



# A column generation approach for solving generation expansion planning problems with high renewable energy penetration



Angela Flores-Quiroz<sup>a</sup>, Rodrigo Palma-Behnke<sup>a,\*</sup>, Golbon Zakeri<sup>b</sup>, Rodrigo Moreno<sup>a,c</sup>

<sup>a</sup> Energy Centre, Faculty of Mathematical and Physical Sciences, University of Chile, Santiago, Chile

<sup>b</sup> Department of Engineering Science, University of Auckland, New Zealand

<sup>c</sup> Department of Electrical and Electronic Engineering, Imperial College London, UK

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## ABSTRACT

The high penetration of renewables envisaged for future power systems will significantly increase the need for flexible operational measures and generation technologies, whose associated investment decisions must be properly planned in the long term. To achieve this, expansion models will need to incorporate unit commitment constraints, which can result in large scale MILP problems that require significant computational resources to be solved. In this context, this paper proposes a novel Dantzig–Wolfe decomposition and a column generation approach to reduce the computational burden and overcome intractability. We demonstrate through multiple case studies that the proposed approach outperforms direct application of commercial solvers, significantly reducing both computational times and memory usage. Using the Chilean power system as a reference case, we also confirm and highlight the importance of considering unit commitment constraints in generation expansion models.

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## 1. Introduction

### 1.1. Motivation: increased need for system flexibility

The high penetration of renewables envisaged for future power systems will significantly increase the requirement for flexible operational measures and generation technologies [1]. This requirement and the associated resources that can provide it must be properly considered in short term system operation and long term investment decisions [2].

In this framework, several papers have studied the impacts of variable renewable generation on the scheduling regime of thermal units, number of startup and shutdown operations of conventional plants, ramping requirements, and reserve requirements [3]. Although most studies focus on operational aspects of renewables, several papers report on impacts on the planned generation capacity mix [4,5], and fundamentally question the ability of traditional planning approaches (e.g. screening curve criteria [6]) to properly incorporate the need for resources that can provide various frequency control services and flexibility to systems with high penetration of renewables. Traditional planning approaches are usually based on a non-chronological representation of the load, ignoring

short term complexity, such as startup costs, minimum operational times, ramp rates, and minimum stable output constraints [4]. However, [5] shows that ignoring short term constraints in generation planning can lead to a suboptimal capacity mix with up to 17% higher operational costs in the case of the Electric Reliability Council of Texas.

### 1.2. Current generation expansion planning approaches

Although generation capacity expansion models have historically ignored short term constraints, recent effort attempted to include them in long term planning [5,7–10]. These were largely focused on solving the generation planning problem based on either heuristic [9,10] or optimization [5,7,8] methods. Heuristic methods such as those developed by Batlle and Rodilla [9] aim to improve the screening curve criteria to include renewable generation and the need for system flexibility. Although some improvements can be obtained with these types of methods, no optimality metrics have been reported.

Regarding optimization methods, [7] proposes stochastic expansion planning to increase system security with high penetration of renewables through integration of fast response (flexible) thermal units. They used a unit commitment model to represent the system operation (although startup costs and minimum up and down time constraints were neglected) and considered a 10-year horizon, where the computational burden was tackled by applying

\* Corresponding author. Tel.: +56 2 2978 4203.

E-mail address: [rodpalma@cec.uchile.cl](mailto:rodpalma@cec.uchile.cl) (R. Palma-Behnke).

## Nomenclature

### Investment variables

$IG_{y,g}$	additional units installed in year $y$ of generator type $g$
$\mathbf{IG}_y$	vector of additional units installed in year $y$
$IG'_{y,g}$	total available units to operate in year $y$ of generator type $g$
$\mathbf{IG}'_y$	vector of total available units to operate in year $y$

### Operational variables

$D_{y,t,g}$	number of shutdowns at hour $t$ in year $y$ for generator type $g$
$LS_{y,t}$	load shedding at hour $t$ in year $y$ [MW]
$P_{y,t,g}$	power supplied by generator type $g$ at hour $t$ in year $y$ [MW]
$Rp_{y,t,g}$	primary reserve of generator type $g$ at hour $t$ in year $y$ [MW]
$Rs_{y,t,g}^{up}$	up secondary reserve provided by generator type $g$ at hour $t$ in year $y$ [MW]
$Rs_{y,t,g}^{down}$	down secondary reserve provided by generator type $g$ at hour $t$ in year $y$ [MW]
$S_{y,t,g}$	number of startups at hour $t$ in year $y$ of generator type $g$
$u_{y,t,g}$	number of committed units at hour $t$ in year $y$ of generator type $g$
$\mathbf{X}_y$	vector of operational variables in year $y$

### Parameters

$\alpha^{VR}$	percentage of the variable generation output covered by secondary reserve
$A_y$	matrix that couples operational and investment decisions in year $y$
$C_{UD}$	unsupplied demand cost [USD/MWh]
$c_{y,g}^{inv}$	investment cost annuity of generator type $g$ in year $y$ [USD/MW]
$\mathbf{c}_y^{inv}$	vector of discounted investment cost in year $y$ [USD/MW]
$\mathbf{c}_y^{op}$	vector of discounted operational cost in year $y$ [USD/MWh]
$C_g^S$	startup cost of generator type $g$ [USD]
$C_{y,g}^{var}$	variable cost of generator type $g$ in year $y$ [USD/MWh]
$L_{y,t,g}$	load at hour $t$ in year $y$ [MW]
$L_y^{\max}$	maximum load in year $y$ [MW]
$J_g^{\max}$	maximum units to be installed of generator type $g$
$NG$	number of generators types
$p_{y,t,g}^{avail}$	available generation at hour $t$ in year $y$ as a proportion of the maximum capacity of one unit of generator type $g$
$p_g^{firm}$	firm capacity of generator type $g$
$p_g^{\max}$	maximum capacity of a unit of generator type $g$ [MW]
$p_g^{\min}$	minimum capacity of a unit of generator type $g$ [MW]
$P_g^S$	maximum output of generator type $g$ when started [MW]
$r$	discount rate
$Rdn_g$	maximum down ramp rate of a unit of generator type $g$ [MW/h]
$RM$	planning reserve margin
$Rp_g^{\max}$	maximum primary reserve capacity of generator type $g$ [MW]
$Rs_g^{\max}$	maximum secondary reserve capacity of generator type $g$ [MW]

$Rrp_{y,t}$	primary reserve requirement at hour $t$ in year $y$ [MW]
$Rrs_{y,t}$	secondary reserve requirement at hour $t$ in year $y$ [MW]
$Rup_g$	maximum up ramp rate of a unit of generator type $g$ [MW/h]
$T$	number of subperiods
$t_g^{off}$	minimum down time of generator type $g$ [h]
$t_g^{on}$	minimum up time of generator type $g$ [h]
$Y$	number of years in the planning horizon

### Operational set and index

$g$	index of generator types
$g \in TH$	set of thermal generators
$g \in F$	set of fast response generators
$g \in VR$	set of variable renewable generators
$t$	index of subperiod
$y$	index of planning year

Benders decomposition. However, under this approach slave sub-problems must be linear and, hence, all integer variables, such as unit commitment states, have to lie in the master problem [11], leading to a high dimensional integer programming (IP) master problem. Consequently, [7] considered planning for a reduced set of fast response gas turbines, with future wind and thermal generation capacity assumed to be fixed.

The unit commitment problem was extended by Ma et al. [8] to control investment decisions in the entire generation mix, minimizing the sum of investment and operational costs. The resulting combined unit commitment and capacity expansion (C-UC-CE) model, was a high dimensional mixed integer linear problem (MILP). Computational burden was limited by considering a small number of units (e.g. 26 units) with a planning horizon of a single year, represented by a reduced number of typical weeks (e.g. 5 weeks). Similarly, [5] proposed a C-UC-CE model that aimed to avoid combinatorial explosion by grouping similar generators into equivalent plants, driving reductions in computational time by approximately a factor of 400. In addition to including unit commitment constraints, [12] represented long term uncertainty through a reduced number of sample weeks for wind and load profiles, and short term wind forecast error was included as an additional amount of operating reserves as a fraction of wind power forecast. Although these methods have reported clear improvements in terms of computational performance, only static planning was performed and further developments are necessary to solve planning problems on realistic sized systems in a multi-stage fashion.

### 1.3. Paper contribution and structure

Development of C-UC-CE models is critical for improving investment decisions in generation infrastructure in future power systems. New methods to tackle the high complexity of such problems are required to obtain fast and timely solutions of large C-UC-CE models. We propose using a Dantzig–Wolfe decomposition together with a Column Generation approach [13], to solve C-UC-CE models in a multi stage fashion with significant reduction in solution times. Additionally, we use concepts and methods from current power system literature to overcome intractability through clustering techniques, tight formulations of the unit commitment problem, and classical screening curves as an initial point for the Column Generation approach. We analyze the benefits of our proposed approach through multiple case studies and demonstrate that C-UC-CE problems can be solved with reduced

timescales and efficient RAM usage. Furthermore, we stress the importance of planning generation capacity through a C-UC-CE model and demonstrate that neglecting short term constraints in planning can result in suboptimal portfolios of generation technologies which may not deliver sufficient flexibility to facilitate integration of renewables.

Section 2 presents the C-UC-CE problem. Section 3 details the proposed Dantzig–Wolfe decomposition together with the Column Generation approach used to solve the C-UC-CE problem. Section 4 presents the results for several test cases, including the Chilean Power System. Section 5 summarizes our conclusions and future work.

## 2. Generation expansion model

As explained in [5], the objective of a C-UC-CE model is to minimize the sum of investment and operational costs for a certain planning horizon. The proposed model considered a single busbar with no transmission constraints and, hence, decisions were made on (i) which generating units to build each year, (ii) which units to commit in a period of time (e.g. an hour), and (iii) how much power each unit generates, simulating the decisions made by a central planner. This approach can be used by utilities for benchmark analysis and by policy makers and regulators to assess and define alternative policies and regulatory frameworks to achieve future targets [14].

For the case of a system with thermal (TH) and variable renewable (VR) generators, the optimization problem is:

**Objective function:** Minimize total investment costs for each year on the planning horizon and operational cost on each period of time,

$$\min \sum_{y=1}^Y \frac{CInv_y}{(1+r)^y} + \sum_{y=1}^Y \frac{CO_y}{(1+r)^y}, \quad (1)$$

where the investment cost in year  $y$  is

$$CInv_y = \sum_{g=1}^{NG} \sum_{l=1}^y c_{l,g}^{inv} IG_{l,g}, \quad (2)$$

and the operational cost for each year is

$$CO_y = \sum_{t=1}^T \left( \sum_{g=1}^{NG} (c_y^{var} P_{y,t,g} + c_g^s S_{y,t,g}) + C_{UD} \cdot LS_{y,t} \right). \quad (3)$$

**Unit clustering:** Incorporating unit commitment decisions into a capacity expansion model leads to a high dimensional mixed integer linear problem. However, we can reduce the complexity by combining similar power plants into a single equivalent generator, following [5] (i.e., if similar generators (with the same technology) are available to be built, we can cluster them into units of a representative plant). Hence, rather than a binary variable for each independent unit, investment and commitment decisions may be represented by integer variables for the cluster of units with upper and lower bounds,

$$0 \leq u_{y,t,g} \leq \sum_{l=1}^y IG_{l,g} \leq I_g^{\max}. \quad (4)$$

Palmitier and Webster [5] show that clustering does not imply significant loss of accuracy but does significantly reduce solution time. Our proposed decomposition can also be applied to the C-UC-CE problem without clustering with the benefit of reducing solution time and overcoming intractability.

**Main constraints:** The solution must meet several technical constraints.

- (a) *Power balance constraint:* Total power generated plus the potential unserved load must be equal to the load at all time periods, i.e.,

$$\sum_{g=1}^{NG} P_{y,t,g} + LS_{y,t} = L_{y,t} \quad \forall y, t \quad (5)$$

- (b) *Operating reserves:* Reserve must be ensured to maintain the security of the power system, such as primary reserve that operates within a few seconds right after the imbalance occurs,

$$\sum_{g \in TH} Rp_{y,t,g} \geq Rrp_{y,t} \quad \forall y, t; \quad (6)$$

and secondary up and down reserve that operates within 30 s and 15 min,

$$\sum_{g \in TH} Rs_{y,t,g}^{up} - \sum_{g \in VR} \alpha^{VR} P_{y,t,g} \geq Rrs_{y,t}^{up} \quad \forall y, t, \quad (7)$$

and

$$\sum_{g \in TH} Rs_{y,t,g}^{down} - \sum_{g \in VR} \alpha^{VR} P_{y,t,g} \geq Rrs_{y,t}^{down} \quad \forall y, t, \quad (8)$$

respectively. Additional secondary reserve must be held for renewables to account for short term uncertainty [12]. The primary reserve requirement  $Rrp_{y,t}$  considers forced outages and the secondary reserve requirements  $Rrs_{y,t}^{up}$  and  $Rrs_{y,t}^{down}$  also consider load variations.

- (c) *Minimum and maximum generation limits:* Generator outputs must be within their limits, i.e., no more than the maximum capacity can be generated (including reserves), and the output must be at least the minimum output required to have stable power production,

$$P_{y,t,g} + Rp_{y,t,g} + Rs_{y,t,g}^{up} \leq u_{y,t,g} P_g^{\max} \quad \forall y, t \quad \forall g \in TH, \quad (9)$$

and

$$u_{y,t,g} P_g^{\min} + Rs_{y,t,g}^{down} \leq P_{y,t,g} \quad \forall y, t \quad \forall g \in TH. \quad (10)$$

- (d) *Reserve capabilities:* The primary and secondary reserves provided by generators are limited to maximum values. The primary reserve limit is based on the frequency response of a generator and the secondary reserve limit is based on its ramping capability [15,16],

$$Rp_{y,t,g} \leq u_{y,t,g} Rp_g^{\max} \quad \forall y, t \quad \forall g \in TH, \quad (11)$$

$$Rs_{y,t,g}^{up} \leq u_{y,t,g} Rs_g^{up \max} \quad \forall y, t \quad \forall g \in TH, \quad (12)$$

and

$$Rs_{y,t,g}^{down} \leq u_{y,t,g} Rs_g^{down \max} \quad \forall y, t \quad \forall g \in TH. \quad (13)$$

- (e) *Startup and shutdown:* Variables are introduced to incorporate startup costs and minimum up and down times,

$$u_{y,t,g} = u_{y,t-1,g} + S_{y,t,g} - D_{y,t,g} \quad \forall y, t \quad \forall g \in TH. \quad (14)$$

- (f) *Ramp rates:* Generators are limited on how fast they can adjust their output power from one period to another,

$$P_{y,t,g} - P_{y,t-1,g} \leq u_{y,t-1,g} Rup_g + S_{y,t,g} P_g^s \quad \forall y, t \quad \forall g \in TH, \quad (15)$$

and

$$P_{y,t-1,g} - P_{y,t,g} \leq u_{y,t-1,g} Rdn_g + D_{y,t,g} P_g^{\max} \quad \forall y, t \quad \forall g \in TH. \quad (16)$$

- (g) *Minimum up and down times*: Once a unit has been switched on, it must run for a minimum time and once a unit has been switched off, it must remain offline for a minimum time,

$$u_{y,t,g} \geq \sum_{\tau=t-t_g^{on}}^t S_{y,\tau,g} \quad \forall y, t \quad \forall g \in TH, \quad (17)$$

and

$$\sum_{l=1}^y IG_{l,g} - u_{y,t,g} \geq \sum_{\tau=t-t_g^{off}}^t D_{y,\tau,g} \quad \forall y, t \quad \forall g \in TH, \quad (18)$$

respectively.

- (h) *Units to be committed*: For thermal generators, the maximum number of units that can be committed is limited by the number of generators that have been built<sup>1</sup>,

$$u_{y,t,g} \leq \sum_{l=1}^y IG_{l,g} \quad \forall y, t \quad \forall g \in TH. \quad (19)$$

- (i) *Renewable generator output*: The power generated by renewable generators is limited by the number of units installed and the availability of the primary source in each period,

$$P_{y,t,g} \leq \left( \sum_{l=1}^y IG_{l,g} \right) P_g^{\max} P_{y,t,g}^{avail} \quad \forall y, t \quad \forall g \in VR. \quad (20)$$

- (j) *Planning reserve margin*: A planning reserve margin has to be considered for the system adequacy,

$$P_g^{firm} \sum_{l=1}^y IG_{l,g} \leq (1 + RM) L_y^{\max} \quad \forall y. \quad (21)$$

The above problem considers that all variables are continuous and equal to or larger than zero, except for  $IG_{y,g}$ ,  $u_{y,t,g}$ ,  $S_{y,t,g}$ , and  $D_{y,t,g}$ , which are integer. Note that the proposed model considers a single busbar with no representation of transmission constraints, as in [5,8,9], to evaluate a first application of the proposed decomposition method on C-UC-CE problems. However, the proposed approach can be expanded to address multiple busbars, with a corresponding increase in the problem size. Accordingly, when solving these types of problems, trade-off between accuracy and complexity must be considered to obtain coherent and practical results.

### 3. Proposed column generation solution approach

The combined unit commitment and capacity expansion planning problem presents significant challenges if fast solutions are required for large input datasets. We propose Dantzig–Wolfe decomposition [17] to reduce computational burden and overcome intractability, by exploiting the particular (block diagonal) structure of the MILP problem formulated in Section 2. A similar approach was developed in [13] to solve the expansion planning problem for a distribution network. We selected Dantzig–Wolfe rather than Benders decomposition, because the latter does not allow integration of integer variables in the slave subproblem, forcing the consideration of both investment and unit commitment decisions at the master problem level, and thereby leading to a high dimensional integer master problem.

#### 3.1. Reformulation of combined unit commitment and capacity expansion problem

In the optimization problem presented in Section 2, (18)–(21) are complicating constraints of the problem that couple the time stages by relating operation variables in year  $y$  with investment decisions in years  $l = 1, \dots, y$ . Hence, we introduce an extra variable,  $IG'_{y,g}$ , to facilitate application of the decomposition.  $IG'_{y,g}$  represents the available units to operate in year  $y$  for generator type  $g$ , and must meet the constraint

$$IG'_{y,g} \leq \sum_{l=1}^y IG_{l,g} \quad \forall y, g. \quad (22)$$

Thus, we can rewrite constraints (18)–(21) in terms of the available units as

$$IG'_{y,g} - u_{y,t,g} \geq \sum_{\tau=t-t_g^{off}}^t D_{y,\tau,g} \quad \forall y, t \quad \forall g \in TH, \quad (23)$$

$$u_{y,t,g} \leq IG'_{y,g} \quad \forall y, t \quad \forall g \in TH, \quad (24)$$

$$P_{y,t,g} \leq IG'_{y,g} P_g^{\max} P_{y,t,g}^{avail} \quad \forall y, t \quad \forall g \in VR, \quad (25)$$

and

$$P_g^{firm} IG'_{y,g} \leq (1 + RM) L_y^{\max} \quad \forall y. \quad (26)$$

The objective function and all other constraints remain the same in the reformulated problem.

#### 3.2. Dantzig–Wolfe decomposition

For the sake of simplicity, we rewrite the reformulated problems in a matrix form, where matrices and vectors are highlighted in bold,

$$\min \sum_{y=1}^Y \sum_{l=1}^y c_l^{\text{inv}T} IG_l + \sum_{y=1}^Y c_y^{\text{op}T} X_y \quad (27a)$$

$$\text{s.t. : } IG'_y \leq \sum_{l=1}^y IG_l \quad \forall y \quad (27b)$$

$$A_y X_y \leq IG'_y \quad \forall y \quad (27c)$$

$$IG'_y \leq I_y^{\max} \quad \forall y \quad (27d)$$

$$X_y \in \mathcal{X}_y \quad \forall y \quad (27e)$$

$$IG'_y, IG_y \in \mathbb{Z}_+^{NG} \quad \forall y \quad (27f)$$

Set  $\mathcal{X}_y$  is the feasible region of operation decisions in year  $y$ . Constraint (27b) couples the time stages by relating available units in year  $y$  with the investment decisions in years  $l = 1, \dots, y$ . If (27b) were relaxed, the investment problem for each year could be solved independently. Hence (27) has a diagonal block structure that can be exploited through the Dantzig–Wolfe decomposition.

The feasible region of available units in year  $y$  can be defined by

$$\mathcal{I}_y = \{IG'_y \in \mathbb{Z}_+^{NG} \mid \exists X_y \in \mathcal{X}_y, A_y X_y \leq IG'_y, IG'_y \leq I_y^{\max}\},$$

where  $\mathcal{I}_y$  is a bounded integer polyhedron since investment options are limited. Therefore, any point in  $\mathcal{I}_y$  can be expressed as an integer

<sup>1</sup> This constraint is redundant (see (4)) but it is presented again for better understanding.

combination of a finite number of integer points,  $\{\widehat{IG}_y^j\}_{j=1}^{K(y)}$ , in  $\mathcal{I}_y$  [18] such that

$$IG'_y = \sum_{j=1}^{K(y)} \lambda_y^j \widehat{IG}_y^j, \quad \sum_{j=1}^{K(y)} \lambda_y^j = 1, \quad \lambda_y^j \in \{0, 1\}.$$

Additionally, for each feasible vector of available units in year  $y$ ,  $\widehat{IG}_y^j$ , there exist at least one associated optimal operational plan,  $\widehat{X}_y^j$ , with operational cost  $c_y^{op \top} \widehat{X}_y^j$ .

We can substitute  $IG'_y$  and  $\mathbf{X}_y$  in (27), to obtain the master problem of the Dantzig Wolfe decomposition,

$$MP : Z_{LP}^{MP} = \min \sum_{y=1}^Y \sum_{l=1}^y c_l^{inv \top} IG_l + \sum_{y=1}^Y \sum_{j=1}^{K(y)} \lambda_y^j c_y^{op \top} \widehat{X}_y^j \quad (28a)$$

$$\text{s.t. : } \sum_{j=1}^{K(y)} \lambda_y^j \widehat{IG}_y^j \leq \sum_{l=1}^y IG_l \quad \forall y \quad (28b)$$

$$\sum_{j=1}^{K(y)} \lambda_y^j = 1 \quad \forall y \quad (28c)$$

$$\lambda_y^j \in \{0, 1\} \quad \forall y, j \quad (28d)$$

$$IG_y \in \mathbb{Z}_+^{NG} \quad \forall y \quad (28e)$$

Constraint (28b) guarantees that the investments are made coherently with the selected vector of available units. Since  $IG_y$  is the vector of additional units installed in year  $y$ , once a unit has been installed it remains available to operate for the following years. Constraints (28c) and (28d) ensure that only one vector of available units, and consequently only one operational plan, is selected for each year.

Since we cannot enumerate all feasible vectors of available units, they must be produced during the solution procedure based on a column generation (CG) approach [19]. CG solves first a linear relaxation version of the master problem containing only an initial feasible vector of available units for each year (the restricted master problem, with objective function  $Z_{LP}^{RMP}$ ). The value of the dual variables is then extracted and used to generate the new column by solving

$$sp(y) : \min \quad c_y^{op \top} \mathbf{X}_y - IG'_y \pi_y - \mu_y \quad (29a)$$

$$\text{s.t. : } \mathbf{X}_y \in \mathcal{X}_y \forall y \quad (29b)$$

$$IG'_y \in \mathbb{Z}_+^{NG} \quad \forall y \quad (29c)$$

for each year,  $y$  [19], the new column is added to the restricted master problem and the CG process is repeated until  $LPgap$  is below a given threshold,

$$LPgap = \frac{Z_{LP}^{RMP} - \left( Z_{LP}^{RMP} + \sum_{y=1}^Y sp(y) \right)}{\left( Z_{LP}^{RMP} + \sum_{y=1}^Y sp(y) \right)}. \quad (30)$$

Since a linear version of the master problem is solved, fractional values can be obtained for integer variables. Hence after the linear master problem has converged, we force integrality in the master problem, and check  $MIPgap$ ,

$$MIPgap = \frac{Z_{LP}^{RMP} - \left( Z_{LP}^{RMP} + \sum_{y=1}^Y sp(y) \right)}{\left( Z_{LP}^{RMP} + \sum_{y=1}^Y sp(y) \right)}, \quad (31)$$

If this gap is below a given threshold, the solution is accepted, otherwise a branch and price procedure is used.

### 3.3. Detailed solution algorithm and implementation

Fig. 1 shows the flowchart of the complete solution procedure.

- **Step 1:** Input data detailing existing units, investment options, hourly load of the base year, load growth rate, fuel prices, investment costs, and renewable generation data.
- **Step 2:** Determine an initial solution of yearly generation capacity mix using the screening curve method [6]. Determine the corresponding operational cost by running a unit commitment that satisfies constraints (5)–(21). This process provides feasible initial columns in a fast and efficient fashion, benefiting from traditional energy based capacity expansion models.
- **Step 3:** Construct the restricted master problem (28) and solve its LP relaxation. Extract the dual prices.
- **Step 4:** Construct the subproblem (29) for each year  $y$  using the dual prices obtained in step 3, and solve them to obtain a feasible vector of available units and operational costs. Parallel computing can be used for this step, since each subproblem requires only information from the master problem and there is no coupling amongst them.
- **Step 5:** If LP gap (30) is less than the threshold value, go to step 6. Else, add the generated columns to the restricted master problem (28) and go to step 3.
- **Step 6:** Solve the integer restricted master problem (28).
- **Step 7:** Check the MIP gap (31), if this is less than the required threshold, the solution has been found. Else, add the generated columns to the restricted master problem (28) and go to step 3.

We implemented an interior point stabilization scheme, as proposed by Rousseau et al. [20], to improve convergence of the CG approach.

## 4. Results and discussion

We compared outcomes from several small scale case studies for a C-UC-CE problem with and without the proposed decomposition approach. To demonstrate the significantly enhanced computational performance of the proposed model, we determined expansion of the generation mix of a realistic power system in Chile (the northern interconnected system) for 2012–2030. We also demonstrate the importance of considering short term constraints in long term generation planning. Models with and without the decomposition approach were implemented on the Levque Cluster at the University of Chile, composed of 66 nodes, each equipped with two 2.67 GHz quad-core Intel Xenon X5550 processors and 24 GB RAM. Both models were implemented in Java and solved using CPLEX 12.5. The master problem was solved using a barrier algorithm with disabled presolve and reductions. An optimality gap of 0.5% was requested.

### 4.1. Typical weeks

To limit the C-UC-CE problem size, and allow comparison with the proposed decomposition approach, the year was represented by a reduced number of typical weeks. This allows consideration of hourly and seasonal variations in demand and wind profiles while limiting the solution time [8].

A representative week was characterized by hourly system load and wind availability. The year was represented by 13 typical weeks, one for each month plus the week where the highest load was observed. For each month, hourly wind and load profiles were

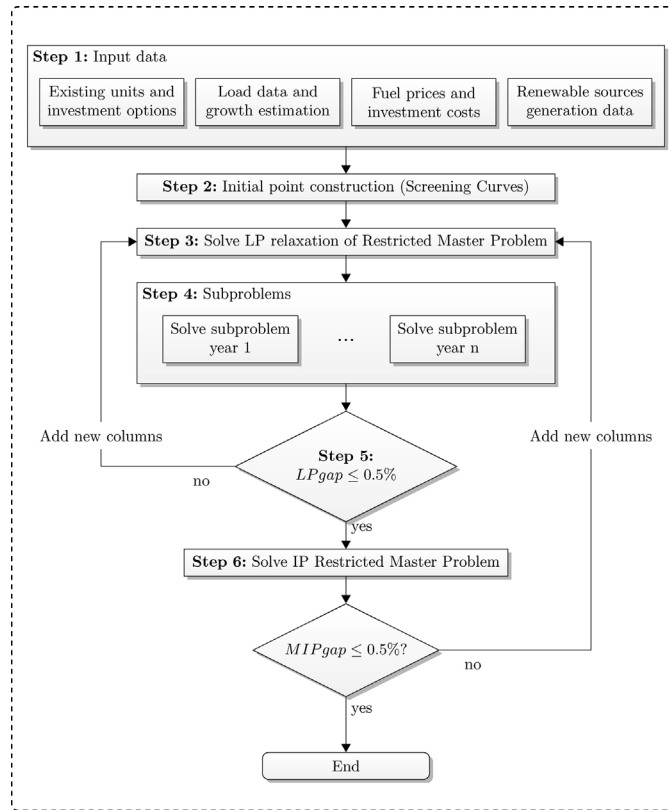
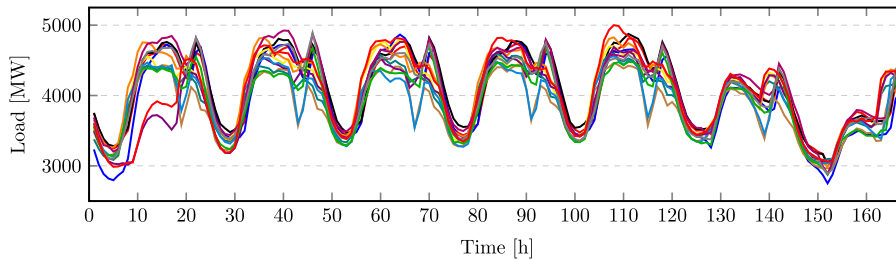
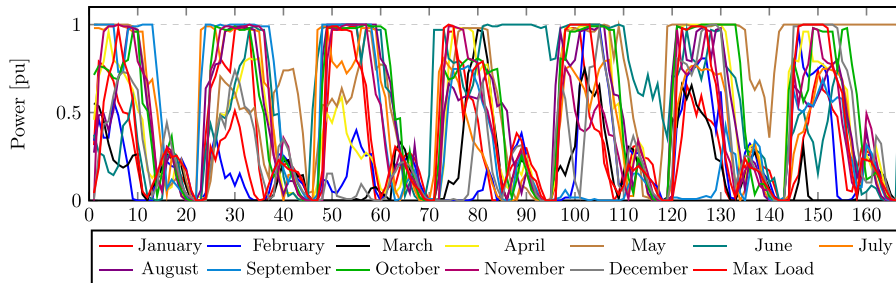


Fig. 1. Flowchart of the proposed planning process.



(a) Representative load profiles



(b) Representative wind profiles

Fig. 2. Representative weekly profiles.

processed using K-Medoids clustering to obtain a representative profile [21]. Fig. 2(a) and (b) shows the representative load and wind profiles, respectively.

We assumed that each representative week was repeated sequentially until completing a month. Thus, for each unit, the commitment state at the initial hour and last hour of the representative

week must be equal [8]. No linkage was considered among different months. In the objective function, the operating cost for each selected week was weighted to reflect the yearly operating cost.

This formulation could be extended to consider seasonal uncertainties following the methodology proposed in [12], where several wind profiles were generated for each seasonal or monthly load

**Table 1**  
Generation investment options.

Generator	Coal	OCGT	Diesel	Fuel oil	Wind
Units	300	100	50	50	10
Max. output [MW]	150	160	200	200	100
Min. output [MW]	100	90	0	0	0

profile. Future research will be directed conducted to solve a stochastic formulation of this problem.

#### 4.2. Small scale case studies

##### 4.2.1. Input data and assumptions

We considered various planning horizons, from 1 to 31 years (2012–2042), to test the performance of (1)–(21) solved using the proposed decomposition approach compared with the same formulation solved through a commercial MIP-solver (CPLEX). Five technologies were considered to be available for expansion (in a greenfield investment mode): coal, open cycle gas turbines (OCGT), diesel, fuel oil, and wind, as shown in Table 1.

Although case studies are meant to be illustrative, we used real 2011 hourly load profiles of the Chilean system, with a peak demand scaled to 5000 MW, a yearly energy of 35.04 TWh, yearly growth rate of 5%, and network losses of 3.5% of demand. We also used real data with 40% capacity factor to represent wind availability through an aggregated hourly wind profile. The 3+5 rule [22] was used for secondary reserve. The investment costs for the different technologies were obtained from the technical study of the Chilean Ministry of Energy [23]. The average fuel prices in year 2012 where obtained from the Chilean Northern System Operator [24] and the price projection is based on the growth rate used in [23].

##### 4.2.2. Multiple time horizons

We ran 31 case studies with increasing time horizons for both problems: (i) the proposed CG approach, and (ii) without decomposition (i.e., original formulation). Fig. 3(a) shows the objective function values (over multiple modelled planning horizons) and demonstrates the accuracy of the proposed CG approach.

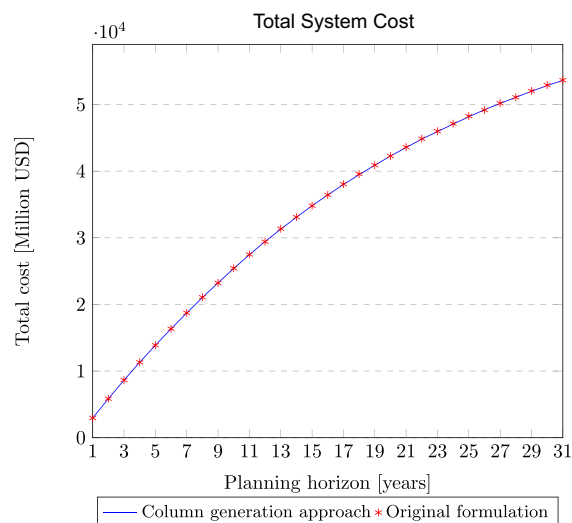
The proposed CG approach becomes significantly faster than the original formulation with increasing planning horizon, as shown in Fig. 3(b). The proposed approach is more than 4 times faster for cases with planning horizon of 31 years. Memory usage is also significantly improved, with the CG approach using a maximum of 1.6 GB of RAM, whereas the problem without decomposition requires  $\approx 7.7$  GB. For the case of 31 years, the original formulation has  $1760.5 \times 10^3$  variables and  $1691.6 \times 10^3$  constraints, while the subproblems have  $56.8 \times 10^3$  variables and  $54.6 \times 10^3$  constraints. It can be seen that each subproblem is considerably smaller and hence the proposed decomposition is critical to tackle larger scale expansion problems with several years in the planning horizon.

For all validations, there was no need to perform a branching procedure when employing the CG approach, because integer solutions were obtained within the desired optimality gap.

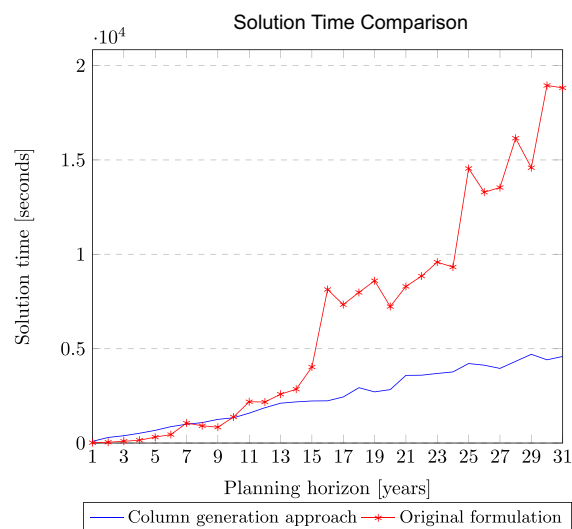
#### 4.3. Large-scale case study: the Chilean (northern) system

##### 4.3.1. Input data and assumptions

We used the Chilean northern interconnected system to test the performance of our long term planning approach. This case presents 47 existing units, clustered in 28 groups. Ten types of generating units were considered for future expansion, as shown in Table 2. We



(a) Comparison of the objective function value of both methodologies for validation cases.



(b) Comparison of the solution time of both methodologies for validation cases.

**Fig. 3.** Computational results.

used the real 2011 hourly load profile of the Chilean system, with yearly energy of 16.21 TWh, peak demand of 2161.8 MW [24], an average growth rate of 5.9% and network losses of 3.5% of demand. We also used real data to represent 3 types of wind units through aggregated availability hourly profiles with capacity factors of 37%, 36%, and 46%. Solar plants were represented through a single availability hourly profile with a capacity factor of 30%, obtained from [25]. The 3+5 rule [22] was used for secondary reserve. The year was represented by 13 typical weeks (one for each month plus one for the week with the peak load). The investment costs for the different technologies were obtained from the technical study of the Chilean Ministry of Energy [23]. The average fuel prices in year 2012 where obtained from the Chilean Northern System Operator [24] and the price projection is based on the growth rate used in [23]. An optimality gap of 0.5% is used.

**Table 2**  
Generation investment options large-scale case.

Generator name	Coal 1	Coal 2	OCGT 1	OCGT 2	Diesel	Fuel oil	Wind 1	Wind 2	Wind 3	Solar
Units	10	30	10	10	10	10	3	100	100	100
Max. output [MW]	250	150	226	150	30	50	100	100	100	100
Min. output [MW]	150	100	160	75	0	0	0	0	0	0

**Table 3**  
Generation investment options: large scale case.

N typical weeks	CG approach				Original formulation	
	Total time	Time MP	N iterations	RAM usage	Total time	RAM usage
1	2.08 h	0.76 seg	19	630 MB	3.01 h	10 GB
2	9.26 h	0.83 seg	22	1 GB	30.37 h	21 GB
5	50.86 h	0.85 seg	23	4 GB	-	Out of memory
13	15 days	1.04 seg	25	9 GB	-	Out of memory

4.3.2. Large scale model performance

The CG algorithm took 15 days to achieve a solution with an optimality gap of 0.49%. The total investment plus operational cost of the system was 14,800 million USD. In contrast, attempting to solve the original formulation without decomposition failed as CPLEX could not find a solution due to limited RAM. The original formulation has  $8921.8 \times 10^3$  variables and  $7584.1 \times 10^3$  constraints, while the subproblems have  $469.6 \times 10^3$  variables and  $399.1 \times 10^3$  constraints.

We performed additional study cases with reduced number of typical weeks to compare the CG based model against an achieved solution using the original formulation, as shown in Table 3. Even for a small number of typical weeks, our proposed approach significantly reduces solution time and memory usage, and the benefits are more significant as the number of typical weeks, and thus size of the problem, increase.

For our proposed approach, the time used solving each master problem is negligible and practically all the solution time is spent in solving the subproblems. Each subproblem was solved sequentially. However these could be solved in parallel on each iteration, which could lead to significant improvements on the solution time.

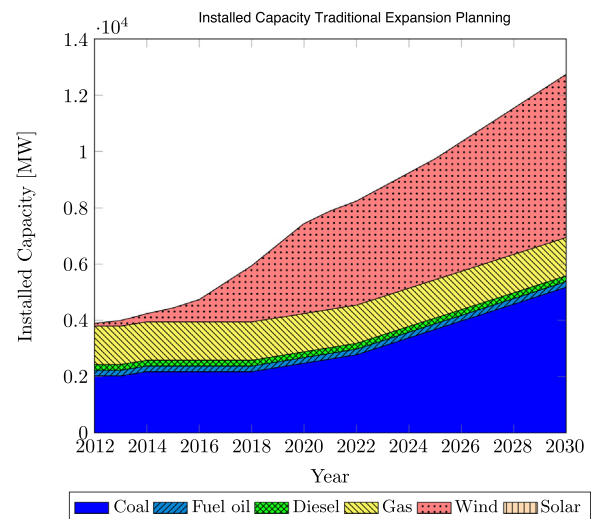
Table 3 shows the trade-off between the number of typical weeks and solution time for the proposed methodology. Modelling the operation with an increased number of weeks could result in unreasonably high solution times, highlighting the importance of a good representation with a reduced number of scenarios [26].

We attempted to solve the case of 13 typical weeks without decomposition in a super computer with 2 TB RAM. A solution could not be found after 30 days runtime.

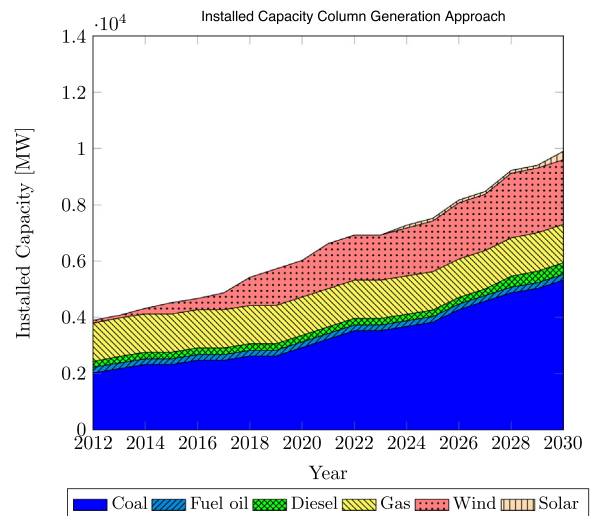
For large scale cases, there were no need to perform a branching procedure during the CG approach since integer solutions were obtained within the desired optimality gap.

4.3.3. Relevance of short term constraints in long term capacity planning

Fig. 4 shows the optimal generation mix for planning methods: (a) without short term constraints (i.e., without (6)–(18), and linear unit commitment state); and (b) with short term constraints, where the most notable difference is the amount of wind installed capacity. Consideration of short term constraints drive more investment in diesel and solar plants. Several experiments were undertaken with different demand profiles and generation parameters (ramp rates, minimum up and down times, etc.), which showed that diesel plants should be installed to increase system flexibility and facilitate connection of further renewables, while solar plants were installed to reduce the daily peak demand.



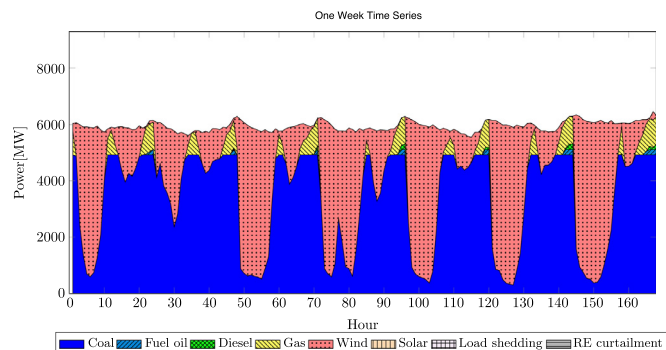
(a) Generation mix for period 2012 - 2030 obtained by traditional capacity expansion planning model



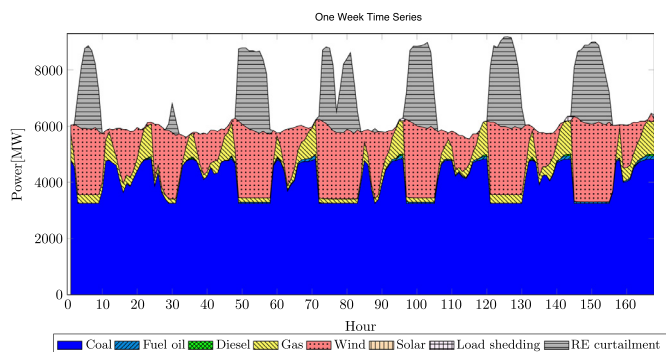
(b) Generation mix for period 2012 - 2030 obtained from C-UC-CE model

**Fig. 4.** Resulting generation mix for both methodologies.

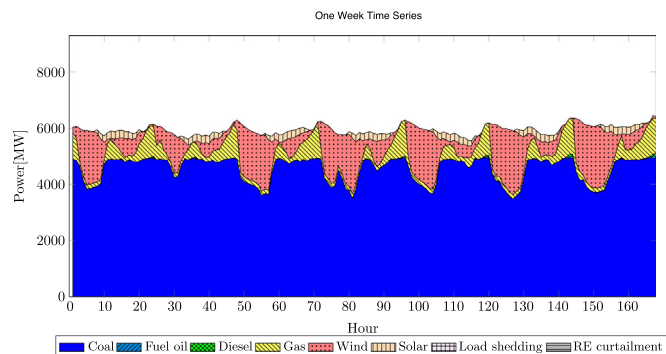




(a) Estimated operation of the generation mix obtained from traditional planning method



(b) Real operation of the generation mix obtained from traditional planning method



(c) Real operation of the generation mix obtained from C-UC-CE model

Fig. 5. One week operation for both methodologies.

Fig. 5 shows the system operation within a week in 2042 associated with investment decisions as illustrated in Fig. 4. Fig. 5(a) shows the solution obtained by the model without short term constraints, where penetration of renewables is  $\approx 46\%$ , and large fluctuations are absorbed by bulk variations of coal plants, which is clearly not feasible. When a proper unit-commitment model is run to obtain real operation of the generation infrastructure shown in Fig. 5(b), large volumes of wind would need to be curtailed to meet minimum output, ramping, and reserve constraints while maintaining an economically efficient operation, as shown in Fig. 5(b). Fig. 5(c) shows the solution obtained from the proposed C-UC-CE model, which includes unit commitment constraints in the expansion problem. Hence the proposed model optimally balances

a portfolio of renewable and conventional generation technologies, considering the opportunity cost of wind curtailments and the alternative cost associated with the necessary measures to provide flexibility. Overall, Fig. 5 demonstrates the need for combined unit commitment and capacity expansion problem models that consider advanced algorithms to solve problems with large datasets.

## 5. Conclusions

This paper develops a multi stage C-UC-CE model which was solved through a novel application of Dantzig–Wolfe decomposition and column generation. The methodology overcomes intractability and significantly reduces computation times.

We analyzed the benefits of our proposed technique through multiple case studies and demonstrated that C-UC-CE problems can be solved in reduced timescales and with efficient RAM usage. Smaller test cases showed up to 4 times increased speed, and approximately one fifth memory usage compared to the original formulation solved by CPLEX MIP. For more realistic case studies based on the Chilean Northern Interconnected System with a planning horizon of 19 years, the proposed CG approach solved the problem significantly faster in every instance with reduced memory usage. In contrast, the MIP solver experienced out of memory errors for the largest datasets. Thus, our proposed methodology is more efficient in solving large scale generation expansion planning problems.

In this paper, the pricing subproblems were solved sequentially. However, they could be solved using parallel computing, obtaining further reductions in solution time. The use of parallel processing will be an avenue of future research. We showed the importance of planning generation capacity through a C-UC-CE model, demonstrating the impact of high penetration of variable renewable energy on short term operation flexibility and investment. Neglecting short term constraints in planning timescales can produce extra costs of  $\approx 7\%$  for the case of the Chilean electricity system between years 2012–2030, since renewables will not be planned together with the necessary generating plants that can deliver flexibility.

Future work includes extending the proposed approach to stochastic generation expansion planning problems, and including hydroelectric power plants and other storage technologies.

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