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DEALING WITH THE RISK OF A POLICY CHANGE: A MULTI-STAGE EXPANSION PLANNING FOR THE ENERGY SECTOR

MEMORIA PARA OPTAR AL TÍTULO DE INGENIERIO CIVIL MATEMATICO

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POR: Giancarlo Andrés Acevedo Echeverría
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PLANIFICACIÓN ÓPTIMA DE GENERACIÓN ELÉCTRICA EN MULTIETAPAS CONSIDERANDO CAMBIOS EN LAS POLÍTICAS DE ENERGÍAS RENOVABLES

Se presenta un modelo para diseñar un plan de acción para la generación eléctrica que incorpora futuro riesgo de cambio en la política de energía renovable. El modelo permite un plan en múltiples etapas, en donde algunas decisiones pueden ser tomadas hoy y otras pospuestas al futuro, cuando la incerteza de un cambio de política es revelada. El modelo es lo suficientemente flexible para considerar diversas potenciales medidas de energías renovables, las cuales pueden ser implementadas por medio de penaltis, impuestos a la emisión de carbono, subsidios u otros. El modelo es resuelto por medio de una descomposición tipo Benders para poder lidiar con un problema de altas dimensiones como la planificación eléctrica de un país. Se muestra que la planificación en múltiples etapas muestras mejorías substanciales en términos de costos y reducción de riesgo.

El modelo es implementado para el SIC (Sistema Interconectado Central de Chile), para 2 etapas o años objetivos, 2025 y 2035, en que se elaboraron 3 casos, el primero en el cual no hay cambios en políticas de energía renovable, el segundo en el cual desde el año 2035 hay un chance de que se imponga una política de impuesto a las emisiones de carbono y finalmente, el último caso en que en el año 2035 se impone una política de 33% de generación mínima renovable no convencional.

Para los 3 casos mencionados anteriormente se tienen distintos portafolios energéticos, donde se puede apreciar el efecto que tiene cada cambio de política para el sistema de generación eléctrica de un país.

El principal objetivo de esta tesis es elaborar un modelo que permita la incorporación del riesgo ante cambios de políticas en el tiempo en la planificación y operación de generación eléctrica.

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OPTIMAL ELECTRICITY GENERATION PLANNING UNDER RENEWABLE POLICY TARGETS

We present a multi-stage model to design the expansion planning for electricity generation to incorporate future –although uncertain– new renewable policy targets. The model allows an expansion planning in multiple stages in which some decision can be taken today, and others postponed to the future, when the uncertainty of new potential policies are eliminated. The model is flexible enough to consider diverse 'potential' renewable policy measures which can be implemented by different mechanisms such as penalties, carbon taxes, subsidies, amongst others. The model is solved by a Benders decomposition algorithm to tackle large dimension problems for a country level planning for electricity generation. We show that a multi-stage planning provide important economic benefits in term of costs and risks reductions.

The model is implemented for the Chilean Central Interconnected System (CIS), for 2 stages or objective year, 2025 and 2035, in which we elaborate 3 different cases, first the system without changes of renewables policy, the second one, the system with a probability of a carbon tax implemented since 2035, and the last one, the model with a 33% policy target.

For the 3 cases mentioned above have different energy portfolios, where you can see the effect each change of policy for the power generation system of a country.

The main objective of this thesis is to develop a model that allows the incorporation of risk from policy changes over time in the planning and operation of electric generation

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1. Introduction

"*BEIJING — China and the United States made common cause on Wednesday against the threat of climate change, staking out an ambitious joint plan to curb carbon emissions as a way to spur nations around the world to make their own cuts in greenhouse gases. The landmark agreement, jointly announced here by President Obama and President Xi Jinping, includes new targets for carbon emissions reductions by the United States and a first-ever commitment by China to stop its emissions from growing by 2030*." (New York Times. November 11th, 2014)

"*With this Clean Power Plan, by 2030, carbon pollution from our power plants will be 32 percent lower than it was a decade ago..... Over the next few years, each state will have the change to put together its own plan for reducing emissions -- because every state has a different energy mix*". (Barack Obama Speech about U.S. announced of the new Clean Power Plan. August 3rd, 2015).

The expansion planning for electricity generation is a complex problem, in which a strategy for the future construction of generation plants is designed, constrained to economic projections (e.g. costs prediction, risk exposures, uncertainty regarding future demands of energy) and operational issues (e.g. stability of the system and security supply). However, in recent years, expansion planning for electricity generation is facing new challenges due to a generalized environmental concern. Many plants of electricity generation, such as those based on fossil-fuels, can induce serious environmental damages including global warming, pollution, acid rain, and rising sea levels. In the last 10 years, an increasing number of countries are committed to reach progressively new renewable policy targets. For instance, the Renewables 2015 Global Status Report, from REN 21, shows that 164 countries have renewable policy targets by 2015 from only 55 countries by early 2005.[1] These renewable policy targets has been (or they will be) implemented by each country through different mechanisms, including strict quotas for renewable generation (with the possibility of removing generation permissions) or by inducing economic incentives (e.g. penalties, carbon tax and subsidies) to induce change in the technologies for electricity production (see, e.g., Stern, 2008; Harstad, 2012a,b; Marron and Toder, 2014). Although a rapid pace of policy adoption can encourage renewable energy development and reduce carbon emissions, it can also be risky if renewable policies are not considered properly in the optimal design for the expansion planning for electricity generation.

[1] REN21 is an international non-profit association made up of members of international organizations (e.g., European commission, UNIDO and the World Bank) and governments (e.g., United Kingdom, Norway and Brazil).

For example, in the case that the electricity generation planning does not take into account a potential future renewable policy target, which can be implemented with severe economic penalties, today it can be economically convenient to install a fossil fuel power plant to expand the energy matrix. Afterwards, if the renewable policy target is implemented, the decision of having installed a technology with high emission factor will increase the cost of the system above the level expected. Thus, given the current environmental concern, 'potential' new (or changes in) renewable policy targets has to be taken into account in an expansion planning for electricity generation.

We propose a novel multi-stage model to design the expansion planning for electricity generation to incorporate future –although uncertain– new renewable policy targets. The model allows an expansion planning in multiple stages in which some decision can be taken today, and others postponed to the future, when the uncertainty of new potential policies are eliminated (i.e. after the future disclosure of the new policy measure from governments and/or regulators). The model is flexible enough to consider diverse 'potential' renewable policy measures which can be implemented by different mechanisms such as penalties, carbon taxes, subsidies, amongst others. The model deliver an optimal allocation of new electricity generation plants, which can operate with different technologies including the use of fossil fuels (e.g. Coal, Oil and Liquefied Natural Gas), conventional renewables (e.g. Hydro and Run-of-River) and non-conventional renewables (e.g. Solar Photovoltaic, Wind, Biomass, Geothermal, Concentrated Solar Power and Small-hydro), for effect of this thesis, we used this classification of technologies although is little different to international community. The model generate an optimal allocation of plants in each stage by taking into account simultaneously: changes in renewable policy targets, expected costs, risk exposures (i.e., prices volatilities, hydrological scenarios and demand growth) and operational issues to assure the electricity supply security(e.g. generation reserves to maintain reliable operation).

As mentioned above, increasing concern for environmental protection has driven countries all over the world to introduce progressively new renewable policy target, which induce a change in the 'rules of the game'. There is also an uncertainty of the 'levels' of future new policy targets and 'how' they will be implemented. For instance, the target adopted in Europe in 2008 is to reach a 20% share of total final energy production from renewables generation by 2020; however it was uncertain the target for 2030. In 2011, the European Renewable Energy Council (EREC) suggested as potential target a 45% share of total final energy from renewables by 2030. However, just in 2014, José Manuel Durão Barroso (President of the European Commission) announced the official European target of 27% of renewable energy generation share by 2030. Moreover, there is uncertainty of the continuity of some incentives, which has been already implemented by governments to induce renewable generation of energy. For example, Alexander Dombrovski (advisor to the President of Ukraine) announced on October 2014 that Ukraine removed its income tax exemption for companies that sell renewable electricity. In addition, in the United States there is uncertainty of the continuity of the Energy Investment Tax Credit (ITC) which will last until December 2016.[2] Consequently, the uncertainty generated by 'potential' changes and/or new energy policies may affect the development of future electricity generation projects.

The model not only considers simultaneously potential changes in renewable policy measures jointly to investment and operational costs, but also it takes into account variables related to security supply and risk exposures. The model includes operational issues in relation to security of electricity supply, since the system must be capable of absorbing changes from the intermittent electricity production of renewable technologies (see, e.g., De Jonghe et al. 2010; and Pérez-Arriaga and Battle, 2012; and Inzunza et al. 2014). For instance, in the case of an electricity generation planning which is mainly composed of renewable technologies which are characterized by an irregular electricity generation, it would probably necessary to install at the same time fossil-fuel plants as reserves, in order to generate electricity in hours when the renewable resource is not available (e.g. electricity generation based on the solar photovoltaic technology does not generate at night, in the absence of solar radiation). In this context, several authors have already focused on security supply in expansion generation. For instance, Huang and Wu (2008), Gotham et al. (2009), and Vithayasrichareon and MacGill (2014) analyze an optimal planning with different quotas of renewable generation, in which the objective is to deliver a secure sustainable electricity system by taking care of the operational aspects of the technologies' allocation. Delarue et al. (2011) include additional operational constraints to allow an efficient absorption of the generation system in cases of intermittent renewable outputs. Recently, given the high growth the growth of renewable generation, De Jonghe et al. (2010),

[2] The ITC reduces federal income taxes for qualified tax-paying owners based on capital investment in renewable energy projects.

Pérez-Arriaga and Battle (2012) and Inzunza et al. (2014) suggest that additional security supply constraints has to be considered to provide more stability to the system.

Our multi-stage planning model maintaining system security levels through scheduling various types of reserves jointly to the provision of various tools and controls to give stability to the system such as: the use of demand side services (customers responding to a signal to change the amount of energy they consume from the power system at a particular time); inclusion of the preservation of system inertia levels (the amount of kinetic energy stored in all spinning turbines and rotors in the system) and spinning and standing reserves (i.e., an optimal reserves allocation of diverse technologies to give stability to the system).

Furthermore, the multi-stage model takes into account risks associated to unexpected changes in economic variables and diverse climate conditions. Historically, even in recent years, the expansion planning for electricity generation has been performed mainly from a perspective of minimizing costs (see, e.g., Neuhoff et al. 2008; Steffen and Weber, 2013; and Eide et al., 2014). However, an expansion planning faces several uncertainties due to the fact that is designed to operate in the 'future'. Thus, future electricity generation will face several risks such as unexpected changes in fossil-fuel prices, changes in the demand growth and different weather conditions. In fact, if we break down electricity production into the different technologies used for generation, each technology can be seen as part of a large portfolio with different assets. Hence, the use of multiple technologies based on renewables and fossil-fuels can induce benefits of diversification and thus reduce levels of risk exposure, given the diverse characteristics and different underlying stochastic processes that describe their costs and generation availability.[3] For instance, on average the highest demand requirement on a day is in general between 12:00 and 14:00 hours, which is exactly the time when photovoltaic generation has the maximum availability for electricity production(in the case of SIC of Chile); hence renewable technologies can also be used to hedge changes in demand. To evaluate the level of these sources of uncertainty, we use the Conditional Value-at-Risk (CVaR). The use of the CVaR is due to two main reasons: firstly, it is a coherent measure of risk (see Artzner, 1999) which has some useful properties for our model such as sub-additivity, monotonicity and positive homogeneity; and secondly, optimization problems with CVaR (thanks to its properties as a coherent measure of risk) can be reduced to linear programming problems (see Rockafellar and Uryasev, 2002).

[3] Portfolio analysis in the energy sector was introduced by Bar-Lev and Katz (1976) and recently by Awerbuch and Berger (2003), Awerbuch (2006), Jansen et al. (2006), Delarue et al. (2011) and Inzunza et al. (2014).

The model is solved by a Benders decomposition algorithm to tackle large dimension problems for a country level planning for electricity generation. The Benders decomposition algorithm consists in dividing the optimization problem in two sub problems called master and slave. The master problem is formulated to optimize the investment decision (i.e. installed capacities per each form of electricity generation) in the present and future stages of the optimal generation planning. The slave problem is formulated to make optimal dispatch decisions 'hour by hour' in the system (i.e. the hourly generation per each plant); and thus take into account generation profiles of renewable technologies and profiles of the demand to take into account dynamics of the electricity consumption over the day. These dispatch decisions represent the operation of the capacity imposed by the first stage. In each iteration, the algorithm guide the master problem based on the dynamics of the slave problem, as it progresses towards an optimal expansion planning.

We implement the multi-stage planning in a real (country level) power system. The aim of this implementation is to present a genuine example of an expansion planning, including different analyses and policy exercises, which can be used as a guide to implement our approach in other regions or countries. We implement the model for the Chilean Central Interconnected System (CIS), where expansion planning is performed in a two-stage decisions problem. The first set of decisions are made in 2015 to build generation plants that will be operative in 2025 (henceforth, stage I). The second set of decisions are made in 2025 for the construction of generation plants that will be operative in 2035 (henceforth, stage II). The generation planning for 2025 consider the current installed capacities which are already based in different generation technologies; while that the generation planning for 2035 is also conditional to the installed capacity 'planned' for 2025. In addition, the decisions made in 2025 (for stage II, objective year 2035) are conditional to the information observed in that year; thus in the second stage it is possible to adjust the expansion planning based on future 'knowledge' which is not currently available. Future information received in 2025 is not only concerning to 'potential' renewable policy changes, but also it is about changes in fossil-fuel prices, future hydrological scenarios and changes in the growth rates for projected demands.

The implementation of our multi-stage planning model in the Chilean CIS is useful to understand the benefits of postponing decisions to the future, when more information is available and uncertainties are reduced. In order, to analyze the benefits of the decision flexibility of our multi-stage approach, we compare our multi-stage planning model with a single-stage planning model in which all expansion decisions are developed today and they cannot be modified. Hence, when we implement the single-stage planning model in the Chilean generation system, all decision are made in 2015 which includes to start building generation plants right now (which will be operative in 2025) and to start the construction of generation plants in 2025 (which will be operative in 2035). The main difference of the single-stage planning in relation to the multi-stage planning is that the former cannot modify decisions in 2025 (decisions are fixed), while that in the later the expansion plan can be modified in 2025 depending of the new information received. We show that, even in a scenario without changes in policy measures, our multi-stage planning model provide important economic benefits in term of costs and risks reductions. Costs and risk reduction are observed since updated information in 2025 about regarding uncertain hydrological scenarios, fossil-fuel prices and demand growth is used in the optimal planning for stage II.

Finally, we perform two policy exercises to analyze the risk of a 'potential' change in renewable policy measures. Firstly, we analyze the effect of a 'potential' future carbon tax of 10 dollars per tones of CO2 under our multi-stage planning model, which is compared to a single-stage expansion plan. This carbon tax is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. We show that economic benefits are larger for the multi-stage planning model than the single stage-setup when the uncertainty of a policy tax is included. For instance, for an average level of risk, there is a 41% of more savings thanks to the flexibility of the multi-stage model when there is an additional uncertainty coming from a potential carbon tax than in the case when there is only uncertainty coming from hydrological scenarios, fossil-fuel prices and demand growth.

Secondly, we analyze the impact of a 'potential' policy target. We analyze the case where there is a policy target of 33% share of total electricity generation from non-conventional renewables by 2035.[4,5] We assume that this policy target is uncertain and it may be announced with a 50% of probability by 2025 and it will be implemented in 2035. In the case that this target is not reached, a penalty of 50 dollars is charged in each hour per megawatt which is not generated through non-

[4] Non-conventional renewable technologies for the Chilean regulation consider all renewable technologies excluding large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts. Large hydropower plants (hydro-reservoirs and run-of-river) are considered as conventional renewable energy sources. This type of energy is the most used in Chile, with a participation of approximately 40% in the current energy matrix.

[5] A potential policy target of a target of 33% share of total electricity generation from non-conventional renewables by 2035 in the Chilean CIS is congruent with the current environmental policy from Government of Chile. Currently, there is already a target of 20% share of total electricity generation from non-conventional renewables by 2025.

conventional renewables. We show that there are also important economic benefits of take into account the risk of a new policy target in the multi-stage expansion model. In addition, we present evidence that an optimal allocation of technologies including non-conventional renewable generation can generate a diversification effect by reducing not only the risk from uncertain hydrological scenarios, but also the financial risks of the system generated by volatility of fossil-fuel prices. Hence, we also shows that economic benefits of the multi-stage are less evident when the optimal planning is designated to reduce the levels of risk, because already for this building plans are considered a high level of non-conventional renewables.

A deficient expansion planning in the energy sector may limit economic growth and human development by affecting key elements of the well-functioning of a country (e.g. industrial production, telecommunications, financial markets, health services and security systems). Thus our multi stage model gives an step ahead in term of flexibility and adaptability to the expansion of the energy matrix, depending of future environmental conditions, political scenarios, operational issues and economic dynamics. The rest of the thesis is organized as follows. Section 2 introduces the multi-stage model to determine expansion planning for electricity generation. Section 3 describes the model implementation. Section 4 presents the main results jointly to policy exercises in relation to changes in renewable policy measures. Section 5 concludes.

2. The Model

Consider an expansion planning for electricity generation in which decisions about the construction of generation plants can be taken at different stages. Hence, some decision can be taken today and others can be postponed to the future in τ_k years when more and useful information is available, where τ_0 is the current time and $k \in \{1,2,\dots,K\}$. For convenience, let the current planning time be equal to zero. There is a set of different technologies $i \in \{1,2,\dots,I\}$ that can be used to build generation plants of electricity, which will take h years to be built.

The generation plants can potentially generate electricity in each hour $j \in \{1,2,\dots,8760\}$ of the year (i.e. 365*24=8760 hours). The potential electricity produced in each hour by the technology i depends of installed capacity of generation in megawatts [MW], cap_i, and on the features of the technology (i.e. solar generation levels are different in summer and in winter , and it can be only obtained when there is sunlight). The installed capacity of generation by the technology i is composed by the direct capacity, c_i, to meet the demand plus the governor reserves, $R_{i,j}^{GR}$, which reflect the capacity in megawatts that is *exclusive* for 'governor' (i.e. a feedback controller) to control system frequency after sudden, large disturbances and thus to keep the security of electricity supply (i.e., $cap_i = c_i + R_{i,j}^{GR}$).

We include the possibility of demand side services (DSS), in which customers responded to signals (e.g. signals can be economic incentives such discounted prices for electricity) to change the amount of energy that they consume from the system power at a particular time. Demand side services can help shift electricity consumption away from peak hours where electricity consumption is high, or enable greater usage of excess electricity generation from renewables, as well as help maximize the use of a smart infrastructure. However, switches in the demand are costly (e.g. additional costs given the discounted prices to generate the change in the demand and/or shifst in the demand to different schedules can induce some social costs). The cost of demand decrease (increase) due to shifts in demand is dc^- (dc^+), in which the amount of changes in demand is D_j^- (D_j^+).

Suppose that there is a set of states $s_k \in \{1,2,\dots,S_k\}$ that will describe the generation features in all years in the period $[\tau_k, \tau_{k+1}]$. The state s_k that will be governing the period between τ_k and τ_{k+1} includes: i) renewable policy measures, $m(s_k)$, such as renewable quotas, potential subsidies and/or penalties for renewable and fossil-fuel technologies; ii) the annuitized investment cost, $INV_i(s_k)$, to install the technology i; iii) the operation and maintenance costs, $VOM_i(s_k)$ per hour of electricity generated by using the technology i; iv) hydrological conditions to evaluate effects of the uncertainty that such condition on the constructions of plants based on hydrological scenarios; and v) the demand, $D_j(s_k)$, per hour [MWh] in a representative year between τ_k and τ_{k+1} which is updated depending of projections of the demand growth. The values $INV_i(s_k)$ and $VOM_i(s_k)$ may be affected by $m(s_k)$, since they can include potential economic incentives (or disincentives) for the generation based on particular types of technologies. In the section Results, we show two examples of changes in the policy renewable (carbon tax and target of generation renewable).

Let $\Gamma(s_k)$ be the possible set of planning decisions (i.e. construction of electricity generation plants, the hourly generation in each plant in a representative year) in the period $[\tau_k, \tau_{k+1}]$ given the state s_k. Suppose that the optimal group of planning decisions is $\tilde{\alpha} \in \Gamma(s_k)$ for the state s_k, $\tilde{\alpha}$ embraces: a) the installed capacity of generation in megawatts [MW] for the technology i denoted by $cap_i(s_k)$, which it has to be taken in τ_{k-1} given the fact that a generation plant take h years to be built and where $cap_i(s_k) = c_i(s_k) + R_{i,j}^{GR}(s_k)$; b) the generation, $g_{i,j}(s_k)$ in megawatts per hour [MWh] in the state s_k for the technology i and the hour j; and c) the demand switches for the demand side services in which $D_j^-(s_k)$ and $D_j^+(s_k)$ are negative and positive shifts in the demand, respectively. In the case that optimal group of planning decisions is $\tilde{\alpha}$ in the state s_k, this group of decisions may generate a lost load, $LL_j(s_k)$, in a given hour j in which there is non-supply of electricity:

$$LL_j(s_k) = D_j(s_k) + D_j^+(s_k) - D_j^-(s_k) - \sum_{i \in I} g_{i,j}(s_k), \tag{1}$$

which is costly since there is a social cost, $voll$, when the demand is not fully covered. The present value of the costs of the system at time τ_k, $C(s_k)$, for the period $[\tau_k, \tau_{k+1}]$ facing the scenario s_k is given by :

$$C(s_k) = \sum_{t=1}^{\tau_{k+1}-\tau_k} \frac{1}{(1+r)^t} \left[\sum_{i \in I} INV_i(s_k) \cdot cap_i(s_k) + \sum_{i \in I} \sum_{j \in J} VOM_i(s_k) \cdot g_{i,j}(s_k) \right. \\ \left. + \sum_{j \in J} D_j^-(s_k) \cdot dc^- + \sum_{j \in J} D_j^+(s_k) \cdot dc^+ + voll \cdot \sum_{j \in J} LL_j(s_k) \right] \tag{2}$$

where r is the annual discount rate for the generation planning. The cost of the system, $C(s_k)$, between τ_k and τ_{k+1} has four components: the investment costs, $INV_i(s_k) \cdot cap_i(s_k)$; the operation and maintenance costs, $VOM_i(s_k) \cdot g_{i,j}(s_k)$; potential additional costs due to shifts in demand, $D_j^-(s_k) \cdot dc^-$ and $D_j^+(s_k) \cdot dc^+$; and the social costs of non-supply of electricity in the hour j, $voll \cdot LL_j(s_k)$.

As a first step, we describe the multi-stage model without taking into account operational constraints. However, afterward, we will include additional constraints to the system to describe realistic conditions of the generation planning and electricity distribution, especially regarding the security of electricity supply.

The optimal allocation in the multi-stage model is based on a portfolio analysis, in which both costs and risks are incorporated in the optimal decision. Portfolio analysis in the energy sector was introduced by Bar-Lev and Katz (1976) and recently used by Awerbuch and Berger (2003), Awerbuch (2006), Jansen et al. (2006), Delarue et al. (2011) and Inzunza et al. (2014). The main difference of our model with previous planning for electricity generation is that we incorporate the possibility of decisions in multiple stages, which is used to reduce the risk of changes in renewable policy targets and risk exposures (i.e., prices volatilities, hydrological scenarios and demand growth). Moreover, and differently from previous studies, we analyze simultaneously in a multistage setup operational aspects in terms of the security of electricity supply, social costs of non-supply, and include several features to characterize properly a country level electricity planning

Therefore, the value of the total planning cost in the year τ_{k-1}, $V(\tau_{k-1})$, in which the new plants will be built and operative in the year τ_k is given by the Bellman equation of the optimization problem expansion planning for electricity generation, which is constrained by a given level of risk:

$$V(\tau_{k-1}) = \min_{\tilde{a} \in \Gamma(s_k)} \frac{1}{(1+r)^{\tau_k - \tau_{k-1}}} \sum_{s_k \in S_k} \left[C(s_k) + \frac{1}{(1+r)^h} V(\tau_k) \right] p(s_k) \qquad (3)$$

s.t

$$\frac{1}{1-\alpha} \sum_{V(\tau_{k-1}) \geq VaR_\alpha} V(\tau_{k-1}) p(s_k) = CVaR^* \qquad (4)$$

Where $p(s_k)$ is the probability of occurrence of the state s_k, and the risk is characterized by the Conditional Value-at-Risk ($CVaR$).

Nevertheless, the constraint in equation (4) is not linear, which makes large dimension optimization problems difficult to solve. Nevertheless, Rockafellar and Uryasev (2002) and Krokhmal et al. (2002) show that a CVaR constraint in a portfolio optimization problem can also be written as a linear programming problem, by adding a set of linear constraints and auxiliary variables (due to the properties of the CVaR given that is a a coherent measure of risk, see Artzner, 1999). Thus, we can re-write equations (3)-(4) as:

$$V(\tau_{k-1}) = \min_{z,\tilde{a}\in\Gamma(s_k)} \frac{1}{(1+r)^h} \sum_{s_k \in S_k} \left[C(s_k) + \frac{1}{(1+r)^h} V(\tau_k) \right] p(s_k) \tag{5}$$

s.t.

$$z + \frac{1}{1-\alpha} \sum_{s_k \in S_k} d_s \cdot p(s_k) \leq CV \tag{6}$$

$$V(\tau_{k-1}) - z \leq d_s \tag{7}$$

Where z is an auxiliary variable that is now part of the optimization problem; while d_s is another auxiliary variable that reflects right deviation of the cost with respect to z. The risk tolerance level in the $CVaR$ is given by CV, which represents the α-CVaR's upper bound of generation portfolio costs.

2.1 Operational Constraints and demand shifting

In terms of operational constraints to maintain the security of electricity supply, it is important to take into account that it is not possible to increase (or to reduce) instantaneously and drastically the generation of electricity when there is a modification in generation conditions. For instance, neither turbines from a large hydro-generation plant nor a coal based generation plant can instantaneously change their level of electricity production, because there is kinetic inertia in both turbines and rotors. Modifications in generation conditions are quite important when renewable generation plants are in the system. For example, there is no electricity generation from plants using solar energy at night; thus other technologies have to be used at night.

Therefore, we impose constraints to ramp rates, in relation to the rates that reflect 'how' quickly a technology i can modify its electricity production

Suppose that the number of online generation units synchronized to the power system is $n_{i,j}(s_k)$ from the technology i, in the hour j under scenario s_k (e.g., the online plants of windmills for wind generation). Let $\overline{P}_i$ ($\underline{P_i}$) be the maximum (minimum) power output of each unit of technology i:

$$n_{i,j}(s_k) \cdot \underline{P_i} \leq g_{i,j}(s_k) \leq n_{i,j}(s_k) \cdot \overline{P}_i \tag{8}$$

$$n_{i,j}(s_k) \cdot \overline{P}_i \leq cap_i(s_k). \tag{9}$$

In terms of ramp rates, which are the rate that a plant changes its output-generation (this rate is expressed in megawatts per hour), we constraint the difference in output-generation between two consecutive hours. Let ρ_i be the ramp rate limit for the technology i, then we add the constraint::

$$\begin{aligned} &g_{i,j}(s_k) - g_{i,j-1}(s_k) \\ &\leq min\{n_{i,j}(s_k), n_{i,j-1}(s_k)\} \cdot \rho_i + (n_{i,j}(s_k) - n_{i,j-1}(s_k)) \cdot \underline{P_i} \end{aligned} \tag{10}$$

$$\begin{aligned} &g_{i,j-1}(s_k) - g_{i,j}(s_k) \\ &\leq min\{n_{i,j}(s_k), n_{i,j-1}(s_k)\} \cdot \rho_i + (n_{i,j-1}(s_k) - n_{i,j}(s_k)) \cdot \underline{P_i}. \end{aligned} \tag{11}$$

Equations (10) (Equation (11)) reflect the constraint that in the ramping-up case (ramping-down case), the change in generation cannot be larger than the ramping capability of the units that are connected during two consecutive hours, plus the output of units connected (disconnected). It is assumed that units are both connected and disconnected at their minimum output, which is a conservative assumption.

The model also considers constraints in terms of demand shifting for demand side services (DSS). Thus, the demand can change in every period under some limits given by:

$$D_j^-(s_k) \leq \overline{ds}^- \cdot D_j(s_k) \tag{12}$$

and

$$D_j^+(s_k) \leq \overline{ds}^+ \cdot D_j(s_k), \tag{13}$$

where $\overline{ds}^-$ and $\overline{ds}^+$ are the maximum proportion of demand in any hour that can be reduced and augmented, respectively. However, the model also imposed that demand changes due to demand shifting are balanced within a time window:

$$\sum_{j \in J_k^D} D_j^+(s_k) - \sum_{j \in J_k^D} D_j^-(s_k) = 0, \tag{14}$$

where K represents a set of days in a year, and J_k^D is a set of hours in a time window of 24 hours.

Constraints for renewable generation

The model constrains the generation according to normalized hourly profiles depending of the technology used for in each plant. Suppose that the generation technology i^W uses wind; thus this technology is constrained by:

$$g_{i^W,j}(s_k) \leq WP_j(s_k) \cdot cap_{i^W}(s_k), \tag{15}$$

Where $WP_j(s_k)$ is the maximum generation output factor –a value between zero and one– for wind which describe the profile for every hour of a representative year in the period $[\tau_k, \tau_{k+1}]$. Suppose that the generation technology i^{SP} is by solar photovoltaic, there is also an upper bound constraint given by:

$$g_{i^{SP},j}(s_k) \leq SPP_j(s_k) \cdot cap_{i^{SP}}(s_k), \tag{16}$$

Where $SPP_j(s_k)$ is the maximum generation output factor for solar photovoltaic for every hour of the year. Similar upper bound restrictions are used to other renewable technologies such as biomass, geothermal, concentrated solar power or small hydro. In relation to run-of-river generation, i^{RR}, we impose that:

$$R_{i^{RR},j}^{spin}(s_k) + g_{i^{RR},j}(s_k) \leq RRP_j(s_k) \cdot cap_{i^{RR}}(s_k) \tag{17}$$

and for hydro-reservoir, i^{HR}, generation:

$$R^{spin}_{i^{HR},j}(s_k) + R^{mech}_{i^{HR},j}(s_k) + g_{i^{HR},j}(s_k) \leq HRP_j(s_k) \cdot cap_{i^{HR}}(s_k) \tag{18}$$

in which $RRP_j(s_k)$ and $HRP_j(s_k)$ are the maximum generation availability for run-of-river and hydro-reservoir technologies, respectively; while $R^{spin}_{i,j,s}$ and $R^{mech}_{i,j,s}$ are decision variables which represent the capacities' headroom in terms of spinning-kinetic reserves and mechanical reserves, respectively; this capacities' headroom is used exclusively to regulate contingencies for primary frequency response (both expressed in megawatts).[6] In addition, and also regarding to plants based on hydro-reservoir, let $v_{i^{HR},j}(s_k)$ denote the volume of stored water in reservoir in hour j under scenario s_k; $inf_{i^{HR},j}(s_k)$ the water inflow per hour; and $sp_{i^{HR},j}(s_k)$ the water lost through spillage. Then, the hydro-reservoir is constrained by:

$$\begin{aligned} v_{i^{HR},j}(s_k) = v_{i^{HR},j-1}(s_k) + inf_{i^{HR},j}(s_k) \cdot cap_{i^{HR}}(s_k) - \frac{g_{i^{HR},j}(s_k)}{\eta} \\ - sp_{i^{HR},j}(s_k) - v_{i^{HR},j}(s_k) \cdot \lambda, \end{aligned} \tag{19}$$

with η is the efficiency of the hydro-technology and λ is a factor used to consider losses of stored water due to evaporation and/or seepage in the reservoir. Finally, the model also considers that there is an upper bound of stored water, $\bar{v}_i$, in a reservoir used for the plants with this technology:

$$v_{i^{HR},j}(s_k) \leq \bar{v}_{i^{HR}} \tag{20}$$

Security of electricity supply constraints and demand side service

The model consider the use of headroom reserves to adjust possible contingencies unexpected changes in the generations of some plants and/or when a plant fails, which is very relevant when electricity plants based on renewable generation are in the system. This contingencies would induce a changes in frequency, and thus to affect the security of the electricity supply. Therefore, the model also has constraints for the dynamics of the primary frequency response. An optimal electrify generation planning is adequate in terms of the primary frequency response if system frequency does not drop below a given limit after any single generation contingency.

[6] In the case of fossil fuels, the upper bound constraints are $R^{spin}_{i,j}(s_k) + R^{mech}_{i,j}(s_k) + g_{i,j}(s_k) \leq FFP_{i,j}(s_k) \cdot cap_i(s_k)$ where $FFP_{i,j}(s_k)$ is also the maximum generation availability for the fossil-fuel technology i in hour j under scenario s_k.

We assume in our model that the system has a feedback controller, called 'governor' that identifies changes in system frequency. The mission of this governor is to turn on some reserves (capacity headroom), which we call 'governor reserves', that are used in the dispatch during contingencies. The governor reserves reflect the capacity that is *exclusive* for governors to control system frequency after sudden large disturbances in the system. Suppose that the optimal set of governor reserves, which are available in hour j under scenario s_k, for each of the different technologies is: $R_{1,j}^{GR}(s_k), R_{2,j}^{GR}, (s_k)\ \dots, R_{i,j}^{GR}(s_k),$.[7,8] Let's assume that there is a large disturbance in system generation, ΔP, measured in megawatts; thus we impose the following constraint for the security supply:

$$\Delta P \leq \sum_{i \in I} R_{i,j}^{GR}\ (s_k). \tag{21}$$

The security supply constraint reflected in equation (21) is a necessary, but not sufficient condition to keep the stability of the system, because it is important to consider the reaction speed of reserves to produce electricity in terms of emergency ramp rates as described in equations (10)-(11).

Suppose that $R_{i,j}^{GR}(s_k)$ is the optimal governor reserve for the technology i, in hour j under scenario s); this reserve is composed $n_{i,j}^{GR}(s_k)$ online units which have a has an emergency ramp rate limit ρ_i. Let impose that governor has a dead band f_{db} (which is the interval of no action when a change in frequency is small) and the frequency cannot to drop below the level f_{MIN}.[9] Let's assume that there is a large change in system generation, ΔP, in which the pre-contingency frequency is f_0. Thus, we include the constraint presented in Chávez et al. (2014), where they show that the 'minimum' governor response emergency ramp rate of the reserves, $\sum_i (n_{i,j,s}^{GR} \cdot \rho_i)$, in order to avoid levels below f_{MIN}, has to respect:

$$\frac{f_0(\Delta P)^2}{4(f_0 - f_{MIN} - f_{db})\left(\sum_i H_i \cdot n_{i,j}^{GR}(s_k) \cdot \overline{P}_i - H_f \cdot \Delta P\right)} \leq \sum_{i \in I} n_{i,j}^{GR}(s_k) \cdot \rho_i \tag{22}$$

---

[7] Spinning-kinetic and mechanical reserves, $R_{i,j,s}^{S}$ and $R_{i,j,s}^{P}$ as explained in equations (17)-(18), are subsets of the governor reserves.

[8] It is important to notice that governor reserves usually are referred to reserves with fossil-fuel technologies; however we want to keep the generality of the model, thus any technology can contribute to the generation reserves.

[9] Thus, in the first instants after a contingency, since there is a dead band, the system frequency is controlled by the inertial response of the own system.

Where H_i is the constant of inertia for technology i, $\overline{P}_i$ is the maximum power output of each unit of technology i, and H_f is the constant of the missing unit that induces the contingency ΔP.[10] Equation (22) reflects a constraint for the security of electricity supply including reserves for all technologies in an aggregated ways. Consequently, to improve the security supply of the system we also include constrains for the each of the reserves that uses different technology, and hence to take into account differences in ramp rates.

Let $t_{MIN,db}^{GR}$ be the time in which the system can recover after a contingency ΔP with the governor reserves: $t_{MIN,db}^{GR} = \Delta P / \sum_i (n_{i,j,s}^{GR} \cdot \rho_i)$. Let $t_{MIN,db}^{GR}$ also be the time that a governor reserve with the technology i can reach its maximum electricity generation: $t_{MIN,db}^{i} = R_{i,j,s}^{GR} / (n_{i,j,s}^{GR} \cdot \rho_i)$. Hence, we impose the constraint that all technologies have to respect that $t_{MIN,db}^{i} \leq t_{MIN,db}^{GR}$ which can be expressed as:

$$\frac{R_{i,j,s}^{GR}}{n_{i,j,s}^{GR} \cdot \rho_i} \leq \frac{\Delta P}{\sum_i n_{i,j,s}^{GR} \cdot \rho_i} \tag{23}$$

Moreover, we limit the amount of reserves that can be provided by each generator as:

$$R_{i,j,s}^{GR} \leq n_{i,j}(s_k) \cdot \overline{P}_i - g_{i,j}(s_k) \tag{24}$$

where $\overline{P}_i$ is defined in equation (8).[11]

To conclude this section, it is important to point out the several of the constraints above non-linear which induce a complex problem when our model is implemented in a country level expansion planning for electricity generation. In the case of convex non-linear constraints, they are linearized by using tangent planes, and for the non-convex equations we define two alternative convex linear programming models that serve as upper and lower bounds to the optimal solution. In Appendix A we describe these simplifications.

[10] In equation (22) , $\sum_i H_i \cdot n_{i,j,s} \cdot \overline{P}_i - H_f \cdot \Delta P$ is equal to the post contingency system kinetic energy.

[11] It is important to mention that the model can also analyze a contingency event by considering the use of demand side services (DSS) where customers may change the amount of energy they consume from the system after being provided with a signal. Hence, instead of analyzing a contingency event ΔP, the model also allows the analysis of the effect of DSS measures by assuming that a contingency is characterized by $\Delta P^* = \Delta P - DR_j^P(s_k)$, where $DR_j^P(s_k)$ is the amount of change in electricity consumption in hour j under scenario s_k, from a DSS perspective which can be also used as a tool for the primary frequency control.

3. Model Implementation

We implement our multi-stage model to plan the expansion in a real –country level– electricity generation system power. As mentioned in the introduction, the objective of this implementation is to provide a concrete example of the use of our model for an optimal expansion plan. This example is useful since it could be used as a guide for the implementation of our model in other countries.

We implement the model to the Chilean Central Interconnected System (CIS). In the Chilean CIS, the current energy matrix is composed by conventional renewables technologies (run of the river and large hydro reservoirs), non-conventional renewables (solar photovoltaic, wind and biomass) and fossil fuels energy (oil, liquefied natural gas and coal). As the model considers a future generation plants, we also include new forms of non-conventional renewables sources such as small hydro, geothermal and concentrated solar power.

The expansion planning is performed in a two-stage decisions problem. In the expansion planning, the first set of decisions are performed in 2015 for the construction of new generation plants which will be operative in 2025 (stage I). The second set of decision are developed in 2025 to build additional generation plants, which will be operative in 2035 (stage II). We select the years 2025 and 2035 due to several reasons. In general countries select new policy target after 10 years from the previous targets because it is necessary this period to adjust the installed energy matrix to reach the new policy goal (e.g. in Europe there is policy target of 20% and 27% share of total final energy from renewables are by years 2020 and 2030, respectively). Periods of 10 years is reasonable time to have the flexibility to build any generation plants, since some of them needs several studies of feasibility, studies of environmental impact, environmental permissions, and the construction per se.[12]

Table 1 shows investment costs, maintenance costs, current installed capacity and projected upper bounds in terms of future new generation plants per each generation technology. It is important to notice that cost values in Table 1 do not include fossil-fuel costs. Investments costs and maintenance costs in Table 1 were obtained from projections described in the report called "Escenarios Energéticos- Chile 2030". The report "Escenarios Energéticos- Chile 2030" was

[12] For instance, the International Energy Agency report that a large hydro reservoir plan can take up 7.5 years only for the construction, which starts after all studies of feasibility, studies of environmental impact, environmental permissions. See: http://www.iea-etsap.org/web/e-techds/pdf/e07-hydropower-gs-gct.pdf.

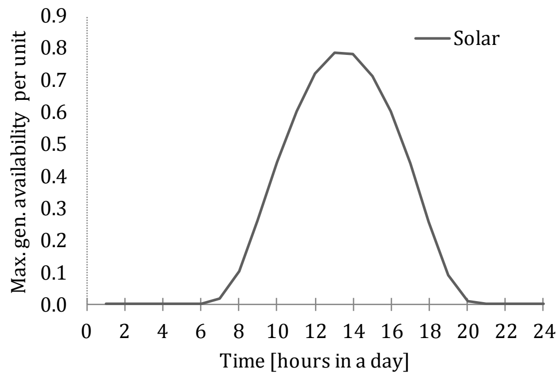
developed by a group of Chilean institutions associated with the electricity generation sector, in which academics participate, where the Chilean Ministry of Energy and the Chilean Ministry of Environmental Affairs are part of the advisory committee. In the implementation for the expansion planning of the Chilean CIS, we assume that these costs are constant between 2025 and 2075. Current installed capacities based on the different generation technologies were taken from the Chilean Association of Electricity Generators (Electricity Bulletin dated in December 2014). Hence, planning decision for 2025 takes considers the current installed capacities; while that generation planning decisions for 2035 is also conditional to the installed capacity 'planned' for 2025. Upper bound capacities reflect the maximum capacities that could be installed by both years 2025 and 2035, which were carefully chosen for each technology depending on the availability of energy sources, taking into account the reality of Chile in terms of economic conditions and development, and the number of projects approved or being studied by the Chilean Ministry of Energy.

Table 1 Investment costs, maintenance costs, and current installed capacity and higher bounds for future installed capacities

The table presents the annuitized investment cost, variable maintenance cost, lifespan, current installed capacity and maximum capacity (higher bound) for each generation technology considered in the model. Costs values do not include fossil-fuel costs. Costs values in this table are taken from research named "Energy Scenarios Chile 2030". The cost parameters are adjusted to the Chilean reality, thus they may differ with international values. Current installed capacities are taken from an Electricity Bulletin emitted by the Chilean Association of Electric Generators and are updated to December 2014. Higher bounds are carefully chosen for each technology depending on energy sources' availability and the number of projects that could be ready for operation before 2035. Biomass technology used in this study is biogas which is used in an integrated gasification combined cycle with a load factor of 85%.

	Annuitized Investment Cost [$/kW-year]	Variable Maintenance Cost [$/MWh]	Current Installed Capacity [MW]	Higher Bound [MW]
Coal	221	4,6	2394	6000
Oil	55	14,7	2303	4000
Reservoir	202	4,6	4053	10770
Wind	188	9,1	634	6150
Solar PV	132	4,4	169	4000
Liq. Nat. Gas (LNG)	93	2,9	2777	4000
Run-of-river	202	4,6	1965	6150
Biomass	241	4,3	504	1540
Geothermal	395	13,1	0	310
Small hydro	303	6,9	350	1230
Solar CSP	463	8,3	0	310

Electricity generation based on renewable energy is intermittent and cannot be completely controlled since is affected by climate conditions; thus we use generation profiles. We obtain data of profiles hour by hour for generation based on solar, wind, small hydro based on climate conditions of a representative year, which were obtained from measurements in different regions of Chile. These profiles were provided by the Chilean Ministry of Energy and University of Chile. For instance, Figure 1 reports the solar generation profile in an average day and in each month of a representative year. Figure 1 shows that between 12:00 and 14:00 hrs are the hours of the day with highest generation availability through solar generation, and in the night it does not produce any energy through this technology; while that January and December are the months with highest solar generation availability thanks to the summer in the southern hemisphere.



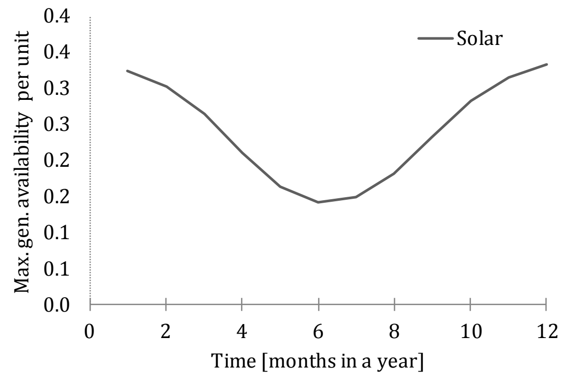
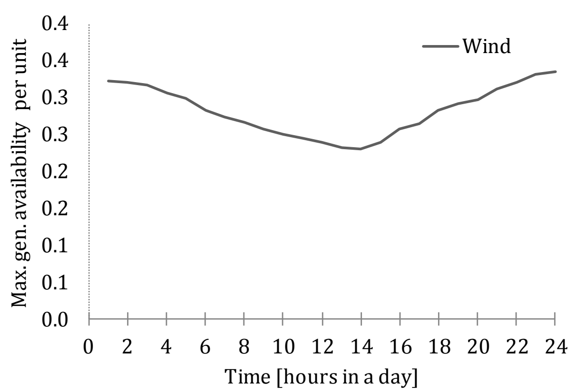


Figure 1. Solar photovoltaic generation profile in an average day and in each month of a representative year.
The left hand side of the figure shows the average profile of the maximum availability of solar photovoltaic generation per hour in an average day. The right hand side of the figure presents the average behavior of the maximum availability of solar photovoltaic generation in each month of a representative year. The solar photovoltaic profile was obtained from data provided by the Chilean Ministry of Energy and University of Chile.

Figure 2 and Figure 3 also present the generation profile in an average day and in each month of a representative year for wind generation and small hydro generation, respectively. Figure 2 report that wind generation availability is highest at night and in January, July and December. In relation to small hydro, Figure 3 shows the behavior of rivers in Chile. In Chile an important part of the rivers receive snowmelt water. Thus, the generation availability increases progressively during the day when the sun melt the snow, and it is also highest in the Chilean summer time (January and December).[13]



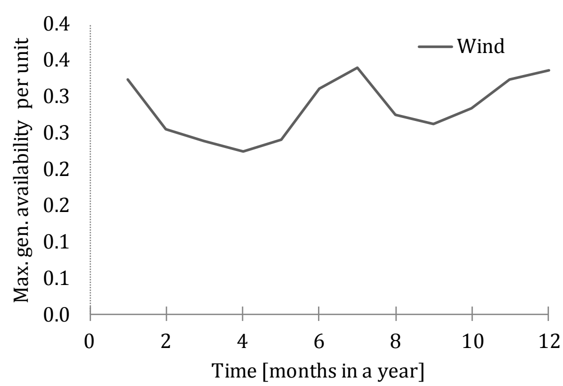


Figure 2. Wind generation profile in an average day and in each month of a representative year.
The left hand side of the figure shows the average profile of the maximum availability of wind generation per hour in an average day. The right hand side of the figure presents the average behavior of the maximum availability of wind generation in each month of a representative year. The wind generation profile was obtained from data provided by the Chilean Ministry of Energy and University of Chile.

[13] The profile of geothermal is not explained since its highest generation availability is almost constant with values around 0.85.

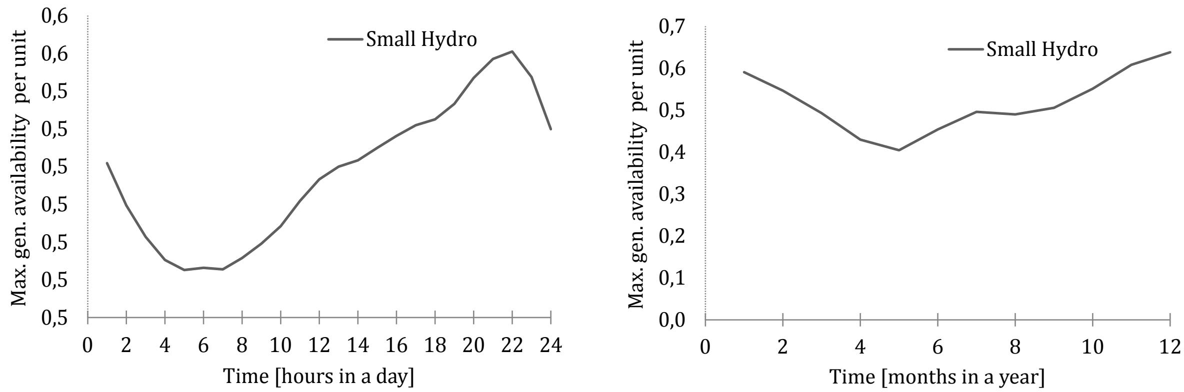


Figure 3 Small hydro generation profile in an average day and in each month of a representative year.

The left hand side of the figure shows the average profile of the maximum availability of small hydro generation per hour in an average day. The right hand side of the figure presents the average behavior of the maximum availability of small hydro generation in each month of a representative year. The small hydro generation profile was obtained from data provided by the Chilean Ministry of Energy and University of Chile.

In relation to generation plants based on both fossil-fuels, they have advantages and disadvantages. Fossil-fuel generation has as drawback the contribution to serious environmental problems due to the important level of CO2 emissions, which is responsible of pollution, global warming and acid rain. Fossil-fuel generation has also associated a financial risk due to the volatility of fossil fuel prices, which can affect directly the generation costs for the system (this price volatility will represent a source of uncertainty, as we will explain in the following paragraphs). However, fossil-fuel generation also has some advantages. Fossil-fuel generation is very effective as reserve to regulate generation in case of potential contingencies and to regulate the intermittency of generation based in renewable energy (e.g. solar or wind generation); thus they are fundamental to maintaining the security of supply electricity. In addition, some of the generations based on fossil-fuel (at present) are cheaper than other non-conventional renewable options.

Conversely, electricity generation based on renewable energy has has also pros and cons. Renewable generation has as disadvantage that the levels of electricity productions vary over time and cannot be modified since they depend of the weather conditions (see Figures 1-3). In addition, some renewable generation technologies have some higher prices than fossil-fuel technologies. Nevertheless, renewable energy production has benefits in terms of low levels of carbon emissions, which generate positive externalities to the environment. In addition, renewable generation has some properties that can be used as a hedge tool against the volatility of fossil-fuel prices. For example, Awerbuch (2006) shows evidence that an optimal allocation of technologies including renewable generation can generate a diversification effect by reducing the financial risks of the system, generated by volatility of fossil-fuel prices. Furthermore, renewable generation has hedging benefits when the demand changes. For instance, we obtain the demand profile hour by hour for a representative year from the Chilean National Commission of Energy, and we observe that it is negatively correlated to the profile of renewable technologies. Figure 4 shows the demand profile of electricity consumption in an average day and in each month of a representative year. Figure 4 shows that there is a high demand between 12:00 and 14:00 (which is the same time when there is highest solar generation availability), and the demand is also high at around 22:00 (the same time where there is highest generation availability with small hydro). In terms of the months of the year, the demand is o high in July when there a high level of wind generation, and it is also high in December when there is elevated generation availability from solar generation and small hydro generation.

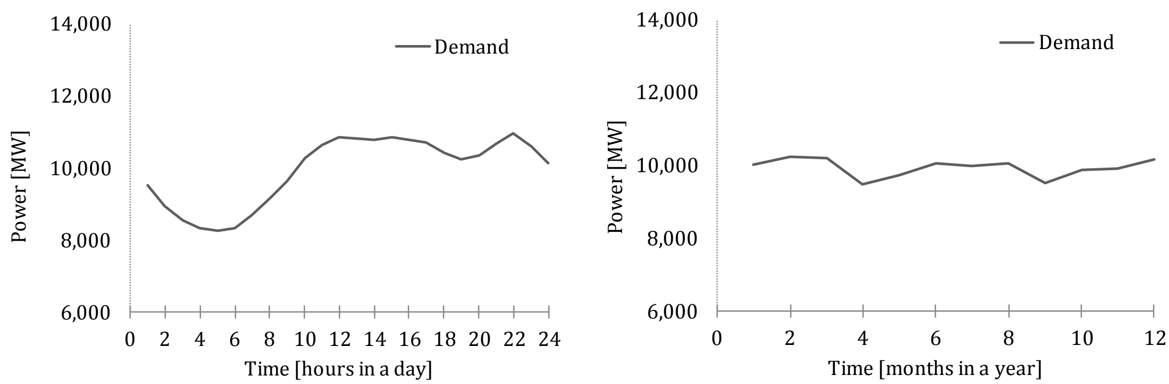


Figure 4 Demand profile of electricity consumption in an average day and in each month of a representative year.
The left hand side of the figure shows the average profile of the demand of electricity consumption per hour in an average day. The right hand side of the figure presents the average behavior of the demand of electricity consumption in each month of a representative year. The demand of electricity consumption profile was obtained the National Commission of Energy which is Associated Chilean Ministry of Energy.

The model is solved to determine optimal portfolios of generation technologies. We optimize the vector of generation capacities in a first stage, and the operation of the proposed generation infrastructure in a second stage, both of which are coordinated by a Benders-based method. The Benders decomposition algorithm allows us to tackle large dimension problems which are the case of the Chilean CISS since it is country level planning. This algorithm consists in splitting the optimization problem in two sub problems called master and slave. The master problem is designed to optimize the investment decision (i.e., generation capacities) in all stages of the optimal expansion planning. The slave problem is formulated to make optimal dispatch decisions hour by hour in the system (i.e. hourly generation per technology), depending of the hourly demand profile and the availability of renewable resources. These dispatch decisions represent the operation of the capacity imposed by the first stage. The Benders' Decomposition-based algorithm used for solving the large-scale model is described in the Appendix B.

In addition, we assume a discount rate of 10% on annual basis, and that after 2035 all generation plants will have a lifespan of 40 years. Thus, using the notation from our model $h = 10$ years, $\tau_0 = 2015$, $\tau_1 = 2025$, $\tau_2 = 2035$ and $\tau_3 = 2075$, where decisions of building new plants are made in τ_0 and τ_1, which will be operative in τ_1 and τ_2, respectively. Additional parameters used in the model implementation are reported in Appendix C.

Sources of uncertainty

In the model there are four sources of uncertainties: fossil fuel prices, hydrology conditions, demand growth rate used for demand projections, and changes in energy policies to incentive renewable generation. Though the use of Monte Carlo simulations we generate 8,000 paths to describe the potential scenarios for the years 2025 and 2035. In these 8,000 scenarios we simulate the market condition hour by hour of years 2025 and 2035, which represent 140,160,000 hours (i.e. 2*8000*365*24) of potential generation conditions that will be used to obtain each evaluation point of expansion for the Chilean CIS.

It is important to notice that in the implementation for the expansion of the Chilean CISS, our multi-stage model allows that decisions made in 2025 (which will be implemented in 2035) are conditional to the information observed in that year; thus in the second stage it is possible to adjust the expansion planning based on future information which is not currently available.

In relation to fossil-fuel price scenarios, they were generated by assuming that fossil-fuel prices follow a geometric Brownian motion:

$$P^i_{t+\Delta t} = P^i{}_t \cdot e^{(\mu_i - \frac{1}{2} \cdot \sigma_i{}^2) \cdot \Delta t + \sigma_i \cdot z^i_{t+\Delta t} \cdot \sqrt{\Delta t}}. \tag{25}$$

We simulate correlated paths by using the y the Cholesky decomposition of the correlation matrix of fossil-fuel returns.[14] The fossil-fuel scenarios are generated by using the historical descriptive statistics of fossil fuel returns, which are presented in Table 2. Historical data of fossil fuel prices to build Table 2 cover the years between 1984 and 2014, and they were obtained from the Energy Information Administration which is part of the U.S. Department of Energy.[15] We use international fossil-fuel prices instead of Chilean values, since the price of commodities are traded internationally.

Table 2 Descriptive statistics of fossil-fuel returns

The table contains statistical data used to elaborate the Monte-Carlo fossil-fuel scenarios. It is assumed that prices follows a geometric Brownian motion, thus statistical parameters of fuel price returns time series such as mean returns, standard deviation, expected price; and correlation coefficients are needed to build the price scenarios for Coal, Liquefied natural gas and Oil (fossil-fuels considered in the model). The correlated random sequences for each fossil-fuel prices are generated by Cholesky decomposition of the correlation matrix. Time series of historical international fossil-fuel prices are obtained from the Energy Information Administration of the US. The period covered is from 1984 to 2014.

	Coal	LNG	Oil
Mean Returns	2%	2%	6%
Standard Deviation Returns	12%	14%	21%
		Correlation Coefficients	
Coal	1	-	-
LNG	0.4	1	-
Oil	0.2	0.7	1

We simulate hydrological scenarios that may affect the security supply of electricity from generation from hydro reservoir and run-of-river. We use historical hydrological data to simulate scenarios, which characterized 50 years (between 1960 to March 2010) of weekly averages for each generator with hydro-reservoir, and monthly averages of water inflows for each run-of-river

[14] Suppose that the number of fossil fuels is I_F, and the Cholesky decomposition of the correlation matrix of fossil-fuel returns is Y. Suppose we generate a vector, $[\varepsilon^1 \ \ \varepsilon^2 \ ... \ \varepsilon^{I_F}]^T$, of random numbers that distribute $N(0,1)$. Then the correlated paths in equation (25) are generated by: $[z^1_{t+\Delta t} \ \ z^2_{t+\Delta t} \ ... \ z^I_{t+\Delta t}]$=Y $\cdot[\varepsilon^1 \ \ \varepsilon^2 \ ... \ \varepsilon^I]^T$.

[15] Fossil-fuel prices were converted to 2014 US dollars using the CPI of the United States.

generator. This hydrological series were obtained the Chile National Commission of. Table 3 presents summary statistics of historical hydrological events that affected the average capacity of generation produced by hydro reservoirs and run-of-river generators. This table present probabilities of occurrence of different levels of capacity factors of generation of electricity, which were used in the simulation of scenarios to characterize the uncertainty of hydro conditions.

Table 3 Descriptive statistics of historical hydrological events

The table presents summary statistics of historical hydrological events that affected the average capacity of generation produced by hydro reservoirs and run-of-river generators. The table present probabilities of occurrence of different levels of capacity factors of generation of electricity. Which were generated from historical data observed over a period of 50 years, from 1960 to 2010. The "Capacity factor" refers to the ratio of the actual output of a power plant over a period of time, to its potential output if it had operated at full nameplate capacity over the same period of time. Capacity factors vary greatly depending on the hydrological condition, thus a dry (wet) year is associated to low (high) capacity factors. The hydrological series used to build the scenarios are extracted from the National Commission of Energy which is part of the Chilean Ministry of Energy.

# of Scenario	1	2	3	4	5	6	7	8	9	10
Probability [p.u.]	0,06	0,06	0,13	0,17	0,13	0,11	0,17	0,08	0,04	0,06
Reservoir average capacity factor	16%	24%	30%	34%	41%	46%	52%	57%	63%	71%
Run-of-river average capacity factor	42%	49%	51%	47%	51%	53%	55%	58%	54%	59%

In the simulation of hydro scenarios, in the case of run-of-river generation, we generate hour by hour profiles using data of inflow data in hourly basis for a representative year (which was provided by the Chilean Ministry of Energy and University of Chile). Thus, hourly inflow data was adjusted depending of the level of run-of-river simulated (through historical simulations) in relation to the average level in the representative year. In the case of hydro-reservoir regeneration, we simulate weekly profiles using data of inflow data in weekly basis also from the year 2012. The weekly hydro-reservoir profiles were also adjusted depending of the level of generation of hydro-reservoir simulations regarding to the average level in the representative year.

We also simulate different demand growth rate conditions, to capture the uncertain of demand projections in 2025 and 2035. We assume that the demand growth rate, g, can be amplifies or reduced by an random factor α. Hence, hence the annual demand growth rate is stochastic and its value is given by αg . In our implementation for the Chilean CIS, we assume that g is equal to 5% which is based on the Chilean projections obtained from the Chilean Ministry of Energy. In the case of α we assume conservative scenarios, in which α has a discrete distribution with support $\{-1.15, -1.07, 1.00, 1.07, 1.15\}$ with a cumulative distribution function $\{0.15, 0.35, 0.65, 0.85, 1.00\}$. We use the demand profile hour by hour for a representative year from the National Commission of Energy (see Figure 4), which is adjusted depending of the growth rate to obtain projected profiles for the years 2025 and 2035.

Moreover, we also simulate potential changes in energy policy measures that may incentive renewable generation. Thus, we perform two policy exercises to capture the uncertainty of a policy change. First, we assume that a 'potential' carbon tax may be announced in 2025 with a given probability, which will be implemented in 2035. Second, we also assume that a 'potential' renewable policy target may be announced in 2025 with a given probability, which will be implemented in 2035, where if the policy target is not reached a penalty has to be paid. We provide a detailed explanation of both policy exercises in Section 4.

4 Results

4.1 Optimal planning under multi-stage decisions without policy measures

As we mentioned in the previous section, we implement our multi-stage planning model in the Chilean Central Interconnected System. Thus, it is important to note that Chile is very rich in hydrological resources due to the geographical location (i.e. beside the Andes mountain range). Thus, diverse forms of generation using hydrological resources are very convenient. In addition, the Andes also allows a high potential of geothermal generation. The long coast of Chile facing the Pacific Ocean is also favorable for wind generation; while that the Atacama Desert presents favorable for solar electricity generation. Therefore, despite that this study represent a concrete implementation of our multi-stage model, some of the results may not be necessarily the same for other countries. However, as explained before, the implementation of our model to the Chilean system power can be used as a guide for potential implementations of our approach to other regions of the world.

Table 4 reports the optimal planning obtained through our multistage model, which there are no policy measures related to the incentives of renewable generation. Therefore, the results in Table 4 represent our base case results. Table 4 shows that run-of-river generation is installed immediately to the maximum possible level in the first stage. Run-of-river generation is installed immediately because this technology is cheap, and in the worst case (in a dry year) its capacity factor of generation is reduced to only 42% (See Table 3), which is still very high compared to hydro reservoir regeneration where in a dry year the capacity factor is reduced massively to 16%.

Table 4 Optimal allocation under multi-stage decisions without policy measures

The table reports the output result of the expansion planning for electricity generation using our multi-stage model, which is implemented Chilean Central Interconnected System (CIS) without imposing any policy measure. The first set of decisions are made in 2015 to build electricity generation plants, which will be operative in 2025 (stage I). The second set of decisions are made in 2025 for the construction of generation plants which will be operative in 2035 (stage II). The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

Optimal allocation under multi-stage decisions without policy measures (Decision made today for stage I // Decision made in stage I for stage II)												
	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	74267		73284		72796		72485		72204		72137	
CVaR [MM $]	91173		92150		93150		94500		96699		98868	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity/Total Capacity (Stage I)											
Coal	2394	10,2%	2394	11,2%	2394	11,5%	2394	11,4%	2394	11,3%	2394	11,2%
Oil	2303	9,8%	2303	10,8%	2303	11,0%	2303	11,0%	2303	10,8%	2303	10,8%
Reservoir	4053	17,3%	4053	19,0%	4053	19,4%	4833	23,0%	5795	27,3%	6032	28,3%
Wind	634	2,7%	634	3,0%	634	3,0%	634	3,0%	634	3,0%	634	3,0%
Solar PV	3082	13,2%	984	4,6%	519	2,5%	169	0,8%	169	0,8%	169	0,8%
LNG	2777	11,8%	2777	13,0%	2777	13,3%	2777	13,2%	2777	13,1%	2777	13,0%
Run-of-river	6150	26,2%	6150	28,8%	6150	29,5%	6150	29,3%	6150	28,9%	6150	28,9%
Biomass	504	2,2%	504	2,4%	504	2,4%	504	2,4%	504	2,4%	504	2,4%
Geothermal	310	1,3%	310	1,5%	310	1,5%	0	0,0%	0	0,0%	0	0,0%
Small hydro	1230	5,2%	1230	5,8%	1230	5,9%	1230	5,9%	524	2,5%	350	1,6%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		24,6%		17,2%		15,3%		12,1%		8,6%		7,8%
% of Renewable (NC) Generation		24,4%		19,1%		17,9%		14,4%		10,0%		8,9%
	Installed Capacity/Total Capacity (Stage II)											
Coal	5087	12,9%	4818	12,5%	4673	12,6%	4254	11,6%	3890	10,9%	3794	10,9%
Oil	2303	5,8%	2303	6,0%	2303	6,2%	2303	6,3%	2303	6,4%	2303	6,6%
Reservoir	9812	24,8%	10266	26,7%	10724	28,8%	10770	29,4%	10770	30,1%	10770	31,0%
Wind	6150	15,6%	5034	13,1%	3489	9,4%	3358	9,2%	2803	7,8%	1887	5,4%
Solar PV	4000	10,1%	4000	10,4%	4000	10,8%	4000	10,9%	4000	11,2%	4000	11,5%
LNG	2777	7,0%	2777	7,2%	2777	7,5%	2777	7,6%	2777	7,8%	2777	8,0%
Run-of-river	6150	15,6%	6150	16,0%	6150	16,5%	6150	16,8%	6150	17,2%	6150	17,7%
Biomass	1540	3,9%	1540	4,0%	1540	4,1%	1540	4,2%	1540	4,3%	1540	4,4%
Geothermal	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,9%	310	0,9%
Small hydro	1230	3,1%	1230	3,2%	1230	3,3%	1230	3,4%	1230	3,4%	1230	3,5%
Solar CSP	145	0,4%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		33,9%		31,5%		28,4%		28,4%		27,6%		25,8%
% of Renewable (NC) Generation		34,2%		31,6%		28,6%		28%		27%		25,6%

Table 4 shows that hydro reservoir generation presents high levels of installed capacity in both stages if we observe the optimal planning with minimum cost (on the right hand side, which is the case with high risk measured by the CVaR). This is due to the fact that hydro reservoir generation is very economical in terms of expected value, though highly risky to very dry years. Hence, installed capacity of hydro reservoir generation is reduced progressively in the optimal planning expansions under lower levels risks (on the left hand side).

As there is a reduction installed capacity of hydro reservoir generation, when we move to the left side of Table 4, there is a substitution effect following priority order. The first technology to reduce risk levels instead of hydro reservoir generation is small-hydro generation. After arriving to the maximum of installed capacity of small-hydro, hydro reservoir generation generations is substituted by geothermal generation, and after that solar photo-voltaic generation is triggered. In the second stage, the demand is covered with wind generation since small-hydro, geothermal and solar photo-voltaic reach the maximum of installed. Therefore, in the first stage the priority order to replace hydro reservoir generation is by generation based on small-hydro, geothermal and then solar photo-voltaic technologies.

In the second stage, wind generation is triggered to cover the projected demand by the year 2035. Coal generation is also installed; however it has 2 purposes: to cover the increase in the projected demand, and to cover (as reserve) the intermittency of renewable generation, especially from wind and solar photovoltaic technologies. In addition, when we move to the left side of Table 4, there is also a substitution effect of hydro reservoir generation by wind generation and coal generation to reduce the levels of risk exposure to dry hydrological scenarios, and when wind generation reach the maximum capacity level (in the planning point A, which is the expansion strategy with minimum level of risk) the generation based on concentrated solar power (CSP) is triggered.

As consequence substitution effect described in the previous paragraph between hydro-reservoir generation and other renewable technologies, there are more non-conventional renewable technologies for generation planning with low level of risk (left hand side) to points with low levels of costs (right hand side). The Chilean government defines non-conventional renewable technologies to all renewable generation except large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts Chilean Non-Conventional Renewable Energy Law (Law 20.257). Electricity generation based on liquid natural gas (LNG) and oil are never triggered (in both stages) due to two reasons: i) very expensive in relation to other technologies; and ii) their price returns have a high level of volatility (see Table 2). The only fossil-fuel generation that is triggered is one based on coal, since it is cheapest fossil fuel technology and it has an annual return volatility of only 12% which is lower than generation based on liquid natural gas (14%) and oil (21%). Therefore, an optimal allocation of technologies including non-conventional renewable generation can generate a diversification effect by reducing not only the risk from uncertain hydrological scenarios, but also the financial risks of the system generated by volatility of fossil-fuel prices. Moreover, since generation based on liquid natural gas and oil are not installed despite that the demand on average has increase a 5% per year, this demand has been covered with other technologies including non-conventional renewable generation, which has a generated a reduction in the participation of fossil-fuel generation, which induce damages to the environment.

It is important to notice that the values of the installed capacity of renewable energy differ from the values of expected renewable generation (In Table 4 and in the following analyses, for some portfolios the former is higher than the later and *vice versa*); although the answer behind these differences is intuitive. In the model, we assumed initial capacities for all technologies, according to the current energy matrix in Chile. However, this assumption does not ensure that the model will decide to generate with that installed capacity. The most extreme case is the oil electricity generation (with 2303 MW of installed capacity). The model does not trigger additional capacity for this technology; furthermore, this technology is set nearly to zero for generation in every hour of the year, because it is not economically convenient –i.e. oil has the highest operating costs of all technologies. In addition, some renewable technologies cannot generate electricity since they depend on weather conditions or time-periods during the day (e.g., solar photovoltaic generation can produce energy in the hours of the day with sunlight). Both effects can produce differences between the installed capacity and expected generation of non-conventional renewable generation in the different portfolios. These considerations can also induce differences within installed capacities. For example, since oil electricity generation does not expand from its current installed capacity, it always has 2303 MW of installed capacity, however the percentage of installed capacity is different across portfolios (in the first stage, portfolio A and portfolio B have a level of 9.8% and 10.8%, respectively).

Similar to Table 4, Table 5 reports the optimal planning without renewable policy measures; however, in the optimal expansion planning is obtained with single-stage model. In the single-stage model all planning decisions (based on projections of the system) are made today and they cannot be modified. Thus, in the implementation for the expansion planning of the Chilean CIS , single-stage planning model are developed in 2015 to start building generation plants right now (which will be operative in 2025) and to start the construction of generation plants in 2025 (which will be operative in 2035). The main difference of the single-stage planning in relation to the multi-stage planning is that the former cannot modify decisions for 2025 (decisions are fixed), while that the later the expansion plan can be modified in 2025 depending of the new information received.

The results presented in Table 5 for the single-stage model has the same qualitative features of the results of the multi-stage model reported in Table 4 (i.e., substitution effects between technologies, order priority of non-conventional renewable generation, and reduction of risk exposure thanks to the use of plants based on renewable energy resources). Most importantly, Table 5 shows that, even in a scenario without changes in policy measures, our multi-stage planning model provide important economic benefits in term of drop in costs and risks. Costs and risk reduction are observed since updated information in 2025 about regarding fossil-fuel prices and demand growth is used in the optimal planning for stage II. The economic benefits in terms of costs and risk reduction can be observed in Figure 5, which describe different optimal allocations without renewable policy measures of plants depending of risk levels under multi-stage decisions (results from Table 5) and single-stage decisions (results from Table 5).

Table 5 Optimal allocation under one-stage decisions without policy measures

The table reports the output result of the expansion planning for electricity generation using a one-stage model, which is implemented Chilean Central Interconnected System (CIS) without imposing any policy measure. In the one-stage planning model, all decision is made in 2015 which includes to build generation plants right now (which will be operative in 2025) and to build generation plants in 2025 (which will be operative in 2035). The main difference of the single-stage planning in relation to the multi-stage planning is that the former cannot modify decisions for 2025 (decisions are fixed), while that the later can modify decision in 2025 depending of the new information received. The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

	Optimal allocation under one-stage decisions without policy measures (Decision made today for stage I and stage II)											
	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	75605		73998		73199		72671		72308		72171	
CVaR [MM $]	91173		91650		92650		94499		96999		99000	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity/Total Capacity (Stage I)											
Coal	2394	10,2%	2394	10,6%	2394	11,5%	2394	11,4%	2394	11,3%	2394	11,3%
Oil	2303	9,8%	2303	10,2%	2303	11,0%	2303	11,0%	2303	10,8%	2303	10,8%
Reservoir	4053	17,3%	4053	17,9%	4053	19,4%	4833	23,0%	5795	27,3%	5795	27,3%
Wind	634	2,7%	634	2,8%	634	3,0%	634	3,0%	634	3,0%	634	3,0%
Solar PV	4000	17,1%	3177	14,0%	519	2,5%	169	0,8%	169	0,8%	169	0,8%
LNG	2777	11,8%	2777	12,3%	2777	13,3%	2777	13,2%	2777	13,1%	2777	13,1%
Run-of-river	6150	26,2%	6150	27,2%	6150	29,5%	6150	29,3%	6150	28,9%	6150	28,9%
Biomass	800	3,4%	504	2,2%	504	2,4%	504	2,4%	504	2,4%	504	2,4%
Geothermal	310	1,3%	310	1,4%	310	1,5%	0	0,0%	0	0,0%	0	0,0%
Small hydro	1230	5,2%	1230	5,4%	1230	5,9%	1230	5,9%	524	2,5%	524	2,5%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		29,7%		25,8%		15,3%		12,1%		8,6%		8,6%
% of Renewable (NC) Generation		29,2%		24,6%		17,9%		14,4%		10,0%		10,0%
	Installed Capacity/Total Capacity (Stage II)											
Coal	5714	14,5%	4686	12,1%	4182	11,1%	4135	11,0%	3851	10,7%	3971	11,6%
Oil	2303	5,8%	2303	5,9%	2303	6,1%	2303	6,1%	2303	6,4%	2303	6,7%
Reservoir	10770	27,3%	10752	27,8%	10747	28,4%	10770	28,7%	10770	29,9%	10770	31,4%
Wind	6150	15,6%	6150	15,9%	6150	16,3%	4357	11,6%	3124	8,7%	1257	3,7%
Solar PV	4000	10,1%	4000	10,3%	4000	10,6%	4000	10,6%	4000	11,1%	4000	11,7%
LNG	2777	7,0%	2777	7,2%	2777	7,3%	2777	7,4%	2777	7,7%	2777	8,1%
Run-of-river	6150	15,6%	6150	15,9%	6150	16,3%	6150	16,4%	6150	17,1%	6150	17,9%
Biomass	1540	3,9%	1540	4,0%	1540	4,1%	1540	4,1%	1540	4,3%	1540	4,5%
Geothermal	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,9%	310	0,9%
Small hydro	1230	3,1%	1230	3,2%	1230	3,3%	1230	3,3%	1230	3,4%	1230	3,6%
Solar CSP	310	0,8%	1	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		34,3%		34,1%		35,0%		30,4%		28,3%		24,3%
% of Renewable (NC) Generation		34,7%		33,7%		33,7%		30%		28%		24%

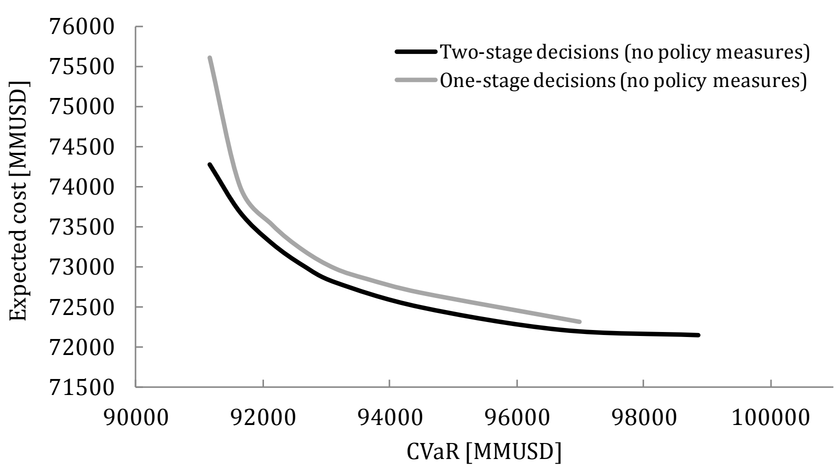


Figure 5 Efficient frontier formed by different optimal allocations of plants depending of risk levels under multi-stage decisions and single-stage decisions (without policy measures). The figure reports the efficient frontier composed by different optimal planning setups depending risk levels, and without any policy measure. The table reports the output result of the expansion planning for electricity generation using our multi-stage model (values from Table 4) and using a single stage model (data from Table 5).

Figure 5 shows the economic benefits of our multi-stage, since there are important reductions in costs for the different levels of risk proxied by the CVaR. Nevertheless, when we observe point on the right hand side of the figure (with high levels of risk but with lowest expected costs), the difference in expected costs decrease between the multi-stage and the single stage planning models. This table shows that the flexibility of the model, even without a possible change politics plays no small role and reaffirms the importance of good planning considering the information that can be obtained in time. Figure 5 also shows that on the left hand side of the figure, the curves are almost flat for both models. This means that the risk of the system can be can be reduced dramatically by just increasing by a little amount the costs with almost no cost increase.

4.2 Optimal allocation under multi-stage decisions with a potential carbon tax

The high level of CO2 due to emissions from fossil fuel combustion is one of the main causes of global warming, which has generated international concern. The climate change has challenging the principles that have guided policymakers, who have turned to induce incentives to renewable energy production. For example, in 2000 the UK government set a target of 10% of electricity supply coming from renewable energy by 2010. After that, in 2008 the European Union committed itself to generating 20% of its energy from renewable sources by 2020. As part of this commitment by 2020, the UK also changed the energy policy to incentive renewable generation to be in line with the European promise.

To promote a sustainable and clean form of energy by encouraging the development of low carbon-emission technologies, some countries have implemented economic incentives such as a carbon taxes or penalties when targets are not reached. An expansion planning for electricity generation should also consider 'future' changes in renewable policy targets, which can be reflected in a future complete new renewable policy target. Thus, we perform a policy exercise to analyze the risk of a change in a renewable policy measure in terms of a 'potential' future carbon tax.

Suppose that with probability $p(s^{q}_{k^*+1})$ in the stage k^* there is an announcement of a renewable policy measure that will be implemented in the stagek^*+1, which reflected in the future states$s^{q}_{k^*+1}$. The potential renewable policy measure is related to a carbon tax of q dollars per tones of CO2. Thus, we re-define the operation and maintenance cost ($\mathrm{VOM}_{i,s}$) variable in order to include a carbon penalization,

$$V\tilde{O}M_i(s_{k^*+1}) = \begin{cases} VOM_i(s_{k^*+1}) + q \cdot em_i & s_{k^*+1} = s^{q}_{k^*+1} \\ VOM_i(s_{k^*+1}) & s_{k^*+1} \neq s^{q}_{k^*+1} \end{cases} \tag{26}$$

Where em_i denotes the carbon emission factor for each technology [tonCO_2/MWh]. By simply replacing $VOM_i(s_{k^*+1})$ with $V\tilde{O}M_i(s_{k^*+1})$, we can incorporate the effect of a carbon tax into the model.

In the implementation to the Chilean CIS, we assume that q is equal to 10 dollars per tones of CO2 produced, and the probability $p(s^{q}_{2})$ of this announcement in 2025 is equal to an 50%. Regarding the carbon emission factors, Table 6 shows the average levels per each technology.

Table 6 Carbon dioxide equivalent emission factors

The table reports the carbon dioxide equivalent emission factors. “Carbon dioxide equivalent” is a term for describing different greenhouse gases in a common unit. For any quantity and type of greenhouse gas, CO_2e means the amount of CO_2 which would have the equivalent environmental impact. Emission factors are related to the technology used to generate electricity from each energy source indicated in the table. The factors shown in the Table are multiplied by the corresponding potential carbon tax [\$/TCO_2e], resulting in extra operating costs which are added to the Variable Operation and Maintenance Costs.

Energy Source	Technology	CO2e Emission factors [TCO_2e/GWh]
Coal	Pulverized Combustion	949
Oil	Diesel fuel	779
LNG	Combined Cycle Gas Turbine	436
Solar PV	Photovoltaic	48
Geothermal	Hydrothermal (steam turbine)	28
Biomass	Integrated gasification combined cycle	24
Solar CSP	Concentrated Solar Power	20
Wind	Onshore wind turbine	11
Reservoir	Conventional (dams)	7
Run-of-river	Run of the river	4
Small hydro	Hydroelectric power < 40MW	4

Table 7 reports the optimal planning obtained through our multistage model, where there is an additional uncertainty generated by a potential carbon tax. Table 7 has a similar qualitative behavior of Table 4; in fact the optimal planning for the first stage is exactly the same. The effect of the potential carbon tax is observed mainly in stage 2 (when the tax may be applied). The potential policy tax generates a direct impact on coal generation, which is reflected in a substitution effect of coal plants by wind generation. For instance, in the point F (minimum cost expansion planning) the amount of coal generation is reduced in relation to Table 4 while that wind generation increase. Nevertheless, coal generation is not reduced drastically, since an important part of the electricity plants based on coal help as reserved to maintain the stability of the system due to the intermittent production that characterized non-conventional electricity generation.

Table 7 Optimal allocation under multi-stage decisions with a potential carbon tax

The table reports the output result of the expansion planning for electricity generation using our multi-stage model, which is implemented Chilean Central Interconnected System (CIS) with a potential carbon tax of 10 dollars per tones of CO2. This carbon tax is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. The first set of decisions are made in 2015 to build electricity generation plants, which will be operative in 2025 (stage I). The second set of decisions is made in 2025 for the construction of generation plants which will be operative in 2035 (stage II). The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

	Optimal allocation under multi-stage decisions with a potential carbon tax (Decision made today for stage I // Decision made in stage I for stage II)											
	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	74620		73429		72987		72753		72601		72552	
CVaR [MM $]	91829		93200		94600		95999		97498		99108	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity/Total Capacity (Stage I)											
Coal	2394	10,3%	2394	11,5%	2394	11,4%	2394	11,3%	2394	11,3%	2394	11,2%
Oil	2303	9,9%	2303	11,0%	2303	11,0%	2303	10,9%	2303	10,8%	2303	10,8%
Reservoir	4053	17,5%	4053	19,4%	4833	23,0%	5420	25,6%	5795	27,3%	6031	28,3%
Wind	634	2,7%	634	3,0%	634	3,0%	634	3,0%	634	3,0%	634	3,0%
Solar PV	2864	12,3%	519	2,5%	169	0,8%	169	0,8%	169	0,8%	169	0,8%
LNG	2777	12,0%	2777	13,3%	2777	13,2%	2777	13,1%	2777	13,1%	2777	13,0%
Run-of-river	6150	26,5%	6150	29,5%	6150	29,3%	6150	29,1%	6150	28,9%	6150	28,9%
Biomass	504	2,2%	504	2,4%	504	2,4%	504	2,4%	504	2,4%	504	2,4%
Geothermal	310	1,3%	310	1,5%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
Small hydro	1230	5,3%	1230	5,9%	1230	5,9%	799	3,8%	524	2,5%	350	1,6%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		23,9%		15,3%		12,1%		10,0%		8,6%		7,8%
% of Renewable (NC) Generation		23,8%		17,9%		14,4%		11,7%		10,0%		8,9%
	Installed Capacity/Total Capacity (Stage II)											
Coal	5116	12,8%	4599	12,0%	4203	11,2%	3806	10,3%	3703	10,3%	3527	10,0%
Oil	2303	5,8%	2303	6,0%	2303	6,1%	2303	6,2%	2303	6,4%	2303	6,5%
Reservoir	10306	25,8%	10744	27,9%	10770	28,7%	10770	29,2%	10770	30,0%	10770	30,4%
Wind	6103	15,3%	4797	12,5%	4235	11,3%	4000	10,8%	3171	8,8%	2830	8,0%
Solar PV	4000	10,0%	4000	10,4%	4000	10,7%	4000	10,8%	4000	11,1%	4000	11,3%
LNG	2777	6,9%	2777	7,2%	2777	7,4%	2777	7,5%	2777	7,7%	2777	7,8%
Run-of-river	6150	15,4%	6150	16,0%	6150	16,4%	6150	16,7%	6150	17,1%	6150	17,4%
Biomass	1540	3,8%	1540	4,0%	1540	4,1%	1540	4,2%	1540	4,3%	1540	4,3%
Geothermal	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,9%	310	0,9%
Small hydro	1230	3,1%	1230	3,2%	1230	3,3%	1230	3,3%	1230	3,4%	1230	3,5%
Solar CSP	145	0,4%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		33,3%		30,9%		30,2%		30,0%		28,5%		28,0%
% of Renewable (NC) Generation		34,1%		31,1%		30,0%		29,6%		28,0%		27,4%

Similar to Table 5, Table 8 reports the optimal planning single-stage model with a potential carbon tax provides important economic benefits in term of drop in costs and risks. As when we compared Table 4 and Table 5, the multi-stage planning model important economic benefits in term of drop in costs and risks in relation to a single stage model. Nevertheless, the economic benefits are larger when the uncertainty of a policy tax is included. For instance, for a level of CVaR of USD$95,500,000,000 the cost saved thanks to the multistage model in relation to the single stage model without any renewable policy measure (with a potential carbon tax) is USD$132,000,000 (USD$186,000,000) which reflect additional savings of 41% when there is an additional uncertainty coming from a potential carbon tax.

Table 8 Optimal allocation under one-stage decisions with a potential carbon tax

The table reports the output result of the expansion planning for electricity generation using a one-stage model, which is implemented Chilean Central Interconnected System (CIS) with a potential carbon tax of 10 dollars per tones of CO2. This carbon tax is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. In the one-stage planning model, all decision is made in 2015 which includes to build generation plants right now (which will be operative in 2025) and to build generation plants in 2025 (which will be operative in 2035). The main difference of the single-stage planning in relation to the multi-stage planning is that the former cannot modify decisions for 2025 (decisions are fixed), while that the later can modify decision in 2025 depending of the new information received. The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

Optimal allocation under one-stage decisions with a potential carbon tax
(Decision made today for stage I and stage II)

	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	75748		73583		73118		72889		72727		72631	
CVaR [MM $]	91829		93200		94600		95999		97498		100035	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity/Total Capacity (Stage I)											
Coal	2394	10,3%	2394	11,5%	2394	11,4%	2394	11,3%	2394	11,3%	2394	11,2%
Oil	2303	9,9%	2303	11,0%	2303	11,0%	2303	10,9%	2303	10,8%	2303	10,8%
Reservoir	4053	17,5%	4053	19,4%	4833	23,0%	5519	26,1%	5791	27,3%	6031	28,3%
Wind	634	2,7%	634	3,0%	634	3,0%	634	3,0%	634	3,0%	634	3,0%
Solar PV	4000	17,2%	568	2,7%	169	0,8%	169	0,8%	169	0,8%	169	0,8%
LNG	2777	12,0%	2777	13,3%	2777	13,2%	2777	13,1%	2777	13,1%	2777	13,0%
Run-of-river	6150	26,5%	6150	29,5%	6150	29,3%	6150	29,1%	6150	28,9%	6150	28,9%
Biomass	681	2,9%	504	2,4%	504	2,4%	504	2,4%	504	2,4%	504	2,4%
Geothermal	310	1,3%	310	1,5%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
Small hydro	1230	5,3%	1230	5,9%	1230	5,9%	726	3,4%	527	2,5%	350	1,6%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		29,5%		15,5%		12,1%		9,6%		8,6%		7,8%
% of Renewable (NC) Generation		28,2%		18,1%		14,4%		11,2%		10,0%		8,9%
	Installed Capacity/Total Capacity (Stage II)											
Coal	5652	14,1%	4255	11,1%	3628	9,7%	3279	8,9%	3693	10,3%	3907	11,0%
Oil	2303	5,8%	2303	6,0%	2303	6,1%	2303	6,2%	2303	6,4%	2303	6,5%
Reservoir	10770	26,9%	10770	28,0%	10770	28,7%	10770	29,2%	10770	30,0%	10770	30,4%
Wind	6150	15,4%	6150	16,0%	6093	16,2%	5891	16,0%	3664	10,2%	1481	4,2%
Solar PV	4000	10,0%	4000	10,4%	4000	10,7%	4000	10,8%	4000	11,1%	4000	11,3%
LNG	2777	6,9%	2777	7,2%	2777	7,4%	2777	7,5%	2777	7,7%	2777	7,8%
Run-of-river	6150	15,4%	6150	16,0%	6150	16,4%	6150	16,7%	6150	17,1%	6150	17,4%
Biomass	1540	3,8%	1540	4,0%	1540	4,1%	1540	4,2%	1540	4,3%	1540	4,3%
Geothermal	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,9%	310	0,9%
Small hydro	1230	3,1%	1230	3,2%	1230	3,3%	1230	3,3%	1230	3,4%	1230	3,5%
Solar CSP	310	0,8%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		33,9%		34,4%		35,1%		35,2%		29,9%		24,1%
% of Renewable (NC) Generation		34,7%		33,7%		33,6%		33,2%		29,0%		24,8%

The behavior of the single stage model is very interesting. Table 8 shows that the substitution effect there is a reduction in the installed capacity of coal and increase in the wind generation However, the amount in which the capacity of coal reduced is smaller than the capacity of wind installed. This, the model install more wind generation 'just in case' which is only used if the carbon tax is implemented. Thus a single stage model generates an inefficient solution, which is improved by a multistage model in which decision can be adjusted. The economic benefits in terms of costs and risk reduction can be observed in Figure 6, which describe different optimal allocations of plants with a potential policy depending of risk levels under multi-stage decisions (results from Table 7) and single-stage decisions (results from Table 8).

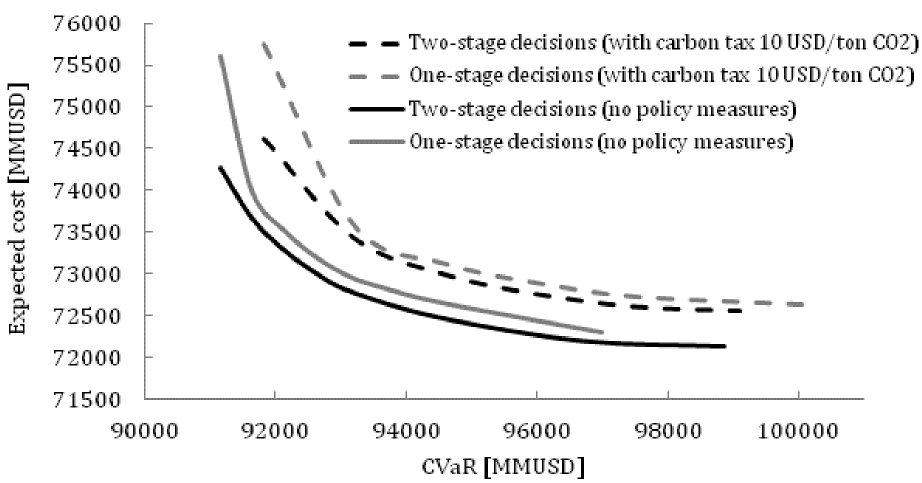


Figure 6 Efficient frontier formed by different optimal allocations of plants depending of risk levels under multi-stage decisions and single-stage decisions (with a potential carbon tax).

The figure reports the efficient frontier composed by different optimal planning setups depending risk levels, with a potential carbon tax of 10 dollars per tones of CO2. This carbon tax is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. The table reports the output result of the expansion planning for electricity generation using our multi-stage model (values from Table 7) and using a single stage model (data from Table 8).

4.3 Optimal allocation under multi-stage decisions with a potential renewable policy target for the year 2035

In responding to climate change and in order to reduce the share of fossil-fuel generation, several governments have enacted (or have changed the level of previous) renewable energy support policies. For instance in Chile, the country of implementation of our multistage model, a change of a previous policy target occurred recently. In 2008, within the Non-Conventional Renewable Energy Law (Law 20.257), the first renewable energy target was established. The goal of this law was to achieve 10% of non-conventional renewable generation by 2024 (non-conventional renewable generation includes all renewable technologies, except large hydro reservoirs and run-of-river with installed capacity larger than 40 megawatts). The subsequent government stressed the importance of further renewable energy production and replaces the figure '10% by 2024' in Law 20.257 with a new requirement of '20% by 2025'.

Therefore, we can use our multi-stage model to evaluate the flexibility benefits of postponing some decisions given the uncertainty of imposing a new policy target. Suppose that with a probability $p(s^{x}_{k^{\diamond}+1})$ in the stage $k^{\diamond}$ there is an announcement of a renewable policy measure that will be implemented in the stage $k^{\diamond}+1$, which reflected in the future state $s^{x}_{k^{\diamond}+1}$. The potential renewable policy measure is concerning renewable policy target of $x_{k^{\diamond}+1}$ proportion of share of total electricity generation that will be generated from renewables by the stage $k^{\diamond}+1$.

$$X(s_{k^{\diamond}+1}) = \begin{cases} pen\left(x_{k^{\diamond}+1} \cdot \sum_{j\in J} D_j(s_{k^{\diamond}+1}) - \sum_{i\in I^{NCR}} \sum_{j\in J} g_{i,j}(s_{k^{\diamond}+1}) \right) & x_{k^{\diamond}+1} > \frac{\sum_{i\in I^{NCR}}\sum_{j\in J} g_{i,j}(s_{k^{\diamond}+1})}{\sum_{j\in J} D_j(s_{k^{\diamond}+1})} \\ 0 & x_{k^{\diamond}+1} \le \frac{\sum_{i\in I^{NCR}}\sum_{j\in J} g_{i,j}(s_{k^{\diamond}+1})}{\sum_{j\in J} D_j(s_{k^{\diamond}+1})} \end{cases} \quad (27)$$

Where pen is a penalty charged in each hour per megawatt which is not generated through non-conventional renewables, while the I^{NCR} is a subset of the electricity generation technologies ($I^{NCR} \subseteq I$) called 'renewables' (or in the Chilean case, non-conventional renewables). The present value of cost of the system, $\tilde{C}(s_{k^{\diamond}+1})$, considering penalties when the policy target is not reached, $X(s_{k^{\diamond}+1})$, for the period $[\tau_{k^{\diamond}+1}, \tau_{k^{\diamond}+2}]$ facing the scenario $s_{k^{\diamond}+1}$ is given by:

$$\tilde{C}(s_{k^\diamond+1}) = \sum_{t=1}^{h} \frac{1}{(1+r)^t} \Bigg[\sum_{i\in I} INV_i(s_{k^\diamond+1}) \cdot cap_i(s_{k^\diamond+1}) + \sum_{i\in I}\sum_{j\in J} VOM_i(s_{k^\diamond+1}) \cdot g_{i,j}(s_{k^\diamond+1}) + \sum_{j\in J} D_j^-(s_{k^\diamond+1}) \cdot dc^- + \sum_{j\in J} D_j^+(s_{k^\diamond+1}) \cdot dc^+ + voll \cdot \sum_{j\in J} LL_j(s_{k^\diamond+1}) + X(s_{k^\diamond+1}) \Bigg] \quad (28)$$

Equation (28) present the total costs including the economic incentives to reach the goal of the renewable policy target. This equation (28) can be used instead of equation (2) in the multi-stage planning model.

In the implementation for the Chilean CIS, as mentioned above in this section, the current renewable generation target is 20% by 2025, but the authorities 'may' suddenly announced a new renewable policy target (e.g., a new policy target by 2035, 10 years after the current policy target). Thus, we perform an exercise in which a new policy target will be announced in 2025 with a level of one third of share of total electricity production will be generated from non-conventional renewables by 2035. This policy target is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. Since this policy exercise concerns policy target, we also impose the Chilean policy target of 20% share of total electricity generation from non-conventional renewables by 2025 (which is already known with certainty by 2015). In the case that any of these targets is not reached, a penalty of 50 dollars is charged per megawatt which is not generated through non-conventional renewables in each hour of a complete year. Therefore, in equation (27), x_1 is equal to 20%, x_2 is equal to 33%, $p(s_1^x)$ is equal to 100% and $p(s_2^x)$ is equal to 50% and pen is equal to 50 dollars per megawatt which is not generated through non-conventional renewables in each hour of a complete year.

Table 9 reports the optimal planning obtained through our multistage model, where there is an additional uncertainty generated by a potential policy target. As explained in Table 4, Table 9 shows that the first policy target (which is known by 2015) is reached in the first stage following the same priority order: firstly, small-hydro generation; secondly, geothermal generation; and thirdly, solar photo-voltaic generation. Then in the second stage, wind generation is installed largely in relation to Table 4, and thus to reach the potential scenarios with the policy target. In addition, some coal is also installed in relation to the stage one which is used mainly as reserve for the intermittently generation produced by plants based on non-conventional renewable energy. It is important to notice that in Table 9, the percentage of non-conventional renewable generation is below the target of 33%, because the multi-stage model only installs additional non-conventional technologies only in the cases when the policy target is implemented in 2035.

Table 9 Optimal allocation under multi-stage decisions with a new potential policy target

The table reports the output result of the expansion planning for electricity generation using our multi-stage model, which is implemented Chilean Central Interconnected System (CIS) with a new policy target of 33% share of total electricity generation from non-conventional renewables by 2035. This policy target is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. Since this policy exercise concerns policy target, we also impose the Chilean policy target of 20% share of total electricity generation from non-conventional renewables by 2025 (which is already known with certainty by 2015). In the case that any of these targets is not reached, a penalty of 50 dollars is charged per megawatt which is not generated through non-conventional renewables in each hour. The first set of decisions are made in 2015 to build electricity generation plants, which will be operative in 2025 (stage I). The second set of decisions is made in 2025 for the construction of generation plants which will be operative in 2035 (stage II). The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

	Optimal allocation under multi-stage decisions with a potential renewable policy target (Decision made today for stage I // Decision made in stage I for stage II)											
	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	75247		73405		72984		72818		72720		72650	
CVaR [MM $]	90747		92000		93000		93999		94999		97326	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity (Stage I)											
Coal	2394	9.8%	2394	11.0%	2394	11.0%	2394	11.0%	2394	11.0%	2394	11.0%
Oil	2303	9.5%	2303	10.6%	2303	10.6%	2303	10.6%	2303	10.6%	2303	10.6%
Hydro	4053	16.6%	4053	18.6%	4053	18.7%	4053	18.7%	4053	18.7%	4053	18.7%
Wind	634	2.6%	634	2.9%	634	2.9%	634	2.9%	634	2.9%	634	2.9%
Solar PV	4000	16.4%	1387	6.4%	1341	6.2%	1341	6.2%	1341	6.2%	1341	6.2%
LNG	2777	11.4%	2777	12.8%	2777	12.8%	2777	12.8%	2777	12.8%	2777	12.8%
Run-of-river	6150	25.3%	6150	28.3%	6150	28.3%	6150	28.3%	6150	28.3%	6150	28.3%
Biomass	504	2.1%	504	2.3%	504	2.3%	504	2.3%	504	2.3%	504	2.3%
Geothermal	310	1.3%	310	1.4%	310	1.4%	310	1.4%	310	1.4%	310	1.4%
Small hydro	1230	5.1%	1230	5.7%	1230	5.7%	1230	5.7%	1230	5.7%	1230	5.7%
Solar CSP	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
% of Renewable (NC) Energy Installed		27.4%		18.7%		18.5%		18.5%		18.5%		18.5%
% of Renewable (NC) Generation		26.6%		20.1%		20.0%		20.0%		20.0%		20.0%
	Installed Capacity (Stage II)											
Coal	5834	14.2%	4706	12.2%	4262	11.3%	3808	10.2%	3612	9.8%	3244	9.0%
Oil	2303	5.6%	2303	6.0%	2303	6.1%	2303	6.2%	2303	6.3%	2303	6.4%
Hydro	10369	25.3%	10036	26.0%	10427	27.6%	10466	28.1%	10471	28.5%	10770	29.8%
Wind	6150	15.0%	5581	14.4%	4725	12.5%	4653	12.5%	4297	11.7%	3836	10.6%
Solar PV	4000	9.8%	4000	10.4%	4000	10.6%	4000	10.7%	4000	10.9%	4000	11.1%
LNG	2777	6.8%	2777	7.2%	2777	7.4%	2777	7.5%	2777	7.6%	2777	7.7%
Run-of-river	6150	15.0%	6150	15.9%	6150	16.3%	6150	16.5%	6150	16.8%	6150	17.0%
Biomass	1540	3.8%	1540	4.0%	1540	4.1%	1540	4.1%	1540	4.2%	1540	4.3%
Geothermal	310	0.8%	310	0.8%	310	0.8%	310	0.8%	310	0.8%	310	0.9%
Small hydro	1230	3.0%	1230	3.2%	1230	3.3%	1230	3.3%	1230	3.4%	1230	3.4%
Solar CSP	299	0.7%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
% of Renewable (NC) Energy Installed		33.0%		32.8%		31.3%		31.5%		31.0%		30.2%
% of Renewable (NC) Generation		34.7%		32.6%		31.0%		30.8%		30.2%		29.3%

Similar to Table 5, Table 10 reports the optimal planning single-stage model with a potential policy target provide important economic benefits in term of drop in costs and risks. However, the economic benefits are larger when the uncertainty of a policy tax is included. For instance, for a level of CVaR of USD$95,500,000,000 the cost saved thanks to the multistage model in relation to the single stage model without any renewable policy measure (with a potential policy target) is USD$132,000,000 (USD$163,000,000) which reflect additional savings of 17% when there is an additional uncertainty coming from a potential carbon tax.

Table 10 shows that is not optimal to have a level below the policy target due to the penalty imposed; for all potential planning (from point A to point F) the policy target is reached. For instance, in the second stage in point F (minimum cost point); there is a massive increase in the capacity of installed wind generation to cover in the potential scenario of an implementation of the policy target by 2035.

Table 10 Optimal allocation under one-stage decisions with a new potential policy target

The table reports the output result of the expansion planning for electricity generation using a one-stage model, which is implemented Chilean Central Interconnected System (CIS) with a new policy target of 33% share of total electricity generation from non-conventional renewables by 2035. This policy target is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. Since this policy exercise concerns policy target, we also impose the Chilean policy target of 20% share of total electricity generation from non-conventional renewables by 2025 (which is already known with certainty by 2015). In the case that any of these targets is not reached, a penalty of 50 dollars is charged per megawatt which is not generated through non-conventional renewables in each hour. In the one-stage planning model, all decision is made in 2015 which includes to build generation plants right now (which will be operative in 2025) and to build generation plants in 2025 (which will be operative in 2035). The main difference of the single-stage planning in relation to the multi-stage planning is that the former cannot modify decisions for 2025 (decisions are fixed), while that the later can modify decision in 2025 depending of the new information received. The table shows the optimal allocation (installed capacity in megawatts) of six different portfolios, named by letters from A to F, constrained by different levels of risk in terms of the CVaR (measured in millions of US dollars). Risk increases from left to right and it is associated with the fossil-fuel price volatility, uncertain hydrological scenarios and changes in the growth rates for projected demands. Percentage of renewable energy installed and renewable generation indicated for each portfolio in the last rows of the table do not consider hydro and run of river technologies since in Chile they are considered as conventional sources of renewable energy.

Optimal allocation under one-stage decisions with a potential renewable policy target (Decision made today for stage I and stage II)												
	Portfolios											
	MIN CVAR										MIN COST	
	A		B		C		D		E		F	
Expected Cost [MM $]	75528		73624		73100		72938		72860		72798	
CVaR [MM $]	91173		92000		93000		94003		95000		96943	
	MW	%	MW	%	MW	%	MW	%	MW	%	MW	%
	Installed Capacity/Total Capacity (Stage I)											
Coal	2394	9,7%	2394	10,6%	2394	11,0%	2394	11,0%	2394	11,0%	2394	11,0%
Oil	2303	9,4%	2303	10,2%	2303	10,6%	2303	10,6%	2303	10,6%	2303	10,6%
Reservoir	4053	16,5%	4053	18,0%	4053	18,7%	4053	18,7%	4053	18,7%	4053	18,7%
Wind	634	2,6%	634	2,8%	634	2,9%	634	2,9%	634	2,9%	634	2,9%
Solar PV	4000	16,2%	2140	9,5%	1341	6,2%	1341	6,2%	1341	6,2%	1341	6,2%
LNG	2777	11,3%	2777	12,3%	2777	12,8%	2777	12,8%	2777	12,8%	2777	12,8%
Run-of-river	6152	25,0%	6152	27,3%	6152	28,4%	6152	28,4%	6152	28,4%	6152	28,4%
Biomass	773	3,1%	504	2,2%	504	2,3%	504	2,3%	504	2,3%	504	2,3%
Geothermal	310	1,3%	310	1,4%	310	1,4%	310	1,4%	310	1,4%	310	1,4%
Small hydro	1230	5,0%	1230	5,5%	1230	5,7%	1230	5,7%	1230	5,7%	1230	5,7%
Solar CSP	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		28,2%		21,4%		18,5%		18,5%		18,5%		18,5%
% of Renewable (NC) Generation		29,0%		22,0%		20,0%		20,0%		20,0%		20,0%
	Installed Capacity/Total Capacity (Stage II)											
Coal	5651	13,7%	4480	11,3%	3921	10,1%	3447	9,0%	3128	8,2%	2691	7,2%
Oil	2303	5,6%	2303	5,8%	2303	5,9%	2303	6,0%	2303	6,1%	2303	6,1%
Reservoir	10770	26,1%	10751	27,1%	10770	27,8%	10770	28,1%	10770	28,3%	10770	28,7%
Wind	6150	14,9%	6150	15,5%	5797	14,9%	5797	15,1%	5797	15,3%	5797	15,4%
Solar PV	4000	9,7%	4000	10,1%	4000	10,3%	4000	10,4%	4000	10,5%	4000	10,6%
LNG	2777	6,7%	2777	7,0%	2777	7,2%	2777	7,2%	2777	7,3%	2777	7,4%
Run-of-river	6152	14,9%	6152	15,5%	6152	15,9%	6152	16,1%	6152	16,2%	6152	16,4%
Biomass	1540	3,7%	1540	3,9%	1540	4,0%	1540	4,0%	1540	4,1%	1540	4,1%
Geothermal	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,8%	310	0,8%
Small hydro	1230	3,0%	1230	3,1%	1230	3,2%	1230	3,2%	1230	3,2%	1230	3,3%
Solar CSP	310	0,8%	0	0,0%	0	0,0%	0	0,0%	0	0,0%	0	0,0%
% of Renewable (NC) Energy Installed		32,9%		33,3%		33,2%		33,6%		33,9%		34,3%
% of Renewable (NC) Generation		34,7%		33,7%		33,0%		33,0%		33,0%		33,0%

The economic benefits in terms of costs and risk reduction can be observed in Figure 7, which describe different optimal allocations of plants with a potential policy target depending of risk levels under multi-stage decisions (results from Table 9) and single-stage decisions (results from Table 10). Figure 7 shows that economic benefits of the multistage are less evident when the optimal planning has a low level of risk (e.g. optimal planning in point a in Table 9 and in Table 10), since already for this building plan are considered a high level of non-conventional renewables since they are useful a hedge instrument against uncertainties from hydrological scenarios and volatilities of fossil-fuel prices.

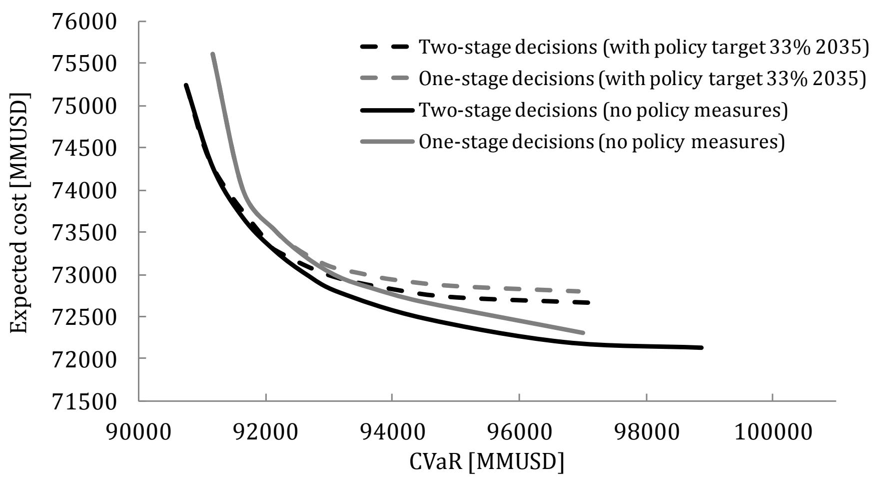


Figure 7 Efficient frontier formed by different optimal allocations of plants depending of risk levels under multi-stage decisions and single-stage decisions (with a new potential policy target).

The figure reports the efficient frontier composed by different optimal planning setups depending risk levels, with a new policy target of 33% share of total electricity generation from non-conventional renewables by 2035. This policy target is uncertain and it may be announced with a 50% of probability in 2025 and it will be implemented in 2035. Since this policy exercise concerns policy target, we also impose the Chilean policy target of 20% share of total electricity generation from non-conventional renewables by 2035 (which is already known with certainty by 2015). In the case that any of these targets is not reached, a penalty of 50 dollars is charged per megawatt which is not generated through non-conventional renewables in each hour of a complete year. The table reports the output result of the expansion planning for electricity generation using our multi-stage model (values from Table 9) and using a single stage model (data from Table 10).

5 Conclusion

In this study, we introduce a novel multi-stage model for optimal electricity generation planning under to deal with the risk of a policy change. Currently, there is a generalized environmental concern. Many plants of electricity generation, such as those based on fossil-fuels, can induce serious environmental damages including global warming, pollution, acid rain, and rising sea levels. In the last 10 years, an increasing number of countries are committed to reach progressively new renewable policy targets. Thus, given the current environmental concern, 'potential' new (or changes in) renewable policy targets has to be taken into account in an expansion planning for electricity generation.

The model allows an expansion planning in multiple stages in which some decision can be taken today, and others postponed to the future, when the uncertainty of new potential policies are eliminated. The model generate an optimal allocation of plants in each stage by taking into account simultaneously: changes in renewable policy targets, expected costs, risk exposures (i.e., prices volatilities, hydrological scenarios and demand growth) and operational issues to assure the electricity supply security. The model is solved by a Benders decomposition algorithm to tackle large dimension problems for a country level planning for electricity generation.

We implement the multi-stage planning in a real (country level) system power. The aim of this implementation is to present a genuine example of an expansion planning, including different analyses and policy exercises, which can be used as a guide to implement our approach in other regions or countries. We implement the model for the Chilean Central Interconnected System (CIS), where expansion planning is performed in a two-stage decisions problem. The first set of decisions is made in 2015 to build generation plants that will be operative in 2025. The second set of decisions is made in 2025 for the construction of generation plants that will be operative in 2035.

We perform two policy exercises to capture the uncertainty of a policy change. First, we assume that a 'potential' carbon tax may be announced in 2025 with a given probability, which will be implemented in 2035. Second, we also assume that a 'potential' renewable policy target may be announced in 2025 with a given probability, which will be implemented in 2035, where if the policy target is not reached a penalty has to be paid.

We show that, even in a scenario without changes in policy measures, our multi-stage planning model provide important economic benefits in term of costs and risks reductions. In addition, we present evidence that an optimal allocation of technologies including non-conventional renewable generation can generate a diversification effect by reducing not only the risk from uncertain hydrological scenarios, but also the financial risks of the system generated by volatility of fossil-fuel prices. Hence, we also shows that economic benefits of the multi-stage for renewable policy targets are less evident when the optimal planning is designated to reduce the levels of risk, because already for this building plans are considered a high level of non-conventional renewables.

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Appendix A: Assumptions and simplifications

Simplification in operation constraints

Constraints (10) and (11) have a common term, $min\{n_{i,j,s}, n_{i,j-1,s}\}$, that uses the minimize function to choose the minimum value between the number of online units of technology i under scenario s in hour j compared to hour j-1. This term can be represented in a linear form by using an auxiliary variable, $n_{i,j,s}^{min}$, and two additional constraints as shown in equations (A1) and (A2):

$$n_{i,j,s}^{min} \leq n_{i,j-1,s} \tag{A1}$$

$$n_{i,j,s}^{min} \leq n_{i,j,s}. \tag{A2}$$

These equations impose an upper bound to $n_{i,j,s}^{min}$ and can replace the minimize function$min\{n_{i,j,s}, n_{i,j-1,s}\}$, thus equations (10) and (11) can be re-written as:

$$g_{i,j,s} - g_{i,j-1,s} \leq n_{i,j,s}^{min} \cdot \rho_i + (n_{i,j,s} - n_{i,j-1,s}) \cdot \underline{P_i} \tag{A3}$$

$$g_{i,j-1,s} - g_{i,j,s} \leq n_{i,j,s}^{min} \cdot \rho_i + (n_{i,j-1,s} - n_{i,j,s}) \cdot \underline{P_i}. \tag{A4}$$

Hence equations (A1)-(A4) rather than (10)-(11) are used in the model.

Simplification in security of supply constraints

Equation (21) is non-linear but convex and thus can be linearized by using tangent planes. To do so, technologies are grouped into two categories according to their emergency ramp rates in order to reduce the number of planes and computational resources used. These two groups are denoted by I^{GS} and I^{GF}, which are sets classified as slow/fast response units for having low/high ramp rate. Additionally, a third group is defined, I^{NG}, which corresponds to technologies that do not participate in primary frequency response (PFR), but are connected through synchronous machines to the system and thus add inertia to it. Units' contribution to inertia (*H*) and their maximum power output ($\bar{P}$) are assumed to be equal among all units.

The numbers of units of the three groups are calculated using equations (A5)-(A7):

$$n_{SL,j,s} = \sum_{i \in I^{GS}} n_{i,j,s} \tag{A5}$$

$$n_{FT,j,s} = \sum_{i \in I^{GF}} n_{i,j,s} \tag{A6}$$

$$n_{NG,j,s} = \sum_{i \in I^{NG}} n_{i,j,s} \tag{A7}$$

Where SL, FT and NG are respectively slow response, fast response and non-governor (synchronous units that are online and do not participate in primary frequency control).

Hence we re-define the region given by equation (22) as that associated with equations (A5)-(A8).

$$\frac{f_0(\Delta P)^2}{4(f_0 - f_{MIN} - f_{db})((n_{NG,j,s} + n_{FT,j,s} + n_{SL,j,s}) \cdot H \cdot \bar{P} - H_f \cdot \Delta P)} \leq \left(n_{SL,j,s} + n_{FT,j,s}\right) \cdot \rho_i'. \tag{A8}$$

The linear form of restriction (A8) is obtained through tangent planes linearization.

Another non-linear constraint is equation (23). This restriction is non-convex, so it has to be treated differently. In this case, upper and lower bounds of the optimal solution are computed. The lower bound is obtained by removing equation (23) from the formulation, leading to a portfolio solution with a lower expected cost of investment and operation but which may violate equation (23) and thus not be technically feasible due to the fact that primary reserves are not allocated correctly within the installed technologies.

The upper bound of the optimal solutions is computed by solving a particular case of the model, in which only one technology may participate in PFR. This simplifies equations because when only one technology saves primary reserve, constraint (24) ends up being a simple linear equality. This is reasonable, because if only one technology participates in primary frequency response, all reserve must be stored in that technology. In this way, we may produce several upper bound solutions by defining various levels of participation from different generation technologies in PFR. Hence, a number of technically feasible suboptimal solutions can be obtained, ultimately selecting that with the lowest gap with respect to the lower bound solution. We found for all case studies analyzed in this paper that the selected technically feasible solutions present less than 0.8% gap. For this reason, all the simulations done in this paper, consider reservoirs for hydro power as the only technology committed to Primary Frequency Response.

Simplification in Operating reserve constraints

The operating reserves security criterion for this study consists of holding reserve for contingency purposes and to protect the system from unpredicted changes in the availability of solar and wind resources. Other renewable energy sources such as Biomass and Geothermal are not included in this analysis because their availability can be predicted accurately (both availability profiles have zero standard deviation).

As defined in Silva (2010), we use a realistic criterion for the representation of operating reserve policies, where reserve amounts required (*Red*) are considered for two purposes; the first is to restore primary frequency control reserves after they have been deployed, ΔP, and the second is to deal with unpredicted changes in variable renewable generation. Thus, the operating reserve requirement must be a function of the contingency magnitude, the non-conventional renewable generation and its installed capacity: $Req = f\left(\Delta\mathrm{P}, \{g_{i,j,s}\}_{i\in I^R}, \{c_i\}_{i\in I^R}\right)$

According to Silva (2010), when uncertainty of renewables forecasts is considered for reserve and these forecast errors are assumed to be non-correlated, normally distributed, with zero mean and a certain standard deviation, the reserve requirement may be quantified as shown in equation (A9), which requires saving 3 times the total standard deviation of the forecast errors.

$$Req = \Delta P + 3 \cdot \sqrt{\sigma_{WIND}^2 + \sigma_{SOLAR}^2}. \tag{A9}$$

Standard deviations of wind and solar forecast errors –w_{ind} and s_{olar} respectively– have to be computed using a certain forecast policy. Wind availability has no clear relationship to hours of the day as solar radiation does, so, a persistent 4 hour ahead forecast is employed to compute its forecast error standard deviation. This methodology produces one parameter that represents the uncertainty for all hours of the year. Solar radiation, on the other hand, is forecast using a day-ahead criterion. Four typical days of radiation are computed (one for each season) and the standard deviation error is calculated for the 24 hours of the day and for every season. The highest of the 4 values computed for every hour is taken as the conservative estimate.

$$Req = \Delta P + 3 \cdot \sqrt{c_{WIND}^2 \cdot \sigma_{WIND}^2 + c_{SOLAR}^2 \cdot \sigma_{SOLAR,j}^2} \tag{A10}$$

In equation (A10), installed capacities are included because forecasts are made in terms of the capacity factor. It can be argued that the above requirement might be too conservative. This is mainly because if, for example, no wind is forecasted for a certain hour, it would not be reasonable to keep reserve for wind purposes. Due to this fact, the following correction is made:

$$Req = \begin{cases} \Delta P + 3 \cdot \sqrt{c_{WIND}^2 \cdot \sigma_{WIND}^2 + c_{SOLAR}^2 \cdot \sigma_{SOLAR,j}^2} & WP_{i,j} \geq 3 \cdot \sigma_{WND} \\ \Delta P + g_{WIND,j,s} + 3 \cdot c_{SOLAR} \cdot \sigma_{SOLAR,j} & WP_{i,j} < 3 \cdot \sigma_{WND} \end{cases} \tag{A11}$$

In this equation, for hours on which the wind capacity factor of the hourly profile used (which is taken as the forecast) is smaller than the total uncertainty ($WP_{i,j} < 3 \cdot \sigma_{WND}$), a deterministic criterion is employed; assuming that in the worst case scenario, all scheduled wind fails to occur. On hours where the wind forecast is sufficiently high, the probabilistic criterion is established.

The same logic may be applied to solar technology. Nonetheless, as an individual standard deviation is computed for every hour, the above correction is not needed for typical zero radiation hours (at night, for example).

As equation (A11) is non-linear, algebra factorization and a first order Taylor series expansion is used to get a linear approximation shown by equation (A12). This linear function is always greater than the original equation (A11), so it is a conservative approximation.

$$Req = \begin{cases} \Delta P + 3 \cdot \left(\frac{1}{\sqrt{2}} \cdot \left(c_{WND} \cdot \sigma_{WND} + c_{SOL} \cdot \sigma_{SOL,j} \right) + \left(1 - \frac{1}{\sqrt{2}} \right) \cdot AV_j \right) & RP_{WND,j} \geq 3 \cdot \sigma_{WN} \\ \Delta P + g_{WND,j,s} + 3 \cdot c_{SOL} \cdot \sigma_{SOL,j} & RP_{WND,j} < 3 \cdot \sigma_{WN} \end{cases} \tag{A12}$$

Where AV_j represents $|c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j}|$. The absolute value function can be easily linearized by equations (A13) and (A14):

$$AV_j \geq c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j} \tag{A13}$$

$$AV_j \geq -\left(c_{WND} \cdot \sigma_{WND} - c_{SOL} \cdot \sigma_{SOL,j} \right). \tag{A14}$$

Finally, to account for all reserves considered in the reserve amounts required, *Red* (spinning and standing reserves), constraint (A15) is added to the model.

$$Req \leq \sum_{i \in I} R^{S}_{i,j,s} + FS \cdot \left(\sum_{i \in I} c_i - n_{i,j,s} \cdot \overline{P}_i \right) + DR^{S}_{j,s}. \tag{A15}$$

Here, $R^{S}_{i,j,s}$ is the capacity headroom in terms of spinning-kinetic reserves used to regulate contingencies as reserves for primary frequency response and the term FS represents the fraction of generation capacity that contributes to operating reserves. The equation (A15) also includes a demand response parameter, $DR^{S}_{j,s}$, to study the effect of responsive loads used in the operating reserve time frame.

Appendix B. Solution Methodology

In this section we explain the solution methodology based on Bender's decomposition algorithm, a technique which is meant to reduce computational complexity of large-scale problems. Here we use a vectors and matrices notation[16] to abbreviate the linear systems. For a better understanding, we explain first the relation between the nomenclature used to explain the model, with the vectors and matrices notation used in this section:

- $\boldsymbol{d}$ is a vector containing investment costs and $\boldsymbol{y}$ is the set of first-stage decision variables (installed capacities): $\mathbf{d}^{\mathrm{T}} \cdot \mathbf{y} \Leftrightarrow \sum_{i \in I} INV_i \cdot c_i$.
- $\mathrm{y}_{\mathrm{s}_1}$ is the set of first-stage decision variables (installed capacities) then s_1 happened: $\mathbf{d}^{\mathrm{T}} \cdot \mathrm{y}_{\mathrm{s}_1} \Leftrightarrow \sum_{i \in I} INV_i \cdot c_{i,s_1}$.
-
- $Q(\boldsymbol{y}, \boldsymbol{c_s}, \boldsymbol{F_s})$ is a function of the decision variable $\boldsymbol{y}$**,** operational costs $\boldsymbol{c_s}$, and $\boldsymbol{F_s}$ which is a matrix of all constraints related with the decision variable of installed capacities. Later $Q(\boldsymbol{y}, \boldsymbol{c_s}, \boldsymbol{F_s})$ will be defined as the second-stage problem.
- $\mathbf{c_s} \Leftrightarrow \sum_{i \in I} \sum_{j \in J} VOM_{i,s} \cdot g_{i,j,s} + \sum_{j \in J} D^{-}_{j,s} \cdot dc^{-} + \sum_{j \in J} D^{+}_{j,s} \cdot dc^{+} + voll \cdot \sum_{j \in J} LL_{j,s}$
- δ Represents the an approximation of Value at Risk (VaR).
- The function denoted by $(\dots\dots\dots)^{+}$ represents the maximum between the expression in parenthesis and zero: $(expr)^{+} = max(expr, 0)$

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[16] Vectors are written as bold, lower-case letters and matrices as bold upper-case letters.

Bender's method coordinates a two-stage stochastic linear programming model to determine the optimum portfolio of generation technologies of a future power system. In the first stage, the investment decision takes place and therefore we minimize total investment and operation costs across a large number of future scenarios, subject to a given level of CVaR.

The first-stage problem is given by:

$$(P1)\ Minimize\ z = \beta(\mathrm{d}^{\mathrm{T}} \cdot \mathrm{y} + \textstyle\sum_{s_1 \in S_1} \mathrm{p}_{s_1} \cdot \mathrm{Q}(\mathrm{y}, \mathrm{c}_{s_1}, \mathrm{F}_{s_1})) + \lambda \textstyle\sum_{s_1 \in S_1} \mathrm{p}_{s_1} \cdot \mathrm{d}_{s_1}^{\mathrm{T}} \cdot \mathrm{y}_{s_1} + \lambda \textstyle\sum_{s_1 \in S_1} \sum_{s_2 \in S_2/s_1} \mathrm{p}_{s_1} \cdot \mathrm{p}_{s_2} \cdot \mathrm{Q}(\mathrm{y}, \mathrm{c}_{s_2}, \mathrm{F}_{s_2}) \quad \text{(B1)}$$

s.t.:

$$\delta + \frac{1}{1-\alpha} \cdot (\textstyle\sum_{s_1 \in S_1} \sum_{s_2 \in S_2/s_1} \mathrm{p}_{s_1} \cdot \mathrm{p}_{s_2} \cdot \big(\beta \mathrm{d}^{\mathrm{T}} \cdot \mathrm{y} + \beta \mathrm{Q}(\mathrm{y}, \mathrm{c}_{s_1}, \mathrm{F}_{s_1}) + \lambda \mathrm{d}_{s_1}^{\mathrm{T}} \cdot \mathrm{y}_{s_1} + \lambda \cdot \mathrm{Q}(\mathrm{y}, \mathrm{c}_{s_2}, \mathrm{F}_{s_2})\ - \delta\big)^{+}) \leq \overline{\mathrm{CVaR}} \quad \text{(B2)}$$

$$\mathrm{y} \in \mathrm{Y} \quad \text{(B3)}$$

$$\mathrm{y}_{s_1} \in \mathrm{Y}, \forall s_1 \in S_1$$

$$\mathrm{y} \leq \mathrm{y}_{s_1}, \forall s_1 \in S_1$$

$$\delta \geq 0 \quad \text{(B4)}$$

where $\boldsymbol{Y}$ is a set of polyhedral constraints that ensure that $\boldsymbol{y}$ corresponds to a feasible solution. Equation (45) represents an upper bound to the CVaR calculated in the left side of the equation. $\overline{CVaR}$ is the maximum allowed portfolio's CVaR.

The second stage represents the operation of the capacity imposed by the first stage (dispatch decisions). The second-stage problem is given by:

For $s_1 \in S_1$

$$\big(\mathrm{P2}_{s_1}\big)\ \mathrm{Q}(\mathrm{y}, \mathrm{c}_{s_1}, \mathrm{F}_{s_1}, \mathrm{E}_{s_1}) = \text{Minimize } \mathrm{c}_{s_1}{}^{\mathrm{T}} \cdot \mathrm{x} \quad \text{(B5.1)}$$

s.t.:

$$\mathrm{F}_{s_1} \cdot \mathrm{y} + \mathrm{E}_{s_1} \cdot \mathrm{x} = \mathrm{h} \quad \text{(B6.1)}$$

$$x \geq 0 \tag{B7.1}$$

Para $s_2 \in S_2/s_1$

$$(P2_{s_2})\ Q(y_{s_1}, c_{s_2}, F_{s_2}, E_{s_2}) = \text{Minimize } c_{s_2}{}^{T} \cdot x \tag{B5.2}$$

s. t.:

$$F_{s_2} \cdot y + E_{s_2} \cdot x = h_2 \tag{B6.2}$$

$$x \geq 0 \tag{B7.2}$$

where $\mathbf{x}$ corresponds to the generation ($\boldsymbol{x} \Leftrightarrow g_{i,j,s}$), thus $Q(\boldsymbol{y}, \boldsymbol{c_s}, \boldsymbol{F_s})$ is the minimization of the operational costs for every scenario s. Matrix $\mathbf{E_s}$ contains all constraints related with the generation of each technology, in particular demands constraints . Therefore, equation (B5) is a matrix-vector expression which summarizes almost all the constraints explained in the model. $\boldsymbol{h}$ is an auxiliary vector to fit the constraints expressed in matrices $\boldsymbol{F_s}$ and $\mathbf{E_s}$.

The master problem (P1) is defined by a convex dominion and a convex objective function. This allows the linearization of non-linear terms through tangent planes, as it is done in the classic Benders' decomposition problem.
Moreover, function $Q(\boldsymbol{y}, \boldsymbol{c_s}, \boldsymbol{F_s})$ has the same structure as the classic slave problem from Benders' decomposition (Benders, 1962), so the same approximation (and cutting planes selection algorithm) can be used for solving this particular problem according to Papavasiliou et al. (2014).

Taking this into account, the master problem can be re-written, including optimality cuts derived from Benders' algorithm as follows:

$$(P1')\ Minimize\ z_L \tag{B8}$$

$$s.t.:$$

$$z_L \geq \beta \cdot d^T \cdot y + \lambda \cdot \sum_{s_1 \in S_1} p_{s_1} \cdot d_{s_1}^T \cdot y_{s_1} \tag{B9}$$

$$z_L \geq \beta \cdot d^T \cdot y + \beta \cdot \sum_{s_1 \in S_1} p_{s_1} \cdot \left(Q\left(\hat{y}^i, c_{s_1}, F_{s_1}, E_{s_1}\right) + (\hat{y}^i - y)^T \cdot {F_{s_1}}^T \cdot {u^i}_{s_1}\right) + \lambda \qquad 1 \leq i \leq k$$

$$\cdot \sum_{s_1 \in S_1} p_{s_1} (d_{s_1}^T \cdot y_{s_1} + \sum_{s_2 \in S_2/s_1} p_{s_2} \cdot \left(Q\left({\hat{y}^i}_{s_1}, c_{s_2}, F_{s_2}, E_{s_2}\right) + \left({\hat{y}^i}_{s_1} - y_{s_1}\right)^T \cdot {F_{s_2}}^T \cdot {u^i}_{s_2}\right)) \tag{B10}$$

$$\boldsymbol{v_{s_1,s_2}} \geq \beta \cdot d^T \cdot y + \beta \cdot Q\left(\hat{y}^i, c_{s_1}, F_{s_1}\right) + \beta \cdot \left(\hat{y}^i - y\right)^T \cdot {F_{s_1}}^T \cdot {u^i}_{s_1} + \lambda \cdot \left(d_{s_1}^T \cdot y_{s_1} + Q\left(\hat{y}^i, c_{s_2}, F_{s_2}\right) + \left({\hat{y}^i}_{s_1} - y_{s_1}\right)^T \cdot {F_{s_2}}^T \cdot {u^i}_{s_2}\right) - \delta \qquad \forall \boldsymbol{s_1} \in \boldsymbol{S_1}, \forall \boldsymbol{s_2} \in \boldsymbol{S_2/s_1}, 1 \leq i \leq k \tag{B11}$$

$$\delta + \frac{1}{1-\alpha} \cdot \sum_{s_1 \in S_1} p_{s_1} \sum_{s_2 \in S_2/s_1} p_{s_2} \boldsymbol{v_{s_1,s_2}} \leq \overline{\mathrm{CVaR}} \tag{B12}$$

$$\boldsymbol{v_{s_1,s_2}} \geq 0 \tag{B13}$$

$$\boldsymbol{y} \in \boldsymbol{Y}, \boldsymbol{y_{s_1}} \in \boldsymbol{Y}, \boldsymbol{y} \leq \boldsymbol{y_{s_1}}, \forall \boldsymbol{s_1} \in \boldsymbol{S_1} \tag{B14}$$

$$\delta \geq 0 \tag{B15}$$

Auxiliary variables v_s are used for obtaining the linear form of constraint (45) and $\boldsymbol{u}^i_s$ are the Lagrange multipliers of $P2_s$ associated with the coupling constraints (i.e. generation capacities), considering the i^{th} investment decision trial $\boldsymbol{\hat{y}^i}$. Constraint (52) is added to avoid unboundedness by setting a lower limit. Using *P1'* and $P2_s$ shown by equations (48)-(58), the following algorithm is proposed:

Step 0: Set $k = 1$. Initialize $\hat{z}_{lower} = -\infty$, $\hat{c}_{lower} = -\infty$ and $\hat{y}^1$ e ${\hat{y}^1}_{s_1} \forall\, s_1 \in \mathbf{S_1}$. Go to step 1.

Step 1: Solve *P1'*. Set $\boldsymbol{\hat{y}^k}$ equal to the optimal first-stage solution and set $\hat{z}_{lower} = \hat{z}_L$ and $\hat{c}_{lower} = \delta + \frac{1}{1-\alpha} \cdot \sum_{s_1 \in S_1} p_{s_1} \sum_{s_2 \in S_2/s_1} p_{s_2} \widehat{v_{s_1,s_2}}$.

Go to step 2.

Step 2: For all $s \in S$, solve P2$_s$ using $\hat{\boldsymbol{y}}^k$ as input. Set $\boldsymbol{u}^k{}_s$ equal to the optimal multipliers of the coupling constraints in equation (49). Set $\hat{z}_{upper} = \beta \cdot d^T \cdot y + \sum_{s_1 \in S_1} p_{s_1} \cdot \beta \cdot Q(\hat{y}^i, c_{s_1}, F_{s_1}, E_{s_1}) + \lambda \cdot \sum_{s_1 \in S_1} p_{s_1} (d_{s_1}^T \cdot y_{s_1} + \sum_{s_2 \in S_2/s_1} p_{s_2} \cdot Q\left(\hat{y}^i{}_{s_1}, c_{s_2}, F_{s_2}, E_{s_2}\right))$ and $\hat{c}_{upper} = CVaR_\alpha(\boldsymbol{c}(\hat{\boldsymbol{y}}^k), \boldsymbol{p})$.

Where $CVaR_\alpha(\boldsymbol{x}, \boldsymbol{\rho})$ is the function that computes the $(1-\alpha)$ percentile conditional value at risk of the cost vector $\boldsymbol{x}$ with the associated probabilities vector $\boldsymbol{\rho}$. $\boldsymbol{c}(\hat{\boldsymbol{y}}^k)$ corresponds to the vector containing the total costs of every scenario $C(s_1, s_2) = \beta \cdot d^T \cdot \hat{y}^k + \beta \cdot Q(\hat{y}^k, c_{s_1}, F_{s_1}, E_{s_1}) + \lambda \cdot d_{s_1}^T \cdot \hat{y}^k{}_{s_1} + \lambda \cdot Q\left(\hat{y}^k{}_{s_1}, c_{s_2}, F_{s_2}, E_{s_2}\right)$ with probability $p_{s_1} \cdot p_{s_2}$, given the first-stage decision $\hat{\boldsymbol{y}}^k, \hat{y}^k{}_{s_1} \forall s_1 \in S_1$ and $\boldsymbol{p}$ is the vector containing scenarios' probabilities totals. Go to step 3.

Step 3: If $|\hat{z}_{upper} - \hat{z}_{lower}| \leq \varepsilon_1$ and $|\hat{c}_{upper} - \hat{c}_{lower}| \leq \varepsilon_2$ then exit with $\hat{\boldsymbol{y}}^k, \hat{\boldsymbol{y}}^k{}_{s_1} \forall s_1 \in S_1$ as the optimal solution. Otherwise, set $k = k + 1$ and go to step 1.

For the purpose of simplicity the addition of Benders' feasibility cuts is not explained, although they might be necessary for obtaining the optimal solution. The addition of these cuts does not vary from the standard procedure done in Benders' classical decomposition algorithm (Benders, 1962).

It is important to underscore that the exit criterion of the algorithm ensures that both expectation and CVAR functions are correctly approximated in the neighborhood of the optimal solution. We used an exit criterion of 1%.

Appendix C: Additional parameter values

The values were selected according to regulation of the power sector in Chile and standard level in the electric power sector. A 399.67 $/MWh value of lost load is used, according to the short-term failure cost reported by the Chilean regulator (National Commission of Energy). Maximum units' outputs is assumed to be 400 MW with an hourly ramp rate (ρ_i) of 40 MW/h for Coal, 240 MW/h for Geothermal and Oil, 200 MW/h for LNG, Biomass and Solar CSP, and 360 MW/h for hydro-electric technologies. Emergency ramp rates (p_i') are those used by Chaves et al. (2014) and equal to 38 MW/s for thermal plants and 8 MW/s for hydro-electric plants.[17] In real power systems, primary frequency response service is provided by a subset of the conventional plants synchronized, which is represented in our model by defining two types of unit per technology: with and without capability to respond to frequency changes, in which the

[17] Emergency ramp rates are the ramp rates of the reserves used by the governor to maintain the security supply.

former presents a slightly higher investment cost that permits identification of the demand for the frequency response service. Also, operation is secured against the outage of a single unit (i.e. 400 MW), under which frequency is not allowed to violate a minimum value of 49.2 Hz from nominal value of 50 Hz (governors' dead-band are assumed to be equal to $\pm$25mHz and units' inertia (H) is equal to 5 s). [18,19] We assume that costs associated with demand services are equal to 1 (2) \$/MW if demand decreases (increases), reservoir seepage and evaporation losses are equal to 0.5% of stored water, and maximum capacity of the reservoir is very high and thus does not constrain hydro's output. Portfolios will be determined by using a α-CVaR with an α of 95%.

[18] Chilean regulator states that under frequency load shedding must take place when system frequency reaches a threshold of 49.2 Hz.
[19] Maximum allowed governors' dead band in Chile.

Table 11 Parameters values

The table contains the parameters values assumed in the model. The table shows the nomenclature of the parameters (symbol), a brief description, the value considered as an input parameter and the corresponding unit of measurement. As the model is implemented for the Chilean Central Interconnected System (CIS), some values are reported by the Chilean regulator while others are taken from references. They respect standards level in the electric power sector. The acronym "p.u." refers to "per unit", expression of quantities as fractions of a defined base unit quantity.

Symbol	Description	Value	Unit
$voll$	Value of lost load	399.67	\$/MWh
$\bar{P}$	Maximum power output of generic unit	400	MW
$\underline{P}_i$	Minimum units output	160 for thermal 40 for hydro	MW
ρ_i	Hourly ramp rate	40 for Coal 240 for Oil, Geothermal 200 for LNG, Biomass, CSP 360 Hydro	MW/h
ρ_i'	Emergency ramp rate	38 for thermal plants 8 for hydro plants	MW/s
f_0	Nominal system frequency	50	Hz
f_{db}	Governors frequency dead band	±25	mHz
f_{MIN}	Minimum frequency allowed	49.2	Hz
H	Inertia constant of generic unit	5	s
λ_i	Factor of losses of stored water due to evaporation and/or seepage in the reservoir	0.0051	p.u.
$\bar{v}_i$	Upper bound of stored water	10,321	MMm^3
η_i	Average inflow-to-power rate	6,840	MWh/m^3
ΔP	Size of largest generation outage	400	MW
$t_{MIN,db}^i$	Deployment time of operating reserves	0.25	h
$DR_{j,s}^S$	Amount of curtailable demand for the operating reserve timeframe	200	MW
$DR_{j,s}^P$	Amount of curtailable demand for the primary frequency control timeframe	200	MW
FS	Fraction of fast start generation capacity that contributes to operating reserves	1	p.u.
dc^-	Cost of demand decrease	1	\$/MW
dc^+	Cost of demand increase	2	\$/MW
$\overline{ds}^-$	Maximum fraction of demand that can be decreased	5%	p.u.
$\overline{ds}^+$	Maximum fraction of demand that can be increased	5%	p.u.
α	CVaR parameter that defines the (1- α)% highest cost scenarios	95%	p.u.
σ_{WND}	Standard deviation of wind forecast errors in all hours	12.8%	p.u.
$\sigma_{SOL,j}$	Standard deviation of solar forecast errors in hour j	0% - 10.6%	p.u.
X	Renewable policy target	20%	p.u.