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Aggregation systems for sales forecasting☆

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ABSTRACT

Sales forecasting consists of calculating the expected sales of a specific product or company. An important issue when dealing with sales forecasting is the calculation of the average sales, usually using the arithmetic mean or the weighted average. This study introduces new methods for calculating the average sales. These methods are two modern aggregation operators: the ordered weighted average, and the unified aggregation operator. The main advantage of this approach is the possibility to deal with uncertain and complex environments in a more complete way. The study develops some key examples through multi-person and multi-criteria techniques. The study also presents a numerical example regarding the calculation of the average sales of a product in a set of countries.

1. Introduction

The average sale is a number that considers a set of numerical sales and provides a representative value by using an averaging technique like the arithmetic mean or the weighted average. The average sale provides averaging results which are very useful in sales forecasting (Dalrymple, 1987; Mentzer & Cox, 1984) in many contexts, including the calculation of the averages sales of a product, company, sector, region, or country (Engle, Granger, & Hallman, 1989; Harrison, 1967).

The literature mentions many aggregation operators that have similar purposes as the weighted average, but in other contexts. A very common operator is the ordered weighted average (OWA) (Yager, 1988; Yager, Kacprzyk, & Beliakov, 2011). OWA is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. OWA is very useful for under or overestimating the information according to the attitudinal character of the decision maker. In this context, OWA permits to construct interval numbers (Merigó, 2012a). Many authors extend and generalize the OWA (Emrouznejad & Marra, 2014; Yager & Kacprzyk, 1997; Yager et al., 2011). Yager and Filev (1999) develop the induced OWA (IOWA) operator, which Merigó and Gil-Lafuente (2009) further generalize. Other authors use different techniques for integrating the OWA operator and the weighted average in the same formulation, including the weighted OWA (Torra, 1997), the hybrid average (Xu & Da, 2003), the importance OWA (Yager, 1998), the immediate weights (Merigó & Gil-Lafuente, 2012; Yager, Engemann, & Filev, 1995), and the OWA weighted average (OWAWA) (Merigó, Engemann, & Palacios-Marqués, 2013). Recently Merigó et al., (2015) present more general aggregation operators that are more flexible and can adapt to a wide range of situations. The authors call these operators unified aggregation operators (UAO). The probabilistic weighted average (PWA) (Merigó, 2012b) is particularly interesting. PWA combines objective and subjective information in the same formulation.

The aim of this study is to analyze the use of modern aggregation operators in the calculation of the average sales to develop better forecasting techniques. To do so, this study presents the OWA sales (OWAS) as a method for calculating the average sales in uncertain environments where the decision maker wants to under or overestimate the information. The main advantage of the OWAS is that this operator can consider any scenario, from the minimum to the maximum sales. Thus, the decision maker and the experts can represent their attitude from the most optimistic to the most pessimistic position. These options give the analysis more flexibility in dealing with the information. The study considers multi-person and multi-criteria techniques. The inclusion of general aggregation operators like the UAO operator allows a complete overview of complex environments.

This study examines some examples of average sales calculation in key situations like the average sales of a product or a company, or the

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average sales of a country or a region. One example is the calculation of the average sales in a set of countries at a world level. These aggregation techniques are very useful for a better representation of the data, and take into account the particular conditions of the problem under study. The main reason for their usefulness is that these aggregation techniques can deal with different sources of information in the same formulation, thus providing a more general approach for the analysis of complex data.

The rest of the study is as follows. Section 2: Review of basic preliminaries. Section 3: Key aggregation techniques for improving the calculation of the average sales. Section 4: Key examples' analysis. Section 5: A numerical example. Section 6: Findings.

2. Aggregation systems

Aggregation systems have increasing popularity in the literature (Beliakov, Pradera, & Calvo, 2007; Grabisch, Marichal, Mesiar, & Pap, 2011). Aggregation systems serve multiple purposes, providing summarized and representative results of a set of data. The most common aggregation systems are the arithmetic mean and the weighted average. With the OWA operator, the researcher does not know the weights (market share or opinions), so he or she aggregates the information according to his or her attitude. Another increasingly popular aggregation operator is the OWA operator (Yager, 1988). Scholars define OWA differently.

Definition 1. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the j^{th} smallest of the a_i.

One of OWA's key advantages is that OWA provides a parameterized family of aggregation operators between the minimum and the maximum. Thus, any scenario from the most optimistic to the most pessimistic one is possible. This option is an advantage because uncertainty means that the future is unpredictable. Therefore, people may have different beliefs about the future depending on their opinion and attitude. The OWA represents these variations mathematically. The use of the minimum and the maximum in the OWA operator yields the classical interval numbers. The advantage is that, apart from these extreme results, OWA allows to consider results closer to the center but with some small degrees of optimism/pessimism. In decision making under uncertainty, OWA integrates the classical methods into a single formulation, being each particular method a specific expression of the OWA operator (Yager, 1988). The pessimistic criteria is found when $w_n = 1$ and $w_j = 0$ for all $j \neq n$. The optimistic criteria if $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The Laplace criteria when $w_j = 1/n$ for all j; and the Hurwicz criteria if $w_1 = \alpha$, $w_n = (1 - \alpha)$, and $w_j = 0$ for all $j \neq 1, n$.

Scholars extend the OWA operator in a wide range of directions. The UAO operator is a general framework for integrating different aggregation operators into a general formulation according to the degree of importance of each concept. This study defines UAO as follows.

Definition 2. A unified aggregation operator of dimension m is a mapping UAO: $R^m \times R^n \rightarrow R$ that has an associated weighting vector C of dimension m, such that:

$$UAO(a_1, \ldots, a_n) = \sum_{h=1}^{m} \sum_{i=1}^{n} C_h w_i^h a_i, \tag{2}$$

where C_h is the weight that each sub-aggregation has in the system, with $C_h \in [0, 1]$, and $\sum_{h=1}^{m} C_h = 1$; w_i^h is the i^{th} weight of the h^{th} weighting vector W with $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$.

The UAO operator includes a wide range of aggregation operators including the weighted average, the OWA, and the probabilistic OWAWA operator (Merigó, 2012a, 2012b). The UAO is more flexible than other operators because UAO can represent different types of weights in the same problem. Some important properties are the following.

- If $C_i = 1$, the system only considers one aggregation.
- If $C_i = 0$, the system does not consider this sub-aggregation.

Note that researchers can extend the UAO operator by adding some other key concepts like distance measures (Zeng, Merigó, & Su, 2013), fuzzy systems (Zeng, Su, & Le, 2012), power aggregations (Wei, Zhao, Wang, & Lin, 2013), Choquet integrals (Belles, Merigó, Guillen, & Santolino, 2014; Wei, Lin, Zhao, & Wang, 2014) and generalized multiple aggregations (Zhou, Chen, & Liu, 2013).

3. New aggregation systems in the average sales

The average sale is an important method for sales forecasting. The literature reports many methods for dealing with sales forecasting (Dalrymple, 1978; Huarng & Yu, 2014). Usually, scholars study the average sale with the arithmetic mean, if all the variables under study are equally important, or with the weighted average, if the variables have different degrees of importance. However, other approaches are feasible (Linares-Mustarós, Merigó & Ferrer-Comalat, 2015). The use of the OWA operator in the average sales yields the ordered weighted average sales (OWAS). OWAS deals with uncertain environments where decision makers do not know the importance of the variables. Instead, the decision maker uses his or her attitudinal character to weight the information. Thus, decision makers can consider any level of sales from the minimum to the maximum one. This option is useful for under or overestimating the sales according to an attitudinal position that a decision maker wants to consider in the analysis. The study defines OWAS as follows for a set of specific sales $S = \{s_1, s_2, \ldots, s_n\}$:

$$OWAS = \sum_{j=1}^{n} w_j S_j, \tag{3}$$

where S_j is the j^{th} smallest of the sales s_i and w_j is the weight such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Note that descending and ascending orders in the OWA operator interrelate by using $w_j = w^*_{n-j+1}$ (Merigó & Gil-Lafuente, 2013). If $w_n = 1$ and $w_j = 0$ for all $j \neq n$, results yield the maximum sale and if $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$, the minimum sale. If $w_j = 1/n$ for all j, the OWAS becomes the classical average sale.

To deal with situations where several experts provide their opinions, researchers can reformulate OWAS to form the multi-person OWAS (MP-OWAS) as follows:

$$MP{-}OWAS = \sum_{x=1}^{y} \sum_{j=1}^{n} e_x w_{jx} S_j, \tag{4}$$

where e_x is the x^{th} weight of each expert such that $e_x \in [0, 1]$ and $\sum_{x=1}^{y} e_x = 1$.

Researchers may include additional criterion to the OWAS like the use of several enterprises and countries forming the multi-person multi-country multi-enterprise OWAS (MPMCME-OWAS). The study defines MPMCME-OWAS as follows:

$$MP{-}MC{-}ME{-}OWAS = \sum_{x=1}^{y} \sum_{r=1}^{q} \sum_{k=1}^{p} \sum_{j=1}^{n} e_x u_{rx} v_{krx} w_{jkrx} S_j, \tag{5}$$

where v_k is the k^{th} weight of each enterprise such that $v_k \in [0, 1]$ and $\sum_{k=1}^{p} v_k = 1$, and u_r is the r^{th} weight of each country such that $u_r \in [0, 1]$ and $\sum_{r=1}^{q} u_r = 1$.

Similarly, by including additional concepts in the analysis, researchers could increase complexity of structures.

Table 1
General formulation for the average sales by using the UAOS operator.

Average price	Formulation
General	$S=\sum_{r=1}^{q}\sum_{h=1}^{m}\sum_{i=1}^{n}u_rC_{hr}w_{ir}^{h}s_{ir}=\sum_{h=1}^{m}C_h\left(\sum_{r=1}^{q}\sum_{i=1}^{n}u_rw_{ir}^{h}s_{ir}\right)$
World	$S=\sum_{h=1}^{m}C_h\left(u_{1-USA}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-CHN}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{200-TUV}\left[\sum_{i=1}^{n}w_{ih200}s_{i200}\right]\right)$
European Union	$S=\sum_{h=1}^{m}C_h\left(u_{1-GER}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-FRA}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{28-MAL}\left[\sum_{i=1}^{n}w_{ih28}s_{i28}\right]\right)$
North America	$S=\sum_{h=1}^{m}C_h\left(u_{1-USA}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-MEX}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{3-CAN}\left[\sum_{i=1}^{n}w_{ih3}s_{i3}\right]\right)$
South America	$S=\sum_{h=1}^{m}C_h\left(u_{1-BRA}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-ARG}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{13-GUY}\left[\sum_{i=1}^{n}w_{ih13}s_{i13}\right]\right)$
Africa	$S=\sum_{h=1}^{m}C_h\left(u_{1-S.A}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-NIG}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{52-S.T}\left[\sum_{i=1}^{n}w_{ih52}s_{i52}\right]\right)$
Asia	$S=\sum_{h=1}^{m}C_h\left(u_{1-CHN}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-JAP}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{41-MLD}\left[\sum_{i=1}^{n}w_{ih41}s_{i41}\right]\right)$
Oceania	$S=\sum_{h=1}^{m}C_h\left(u_{1-AUS}\left[\sum_{i=1}^{n}w_{ih1}s_{i1}\right]+u_{2-N.Z}\left[\sum_{i=1}^{n}w_{ih2}s_{i2}\right]+\ldots+u_{14-TUV}\left[\sum_{i=1}^{n}w_{ih14}s_{i14}\right]\right)$

CHN = China; TUV = Tuvalu; GER = Germany; FRA = France; MAL = Malta; MEX = Mexico; CAN = Canada; BRA = Brazil; ARG = Argentina; GUY = Guyana; S.A = South Africa; NIG = Nigeria; S.T. = Sao Tome and Principe; JAP = Japan; MLD = Maldivas; AUS = Australia; N.Z. = New Zealand.

A more general formulation can consider different sources of information when dealing with uncertain environments, thus adapting in a more flexible way to different complex situations. To do so, this study uses the UAO operator obtaining the UAO sales (UAOS), defining UAOS as follows for a set of sales $S = \{s_1, s_2, \ldots, s_n\}$:

$$UAOS=\sum_{h=1}^{m}\sum_{i=1}^{n}C_hw_i^hs_i, \tag{6}$$

where s_i is the i^{th} sale decision makers consider, C_h is the weight of each sub aggregation with $C_h \in [0, 1]$ and $\sum_{h=1}^{m} C_h = 1$, w_i^h is the i^{th} weight of the h^{th} weighting vector W with $w_i^h \in [0, 1]$, and $\sum_{i=1}^{n} w_i^h = 1$.

The inclusion of experts' opinions can extend UAOS as follows:

$$MP-UAOS=\sum_{x=1}^{y}\sum_{h=1}^{m}\sum_{i=1}^{n}e_xC_{hx}w_{ix}^{h}s_i, \tag{7}$$

where e_x is the x^{th} weight of each expert such that $e_x \in [0, 1]$ and $\sum_{x=1}^{y} e_x = 1$; and the use of companies and countries yields the following expression:

$$MP-MC-ME-UAOS=\sum_{x=1}^{y}\sum_{r=1}^{q}\sum_{k=1}^{p}\sum_{h=1}^{m}\sum_{i=1}^{n}e_xu_{rx}v_{krx}C_{hrkx}w_{ikrx}^{h}s_{ikrx}, \tag{8}$$

where v_k is the k^{th} weight of each enterprise such that $v_k \in [0, 1]$ and $\sum_{k=1}^{p} v_k = 1$, and u_r is the r^{th} weight of each country such that $u_r \in [0, 1]$ and $\sum_{r=1}^{q} u_r = 1$.

The UAOS includes a wide range of particular cases (Merigó et al., 2015) like the weighted average, the OWA operator, and the OWAWA operator. UAOS can deal with many situations that may include probabilities and the attitudinal character of the decision maker.

Dealing with complex information with the UAOS operator requires considering any result that may move from the minimum to the maximum. Thus, any result accomplishes the boundary condition such that:

$$Min\{s_i\} \leq UAOS \leq Max\{s_i\}, \tag{9}$$

where UAOS is the average sale of the aggregation system. Note that this condition applies only if t-norms and t-conorms do not apply.

For further reading on properties and particular cases of these aggregation operators, see Merigó (2012a) and Merigó, Palacios-Marqués, and Benavides-Espinosa (2015).

4. Example of average sales

The study analyzes average sale by using a wide range of aggregation operators. This section presents key examples of real world problems where the new approach on average sale could apply. Table 1 presents some general examples of supranational regions where researchers could study the average sales. Note that these examples appear when using multi-expert information although researchers can use general structures that consider several companies.

Within each country, researchers may develop a similar analysis taking into account provinces, cities, and towns. Thus, for the USA an additional level of aggregation could apply with 50 states and cities that those states comprise.

Another interesting example is the European Union. In this case, researchers should consider 28 countries forming 28 sub-aggregations. Table 2 presents an additional level of aggregation following the EU aggregation. Here, the study analyzes each country individually, taking into account the provinces that form the country. Although the example shows only some representative countries, studies can consider all of them.

Table 2 shows that the aggregation of the EU sales depends on the average sales in each country that in turn depends on the average sales of each province.

5. Numerical example

This section presents a real world application of the UAOS operator. Assume a company that is analyzing the average sales of one of its leading products worldwide. Three experts assess the information and provide their own opinions depending on uncertain environmental changes that can occur. The study calculates the average sales in terms of sales by millions of inhabitants. That is, for each region, sales are divided by the total population and multiplied by one million. This operation shows the intensity of the sales in each region. The company has a strong position in the EU but also operates in many countries worldwide. To provide a full picture, this example shows the formation of the EU average sales to observe a possible fragmentation of each region. Next, the study considers the world average sales of the product. A similar decomposition can apply to any other country. For simplification reasons, the study does not show everything.

First, the study examines the EU average sales assuming that the first expert has a degree of importance of 50% while the second one has 30% and the third one 20%. The initial market share has an importance of 40%, and the subjective beliefs formed with the UAO operator, a degree

Table 2
Average sales by using the UAOS operator for some countries of the EU.

Average price	Formulation
Germany	$S=\sum_{h=1}^{m} C_h\left(u_{1-N.R}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-BAV}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{16-BRE}\left[\sum_{i=1}^{n} w_{ih16}s_{i16}\right]\right)$
France	$S=\sum_{h=1}^{m} C_h\left(u_{1-I.F}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-R.A}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{22-COR}\left[\sum_{i=1}^{n} w_{ih22}s_{i22}\right]\right)$
UK	$S=\sum_{h=1}^{m} C_h\left(u_{1-ENG}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-SCO}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{4-N.I}\left[\sum_{i=1}^{n} w_{ih4}s_{i4}\right]\right)$
Italy	$S=\sum_{h=1}^{m} C_h\left(u_{1-LOM}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-CAM}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{20-A.V}\left[\sum_{i=1}^{n} w_{ih20}s_{i20}\right]\right)$
Spain	$S=\sum_{h=1}^{m} C_h\left(u_{1-AND}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-CAT}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{17-L.R}\left[\sum_{i=1}^{n} w_{ih17}s_{i17}\right]\right)$
Poland	$S=\sum_{h=1}^{m} C_h\left(u_{1-MAS}\left[\sum_{i=1}^{n} w_{ih1}s_{i1}\right]+u_{2-SIL}\left[\sum_{i=1}^{n} w_{ih2}s_{i2}\right]+\ldots+u_{16-LUB}\left[\sum_{i=1}^{n} w_{ih16}s_{i16}\right]\right)$

N.R = North Rhine-Westphalia; BAV = Bavaria; BRE = Bremen; I.F = Ile-de-France; R.A = Rhone-Alpes; COR = Corsica; ENG = England; SCO = Scotland; N.I = Northern Ireland; LOM = Lombardy; CAM = Campania; A.V = Aosta Valley; AND = Andalusia; CAT = Catalonia; L.R = La Rioja; MAS = Masovian; SIL = Silesian; LUB = Lubusz.

of 60%. The calculation shows two different perspectives. The first one calculates the average sales of each expert and then integrates the results. The second perspective integrates the opinions of the experts, and then, calculates the average sales. By using simple weighted averages appearing in the example, both perspectives provide the same result. However, when dealing with OWA operators, the results of each perspective may differ. For simplicity, this study does not consider the OWA operator. Table 3 presents the average sales for the EU.

Observe that the company has a strong position in the EU but does not sell the product in many small countries. In terms of intensity, the highest sales are in Denmark while the smallest ones are in Austria. Note that in absolute terms, big countries tend to have bigger sales because they have more population.

In the world average sales of this product for this specific company, the study considers EU countries together. The average sale results from the UAOS operator appearing in Table 3. Table 4 shows the results.

Results show that the average sales for one million inhabitants move between 74.8 and 590.9. The classical average result is 311.11 while the UAOS new result is 297.72. Note that results differ because this method considers complexities that classical aggregations ignore. Therefore, the study uses the final result in the UAOS operator. The advantage of this new approach is that the classical arithmetic mean provides a neutral result, which is not always efficient. Thus, the UAOS is more practical because UAOS can move around a central value but with small changes that adapt better to the expected beliefs for the future.

This example considers only one period of time because of space issues. However, future research could consider different periods of time where the classical time series models in the literature could apply. In this case, the classical models usually apply the arithmetic mean or the weighted average while the new approach can use the UAOS (and also the OWAS), bringing all the advantages this study explains.

A key advantage of this analysis is that this approach shows the intensity of sales in each country, thus showing where the company has a strong position. The opinion of several experts increases the robustness of the information, allowing more accurate forecasts of the average sales. The UAOS operator can deal with the opinion of three experts that form their results (opinion) according to the initial market share of each country and their own subjective beliefs about the future regarding each country's importance in the determination of the total sales.

6. Conclusions

This study presents a new approach for the calculation of the average sale in sales forecasting. The main contribution focuses on the use of modern aggregation systems in the average sales to include complex environments and multiple sources of information in the analysis. This

Table 3
EU average sales.

	Expert 1			Expert 2			Expert 3			Collective results		
Country	AS	MS	UAO	AS	MS	UAO	AS	MS	UAO	AS	MS	UAO
Austria	158	0.02	0.02	157	0.02	0.02	154	0.02	0.01	157	0.02	0.02
Belgium	197	0.02	0.02	184	0.02	0.02	190	0.02	0.01	192	0.02	0.02
Czech Rep.	248	0.02	0.02	260	0.02	0.03	253	0.02	0.02	253	0.02	0.02
Denmark	538	0.04	0.02	515	0.04	0.03	527	0.04	0.02	529	0.04	0.02
France	420	0.14	0.15	425	0.14	0.15	427	0.14	0.16	423	0.14	0.15
Germany	375	0.15	0.16	373	0.15	0.15	377	0.15	0.14	375	0.15	0.15
Hungary	289	0.03	0.02	291	0.03	0.03	294	0.03	0.02	291	0.03	0.02
Ireland	462	0.02	0.02	458	0.02	0.02	459	0.02	0.01	460	0.02	0.02
Italy	487	0.14	0.14	492	0.14	0.13	487	0.14	0.13	489	0.14	0.14
Netherlands	387	0.05	0.06	385	0.05	0.05	389	0.05	0.06	387	0.05	0.06
Poland	268	0.07	0.08	266	0.07	0.07	269	0.07	0.08	268	0.07	0.08
Portugal	324	0.03	0.02	325	0.03	0.02	328	0.03	0.04	325	0.03	0.02
Spain	386	0.09	0.09	384	0.09	0.11	387	0.09	0.11	386	0.09	0.10
Sweden	470	0.05	0.04	473	0.05	0.03	475	0.05	0.04	472	0.05	0.04
UK	391	0.13	0.14	393	0.13	0.14	392	0.13	0.15	392	0.13	0.14
EU average	392			392			393			392		
EU Min	158			157			154			157		
EU UAO	390			389			393			390		
EU Max	538			515			527			529		
Total sales												

AS = average sales for 1.000.000 inhabitants; MS = initial market share; UAO = subjective belief of initial market share conditioned by future changes (includes several weights).

Table 4
World average sales.

Country	Expert 1 AS	Expert 1 MS	Expert 1 UAO	Expert 2 AS	Expert 2 MS	Expert 2 UAO	Expert 3 AS	Expert 3 MS	Expert 3 UAO	Collective results AS	Collective results MS	Collective results UAO
EU	392	0.1700	0.1300	392	0.1700	0.1200	393	0.1700	0.1500	392	0.1700	0.1310
USA	425	0.1900	0.1400	430	0.1900	0.1300	426	0.1900	0.1500	427	0.1900	0.1390
Canada	370	0.0200	0.0200	365	0.0200	0.0200	372	0.0200	0.0200	369	0.0200	0.0200
Mexico	278	0.0300	0.0300	279	0.0300	0.0400	283	0.0300	0.0300	279	0.0300	0.0330
Brazil	250	0.0400	0.0500	253	0.0400	0.0500	257	0.0400	0.0400	252	0.0400	0.0480
Argentina	314	0.0200	0.0100	316	0.0200	0.0100	318	0.0200	0.0100	315	0.0200	0.0100
Colombia	201	0.0100	0.0100	204	0.0100	0.0100	202	0.0100	0.0100	202	0.0100	0.0100
Venezuela	195	0.0100	0.0100	196	0.0100	0.0100	198	0.0100	0.0100	196	0.0100	0.0100
Peru	176	0.0050	0.0050	174	0.0050	0.0050	179	0.0050	0.0050	176	0.0050	0.0050
Chile	319	0.0050	0.0050	327	0.0050	0.0050	324	0.0050	0.0050	322	0.0050	0.0050
Norway	587	0.0050	0.0050	596	0.0050	0.0050	593	0.0050	0.0050	591	0.0050	0.0050
Switzerland	469	0.0050	0.0050	463	0.0050	0.0050	470	0.0050	0.0050	467	0.0050	0.0050
Russia	305	0.0500	0.0600	320	0.0500	0.0700	318	0.0500	0.0600	312	0.0500	0.0630
Turkey	285	0.0300	0.0300	283	0.0300	0.0200	284	0.0300	0.0300	284	0.0300	0.0270
Saudi Arabia	350	0.0100	0.0100	352	0.0100	0.0100	359	0.0100	0.0100	352	0.0100	0.0100
Iran	172	0.0100	0.0200	174	0.0100	0.0100	175	0.0100	0.0100	173	0.0100	0.0150
India	138	0.0500	0.0600	147	0.0500	0.0700	140	0.0500	0.0600	141	0.0500	0.0630
Pakistan	125	0.0100	0.0200	126	0.0100	0.0200	128	0.0100	0.0100	126	0.0100	0.0180
Bangladesh	110	0.0100	0.0200	112	0.0100	0.0200	110	0.0100	0.0100	111	0.0100	0.0180
Indonesia	168	0.0300	0.0300	175	0.0300	0.0400	170	0.0300	0.0400	171	0.0300	0.0350
Singapore	456	0.0050	0.0050	460	0.0050	0.0050	458	0.0050	0.0050	458	0.0050	0.0050
Vietnam	174	0.0100	0.0100	173	0.0100	0.0200	175	0.0100	0.0100	174	0.0100	0.0130
Philippines	196	0.0100	0.0200	192	0.0100	0.0100	194	0.0100	0.0100	194	0.0100	0.0150
Thailand	185	0.0100	0.0100	182	0.0100	0.0200	183	0.0100	0.0100	184	0.0100	0.0130
China	236	0.0900	0.1100	239	0.0900	0.1100	242	0.0900	0.1000	238	0.0900	0.1080
Japan	375	0.0600	0.0500	371	0.0600	0.0500	372	0.0600	0.0500	373	0.0600	0.0500
South Korea	294	0.0200	0.0200	293	0.0200	0.0200	295	0.0200	0.0600	294	0.0200	0.0280
Taiwan	268	0.0100	0.0100	274	0.0100	0.0100	275	0.0100	0.0100	271	0.0100	0.0100
Australia	359	0.0100	0.0100	372	0.0100	0.0100	363	0.0100	0.0100	364	0.0100	0.0100
New Zealand	340	0.0050	0.0050	342	0.0050	0.0050	345	0.0050	0.0050	342	0.0050	0.0050
South Africa	215	0.0200	0.0200	209	0.0200	0.0200	210	0.0200	0.0200	212	0.0200	0.0200
Nigeria	123	0.0100	0.0200	126	0.0100	0.0200	124	0.0100	0.0100	124	0.0100	0.0180
Egypt	194	0.0100	0.0100	196	0.0100	0.0100	195	0.0100	0.0100	195	0.0100	0.0100
Argelia	210	0.0050	0.0050	213	0.0050	0.0050	211	0.0050	0.0050	211	0.0050	0.0050
Sudan	095	0.0050	0.0100	092	0.0050	0.0050	095	0.0050	0.0050	094	0.0050	0.0080
Ethiopia	074	0.0050	0.0100	076	0.0050	0.0050	075	0.0050	0.0050	075	0.0050	0.0080
Angola	140	0.0050	0.0050	142	0.0050	0.0050	143	0.0050	0.0050	141	0.0050	0.0050
World average	310			313			312			311		
World Min	74			76			75			75		
World UAO	296			296			306			298		
World Max	587			596			593			591		

AS = average sales for 1,000,000 inhabitants; MS = initial market share; UAO = unified aggregation operator.

study also suggests the use of the OWA operator to consider the attitudinal character and the UAO operator as a general framework that can adapt to a wide range of situations. The main advantage of the OWAS and UAOS is the option to represent the information in a more complete way taking into account any scenario from the minimum to the maximum and without losing information in the problem.

The study presents simple, real examples including the calculation of the world average sales, the EU average sales, and some national cases. This approach considers complex environments where several experts opine. This study includes a numerical example regarding the calculation of the world average sales of a product. Countries' size varies; thus, the analysis uses per capita terms.

When calculating the average sales, future research should consider induced and generalized aggregation operators. Studies could focus on the use of imprecise information such as interval or fuzzy numbers. A need exists for some other examples of these approaches, giving a special focus to the average sales at company level.

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