Pollution–income dynamics

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HIGHLIGHTS

- We examine a dynamic model of the pollution–income relationship.
- We integrate scale, composition and technique effects of growth on pollution.
- Both inter-temporal and temporal elasticities of substitution are considered.
- We derive general conditions for the environmental Kuznets curve.
- Conditions for eluding the limits to growth are stricter than previously argued.

ARTICLE INFO

ABSTRACT

This paper integrates the scale, composition, and technique effects of economic growth on pollution using a multi-output endogenous growth framework. Under certain empirically verifiable parameter conditions economic growth is not sustainable, even under an optimal pollution tax.

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1. Introduction

We examine the relationship between pollution and income in a dynamic general equilibrium framework with endogenous growth in a multi-output context. Previous theoretical literature has assumed a single final good thus ignoring the output composition effect and has often modeled production using a Cobb–Douglas specification (e.g. López, 1994; Stokey, 1998; Andreoni and Levinson, 2001; Johansson and Kriström, 2007). However, empirical evidence shows that the structure of consumption, not merely its level, is important in affecting the pollution–income relationship (Grossman and Krueger, 1995), and the Cobb–Douglas specification is often rejected (Chirinko, 2008). Figueroa and Pasten

(2013) is one of the few analyses that allow for more general functional forms for consumer and production technologies. While their analysis constitutes a generalization of earlier models, it is static because output expansion is assumed to be exogenous and it considers only one final good.

A conclusion of the theoretical literature is that the so-called environmental Kuznets process (EKC), where pollution first increases with income but reaches a turning point beyond which it secularly declines, is plausible. That is, the limits to growth would overcome. Below we show that this optimistic conclusion requires rather stringent assumptions often ignored by the literature.

2. The model

The economy produces two goods: a clean and a dirty one. The dirty good production generates pollution while production of the clean good involves no pollution. Let k denote the total man-made composite clean input at time t, which may include human and physical capital, which is distributed between the clean and dirty

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industries. Let k_d denote the amount of capital employed in the dirty sector and x be pollution emissions. Following López (1994), and Copeland and Taylor (2005), we regard pollution as a factor of production. Let $F(k_d, x)$ represents the production technology of the dirty good. Assuming a constant elasticity of substitution (CES) function,

$$y_d = F(k_d, x) = \left[\alpha k_d^{-\frac{1-\omega}{\omega}} + (1-\alpha)x^{-\frac{1-\omega}{\omega}} \right]^{-\frac{1-\omega}{\omega}}, \qquad (1)$$

where $\omega > 0$ represents the elasticity of substitution between capital and pollution, and $0 < \alpha < 1$ is a fixed parameter. The dirty sector produces only final goods.

The output of the clean good is assumed to depend only on the capital input and is governed by the linear technology:

$$y_c = A(k - k_d). \qquad (2)$$

This sector produces the final good and new capital. If we normalize the price of the clean good to unity ($p_c = 1$), the economy's budget constraint is,

$$\dot{k} = A(k - k_d) + pF(k_d, x) - c - \delta k, \qquad (3)$$

where p is the (relative) price of the dirty good, $c \equiv c_c + pc_d$ is the total consumption expenditure in units of the clean good, δ is the rate of capital depreciation, and $\dot{k} \equiv dk/dt$. The sum of the first two terms on the right-hand side of (3) represents the income of the economy. The gross capital accumulation, $\dot{k} + \delta k$, is equal to net savings (income less consumption).[1]

The consumer's indirect utility function is:

$$u = \frac{1}{1-a} \left(\frac{c}{e(1, p)} \right)^{1-a},$$

where c denotes the total consumption expenditure, $e(1, p)$ is the unit (dual) expenditure function or cost-of-living index. The parameter a is the elasticity of marginal utility (EMU) and $u(c, p)$ is assumed to be increasing and strictly concave in c.[2]

The consumer's underlying preferences are described by a CES utility function so that the unit expenditure function is given as

$$e(1, p) = \left[\gamma_c + \gamma_d p^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where $\sigma > 0$ is the elasticity of substitution between the dirty good and clean good, and $\gamma_c > 0$ and $\gamma_d > 0$ are fixed parameters. The optimal level of c is determined by the inter-temporal optimization, as detailed below. We assume that the environmental damage is separable from consumption in the welfare function, and can be represented as $v(x) = \frac{x^{1+\eta}}{1+\eta}$, where $\eta > 0$ is a fixed parameter. Then the consumer's instantaneous welfare is:

$$U \equiv \frac{1}{1-a} \left(\frac{c}{e(1, p)} \right)^{1-a} - \frac{x^{1+\eta}}{1+\eta}.$$

We assume a fixed discount rate, ρ. If the government optimally regulates pollution, the economy behaves "as if" it maximizes the present discounted value of welfare,

$$\max_{c, x} \int_0^\infty \left\{ \frac{1}{1-a} \left(\frac{c}{e(1, p)} \right)^{1-a} - \frac{x^{1+\eta}}{1+\eta} \right\} \exp(-\rho t) dt,$$

subject to the budget constraint (3), and the initial condition $k = k_0$. The government imposes an optimal pollution tax and reimburses the tax revenue as a lump-sum to consumers. The above optimization implies the following current-value Hamiltonian,

$$H = \frac{1}{1-a} \left(\frac{c}{e(1, p)} \right)^{1-a} - \frac{x^{1+\eta}}{1+\eta}$$
$$+ \lambda \left[A(k - k_d) + pF(k_d, x) - c - \delta k \right],$$

where λ is the shadow price of capital.

The following first-order conditions are necessary:

$$e(1, p)^{a-1} c^{-a} = \lambda, \qquad (4)$$

$$pF_1(k_d, x) - A = 0, \qquad (5)$$

$$-v_1(x) + \lambda pF_2(k_d, x) = 0, \qquad (6)$$

$$\dot{\lambda}/\lambda = -[A - \rho - \delta] \equiv -M, \qquad (7)$$

$$\dot{k} = A(k - k_d) + pF(k_d, x) - c - \delta k, \qquad (8)$$

$$\lim_{t \to \infty} \lambda k(t) e^{-\rho t} = 0, \qquad (9)$$

where a subscript number reflects the first derivative with respect to the corresponding argument in functions of more than one variable. From (6), the optimal pollution tax is equal to the marginal rate of substitution between the pollution and consumption expenditure, $\tau \equiv v_1(x)/\lambda$.

Defining the share of the dirty good in the consumption expenditure as $s(p) \equiv \frac{pc_d}{pc_d + c_c}$, and the share of capital in the output value of the dirty good as $S_k \equiv \frac{k_d F_1(k_d, x)}{F(k_d, x)}$, the CES specifications imply that,

$$s(p) = \frac{\gamma_d}{\gamma_c p^{\sigma-1} + \gamma_d}; \qquad S_k = \alpha \left[(1-\alpha) \left(\frac{k_d}{x} \right)^{\frac{1-\omega}{\omega}} + \alpha \right]^{-1}.$$

Using Roy's identity, the consumer demand for the dirty good is $c_d = \frac{\gamma_d p^{-\sigma} c}{\gamma_c + \gamma_d p^{1-\sigma}}$. Using (4) and (7) yield the growth rate of c_d,

$$\hat{c}_d = \frac{1}{a} M - \left[\frac{s(p)}{a} + (1 - s(p))\sigma \right] \hat{p}. \qquad (10)$$

The market clearing condition for the dirty good is:

$$F(k_d, x) = \frac{\gamma_d p^{-\sigma} c}{\gamma_c + \gamma_d p^{1-\sigma}}. \qquad (11)$$

The rate of growth of its production is:

$$\hat{F}(k_d, x) = S_k \left(\frac{\hat{k_d}}{x} \right) + \hat{x}. \qquad (12)$$

Since the market for the dirty good must clear at all points in time the growth rates of production and demand for the dirty good must be equal. Hence, using (10) and (12) we have,

$$z\hat{p} + S_k \left(\frac{\hat{k_d}}{x} \right) + \hat{x} = \frac{M}{a}, \qquad (13)$$

where $z \equiv \frac{s(p)}{a} + (1 - s(p))\sigma > 0$.

From (5), we also have that $\hat{p} + \hat{F}_1(k_d, x) = 0$, which using the CES function implies,

$$\hat{p} - \frac{1}{\omega} (1 - S_K) \left(\frac{\hat{k_d}}{x} \right) = 0. \qquad (14)$$

Finally, differentiating (6) with respect to time, we obtain

$$-\eta \hat{x} + \hat{p} + \frac{1}{\omega} S_K \left(\frac{\hat{k_d}}{x} \right) = M. \qquad (15)$$

The equation system (13)–(15) simultaneously solves for the three endogenous variables $\hat{p}$, $\left(\frac{\hat{k_d}}{x}\right)$, and $\hat{x}$,

$$\hat{p} = \frac{M(1 - S_k)\left[\frac{\eta}{a} + 1\right]}{|W|} > 0, \qquad (16)$$

$$\left(\frac{\hat{k_d}}{x}\right) = \frac{M\left[\frac{\eta}{a} + 1\right]\omega}{|W|} > 0, \qquad (17)$$

$$\hat{x} = \frac{MT(t)}{|W|}, \qquad (18)$$

where $T(t) \equiv (1 - s(t) + s(t)S_k(t))\left(\frac{1}{a} - \sigma\right) + (\sigma - \omega)S_k(t)$ and $|W| \equiv [(1 - S_k(t))(1 + z(t)\eta) + S_k(t)] + \eta S_k(t)\omega > 0$.

Eq. (16) implies that the price of dirty goods continuously increases over time if the economy has growth potential ($A > \rho + \delta$ or, equivalently, if $M > 0$). The price of the dirty good depends on the pollution tax ($v_1(x)/\lambda$), which is increasing over time because, as we show below, λ falls as total consumption increases. The increasing price of the dirty good, in turn, causes the so-called composition effect, that is, consumers increase the clean-good to dirty-good consumption ratio.

Eq. (17) corresponds to the technique effect. Thus, (16) and (17) imply that the economy must rely on both the output composition and technique effects as a way to counter the scale effect caused by economic growth. The net change of pollution, which is described by (18) is ambiguous and critically dependent on the dynamics of the function $T(t)$.

Proposition 1 shows that the dynamic path described by (16)–(18) implies a positive rate of consumption growth despite that the pollution tax is continuously increasing over time.

Proposition 1. (i) *The growth rate of real consumption is:* $\left(\frac{\hat{c}}{e}\right) = \frac{1}{a}\left[M - s(p)\hat{p}\right]$, *where $\hat{p}$ is given by* (16). (ii) *The rate of growth of real consumption remains positive throughout the growth path.*

Proof. See Appendix.

3. The EKC

Proposition 2 summarizes the conditions for an EKC,

Proposition 2. *If $\omega = 1$ (Cobb–Douglas production) then an EKC exists if and only if $1 > \frac{(1/a) - \alpha}{1 - \alpha} > \sigma$ when $a > 1$ and $1 < \frac{(1/a) - \alpha}{1 - \alpha} < \sigma$ when $a < 1$ (where $\alpha \equiv S_k$). If $\omega \neq 1$ and either $\omega > 1$ or $\sigma > 1$ (but not both), then an EKC emerges if only if $\sigma < 1/a < \omega$ or $\omega < 1/a < \sigma$.*

Proof. See Appendix.

As Proposition 1 shows, when $a \equiv EMU < 1$ the economy will tend to grow faster than when $EMU > 1$. Proposition 2 implies that when the economy grows fast (e.g., $EMU < 1$) an EKC can be achieved if and only if either consumers or producers exhibit a high degree of flexibility. Moreover, from the definition of the function $T(t)$ in (18), it follows that if $\omega < \sigma < 1/a$ pollution is increasing along the complete growth path and therefore EKC does not exist and economic growth cannot be environmentally sustainable.

4. Conclusion

This is the first paper to integrate the scale, composition, and technique effects of economic growth on pollution using a rigorous endogenous growth framework. We show that, under certain empirically verifiable parameter conditions described in Proposition 2, economic growth cannot be sustainable even if an optimal pollution tax profile is applied. In particular, if the production and consumption elasticities of substitution are both lower than the inverse of the elasticity of marginal utility then pollution increases cannot be arrested by an optimal pollution tax. In this case an EKC may arise only if the pollution tax is complemented with other environmental policies.

We show that if the elasticity of substitution between dirty and clean inputs is much less than unity, as often reported in the empirical literature, the feasibility of sustainable growth hinges greatly on the size of the consumption composition effect, an effect consistently neglected in the theoretical literature.

Appendix

Proof of Proposition 1. (i) By Roy's identity, $c_d = \frac{c}{e(1,p)}e_2(1, p)$.

Using Shephard's lemma, $\hat{e}(1, p) = \frac{pe_2}{e}\hat{p} = s(p)\hat{p}$. Therefore, $\left(\frac{\hat{c}}{e}\right) = \hat{c} - \hat{e} = \frac{1}{a}\left[M - s(p)\hat{p}\right]$.

(ii) $\hat{c} > 0$ if $\hat{p} < M/s(p)$. Using (16),

$$\hat{p} = \frac{(1/\omega)M(1 - S_k)\left[(\eta/a) + 1\right]}{(1/\omega)\left[(1 - S_k)(1 + z\eta) + S_k\right] + \eta S_k} < M/s(p).$$

Rearranging,

$$(1 - S_k)\left(\frac{\eta}{a} + 1\right)s(p)$$
$$< [(1 - S_k)(1 + z\eta)] + S_k + \eta S_k \omega. \qquad (A.1)$$

Since, $(S_k + \eta S_k \omega) > 0$ and $z \equiv \frac{s(p)}{a} + (1 - s(p))\sigma$, (A.1) is satisfied if

$$\frac{\eta s(p)}{a} + s(p) < 1 + \frac{\eta s(p)}{a} + (1 - s(p))\sigma\eta. \qquad (A.2)$$

(A.2) holds if $0 < (1 - s(p))(1 + \sigma\eta)$, which is always true.

Proof of Proposition 2. Define

$$H(t) \equiv \frac{1 - s(t) + s(t)S_k(t)}{S_k(t)}. \qquad (A.3)$$

Using (A.3) in (18),

$$T(t) \equiv [H(t) - \chi]\left(\frac{1}{a} - \sigma\right)S_k(t), \qquad (A.4)$$

where $\chi \equiv \frac{\omega - \sigma}{(1/a) - \sigma}$.

• Case $\omega \neq 1$. From (18), $\frac{d\ln x}{dt} = 0$ if and only if $T(t) = 0$. That is, if

$$H(t) = \chi. \qquad (A.5)$$

From (A.3), $H(t) > 1$. Hence, $\chi > 1$, which implies that either

$$\omega > (1/a) > \sigma \qquad (A.6)$$

or

$$\sigma > (1/a) > \omega \qquad (A.7)$$

is a necessary condition for EKC.

When either $\omega > 1$ or $\sigma > 1$ the function $H(t)$ is monotonic. To see this, note from (A.3) that $H(t)$ is decreasing in s and increasing in S_k. From (10), if $\omega > 1$ and $\sigma < 1$, then S_k and s are both increasing and therefore $\partial H(t)/\partial t < 0$ for all t; if $\omega < 1$ and $\sigma > 1$, then S_k and s are both decreasing and therefore $\partial H(t)/\partial t > 0$ for all t. Moreover, $1 < H(t) < \infty$. Hence, condition (A.5) holds at a unique time, t^*.

Sufficient condition for an EKC is that T must go from positive to negative at t^*. For $\omega > 1$ and $\sigma < 1$, $H'(t) < 0$ meaning that $T(t^* - \upsilon) < T(t^*) < T(t^* + \upsilon)$ for any $\upsilon > 0$ if $1/a - \sigma > 0$, which corresponds to the necessary condition (A.6). For $\omega < 1$ and $\sigma > 1$, $H'(t) > 0$ meaning that $T(t^* - \upsilon) < T(t^*) < T(t^* + \upsilon)$ for any $\upsilon > 0$ if $1/a - \sigma < 0$, which corresponds to the necessary

condition (A.7). Either (A.6) or (A.7) is therefore necessary and sufficient condition for EKC.

● **Case** $\omega = 1$. S_k becomes constant equal to $0 < \alpha < 1$ and

$$H(t) \equiv \frac{1 - s(t) + \alpha s(t)}{\alpha} > 1, \qquad (A.8)$$

which is monotonic over time. From Eq. (18),

$$\frac{T(t)}{\alpha} = H(t)\left(\frac{1}{a} - \sigma\right) + (\sigma - 1). \qquad (A.9)$$

Necessary condition for a turning point is that $T = 0$ in which case $\partial \ln x / \partial t = 0$:

$$H(t^*)\left(\frac{1}{a} - \sigma\right) + (\sigma - 1) = 0, \qquad (A.10)$$

where t^* is the turning point. (A.5) and (A.7) imply,

$$H(t^*) = \frac{1 - \sigma}{(1/a) - \sigma} > 1. \qquad (A.11)$$

Thus, the inequality (A.11) requires:

$$1 > 1/a > \sigma \qquad (A.12)$$

or alternatively

$$1 < 1/a < \sigma. \qquad (A.13)$$

Also, since $H(t)$ is continuous and monotonic, the solution to (A.8) is unique and must satisfy the following condition,

$$1 < H(t^*) < 1/\alpha. \qquad (A.14)$$

To see this, use (A.8) noting that $\partial H / \partial s < 0$, H decreases over time from $1/\alpha$ towards 1 if $1 > 1/a > \sigma$ and H increases over time from 1 towards $1/\alpha$ if $1 < 1/a < \sigma$.

Using (A.14), condition (A.12) implies that

$$a > 1 > \frac{(1/a) - \alpha}{1 - \alpha} > \sigma \qquad (A.15)$$

and (A.13) implies that

$$a < 1 < \frac{(1/a) - \alpha}{1 - \alpha} < \sigma. \qquad (A.16)$$

From (A.5) it follows that for the turning point to go from $T > 0$ ($\partial \ln x / \partial t > 0$) to $T < 0$ ($\partial \ln x / \partial t < 0$) as needed by the EKC, it must satisfy the following conditions:

For both $1 > 1/a > \sigma$ and $1 < 1/a < \sigma$,

$$\frac{\partial H(t)}{\partial s(t)} \frac{\partial s(t)}{\partial t}\left(\frac{1}{a} - \sigma\right) = \left(\frac{\alpha - 1}{\alpha}\right)\frac{\partial s(t)}{\partial t}\left(\frac{1}{a} - \sigma\right) < 0. \quad (A.17)$$

Since $\frac{\partial s(t)}{\partial t} > 0$ if $\sigma < 1$ and $\frac{\partial s(t)}{\partial t} < 0$ if $\sigma > 1$, both (A.15) and (A.16) meet the sufficient condition (A.17). That is, (A.15) and (A.16) are necessary and sufficient for an EKC.

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