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Spatiotemporal Complexity of a City Traffic Jam

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We model city traffic by a cellular automaton. It consist of a row of interacting cars through a sequence of traffic lights, synchronized by a propagating green signal. We start the system from an initial jammed condition of low density, which shows the same scaling laws previously found [1]. However, for large initial jammed densities, the emergence of spatial variability in the behavior of the cars along the sequence of traffic lights, produce the breakdown of the scaling laws. This spatial disorder corresponds to a different attractor of the system, from that of the small initial jammed densities. As we include velocity perturbations in the dynamics of the cars, all these attractors collapse to a single noisy attractor for all initial jammed densities. However, this emergent state, shows what seems a stochastic resonance in which the average traffic velocity increases with respect to that of the system without noise.

Keywords: Traffic jam; traffic dynamics; traffic signal; emergent state; fluctuation; stochastic resonance

1 INTRODUCTION

General traffic and pedestrian flows [2–8] , provides interesting problems for reaseach which may impact our societies and economies [9]. In these scenarios usually we find emergent behaviors [10–13], jamming transitions,

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and chaos [14–16]. Here, we extend previous studies [1, 17] regarding the city traffic as a number of interacting particles moving through a sequence of traffic lights, a system which has many nontrivial features [18–24].

In previous studies, the critical behavior around resonances, was shown to be robust [1, 14]. We showed that the dynamics is resilient to the street length distribution and to the initial car density for an unjammed situation. To achieve a resonant dynamics in this system, we synchronize traffic lights [14, 18], for which, we had explored two possibilities (a) the traveling time between traffic signals is the same as the period of the signals (synchronized phase strategy), and (b) the average speed of the car is the same as the propagating green signal in the traffic lights array (*greenwave* strategy). The detailed study of the characteristics of synchronized traffic flow, is an ongoing researh [25, 26]. The resonances that appear under the *greenwave* strategy in the absence of noise (random perturbations), [1, 14], do not depend on the distance distribution between traffic lights, nor on the finite braking and accelerating capabilities of the car. When a stochastic perturbation is introduced, the resonance move and reshape, what evoke a standard phase transition. If we already start with a traffic jam, it was found a stochastic resonant behavior [17], which result in an increase of the cars average flow speed, relative to situations without perturbations. In this study, we will deepen on this new global state, by observing how the scaling laws close to resonance change, for different perturbation levels and jammed initial conditions. We give further characterization of the resonant behavior changes as we increase the initial number of interacting cars that jam the system. For the purpose of this work, we concentrate on a very simplified cellular automaton (CA) model since we know that the critical behavior, close enough to the resonance, is almost unsensitive to these details (e.g., finite braking and accelerating capabilities, etc. [14, 18]).

2 MODEL

We use a CA model to study the flow of an initially jammed row of cars, moving through a sequence of traffic lights. In our model, we consider a street of length L_{tot} with N_s traffic lights. The length L_n between the n^{th} and $(n+1)^{\text{th}}$ traffic light is divided in $N_{L_n} = L_n/\ell$ cells of length ℓ. The time it takes a car to move to the next cell, namely τ, is the automaton evolution time step. A car will move to the next cell only if the following conditions are satisfied:

- No other car is stopped in the next cell.
- If the current cell has a traffic light, it must show a green signal.

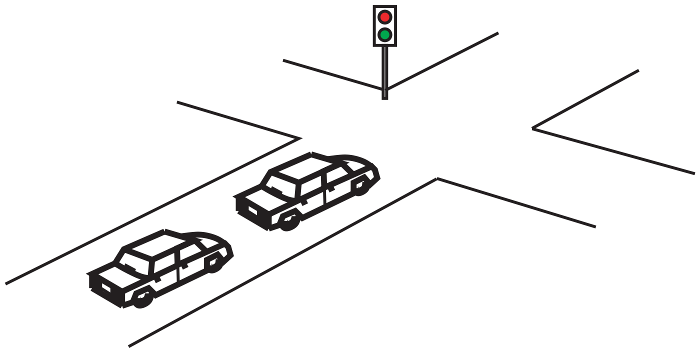

FIGURE 1
Possible system state, with a car stopped at a traffic light with a traffic jam ahead, in which case the car will not move, even if the signal is green.

- If the current cell, indexed j, has a traffic light, there must be no car stopped at the $(j+2)^{\text{th}}$ cell, as the drivers avoid stopping at the $(j+1)^{\text{th}}$ cell because that would block the intersection, as shown in Figure 1, which is forbidden by law.
- The cars are affected by velocity perturbations, characterized by the parameter r, which represents a random probability that a car may stop in the next time step, even when it is allowed to move by the previous rules.

For now, we will assume $r = 0$. The only possible values for the velocity of the cars are $v_{\text{max}} = \ell/\tau$, which corresponds to one cell per time step, and 0. We are also assuming that the cars cannot pass each other. Figure 1 shows a schematic representation of a state of the system at a particular time. Occupied cells are represented by a car, and empty otherwise.

The switching of the n^{th} traffic light, form green to red and vice-versa, is given by the periodic function $f_n(t) = \sin(\omega_n t + \phi_n)$. When $f_n(t) > 0$ the traffic light is green, and the cars at the intersection can move to the next unoccupied cell. If $f_n(t) < 0$ a red traffic light stops the motion of the vehicles approaching to it. Here, ω_n represents the frequency of the n^{th} traffic light, since for simplicity we are considering that all traffic lights have the same period T, *i.e.* $\omega_n = 2\pi/T \ \forall n$. The phase ϕ_n takes into account the fact that each traffic light will have a given time delay with respect to the previous one. For the case of the *greenwave* strategy studied here, a green pulse propagates through the sequence of traffic lights with velocity v_{wave}, which can be positive or negative. Therefore, the phase ϕ_n will be given by $\phi_n = -(\omega/v_{\text{wave}}) \sum_{j=0}^{n} L_j$, since $\sum_{j=0}^{n} L_j$ is the position of the n^{th} traffic light. We also define $\alpha = v_{\max}/v_{\text{wave}}$, to compare the green wave speed with the cars maximum speed. Therefore, the traffic light control function is given by

$$f_n(t,\alpha) = \sin\left[\frac{2\pi}{T}\left(t - \frac{\alpha}{v_{\max}}\sum_{j=0}^{n} L_j\right)\right]. \tag{1}$$

If we assume that (a) the distance between traffic lights is about $L = 250$ m, (b) the length of each cell is $\ell = 10$ m, the cruising velocity is $v_{\max} = 10$ m/s (36 km/h), then each time step corresponds to $\tau = 1$ s. Let us note that these values are consistent with the car having equal accelerating and braking capabilities of $a = v_{\max}^2/2\ell = 5$ m/s^2. For the traffic light period we will use $T = 60$ s.

3 TRAFFIC JAM

We will consider first a sequence of $N_s = 100$ equidistant traffic lights, separated by a distance $L_n = L$. For the purpose of reproducing an initial jammed state we consider the simplest possibility, namely, an initial condition with a fixed number J_N of cars stopped at each traffic light. Therefore, the initial traffic jam length is J_N. The cars enter the first simulation cell at a rate $1/f$, that means we inject cars every f time steps, unless the first cell is occupied. We are interested in a jammed situation, so that we will use $f = 1$ throughout, which implies that we inject a new car into the system every time that an empty spot is available at the first simulation cell.

Now we consider a simulation with parameters relevant for city situations, namely $L = 25\,\ell$ (250 m). Let us assume no random velocity perturbations, as described above, and consider initial conditions with a fixed $J_N \in [0, N_L]$, such that if $J_N = 0$ we have an empty street, and for $J_N = N_L$ we have a street without empty cells. To remove the transient behavior, we let the system evolve for a time $10^4\,T$. To compute the statistics, we follow the dynamics for an additional amount of time equal to $10^4\,T$. With this data, we compute the average speed of the cars (total distance traveled divided by total travel time) between the 20$^{\text{th}}$ and the $(N_s - 20)^{\text{th}}$ traffic light, where we are not considering the first and the last 20 traffic lights to avoid boundary effects. It is important to keep in mind that for the given traffic light period T, a traffic light can evacuate at most $T/4 = 15$ cars during a green light period, depending on the number of cars stopped at the following traffic light. The results are shown in Figure 2(a), where the normalized average speed is described as a function of the green wave speed parameter $\alpha = v_{\max}/v_{\text{wave}}$.

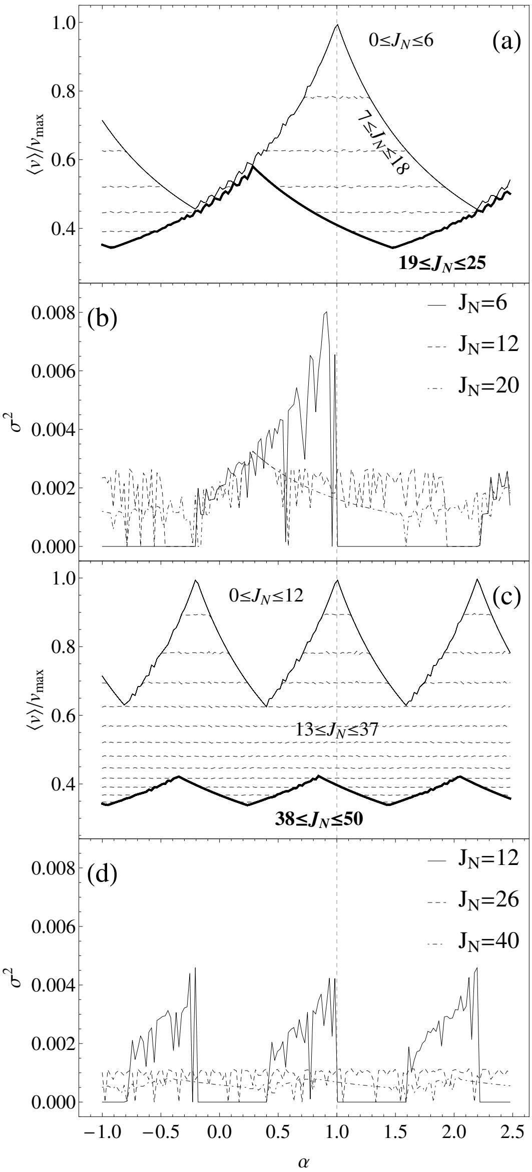

FIGURE 2
Averaged speed of the cars normalized to $v_{\max}$ for the full range of initial conditions $J_N \in [0, N_L]$ as a function of α, for (a) $L = 25\,\ell$ and (c) $L = 50\,\ell$. The thin line corresponds to the $J_N \le N_L/4$ superimposed curves, and represent the resonant solution with $\langle v \rangle / v_{\max} = 1$ at $\alpha = 1$ studied in Ref. [1]. The dashed lines correspond to $N_L/4 < J_N < 3N_L/4$ with increasing J_N values from top to bottom. The thick line corresponds to the superimposed $3N_L/4 \le J_N < N_L$ curves. The standard deviation of $\langle v \rangle / v_{\max}$ for (b) $L = 25\,\ell$ and (c) $L = 50\,\ell$. It is computed for three values of J_N.

As shown in Figure 2(a), for a range of initial conditions, namely $J_N \leq N_L/4 \approx 6$, the system converges to the already known dynamics of cars with an empty initial condition ($J_N = 0$). This situation has a resonant average normalized speed $\langle v \rangle / v_{\max} = 1$ at $\alpha = 1$, as was shown in Ref. [1]. In general there are other resonances for other values of α. In these softly jammed cases, the traffic jam will dissolve after some time. However, there exists initial conditions with $J_N > N_L/4 \approx 6$ for which the system saturates and it is unable to remove the initial traffic jam. We observe an emergent phenomenon for $6 \approx N_L/4 < J_N < 3N_L/4 \approx 19$, where there exists a range of α values in which the average speed is constant, with the car behavior being independent of the green wave speed. This is an emergent state, as the traffic lights do not seem to affect the average speed of the cars, except for producing the jamming of cars during the red phase of the traffic light. This jammed state has its own dynamics and cannot be altered by our control strategy by means of the traffic light phase. This dynamic is expected since the road has a surplus of vehicles which cannot be evacuated due to the limited outflow.

We can see in Figure 2(a) and (c) that for $J_N > 3N_L/4 \approx 18$, the constant average speed region described above disappears and all these initial conditions converge to the same curve with a resonance that depends on the value of L, namely, $\alpha \approx 0.3$ with $< v > / v_{\max} \approx 0.55$ for $L = 25\,\ell$. In Figure 2(c) we observe the same type of dynamics for a longer street length $L = 50\,\ell$. For $J_N \leq N_L/4 = 12.5$ we observe the standard resonant behavior as if the roads were empty ahead, with the traffic lights being able to clear the initial traffic jam. For $12.5 = N_L/4 < J_N < 3N_L/4 = 37.5$ we obtain an emergent phenomenon with a range of α values in which the average speed is constant with the car behavior being independent of the greenwave speed. For $J_N > 3N_L/4 = 37.5$ the constant average speed disappears and all the initial conditions converge to the same curve with a resonance at $\alpha \approx 0.8$ with $< v > / v_{\max} \approx 0.4$.

Let's note that a lower value of α at resonance is expected in a jammed situation, because at a given traffic light we have to wait for the time required to evacuate the cars at the next traffic light before we can make it through. This effect becomes even more apparent in the increase of the average speed close to $\alpha \approx -1$ in Figure 2(a), defining an effective anti-greenwave strategy. Hence for some highly jammed situations, it may be convenient to apply a anti-greenwave strategy to increase the traffic flow of the system.

We will now characterize some of the complexities of the emergent states that appear in this system. In Figure 2(c), we show the standard deviation of $\langle v \rangle / v_{\max}$ for the case $L = 25\,\ell$, $J_N = 6$, 12, and 20, that represent the three regimes described above. We note that deviation depends on α, and decreases as we increase J_N. The same occurs for the case of $L = 50\,\ell$, as shown in Figure 2(d).

We can characterize the complex nature of the traffic jam by analyzing the travel time of the cars between traffic lights, and in particular studying its spatial dependence. For that, we can define $\Delta t_{n,j}$ as the time taken by the j^{th} car to travel from the n^{th} to the $(n+1)^{\text{th}}$ traffic light. First, we average over all the cars passing from the n^{th} to the $(n+1)^{\text{th}}$ traffic light in the simulation, and then we average over the N_s traffic lights. Hence, we compute the average time travel between traffic lights, namely Δt, which can be normalized by the cruising time $T_c = L/v_{\max}$ assuming an empty road. In Figure 3(a) we show $\Delta\zeta = \Delta t/T_c$ for $L = 25\,\ell$ as a function of α, which is consistent with the pattern of Figure 2. In the jammed situations, namely $J_n > N_L/4$, we clearly see a large increase in the average travel time. For a longer street length, namely $L = 50\,\ell$, we show the same results in Figure 3(b). In general,

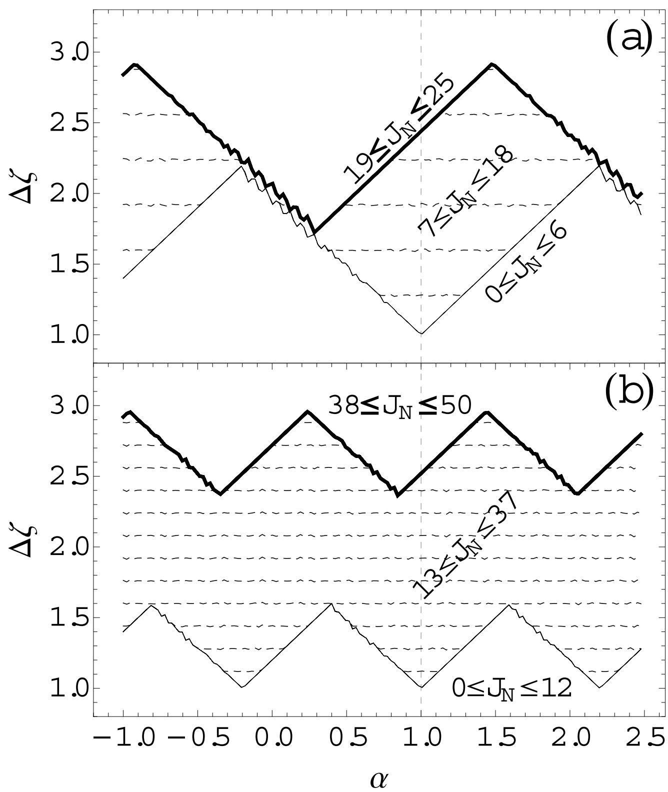

FIGURE 3
Averaged normalized travel time between traffic lights $\Delta\zeta = \Delta t/T_c$ for the full range of initial conditions $J_N \in [0, N_L]$ as a function of α, for (a) $L = 25\,\ell$ and (b) $L = 50\,\ell$. The label of the curves follow the description used in Figure 2(a), but with the dashed lines increasing in J_N value from bottom to top.

we observe the same convergence to an attractor for $J_N < N_L/4$ and $J_N > 3N_L/4$, however, the two attractors no longer collide for certain values of α. In this particular case we find again the emergent states for $N_L/4 \leq J_N \leq 3N_L/4$ with an averaged speed that remains constant for a range of α values, however, now there are initial jammed states for which the averaged speed becomes constant and independent of α, for all values of α.

The average travel time is a periodic function of time, due to the periodicity of the traffic light control function. In the emergent state, for $N_L/4 < J_N < 3N_L/4$, the following relation applies, namely

$$\frac{\langle v \rangle}{v_{\max}} = \frac{N_L}{4J_N}. \tag{2}$$

Therefore, the average speed in this emergent state is inversely proportional to J_N/N_L, the initial traffic jam fraction. This behavior is related to the average conserved current $\rho\langle v \rangle$. Let us note that we can define the density as the number of cars divided by the street length in cell units, which results in the fraction $\rho = J_N/N_L$. Observe that for the m^{th} car, in a row of cars waiting at a traffic light, it takes $2m$ time steps to cross the traffic light after it becomes green if the road ahead is empty, therefore, at most $T/4$ cars are able to cross the traffic light during a given traffic light period. In particular, this occurs at the street end, therefore, the number of cars flowing out of the system is also $T/4$, which leads us to the rate 1/4 cars per time step. Hence we obtain the current $\rho\langle v \rangle = v_{\max}/4$, from which Eq. (2) arises. Although, this result was suggested in Ref. [1], we should recall that this is only valid for the emergent state that appears for $N_L/4 < J_N < 3N_L/4$ in a limited range of α in the case of $L = 25\,\ell$, and in the full range of α for some values of J_N in the $L = 50\,\ell$ case.

Another way to characterize the behavior of the traffic jam that occurs in these regions, is to define the traffic jam number Y_n as the number of contiguous cars stopped at the n^{th} traffic light at the moment it switches to green. We can average this variable over a long time and over all the N_s traffic lights to obtain the average jam number Y as a function of α and J_N. For graphical purposes it becomes convenient to graph the normalized average traffic jam length defined as $Y_L = Y\ell/L$, which is shown in Figure 4(a) for $L = 25\,\ell$. Qualitatively we note the inverse relation between the average velocity and the traffic jam length for $\alpha < 1$, however, in the region where the average speed is independent of α we note that the average traffic jam length varies non-trivially with α. For $J_N > N_L/4 = 6.25$ the traffic jam length increases with $\alpha > 1$ (decreasing v_{wave}), because the *greenwave* takes longer to travel from light to light, and more cars have to stop at the next traffic light. Also, it is important to note that the maximum normalized length that a traffic

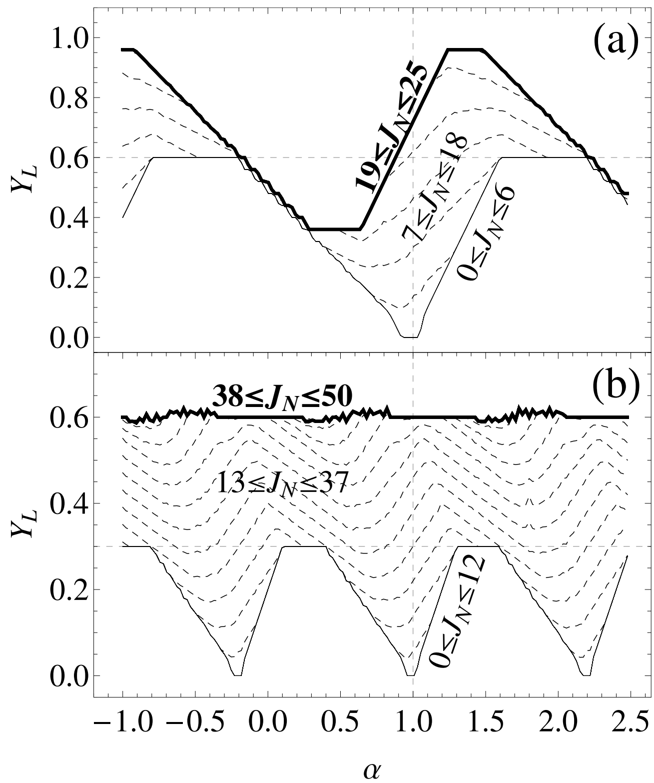

FIGURE 4
Normalized averaged traffic jam length Y_L for the full range of initial conditions $J_N \in [0, N_L]$ as a function of α for (a) $L = 25\,\ell$ and (b) $L = 50\,\ell$. The label of the curves follow the description used in Figure 2(a), but with the dashed lines increasing in J_N value from bottom to top. The maximum normalized length that a traffic light can evacuate, assuming an empty situation ahead, is $Y_L = 15/25 = 0.6$ for $L = 25\,\ell$ and $Y_L = 15/50 = 0.3$ for $L = 50\,\ell$, which are shown as horizontal line for Y_L on the figures.

light can evacuate, assuming an empty situation ahead, is $Y_L = 15/25 = 0.6$. Hence for $J_N > N_L/4 = 6.25$ there are values of α for which the system is completely jammed, in the sense that traffic lights are not able to evacuate all the jammed cars during one light period. As expected, we understand the region $J_N \leq N_L/4$, as the region in which the system is able to evacuate all the cars from the traffic light.

For comparison, the case of $L = 50\,\ell$ is shown in Figure 4(b). The maximum normalized length that a traffic light can evacuate, assuming an empty situation ahead, is $Y_L = 15/50 = 0.3$, so that for $J_N > N_L/4 = 12.5$ the system is completely jammed for all values of α, in the sense that traffic lights are not able to evacuate all the jammed cars during one light period.

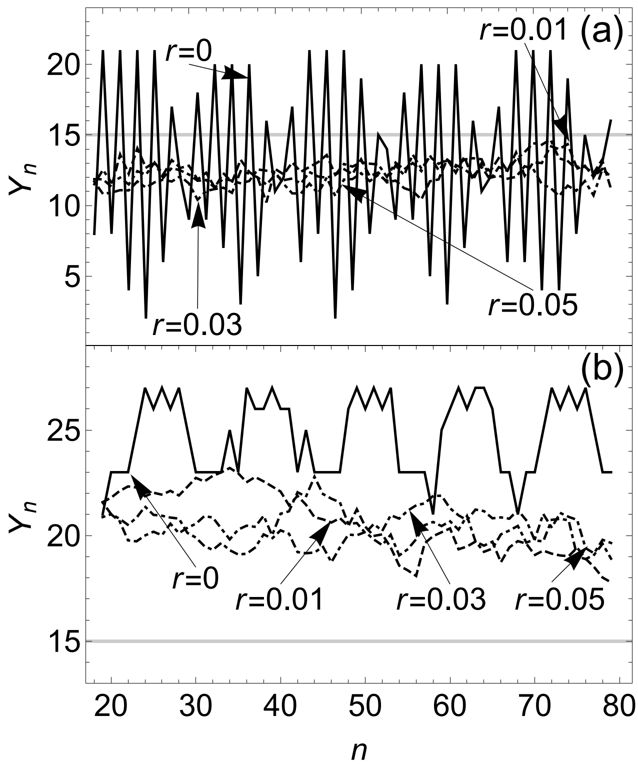

FIGURE 5
Traffic light jam number at each traffic light, Y_n for $\alpha = 1.6$, as a function of n (traffic light index). (a) $L = 25\,\ell$ and $J_N = 10$, and (b) $L = 50\,\ell$ for $J_N = 20$. We plot for $r = 0$, 0.01, 0.03, and 0.05.

Clearly this emergent state with an average traffic jam length for a range of values of α, that occurs for $N_L/4 < J_N < 3N_L/4$, requires further attention. Hence, we now analyze the normalized spatial dependence of the traffic jam number at each traffic light, given by Y_n, with n as the traffic light index. In Figure 5(a) we display Y_n for $J_N = 10$, $L = 25\,\ell$, and a fixed green wave speed $\alpha = 1.6$. In this particular case Y_n has a spatially dependent profile, which is not spatially periodic. It is important to mention that since the light control function is periodic, there are certain α values for which Y_n become spatially periodic. This occurs when α satisfies a spatial integer period $\Delta n = m\,T\,v_{\max}/L\alpha$ for some integer m. The situation of $L = 50\,\ell$ is shown in Figure 5(b) for $J_N = 20$. Again we see the spatially disorder system, with a varying number of jammed cars at each traffic light.

We can represent the non-trivial spatial dependence in the traffic jam length at each traffic light by counting the number of different values of Y_n

that we observe along the road, for a given value of α. For example, in the case of Figure 5 we have 5 different values of Y_n. To characterize the complexity in the spatial profile of the jammed length Y_n between traffic lights we define the normalized jammed length spatial entropy

$$\widehat{D}(\alpha) \propto -\sum_{Y} P(Y) \ln\ P(Y),$$

produced by the distribution $P(Y)$ of traffic jam lengths. This measure, normalized by the reference value $\ln N_L$, is shown in Figure 6(a) as a function of α for $L = 25\,\ell$. A relatively large value of $\widehat{D}(\alpha)$ represents an increased spatial complexity. We can observe that the regions where the averaged speed is independent of α corresponds precisely to situations of increased spatial

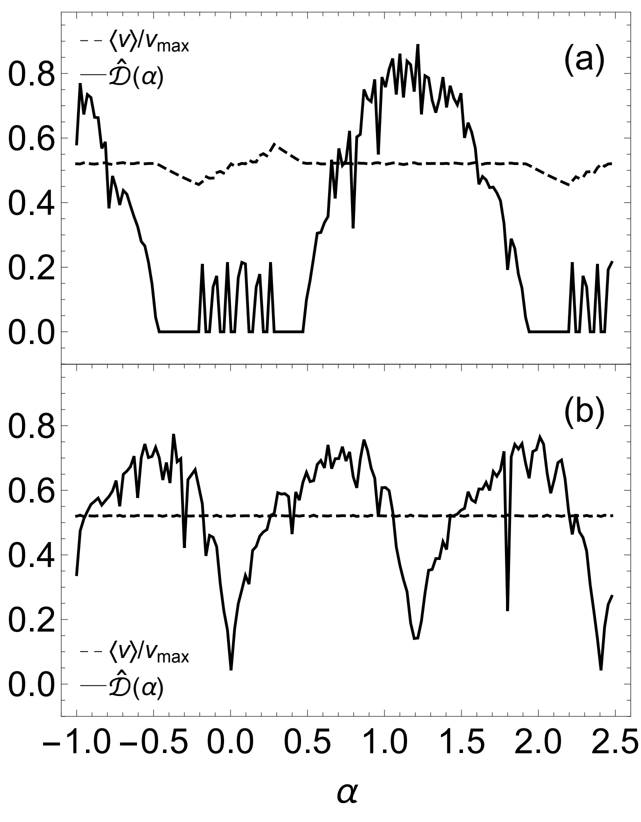

FIGURE 6
Normalized spatial entropy $\widehat{D}$ as a function of α. (a) $J_N = 10$ for $L = 25\,\ell$, and (b) $J_N = 20$ for $L = 50\,\ell$. The normalization is done respect to $\ln N_L$. The dashed line corresponds to the average value of the normalized speed from Figure 5(a) and (b), respectively.

complexity. This is expected since the *greenwave* tends to synchronize the cars in a given cluster. Therefore, this increased complexity is due to desynchronization between the cars and the *greenwave* caused by a large occupation level.

For the region with constant average speed, the distribution of traffic jam length, and the corresponding time travel between traffic lights, is highly inhomogeneous, and even when it is constant on average, the standard deviation tends to be higher in this range. To explain this peculiar behavior let us consider again Figure 5, in which we have plotted the spatial variation of the traffic jam length at each traffic light over the street length for a fixed α. In terms of the normalized velocity α, there is a correspondence between the constant averaged speed zone with the region where the spatial complexity increases. This correspondence tells us the true nature of the dynamics of the system. For $J_N < N_L/4$ the traffic is undersaturated and the initial jammed condition dissolves as time passes. For $J_N > 3N_L/4$ the traffic is oversaturated and there are much more cars trying to cross a traffic light than the ones exiting from the next traffic light, so that all of the initial conditions converge to the same attractor. For $N_L/4 < J_N < 3N_L/4$ there is a range for α values in which the system is not fully saturated, nor is able to dissolve the initial jammed condition. At the same time, the travel time increases linearly with J_N as the cars are being stopped at every traffic light, following the dynamic imposed by the traffic light control function. In this case, this condition is determined by the function $f_n(t,\alpha) \sim \sin(n\alpha)$ leading to a large standard spatial deviation of the averaged travel time. As the average $\langle f_n(t,\alpha)\rangle_n$ converges to 0 with increasing N_s, this results in an averaged travel time and averaged speed that are constant in that range.

The behavior described here is robust against perturbations to the jam length J_N, as shown in Figure 7. We have taken an initial traffic jam length for the n^{th} traffic light as $J_{N,n} = J_N(1+\delta_n)$, in which δ_n is a random integer value in the range $[-r_J, r_J]$, for a given value of $r_J < 1$. We observe in the figure that the average normalize velocity, and hence the dynamics, is not affected significantly by these spatial perturbations to the length of the initial traffic jam. Even for large perturbations ($r_N = 0.5$) the averaged speed is very similar to the case of an uniform initial condition with $J_N = \langle J_{N,n}\rangle$. In this sense the behavior is universal and robust.

4 RANDOM VELOCITY PERTURBATIONS

Realistic traffic situations are characterized by a distribution of lengths, speeds, etc. Furthermore, the driving dynamics of a given car is not deterministic, hence, in order to approximate a more realistic situation we introduce

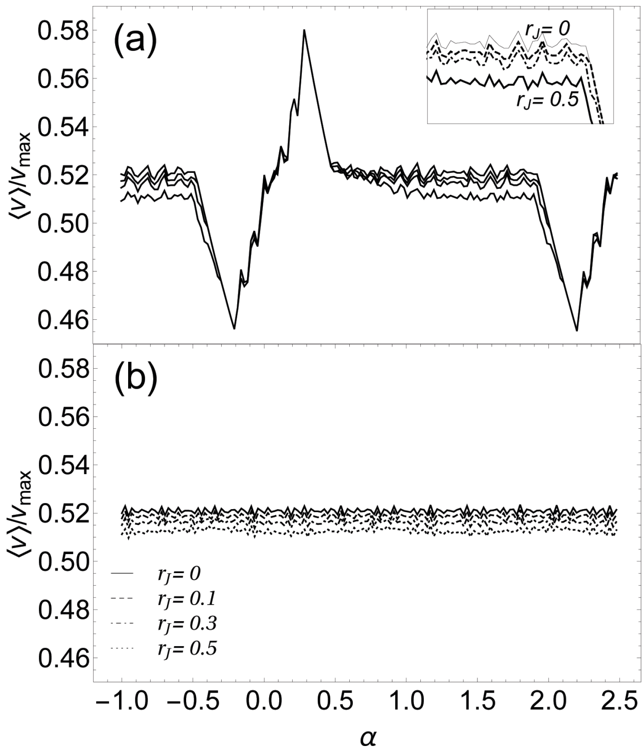

FIGURE 7
Normalized averaged speed comparison for $r_J = 0.0,\ 0.1,\ 0.3,\ 0.5$ at (a) $J_N = 12$ and (b) $J_N = 24$.

velocity perturbations in our simulations, through the parameter r, which represents the random probability that a car does not move at a given time step, even if all the other conditions to do are satisfied. For example, the case $r = 0.01$ corresponds to having a 1% probability of stopping at the next time step.

The simulation was done with the same parameters as in Figure 2(a), but for $r = 0.01,\ 0.03,\ 0.05$. The plot for a full range of initial conditions is shown in Figure 8. For $r = 0$, the same curves as in Figure 2 are obtained, each curve corresponding to an initial condition. However, for a given nonzero value of r, all initial initial conditions yield essentially the same curve. We also observe that the constant zones, seen before for the $r = 0$ case, disappear. All the attractors for $r = 0$ now converge to a statistically equivalent system when $r \neq 0$ for all initial jammed densities. The standard deviations, represented by the error bars included in the figure, are also small.

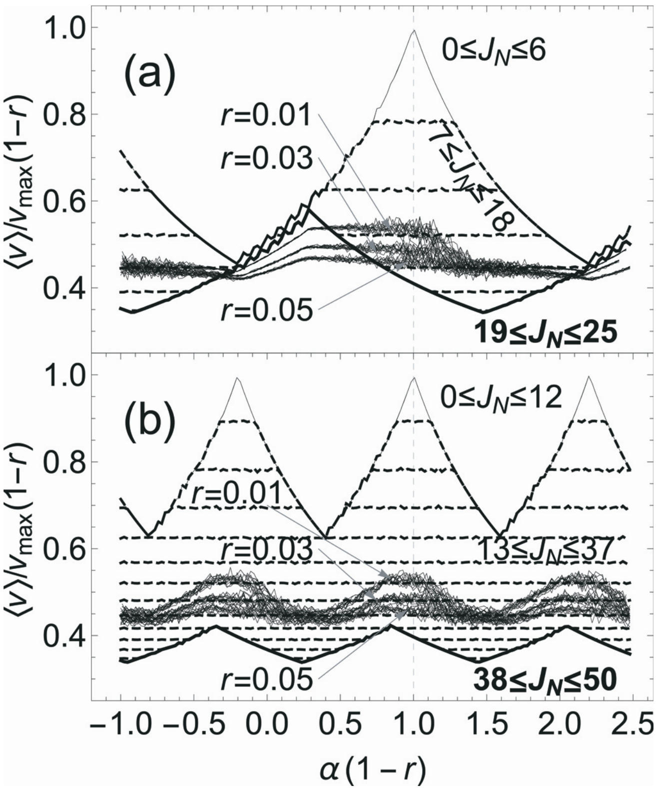

FIGURE 8
Averaged speed of the cars normalized by $v_{\max}$ for the full range of initial jammed conditions $J_N \in [0, 20]$ as a function of α. We show a comparison between $r = 0$, $r = 0.01$, $r = 0.03$, and $r = 0.05$. The label of the curves follow the description used in Figure 2(a), with the dashed lines increasing in J_N value from top to bottom.

As shown in Ref. [1] the introduction of this noise changes the average velocity of an isolated car as $v_{\max} \rightarrow v_{\max}(1 - r)$, assuming an empty road ahead without cars or traffic lights. Thus, to compare the different curves produced by the different results of r, we use this normalization for $v_{\max}$ in both axis of Figure 8. The fact that the curves remain different for different values of r, suggests that the resulting dynamics is a consequence of collective effects. By analyzing this emergent state, it is possible to see that it represents the dynamics of a traffic jam, with well defined pulses of about $T/4$ cars propagating with velocity $v_{\max}(1 - r)$, and in which a given pulse drops a number of cars at a given traffic light, and picks up the cars left by the pulse propagating ahead at the next traffic light. The pulses are produced by the traffic lights, but there is a range of α ($0.5 \leq \alpha \leq 1.0$) values in which the average speed does not change, hence the traffic lights just generate the pulses, but do not seem to affect the dynamics in other ways [1].

In Figure 8 we notice a very counter intuitive effect of the velocity perturbations. We see that for a range of initial jammed conditions and α values, the introduction of velocity perturbations increases the averaged speed relative to the case $r = 0$, when $J_N > 3N_L/4$. Hence, the velocity perturbations not only add stability to the system, but in some cases, improve the flow. The increase occurs in the range of α values for which there is a constant average velocity in the situation with no velocity perturbations, i.e., in the emergent state region. Hence, this behavior is similar to a stochastic resonance in which the average traffic speed increases with respect to that of the system without noise for some initial jammed densities. It is not very difficult to realize that the stochastic resonance occurs at each traffic light, as we introduce the velocity perturbations. In Figs. 5(a) and (b) we show that for $r = 0$ there are spatial situations in which $Y_n > 15$, the largest number of cars that can be evacuated during a light period. We see that for $r > 0$ the spatial variation of Y_n gets smoothed out with an average value that is lower than $\langle Y_n \rangle < 15$, so that in principle the traffic light could evacuate all cars, if the next traffic light could allow it. Hence, the introduction of velocity perturbations reduces the spatial variability (and hence the standard deviation mentioned above) of the number of cars at each traffic light. This reduction on the spatial variability of Y_n, due to its nonlinear effect on the travel time, generates the stochastic resonance phenomena. For $L = 25\,\ell$ the system organizes itself so that the average number of cars at the traffic light becomes $\langle Y_n \rangle < 15$. However, this condition cannot occur for $L = 50\,\ell$, so the velocity reduces quite significantly compared to the $L = 25\,\ell$ case.

This essentially brings the question as to what is the appropriate period that the traffic lights in a given city must have to optimize the average flow speed. In general our results suggest that $T/4$ should be about the time to evacuate all cars that are expected to stop between traffic lights, which in our case should be about $L\tau/2\ell$. But of course other complexities of city traffic must be considered. These results may be relevant in practical applications of city traffic and will be further developed elsewhere.

5 CONCLUSIONS

We have studied the critical behavior close to the resonance in the normalized velocity $\alpha \sim 1$, for a *greenwave* strategy using a CA model for city traffic, that includes car interactions and traffic lights. A given car density is produced by the effect of the $1/f$ parameter, which measures the frequency with which cars enter the street. In this manuscript we used $f = 1$ because we are interested in jammed situations. Some detailed effects that are relevant for car dynamics in cities are missing in this CA model, such as the finite accelerating and braking capabilities of the cars, which can be improved by allowing multiple velocity states [2]. In spite of this, the statistical behavior of the cars displays nontrivial dynamics, even for the case of unperturbed velocities ($r = 0$). In this case, the car dynamics display spatial complexity in their travel time between traffic lights under initial jammed conditions, a result that produces an average speed independent of α for a range of wave velocities and initial jammed densities. Hence, for some initial jammed densities, namely $J_N > N_L/4$ the dynamics converge in time to a different attractor from the one studied previously [1, 14, 18], which started from an empty street as initial condition.

When cars are allowed to vary their velocities in a random manner (modeled in our case as a probability of not moving in a given time step), the behavior around the resonance changes, and all the systems with different initial jammed conditions converge in time to an equivalent statistical state, represented by an emergent state that has some of the properties of traffic jams in cities. This emergent state, for a range of α values, is caused mainly by the traffic light switching, but once it is established, the light switching is not relevant for the car dynamics anymore. This resembles a classical gas, where collisions establish and maintain an equilibrium state, but do not otherwise affect macroscopic thermodynamic quantities.

As we introduce velocity perturbations, we notice the occurrence of a phenomenon that resembles a stochastic resonance producing an increase in the traffic flow, as characterized by the average speed, for a certain range of *greenwave* speeds and initial jammed conditions ($J_N > 3N_L/4$). Hence, the noise is able to unjam the system, and increase the average flow. But, let us note that the real system already includes this "optimization", and further studies in the earlier stages of the jam formation, will be necessary to determine if some kind of control could be implemented at the lights (random perturbations from some distribution) to fluidize even more the stream. The result found in [17], extended and commented here, propose a model for one of the internal mechanisms of a traffic jam, describing how random perturbations are an essential part of them and suggesting why we should pursue farther on this track. This counter-intuitive result could have applications in city traffic management.

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