir average capacity factor	16%	24%	30%	34%	41%	46%	52%	57%	63%	71%
Run-of-river average capacity factor	42%	49%	51%	47%	51%	53%	55%	58%	54%	59%

**Table 2. Hydrological scenarios.**

The table presents summary statistics of ten hydrological scenarios for the Chilean generation system. These scenarios have different probabilities of occurrence and are characterized by hourly capacity factor profiles, generated from historical data observed over a period of 50 years, from 1960 to 2010. The "Capacity factor" refers to the ratio of the actual output of a power plant over a period of time, to its potential output if it had operated at full nameplate capacity over the same period of time. The electricity generation planning is modeled for a representative day of a year on an hour by hour basis, thus 24 capacity factors are averaged for each hydrological scenario and reported in the table. Two hydro resources are considered in the model: reservoir and run of river. Capacity factors vary greatly depending on the hydrological condition, thus a dry (wet) year is associated to low (high) capacity factors. The hydrological series used to build the scenarios are extracted from the National Commission of Energy which is part of the Chilean Ministry of Energy.

We also generate fossil-fuel price scenarios through Monte Carlo simulations using the Cholesky decomposition of the correlation matrix, in order to capture the dynamics of price correlations. We take historical international fossil-fuel prices on an annual basis between 1984 and 2014 to

<sup>9</sup> Water inflow series are extracted from National Commission of Energy, and due to the extension of this data, they are available from the authors upon request.

obtain the variance-covariance matrix of price returns. Fossil-fuel prices were obtained from the Energy Information Administration which is part of the U.S. Department of Energy. These prices were converted to 2014 US dollars using the CPI of the United State. We use international fossil-fuel prices rather than Chilean values, since the price of commodities in electricity generation are traded internationally. Thus, we assume that prices follow a geometric Brownian motion

$$P_{t+\Delta t}^i = P_t^i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2) \cdot \Delta t + \sigma_i \cdot z_{t+\Delta t}^i \cdot \sqrt{\Delta t}} \quad (10)$$

where we generate correlated random sequences for each fossil-fuel price by the Cholesky decomposition of the correlation matrix. If the Cholesky decomposition of the correlation matrix is  $Y$ , then the random values for  $[z_{t+\Delta t}^1 \ z_{t+\Delta t}^2 \ \dots \ z_{t+\Delta t}^l]^T$  are equal to  $Y \cdot [\varepsilon^1 \ \varepsilon^2 \ \dots \ \varepsilon^l]^T$ , where  $\varepsilon^i$  are random numbers that distribute  $N(0,1)$ . Table 3 presents summary statistics of fossil-fuel prices and returns.

	Coal	LNG	Oil
Mean Returns	2%	2%	6%
Standard Deviation Returns	12%	14%	21%
Expected Price (2025) [\$/MWh]	43	107	204
Correlation Coefficients			
Coal	1	-	-
LNG	0,4	1	-
Oil	0,2	0,7	1

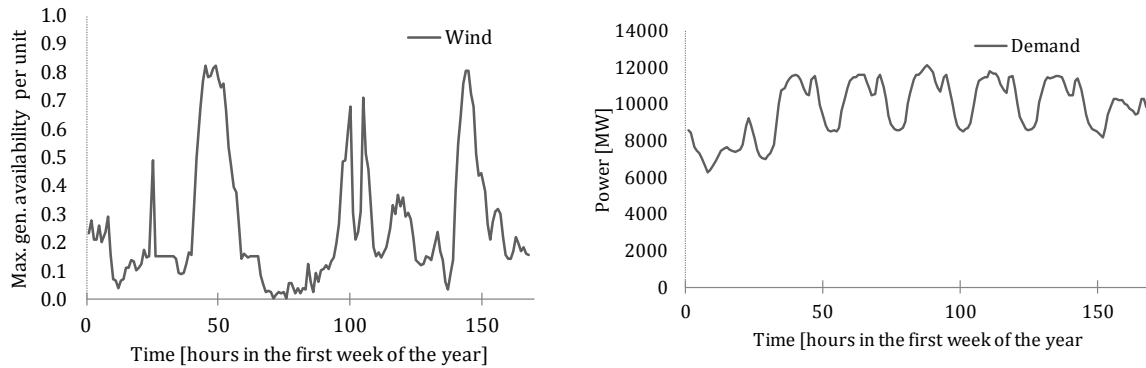
**Table 3. Statistical data of fuel price returns time series.**

The table contains statistical data used to elaborate 100 Monte-Carlo fossil-fuel scenarios. It is assumed that prices follows a geometric Brownian motion, thus statistical parameters of fuel price returns time series such as mean returns, standard deviation, expected price; and correlation coefficients are needed to build the price scenarios for Coal, Liquefied natural gas and Oil (fossil-fuels considered in the model). The correlated random sequences for each fossil-fuel prices are generated by Cholesky decomposition of the correlation matrix. Time series of historical international fossil-fuel prices are obtained from the Energy Information Administration of the US. The period covered is from 1984 to 2014.

The amount of potential energy that can be produced in Chile is on average constant per year for wind, solar and geothermal generation, which is not the case for hydrological scenarios (e.g., we could have dry years). Similarly, the expected level of annual demand does not have a high variability. Therefore, we focus on uncertainty for hydro and fossil fuel prices that can significant change from one year to another. Additionally, we consider a small case study on hour by hour basis where generation and demand profiles present certain level of variability. Thus, we analyzed those profiles and chose carefully a representative day of the year in order to work with a sample that may not affect the generation planning.

Figure 2 shows the wind profile and demand profile per hour in the first week of 2012. We can see in Figure 2 that hour by hour profiles can provide an important source of variability, which can interact with both hydrological scenarios and fossil-fuel price uncertainty. The profiles in 2012 for wind, solar, geothermal and electricity demand were obtained from data provided by the

Chilean Ministry of Energy and University of Chile. In the case of the demand profiles, their values are scaled up in order to meet 2025 projections undertaken by the Chilean Ministry of Energy.<sup>10</sup>



**Figure 2. Wind profile and Demand profile per hour in the first week of 2012.**

The left hand side of the figure shows the profile of the maximum availability of wind generation per hour in the first week of 2012 (24\*7=168 hours). This profile was obtained from data provided by the Chilean Ministry of Energy and University of Chile. The right hand side of the figure shows the electricity demand per hour also in the first week of 2012 reported by the National Commission of Energy, but their values are scaled up in order to meet 2025 projections undertaken by the Chilean Ministry of Energy.

### 3.4 Costs and current installed capacities for different generation technologies.

The costs associated to the different technologies were taken from projections presented in the report called "Escenarios Energéticos- Chile 2030" which was developed by a group of Chilean institutions associated with the electricity generation sector, in which academics participate, where the Chilean Ministry of Energy and the Chilean Ministry of Environmental Affairs are part of the advisory committee of this report. Current installed capacities were taken from an Electricity Bulletin dated December 2014 from the Chilean Association of Electricity Generators. Maximum capacities (higher bounds) that could be installed by 2025 were carefully chosen for each technology depending on the availability of energy sources, and the number of projects approved or being studied by the Chilean Ministry of Energy that could be ready for operations before 2025.<sup>11</sup> Table 4 presents the parameters assumed in this study in terms of costs, current installed capacities, and maximum capacities. Additional parameters used in the model implementation are reported in Appendix 4.

<sup>10</sup> In the case of fossil-fuel generation (coal, oil and liquefied natural gas) and biomass, we do not consider any profile since these technologies are used as required since they only consume the respective combustible.

<sup>11</sup> Biomass technology used in this study is biogas which is used in an integrated gasification combined cycle with a load factor of 85%.

	Annuitized Investment Cost	Variable Maintenance Cost	Lifespan	Current Installed Capacity	Higher Bound
	[\$/kW-year]	[\$/MWh]	[Years]	[MW]	[MW]
Coal	221	5 <sup>(a)</sup>	24	2394	6000
Oil	55	15 <sup>(a)</sup>	24	2303	3000
Hydro	202	5	50	4053	7000
Wind	188	9	20	634	4000
Solar PV	132	4	25	169	2600
Liq. Nat. Gas (LNG)	93	3 <sup>(a)</sup>	24	2777	3000
Run-of-river	202	5	50	1965	4000
Biomass	241	4 <sup>(a)</sup>	24	504	1000
Geothermal	395	13	24	0	200
Small hydro	303	7	50	350	800
Solar CSP	463	8	25	0	200

<sup>a</sup> Does not include fuel costs.

#### **Table 4. Generation Input Data.**

The table presents the annuitized investment cost, variable maintenance cost, lifespan, current installed capacity and maximum capacity (higher bound) for each generation technology considered in the model. Annuitized investment cost are calculated using a 10% discount rate over a time period equal to the lifespan. Costs parameters and lifespan shown in this table are taken from research named "Energy Scenarios Chile 2030". The cost parameters are adjusted to the Chilean reality, thus they may differ with international values. Current installed capacities are taken from an Electricity Bulletin emitted by the Chilean Association of Electric Generators and are updated to December 2014. Higher bounds are carefully chosen for each technology depending on energy sources' availability and the number of projects that could be ready for operation before 2025.

## 4. Results

### 4.1 Renewable energy planning with policy targets

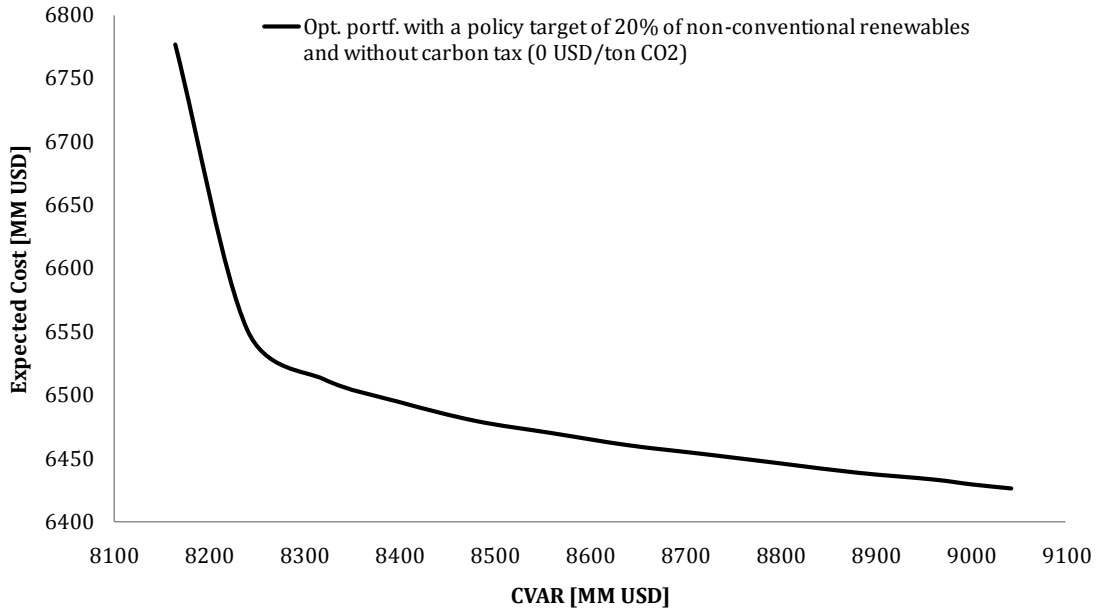
In this subsection, we present the results of our model implementation in relation to the effects of renewable policy target constraints. We consider  $X = 20$  in equation (8) to reach the Chilean policy goal. However, in the Chilean case 'renewables' represent a subset of non-conventional renewables as explained before. Hence, in the implementation for the Chilean generation planning by 2025, the policy target constraint in equation (8) only considers generation from non-conventional renewables.

We present the results of different optimal portfolios which have diverse levels of risk in terms of the *CVaR*. Table 5 reports the planning investment decisions (installed capacity) and the corresponding distribution of technologies for six different portfolios (named by letters from A to F). Risk increases from left to right. The last rows show the percentage accounted for by non-conventional renewable energy sources in terms of installed capacity and expected average generation. Figure 3 shows the cost-risk efficient frontier of all portfolios in Table 5.

Opt. portf. with a policy target of 20% of non-conventional renewables and without carbon tax (0 USD/ton CO2)												
	A		B		C		D		E		F	
Expected Cost [MM \$]	6777		6512		6479		6460		6438		6425	
CVaR [MM \$]	8165		8320		8480		8640		8880		9042	
Installed Capacity												
	MIN CVAR								MIN COST			
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
Coal	4996	18,7%	4975	21,3%	4968	21,4%	4601	19,6%	4142	17,0%	3872	15,6%
Oil	2303	8,6%	2303	9,9%	2303	9,9%	2303	9,8%	2303	9,5%	2303	9,3%
Hydro	4053	15,2%	4053	17,4%	4253	18,3%	4932	21,0%	6221	25,6%	7000	28,2%
Wind	4000	15,0%	634	2,7%	634	2,7%	634	2,7%	634	2,6%	634	2,6%
Solar PV	2600	9,7%	2600	11,1%	2600	11,2%	2600	11,1%	2600	10,7%	2600	10,5%
LNG	2777	10,4%	2777	11,9%	2777	12,0%	2777	11,8%	2777	11,4%	2777	11,2%
Run-of-river	4000	15,0%	4000	17,1%	4000	17,3%	4000	17,0%	4000	16,4%	4000	16,1%
Biomass	1000	3,7%	1000	4,3%	648	2,8%	648	2,8%	648	2,7%	648	2,6%
Geothermal	200	0,7%	200	0,9%	200	0,9%	200	0,9%	200	0,8%	200	0,8%
Small hydro	800	3,0%	800	3,4%	800	3,5%	800	3,4%	800	3,3%	800	3,2%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,2%</b>		<b>22,4%</b>		<b>21,1%</b>		<b>20,8%</b>		<b>20,1%</b>		<b>19,7%</b>	
<b>% of Renewable Generation</b>	<b>31,8%</b>		<b>22,8%</b>		<b>20,0%</b>		<b>20,0%</b>		<b>20,0%</b>		<b>20,0%</b>	

**Table 5. Portfolios that meet the Renewable energy policy target.**

The table reports the output result of the Chilean electricity generation planning by 2025, under a policy target of 20% of non conventional renewables and without carbon tax. The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the *CVaR* (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility and uncertain hydrological scenarios. In total, 1000 full year-scenarios are generated to reflect potential conditions of an average day in year 2025. The table also reports the corresponding share of total installed capacity for every technology inside a portfolio. Expected cost of the portfolios corresponds to the sum of annuitized investment cost and operating cost related to the annual maintenance of the system (measured in millions of US dollars). Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy. The table is associated with Figure 3.



**Figure 3. Efficient frontier formed by portfolios with a policy target of 20% of non conventional renewables but without considering carbon tax.**

The figure reports the efficient frontier composed by portfolios of Table 5. While Minimizing Costs, the model was constrained by different CVaR limits to draw the Pareto boundary between minimum expected cost and minimum risk solutions. The axis values are displayed in millions of US dollars.

Table 5 shows that the policy target constraint in equation (8) is active for portfolios C-F, since the percentage of non-conventional renewable generation reaches exactly 20%. However, the distribution of the different technologies is not homogeneous, since portfolios C-F also have different levels of risk exposure. This analysis is useful in understanding that there is not an optimal, single predictable technology when costs, risks and security supply are simultaneously added to the planning framework under renewable policy targets.

It is important to notice that the values of the installed capacity of renewable energy differ from the values of expected renewable generation (In Table 5 and in the following analyses, for some portfolios the former is higher than the later and *vice versa*); although the answer behind these differences is intuitive. In the model, we assumed initial capacities for all technologies, according to the current energy matrix in Chile. However, this assumption does not ensure that the model will decide to generate with that installed capacity. The most extreme case is the oil electricity generation (with 2303 MW of installed capacity). The model does not trigger additional capacity for this technology; furthermore, this technology is set nearly to zero for generation in every hour of the represented day, because it is not economically convenient –i.e. oil has the highest operating costs of all technologies. In addition, some renewable technologies cannot generate electricity since they depend on weather conditions or time-periods during the day (e.g., solar photovoltaic generation can produce energy in the hours of the day with sunlight). Both effects can produce differences between the installed capacity and expected generation of non-conventional renewable generation in the different portfolios. These considerations can also induce differences within installed capacities. For example, since oil electricity generation does not expand from its current installed capacity, it always has 2303 MW of installed capacity, however the percentage of installed capacity is different across portfolios (portfolio E and portfolio F have a level of 9.5% and 9.3%, respectively).

To explain the above idea clearly, let us take a portfolio with two different technologies, each one with 1 MW of installed capacity, meaning a fifty-fifty share. If the demand is 2 MWh, then each technology will generate 1 MWh to satisfy the demand and they will have the same share of installed capacity and generation (both 50%). Now, if the demand is only 1 MWh, then it is not clear which technology will be preferred to generate. If one technology is much convenient than the other, then it will generate exactly 1 MWh to satisfy the demand by its own. In this case the share of installed capacity remains 50%-50%, but the share of generation will be 100% for one technology and 0% for the other. This example illustrates the difference between % of installed capacity and generation in portfolios.

Table 5 presents evidence that as the level of risk is reduced, the use of renewables increases. In fact, the policy target constraint in equation (8) is not active in the portfolio with the lowest level of risk (portfolio A). In portfolio A it is 'economically optimal' to have large levels of capacity installed (32.2%) and generation (31.8%) of non-conventional renewables, which can be reached 'without' any policy target. This is a key point, because in a situation in which risk reductions of the generation system are more important than cost cuts, a significant level of non-conventional renewable technology can be economically justified under a portfolio analysis. The higher penetration of renewables reduce the CVaR value by US\$877 millions from portfolio F to portfolio A, but the increase in renewable generation results in a more expensive portfolio in terms of expected cost (portfolio A is US\$351 millions more expensive than portfolio F).

Table 5 reports that hydro reservoir generation is reduced as portfolios with lower risks are considered. Although hydro-electric plants are the most economic investment option and therefore preferred in the minimum cost solution, it drives risk exposure due to uncertainty in hydro production. In fact, in a dry year hydro-electric plants can produce only at 20% of their maximum load factor, while in a humid year production increases up to 60% of maximum load factor. Since lack of hydro production that can occur in dry years, the high penetration of hydro can significantly increase the cost of operation (including cost of unsupplied demand).

Interestingly, Table 5 reports that as hydro power capacity shrinks, coal capacity grows. As the portfolios have lower risk, less hydro-electrical capacity is installed and coal generation become more attractive, since it is used to maintain a reasonable security supply due to intermittent renewable generation. The percentage of coal installed is higher in portfolio A (18.7%) than in Portfolio F (15.6%).

Table 5 shows that some technologies barely expand from their current installed capacity (Oil, LNG and Solar CSP), while other technologies reach the upper bound (geothermal, small hydro, run-of-river and solar PV). In the case of Oil, LNG and Solar CSP, they are not activated in terms of new allocation, since they are not economically convenient; these technologies are highly expensive. In the case of geothermal, small hydro, run-of-river and solar PV, they are technologies that appear as attractive options independently of the portfolio's risk level. They are always fully used since they reduce exposure to fuel price volatility. Furthermore, geothermal, small hydro and run-of-river technologies have high capacity factors (over 50%) and relatively stable profiles in terms of hour by hour changes during the year. The installed capacity of solar PV is also justified by the high correlation between solar and demand profiles, as shown in Figure 1.



## 4.2 Renewable energy planning without policy targets

In this subsection, we show the same portfolio analysis as in Table 5, but now we do not impose any policy target constraint in the model – i.e. we assume that  $X = 0$  in equation (8). This new exercise is presented in Table 6. If we compare Table 5 and Table 6, we can notice that portfolios A and B are the same, in that they already meet the target of renewable generation, and there is no need to change their composition (in portfolios A and B the constraint in equation (8) is not active in Table 5), since they reach the policy target thanks to risk reduction considerations. However, when *CVaR* is not constrained sufficiently, we can see in Table 6 that portfolios C to F are short of non-conventional renewable generation regarding the target of 20% imposed by the Chilean government.

Opt. portf. without a policy target and without carbon tax (0 USD/ton CO2)												
	A		B		C		D		E		F	
Expected Cost [MM \$]	6777		6512		6478		6457		6435		6421	
CVaR [MM \$]	8165		8320		8480		8640		8880		9062	
Installed Capacity												
	MIN CVAR								MIN COST			
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
Coal	4996	18,7%	4975	21,3%	5124	22,3%	4752	20,3%	4280	17,6%	3972	16,0%
Oil	2303	8,6%	2303	9,9%	2303	10,0%	2303	9,8%	2303	9,5%	2303	9,3%
Hydro	4053	15,2%	4053	17,4%	4053	17,6%	4865	20,8%	6223	25,6%	7000	28,2%
Wind	4000	15,0%	634	2,7%	634	2,8%	634	2,7%	634	2,6%	634	2,6%
Solar PV	2600	9,7%	2600	11,1%	2600	11,3%	2600	11,1%	2600	10,7%	2600	10,5%
LNG	2777	10,4%	2777	11,9%	2777	12,1%	2777	11,8%	2777	11,4%	2777	11,2%
Run-of-river	4000	15,0%	4000	17,1%	4000	17,4%	4000	17,1%	4000	16,4%	4000	16,1%
Biomass	1000	3,7%	1000	4,3%	504	2,2%	504	2,2%	504	2,1%	504	2,0%
Geothermal	200	0,7%	200	0,9%	200	0,9%	200	0,9%	200	0,8%	200	0,8%
Small hydro	800	3,0%	800	3,4%	800	3,5%	800	3,4%	800	3,3%	800	3,2%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,2%</b>		<b>22,4%</b>		<b>20,6%</b>		<b>20,2%</b>		<b>19,5%</b>		<b>19,1%</b>	
<b>% of Renewable Generation</b>	<b>31,8%</b>		<b>22,8%</b>		<b>18,9%</b>		<b>18,9%</b>		<b>18,9%</b>		<b>18,9%</b>	

**Table 6. Expected costs, CVaR and installed capacity.**

The table presents the same report as in Table 5 but without a policy target. The table reports the optimal allocation (installed capacity in megawatts) for the Chilean electricity generation system by the representative day of year 2025. Six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars) are shown in the Table. Risk increases from left to right and it is associated with the fossil-fuel price volatility and uncertain hydrological scenarios. In total, 1000 full year-scenarios are generated to reflect potential conditions of year 2025. The table also reports the corresponding share of total installed capacity for every technology inside a portfolio. Expected cost of the portfolios corresponds to the sum of annuitized investment cost and operating cost related to the annual maintenance of the system (measured in millions of US dollars). Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

Table 6 shows that when the policy target is imposed (Table 5), biomass generation is the preferred technology for reaching the renewable Chilean goal by 2025. In portfolios C to F there is an increase in the installed capacity of biomass, from 504 MW (in Table 6) to 648 MW (in Table 5). Biomass is the preferred technology as it is more economically optimal than the others. Biomass has a high level for the maximum generation availability in relation to other non-conventional renewables (Biomass: 0.85, Wind: 0.28 and Solar CSP: 0.51), and relatively low investment costs (less than Solar CSP). Once the renewable policy target is accomplished (in

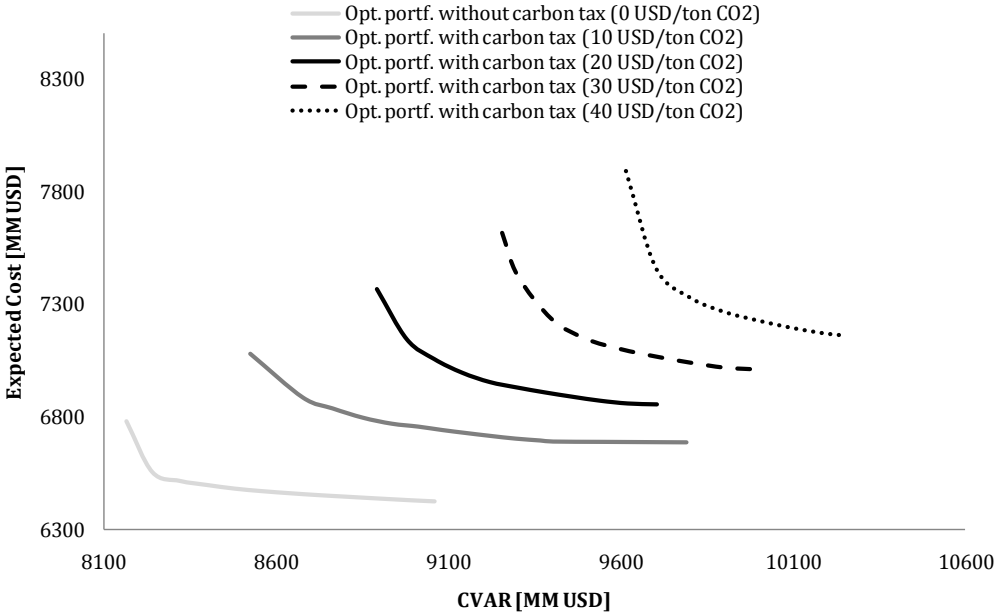
portfolios C to F in Table 5) by the use of biomass technology, additional biomass generation is triggered only in portfolios with low levels of risk exposure (portfolios A and B). This can be noted by comparing portfolio C with B. In portfolio C, biomass installed capacity is just enough to meet the renewable generation target of 20% and then, in portfolio B, it expands and reaches the upper bound.

Another difference between Table 5 and Table 6 is the change in allocation of hydro and coal. For portfolios C to F, the investing decision is driven by similar consideration, as explained in the previous section: as risk is constrained in portfolios, less hydro-electric capacity is installed and Coal capacity grows. However, the installed capacities are different due to the policy target constraint in Table 5. For instance, the minimum hydro-electric capacity is now reached in portfolio C in Table 6, instead of portfolio B in Table 5. This causes an interesting effect: once hydro-electric capacity is at its minimum, the increase in Coal capacity makes portfolio risk rise again due to its price volatility, thus, between portfolios C to B we can see an unexpected reduction in Coal capacity, while Biomass is fully installed to hedge against coal price uncertainties.

### 4.3 Policy Analysis

In subsection 2.2 we presented the inclusion of carbon emissions cost in the objective function. In the next exercise we show a sensitivity analysis in which different levels of carbon tax were applied.

Figure 4 presents the efficient frontiers of the sensitivity analysis, and Table 7 shows the composition of the portfolios. In Table 7 and Figure 4 we do not impose any policy target constraint –i.e.  $X = 0$  in equation (8). Thus, the levels of non-conventional renewables are exclusively driven by changes in the renewable policy regarding different values of a potential carbon tax.



**Figure 4. Sensitivity analysis of the efficient frontier with different levels of carbon costs.** The figure reports the efficient frontiers composed by portfolios of Table 7. Each curve is determined by different levels of carbon emissions penalization (0, 10, 20, 30 and 40 U.S dollars per tones of CO<sub>2</sub>e

emitted). While Minimizing Costs, the model was constrained by different CVaR limits to draw the Pareto boundary between minimum expected cost and minimum risk solutions. The axis values are displayed in millions of US dollars.

Opt. portf. without policy targets constraints but with different carbon taxes												
	A		B		C		D		E		F	
	MIN CVAR										MIN COST	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
Opt. portf. without policy targets constraints and without carbon tax (0 USD/ton CO2)												
Expected Cost [MM \$]	6777		6512		6478		6457		6435		6421	
CVaR [MM \$]	8165		8320		8480		8640		8880		9062	
Installed Capacity												
Coal	4996	18,7%	4975	21,3%	5124	22,3%	4752	20,3%	4280	17,6%	3972	16,0%
Hydro	4053	15,2%	4053	17,4%	4053	17,6%	4865	20,8%	6223	25,6%	7000	28,2%
Wind	4000	15,0%	634	2,7%	634	2,8%	634	2,7%	634	2,6%	634	2,6%
Biomass	1000	3,7%	1000	4,3%	504	2,2%	504	2,2%	504	2,1%	504	2,0%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,2%</b>		<b>22,4%</b>		<b>20,6%</b>		<b>20,2%</b>		<b>19,5%</b>		<b>19,1%</b>	
<b>% of Renewable Generation</b>	<b>31,8%</b>		<b>22,8%</b>		<b>18,9%</b>		<b>18,9%</b>		<b>18,9%</b>		<b>18,9%</b>	
Opt. portf. without policy targets constraints but with carbon tax (10 USD/ton CO2)												
Expected Cost [MM \$]	7077		6835		6755		6713		6687		6685	
CVaR [MM \$]	8527		8766		9014		9224		9550		9793	
Installed Capacity												
Coal	5006	18,7%	4029	15,4%	3147	11,3%	2743	10,0%	3302	13,4%	2904	12,0%
Hydro	4053	15,2%	5587	21,3%	7000	25,2%	7000	25,5%	7000	28,4%	7000	28,9%
Wind	4000	15,0%	2890	11,0%	4000	14,4%	4000	14,6%	634	2,6%	634	2,6%
Biomass	1000	3,7%	1000	3,8%	1000	3,6%	1000	3,6%	1000	4,1%	1000	4,1%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,2%</b>		<b>28,6%</b>		<b>30,9%</b>		<b>31,4%</b>		<b>21,3%</b>		<b>21,6%</b>	
<b>% of Renewable Generation</b>	<b>31,8%</b>		<b>28,9%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>22,8%</b>		<b>22,8%</b>	
Opt. portf. without policy targets constraints but with carbon tax (20 USD/ton CO2)												
Expected Cost [MM \$]	7369		7059		6956		6910		6874		6857	
CVaR [MM \$]	8894		9060		9221		9380		9540		9705	
Installed Capacity												
Coal	5067	18,9%	3974	13,9%	3402	12,1%	3024	10,9%	2700	9,9%	2394	8,8%
Hydro	4053	15,1%	6850	24,0%	7000	24,9%	7000	25,3%	7000	25,6%	7000	25,9%
Wind	4000	14,9%	4000	14,0%	4000	14,2%	4000	14,4%	4000	14,6%	4000	14,8%
Biomass	1000	3,7%	1000	3,5%	1000	3,6%	1000	3,6%	1000	3,7%	1000	3,7%
Solar CSP	0	0,0%	0	0,0%	5	0,0%	5	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,1%</b>		<b>30,2%</b>		<b>30,6%</b>		<b>31,1%</b>		<b>31,4%</b>		<b>31,8%</b>	
<b>% of Renewable Generation</b>	<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>	
Opt. portf. without policy targets constraints but with carbon tax (30 USD/ton CO2)												
Expected Cost [MM \$]	7619		7428		7146		7068		7018		7011	
CVaR [MM \$]	9255		9300		9500		9700		9900		9983	
Installed Capacity												
Coal	4805	18,0%	4377	15,9%	3550	12,6%	2997	10,8%	2559	9,4%	2394	8,8%
Hydro	4053	15,2%	5403	19,6%	7000	24,8%	7000	25,3%	7000	25,7%	7000	25,9%
Wind	4000	15,0%	4000	14,5%	4000	14,2%	4000	14,5%	4000	14,7%	4000	14,8%
Biomass	1000	3,7%	1000	3,6%	1000	3,5%	1000	3,6%	1000	3,7%	1000	3,7%
Solar CSP	200	0,7%	116	0,4%	7	0,0%	0	0,0%	0	0,0%	0	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,9%</b>		<b>31,6%</b>		<b>30,5%</b>		<b>31,1%</b>		<b>31,6%</b>		<b>31,8%</b>	
<b>% of Renewable Generation</b>	<b>32,7%</b>		<b>32,3%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>	
Opt. portf. without policy targets constraints but with carbon tax (40 USD/ton CO2)												
Expected Cost [MM \$]	7890		7460		7267		7195		7170		7162	
CVaR [MM \$]	9615		9703		9901		10100		10200		10259	
Installed Capacity												
Coal	4869	18,2%	4215	14,5%	3286	11,7%	2762	10,1%	2523	9,3%	2394	8,8%
Hydro	4053	15,1%	7000	24,1%	7000	25,0%	7000	25,5%	7000	25,7%	7000	25,8%
Wind	4000	14,9%	4000	13,7%	4000	14,3%	4000	14,6%	4000	14,7%	4000	14,8%
Biomass	1000	3,7%	1000	3,4%	1000	3,6%	1000	3,6%	1000	3,7%	1000	3,7%
Solar CSP	200	0,7%	200	0,7%	8	0,0%	0	0,0%	7	0,0%	7	0,0%
<b>% of Renewable Energy Installed</b>	<b>32,8%</b>		<b>30,2%</b>		<b>30,8%</b>		<b>31,3%</b>		<b>31,6%</b>		<b>31,8%</b>	
<b>% of Renewable Generation</b>	<b>32,8%</b>		<b>32,7%</b>		<b>31,9%</b>		<b>31,8%</b>		<b>31,8%</b>		<b>31,8%</b>	

**Table 7. Optimal portfolios with different levels of carbon costs.**

The table presents the same report as in Table 5 and Table 6 but for different carbon taxes without policy targets. The carbon tax is based on the emission factors presented in Table 1 and it is added to Variable Operation and Maintenance Costs. This table contains a sensitivity analysis for different levels of carbon emissions penalization (0, 10, 20, 30 and 40 U.S dollars per tones of CO<sub>2</sub>e emitted). Some technologies are hidden from the table because they did not present change in installed capacity from one portfolio to another. The table reports the optimal allocation (installed capacity in megawatts) for the Chilean electricity generation system by the year 2025. The table is associated with Figure 4.

As explained previously, some technologies do not change in the different portfolios because either they do not expand from their current installed capacity or they reach the upper bound (Oil, LNG, Geothermal, Small hydro, Run-of-river and Solar PV). This also happens in the following analyses in this study. Therefore, henceforth, we will focus the analysis on technologies that vary from one portfolio to another: Coal, Hydro, Wind, Biomass and Solar CSP.

As can be seen in the previous subsection (Table 6), in cases where a carbon tax was not considered, biomass and wind reach their maximum installed capacity only when risk is sufficiently constrained. Here, in Table 7, we notice that a carbon tax value of US\$10/TCO<sub>2</sub> is enough to encourage biomass to expand to the upper bound independently of the portfolio's risk level. The same is true for wind, when a carbon tax value of US\$20/TCO<sub>2</sub> is considered.

In the other cases in Table 7, when the carbon tax value is increased up to US\$30/TCO<sub>2</sub> and US\$40/TCO<sub>2</sub>, the composition of the portfolio does not present major changes because wind and biomass technologies have already reached the upper bound of installed capacity. It is worth mentioning that solar CSP is triggered in the minimum risk portfolio (where 200MW of capacity is installed). To some extent, hydro is also fully installed with the exception of minimum risk portfolios. The role of coal technology is to fill the shortfall in generation and to contribute to security of supply. With a tax of US\$40/TCO<sub>2</sub>, solar CSP is not only triggered and fully installed in portfolio A (the minimum risk portfolio), but also in portfolio B, which is a less risk constrained portfolio. These results present evidence of the effect of carbon tax on encouraging renewable energy development as an alternative to accomplish 20% of renewable generation. A penalty of US\$10/TCO<sub>2</sub> is enough to reach the policy target, achieving 22.8% of renewable generation when just a pure cost reduction is considered (portfolio F). If we combine the effect of carbon tax and risk reduction, the maximum participation of renewables is justified with a US\$40/TCO<sub>2</sub> tax, which results in 32.8% of renewable generation.

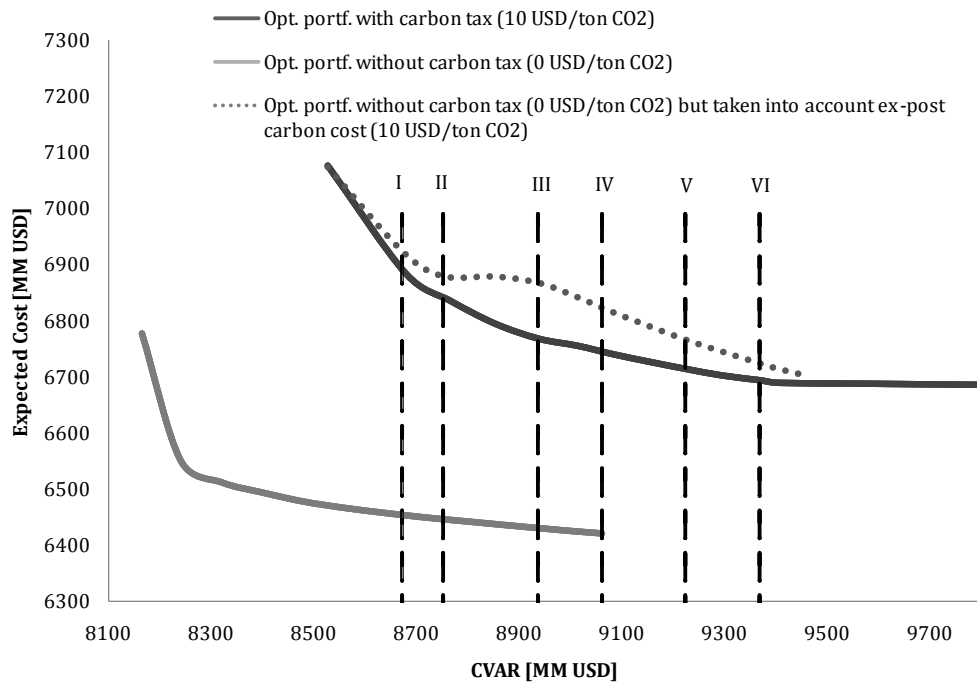
Increasing the levels of carbon tax will naturally increase the expected costs of portfolios (in part, because the carbon tax is a cost component of the variable generation cost). We notice that, for every US\$10 extra in carbon emissions penalization, the expected cost increases on average by 3%. This percentage varies in from 1.6% to 5.2%, equivalent to US\$ 121 - US\$ 369.

Finally, we perform a policy exercise to evaluate how important it is to take into account a carbon tax in optimal generation planning (which is reported in Table 8 and Figure 5), compared with a case where the planner neglects the tax and the operator optimizes the use of infrastructure penalizing carbon emissions. In this context, we evaluate the effect of a sudden carbon tax (e.g., due to a change in renewable policy in subsequent governments in a country that is concerned about global warming), which may be imposed after planning and building the plants for the different technology allocations.

		Opt. portf. with and without considering a carbon tax					
		I	II	III	IV	V	VI
CVaR		8673	8751	8938	9062	9223	9369
Expected Cost		Opt. portf. without carbon tax (0 USD/ton CO <sub>2</sub> )					
		6454	6447	6431	6421	-	-
Expected Cost		Opt. portf. with carbon tax (10 USD/ton CO <sub>2</sub> )					
		6883	6837	6767	6739	6713	6693
Expected Cost		Opt. portf. without carbon tax (0 USD/ton CO <sub>2</sub> ) but taken into account ex-post carbon cost (10 USD/ton CO <sub>2</sub> )					
		6923	6879	6867	6816	6766	6723

**Table 8. Expected cost and CVaR of Optimal portfolios.**

The Table reports the expected costs and risks (in terms of CVaR) of six portfolios (I to VI) related to three different generation planning conditions: without carbon tax, with a carbon tax of 10 USD/ton, and when no carbon tax is considered but a carbon tax of 10 USD/ton is 'unexpectedly' imposed after building the generation plants. When no carbon tax is considered, portfolios V and VI do not have a feasible point associated with the corresponding level of CVaR (see Figure 5). Expected cost of the portfolios corresponds to the sum of annuitized investment cost and operating cost related to the annual maintenance of the system. All values in the table are in millions of US dollars. The table is associated with Figure 5.



**Figure 5. Efficient frontier with and without carbon costs.**

The figure shows the efficient frontiers formed by portfolios that take into account a carbon penalization of 0 and 10 U.S dollars per tones of CO<sub>2</sub>e emitted. The dotted curve is formed by portfolios which have the same composition as in light-grey curve (without carbon tax) but an ex post carbon tax of US\$10/TCO<sub>2</sub>e is considered. No policy target constraint was imposed in this exercise. The shape of the dotted curve is related to Carbon installed capacity, the technology with the highest CO<sub>2</sub> emission factor. The Figure shows the CVaR levels associated with portfolios I to VI of Table 8. The axis values are displayed in millions of US dollars.

In Table 8 and Figure 5, we do not impose any policy target constraint, hence  $X = 0$  in equation (8), since we want to isolate the effect of an 'unexpected' carbon tax in the analysis. Table 8 and Figure 5 report the costs and risks of three cases: i) when a carbon tax of 0 USD/ton CO<sub>2</sub> is considered in the generation planning, i.e. there is no carbon tax in the system and the results are equivalent to Table 6; ii) when a carbon tax of 10 USD/ton CO<sub>2</sub> is considered in the planning of portfolios, which is equivalent to Table 7; and iii) when a carbon tax of 0 USD/ton CO<sub>2</sub> is considered in optimal generation, but a carbon tax of 10 USD/ton is 'unexpectedly' imposed after designing the planning and after building the new generation plants. Thus, in the third case, the carbon tax is not considered in the optimization problem of the generation planning, but this tax is considered in the ex-post real costs. We choose a carbon tax of US\$10/TCO<sub>2</sub> because this penalty would encourage renewable generation to overcome the policy target of 20% by 2025.

The plot presented in Figure 5 shows that the curve corresponding to cases of unexpected changes in the renewable policy (dotted curve) does not follow a typical efficient frontier shape as in Figures 3 and 4. This can be explained by the fact that the dotted curve was not built through optimizations reflected in equations (6)-(9). The curve shape of the unexpected changes in the renewable policy is related to the installed capacity of Coal. The technology allocation of the dotted curve and the light-grey curve are the same, and equal to the allocation of Table 6. As shown in Table 6, the coal capacity reaches its maximum in portfolio C, which is equivalent to portfolio III of the dotted curve. In this context, the coal technology has the highest CO<sub>2</sub> emission factor, meaning that there is an unexpected increase in the operating costs of the system caused by the new carbon tax. Thus, Table 8 (last two rows) reports that portfolio III presents the highest gap in terms of expected cost between the grey and dotted curve (a difference in amount of US\$ 100 MM).

For extreme portfolios (I and II or V and VI) we can notice in Figure 5 that the gap between curves is shrinking. This can be explained by two reasons (see Table 6): for portfolios with limited CVaR level (I and II), renewable technologies capacity is installed which lowered the share of Coal in the composition of portfolios. This means that the effect of an unexpected increase in the operating costs of the system caused by the new carbon tax is diminished with the presence of technologies with low CO<sub>2</sub> emission factors. On the other hand, when the minimization of cost is considered (portfolios V and VI) something similar happens. The share of Coal is also reduced in the composition of portfolios, but now it is explained by the installed capacity of Hydro (a cheap technology that also has low CO<sub>2</sub> emission factor).

## 5. Conclusion

In this study, we introduce a novel model for optimal electricity generation planning under renewable policy targets. The model is based on a portfolio approach that takes into account costs, risks and the security of electricity supply. We show that as the objective is more to do with risk reduction than minimizing costs, there is a high renewable capacity installed because it reduces exposure to fuel price volatility, uncertain hydrological scenarios and hourly demand changes.

We conclude that the portfolios obtained with a carbon tax are different to the ones obtained by imposing a 20% lower-bound level of non-conventional renewable generation (i.e. included as a constraint in the optimization model). For example, we show that concentrating solar power (CSP) is installed with a carbon tax, which does not happen if the tax is not imposed. Furthermore, we show that there is not an optimal single-renewable technology to be used under policy targets in an efficient electricity generation planning, but a portfolio of them. Optimal allocations of 'clean-sustainable' forms of electricity generation depend on generation costs, and on their hedge features to different sources of uncertainty. Optimal allocations also depend on whether policy targets are reached by strong governmental laws or by economic incentives such as carbon taxes. Moreover, we show that a level of carbon tax of US\$10 per ton of CO<sub>2</sub> can induce a level of 20% of generation with non-conventional renewable technologies for all planning portfolios, independent of the level of risk (under the cost assumptions used in the model). A more expensive tax would only increase operating costs without any clear contribution to developing renewable energy production.

The model is simple and intuitive and can be implemented in different countries since it is solved in a two-stage stochastic programming setup, which is coordinated by a Benders-based method. Hence, our model can manage large dimension problems for national level electricity generation planning. However, if we want to extend the scope of the analysis by adding more variables and scenarios, it is important to consider that there is a trade-off between extension of the model and computational resources (the algorithm has to converge in reasonable times). The efficiency of the solution methodology is the key factor for solving large optimization problems and shortening simulation times. A suggestion for further work is to reduce the running time of the model, either through an improvement of the algorithm or through the implementation of cloud computing resources for solving such large-scale optimization problems. This enable researchers to run more cases to analyze more scenarios quickly and to perform further sensitivity analysis. Other suggestion is to give flexibility to the model by adding two or more stages of decision making. The results presented in this work showed an optimal portfolio for the year 2025, but there is no insight regarding when to take the investment decision. A multistage model that recognize the option value of the decision under uncertainty would be valuable for planners. For instance, a more flexible model could be useful to analyze future policy changes that may affect the optimal generation planning.

Finally, we want to expose some interesting issues that remain to be addressed. For example, determining optimal subsidy or tax requirements to reach a specific renewable energy target. It would be useful to justify, based on an optimization model, the subsidy or tax amount that countries may apply in the energy sector. A complementary methodology for quantifying economic incentives could simplify the task and add foundation to the decision taken by planners.

Additionally, the changing energy mix will create new challenges to maintaining a secure and stable Transmission System. The inclusion of costs associated with electricity transmission could also be a proposition for further work. Professor David Newbery (see Newbery, 2011) suggests that the subsidy (feed-in tariff) for wind farms needs to be highly location-specific. This means that a renewable generation plant that is located far from the grid may receive more benefits than those power plants that can easily be connected to transmission lines. Thus, adding transmission costs to the model gives it more robustness and adds realism to the analysis. These recommendations have been left for future research.



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## 7. Appendix

### Appendix 1. Model

This appendix continues with the model presented in Section 2. It contains the equations developed by Bernales, Moreno, Rudnick and Inzunza; 2014, that were not modified. They make the model more realistic in terms of the following: provision of various generation frequency controls (to manage the flow of alternating current power from multiple generators through the network); the use of demand side services (customers responding to a signal to change the amount of energy they consume from the power system at a particular time); inclusion of the preservation of system inertia levels (the amount of kinetic energy stored in all spinning turbines and rotors in the system) and spinning and standing reserves (i.e., an optimal reserves allocation of diverse technologies to give stability to the system).

#### Demand dispatch constraints

Let  $D_j$  denote the demand per hour [MWh], and as mentioned before  $D_{j,s}^+$  ( $D_{j,s}^-$ ) is the demand increase (decrease) in the hour  $j$  under scenario  $s$  due to shifts in demand. The model ensures that generation and load are balanced in every hour of the representative day of year  $YY$ , including a load that can be curtailed or lost under extreme scenarios (e.g., due to dry inflows and changes in demand); thus we impose that:

$$\sum_{i \in I} g_{i,j,s} = D_j + D_{j,s}^+ - D_{j,s}^- - LL_{j,s}. \quad (11)$$

We limit the amount of flexible demand in every period to be lower than (or equal to) a percentage of the demand, which is represented by:

$$D_{j,s}^- \leq \overline{ds}^- \cdot D_j \quad (12)$$

and

$$D_{j,s}^+ \leq \overline{ds}^+ \cdot D_j, \quad (13)$$

where  $\overline{ds}^-$  and  $\overline{ds}^+$  reflect the maximum fraction of demand in any hour that can be decreased and increased, respectively. The model ensures that changes in load due to flexible demand are balanced within a time window, e.g. 24 hours, which is given by:

$$\sum_{j \in J_k^D} D_{j,s}^+ - \sum_{j \in J_k^D} D_{j,s}^- = 0, \quad (14)$$

where  $K$  represents a set of days in a year, and  $J_k^D$  is a set of hours in a particular day (subset of  $J$ ).

## Constraints for renewable generation

The model constrains their production and reserve provision according to normalized hourly profiles. Suppose that the  $i$  generation technology is through wind, and this technology is constrained by:

$$g_{i,j,s} \leq WP_{i,j,s} \cdot c_i, \quad (15)$$

while in the case of generation through solar photovoltaic technology there is an upper bound constraint given by:

$$g_{i,j,s} \leq SP_{i,j,s} \cdot c_i, \quad (16)$$

where  $WP_{i,j,s}$  and  $SP_{i,j,s}$  are the maximum generation output factor –a value between zero and one– for wind and solar photovoltaic, respectively, computed for every hour of the year. Similar upper bound constraints can be applied to other non-conventional renewable technologies such as biomass, geothermal, concentrated solar power or small hydro. Regarding run-of-river generation, we impose that:

$$R_{i,j,s}^S + g_{i,j,s} \leq RRP_{i,j,s} \cdot c_i \quad (17)$$

and for hydro-reservoir generation:

$$R_{i,j,s}^S + R_{i,j,s}^P + g_{i,j,s} \leq HRP_{i,j,s} \cdot c_i \quad (18)$$

in which  $RRP_{i,j,s}$  ( $HRP_{i,j,s}$ ) is the maximum generation availability for run-of-river (hydro-reservoir) technology, while  $R_{i,j,s}^S$  and  $R_{i,j,s}^P$  are decision variables which represent the capacities' headroom in terms of spinning-kinetic reserves and mechanical reserves, respectively; this capacities' headroom is used exclusively to regulate contingencies for primary frequency response (both expressed in megawatts).<sup>12</sup>

Moreover, in a hydro reservoir used for the technology  $i$  in hour  $j$  under scenario  $s$ , let  $v_{i,j,s}$  denote the volume of stored water in reservoir;  $inf_{i,j,s}$  the water inflow per hour; and  $sp_{i,j,s}$  the water lost through spillage. Then the hydro reservoir should respect:

$$v_{i,j,s} = v_{i,j-1,s} + inf_{i,j,s} - \frac{g_{i,j,s}}{\eta_i} - sp_{i,j,s} - v_{i,j,s} \cdot \lambda_i \quad (19)$$

where  $\eta_i$  is the efficiency of the hydro-technology  $i$  given the generation  $g_{i,j,s}$ , and  $\lambda_i$  is a factor used to consider losses of stored water due to evaporation and/or seepage in the reservoir. In addition, if we know that there is an upper bound of stored water in a reservoir used for the technology  $i$ ,  $\bar{v}_i$ , then:

$$v_{i,j,s} \leq \bar{v}_i \quad (20)$$

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<sup>12</sup> In the case of fossil fuels, the upper bound constraints are  $R_{i,j,s}^S + R_{i,j,s}^P + g_{i,j,s} \leq FFP_{i,j,s} \cdot c_i$  where  $FFP_{i,j,s}$  is also the maximum generation availability for the fossil-fuel technology  $i$  in hour  $j$  under scenario  $s$ .

## Operational Constraints

As a first step for the security supply constraints, we have to impose restrictions to ramp rates –i.e. the rate that a technology generation changes its output-generation, which is expressed in megawatts per hour. For instance, neither turbines from a large hydro-generation plant nor a coal based generation plant can instantaneously change their level of electricity production, because there is kinetic inertia in both turbines and rotors. To properly constrain ramp rates, we need to determine the number of online units of the technology  $i$ ,  $n_{i,j,s}$ , in hour  $j$  under scenario  $s$  that are synchronized to the power system (e.g., the online units of windmills for wind generation). Let's assume that  $\bar{P}_i$  ( $\underline{P}_i$ ) is the maximum (minimum) power output of each unit of technology  $i$ :

$$n_{i,j,s} \cdot \underline{P}_i \leq g_{i,j,s} \leq n_{i,j,s} \cdot \bar{P}_i \quad (21)$$

$$n_{i,j,s} \cdot \bar{P}_i \leq c_i. \quad (22)$$

To account for the limited ramping capabilities (i.e. 'how' quickly a technology  $i$  can modify its electricity production), we impose a limit to the difference in output between two consecutive hours. Suppose that  $\rho_i$  is the ramp rate limit (in megawatts per hour) for the technology  $i$ , then we can impose that:

$$g_{i,j,s} - g_{i,j-1,s} \leq \min\{n_{i,j,s}, n_{i,j-1,s}\} \cdot \rho_i + (n_{i,j,s} - n_{i,j-1,s}) \cdot \underline{P}_i \quad (23)$$

$$g_{i,j-1,s} - g_{i,j,s} \leq \min\{n_{i,j,s}, n_{i,j-1,s}\} \cdot \rho_i + (n_{i,j-1,s} - n_{i,j,s}) \cdot \underline{P}_i. \quad (24)$$

These constraints state that in the ramping-up (ramping-down) case, this difference cannot be higher than the ramping capability of the units that are connected during both hours, plus the output of units connected (disconnected) to meet the desired final generation level. It is assumed that units are both connected and disconnected at their minimum output.<sup>13</sup>

## Security of supply constraints

In the model, we need to have some headroom to regulate potential contingencies (changes in frequency), which is very relevant when we are using renewable generation (see, e.g., Chávez et al., 2014; and Inzunza et al. 2014). Thus, we include dynamics for primary frequency response in contingency scenarios in the model, in order to ensure the security of supply in the operation portfolios. Primary frequency response will be defined to be adequate if system frequency does not drop below a given limit after any single generation contingency.

In the model, the system has a feedback controller, called 'governor' that senses changes in system frequency. The goal of the 'governor' is to activate some reserves (capacity headroom), which we call 'governor reserves', that are used in the dispatch during scarcity events such as primary frequency response. The governor reserves, in megawatts, reflect the capacity that is *exclusive* for governors to control system frequency after sudden, large disturbances. Suppose that there is a set of governor reserves for primary frequency response  $\{R_{1,j,s}^{GR}, R_{2,j,s}^{GR}, \dots, R_{i,j,s}^{GR}\}$ , which uses different technologies in hour  $j$  under scenario  $s$ .<sup>14,15</sup> Let's assume that there is a large

<sup>13</sup> It is also assumed that units connected at certain hours remain at their minimum output level until the next dispatch period, which is a conservative assumption.

<sup>14</sup> Spinning-kinetic and mechanical reserves,  $R_{i,j,s}^S$  and  $R_{i,j,s}^P$  as explained in equations (16)-(17), are subsets of the governor reserves.

change in system generation,  $\Delta P$ , measured in megawatts; thus we impose the following constraint for the security supply:

$$\Delta P \leq \sum_{i \in I} R_{i,j,s}^{GR} \quad (25)$$

The constraint reflected in equation (25) is a necessary, but not sufficient condition, since we have to include two key points in the system constraints: a) the relation between changes in the system frequency and the reaction speed of reserves to produce electricity in terms of emergency ramp rates; and b) how to include constraints for each technology to take into account the different ramp rates that technologies have as part of their intrinsic characteristics.

Before analyzing additional constraints for security supply, it is important to mention that we can also analyze a contingency event from the demand side of services (in which customers may change the amount of energy they consume from the system after being provided with a signal). Hence, instead of analyzing a contingency event  $\Delta P$ , we can analyze  $\Delta P^* = \Delta P - DR_{j,s}^P$ , where  $DR_{j,s}^P$  is the amount of curtailed demand in hour  $j$  under scenario  $s$ , from a demand side perspective which can be also used as a tool for the primary frequency control.

Suppose that the governor reserves  $R_{i,j,s}^{GR}$  (for the technology  $i$ , in hour  $j$  under scenario  $s$ ) has an emergency ramp rate limit  $\rho_i'$  (in megawatts per seconds), the number of online units of this reserve is  $n_{i,j,s}^{GR}$  and the governor has a dead band  $f_{db}$  (which is the interval of no action when a change in frequency is small).<sup>16, 17</sup> Let's assume that there is a large change in system generation,  $\Delta P$ , in which the pre-contingency frequency is  $f_0$ , and we do not want the frequency to drop below the level  $f_{MIN}$ . Chávez et al. (2014) show that the 'minimum' governor response emergency ramp rate of the reserves,  $\sum_i (n_{i,j,s}^{GR} \cdot \rho_i')$ , in order to avoid levels below  $f_{MIN}$ , has to respect:

$$\frac{f_0 (\Delta P)^2}{4(f_0 - f_{MIN} - f_{db})(\sum_i H_i \cdot n_{i,j,s} \cdot \bar{P}_i - H_f \cdot \Delta P)} \leq \sum_{i \in I} n_{i,j,s}^{GR} \cdot \rho_i' \quad (26)$$

where  $H_i$  is the inertia constant (measured in hours) for all units connected with technology  $i$ ,  $\bar{P}_i$  is the maximum power output of each unit of technology  $i$ , and  $H_f$  is the inertia constant of the missing unit that induces the contingency  $\Delta P$ . Therefore,  $\sum_i H_i \cdot n_{i,j,s} \cdot \bar{P}_i - H_f \cdot \Delta P$  is equal to the post contingency system kinetic energy.

The model also constrains each technology for security supply, in order to take into account different ramp rates. The time,  $t_{MIN,db}$ , in which the system can recover after a contingency  $\Delta P$  with the 'minimum' governor reserve response emergency ramp rate,  $\sum_i (n_{i,j,s}^{GR} \cdot \rho_i')$ , can be expressed as:  $t_{MIN,db}^{GR} = \Delta P / \sum_i (n_{i,j,s}^{GR} \cdot \rho_i')$ . In addition, the time,  $t_{MIN,db}^i$ , that a governor reserve with the technology  $i$ , can reach its maximum electricity generation is given by:  $t_{MIN,db}^i =$

<sup>15</sup> It is important to notice that governor reserves usually are referred to reserves with fossil-fuel technologies; however we want to keep the generality of the model, thus any technology can contribute to the generation reserves.

<sup>16</sup> Thus, in the first instants after a contingency, since there is a dead band, the system frequency is controlled by the inertial response of the own system.

<sup>17</sup> Emergency ramp rates are,  $\rho_i'$ , which have under a contingency a quick reaction speed to maintain the control system frequency.



$R_{i,j,s}^{GR}/(n_{i,j,s}^{GR} \cdot \rho_i)$ . Consequently, we can impose a rule stating that all technologies have to respect that  $t_{MIN,db}^i \leq t_{MIN,db}^{GR}$ , hence we can write that:

$$\frac{R_{i,j,s}^{GR}}{n_{i,j,s}^{GR} \cdot \rho_i} \leq \frac{\Delta P}{\sum_i n_{i,j,s}^{GR} \cdot \rho_i'} \quad (27)$$

Additionally, reserves must be linked to the headroom of all technologies; so we limit the amount of reserves that can be provided by each generator as:

$$R_{i,j,s}^{GR} \leq n_{i,j,s}^{GR} \cdot \bar{P}_i - g_{i,j,s} \quad (28)$$

Some of the constraints shown above still make the optimization problem non-linear and complex to solve, even after including the linearization of the *CVaR*. The non-linear and convex equations are linearized by using tangent planes, and for the non-convex equations we define two alternative convex linear programming models that serve as upper and lower bounds to the optimal solution. In Appendix 2 we describe these simplifications.

## Appendix 2. Assumptions and simplifications

### Simplification in operation constraints

Constraints (23) and (24) have a common term,  $\min\{n_{i,j,s}, n_{i,j-1,s}\}$ , that uses the minimize function to choose the minimum value between the number of online units of technology  $i$  under scenario  $s$  in hour  $j$  compared to hour  $j-1$ . This term can be represented in a linear form by using an auxiliary variable,  $n_{i,j,s}^{min}$ , and two additional constraints as shown in equations (29) and (30):

$$n_{i,j,s}^{min} \leq n_{i,j-1,s} \quad (29)$$

$$n_{i,j,s}^{min} \leq n_{i,j,s} \quad (30)$$

These equations impose an upper bound to  $n_{i,j,s}^{min}$  and can replace the minimize function  $\min\{n_{i,j,s}, n_{i,j-1,s}\}$ , thus equations (23) and (24) can be re-written as:

$$g_{i,j,s} - g_{i,j-1,s} \leq n_{i,j,s}^{min} \cdot \rho_i + (n_{i,j,s} - n_{i,j-1,s}) \cdot \underline{P}_i \quad (31)$$

$$g_{i,j-1,s} - g_{i,j,s} \leq n_{i,j,s}^{min} \cdot \rho_i + (n_{i,j-1,s} - n_{i,j,s}) \cdot \underline{P}_i \quad (32)$$

Hence equations (29)-(32) rather than (23)-(24) are used in the model.

### Simplification in security of supply constraints

Equation (26) is non-linear but convex and thus can be linearized by using tangent planes. To do so, technologies are grouped into two categories according to their emergency ramp rates in order to reduce the number of planes and computational resources used. These two groups are denoted by  $I^{GS}$  and  $I^{GF}$ , which are sets classified as slow/fast response units for having low/high ramp rate. Additionally, a third group is defined,  $I^{NG}$ , which corresponds to technologies that do not participate in primary frequency response (PFR), but are connected through synchronous machines to the system and thus add inertia to it. Units' contribution to inertia ( $H$ ) and their maximum power output ( $\bar{P}$ ) are assumed to be equal among all units.

The numbers of units of the three groups are calculated using equations (33)-(35):

$$n_{SL,j,s} = \sum_{i \in I^{GS}} n_{i,j,s} \quad (33)$$

$$n_{FT,j,s} = \sum_{i \in I^{GF}} n_{i,j,s} \quad (34)$$

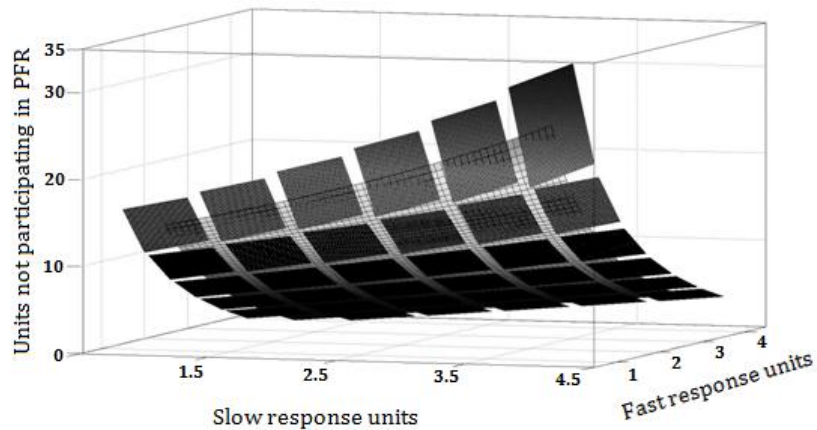
$$n_{NG,j,s} = \sum_{i \in I^{NG}} n_{i,j,s} \quad (35)$$

where SL, FT and NG are respectively slow response, fast response and non governor (synchronous units that are online and do not participate in primary frequency control).

Hence we re-define the region given by equation (26) as that associated with equations (33)-(36).

$$\frac{f_0(\Delta P)^2}{4(f_0 - f_{MIN} - f_{db})(n_{NG,j,s} + n_{FT,j,s} + n_{SL,j,s}) \cdot H \cdot \bar{P} - H_f \cdot \Delta P} \leq (n_{SL,j,s} + n_{FT,j,s}) \cdot \rho'_i. \quad (36)$$

The linear form of restriction (36) is obtained through tangent planes linearization as shown in Figure 6.



**Figure 6. Linearization of equation (36) through tangent planes.**

Equation (36) is convex, so it may be linearized by using tangent planes. In order to reduce the number of planes and computational resources when solving, technologies participating in primary frequency response (PFR) are grouped according to their emergency ramping rate: Fast and slow ramping technologies. Technologies within the same group are assumed to have the same emergency ramping parameter. A third group is distinguished, which are technologies that do not participate in PFR but are connected through synchronous machines to the system and add inertia to it. For this purpose, inertia parameters of units and their maximum power output are assumed equal to all units. Without any loss of generality, i.e. technologies that do not participate in PFR,  $n_{NG,j,s}$ , was chosen as the dependent variable during the linearization, so planes are presented as lower bounds of this variable.

Another non-linear constraint is equation (27). This restriction is non-convex, so it has to be treated differently. In this case, upper and lower bounds of the optimal solution are computed. The lower bound is obtained by removing equation (27) from the formulation, leading to a portfolio solution with a lower expected cost of investment and operation but which may violate equation (27) and thus not be technically feasible due to the fact that primary reserves are not allocated correctly within the installed technologies.

The upper bound of the optimal solutions is computed by solving a particular case of the model, in which only one technology may participate in PFR. This simplifies equations because when only one technology saves primary reserve, constraints (25) and (27) end up being a simple linear equality. This is reasonable, because if only one technology participates in primary frequency response, all reserve must be stored in that technology. In this way, we may produce several upper bound solutions by defining various levels of participation from different generation technologies in PFR. Hence, a number of technically feasible suboptimal solutions can be obtained, ultimately selecting that with the lowest gap with respect to the lower bound solution. We found for all case studies analyzed in this work that the selected technically feasible solutions present less than 0.8% gap. For this reason, all the simulations done in this work, consider reservoirs for hydro power as the only technology committed to Primary Frequency Response.

### **Simplification in Operating reserve constraints**

The operating reserves security criterion for this study consists of holding reserve for contingency purposes and to protect the system from unpredicted changes in the availability of solar and wind resources. Other renewable energy sources such as Biomass and Geothermal are not included in this analysis because their availability can be predicted accurately (both availability profiles have zero standard deviation).

As defined in Silva (2010), we use a realistic criterion for the representation of operating reserve policies, where reserve amounts required ( $Req$ ) are considered for two purposes; the first is to restore primary frequency control reserves after they have been deployed,  $\Delta P$ , and the second is to deal with unpredicted changes in variable renewable generation. Thus, the operating reserve requirement must be a function of the contingency magnitude, the non-conventional renewable generation and its installed capacity:  $Req = f(\Delta P, \{g_{i,j,s}\}_{i \in I^R}, \{c_i\}_{i \in I^R})$

According to Silva (2010), when uncertainty of renewables forecasts is considered for reserve and these forecast errors are assumed to be non-correlated, normally distributed, with zero mean and a certain standard deviation, the reserve requirement may be quantified as shown in equation (37), which requires saving 3 times the total standard deviation of the forecast errors.

$$Req = \Delta P + 3 \cdot \sqrt{\sigma_{WIND}^2 + \sigma_{SOLAR}^2}. \quad (37)$$

Standard deviations of wind and solar forecast errors  $-\sigma_{WIND}$  and  $\sigma_{SOLAR}$  respectively– have to be computed using a certain forecast policy. Wind availability has no clear relationship to hours of the day as solar radiation does, so, a persistent 4 hour ahead forecast is employed to compute its forecast error standard deviation. This methodology produces one parameter that represents the uncertainty for all hours of the year. Solar radiation, on the other hand, is forecast using a day-ahead criterion. Four typical days of radiation are computed (one for each season) and the standard deviation error is calculated for the 24 hours of the day and for every season. The highest of the 4 values computed for every hour is taken as the conservative estimate.

$$Req = \Delta P + 3 \cdot \sqrt{c_{WIND}^2 \cdot \sigma_{WIND}^2 + c_{SOLAR}^2 \cdot \sigma_{SOLAR,j}^2} \quad (38)$$

In equation (38), installed capacities are included because forecasts are made in terms of the capacity factor. It can be argued that the above requirement might be too conservative. This is mainly because if, for example, no wind is forecasted for a certain hour, it would not be reasonable to keep reserve for wind purposes. Due to this fact, the following correction is made:

$$Req = \begin{cases} \Delta P + 3 \cdot \sqrt{c_{WIND}^2 \cdot \sigma_{WIND}^2 + c_{SOLAR}^2 \cdot \sigma_{SOLAR,j}^2} & WP_{i,j} \geq 3 \cdot \sigma_{WND} \\ \Delta P + g_{WIND,j,s} + 3 \cdot c_{SOLAR} \cdot \sigma_{SOLAR,j} & WP_{i,j} < 3 \cdot \sigma_{WND} \end{cases} \quad (39)$$

In this equation, for hours on which the wind capacity factor of the hourly profile used (which is taken as the forecast) is smaller than the total uncertainty ( $WP_{i,j} < 3 \cdot \sigma_{WND}$ ), a deterministic criterion is employed, assuming that in the worst case scenario, all scheduled wind fails to occur. On hours where the wind forecast is sufficiently high, the probabilistic criterion is established. The same logic may be applied to solar technology. Nonetheless, as an individual standard deviation is computed for every hour, the above correction is not needed for typical zero radiation hours (at night, for example).

As equation (39) is non-linear, algebra factorization and a first order Taylor series expansion is used to get a linear approximation shown by equation (40). This linear function is always greater than the original equation (39), so it is a conservative approximation.

$$Req = \begin{cases} \Delta P + 3 \cdot \left( \frac{1}{\sqrt{2}} \cdot (c_{WND} \cdot \sigma_{WND} + c_{SOL} \cdot \sigma_{SOL,j}) + \left(1 - \frac{1}{\sqrt{2}}\right) \cdot AV_j \right) & RP_{WND,j} \geq 3 \cdot \sigma_{WND} \\ \Delta P + g_{WND,j,s} + 3 \cdot c_{SOL} \cdot \sigma_{SOL,j} & RP_{WND,j} < 3 \cdot \sigma_{WND} \end{cases} \quad (40)$$

where  $AV_j$  represents  $|c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j}|$ . The absolute value function can be easily linearized by equations (41) and (42):

$$AV_j \geq c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j} \quad (41)$$

$$AV_j \geq -(c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j}). \quad (42)$$

Finally, to account for all reserves considered in the reserve amounts required,  $Req$  (spinning and standing reserves), constraint (43) is added to the model.

$$Req \leq \sum_{i \in I} R_{i,j,s}^S + FS \cdot \left( \sum_{i \in I} c_i - n_{i,j,s} \cdot \bar{P}_i \right) + DR_{j,s}^S. \quad (43)$$

Here,  $R_{i,j,s}^S$  is the capacity headroom in terms of spinning-kinetic reserves used to regulate contingencies as reserves for primary frequency responses and the term  $FS$  represents the fraction of generation capacity that contributes to operating reserves. The equation (43) also includes a demand response parameter,  $DR_{j,s}^S$ , to study the effect of responsive loads used in the operating reserve time frame.

### Appendix 3. Solution Methodology

In this section we explain the solution methodology based on Bender's decomposition algorithm, a technique which is meant to reduce computational complexity of large-scale problems. Here we use a vectors and matrices notation<sup>18</sup> to abbreviate the linear systems, which is related to the equations of the model presented in section 2. For a better understanding, we explain first the relation between the nomenclature used to explain the model, with the vectors and matrices notation used in this section:

- $\mathbf{d}$  is a vector containing investment costs and  $\mathbf{y}$  is the set of first-stage decision variables (installed capacities):  $\mathbf{d}^T \cdot \mathbf{y} \Leftrightarrow \sum_{i \in I} INV_i \cdot c_i$ .
- $Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s)$  is a function of the decision variable  $\mathbf{y}$ , operational costs  $\mathbf{c}_s$ , and  $\mathbf{F}_s$  which is a matrix of all constraints related with the decision variable of installed capacities. Later  $Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s)$  will be defined as the second-stage problem.
- $\mathbf{c}_s \Leftrightarrow \sum_{i \in I} \sum_{j \in J} VOM_{i,s} \cdot g_{i,j,s} + \sum_{j \in J} D_{j,s}^- \cdot dc^- + \sum_{j \in J} D_{j,s}^+ \cdot dc^+ + voll \cdot \sum_{j \in J} LL_{j,s}$
- $\delta$  represents the Value at Risk (VaR).
- The function denoted by  $(\dots)^+$  represents the maximum between the expression in parenthesis and zero:  $(expr)^+ = \max(\overline{expr}, 0)$

Bender's method coordinates a two-stage stochastic linear programming model to determine the optimum portfolio of generation technologies of a future power system. In the first stage, the investment decision takes place and therefore we minimize total investment and operation costs across a large number of future scenarios, subject to a given level of CVaR.

The first-stage problem is given by:

$$(P1) \text{ Minimize } z = \mathbf{d}^T \cdot \mathbf{y} + \sum_{s \in S} p_s \cdot Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s) \quad (44)$$

<sup>18</sup> Vectors are written as bold, lower-case letters and matrices as bold upper-case letters.

$$\begin{aligned}
& s. t.: \\
\delta + \frac{1}{1-\alpha} \cdot \sum_{s \in \mathcal{S}} p_s \cdot (\mathbf{d}^T \cdot \mathbf{y} + Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s) - \delta)^+ & \leq \overline{CVaR} \quad (45)
\end{aligned}$$

$$\mathbf{y} \in \mathbf{Y} \quad (46)$$

$$\delta \geq 0 \quad (47)$$

where  $\mathbf{Y}$  is a set of polyhedral constraints that ensure that  $\mathbf{y}$  corresponds to a feasible solution. Equation (45) represents an upper bound to the CVaR calculated in the left side of the equation.  $\overline{CVaR}$  is the maximum allowed portfolio's CVaR.

The second stage represents the operation of the capacity imposed by the first stage (dispatch decisions). The second-stage problem is given by:

$$(P2_s) \quad Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s) = \text{Minimize } \mathbf{c}_s^T \cdot \mathbf{x} \quad (48)$$

$$\begin{aligned}
& s. t.: \\
\mathbf{F}_s \cdot \mathbf{y} + \mathbf{E} \cdot \mathbf{x} & = \mathbf{h} \quad (49)
\end{aligned}$$

$$\mathbf{x} \geq \mathbf{0} \quad (50)$$

where  $\mathbf{x}$  corresponds to the generation ( $\mathbf{x} \Leftrightarrow g_{i,j,s}$ ), thus  $Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s)$  is the minimization of the operational costs for every scenario  $s$ . Matrix  $\mathbf{E}$  contains all constraints related with the generation of each technology. Therefore, equation (49) is a matrix-vector expression which summarizes almost all the constraints explained in the model.  $\mathbf{h}$  is an auxiliary vector to fit the constraints expressed in matrices  $\mathbf{F}_s$  and  $\mathbf{E}$ .

The master problem (P1) is defined by a convex dominion and a convex objective function. This allows the linearization of non-linear terms through tangent planes, as it is done in the classic Benders' decomposition problem.

Moreover, function  $Q(\mathbf{y}, \mathbf{c}_s, \mathbf{F}_s)$  has the same structure as the classic slave problem from Benders' decomposition (Benders, 1962), so the same approximation (and cutting planes selection algorithm) can be used for solving this particular problem according to Papavasiliou et al. (2014).

Taking this into account, the master problem can be re-written, including optimality cuts derived from Benders' algorithm as follows:

$$(P1) \quad \text{Minimize } z_L \quad (51)$$

$$s. t.:$$

$$z_L \geq \mathbf{d}^T \cdot \mathbf{y} \quad (52)$$

$$z_L \geq \mathbf{d}^T \cdot \mathbf{y} + \sum_{s \in S} p_s \cdot (Q(\hat{\mathbf{y}}^i, \mathbf{c}_s, \mathbf{F}_s) + (\hat{\mathbf{y}}^i - \mathbf{y})^T \cdot \mathbf{F}_s^T \cdot \mathbf{u}^i_s) \quad 1 \leq i \leq k \quad (53)$$

$$v_s \geq \mathbf{d}^T \cdot \mathbf{y} + Q(\hat{\mathbf{y}}^i, \mathbf{c}_s, \mathbf{F}_s) + (\hat{\mathbf{y}}^i - \mathbf{y})^T \cdot \mathbf{F}_s^T \cdot \mathbf{u}^i_s - \delta \quad \forall s \in S, 1 \leq i \leq k \quad (54)$$

$$\delta + \frac{1}{1 - \alpha} \cdot \sum_{s \in S} p_s \cdot v_s \leq \overline{CVaR} \quad (55)$$

$$v_s \geq 0 \quad (56)$$

$$\mathbf{y} \in \mathcal{Y} \quad (57)$$

$$\delta \geq 0 \quad (58)$$

Auxiliary variables  $v_s$  are used for obtaining the linear form of constraint (45) and  $\mathbf{u}^i_s$  are the Lagrange multipliers of  $P2_s$  associated with the coupling constraints (i.e. generation capacities), considering the  $i^{th}$  investment decision trial  $\hat{\mathbf{y}}^i$ . Constraint (52) is added to avoid unboundedness by setting a lower limit. Using  $PI'$  and  $P2_s$  shown by equations (48)-(58), the following algorithm is proposed:

**Step 0:** Set  $k = 1$ . Initialize  $\hat{z}_{lower} = -\infty$ ,  $\hat{c}_{lower} = -\infty$  and  $\hat{\mathbf{y}}^1$ . Go to step 1.

**Step 1:** Solve  $PI'$ . Set  $\hat{\mathbf{y}}^k$  equal to the optimal first-stage solution and set  $\hat{z}_{lower} = \hat{z}_L$  and  $\hat{c}_{lower} = \hat{\delta} + 1/(1 - \alpha) \cdot \sum_{s \in S} p_s \cdot \hat{v}_s$ . Go to step 2.

**Step 2:** For all  $s \in S$ , solve  $P2_s$  using  $\hat{\mathbf{y}}^k$  as input. Set  $\mathbf{u}^k_s$  equal to the optimal multipliers of the coupling constraints in equation (49). Set  $\hat{z}_{upper} = \mathbf{d}^T \cdot \hat{\mathbf{y}}^k + \sum_{s \in S} p_s \cdot Q(\hat{\mathbf{y}}^k, \mathbf{c}_s, \mathbf{F}_s)$  and  $\hat{c}_{upper} = CVaR_\alpha(\mathbf{c}(\hat{\mathbf{y}}^k), \mathbf{p})$ .

Where  $CVaR_\alpha(\mathbf{x}, \mathbf{p})$  is the function that computes the  $(1 - \alpha)$  percentile conditional value at risk of the cost vector  $\mathbf{x}$  with the associated probabilities vector  $\mathbf{p}$ .  $\mathbf{c}(\hat{\mathbf{y}}^k)$  corresponds to the vector containing the total costs of every scenario computed when solving  $P2_s$  slaves, given the first-stage decision  $\hat{\mathbf{y}}^k$  and  $\mathbf{p}$  is the vector containing scenarios' probabilities. Go to step 3.

**Step 3:** If  $|\hat{z}_{upper} - \hat{z}_{lower}| \leq \varepsilon_1$  and  $|\hat{c}_{upper} - \hat{c}_{lower}| \leq \varepsilon_2$  then exit with  $\hat{\mathbf{y}}^k$  as the optimal solution. Otherwise, set  $k = k + 1$  and go to step 1.

For the purpose of simplicity the addition of Benders' feasibility cuts is not explained, although they might be necessary for obtaining the optimal solution. The addition of these cuts does not vary from the standard procedure done in Benders' classical decomposition algorithm (Benders, 1962).

It is important to underscore that the exit criterion of the algorithm ensures that both expectation and CVAR functions are correctly approximated in the neighborhood of the optimal solution. We used an exit criterion of 1%.

#### Appendix 4. Additional parameter values

The values were selected according to regulation of the power sector in Chile and standard level in the electric power sector. A 399.67 \$/MWh value of lost load is used, according to the short-term failure cost reported by the Chilean regulator (National Commission of Energy). Maximum units' outputs is assumed to be 400 MW with an hourly ramp rate ( $\rho_i$ ) of 40 MW/h for Coal, 240 MW/h for Geothermal and Oil, 200 MW/h for LNG, Biomass and Solar CSP, and 360 MW/h for hydro-electric technologies. Emergency ramp rates ( $\rho'_i$ ) are those used by Chaves et al. (2014) and equal to 38 MW/s for thermal plants and 8 MW/s for hydro-electric plants.<sup>19</sup> In real power systems, primary frequency response service is provided by a subset of the conventional plants synchronized, which is represented in our model by defining two types of unit per technology: with and without capability to respond to frequency changes, in which the former presents a slightly higher investment cost that permits identification of the demand for the frequency response service. Also, operation is secured against the outage of a single unit (i.e. 400 MW), under which frequency is not allowed to violate a minimum value of 49.2 Hz from nominal value of 50 Hz (governors' dead-band are assumed to be equal to  $\pm 25$  mHz and units' inertia (H) is equal to 5 s).<sup>20,21</sup> We assume that costs associated with demand services are equal to 1 (2) \$/MW if demand decreases (increases), reservoir seepage and evaporation losses are equal to 0.5% of stored water, and maximum capacity of the reservoir is very high and thus does not constrain hydro's output. Portfolios will be determined by using a  $\alpha$ -CVaR with an  $\alpha$  of 95%.

Symbol	Description	Value	Unit
$voll$	Value of lost load	399.67	\$/MWh
$\bar{P}$	Maximum power output of generic unit	400	MW
$\underline{P}_i$	Minimum units output	160 for thermal 40 for hydro 40 for Coal	MW
$\rho_i$	Hourly ramp rate	240 for Oil, Geothermal 200 for LNG, Biomass, CSP 360 Hydro	MW/h
$\rho'_i$	Emergency ramp rate	38 for thermal plants 8 for hydro plants	MW/s
$f_0$	Nominal system frequency	50	Hz
$f_{db}$	Governors frequency dead band	$\pm 25$	mHz

<sup>19</sup> Emergency ramp rates are the ramp rates of the reserves used by the governor to maintain the security supply.

<sup>20</sup> Chilean regulator states that under frequency load shedding must take place when system frequency reaches a threshold of 49.2 Hz.

<sup>21</sup> Maximum allowed governors' dead band in Chile.



$f_{MIN}$	Minimum frequency allowed	49.2	Hz
$H$	Inertia constant of generic unit	5	s
$\lambda_i$	Factor of losses of stored water due to evaporation and/or seepage in the reservoir	0.0051	p.u.
$\bar{v}_i$	Upper bound of stored water	10,321	MMm <sup>3</sup>
$\eta_i$	Average inflow-to-power rate	6,840	MWh/m <sup>3</sup>
$\Delta P$	Size of largest generation outage	400	MW
$t_{MIN,db}^i$	Deployment time of operating reserves	0.25	h
$DR_{j,s}^S$	Amount of curtailable demand for the operating reserve timeframe	200	MW
$DR_{j,s}^P$	Amount of curtailable demand for the primary frequency control timeframe	200	MW
$FS$	Fraction of fast start generation capacity that contributes to operating reserves	1	p.u.
$dc^-$	Cost of demand decrease	1	\$/MW
$dc^+$	Cost of demand increase	2	\$/MW
$\overline{ds}^-$	Maximum fraction of demand that can be decreased	5%	p.u.
$\overline{ds}^+$	Maximum fraction of demand that can be increased	5%	p.u.
$\alpha$	CVaR parameter that defines the (1- $\alpha$ )% highest cost scenarios	95%	p.u.
$\sigma_{WND}$	Standard deviation of wind forecast errors in all hours	12.8%	p.u.
$\sigma_{SOL,j}$	Standard deviation of solar forecast errors in hour j	0% - 10.6%	p.u.
$X$	Renewable policy target	20%	p.u.

**Table 9. Parameters values.**

The table contains the parameters values assumed in the model. The table shows the nomenclature of the parameters (symbol), a brief description, the value considered as an input parameter and the corresponding unit of measurement. As the model is implemented for the Chilean Central Interconnected System (CIS), some values are reported by the Chilean regulator while others are taken from references. They respect standards level in the electric power sector. The acronym "p.u." refers to "per unit", expression of quantities as fractions of a defined base unit quantity.