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Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory

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ABSTRACT

The aim of the paper is to develop new aggregation operators using Bonferroni means, OWA operators and some distance measure. We introduce the BON-OWAAC and BON-OWAIMAM operators. We are able to include coefficient adequacy and the maximum and minimum levels in the same formulation with Bonferroni means and an OWA operator. The main advantages of using these operators are that they allow consideration of continuous aggregations, multiple comparisons between each argument and distance measures in the same formulation. An application is developed using these new algorithms in combination with Moore's families and Galois lattices to solve group decision-making problems. The professional and personal interests of the entrepreneurs who share co-working spaces are taken as an example for establishing relationships and groups. According to the professional and personal profile affinities for each entrepreneur, the results show dissimilarity and fuzzy relationships and the maximum similarity sub-relations to establish relationships and groups using Moore's families and Galois lattice. Finally, this new type of distance family can be used for applications in areas such as sports teams, strategy marketing and teamwork.

1. Introduction

Entrepreneurial activity requires several conditions to foster creative and innovative environments. According to Lee et al. [28], entrepreneurial activity requires a scenario in which creativity, diversity and innovation are encouraged and valued. One of the main strategies for creating such entrepreneurial environments is co-working space. Co-working spaces are creative and energetic places that provide support (moral, emotional, professional, and financial) and facilities (infrastructure) to enable entrepreneurs to start and grow their businesses for small firms, freelancers and start-ups [4,9]. In addition, these spaces use scale economies, promote members through social media, share knowledge and—given the chance to collaborate—provide opportunities to develop systemic solutions and grant the ability to identify opportunities among their members [40,42,43]. Likewise, these workplaces are characterized by their purpose and model, which are categorized into co-working space, incubators, accelerators, and service offices and are classified by target audience, operating mode, relation to other organizations and size [9]. Hence, for entrepreneurs to take advantage of their working spaces, they should be able to choose the place that best suits their professional needs, personal interests, skills, capabilities and knowledge. Because the members who belong to these workplaces have profiles and varied interests, which are linked by relationship, similarities and affinities, this promotes faster collaboration, knowledge transfer and the detection of entrepreneurial opportunities. Typically, these processes take considerable time before the members of co-working spaces can meet and share experiences, information, and knowledge depending on personality and attitude.

The literature shows that there are a wide range of methods that allow aggregating information [23,24,36]; these models create great types of relationships and obtain representative values of the aggregated information. One of the most popular models is the ordered weighted averaging OWA operator [55], which has been developed largely from an extension in combination with the mathematical models of others. One of these types of models includes selection indexes [37]. These are known as ordered weighted averaging distance (OWAD) [12,37], ordered weighted averaging adequacy coefficient (OWAAC) operator and ordered weighted averaging index of maximum and minimum level (OWAIMAM) operator [38], which allow the aggregation of information through compar-

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ison between two elements to reach a representative value. Likewise, a new aggregation operator is proposed by using Bonferroni means (BM) [6], which allow multiple comparisons between input arguments and the capture of their interrelationships. Yager [57] combined the OWA operator with BM, proposing a new aggregation operator called BON-OWA, and suggested a generalization of this operator. This new aggregation operator incited the curiosity of the scientific community, driving multiple authors to begin studying and developing new models based on this [2,18,22,50,58].

The aim of this paper is to develop a new mathematical application based on Bonferroni means, the OWA operator and some distance measurement. This application consists of BM in combination with OWAIMAN and OWAAC operators. The main advantage of this proposition is that it allows the use of continuous aggregations, multiple comparisons between each argument and distance measures in the same formulation. Likewise, these new methods are combined with Moore's families and Galois Lattice to solve group decision-making problems. These methods are used as a previous step to apply Moore's families and Galois Lattice algorithm to obtain different distances between a set of elements and to gather each element according to relationship similarities and affinities. A numerical example is developed to demonstrate the usefulness of the new proposition. This application is focused on the establishment of relationships and the groups of affinities between an entrepreneur who wishes to belong to a co-working space according to professional needs, personal interests, skills, capabilities and knowledge in comparison to each individual who already belongs to a co-working space. These algorithms allow the aggregation of information obtained in a single value representative of the information according to the personal parameters of each entrepreneur.

The structure of this paper is as follows. In Section 2, basic concepts of Bonferroni means, OWA operators and distance measures are briefly reviewed, and a new proposal is presented. In Section 3, the decision-making approach is explained. In Section 4, the numerical application processes for gathering affinities using BON-OWAD, BON-OWAAC, Moore's families and Galois Lattice are displayed. In Section 5, a numerical example of the new method is given, focusing on co-working in entrepreneurship. In Section 6, a summary and the main conclusion are presented.

2. Preliminaries

In this section, we briefly review Bonferroni means, OWA operators, BON-OWA, distance measures and OWAD to develop new tools based on distance measures in combination with Bonferroni means and OWA operators.

2.1. Bonferroni means

The Bonferroni mean [6] is another type of mean that can be used in the aggregation process to present information. Recently several authors have used the Bonferroni mean with OWA operators [2,57], uncertain data [51], linguistic variables [29,48], intuitionistic information [7,49,54] and hesitant representation [65,66]. It can be defined by using the following expression:

$$B\left(a_1, a_2, \dots, a_n\right) = \left(\frac{1}{n}\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q\right)^{\frac{1}{r+q}}, \tag{1}$$

where r and q are parameters, such that $r, q \geq 0$ and the arguments $a \geq 0$. By rearranging the terms [57], it can also be formulated in the following way:

$$B\left(a_1, a_2, \dots, a_n\right) = \left(\sum_{i=1}^{n} a_i^r\left(\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q\right)\right)^{\frac{1}{r+q}}. \tag{2}$$

2.2. OWA operators

2.2.1. OWA operator

The OWA operator [55] provides a parameterized class of mean type of aggregation operators. It can be defined as follows.

Definition 1. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA\left(a_1, a_2, \dots, a_n\right) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the jth largest of the a_j.

2.2.2. Bonferroni OWA

The Bonferroni OWA [57] is a mean type aggregation operator. It can be defined by using the following expression:

$$BON-OWA\left(a_1, \dots, a_n\right) = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r OWA_W\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{4}$$

where $OWA_W(V^i) = (\frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} a_j^q)$, with (V^i) being the vector of all a_j, except a_i, and w being an $n-1$ vector W_i associated with A_i, whose components w_{ij} are the OWA weights. Let W be an OWA weighting vector of dimension $n-1$ with components $w_i \in [0, 1]$ when $\sum_i w_i = 1$. We can then define this aggregation as $OWA_W(V^i) = (\sum_{j=1}^{n-1} w_i a_{\pi_i(j)})$, where $a_{\pi_i(j)}$ is the largest element in the tuple V^i and $w_i = \frac{1}{n-1}$ for all i. Thus, we have observed that this aggregation is equal at the original case. Furthermore, according to Yager [57], the weight vector w_i can be stipulated by different methods. One approach is to directly specify the vector W. Another is using O'Hagan's [41] approach for maximizing the entropy $-\sum_{j=1}^{n-1} w_j \ln(w_j)$ subject to a degree of orness $\sum_{j=1}^{n-1} w_j \frac{n-j}{n-1} = \alpha$, $\sum_{j=1}^{n-1} w_j = 1$, $0 \leq w_j \leq 1$. Another approach is via BUM function f, in which we obtain $w_j = f(\frac{j}{n-1}) - f(\frac{j-1}{n-1})$. Based on this method, another approach is developed, which starts with a parameterized family of BUM functions and defines the desired aggregation by specifying the value-associated parameter [57]. Another parameter function is $f(x) = x^r$ for r > 0, where from r we obtain a particular function. The attitudinal character is such that $\alpha = \frac{1}{r+1}$, and if we specify α, we can obtain $r = \frac{1-\alpha}{\alpha}$ [57].

2.3. Distance measures

2.3.1. Hamming distance

The Hamming distance [17] is a useful technique for calculating the differences between two elements, two sets, etc. In fuzzy set theory, it can be useful, for example, for the calculation of distances between fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. For two sets A and B, the weighted Hamming distance can be defined as follows.

Definition 2. A weighted Hamming distance of dimension n is a mapping d_{WH}: $R^n x R^n \to R$ that has an associated weighting vector

W of dimension n with the sum of the weights being 1 and $w_j \in [0, 1]$, such that

$$d_{WH}(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{i=1}^{n} w_i |x_i - y_i|, \tag{5}$$

where x_i and y_i are the ith elements of sets X and Y.

2.3.2. Adequacy coefficient

The adequacy coefficient [26,27] is an index used for calculating the differences between two elements, two sets, etc. Although similar to the Hamming distance, it differs because the adequacy coefficient neutralizes the result when the comparison shows that the real element is higher than that of the ideal one. For two sets A and B, the weighted adequacy coefficient can be defined as follows.

Definition 3. A weighted adequacy coefficient of dimension n is a mapping K: $[0, 1]^n x[0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W of dimension n with the sum of the weights 1 and $w_j \in [0, 1]$, such that

$$K(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{i=1}^{n} w_i [1 \wedge (1 - x_i + y_i)], \tag{6}$$

where x_i and y_i are the ith elements of sets X and Y.

2.3.3. Maximum and minimum level index

The index of maximum and minimum level is an index that unifies Hamming distance and adequacy coefficient in the same formulation [14,15]. For two sets A and B, the weighted index of maximum and minimum level can be defined as follows.

Definition 4. An AWIMAM of dimension n is a mapping K: $[0, 1]^n x[0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W of dimension n with the sum of the weights 1 and $w_j \in [0, 1]$, such that

$$\eta(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{u} w_i(u) * |x_i(u) - y_i(u)| + \sum_{v} w_i(v) * [0 \vee x_i(v) - y_i(v)], \tag{7}$$

where x_i and y_i are the ith arguments of sets X and Y, u and v are the numbers of arguments to be treated with the Hamming distance and the adequacy coefficient, respectively, and $u + v = n$.

2.4. OWA operator and distance measures

The OWAD operator [12,37,52] is an aggregation operator that uses OWA operators and distance measures in the same formulation. It can be defined as follows for two sets X and Y.

Definition 4. An OWAD operator of dimension n is a mapping $OWAD$: $R^n x R^n \rightarrow R$ that has an associated weighting vector W, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWAD(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j D_j, \tag{8}$$

where D_j represents the jth largest of $|x_j - y_j|$.

The OWAAC operator [13,37] is an aggregation operator that uses the adequacy coefficient and the OWA operator in the same formula. It can be defined as follows for two sets X and Y.

Definition 5. An OWAAC operator of dimension n is a mapping $OWAAC$: $[0, 1]^n x[0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that

$$OWAAC(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j, \tag{9}$$

where K_j represents the jth largest of $[1 \wedge (1 - x_i + y_i)]$.

The OWAIMAM operator [38] is an aggregation operator that uses the Hamming distance, the adequacy coefficient and the OWA operator in the same formulation. It can be defined as follows.

Definition 7. An OWAIMAM operator of dimension n, is a mapping $OWAIMAM$: $[0, 1]^n x[0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $w_j \in [0, 1]$ and the sum of the weights is equal to 1, such that

$$OWAIMAM(\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j, \tag{10}$$

where K_j represents the jth largest of all $|x_i - y_i|$ and $[0 \vee (x_i - y_i)]$ and is formed when comparing sets X and Y. Note that for each pair of arguments $\{x_i, y_i\}$, we should define which of them will be analyzed with the individual distance $|x_i - y_i|$ and which ones with the norm $[0 \vee (x_i - y_i)]$.

2.5. Bonferroni distances

In this section, we briefly review Bonferroni means and distance measures. For working with distance measures using Bonferroni means, arguments a_i and a_j are two sets of variables instead of one, but the methodology remains the same. For this study, we focus on BD and BON-OWAD concepts, which are defined as follows.

Definition 6. The Bonferroni distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ is given by

$$BD(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{i=1}^{n} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1 \\ j\neq i}}^{n} d_j^q\right)\right)^{\frac{1}{r+q}}, \tag{11}$$

where d_i and d_j are individuals, such that $d_i = |x_i - y_i|$ and $d_j = |x_j - y_j|$.

Definition 7. A BON-OWAD distance for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ is given by

$$BON - OWAD(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{k=1 \\ j\neq i}}^{n} D_i^r OWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{12}$$

where $OWAD_{w_i}(V^i) = (\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} D_j^q)$ with (V^i) being the vector of all $|x_j - y_j|$ except $|x_i - y_i|$, and w_i being an $n-1$ vector W_i associated with A_i whose components w_{ij} are the OWA weights. Likewise, D_i is the *ith* smallest of the individual distances $|x_i - y_i|$.

3. Bonferroni means with the adequacy coefficient and the index of maximum and minimum levels

The adequacy coefficient was proposed by Kaufmann & Gil-Aluja [26,27], and the OWAAC operator was proposed by Gil-Lafuente & Merigó [13] and Merigó & Gil-Lafuente [37]; the combination of both allow the aggregation of information through the comparison of two elements with the feature that it neutralizes the result when the comparison gives a real element that is higher than the ideal one.

Proposition 1. *The Bonferroni adequacy coefficient for two sets* $X = \{x_1, x_2, \dots, x_n\}$ *and* $Y = \{y_1, y_2, \dots, y_n\}$ *is given by*

$$BAC(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = \left(\sum_{i=1}^{n} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} d_j^q\right)\right)^{\frac{1}{r+q}}, \quad (13)$$

where d_i *and* d_j *are the individual differences such that* $d_i = [1 \wedge (1 - x_i + y_i)]$ *and* $d_j = [1 \wedge (1 - x_j + y_j)]$.

Proposition 2. *A BON-OWAAC distance for two sets* $X = \{x_1, x_2, \dots, x_n\}$ *and* $Y = \{y_1, y_2, \dots, y_n\}$ *is given by*

$$BON - OWAAC(\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{i=1\\ j\neq i}}^{n} D_i^r\, OWAAC_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \quad (14)$$

where $OWAAC_{w_i}(V^i) = (\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q)$, *with* (V^i) *being the vector of all* $1 \wedge (1 - x_j + y_j)$ *except* $1 \wedge (1 - x_i + y_i)$, *and* w_i *being an* $n-1$ *vector* W_i *associated with* A_i *whose components* w_{ij} *are the OWA weights. Likewise,* D_i *is the* ith *smallest of the individual distance* $[1 \wedge (1 - x_i + y_i)]$.

Furthermore, BON-OWAAC has the following properties: 1) $\mathrm{BON-OWAAC}^{r,q}(0,\ 0, \dots, 0) = 0$; 2) $\mathrm{BON-OWAAC}^{r,q}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = \mathrm{ac}$, if $\mathrm{ac}_k = \mathrm{ac}$, for all i; 3) $\mathrm{BON-OWAAC}^{r,q}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) \geq \mathrm{BON-OWAAC}^{r,q}(\mathrm{ad}_1,\ \dots, \mathrm{ad}_n)$, i.e., $\mathrm{BON-OWAAC}^{r,q}$ is monotonic, if $\mathrm{ac}_i \geq \mathrm{ad}_i$, for all i; 4) $\min\{ac_i\} \leq \mathrm{BON-OWAAC}^{r,q}(\mathrm{ac}_1,\ \dots, \mathrm{ac}_n) \leq \max\{ac_i\}$. In addition, if $q = 0$, then $\mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n}\sum_{i=1}^{n} D_i^r)^{1/r}$. If $r = 2$ and $q = 0$, then BON-OWAAC reduces to the square mean distance: $\mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n}\sum_{i=1}^{n} D_i^2)^{1/2}$. If $r = 1$ and $q = 0$, then BON-OWAAC reduces to the average distance: $\mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = \frac{1}{n}\sum_{i=1}^{n} D_i$. If $r \to +\infty$ and $q = 0$, then BON-OWAAC reduces to the max operator: $\lim_{r\to+\infty} \mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = \max\{D_i\}$; if $r \to 0$ and $q = 0$, then BON-OWAAC reduces to the geometric mean distance: $\lim_{r\to 0} \mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\prod_{i=1}^{n} D_i)^{1/n}$. If $r = q = 1$, then BON-OWAAC reduces to the following expression: $\mathrm{BON-OWAAC}^{1,1}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n(n-1)})\sum_{\substack{i,\ j=1\\ i\neq j}}^{n} D_i D_j$.

To understand the BON-OWAAC numerically, let us present a simple example.

Example 1. Let $X = (0.2,\ 0.5,\ 0.4)$ and $Y = (0.5,\ 0.1,\ 0.7)$ be two sets of arguments. w_i is the weighting vector of the argument $|x_i - y_i|$ associated with A_i with components v_{ij}. Here, we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$ and $\alpha_3 = 0.5$. We take $r = q = 0.5$. In addition, $V^1 = (1 \wedge (1 - 0.5 + 0.1))$ *and* $(1 \wedge (1 - 0.4 + 0.7))$, $V^2 = (1 \wedge (1 - 0.2 + 0.5))$ *and* $(1 \wedge (1 - 0.4 + 0.7))$ and $V^3 = (1 \wedge (1 - 0.2 + 0.5))$ *and* $(1 \wedge (1 - 0.5 + 0.1))$. Using this, we obtain:

$$OWAAC_{v1}\left(V^1\right) = 0.4 \times ((1 \wedge (1 - 0.5 + 0.1)) + (1 \wedge (1 - 0.4 + 0.7))) = 0.64$$

$$OWAAC_{v2}\left(V^2\right) = 0.3 \times ((1 \wedge (1 - 0.2 + 0.5)) + (1 \wedge (1 - 0.4 + 0.7))) = 0.60$$

$$OWAAC_{v3}\left(V^3\right) = 0.5 \times ((1 \wedge (1 - 0.2 + 0.5)) + (1 \wedge (1 - 0.5 + 0.1))) = 0.95$$

$$BON - OWAAC = \left(\frac{1}{3} \times \begin{pmatrix} ((1 \wedge (1 - 0.2 + 0.5)) \times 0.64) \\ +((1 \wedge (1 - 0.5 + 0.1)) \times 0.60) \\ +((1 \wedge (1 - 0.4 + 0.7)) \times 0.95) \end{pmatrix}\right)^{\frac{1}{0.5+0.5}} = 0.806$$

The maximum and minimum level index was proposed by [14,15], and OWAIMAN was proposed by [38]. These operators allow aggregating information through the comparison between the elements of two sets. Moreover, the IMAM index unifies the Hamming distance and the adequacy coefficient in the same formulation.

Proposition 3. *The Bonferroni index of maximum and minimum level for two sets* $X = \{x_1, x_2, \dots, x_n\}$ *and* $Y = \{y_1, y_2, \dots, y_n\}$ *is given by*

$$BIMAM\ (\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = \left(\sum_{i=1}^{n} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} d_j^q\right)\right)^{\frac{1}{r+q}}, \quad (15)$$

where d_i *and* d_j *are the individuals such that* $d_i = |x_i - y_i|$ *and* $[0 \vee (x_i, y_i)]$, *and* $d_j = [|x_j - y_j|$ *and* $[0 \vee (x_j, y_j)]$.

Proposition 4. *A BON-OWAIMAM distance for two sets* $X = \{x_1, x_2, \dots, x_n\}$ *and* $Y = \{y_1, y_2, \dots, y_n\}$ *is given by*

$$BON - OWAIMAM\ (\langle x_1, y_1\rangle, \dots, \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{i=1\\ j\neq i}}^{n} D_i^r\ OWAIMAM_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \quad (16)$$

where $OWAIMAM_{w_i}(V^i) = (\frac{1}{1-n}\sum_{\substack{j=1\\ j\neq i}}^{n} D_j^q)$, *with* (V^i) *being the vector of all* $[|x_j - y_j|$ *and* $[0 \vee (x_j, y_j)]$ *except* $|x_i - y_i|$ *and* $[0 \vee (x_i, y_i)]$, *and* w_i *being an* $n-1$ *vector* W_i *associated with* A_i *whose components* w_{ij} *are the OWA weights. Likewise,* D_i *is the* ith *smallest of the individual distances* $|x_i - y_i|$ *and* $[0 \vee (x_i, y_i)]$. *Furthermore, BON-OWAIMAM has the following properties: 1)* $\mathrm{BON-OWAIMAM}^{r,q}(0,\ 0, \dots, 0) = 0$; *2)* $\mathrm{BON-OWAIMAM}^{r,q}(\mathrm{a},\ \mathrm{a}, \dots, \mathrm{a}) = \mathrm{a}$, if $\mathrm{a}_k = \mathrm{a}$, for all i; *3)* $\mathrm{BON-OWAIMAM}^{r,q}(\mathrm{a}_1,\ \mathrm{a}_2, \dots, \mathrm{a}_n) \geq \mathrm{BON-OWAIMAM}^{r,q}(\mathrm{d}_1,\ \mathrm{d}_2, \dots, \mathrm{d}_n)$, i.e., $\mathrm{BON-OWAIMAM}^{r,q}$ is monotic, if $\mathrm{a}_i \geq \mathrm{d}_i$, for all i; *4)* $\max_i\{a_i\} \leq \mathrm{BON-OWAIMAM}^{r,q}(\mathrm{a}_1,\ \mathrm{a}_2, \dots, \mathrm{a}_n) \leq \min\{ac_i\}$. *In addition, if* $q = 0$, *then* $\mathrm{BON-OWAIMAM}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n}\sum_{i=1}^{n} D_i^r)^{1/r}$. *If* $r = 2$ and $q = 0$, *then BON-OWAIMAM reduces to the square mean distance:* $\mathrm{BON-OWAIMAM}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n}\sum_{i=1}^{n} D_i^2)^{1/2}$. *If* $r = 1$ and $q = 0$, *then BON-OWAIMAM reduces to the average distance:* $\mathrm{BON-OWAIMAM}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = \frac{1}{n}\sum_{i=1}^{n} D_i$. *If* $r \to +\infty$ and $q = 0$, *then BON-OWAIMAM reduces to the* max *operator:* $\lim_{r\to+\infty} \mathrm{BON-OWAIMAM}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = \max\{D_i\}$; *if* $r \to 0$ and $q = 0$, *then BON-OWAIMAM reduces to the geometric mean distance:* $\lim_{r\to 0} \mathrm{BON-OWAAC}^{r,0}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\prod_{i=1}^{n} D_i)^{1/n}$. *If* $r = q = 1$, *then BON-OWAIMAM reduces to the following expression:* $\mathrm{BON-OWAIMAM}^{1,1}(\mathrm{ac}_1,\ \mathrm{ac}_2, \dots, \mathrm{ac}_n) = (\frac{1}{n(n-1)})\sum_{\substack{i,\ j=1\\ i\neq j}}^{n} D_i D_j$.

To understand the BON-OWAIMAM numerically, let us present a simple example.

Example 2. Let $X = (0.2,\ 0.5,\ 0.4)$ and $Y = (0.5,\ 0.1,\ 0.7)$ be two sets of arguments. w_i is the weighting vector of the argument $|x_i - y_i|$ associated with A_i with components v_{ij}. Here, we shall let $\alpha_1 = 0.4$, $\alpha_2 = 0.3$ and $\alpha_3 = 0.5$. We take $r = q = 0.5$.

In addition: $V^1 = (|0.5 - 0.1|)$ *and* $(1 \wedge (1 - 0.4 + 0.7))$, $V^2 = (|0.2 - 0.5|)$ *and* $(1 \wedge (1 - 0.4 + 0.7))$ and $V^3 = (|0.2 - 0.5|)$ *and* $(1 \wedge (1 - 0.5 + 0.1))$. Using this, we obtain

$$OWAIMAM_{v1}\left(V^1\right) = 0.4 \times ((|0.5 - 0.1|) + (1 \wedge (1 - 0.4 + 0.7))) = 0.56$$

$$OWAIMAM_{v2}\left(V^2\right) = 0.3 \times ((|0.2 - 0.5|) + (1 \wedge (1 - 0.4 + 0.7))) = 0.39$$

$$OWAIMAM_{v3}\left(V^3\right) = 0.5 \times ((|0.2 - 0.5|) + (1 \wedge (1 - 0.5 + 0.1))) = 0.45$$

$$BON - OWAIMAM = \left(\frac{1}{3} \times \left(\begin{array}{l}((|0.2 - 0.5|) \times 0.56) \\ +((1 \wedge (1 - 0.5 + 0.1)) \times 0.39) \\ +((|0.4 - 0.7|) \times 0.45)\end{array}\right)\right)^{\frac{1}{0.5+0.5}} = 0.423$$

These OWA operators are commutative, monotonic, non-negative and reflexive. They are commutative from the OWA perspective because $f(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = f(\langle c_1, d_1\rangle, \ldots, \langle c_n, d_n\rangle)$, where $(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle)$ is any permutation of the arguments $(\langle c_1, d_1\rangle, \ldots, \langle c_n, d_n\rangle)$. They are also commutative from the distance measure perspective because $f(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = f(\langle y_1, x_1\rangle, \ldots, \langle y_n, x_n\rangle)$. They are monotonic because if $|x_i - y_i| \geq |c_i - d_i|$ for all i, then $f(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) \geq f(\langle c_1, d_1\rangle, \ldots, \langle c_n, d_n\rangle)$. Non-negativity is also always accomplished—that is, $f(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) \geq 0$. Finally, they are also reflexive because $f(\langle x_1, x_1\rangle, \ldots, \langle x_n, x_n\rangle) = 0$.

Another issue to consider is the different measures used in the OWA literature for characterizing the weighting vector [34]. As mentioned above, the weighting vector can be stipulated by a number of methods. We consider the entropy of dispersion, the balance operator, the divergence of W and the degree of orness [34,55]. The entropy of dispersion is defined as follows:

$$H(W) = -\left(\frac{1}{n}\sum_{i} \ln(w_i)\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_i \ln(w_i)\right)\right)^{\frac{1}{r+q}}. \tag{17}$$

For the balance operator, we obtain

$$Bal(W) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1-2_i}{n-1}\right)\left(\sum_{\substack{j=1 \\ j\neq i}}^{n}\left(\frac{n+1-2_j}{n-1}\right)w_i\right)\right)^{\frac{1}{r+q}}. \tag{18}$$

For the divergence of W, we obtain

$$Div(W) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n-i}{n-1} - \alpha(W)\right)^2 \times \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_i\left(\frac{n-j}{n-1} - \alpha(w)\right)^2\right)\right)^{\frac{1}{r+q}}. \tag{19}$$

For the degree of orness, we obtain

$$\alpha(W) = \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n-i}{n-1}\right)\left(\sum_{\substack{j=1 \\ j\neq i}}^{n} w_i\left(\frac{n-j}{n-1}\right)\right)\right)^{\frac{1}{r+q}}. \tag{20}$$

Further extensions to the BON-OWAAC and BON-OWAIMAM could be developed following the current developments on aggregation operators [1,59,62]. For example, we could introduce approaches that work with a unified framework between the OWA operator and the weighted average [32,44,53]. Thus, we would obtain the Bonferroni weighted OWA adequacy coefficient (BON-WOWAAC) and the Bonferroni weighted OWAIMAM (BON-WOWAIMAM) operator. The main advantage of this approach is that it can consider the classical Bonferroni aggregation and the attitudinal character characteristic of the decision maker at the same time. In addition, it can be reduced to any of them if needed. They are formulated as follows. Observe that we proceed using the formulas of Merigó [32].

Proposition 5. *A BON-WOWAAC operator for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ is given by*

$$BON - WOWAAC(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \beta + \left(\sum_{i=1}^{n} d_i^r\left(\frac{1}{1-n}\sum_{\substack{j=1 \\ j\neq i}}^{n} d_j^q\right)\right)^{\frac{1}{r+q}} + (1-\beta) \times \left(\frac{1}{n}\sum_{\substack{i=1 \\ j\neq i}}^{n} D_i^r\, OWAAC_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{21}$$

where $\beta \in [0, 1]$, the first part of the equation represents the BAC operator, and the second part is the BON-OWAAAC operator.

Observe that if $\beta = 1$, we obtain the BAC operator, and if $\beta = 0$, we obtain the BON-OWAAC operator. The more the weight of β is located at the top, the more we use the BAC operator, and vice versa.

Proposition 6. *A BON-WOWAIMAM operator for two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ is given by*

$$BON - WOWAIMAM(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \beta + \left(\sum_{i=1}^{n} d_i^r\left(\frac{1}{1-n}\sum_{\substack{j=1 \\ j\neq i}}^{n} d_j^q\right)\right)^{\frac{1}{r+q}} + (1-\beta) \times \left(\frac{1}{n}\sum_{\substack{i=1 \\ j\neq i}}^{n} D_i^r\, OWAIMAM_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{22}$$

where $\beta \in [0, 1]$, the first part of the equation represents the BIMAM operator, and the second part the BON-OWAIMAM operator.

Note again that if $\beta = 1$, we obtain the BIMAM, and if $\beta = 0$, we obtain the BON-OWAIMAM.

4. Bonferroni distances with the ordered weighted moving average

The moving average (MA) is a useful technique that allows the representation of dynamic information because the average moves towards some part of the whole sample. The introduction of MA in aggregation theory has allowed aggregating information that moves towards a set of imprecise arguments. Yager [56] and Merigó & Yager [39] have proposed the use of moving average with OWA operators. Recently, new concepts about moving average and OWA operators have been introduced, such as moving average with probabilistic weighted average [33], generalized moving average, distance measures and OWA operators [39], and linguistic moving aggregation operators [35]. Thus, the moving average and ordered weighted moving average are defined as follows:

Definition 8. Moving average permits the consideration of the results of some part of the sample and the ability to make comparisons when modifying the partial sample selected.

$$MA(a_{1+t}, a_{2+t}, ..., a_{m+t}) = \frac{1}{m}\sum_{i=1+t}^{m+t} a_i, \tag{23}$$

where t indicates the movement conducted in the average from the initial analysis.

Definition 9. OWMA of dimension m is a mapping OWMA: $R^m \rightarrow R$ that has an associated weighting vector W of dimension m, with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0, 1]$, such that

$$OWMA(a_{1+t}, a_{2+t}, ..., a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{24}$$

where b_j is the jth largest argument of the a_i, m is the total number of arguments considered from the whole sample, and t indicates the movement in the average from the initial analysis.

With the approach used in Definitions 8 and 9, we can extend the operators presented in Section 3 with moving averages forming the moving Bonferroni distance, moving BON-OWA, moving Bonferroni adequacy coefficient, moving BON-OWAAC, moving Bonferroni IMAM and moving BON-OWAIMAM operators. Following Eqs. (11) and (23), we can formulate the moving Bonferroni distance (MBD) as follows:

$$MBD\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{i=1+t}^{n+t} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1+t\\ j\neq i}}^{n+t} d_j^q\right)\right)^{\frac{1}{r+q}}, \tag{25}$$

By using Eqs. (12) and (24), we can formulate the moving BON-OWAD (BON-OWMAD) operator in the following way:

$$BON - OWMAD\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{i=1+t\\ j\neq i}}^{n+t} D_i^r OWAD_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{26}$$

The moving Bonferroni adequacy coefficient (MBAC) is formulated as follows by using Eqs. (13) and (23):

$$MBAC\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\sum_{i=1+t}^{n+t} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1+t\\ j\neq i}}^{n+t} d_j^q\right)\right)^{\frac{1}{r+q}}, \tag{27}$$

Using Eqs. (14) and (24), we can construct the moving BON-OWAAC (BON-OWMAAC) operator as:

$$BON - OWMAAC\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{i=1+t\\ j\neq i}}^{n+t} D_i^r OWAAC_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{28}$$

By using Eqs. (15) and (23), we can formulate the moving Bonferroni IMAM (MBIMAM) operator in the following way:

$$MBIMAM\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\sum_{i=1+t}^{n+t} d_i^r \left(\frac{1}{1-n}\sum_{\substack{j=1+t\\ j\neq i}}^{n+t} d_j^q\right)\right)^{\frac{1}{r+q}}, \tag{29}$$

The moving BON-OWAIMAM (BON-OWMAIMAM) operator follows Eqs. (16) and (24) and is constructed as follows:

$$BON - OWMAIMAM\ (\langle x_1, y_1\rangle, ..., \langle x_n, y_n\rangle) = \left(\frac{1}{n}\sum_{\substack{i=1+t\\ j\neq i}}^{n+t} D_i^r OWAIMAM_{w_i}\left(V^i\right)\right)^{\frac{1}{r+q}}, \tag{30}$$

Similarly, we could construct many other moving Bonferroni means following the current aggregation operator literature [1,35,59].

5. Group decision-making problems with Galois lattices and Bonferroni means

Moore's families and Galois lattices [5,10,11,45] are used in combination with BON-OWAAC and BON-OWAIMAM operators in the establishment of relationships and affinities according to the comparison of personal interests, skills, capabilities and knowledge of each individual. These algorithms allow the aggregation of information obtained in a single representative value of the information according to the personality parameters and the attitude parameters of each individual. Furthermore, Moore's families and Galois lattices allow relating and grouping of these characteristics according to relationship similarities and affinities.

The introduction of Bonferroni means and BON-OWAAC and BON-OWAIMAM can reflect the degree of different skills, capabilities, knowledge and personal interest necessary to define possible groups within co-working spaces according to personality and attitude using Moore's families and Galois Lattices. Several models that use Bonferroni means have been developed and applied to multiple attribute decision-making (MADM), multicriteria decision-making (MCDM) and multiple attribute grouping decision-making (MAGDM), including linguistic variables [25,30,31], intuitionistic fuzzy information [8,20,46,63,64], geometric operators [16,50], Atanassov orthopairs [3,21,60] and hesitant representation [19,47,61]. To show how this tool works with Moore's families and Galois Lattices and how its algorithms can be applied, we describe the decision making process as follows.

Step 1. Analyze and determine the personal interests, skills, capabilities and knowledge of each entrepreneur Theoretically, this would be represented as: $C = \{C_1, C_2, C_i, \ldots, C_n\}$, where C_i is the ith characteristic of the members belonging to the co-working space to be considered.

Step 2. Establishment of professional needs, personal interests, skills, capabilities and knowledge for an entrepreneur who wants to belong to a co-working space, where P is the ideal creative characteristics expressed by a fuzzy subset.

Step 3. Establishment of professional needs, personal interests, skills, capabilities and knowledge for each entrepreneur who already belongs to a co-working space with K =

$1,\ 2,\ \ldots, m$; where P_k is the kth entrepreneur expressed by a fuzzy subset; C_i is the ith set of personal interests, skills, capabilities and knowledge to be considered; and $l_i^{(k)} \in [0,1]; i = 1,\ 2,\ \ldots, n$ is the value between 0 and 1 for the ith set of personal interests, skills, capabilities and knowledge of the kth entrepreneur.

Step 4. A comparison of the professional needs, personal interests, skills, capabilities and knowledge of each member is considered through the use of the BON-OWAAC, BAC, BON-OWAIMAM and BIMAM operators. In this step, we express numerically the distance between professional needs, personal interests, skills, capabilities and knowledge of an entrepreneur who wishes to belong to a co-working space with those of each entrepreneur who already belongs to one. In this sense, we use the new proposition based on BON-OWA and OWAAC and OWAIMAM operators.

Step 5. Group entrepreneurs according to the maximum similarity and sub-relations of their professional needs, personal interests, skills, capabilities and knowledge levels using Moore's families and Galois lattice. These algorithms are used by Gil-Aluja [11] and are composed of five steps, which will be explained in the following steps.

Step 5.1. From the similarity relation $[\tilde{S}]$ obtained by the comparison explained in the step 4, we found a dissimilarity fuzzy relation $[\tilde{D}]$ through its complement, where $[\tilde{D}] = |1 - [\tilde{S}]|$. From $[\tilde{D}]$ we found a Boolean relation through the determined α level, where $\alpha = n$. Thus, we obtain a symmetric and reflexive matrix.

Step 5.2. Based on a Boolean matrix, we consider it as a starting point for Moore's families. We obtain right connection B^+ and left connection B^- to establish more families. The "connection to the right" B^+, the subset elements of E_1, such that for every $A \in \Pi(E_1)$, B^+ are the successors of all elements belonging to A, which is given by: $\forall\ x \in A\ :\ B^+A = \{y \in E_1/(y,x) \in [B]\}$, where $B^+\emptyset = E_1$. From its definition the following expression is given: $\forall\ x \in A \in \Pi(E_1)\ :\ B^+A = \bigcap_{x \in A} B^+\{x\}$. The connection to the left, B^-, the subset elements of E_1, such that for every $A \in \Pi(E_1)$, B^- is the successor of all elements belonging to A, which is given by: $\forall\ x \in A\ :\ B^-A = \{y \in E_1/(y,x) \in [B]\}$, where $B^-\emptyset =$. By definition, the following expression is given: $\forall\ x \in A \in \Pi(E_1)\ :\ B^-A = \bigcap_{x \in A} B^-\{x\}$. Because B^+ and B^- come from the fuzzy relationship $\tilde{R}$, the closures of Moore $\Pi(E_1)$ are given by $M^{(1)} = B^- \circ B^+$, $M^{(2)} = B^+ \circ B^-$, where $\circ$ is the max–min composition. However, if the matrix is a square, $B^+ = B^- = B^*$; i.e., we must find only a unique connection.

Step 5.3. Based on Moore closing, we establish the closure subsets $\Pi(E_1)$ coming from closure $M^{(1)}$ and $M^{(2)}$, which are given by $\Gamma(E, M^{(1)}) = \bigcup_{A \subset \Pi(E_1)} B^+A$ and $\Gamma(E, M^{(2)}) = \bigcup_{A \subset \Pi(E_1)} B^-A$. Thus, $\bigcup_{A \subset \Pi(E_1)} B^+A = \{A, B, C, \ldots, M, AB, AC, BC, \ldots, MM, E_1\}$ and $\bigcup_{A \subset \Pi(E_1)} B^-A = \{\emptyset, a, b, c, \ldots, m, ab, ac, bc, \ldots, mm,\ E_1\}$. Thus, the families of closed elements $\Gamma(E_2,\ M^{(1)})$ and $\Gamma(E_1,\ M^{(2)})$ are associated by the same cardinal: $car.\Gamma(E_2, M^{(1)}) = car.\Gamma(E_1, M^{(2)})$. Note that these families constitute an isomorphic lattice.

Step 5.4. Based on the Moore's families obtained, we build a Galois lattice. A Galois lattice is an algebraic structure that allows making clusters based on affinities. $\Pi(E_1)$ and $\Pi(E_2)$, the power sets of E_1 and E_2, have established the ordered relationship [10] given by two steps. First, $\forall\ X,\ X' \in \Pi(E_1),\ \forall\ Y, Y'\Pi(E_2)$, i.e., $((X,\ Y) \leq (X',\ Y')) \Leftrightarrow (X \supset X',\ Y \subset Y')$. Second, $\forall\ X,\ X' \in \Pi(E_1),\ \forall\ Y, Y'\Pi(E_2)$, i.e., $((X,\ Y) \geq (X',\ Y')) \Leftrightarrow (X \supset X',\ Y \subset Y')$. Thus, we obtain a graph where it is possible to identify affinities between each entrepreneur within co-working spaces.

Step 6. Decisions are adopted according to the results found in the previous steps and should guide the decision about which types of creative groups are required. Noticeably, the decision is based on choosing the group of entrepreneurs that best fits the decision-makers' interests.

6. Application of decision-making in the formation of creative groups

In this section, we present an application to understand the new approach suggested above. The methodology uses the BON-OWAAC and BON-OWAIMAM operators' in-group decision-making problems with Moore's families and Galois Lattice. The main advantages of using this operator are that it allows the considering continuous aggregations, multiple comparisons between each argument and distance measures in the same formulation according to the ordered position of each argument. In addition, using Moore's families and Galois lattice, we can collect parameters according to its affinities. The application is focused on comparing professional needs, personal interests, skills, capabilities and knowledge of an individual who wishes to belong to a co-working space with those of an individual already belonging to one. The new method proposed allows a continuous comparison and grouping among its alternatives. The approach design comprises six steps, which are presented as follows:

6.1. Decision making approach

Step 1. It has been assumed that the decision-maker wishes to link professional needs, personal interests, skills, capabilities and knowledge similarities between entrepreneurs who belong to a co-working space with a new member and to gather information according to affinities. The manager chooses five entrepreneurs who belong to a co-working space E_1, E_2, E_3, E_4 and E_5, with different levels of professional needs, personal interests, skills, capabilities and knowledge (see Table 1). Each characteristic of the group of individuals is considered a property.

Step 2. It has been assumed that the decision-maker has established the professional needs, personal interests, skills, capabilities and knowledge characteristics of each entrepreneur in a professional and personal profile (PPP) (see Table 2). These characteristics are inherent for each individual according to their own intellectual, personality and motivational traits, which are shaped by education and social and professional environments.

Step 3. It has been assumed that the decision-maker has fixed the level of each professional and personal characteristics for each entrepreneur who belongs to each co-working space (E_1 to E_5) and the new member (E_0). Here, each level could be composed by objective or subjective information according to prior evaluation. It shows the level of professional and personal interests for each entrepreneur, creating a professional and personal profile composed of 36 characteristics (see Table 3).

Step 4. To make a technical comparison between each level of professional and personal interests for each entrepreneur, the BON-OWAAC and BON-OWAIMAM operators are used as a starting point. Furthermore, we also consider HD, AC, IMAM, OWAD, OWAAC, OWAIMAM, BD, BAC, BIMAM and BON-OWAD. The main idea is to make continuous comparisons to create groups with Moore's families and the Galois lattice algorithm. The weighting vector $w =$ 0.8, 0.8, 0.5, 0.9, 0.5, 0.9, 0.7, 0.8, 0.6, 0.8, 0.5, 0.7, 0.2, 0.2, 0.4, 0.7, 0.1, 1.0, 1.0, 0.9, 0.6, 0.4, 0.7, 0.8, 0.7, 0.1, 0.8, 0.7, 1.0, 0.5, 0.8, 0.9,

Table 1
Group of creative individuals.

E_1	Entrepreneur who works in the development of mobile applications
E_2	Entrepreneur who works in the design of web pages
E_3	Entrepreneur who works in arts, images and illustrations
E_4	Entrepreneur who works in accounting and financial advice for small businesses
E_5	Entrepreneur who works in marketing and branding advice for small businesses

Table 2
Professional and personal profile (PPP).

	Professional needs		Personal interests		Skills		Capabilities		Knowledge
C_1	Accounting	C_8	New business	C_{15}	Programing	C_{22}	Creative	C_{29}	Accounting
C_2	Programing	C_9	Arts	C_{16}	Informatics	C_{23}	Drawing	C_{30}	Finance
C_3	Finance	C_{10}	Internet	C_{17}	Designing	C_{24}	Rhetoric	C_{31}	Marketing
C_4	Branding	C_{11}	Mobile applications	C_{18}	Writing	C_{25}	Negotiation	C_{32}	Arts and illustration
C_5	Relationships	C_{12}	Sports	C_{19}	Analysis	C_{26}	Leadership	C_{33}	Informatics
C_6	Marketing	C_{13}	Music	C_{20}	Software	C_{27}	Persuasion	C_{34}	Branding
C_7	Design	C_{14}	Lecture	C_{21}	Social networks	C_{28}	Teamwork	C_{35}	Programing
								C_{36}	Strategy

Table 3
Level of professional and personal interests of each entrepreneur.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
E_0	0.5	0.4	0.5	1.0	1.0	0.8	0.4	0.6	0.1	0.4	0.1	1.0	0.5	0.5	0.2	0.4	0.4	1.0
E_1	0.7	0.5	0.5	0.5	0.7	0.6	0.8	0.1	0.5	1.0	1.0	0.5	0.2	0.5	1.0	1.0	0.9	0.6
E_2	0.3	0.2	0.8	1.0	0.5	0.6	1.0	0.3	0.8	1.0	0.7	0.5	0.5	0.5	1.0	1.0	0.6	0.6
E_3	0.3	0.2	0.8	1.0	0.5	0.6	1.0	0.3	0.8	1.0	0.7	0.5	0.5	0.5	1.0	1.0	0.6	0.6
E_4	0.5	0.8	0.2	0.3	1.0	0.6	1.0	1.0	1.0	0.6	0.7	0.5	0.8	0.5	0.3	0.5	1.0	1.0
E_5	1.0	0.5	1.0	0.5	0.7	0.9	0.1	0.2	0.5	0.8	0.5	0.5	0.6	0.5	0.3	0.5	0.7	0.5

	C_{19}	C_{20}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	C_{27}	C_{28}	C_{29}	C_{30}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	C_{36}
E_0	0.6	0.4	1.0	0.9	0.2	0.5	0.8	0.8	0.7	1.0	0.5	0.5	0.8	0.4	0.4	1.0	0.2	0.8
E_1	0.6	1.0	1.0	1.0	0.3	0.6	0.5	0.8	0.4	0.5	0.4	0.5	1.0	0.4	1.0	0.2	1.0	0.4
E_2	0.5	1.0	1.0	0.5	0.3	0.9	0.5	0.8	1.0	0.8	0.4	0.6	0.7	1.0	1.0	0.8	1.0	0.4
E_3	0.5	0.4	0.8	1.0	1.0	0.5	0.5	0.8	0.4	0.9	0.4	0.5	0.4	1.0	0.5	0.4	0.2	0.3
E_4	1.0	0.3	0.1	0.5	0.2	0.6	1.0	0.8	0.5	0.6	1.0	1.0	0.6	0.4	0.5	0.5	0.2	0.7
E_5	1.0	0.4	1.0	0.5	0.2	1.0	1.0	0.8	1.0	0.7	0.5	0.6	1.0	0.3	0.5	1.0	0.3	0.8

1.0, 0.3, 0.8, 0.2 is used in OWAD, OWAAC, OWAIMAM, BON-OWAD, BON-OWAAC and BON-OWAIMAM. Likewise, we consider that $r = q = 0.5$ for all Bonferroni operators.

Step 5. Entrepreneurs are related and grouped according to the similarity of relations and affinities of their professional needs, personal interests, skills, capabilities and knowledge levels using Moore's families and the Galois lattice algorithm**.**

Step 6. Decisions are adopted according to the results found in the previous steps.

6.2. Results

The following section shows the main results of the application. These results are structured as it follows: The new methods of BON-OWAAC and BON-OWAIMAM were implemented to obtain the similarity of relationships to use Moore's families and the Galois lattice for establishing groups of affinities. Likewise, we implemented other methods such as HD, AC, IMAM, OWAD, OWAAC, OWAIMAM, BD, BAC, BIMAM and BON-OWAD to show the versatility of these methods for establishing these types of relationships. Moreover, it is important to mention that the proposed method has specific characteristics. For BON-OWAAC, the differences between two sets are established by a threshold in the comparison process when one set is higher than the other, so the results are equal from this point. For BON-OWAIMAM, the difference between two sets is established by comparison by unifying the Hamming distance and the adequacy coefficient in the same formulation. These methods allow establishing a continuous comparison and interrelationship between the professional needs, personal interests, skills, capabilities and knowledge of an individual who wishes to belong a to co-working space with those of each individual who already belongs to one. Based on the comparison results, we found dissimilarity fuzzy relationships (see Table 4), from which a Boolean relationship is found through a determined α level.

Thus, we carried out the sum of the products in minimum terms to obtain the complement for each term and to establish the similarity relations and Moore's families (see Table 5). Finally, we built a Galois lattice using Moore's families as a starting point (see Fig. 1). We have used 12 different distance techniques in which dissimilarity fuzzy relations are expressed in max terms such as HD, OWAD, BD, and BON-OWAD and min terms such as AC, OWAAC, BAC, and BON-OWAAC. It is possible that some methods show similar results, although the existing differences are defined by each characteristic. Likewise, it is noted that the results obtained by BON-OWA operators allowed us to make multiple comparisons between input arguments and to capture the interrelationships.

In Table 5, we display Boolean matrices only for BON-OWAD, BON-OWAAC and BON-OWAIMAM based on similarity relations. The maximum similarity sub-relations for each group can be expressed in max ($\geq$) or min ($\leq$) terms; i.e., in some cases, the best results are near 1, and in other cases, the best results are near 0. Thus, BON-OWAD is considered in max terms ($\alpha \geq 0{,}76$), and BON-OWAAC and BON-OWAIMAM are considered in min terms ($\alpha \leq 0{,}10$ and $\alpha \leq 0{,}28$). This situation is given by the specific char-

Table 4
Dissimilarity fuzzy relations based on several technical comparisons.

HD	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.66	0.67	0.72	0.72	0.78
E_2	0.66	1.00	0.81	0.70	0.69	0.66
E_3	0.67	0.81	1.00	0.68	0.65	0.72
E_4	0.72	0.70	0.68	1.00	0.67	0.70
E_5	0.72	0.69	0.65	0.67	1.00	0.74
E_6	0.78	0.66	0.72	0.70	0.74	1.00

OWAD	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.78	0.77	0.83	0.81	0.85
E_2	0.78	1.00	0.88	0.79	0.80	0.78
E_3	0.77	0.88	1.00	0.79	0.74	0.79
E_4	0.83	0.79	0.79	1.00	0.77	0.80
E_5	0.81	0.80	0.74	0.77	1.00	0.82
E_6	0.85	0.78	0.79	0.80	0.82	1.00

BD	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.67	0.75	0.70	0.70	0.76
E_2	0.67	1.00	0.82	0.69	0.68	0.71
E_3	0.75	0.82	1.00	0.68	0.66	0.73
E_4	0.70	0.69	0.68	1.00	0.67	0.72
E_5	0.70	0.68	0.66	0.67	1.00	0.75
E_6	0.76	0.71	0.73	0.72	0.75	1.00

BON-OWAD	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.67	0.76	0.70	0.69	0.76
E_2	0.67	1.00	0.82	0.69	0.67	0.70
E_3	0.76	0.82	1.00	0.68	0.65	0.73
E_4	0.70	0.69	0.68	1.00	0.67	0.72
E_5	0.69	0.67	0.65	0.67	1.00	0.74
E_6	0.76	0.70	0.73	0.72	0.74	1.00

AC	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1	0.14	0.11	0.12	0.15	0.07
E_2	0.14	1.00	0.07	0.16	0.19	0.15
E_3	0.11	0.07	1.00	0.19	0.24	0.15
E_4	0.12	0.16	0.19	1.00	0.19	0.13
E_5	0.15	0.19	0.24	0.19	1.00	0.08
E_6	0.07	0.15	0.15	0.13	0.08	1.00

OWAAC	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.44	0.43	0.43	0.45	0.40
E_2	0.44	1.00	0.39	0.47	0.48	0.46
E_3	0.43	0.39	1.00	0.49	0.53	0.47
E_4	0.43	0.47	0.49	1.00	0.48	0.44
E_5	0.45	0.48	0.53	0.48	1.00	0.41
E_6	0.40	0.46	0.47	0.44	0.41	1.00

BAC	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.14	0.09	0.16	0.17	0.08
E_2	0.14	1.00	0.07	0.17	0.19	0.12
E_3	0.09	0.07	1.00	0.19	0.22	0.12
E_4	0.16	0.17	0.19	1.00	0.20	0.11
E_5	0.17	0.19	0.22	0.20	1.00	0.08
E_6	0.08	0.12	0.12	0.11	0.08	1.00

BON-OWAAC	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.15	0.10	0.18	0.19	0.09
E_2	0.15	1.00	0.08	0.19	0.21	0.13
E_3	0.10	0.08	1.00	0.21	0.23	0.13
E_4	0.18	0.19	0.21	1.00	0.21	0.12
E_5	0.19	0.21	0.23	0.21	1.00	0.10
E_6	0.09	0.13	0.13	0.12	0.10	1.00

IMAM	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.43	0.39	0.40	0.44	0.45
E_2	0.43	1.00	0.43	0.38	0.34	0.42
E_3	0.39	0.43	1.00	0.42	0.38	0.44
E_4	0.40	0.38	0.42	1.00	0.39	0.41
E_5	0.44	0.34	0.38	0.39	1.00	0.38
E_6	0.45	0.42	0.44	0.41	0.38	1.00

OWAIMAM	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.64	0.61	0.63	0.64	0.65
E_2	0.64	1.00	0.64	0.61	0.58	0.63
E_3	0.61	0.64	1.00	0.64	0.60	0.64
E_4	0.63	0.61	0.64	1.00	0.61	0.63
E_5	0.64	0.58	0.60	0.61	1.00	0.59
E_6	0.65	0.63	0.64	0.63	0.59	1.00

BIMAM	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.30	0.25	0.31	0.33	0.29
E_2	0.30	1.00	0.27	0.29	0.27	0.27
E_3	0.25	0.27	1.00	0.32	0.29	0.28
E_4	0.31	0.29	0.32	1.00	0.30	0.26
E_5	0.33	0.27	0.29	0.30	1.00	0.25
E_6	0.29	0.27	0.28	0.26	0.25	1.00

BON-OWAIMAM	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1.00	0.31	0.26	0.33	0.34	0.30
E_2	0.31	1.00	0.28	0.31	0.29	0.28
E_3	0.26	0.28	1.00	0.32	0.29	0.27
E_4	0.33	0.31	0.32	1.00	0.31	0.28
E_5	0.34	0.29	0.29	0.31	1.00	0.26
E_6	0.30	0.28	0.27	0.28	0.26	1.00

Table 5
Maximum similarity sub-relations for conformed creative groups through several α levels.

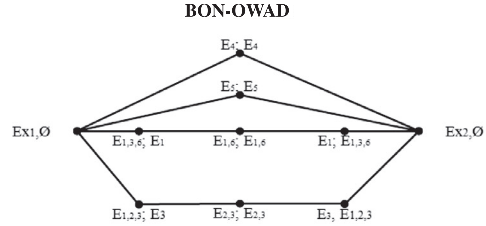
BON-OWAD						
$\alpha \geq 0.76$	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1	0	1	0	0	1
E_2	0	1	1	0	0	0
E_3	1	1	1	0	0	0
E_4	0	0	0	1	0	0
E_5	0	0	0	0	1	0
E_6	1	0	0	0	0	1

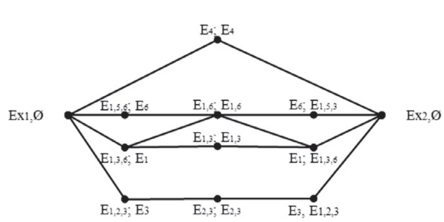
BON-OWAAC						
$\alpha \leq 0.10$	E_1	E_2	E_3	E_4	E_5	E_6
E_1	1	0	1	0	0	1
E_2	0	1	1	0	0	0
E_3	1	1	1	0	0	0
E_4	0	0	0	1	0	0
E_5	0	0	0	0	1	1
E_6	1	0	0	0	1	1

BON-OWAIMAM						
$\alpha \leq 0.28$	P_1	P_2	P_3	P_4	P_5	P_6
E_1	1	0	1	0	0	0
E_2	0	1	1	0	0	1
E_3	1	1	1	0	0	1
E_4	0	0	0	1	0	1
E_5	0	0	0	0	1	1
E_6	0	1	1	1	1	1

Moore's Families

BON-OWAD	BON-OWAAC	BON-OWAIMAM
$E_1 \rightarrow E_{1,3,6}$	$E_1 \rightarrow E_{1,3,6}$	$E_6 \rightarrow E_{2,3,4,5,6}$
$E_3 \rightarrow E_{1,2,3}$	$E_3 \rightarrow E_{1,2,3}$	$E_3 \rightarrow E_{1,2,3,6}$
$E_4 \rightarrow E_4$	$E_6 \rightarrow E_{1,5,6}$	$E_{1,3} \rightarrow E_{1,3}$
$E_5 \rightarrow E_5$	$E_4 \rightarrow E_4$	$E_{4,5} \rightarrow E_{4,5}$
$E_{1,6} \rightarrow E_{1,6}$	$E_{1,3} \rightarrow E_{1,3}$	$E_{5,6} \rightarrow E_{5,6}$
$E_{2,3} \rightarrow E_{2,3}$	$E_{1,6} \rightarrow E_{1,6}$	$E_{2,3,6} \rightarrow E_{2,3,6}$
	$E_{2,3} \rightarrow E_{2,3}$	





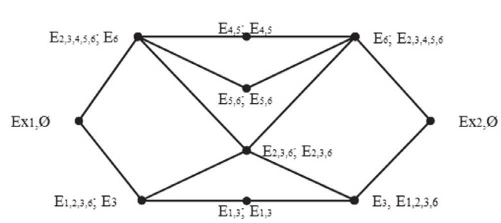


Fig. 1. Moore's families and Galois lattices for BON-OWAD, BON-OWAAC and BON-OWAIMAM.

acteristics of each method. We have analyzed groups formed by the newly proposed BON-OWA operators. Because we have used different methods, the number and type of groups are different. However, in several cases, the results can coincide between different methods depending on the type of decision. In that case, we note that the type and number of groups are different, which is evident in the number of families formed by each BON-OWA operator and their established relationship. Hence, the existence of interrelationships between each individual depends on the method used.

Based on the maximum similarity sub-relations, we have obtained Moore's families for BON-OWAD, BON-OWAAC and BON-

OWAIMAM. Thus, we have built the Galois lattices for each BON-OWA proposed to establish the relationship among all of the studied cases (Fig. 1). Hence, each of these lattices shows a different interrelationship between each of the entrepreneurs. On the one hand, we have observed that BON-OWAD and BON-OWAAC lattices have a similar structure although the shapes in which relations are established are different. On the other hand, we have also observed that the BON-OWAIMAM lattice has a different structure, which, given by this operator, has the characteristics of both HD and AC in the same formulation. Furthermore, it is important to note that this operator can apply HD or AC according to preference or needs of the decision-maker.

Each of these lattices shares specific characteristics according to the professional needs, personal interests, skills, capabilities and knowledge affinities of each entrepreneur. Because these lattices have been established by these affinities, the potential for establishing synergies, improving understanding and enhancing their skills and cooperation would be more effective. In addition, we can observe and detect the specific relationships, highlighting a key entrepreneur who shares affinities with a specific group of entrepreneurs or small groups of entrepreneurs. However, it is important to highlight that groups can change according to the criteria of the decision-maker, prior needs and importance of characteristics. Thus, these methods can adapt within the timeline.

7. Conclusions

We have studied OWA operators, some distance measures and Bonferroni means to propose new aggregation operators. We have introduced new aggregation operators using AC and IMAM in the same formulation with Bonferroni means and the OWA operator. The methods introduced are called BON-OWAAC and BON-OWAIMAM. The main advantages of using these operators are that they allow considering continuous aggregations, multiple comparisons between each argument and distance measures in the same formulation. In addition, each method has specific advantages. For BON-OWAAC, the differences between two sets are established by a threshold in the comparison process when one set is higher than the other, so the results are equal from this point. For BON-OWAIMAM, the differences between two sets are established using characteristics of both HD and AC in the same formulation. Likewise, we have obtained other methods such as BAC, BIMAM, BON-WOWAAC and BON-WOWAIMAM. Thus, we obtain a new group of distance families, which allows the importance and interrelationship of each distance to be analyzed. Moreover, we have also extended these operators with moving averages to aggregate time series and related issues.

We have developed an application focused on comparing the professional needs, personal interests, skills, capabilities and knowledge of an entrepreneur who wishes to belong to a co-working space with those of an entrepreneur who already belongs to one. We have used this new group of distance families in combination with Moore's families and Galois lattices to connect co-working entrepreneur members according to their interests and capabilities. New methods are used as a previous step to apply Moore's families and the Galois lattices to obtain different distances between a set of elements, to gather each entrepreneur according to the maximum similarity sub-relations and to establish relationships according to their affinities. First, we have found dissimilar fuzzy relations to establish a Boolean relation through a determined α level. Second, we have used 12 different distance techniques in which similarity relationships are expressed in max terms and min terms. Finally, we have shown maximum similarity sub-relations for BON-OWA operators only. We have obtained Moore's families and built Galois lattices to represent an established interrelationship between each entrepreneur. Each of these lattices shares specific characteristics according to the professional and personal profile affinities of each entrepreneur.

The versatility of this algorithm has been highlighted within a relationship and gathering context, which is focused on the professional and personal interests of entrepreneurs in co-working places. The main implications of using both operators are that they can help enhance the creation of synergies, improve understanding and strengthen the skills and cooperation among individuals. Likewise, with better analysis and interpretation of subjective information obtained from the personal characteristics of individuals, it also helps create more efficient teamwork. Furthermore, these new algorithms can be used in different fields. First, they are used in areas such as sports teams, strategy marketing and entrepreneurship. Second, they allow the aggregation of objective and subjective information from different sources. Third, they allow consideration of continuous aggregations and multiple comparisons between each argument.

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