OWA Operators in Portfolio Selection

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Abstract Portfolio choice is the process of selecting the optimal proportion of various assets. One of the most well-known methods is the mean-variance approach developed by Harry Markowitz. This paper introduces the ordered weighted average (OWA) in the mean-variance model. The key idea is that the mean and the variance can be extended with the OWA operator being able to consider different degrees of optimism or pessimism in the analysis. Thus, this method can adapt to a wide range of scenarios providing a deeper representation of the available information from the most pessimistic situation to the most optimistic one.

Keywords Portfolio selection · Ordered weighted average · Mean · Variance

1 Introduction

Portfolio selection is a theory of finance that aims to maximize the expected return of a portfolio considering a specific level of portfolio risk or minimize the risk for a given amount of expected return. The objective is to select a set of investment assets that have collectively a lower risk than any individual asset. Initially, it was introduced by Markowitz with the development of a mean-variance portfolio selection approach (Markowitz 1952). This model was able to find the optimal portfolio but it needed a lot of calculations. In the 50s and 60s this was a significant problem because the computers were not very strong and not able to make huge

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calculations. Therefore, it was not easy to deal with Markowitz's theory. In order to solve this weakness, Sharpe (1963) suggested a new approach that simplified Markowitz's model a lot. He found that most of the investment assets were subject to similar conditions. Thus, he introduced a diagonal model based on regression techniques that could find a solution with a substantial lower number of calculations than Markowitz's model.

During the last decades, Markowitz's approach has received increasing attention due to the development of computers that can make a lot of calculations in a short period of time. A key example is the special issue published in the European Journal of Operational Research celebrating the 60th anniversary of Markowitz's model (Zopounidis et al. 2014) that published several overviews regarding the newest developments in the field (Kolm et al. 2014; Markowitz 2014). Moreover, many other contributions have appeared during the last years in a wide range of journals (Garlappi et al. 2007; Maccheroni et al. 2013; Wu et al. 2014).

A fundamental issue in Markowitz's mean-variance framework is the aggregation of the mean and the variance with the arithmetic mean or the weighted average. However, it is possible to use other aggregation operators for this purpose (Beliakov et al. 2007; Grabisch et al. 2011). A well-known aggregation operator that aggregates the information according to the attitudinal character of the decision maker is the ordered weighted average (OWA) (Yager 1988; Yager et al. 2011). The OWA operator has been extended and generalized by a lot of authors (Emrouznejad and Marra 2014). Yager and Filev (1999) developed a more general framework that used order inducing variables in the reordering process. Fodor et al. (1995) introduced the quasi-arithmetic OWA operator by using quasi-arithmetic means in the analysis. Merigó and Gil-Lafuente (2009) presented the induced generalized OWA operator as an integration of the induced and quasi-arithmetic approaches. Some other studies have focused on the unification between the probability and the OWA operator (Engemann et al. 1996; Merigó 2012) and some other ones on the weighted average and the OWA operator (Merigó et al. 2013; Torra 1997; Xu and Da 2003). Recently, some further generalizations have been developed in this direction with the integration of the probability, the weighted average and the OWA operator in the same formulation (Merigó et al. 2012).

The aim of this paper is to introduce a mean-variance portfolio approach by using the OWA operator in the aggregation of the expected returns and risks. The key idea is to replace the mean and the variance used in Markowitz's model by the OWA operator and the variance-OWA (Var-OWA) (Yager 1996). Note that the OWA and the Var-OWA are extensions of the classical mean and the variance by considering the attitudinal character of the decision maker and any scenario that may occur from the most pessimistic to the most optimistic one. The key motivation for using the OWA operator instead of the arithmetic mean or the weighted average is that for uncertain environments where the available information is very complex, many times it is not possible to assign weights or probabilities to the expected results. Therefore, the expected value and the variance cannot be calculated using the common formulations and a different framework is needed. The main assumption of the OWA operator is that the information can be weighted according to the attitudinal character of the decision maker instead of the possibilities or probabilities that each state of nature or criteria will occur. The motivation for doing so is that often the probabilities or weights of the future events are not available and a different methodology is needed for aggregating the information. The OWA aggregates the information under or overestimating it according to the specific attitude of the decision maker. Additionally, it also considers any result that can occur from the minimum to the maximum providing a complete representation of the information that does not loose information in the analysis.

Markowitz' approach is reformulated by using OWA operators in the mean and the variance. Some key properties of the conceptual implications of this approach are studied including the measures for the characterization of the OWA operator (Yager 1988) and some additional extensions suggested in this paper. A wide range of particular types of OWA operators are also considered in order to see some specific attitudes that the decision maker may adopt.

The rest of the paper is organized as follows. Section 2 presents some basic preliminaries concerning the OWA operator and Markowitz's portfolio model. Section 3 introduces the use of the OWA in the mean-variance portfolio methodology. Section 4 summarizes the main results and conclusions of the paper.

2 Preliminares

2.1 The OWA Operator

The ordered weighted average (OWA) is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum (Yager 1988). In decision making under uncertainty, it is very useful for taking decisions with a certain degree of optimism or pessimism. It generalizes the classical methods into a single formulation including the optimistic, pessimistic, Laplace and Hurwicz criterion. It can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWA\,(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where b_j is the jth largest of the a_i.

An important issue when dealing with the OWA operator is the reordering process. In definition 1 the reordering has been presented in a descending way although it is also possible to consider an ascending order by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the descending OWA and w^*_{n-j+1} the jth weight of the ascending OWA operator. Moreover, it is also possible to adapt the ordering of the arguments to the weights and vice versa (Yager 1988). Note that the OWA is commutative, monotonic, bounded and idempotent.

In order to characterize the weighting vector of an OWA aggregation, Yager (1988) suggested the *degree of orness* and the *entropy of dispersion*. The degree of orness measures the tendency of the weights to the minimum or to the maximum. It is formulated as follows:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} \right). \tag{2}$$

As we can see, if $\alpha(W) = 1$, the weighting vector uses the maximum and if $\alpha(W) = 0$, the minimum. The more of the weights located to the top, the higher is α and vice versa.

The entropy of dispersion is an extension of the Shannon entropy when dealing with OWA operators. It is expressed as:

$$H(W) = -\left(\sum_{j=1}^{n} w_j \ln(w_j) \right). \tag{3}$$

Observe that the highest entropy is found with the arithmetic mean ($H(W) = \ln(n)$) and the lowest one when selecting only one result such as the minimum or the maximum because in this case the entropy is 0.

The OWA operator includes the classical methods for decision making under uncertainty as particular cases. The optimistic criteria is found if $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$. The pessimistic (or Wald) criteria is obtained when $w_n = 1$ and $w_j = 0$, for all $j \neq n$. The Laplace criteria is formed if $w_j = 1/n$, for all a_i. And the Hurwicz criteria is obtained when $w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$ for all $j \neq 1, n$. For further information on other cases of the OWA operator see Merigó and Gil-Lafuente (2009) and Yager (1993).

2.2 Portfolio Selection with Markowitz Approach

Consider a portfolio formed by m individual assets, so that r_i^k represents asset k's return at state of nature i for all $k = 1, 2, \ldots, m$, and $i = 1, 2, \ldots, n$. States of nature are distributed according to the probability vector $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$. so that $\pi_i \in [0, 1]$ for all $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \pi_i = 1$.

Let $X = (x_1, x_2, \ldots, x_n)$ be the vector of wealth proportions invested in each individual asset of the portfolio so that $x_k \in [0, 1]$ for all $k = 1, 2, \ldots, m$, and $\sum_{k=1}^{m} x_k = 1$. The mean return of asset k is then computed as

$$E(r^k) = \sum_{i=1}^{n} \pi_i r_i^k, \tag{4}$$

and the mean return portfolio is hence given by

$$E(r^p; x) = \sum_{k=1}^{m} x_k E(r^k), \tag{5}$$

as the expectation operator E is linear.

Moreover, the covariance between assets j and k is given by

$$\begin{aligned} COV(r^j, r^k) = \sum_{i=1}^{n} E(r_i^j - E(r^j))E(r_i^k - E(r^k)) \\ = \sum_{i=1}^{n} \pi_i (r_i^j - E(r^j))(r_i^k - E(r^k)), \end{aligned} \tag{6}$$

and the variance of the portfolio can then be computed as follows:

$$V(r^p; x) = \sum_{j=1}^{m} \sum_{k=1}^{n} x_j x_k COV(r^j, r^k), \tag{7}$$

given the linearity of the operator E.

A key aspect in Markowitz methodology is to characterize the *efficient frontier*, which collects all the pairs that yield the maximum mean return portfolio for a given level of risk (the portfolio variance or standard deviation), or alternatively, all the pairs representing the minimum portfolio risk for a given level of mean return portfolio. From this duality, two practical methodologies emerge to construct the efficient frontier: a quadratic or a parametric programming model. Both of them are summarized in Table 1.

Table 1 Quadratic and parametric programming in Markowitz portfolio selection approach. $V*$ and $E*$ represent a given level of portfolio variance and portfolio mean return, respectively

	Program 1	Program 2
Objective function	$\max_x E(r^p; x)$	$\min_x V(r^p; x)$
Parametric constraints	$V(r^p; x) = V*$	$E(r^p; x) = E*$
Budget constraints	$\sum_{k=1}^{m} x_k = 1$	$\sum_{k=1}^{m} x_k = 1$
Non negativity	$\forall x_k \in [0, 1]$	$\forall x_k \in [0, 1]$

An important result coming from portfolio choice analysis is the investor's wealth allocation that allows him to bear the minimum level of risk, i.e., the so-called *minimum-variance portfolio*. This is a relevant result, especially for a too risk-averse investor who wants to minimize the variability of his position irrespective if this portfolio yields a quite low expected return. In formal terms, let us define $\bar{x}$ the minimum-variance portfolio, as follows:

$$\bar{x} = \arg\min_{x} V(r^p; x), \tag{8}$$

where $\bar{X} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m)$ is so that $\sum_{k=1}^{m} \bar{x}_k = 1$ for all $\bar{x}_k \in [0, 1]$.

3 The OWA Operator in Portfolio Selection

Markowitz's approach is based on the use of the mean and the variance. These techniques are usually studied with an arithmetic mean or a weighted average (or probability). However, many times the degree of uncertainty is more complex and it is necessary to represent the information in a deeper way. In this case, more general aggregation operators are needed in order to assess the information properly. A practical technique for doing so is the OWA operator because it provides a parameterized group of aggregation operators between the minimum and the maximum. Moreover, it is able to represent the *attitudinal* character of the decision maker in the specific problem considered. Therefore, the main advantage of this approach is that it represents the problem in a more complete way because it can consider any scenario from the most pessimistic to the most optimistic one and select the situation that is in closest accordance with the inverstor's interests.

In order to revise Markowitz's approach with the OWA operator, it is necessary to change the formulas used to compute the portfolio's mean return and risk, that is, Eqs. (5) and (7) of Sect. 2. Note that this is suggested for situations with high levels of uncertainty where probabilistic information is not available. In particular, when:

(1) The return of any asset is uncertain and cannot be assessed with probabilities. Therefore, the expected value cannot be used. In these uncertain environments, the Mean-OWA operator may become an alternative method for aggregating the information of the assets.
(2) The risk of any asset cannot be measured with the usual variance because probabilities are unknown. However, it is possible to represent it with the Variance-OWA (Merigó 2012; Yager 1996).

3.1 Asset Return and Risk Using OWA

One of the key ideas of this approach is to introduce a model that can adapt better to uncertain environments because Markowitz's approach is usually focused on risky environments. Note that with the expected value it is possible to consider subjective and objective probabilities but it is not possible to assess situations without any type of probabilities. An alternative for dealing with these situations is the OWA operator that aggregates the information according to the attitudinal character of the investor.

Definition 2. Let r_i be an asset's return at state of nature i for $i = 1, 2, \ldots, n$. The Mean-OWA operator can then be represented as follows:

$$E_{OWA} = OWA(r_1, r_2, \ldots, r_n) = \sum_{j=1}^{n} w_j q_j, \tag{9}$$

where q_j is the jth largest of the r_i.

Next, let us look into the other main perspective when dealing with portfolio selection. The analysis of risk in Markowitz's approach is based on the use of the variance measure. In this paper, we have suggested to use the Variance-OWA as a measure of risk. The main advantage is that this formulation is more general than the classical variance because it provides a parameterized family of variances between the minimum and the maximum one. Thus, it gives a better representation of the problem and selects the specific result that is in closest accordance with the attitude of the decision maker. Following Yager (1996) and Merigó (2012), an asset variance when using the OWA operator can be formulated as follows.

Definition 3. Given an asset with expected returns $(r_1, r_2, \ldots, r_n)$, let us define s_i as:

$$s_i = (r_i - E_{OWA})^2$$

For $i = 1, 2, \ldots, n$, where E_{OWA} is defined according to (7). The Variance-OWA can then be defined as follows:

$$V_{OWA} = OWA(s_1, s_2, \ldots, s_n) = \sum_{j=1}^{n} w_j t_j, \tag{10}$$

where t_j is the jth smallest of the s_i.

Observe that here we use an ascending order because usually it is assumed that a lower risk represents a better result and thus it should appear first in the aggregation. Similarly to the case of expected returns, with the OWA approach we can consider any risk from the minimum to the maximum one by using $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$ (minimum variance) and $w_n = 1$ and $w_j = 0$ for all $j \neq n$ (maximum variance).

3.2 Portfolio Mean Return and Risk Using OWA

Consider now a portfolio in which it can be combined m individual assets, so that r_i^k represents asset k's return at state of nature i for all $k = 1, 2, \ldots, m$, and $i = 1, 2, \ldots, n$. Let $X = (x_1, x_2, \ldots, x_m)$ be the vector of wealth proportions invested in each individual asset of the portfolio so that $x_k \in [0, 1]$ for all $k = 1, 2, \ldots, m$, and $\sum_{k=1}^{m} x_k = 1$. Moreover, let us define r_i^p, the portfolio's return at state of nature i, as:

$$r_i^p = \sum_{k=1}^{m} x_k r_i^k$$

for all $i = 1, 2, \ldots, n$. The portfolio mean-OWA is then given by

$$E_{OWA}(r^p; x) = E_{OWA}(r_1^p, r_2^p, \ldots, r_n^p), \tag{11}$$

or equivalently, using definition given by (9), it becomes

$$E_{OWA}(r^p; x) = E_{OWA}(r_1^p, r_2^p, \ldots, r_n^p) = \sum_{j=1}^{n} w_j q_j^p, \tag{12}$$

where q_j^p is the jth largest of the r_i^p.

Also, by applying (11) or (12), we can define s_i^p as:

$$s_i^p = (r_i^p - E_{OWA}(r^p; x))^2, \tag{13}$$

for $i = 1, 2, \ldots, n$. The portfolio variance-OWA can then be formulated as follows:

$$V_{OWA}(r^p; x) = V_{OWA}(r_1^p, r_2^p, \ldots, r_n^p), \tag{14}$$

and it can be calculated as

$$V_{OWA}(r^p; x) = OWA(s_1^p, s_2^p, \ldots, s_n^p) = \sum_{j=1}^{n} w_j t_j^p,$$

where t_j^p is the jth smallest of the s_i^p.

Once this initial information is calculated, the rest of the approach follows Markowitz methodology where a quadratic or parametric programming model is used (see Table 2).

Table 2 Quadratic and parametric programming in portfolio selection using OWA. V^* and E^* represent a given level of portfolio variance OWA and portfolio mean-OWA return, respectively

	Program 1	Program 2
Objective function	$\max_x E(r^p; x)$	$\min_x V(r^p; x)$
Parametric constraints	$V_{OWA}(r^p; x) = V^*$	$E_{OWA}(r^p; x) = E^*$
Budget constraints	$\sum_{k=1}^{m} x_k = 1$	$\sum_{k=1}^{m} x_k = 1$
Non negativity	$\forall\, x_k \in [0, 1]$	$\forall\, x_k \in [0, 1]$

As in the Markowitz approach, we define $\bar{x}_{OWA}$, the minimum-variance-OWA portfolio, as follows

$$\bar{x}_{OWA} = \arg\min_{x} V_{OWA}(r^p; x), \tag{16}$$

where $\bar{x}_{OWA} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m)$ is so that $\sum_{k=1}^{m} \bar{x}_k = 1$ for all $\bar{x}_k \in [0, 1]$.

3.3 Investor's Criteria for Mean-OWA

Observe that with the OWA operator, the expected returns are studied considering any scenario from the minimum to the maximum one. Thus, the decision maker does not lose any information in this initial stage. Once he selects a specific attitude, he opts for a specific result and decision although he still knows any extreme situation that can occur in the problem. This can be proved analyzing some key particular cases of the OWA aggregation including the minimum, the maximum and the arithmetic mean:

- If $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$, the OWA becomes the maximum (or optimistic criteria).
- If $w_n = 1$ and $w_j = 0$ for all $j \neq n$, it is formed the minimum (or pessimistic criteria).
- If $w_j = 1/n$ for all j, it becomes the arithmetic mean (Laplace criteria).

By looking to the maximum and the minimum, it is proved that the OWA operator accomplishes the boundary condition:

$$\text{Min}\{a_i\} \leq f(a_1, a_2, \ldots, a_n) \leq \text{Max}\{a_i\}. \tag{17}$$

Some other interesting particular cases of the OWA operator are the following:

- Hurwicz criteria: If $w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$, for all $j \neq 1, n$.
- Step-OWA: $w_k = 1$ and $w_j = 0$ for all $j \neq k$. Note that if $k = 1$ we get the maximum and if $k = n$, the minimum.
- Median-OWA: If n is odd we assign $w_{(n+1)/2} = 1$ and $w_j = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_j = 0$ for all others.
- Olympic-OWA: When $w_1 = w_n = 0$ and for all others $w_j = 1/(n - 2)$.
- S-OWA: If $w_1 = (1/n)(1 - (\varepsilon + \delta)) + \varepsilon$, $w_n = (1/n)(1 - (\varepsilon + \delta)) + \delta$, $w_j = (1/n)(1 - (\varepsilon + \delta))$ for $j = 2$ to $n - 1$, where $\varepsilon, \delta \in [0, 1]$ and $\varepsilon + \delta \leq 1$.
- Centered-OWA: If it is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. Strongly decaying when $i < j \leq (n + 1)/2$ then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$.

For further reading on other particular types of OWA operators see Merigó and Gil-Lafuente (2009) and Yager (1993).

4 Conclusions

The Markowitz mean-variance portfolio selection approach can be extended by using the OWA operator. The main advantage of this new formulation is that it can represent uncertain environments by using the attitudinal character of the decision maker. Thus, it aggregates the information considering the degree of optimism or pessimism that an individual has in a specific problem. This method assumes that probabilistic data is not available because the environment shows a high degree of uncertainty. The mean return and the risk of any asset have been studied with OWA operators instead of using the classical weighted and arithmetic mean. By doing so, the investor gets a more complete view of the problem because he can consider any scenario from the minimum to the maximum and select the one closest to his interests. Some particular cases have been studied in order to see how the aggregation process might be seen under different perspectives. Among others, the classical methods for decision making under uncertainty have been considered since the OWA operator includes them as special cases. That is, the optimistic, pessimistic, Laplace and Hurwicz criterion.

The degree of optimism and pessimism has been studied under a new formulation that takes into account the specific values of the arguments. Thus, rather than only considering the degree of orness through weights, it is also possible to consider the position of the arguments which also conditions the optimism or pessimism of the aggregation. Some numerical examples have also been presented in order to understand numerically the new approach. These examples have been developed considering different types of OWA positions that can be used in the decision and aggregation process including the optimistic, pessimistic, Olympic, Laplace and Hurwicz criterion. Various results emerge from this numerical exercise. First, the isolated ordering effect of OWA suggests that this operator induces more optimism *only* on the inefficient portfolio region, and thus, both efficient frontiers (Markowitz and OWA) are identical. Second, the combined ordering and weighting effect of OWA suggests that the minimum-risk portfolio is the *same* in both types of frontiers. This result is robust to different attitudinal characters of investors. Third, an optimistic investor faces better return-risk profile, so he expects, for a given level of either return or risk, a higher utility than a Markowitzian investor. This dominance gets exacerbated as the optimism degree increases, leading to an increasing polarization of portfolio choice in favor of the individual asset with the best return profile. Fourth, a pessimistic investor faces a worse return-risk trade-off, so he expects a lower utility than a Markowitzian investor. As a consequence, he always selects the minimum risk portfolio irrespective of his risk-aversion degree. Fifth, in the case of moderate, Laplace-type, and extremist investor, the results in general are more ambiguous as they depend on the specific probabilistic and weighting vectors under consideration.

In future research, further developments can be developed by using other extensions and generalizations of the OWA operator in Markowitz mean-variance portfolio selection model including the probabilistic OWA operator (Merigó 2012) and induced aggregation operators (Merigó and Gil-Lafuente 2009; Yager and Filev 1999). Moreover, some other portfolio selection methods can be considered with the OWA operator including the Sharpe model, the CAPM and the APT. Lastly, the approach here proposed can be seen as a starting point to provide alternative explanations to some controversial results in financial economics such as the two-fund separation puzzle.

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