

Credit Market Segmentation, Essentiality of Commodities, and Supermodularity

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CREDIT MARKET SEGMENTATION, ESSENTIALITY OF COMMODITIES, AND SUPERMODULARITY

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ABSTRACT. We consider incomplete market economies where agents are subject to price-dependent trading constraints compatible with credit market segmentation. Equilibrium existence is guaranteed when either commodities are essential, i.e, indifference curves through individuals' endowments do not intersect the boundary of the consumption set, or utility functions are concave and supermodular. Since we do not require the smoothness of mappings representing preferences, financial promises, or trading constraints, our approach is compatible with the existence of ambiguity-adverse agents, non-recourse collateralized loans, or income-dependent thresholds determining the access to credit.

KEYWORDS. Credit Market Segmentation - Essential Commodities - Supermodularity

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1. INTRODUCTION

In the last years the theory of general equilibrium with incomplete financial markets was extended to environments where agents are subject to endogenous portfolio constraints.¹ In one strand of this literature it is assumed that financial trade is restricted by price-dependent inequality constraints determined by well-behaved functions, allowing to analyze equilibrium existence in smooth economies with techniques of differential topology (cf. Carosi, Gori, and Villanacci (2009), Gori, Pireddu, and Villanacci (2014), Hoelle, Pireddu, and Villanacci (2016)). Alternatively, there are models that focus on trading constraints determined by arbitrary set-valued mappings, ensuring equilibrium existence with fixed-point techniques under either impatience conditions on preferences (cf. Seghir and Torres-Martínez (2011), Pérez-Fernández (2013), Cea-Echenique and Torres-Martínez (2014)) or super-replication properties that guarantee the fully hedge of segmented assets' promises (cf. Cea-Echenique and Torres-Martínez (2014, 2016)).

We contribute to this growing literature with two results of equilibrium existence in economies where agents are subject to arbitrary price-dependent trading constraints compatible with credit market segmentation. Our findings highlight the role of essential commodities and supermodular utility functions to bound asset prices, in order to prove equilibrium existence without requiring the smoothness of mappings representing preferences, financial promises, or trading constraints. In particular, our approach is compatible with the existence of ambiguity-adverse agents, non-recourse collateralized loans, or income-dependent thresholds determining the access to credit.

It is worth noting that, under traditional hypotheses on primitives, if prices and allocations are endogenously bounded, then a competitive equilibrium can be found by applying fixed point methods. However, the presence of segmentation in credit markets may prevent us to induce upper bounds on asset prices just by normalizing it. Indeed, a normalization of prices may generate discontinuities on individuals' demands when not all agents have access to short-sale financial contracts. Therefore, to find these upper bounds we include additional assumptions.

Our first result of equilibrium existence focuses on the essentiality of commodities, assuming that indifference curves through individuals' endowments do not intersect the boundary of the consumption set. Under this requirement, any market feasible and individually optimal allocation is uniformly bounded away from zero. Thus, small losses on assets' deliveries do not compromise the feasibility of consumption. Furthermore, the continuity, convexity, and locally non-satiability of

¹Classical models with exogenous portfolio constraints focus on restrictions determined by convex and closed sets, ensuring equilibrium existence under non-redundancy conditions over the financial structure and/or financial survival requirements (cf. Cass (1984, 2006), Siconolfi (1989), Balasko, Cass, and Siconolfi (1990), Polemarchakis and Siconolfi (1997), Cass, Siconolfi, and Villanacci (2001), Angeloni and Cornet (2006), Won and Hahn (2007, 2012), Aouani and Cornet (2009, 2011), Cornet and Gopalan (2010)).

preferences guarantee that the effects on welfare generated by these small losses may be offset by an increment on first-period consumption. Therefore, by non-arbitrage, the cost of these losses is naturally bounded from above by the cost of the bundles that offset them, inducing natural upper bounds on asset prices.

Notice that, when commodities are essential, our methodology to bound asset prices depends on a property of economies with continuous, locally non-satiated, and strictly convex preferences: agents with interior consumption may offset the welfare costs associated to small reductions on segmented assets' deliveries by increasing the demand of commodities at first period. Although related in spirit, this property is weaker than the super-replication assumption imposed by Cea-Echenique and Torres-Martínez (2016). Indeed, they assume that agents may offset wealth losses associated to any reduction on segmented assets' deliveries by increasing either the consumption of durable commodities or the investment on unsegmented financial contracts.

Our second result of equilibrium existence concentrates on economies where preferences can be represented by concave and supermodular utility functions. Under these requirements, we show that individuals' marginal rates of substitution between current and future earnings are bounded, implying that discounted values of assets' deliveries are bounded too. Hence, to find an upper bound for the price of a traded segmented asset, it is sufficient to bound the shadow-prices of non-negativity constraints of second-period consumption. For this reason, we also assume that segmented contracts enlarge the set of states of nature where financial transfers are available, implying that any agent investing in a segmented contract demands a positive consumption at the states of nature where she receives its deliveries. As in the previous approach, the existence of endogenous upper bounds for asset prices allow us to prove equilibrium existence by traditional fixed-point techniques.

The characteristics of a standard economy are described in the next section. In Section 3 we develop our first approach on equilibrium existence, assuming that commodities are essential. Section 4 is devoted to economies where agents have supermodular utility functions. The proof of our main results are contained in Appendices A and B.

2. STANDARD ECONOMIES

We consider a two-period financial market economy where agents are subject to personalized trading constraints as in Cea-Echenique and Torres-Martínez (2016). There is no uncertainty at the first period and a finite set S of states of nature can be attained at the second period. There is a finite set \mathcal{L} of perfectly divisible commodities available for trade at prices $p = (p_s)_{s \in S}$, where $\mathcal{S} := \{0\} \cup S$ denotes the set of states of nature in the economy. First-period consumption is subject to transformations through time, which are described by linear technologies $(Y_s)_{s \in S}$. Financial

markets are characterized by a finite set \mathcal{J} of contracts, which are traded at the first period and make contingent promises $(R_{s,j}(p))_{(s,j) \in \mathcal{S} \times \mathcal{J}}$ at second period.

Since we focus on the analysis of equilibrium existence, we consider the following space of commodity and asset prices $\mathbb{P} := \{((p_s)_{s \in \mathcal{S}}, q) \in \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} \times \mathbb{R}_+^{\mathcal{J}} : \|(p_0, q)\|_{\Sigma} \neq 0 \wedge (p_s)_{s \in \mathcal{S}} \in \mathcal{P}\}$, where $\mathcal{P} := \{(p_s)_{s \in \mathcal{S}} \in \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} : \|p_0\|_{\Sigma} \leq 1 \wedge \|p_s\|_{\Sigma} = 1, \forall s \in \mathcal{S}\}$. In addition, let $\mathbb{E} := \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} \times \mathbb{R}^{\mathcal{J}}$ be the space of physical and financial positions.

There is a finite set \mathcal{I} of agents, where each i has a utility function $V^i : \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} \rightarrow \mathbb{R} \cup \{-\infty\}$ with an effective domain $\mathbb{X}^i := \{x \in \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} : V^i(x) > -\infty\}$, and endowments $(w_s^i)_{s \in \mathcal{S}} \in \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}}$ which allows to consume the bundle $W^i = (W_s^i)_{s \in \mathcal{S}} := (w_0^i, (w_s^i + Y_s w_0^i)_{s \in \mathcal{S}})$. In addition, trade is restricted in such a way that each agent i is subject to price-dependent personalized constraints described by a correspondence $\Phi^i : \mathcal{P} \rightarrow \mathbb{E}$. This framework allows us to consider thresholds giving exclusive access to credit to either low or high income agents, bounds on the amount of debt as a function of future or current income, or collateral constraints, for instance.²

Hence, given prices $(p, q) \in \mathbb{P}$, each agent i chooses a vector $(x^i, z^i) \in \mathbb{E}$ in the set $C^i(p, q)$ of trading admissible and budget feasible allocations:

$$(x^i, z^i) \in \Phi^i(p); \quad p_0 x_0^i + q z^i \leq p_0 w_0^i; \quad p_s x_s^i \leq p_s (w_s^i + Y_s x_0^i) + \sum_{j \in \mathcal{J}} R_{s,j}(p) z_j^i, \quad \forall s \in \mathcal{S}.$$

DEFINITION. A competitive equilibrium is given by a vector $[(\bar{p}, \bar{q}), (\bar{x}^i, \bar{z}^i)_{i \in \mathcal{I}}] \in \mathbb{P} \times \mathbb{E}^{\mathcal{I}}$ such that:

- (i) For each agent i , $(\bar{x}^i, \bar{z}^i) \in \operatorname{argmax}_{(x^i, z^i) \in C^i(\bar{p}, \bar{q})} V^i(x^i)$.
- (ii) Physical and financial markets clear, i.e., $\sum_{i \in \mathcal{I}} (\bar{x}^i - W^i, \bar{z}^i) = 0$.

Standard economies are characterized by the following hypotheses, which allow us to ensure that optimal individual allocations are well behaved functions of prices.

ASSUMPTION A

(i) For each $i \in \mathcal{I}$, V^i is continuous and strictly quasi-concave on $\mathbb{X}^i \supseteq \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$.

(ii) For any agent $i \in \mathcal{I}$ the following property of locally non-satiability holds:

$$\forall x \in \mathbb{X}^i, \forall s \in \mathcal{S}, \forall \epsilon > 0, \exists y \in \mathbb{R}^{\mathcal{L}} : \quad \|y\|_{\Sigma} < \epsilon, \quad ((x_k)_{k \neq s}, x_s + y) \in \mathcal{A}^i(x),$$

where $\mathcal{A}^i(x) := \{x' \in \mathbb{X}^i : V^i(x') > V^i(x)\}$.

(iii) For each $(s, l) \in \mathcal{S} \times \mathcal{L}$, there is an agent whose utility function is strictly increasing in $x_{s,l}$.

²See Seghir and Torres-Martínez (2011) or Cea-Echenique and Torres-Martínez (2016) for more detailed examples.

³We say that V^i is strictly quasi-concave if and only if it is quasi-concave and given $x, y \in \mathbb{X}^i$ such that $V^i(x) \neq V^i(y)$ we have that $V^i(\lambda x + (1 - \lambda)y) > \min\{V^i(x), V^i(y)\}$, $\forall \lambda \in (0, 1)$.

We denote the set of *segmented financial contracts* by

$$\mathcal{K} := \left\{ k \in \mathcal{J} : \exists p \in \mathcal{P}, \forall \delta > 0, -\delta e_k \notin \bigcap_{i \in \mathcal{I}} \Phi^i(p) \right\},$$

where $e_k \in \mathbb{E}$ is the allocation composed by just one unit of contract k .

ASSUMPTION B

For each agent $i \in \mathcal{I}$, given $p \in \mathcal{P}$ and $(x^i, z^i) \in \Phi^i(p)$,

- (i) Φ^i is lower hemicontinuous, with convex values and closed graph relative to $\mathcal{P} \times \mathbb{E}$.
- (ii) $\Phi^i(p) + (\mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} \times \mathbb{R}_+^{\mathcal{J}}) \subseteq \Phi^i(p)$ and $(0, 0) \in \Phi^i(p)$.
- (iii) $(x^i, z^i) - \alpha e_k \in \Phi^i(p)$, $\forall k \in \mathcal{K}$, $\forall \alpha \in [0, \max\{z_k^i, 0\}]$.
- (iv) For any $y \in \mathbb{R}_+^{\mathcal{L}} \times \mathbb{R}^{\mathcal{L} \times \mathcal{S}}$ such that $x^i + y \geq 0$, we have that $(x^i + y, z^i) \in \Phi^i(p)$.

Assumption B(i) guarantees that trading constraints are determined by well behaved correspondences: any $(x^i, z^i) \in \Phi^i(p)$ can be approximated by plans that are admissible at prices near p ; convex combinations of trading admissible plans are also trading admissible; and any convergent sequence of prices and trading admissible plans has a trading admissible limit. Under Assumption B(ii) there are no restrictions for incrementing either investment or consumption when the amount of debt does not increase. It also guarantees that free disposal of physical endowments is allowed and there is no obligation to trade assets. Assumption B(iii) ensures that long-positions in segmented contracts can be reduced without compromising trading admissibility, i.e., segmented contracts cannot serve as financial collateral. Finally, B(iv) implies that second-period consumption is not subject to trading constraints other than budgetary admissibility.

ASSUMPTION C

Financial contracts' promises are continuous functions of commodity prices. Also, for every $p \in \mathcal{P}$ such that $p \gg 0$ we have that $(R_{s,j}(p))_{s \in \mathcal{S}} \neq 0$, $\forall j \in \mathcal{J}$.

Let $\Lambda : \mathcal{P} \rightarrow \mathbb{E}^{\mathcal{I}}$ be the set-valued mapping that associates commodity prices p with trading admissible allocations $((x^i, z^i))_{i \in \mathcal{I}} \in \prod_{i \in \mathcal{I}} \Phi^i(p)$ satisfying market feasibility, $\sum_{i \in \mathcal{I}} (x^i - W^i, z^i) = 0$, and second-period budget constraints, $p_s x_s^i = p_s (w_s^i + Y_s x_0^i) + \sum_{j \in \mathcal{J}} R_{s,j}(p) z_j^i$, $\forall (i, s) \in \mathcal{I} \times \mathcal{S}$.

ASSUMPTION D

The set $\{(p, a) \in \mathcal{P} \times \mathbb{E}^{\mathcal{I}} : p \gg 0 \wedge a \in \Lambda(p)\}$ is bounded.

When individual's aggregated endowments are interior points of the consumption space, i.e., $(W^i)_{i \in \mathcal{I}} \gg 0$, and Assumptions A(iii), B(i), and B(ii) hold, the requirement above is weaker than the following non-redundancy condition: *for each agent $i \in \mathcal{I}$, and for every price $p \in \mathcal{P}$,*

$$\left\{ z \in \mathbb{R}^{\mathcal{J}} \setminus \{0\} : \sum_{j \in \mathcal{J}} R_{s,j}(p)z = 0 \quad \wedge \quad (W^i, \delta z) \in \Phi^i(p), \forall s \in S, \forall \delta > 0 \right\} = \emptyset.^4$$

Hence, Assumption D holds when agents do not have access to unbounded sequences of portfolios that do not generate transfers at second period. Notice that, trading constraints make this requirement compatible with the existence of financial contracts with collinear vectors of promises.⁵

The following example shows that standard economies may not have equilibria.

EXAMPLE 1. Consider an economy without uncertainty, where there is a perishable commodity and a real Arrow security. There are two agents, a and b , characterized by $V^a(x_0, x_1) = V^b(x_0, x_1) = 4x_0(1 + x_0)^{-1} + \sqrt{x_1}$, $(w^a, w^b) = ((1, 0), (1, 1))$, $\Phi^a(p, q) = \mathbb{R}_+^2 \times \mathbb{R}$, and $\Phi^b(p, q) = \mathbb{R}_+^2 \times \mathbb{R}_+$. Since agent a does not have initial resources at second period, she invests in the financial contract independently of prices. This is incompatible with the existence of equilibria, as agent b cannot short-sale promises. \square

3. ESSENTIALITY OF COMMODITIES

In the previous section we introduced assumptions that allow us to endogenously bound individual allocations in order to prove equilibrium existence by following traditional fixed point techniques. However, the presence of credit market segmentation may prevent us to induce upper bounds on financial prices by normalizing them jointly with commodity prices, as this normalization may induce discontinuities in choice set correspondences. Therefore, we need to obtain upper bounds on segmented asset prices without including frictions on individual decisions. With this objective in mind, we introduce a property satisfied by all standard economies and which ensures that, by increasing first-period consumption, investors may offset the *negative effects on welfare* generated by *small reductions* on segmented contract promises.

⁴This result is a consequence of Cea-Echenique and Torres-Martínez (2016, Proposition on page 22). Although they only affirm that the non-redundancy condition above implies that $\{(p, q), a\} \in \mathbb{P}' \times \mathbb{E}^{\mathcal{I}} : (p, q) \gg 0 \wedge a \in \Omega(p, q)\}$ is bounded for any compact set $\mathbb{P}' \subseteq \mathbb{P}$, where $\Omega(p, q) := \Lambda(p) \cap \prod_{i \in \mathcal{I}} C^i(p, q)$, first-period budget constraints do not have any role in the proof of their result.

⁵As was pointed out by Cea-Echenique and Torres-Martínez (2016), the non-redundancy condition above generalizes the hypothesis introduced by Siconolfi (1989, Assumption A5) in the context of two-period economies with nominal asset markets and exogenous portfolio constraints.

COMPENSATION OF SMALL LOSSES IN SEGMENTED MARKETS

For each $i \in \mathcal{I}$ and $x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$, there exists $(\varepsilon^i(x), \tau^i(x)) \in \mathbb{R}_{++} \times \mathbb{R}_+^{\mathcal{L}}$, which continuously vary with x , and there are net trades $(\theta_s^i(p, x))_{s \in \mathcal{S}} \in \mathbb{R}^{\mathcal{L} \times \mathcal{S}}$ for any $p \in \mathcal{P}$ with $p \gg 0$ such that

$$p_s \theta_s^i(p, x) \leq -\varepsilon^i(x) \sum_{k \in \mathcal{K}} R_{s,k}(p), \quad \forall s \in \mathcal{S},$$

and $(x_0 + \tau^i(x), (x_s + \theta_s^i(p, x))_{s \in \mathcal{S}}) \in \mathcal{A}^i(x)$.

Hence, when the compensation of small losses in segmented markets holds, an agent i that demands an interior plan of consumption $x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$ may offset the negative effects on her welfare generated by a percentage reduction $\varepsilon^i(x)$ on deliveries of segmented assets. She can make it by increasing her demand at first period to $x_0 + \tau^i(x)$ and changing her second-period consumption through net trades $(\theta_s^i(p, x))_{s \in \mathcal{S}}$. Notice that this property is weaker than the super-replication condition imposed by Cea-Echenique and Torres-Martínez (2016), which requires that *negative effects on wealth* associated to *any reduction* on segmented contract promises can be offset with an increment on either consumption of non-perishable commodities and/or investment on unsegmented contracts.

The next result shows that the property above is a mild condition.

PROPOSITION 1. *The compensation of small losses in segmented markets holds in any economy satisfying Assumptions A(i), A(ii), and C.*

Proof. Given $i \in \mathcal{I}$, let $G^i : [0, 0.5] \times \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}} \rightarrow \mathbb{R}$ be the mapping defined by

$$G^i(\delta, x) := V^i \left(x_0 + a^i(x), \left(x_s - \delta \min_{(s,l) \in \mathcal{S} \times \mathcal{L}} x_{s,l}(1, \dots, 1) \right)_{s \in \mathcal{S}} \right) - V^i(x),$$

where $a^i(x) = \operatorname{argmax}_{a \in \mathbb{R}_+^{\mathcal{L}} : \|a\|_{\Sigma} \leq 1} V^i(x_0 + a, (x_s)_{s \in \mathcal{S}})$. Notice that, as V^i is continuous and strictly quasi-concave, the Berge's Maximum Theorem ensures that a^i is a continuous function. Hence, G^i is well-defined and continuous. Also, the local non-satiation on first-period consumption implies that $G^i(0, x) > 0, \forall x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$. It follows by the strict quasi-concavity of V^i that the correspondence $\Omega^i : \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}} \rightarrow [0, 0.5]$ characterized by $\Omega^i(x) := \{\delta \in [0, 0.5] : G^i(\delta, x) \geq 0.9G^i(0, x)\}$ is continuous and has non-empty and compact values. By the Berge's Maximum Theorem once again, the function $\widehat{\delta}^i : \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}} \rightarrow [0, 0.5]$ defined by $\widehat{\delta}^i(x) = \operatorname{argmax}_{\delta \in \Omega^i(x)} \delta$ is continuous and strictly positive. Notice that, by construction, $G^i(\widehat{\delta}^i(x), x) > 0, \forall x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$. Therefore, if we define

$$\varepsilon^i(x) := \widehat{\delta}^i(x) \frac{\min_{(s,l) \in \mathcal{S} \times \mathcal{L}} x_{s,l}}{1 + \max_{(p,s) \in \mathcal{P} \times \mathcal{S}} \sum_{k \in \mathcal{K}} R_{s,k}(p)}, \quad \forall x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}},$$

then the requirements of the compensation of small losses in segmented markets are satisfied by $(\tau^i(x), (\theta_s^i(p, x))_{s \in S}) = (a^i(x), (-\widehat{\delta}^i(x) \min_{(k,l) \in S \times \mathcal{L}} x_{k,l}(1, \dots, 1))_{s \in S})$. \square

When optimal consumption bundles are both market feasible and *uniformly bounded away from zero*, the compensation of small losses in segmented markets induce endogenous upper bounds for segmented asset prices. The main idea is the following: if $(x^i, z^i) \in C^i(p, q)$ is a market feasible optimal choice for agent i at prices (p, q) such that $x^i \gg 0$ and $z_k^i > 0$, then the price of k is bounded by $p_0 \tau^i(x^i) / \varepsilon^i(x^i)$.⁶ Since $p_0 \tau^i(x^i) / \varepsilon^i(x^i)$ is well defined and varies continuously with $(p, x^i) \in \mathcal{P} \times \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$, to obtain an upper bound for q_k is sufficient to ensure that market feasible optimal consumption allocations belong to a compact subset of $\mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$. Since market feasible allocations are bounded from above as a consequence of the scarcity of commodities, our focus is on the existence of positive lower bounds.

Thus, we require the *essentiality of commodities*, in the sense that indifference curves through individual's aggregated endowments do not intersect the boundary of the consumption set, a property that implies that $W^i \gg 0, \forall i \in \mathcal{I}$.⁷

THEOREM 1 (EQUILIBRIUM EXISTENCE)

Any standard economy has a competitive equilibrium when $V^i(W^i) > V^i(y), \forall i \in \mathcal{I}, \forall y \in \partial \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}}$.

As a byproduct of Theorem 1, we can guarantee the absence of robust examples of non-existence of equilibria when standard economies are parametrized by preferences and endowments.

REMARK (GENERIC EXISTENCE OF EQUILIBRIA IN STANDARD ECONOMIES)

Let \mathcal{G} be the set of standard economies parametrized by preferences and endowments. Given $\mathcal{E} = (V^i, w^i)_{i \in \mathcal{I}}$ and $\widetilde{\mathcal{E}} = (\widetilde{V}^i, \widetilde{w}^i)_{i \in \mathcal{I}}$ in \mathcal{G} , for each $n \in \mathbb{N}$ consider the pseudometric

$$\rho_n(\mathcal{E}, \widetilde{\mathcal{E}}) = \max_{i \in \mathcal{I}} \max_{x \in [n^{-1}, n]^{S \times \mathcal{L}}} |V^i(x) - \widetilde{V}^i(x)| + \max_{i \in \mathcal{I}} \|w^i - \widetilde{w}^i\|_{\Sigma}.$$

⁶If this is not the case, agent i may reduce her long position on k in $\rho \varepsilon^i(x^i) < z_k^i$ units, with $\rho \in (0, 1)$. With this operation she obtains resources to buy the compensation bundle $\rho \tau^i(x^i)$ without compromising the feasibility of second-period consumption. A contradiction with the optimality of her choices (see Lemma 1 in Appendix A).

⁷The essentiality of commodities was also imposed in smooth models of incomplete markets with price-dependent portfolio constraints, through the following requirement: for any agent $i \in \mathcal{I}$, and for each $\underline{x} \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}}$, $\{x \in \mathbb{R}_{++}^{\mathcal{L} \times \mathcal{S}} : V^i(x) \geq V^i(\underline{x})\}$ is a closed set (cf. Carosi, Gori and Villanacci (2009), Gori, Pireddu and Villanacci (2013), Hoelle, Pireddu, and Villanacci (2016)). Under strict monotonicity of preferences, this condition is stronger than our hypothesis.

Let $\mathcal{G}_n \subset \mathcal{G}$ be the set of economies $\mathcal{E} = (V^i, w^i)_{i \in \mathcal{I}}$ such that

$$V^i(W^i) > V^i(y), \quad \forall i \in \mathcal{I}, \forall y \in \partial \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} \cap [0, n]^{\mathcal{L} \times \mathcal{S}}.$$

Notice that \mathcal{G}_n is an open and dense subset of \mathcal{G} in the topology induced by ρ_n .⁸ In addition, although economies in \mathcal{G}_n do not necessarily satisfy the conditions of Theorem 1, the proof of this result implies that, for n large enough, any economy in \mathcal{G}_n has a non-empty set of equilibria.⁹ Thus, we find an open and dense subset of standard economies that have equilibria. \square

Previous results of equilibrium existence in markets where agents are subject to personalized trading constraints impose different kind of hypotheses in order to find endogenous upper bounds for financial prices. These requirements are summarized in the following assumption.

ASSUMPTION E

Assume that $(W^i)_{i \in \mathcal{I}} \gg 0$ and one of the following conditions hold:

(i) For any $(p, q) \in \mathbb{P}$, $\{i \in \mathcal{I} : \exists (x^i, z^i) \in \Phi^i(p), p_0 \cdot w_0^i - q \cdot z^i > 0\} = \mathcal{I}$.

(ii) There is a non-empty subset of agents $\mathcal{I}^* \subseteq \mathcal{I}$ such that:

- For each $i \in \mathcal{I}^*$, V^i is strictly increasing and finite in $\mathbb{R}_+^{\mathcal{L} \times \mathcal{S}}$.
- $\forall (\rho, x) \in (0, 1) \times \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}}, \exists \nu(\rho, x) \in \mathbb{R}_+^{\mathcal{L}} : (x_0 + \nu(\rho, x), (\rho x_s)_{s \in \mathcal{S}}) \in \mathcal{A}^i(x), \forall i \in \mathcal{I}^*$.
- For each $k \in \mathcal{K}$ there exists $z \in \mathbb{R}_+^{\mathcal{J}}$ such that $z_k > 0$ and $-(0, z) \in \bigcup_{i \in \mathcal{I}^*} \Phi^i(p), \forall p \in \mathcal{P}$.

(iii) There exists $(\tilde{x}_0, \tilde{z}) \in \mathbb{R}_+^{\mathcal{L}} \times \mathbb{R}_+^{\mathcal{J} \setminus \mathcal{K}}$ such that,

$$\sum_{k \in \mathcal{K}} R_{s,k}(p) \leq p_s Y_s \tilde{x}_0 + \sum_{j \in \mathcal{J} \setminus \mathcal{K}} R_{s,j}(p) \tilde{z}_j, \quad \forall s \in \mathcal{S}, \forall p \in \mathcal{P}.$$

Assumption E(i) is a *financial survival* condition, which requires that independently of prices all agents have access to some amount of liquidity at the first period. This hypothesis is incompatible with credit market segmentation and holds when $\mathcal{K} = \emptyset$.¹⁰ Assumption E(ii) was required by Seghir and Torres-Martínez (2011) to guarantee that *any reduction* on future consumption can be offset by

⁸ \mathcal{G}_n is open as a consequence of the continuity of utility functions and the definition of ρ_n . On the other hand, given $\mathcal{E} = (V^i, w^i)_{i \in \mathcal{I}} \in \mathcal{G}$, for any $m \in \mathbb{N}$ consider the standard economy $\tilde{\mathcal{E}}^m = (\tilde{V}^{i,m}, w^{i,m})_{i \in \mathcal{I}}$ characterized by

$$V^{i,m}(x) = V^i(x) + \frac{1}{m} \sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{L}} \ln(x_{s,l}), \quad \tilde{w}^{i,m} = w^{i,m} + \frac{1}{m} (1, \dots, 1), \quad \forall i \in \mathcal{I}.$$

It follows that $\{\tilde{\mathcal{E}}^m\}_{m \in \mathbb{N}} \subset \mathcal{G}_n$ and converges to \mathcal{E} as m increases.

⁹Following the notation of Appendix A, it is sufficient to consider $n > 3\overline{W}$.

¹⁰In the same spirit, Aouani and Cornet (2009, Assumption FN2) and Aouani and Cornet (2011, Assumption FS) require that agents have access to short-sell promises at non-arbitrage prices.

an increment on first-period demand.¹¹ Finally, Assumption E(iii) was imposed by Cea-Echenique and Torres-Martínez (2016) to ensure that deliveries of segmented contracts can be *super-replicated* by a portfolio composed by durable commodities and/or unsegmented contracts. Thus, losses in segmented markets can be fully hedged.

The following example shows that Theorem 1 is a new channel to prove equilibrium existence, as there are standard economies where commodities are essential and Assumption E does not hold.

EXAMPLE 2. Consider an economy with two states of nature at second period, $S = \{u, d\}$. There is only one perishable commodity, $\mathcal{L} = \{l\}$. Financial markets are characterized by a complete set of real Arrow securities, $\mathcal{J} = \{j_u, j_d\}$. There are two agents, $\mathcal{I} = \{a, b\}$, with identical preferences,

$$V^a(x_0, x_u, x_d) = V^b(x_0, x_u, x_d) = \left(\frac{x_0}{1 + x_0} \right)^{0.5} x_u^{0.25} x_d^{0.25},$$

and endowments satisfying $(w^a, w^b) \gg 0$. However, only a may short-sale j_u , because $\Phi^a(p_0) = \mathbb{R}_+^3 \times [-1, +\infty) \times [-1, +\infty)$ and $\Phi^b(p_0) = \mathbb{R}_+^3 \times [0, +\infty) \times [-1, +\infty)$.

Although this economy is standard and commodities are essential, Assumption E does not hold. Indeed, E(i) is not satisfied as j_u is segmented, while E(ii) fails for $x = (4, 4, 4)$ and $\rho = 0.25$. Also, E(iii) does not hold because the commodity is perishable and the unsegmented contract does not make promises at u . \square

4. SUPERMODULARITY

We develop a result of equilibrium existence in standard economies with one commodity at each state of nature and where preferences are represented by concave and supermodular utility functions, a framework compatible with risk-averse expected utility maximizers.

ASSUMPTION F

- (i) *There is only one commodity at each state of nature. For each $i \in \mathcal{I}$, $\mathbb{X}^i = \mathbb{R}_+^S$ and $(w_s^i)_{s \in S} \gg 0$.*
- (ii) *$(V^i)_{i \in \mathcal{I}}$ are concave and supermodular on an open set containing \mathbb{R}_+^S .*

For any standard economy satisfying Assumption F, individuals' marginal rates of substitution between current and future earnings are bounded, inducing upper bounds for the discounted value of assets' deliveries (see Appendix B for detailed arguments). Thus, to find an upper bound for

¹¹The framework of Seghir and Torres-Martínez (2011) only consider financial trading constraints that depend on consumption allocations. However, it can be extended to include price-dependent trading constraints (cf. Pérez-Fernández (2013), Cea-Echenique and Torres-Martínez (2014)). Assumption E(ii) includes the requirements considered by these extensions.

the price of a traded segmented asset, it is sufficient to avoid frictions associated to investors' non-negative constraints of second-period consumption. To attempt this objective we introduce the following hypothesis.

ASSUMPTION G

For each agent $i \in \mathcal{I}$, given $p \gg 0$ and $(x^i, z^i) \in \Phi^i(p)$,

$$\forall s \in S, \forall k \in \mathcal{K} : \quad [R_{s,k}(p) \neq 0 \wedge z_k^i > 0] \implies \sum_{j \in \mathcal{J}} R_{s,j}(p) z_j^i \geq 0.$$

Notice that this assumption and the interiority of endowments imply that an agent investing in a segmented contract demands a positive consumption at states of nature where she receives its deliveries. Also, B(ii) and G guarantee that $(R_{s,k}(p)R_{s,j}(p))_{s \in S} = 0$, $\forall p \gg 0$, $\forall k \in \mathcal{K}$, $\forall j \in \mathcal{J} \setminus \mathcal{K}$.¹² Thus, segmented contracts do not improve the opportunities of risk sharing at states of nature where unsegmented assets make promises, but reduce the incompleteness of financial markets by adding new states of nature to the support of the space of transfers. In particular, Assumption G holds when financial contracts are non-redundant Arrow securities.

THEOREM 2 (EQUILIBRIUM EXISTENCE)

Under Assumptions F and G, every standard economy has a competitive equilibrium.

The following example illustrates how this alternative result of equilibrium existence complements the previous findings of the literature.

EXAMPLE 3. Consider a standard economy with $S = \{u, d\}$. There is only one perishable commodity and one financial contract that promises one unit of the commodity at $s = u$. There are two agents, $\mathcal{I} = \{a, b\}$, characterized by utility functions $V^a(x_0, x_u, x_d) = V^b(x_0, x_u, x_d) = \frac{x_0}{1+x_0} + \frac{1}{4} \left(\frac{x_u}{1+x_u} + \frac{x_d}{1+x_d} \right)$, endowments $(w^a, w^b) = ((1, 0.5, 1), (1, 1, 0.5))$, and trading constraints $\Phi^a(p_0) = \mathbb{R}_+^3 \times [0, +\infty)$ and $\Phi^b(p_0) = \mathbb{R}_+^3 \times [-1, +\infty)$. In this context, Assumption E does not hold by identical arguments to those made in Example 2. Even more, commodities are not essential, as $V^a(4, 0, 0) > V^a(w^a)$. However, an equilibrium exists as a consequence of Theorem 2.¹³ \square

¹²Given $p \gg 0$, suppose that there is $s \in S$ and $(k, j) \in \mathcal{K} \times (\mathcal{J} \setminus \mathcal{K})$ such that $R_{s,k}(p)R_{s,j}(p) > 0$. Then, there exists $\delta > 0$ such that $-\delta e_j \in \Phi^i(p)$ for all agent $i \in \mathcal{I}$. Moreover, Assumption B(ii) implies that $\alpha e_k - \delta e_j \in \Phi^i(p)$, $\forall i \in \mathcal{I}$, $\forall \alpha > 0$. This is incompatible with Assumption G.

¹³It is not difficult to verify that $(\bar{p}, \bar{q}) = ((1, 1, 1), 16/49)$ are equilibrium prices associated to optimal decisions $((\bar{x}_0^a, \bar{x}_u^a, \bar{x}_d^a, \bar{z}^a), (\bar{x}_0^b, \bar{x}_u^b, \bar{x}_d^b, \bar{z}^b)) = ((59/63, 25/36, 1, 7/36), (67/63, 29/36, 0.5, -7/36))$.

5. CONCLUDING REMARKS

In the context of general equilibrium models with incomplete markets, we provide two results of equilibrium existence when agents are subject to personalized price-dependent trading constraints compatible with credit market segmentation. Instead of impatience conditions on preferences or super-replication properties ensuring the fully hedge of segmented contract promises, we focus on either the essentiality of commodities or the supermodularity of utility functions.

We presume that our methodologies to find endogenous upper bounds for asset prices can be easily extended to economies with more than two periods and long-lived assets. However, the extension to infinite horizon economies is more demanding: additional restrictions on trading constraints may be necessary to avoid Ponzi schemes and the effect of rational asset pricing bubbles may compromise the existence of uniform upper bounds for asset prices.

APPENDIX A: PROOF OF THEOREM 1

The proof of Theorem 1 follows analogous steps to those made in the proof of equilibrium existence by Cea-Echenique and Torres-Martínez (2016). Given $M \in \mathbb{N}$, consider the truncated space of prices $\mathbb{P}(M) := \{(p, q) \in \mathbb{P} : \|(p_0, (q_j)_{j \in \mathcal{J} \setminus \mathcal{K}})\|_{\Sigma} = 1 \wedge q_k \in [0, M], \forall k \in \mathcal{K}\}$.

Let $\mathbb{K} := [0, 3\bar{W}]^{\mathcal{L} \times \mathcal{S}} \times [-2\bar{\Lambda}, 2\#\mathcal{I}\bar{\Lambda}]^{\mathcal{J}}$, where

$$\begin{aligned} \bar{W} &:= 1 + \sum_{i \in \mathcal{I}} \|W^i\|_{\Sigma} + \max_{(p, s) \in \mathcal{P} \times \mathcal{S}} \left(\#\mathcal{J} \#\mathcal{I} 2\bar{\Lambda} \sum_{j \in \mathcal{J}} R_{s,j}(p) \right); \\ \bar{\Lambda} &:= \sup_{p \in \mathcal{P} : p \gg 0} \sup_{a \in \Lambda(p)} \|a\|_{\Sigma}. \end{aligned}$$

Notice that, $\bar{\Lambda}$ is finite as a consequence of Assumption D. Since indifference curves through individual's endowments do not intersect the boundary of the consumption set, there exists $\rho > 0$ such that, if $x \in [0, 3\bar{W}]^{\mathcal{L} \times \mathcal{S}}$ and $V^i(x) \geq V^i(W^i)$ for some $i \in \mathcal{I}$, then $x_{s,l} \geq \rho$, $\forall (s, l) \in \mathcal{S} \times \mathcal{L}$.¹⁴

Let $\Psi_M : \mathbb{P}(M) \times \mathbb{K}^{\mathcal{I}} \rightarrow \mathbb{P}(M) \times \mathbb{K}^{\mathcal{I}}$ be the correspondence given by

$$\Psi_M(p, q, (x^i, z^i)_{i \in \mathcal{I}}) = \mu_M((x^i, z^i)_{i \in \mathcal{I}}) \times \prod_{i \in \mathcal{I}} \phi^i(p, q),$$

where

$$\begin{aligned} \mu_M((x^i, z^i)_{i \in \mathcal{I}}) &:= \operatorname{argmax}_{(p, q) \in \mathbb{P}(M)} \sum_{s \in \mathcal{S}} p_s \sum_{i \in \mathcal{I}} (x_s^i - W_s^i) + q \sum_{i \in \mathcal{I}} z^i; \\ \phi^i(p, q) &:= \operatorname{argmax}_{(x^i, z^i) \in C^i(p, q) \cap \mathbb{K} : x_{s,l}^i \geq 0.5\rho, \forall (s, l) \in \mathcal{S} \times \mathcal{L}} V^i(x^i), \quad \forall i \in \mathcal{I}. \end{aligned}$$

It follows from Lemma 1 of Cea-Echenique and Torres-Martínez (2016, page 24) that for any agent $i \in \mathcal{I}$ the choice set correspondence $C^i : \mathbb{P}(M) \rightarrow \mathbb{E}$ is lower hemicontinuous with closed graph and non-empty

¹⁴Indeed, if this property is not satisfied, then the compactness of $[0, 3\bar{W}]^{\mathcal{L} \times \mathcal{S}}$ ensures that there is a sequence $\{x(m)\}_{m \in \mathbb{N}}$, with $V^i(x(m)) \geq V^i(W^i)$ for every m , which converges to $\partial \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}}$. Thus, the continuity of preferences and the essentiality of commodities implies that $V^i(x(m)) < V^i(W^i)$ for m high enough, which is a contradiction.

and convex values. Therefore, identical arguments to those made by Cea-Echenique and Torres-Martínez (2016, Lemma 2, page 25) guarantee that the set of fixed points of Ψ_M is non-empty.

Let

$$\mathcal{X} := \left\{ (x^i)_{i \in \mathcal{I}} \in \prod_{i \in \mathcal{I}} \mathbb{R}_+^{\mathcal{L} \times \mathcal{S}} : \sum_{i \in \mathcal{I}} x_0^i \leq \sum_{i \in \mathcal{I}} w_0^i \quad \wedge \quad \max_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}} \|x_s^i\|_{\Sigma} \leq 2\overline{W} \right\}.$$

LEMMA 1. *Under Assumptions A(i), B(iii), and C, let $(\bar{p}, \bar{q}, (\bar{x}^i, \bar{z}^i)_{i \in \mathcal{I}})$ be a fixed point of Ψ_M such that $\bar{p} \gg 0$ and $(\bar{x}^i)_{i \in \mathcal{I}} \in \mathcal{X}$. Then, for any $k \in \mathcal{K}$ we have that*

$$\sum_{i \in \mathcal{I}} \bar{z}_k^i > 0 \quad \implies \quad \bar{q}_k < \bar{Q} := \max_{i \in \mathcal{I}} \max_{x \in [0, 3\overline{W}]^{\mathcal{L} \times \mathcal{S}} : x_{s,l} \geq 0.5\rho, \forall (s,l) \in \mathcal{S} \times \mathcal{L}} \frac{\|\tau^i(x)\|_{\Sigma}}{\varepsilon^i(x)},$$

where $(\varepsilon^i, \tau^i)_{i \in \mathcal{I}}$ implements the compensation of small losses in segmented markets.

Proof. Suppose that there is excess of demand for $k \in \mathcal{K}$. Let h be an agent such that $\bar{z}_k^h > 0$. Since $\bar{p} \gg 0$ and $(\bar{x}_s^h)_{s \in \mathcal{S}} \gg 0$, there is $(\theta_s^h(\bar{p}, \bar{x}^h))_{s \in \mathcal{S}}$ in $\mathbb{R}^{\mathcal{L} \times \mathcal{S}}$ satisfying

$$\bar{p}_s \theta_s^h(\bar{p}, \bar{x}^h) \leq -\varepsilon^h(\bar{x}^h) \sum_{j \in \mathcal{K}} R_{s,j}(\bar{p}), \quad \forall s \in \mathcal{S},$$

such that $V^h(\bar{x}^h + (\tau^h(\bar{x}^h), (\theta_s^h(\bar{p}, \bar{x}^h))_{s \in \mathcal{S}})) > V^h(\bar{x}^h)$. Since $(\bar{x}^i)_{i \in \mathcal{I}} \in \mathcal{X}$, there exists $\nu \in (0, 1]$ such that $0 < \nu \varepsilon^h(\bar{x}^h) \leq \bar{z}_k^h$, $\bar{x}_0^h + \nu \tau^h(\bar{x}^h) \in [0, 3\overline{W}]^{\mathcal{L}}$, and $\bar{x}_s^h + \nu \theta_s^h(\bar{p}, \bar{x}^h) \in [0, 3\overline{W}]^{\mathcal{L}}$, $\forall s \in \mathcal{S}$. Hence, it follows from Assumption B(iii) that agent h can reduce her investment in k from \bar{z}_k^h to $\bar{z}_k^h - \nu \varepsilon^h(\bar{x}^h)$. As the strictly quasi-concavity of V^h implies that $V^h(\bar{x}^h + \nu(\tau^h(\bar{x}^h), (\theta_s^h(\bar{p}, \bar{x}^h))_{s \in \mathcal{S}})) > V^h(\bar{x}^h)$, the amount of resources that she obtains from this reduction, $\bar{q}_k \nu \varepsilon^h(\bar{x}^h)$, cannot be sufficient to buy the bundle $\nu \tau^h(\bar{x}^h)$. Since $(\bar{p}, \bar{q}) \in \mathbb{P}(M)$, we conclude that $\bar{q}_k < \bar{Q}$. \square

For any standard economy where commodities are essential the results above imply that, by identical arguments to those made in Cea-Echenique and Torres-Martínez (2016, Lemma 4, page 26), given $M > \bar{Q}$ the fixed points of Ψ_M are competitive equilibria. This concludes the proof of equilibrium existence.

APPENDIX B: PROOF OF THEOREM 2

It is sufficient to adapt the proof of Theorem 1 avoiding the lower bounds ρ in the definition of the best-reply correspondence Ψ_M and replacing Lemma 1 by the following result:

LEMMA 2. *Under Assumptions A(i), B(iii), C, F, and G, let $(\bar{p}, \bar{q}, (\bar{x}^i, \bar{z}^i)_{i \in \mathcal{I}})$ be a fixed point of Ψ_M such that $\bar{p} \gg 0$ and $(\bar{x}^i)_{i \in \mathcal{I}} \in \mathcal{X}$. Then, there exists $\widehat{Q} > 0$ independent of M such that,*

$$\forall k \in \mathcal{K} : \quad \sum_{i \in \mathcal{I}} \bar{z}_k^i > 0 \quad \implies \quad \bar{q}_k < \widehat{Q}.$$

Proof. Since there is only one commodity at each state of nature, (A1)(ii) implies that $(V^i)_{i \in \mathcal{I}}$ are strictly increasing. Suppose that there is excess of demand for a segmented contract k and let h be an agent such

that $\bar{z}_k^h > 0$. Since $(w_s^h)_{s \in S} \gg 0$, it follows from Assumption G that $\bar{x}_s^h > 0$ when $R_{s,k}(\bar{p}) > 0$. Hence, Kuhn-Tucker's Theorem guarantees that there are multipliers $(\lambda_s^h)_{s \in S} \gg 0$ such that,

$$\begin{aligned} \lambda_0^h \bar{p}_0 &\geq V^h(\bar{x}^h + e_0) - V^h(\bar{x}^h) + \sum_{s \in S} \lambda_s^h Y_s, & \lambda_0^h \bar{q}_k &= \sum_{s \in S} \lambda_s^h R_{s,k}(\bar{p}), \\ \lambda_s^h &\leq \frac{V^h(\bar{x}^h) - V^h(\bar{x}^h - \delta e_s)}{\delta}, & \forall \delta &\in (0, \bar{x}_s^h], \forall s \in S : R_{s,k}(\bar{p}) > 0, \end{aligned}$$

where $e_s \in \mathbb{R}_+^S$ is characterized by $e_{s,s} = 1$ and $e_{s,s'} = 0$ for all $s' \neq s$, and $e_{-s} := \sum_{s' \neq s} e_{s'}$. Notice that, the inequality in the first condition follows from B(ii), while the equality in the second condition is a consequence of B(ii) and B(iii). Assumptions B(ii) and B(iv) guarantee the last condition above.

Since $\bar{p}_0 \in (0, 1]$ and $(\bar{x}^i)_{i \in \mathcal{I}} \in \mathcal{X}$, it follows from F(ii) that for any $\delta > 0$ low enough,

$$\begin{aligned} \bar{q}_k &= \sum_{s \in S : R_{s,k}(\bar{p}) > 0} \frac{\lambda_s^h}{\lambda_0^h} R_{s,k}(\bar{p}) \\ &\leq \sum_{s \in S} \frac{V^h(\bar{x}^h) - V^h(\bar{x}^h - \delta e_s)}{\delta (V^h(\bar{x}^h + e_0) - V^h(\bar{x}^h))} R_{s,k}(\bar{p}) \\ &\leq \sum_{s \in S} \frac{V^h(3\bar{W}e_{-s} + \bar{x}_s^h e_s) - V^h(3\bar{W}e_{-s} + (\bar{x}_s^h - \delta)e_s)}{\delta (V^h((3\bar{W} + 1)e_0) - V^h(3\bar{W}e_0))} R_{s,k}(\bar{p}). \end{aligned}$$

where the last inequality follows from the supermodularity and concavity of V^h .

The concavity of V^h in an open set containing \mathbb{R}_+^S implies that the superdifferential $\partial V^h(x)$ is non-empty and compact for any $x \in [0, 3\bar{W}]^S$. Furthermore, as V^h is continuous and strictly increasing in second-period consumption, for each $s \in S$ we have that $\max_{x \in [0, 3\bar{W}]^S} \max_{\mu \in \partial V^h(x)} \mu \cdot e_s < +\infty$.

Therefore,

$$\begin{aligned} \bar{q}_k &\leq \sum_{s \in S} \frac{\max_{x \in [0, 3\bar{W}]^S} \max_{\mu \in \partial V^h(x)} \mu \cdot e_s}{V^h((3\bar{W} + 1)e_0) - V^h(3\bar{W}e_0)} R_{s,k}(\bar{p}) \\ &\leq \hat{Q} := \max_{i \in \mathcal{I}} \max_{j \in \mathcal{J}} \max_{p \in \mathcal{P}} \sum_{s \in S} \frac{\max_{x \in [0, 3\bar{W}]^S} \max_{\mu \in \partial V^i(x)} \mu \cdot e_s}{V^i((3\bar{W} + 1)e_0) - V^i(3\bar{W}e_0)} R_{s,j}(p). \end{aligned}$$

□

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