Should governments provision against fiscal mismatch?: New evidence for fiscal sustainability

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Abstract

Governments spend a significant share of non-traded goods, which become disproportionally more expensive as economies grow (e.g. Balassa-Samuelson effect). In fact, government inflation increased twice above average inflation in countries like Finland and US. This paper shows a novel and simple model of how economic growth impacts fiscal sustainability. Also, we empirically explore the potential magnitudes behind this phenomenon. The main result is that governments should save for the future mismatch. As in standard models of hedging, the size of this provision depends on the product of the differential price (inflation) times the net exposure, measured as the difference of elasticities of fiscal revenue and expenditures respect to price. Besides, while tax systems focused on tradable goods make more efforts to maintain fiscal sustainability through of high tax rate, non-tradable good sector tend to display higher saving for this "government price risk" when the objective is to maximize welfare. From an empirical perspective, with a panel of 28 high-middle income countries, we show a mismatch at least for the last 20 years with an increasing trend over time. In general, an increase in 1% in GDP growth implies a 0.21% - 0.36% in mismatch on average. Besides, consumption taxes would be more mismatched. Instead, corporate taxes would not have problems to finance spending giving importance of how this effect should be incorporated in the practical analysis, either in the discussion of tax or fiscal policy.

Keywords: Goverment, Prices, Fiscal sustainability, Exposure.
JEL Classification: E31, H30, H41, H50, H62

Contents

1 Introduction

Government prices grow more than the economy prices. Engel & Wagner (2016) shows that a divergence between the government inflation (P_g) and GDP inflation (P_y) could be a first-order macroeconomic phenomenon for most develop and developing countries. However, this effect on government prices necessarily has an impact on their fiscal sustainability? Obviously, a permanent increase in expenditure necessarily needs an increase in future income to pay this expense, i.e. should be consistent with the balance of the intertemporal government budget constraint (Burnside, 2005). However, it is not clear that is satisfying for the imminent bias towards non-tradable government spending (Edwards, 1989; Froot & Rogoff, 1991; De Gregorio, Giovannini & Wolf, 1994) which is affected by the growth of government prices beyond that prices of the economy.

From the theoretical point of view, this phenomenon can be understood through two lines, one from the international economy literature with Balassa-Samuelson (1964) effect, which produces a differentiated impact on non-tradable prices over tradable. Also, other from Baumol's cost disease (1967) phenomenon which explains the increase in the cost of non-tradable goods due to the differential of productivity between sectors. Both account for this fact but from a different perspective. The intention of this paper is to connect this classical literature with the ability of the government to finance its spending. For example, using pension's jargon, promising to fund teachers for forever is a type of in-kind defined benefit, which cannot be sustainably financed with just a defined contribution in dollars. For this, the government has to take into account the long-run dynamics of the prices of inputs, because in the future the same teachers would be more expensive in real dollars due of Balassa-Samuelson effect or Baumol's cost disease. Therefore, from an economic point of view is important to answer whether government through its tax structure can provide a natural hedge, because if this does not happen, there will be a mismatch, taking a direct impact on fiscal sustainability. This mismatch between what government collects of tax revenues and what governments have to spend, in turn, is related to Olivera-Tanzi effect (1967; 1978), in which governments revenues had a net exposure to inflation, and literature of Sovereign Wealth Fund (SWF). However, the motivation for SWF for government's savings is not risking sharing by reducing consumption volatility due to uncertain shocks, but of an inherent risk of the economy given the increase in the price differential between non-tradable goods on tradable.

For this and with the intention of studying underlying mechanism we develop a novel and simple model of how economic growth impacts fiscal sustainability. In specific, we explain the optimal response of the government in different tax structure context to determinate how this differential price effect affects the optimal tax rate and fiscal saving rule. The main message is that the relative price dynamics corresponds to an inherent risk for the government. As standard models of hedging, the size of optimal fiscal saving rule or SWF to provision for this effect depends on the product of this differential price (inflation) times the net exposure, measured as the difference of elasticities of fiscal revenue and expenditures respect to price. The model shows that the optimal decision of the government implies a higher tax rate for the tradable tax structure, but "Sustainability Fund" is greater in the structure of non-tradable due to the intensity of consumption, allowing to save more for the future without generating distortions in the optimal decisions of the agents. So the exposure would be much greater for the case of the non-tradable system, making the government have a greater capacity to finance this intensive expenditure on non-tradable.

Also, we explore the potential magnitudes behind this phenomenon with a reduced form of exposure of the model. Empirically, we elaborate a general framework that decomposes the impact of growth in government in three effects. Tax buoyancy (Haughton, 1998), Balassa-Samuelson effect and Wagner's law effect (Wagner, 1911; Abizadeh & Gray, 1985; Akitoby et al., 2006; Magazzino et al., 2015). The first measure is how tax revenues vary with change in GDP. Second, explain the price dynamics. Third, explains increasing government activity due to economic growth. Under this scheme, a possible mismatch of tax structure can be a result of economic growth if revenues are not able to finance spending and prices via Balassa-Samuelson and Wagner's law effect. In specific, we use a panel of 28 high-middle income countries from 1980 to 2014. Evidence shows a mismatch of taxes respects to expenditures at least for the last 20 years with an increasing trend over time. An increase in 1% on economic growth implies a 0.21% - 0.36% of mismatch on average. This effect is relatively high considering literature related to fiscal sustainability and tax measures as buoyancy (Belinga et al., 2014) since it would not be enough to have a greater elasticity than one to satisfy with the fiscal commitments, but between 1.21 - 1.36. Moreover, if it analyzed the types of taxes, consumption taxes would be more exposed. Instead, corporate taxes would not have problems with the finance spending. The last results would make sense with the model proposed since more intensive structures in non-tradable goods allow a higher source of income than a biased towards tradable goods systems such as consumption tax source.

These results suggest important implications for policy since fiscal rules and measures of fiscal sustainability may also need to include hedging of the stream of expenditures and revenues against the potential increase in non-traded prices. For example, the trajectory of the debt-to-GDP ratio (Blanchard et al., 1991) should include the differential effect of prices affecting the solvency that the government wants to maintain over its debt. The fact that the economy grows not only serves to keep the debt constant as this same will affect the price differential by increasing it. On the other hand, government's "Sovereign Wealth Fund" (SWF) not only depends on elements that carry with them an uncertain component that allows reducing the volatility. In this sense, a permanent saving is required (as "Sustainability Fund") due to the natural increase in the price of the government goods, changing the nature of this type of financial management.

The rest of the paper is structured as follows. Section 2 review the main stylized fact that motivates this research. Section 3 the main literature. Section 4 develop the theoretical model that describe the principal mechanism and conclusions to incorporate relative price dynamics. Section 5 explains the data, descriptive statistics and empirical analysis of potential mismatch of tax structure. In section 6 the main results will be analyzed and discussed through the fiscal policy implications. Section 7 concludes with some remarks, limitations and future research.

2 Stylized facts

The main stylized fact behind this paper is how government prices grow more than the economy prices. A quick look at the data used Engel & Wagner (2016)[1] immediately shows that a divergence between the Government Price Deflator (P_g) and GDP Deflator (P_y) could be a first-order macroeconomic phenomenon, in particular, or OECD countries of Figure 1. In the last half a century the Government Deflator in Finland increased twice as fast the CPI. For Italy, it grew 65% faster than CPI. Moreover, this fact is relevant for the US and the majority of countries in our sample.

In another hand, in recent years countries that suffered severe economic contractions like Greece also had an effect, but in the opposite sense. Notably, Germany remains particularly "competitive", within the meaning that the government deflator does not rise much faster than overall CPI. Besides, through an econometric analysis with a panel of 56 middle income and developed economies, it is shown that when an economy grows, then the Government Price Deflator is on average rising faster than GDP deflator. In particular, it was found an elasticity of P_g/P_y to GDP changes of 0.1 to 0.3; meaning that a 10% growth ends up in some 1-3% additional inflation for government inputs.

This stylized fact provides an important question from the government's point of view. Will it be affected by the price dynamics that it faces? What this paper tries to analyze is precisely that, if the prices of the goods that the government spends and consumes have an impact on the form of how to finance the expenditure through taxes and what should be the optimal fiscal policy in this context.

[1]This working paper corresponds to a previous phase of this research and in which it a participated as a research assistant.

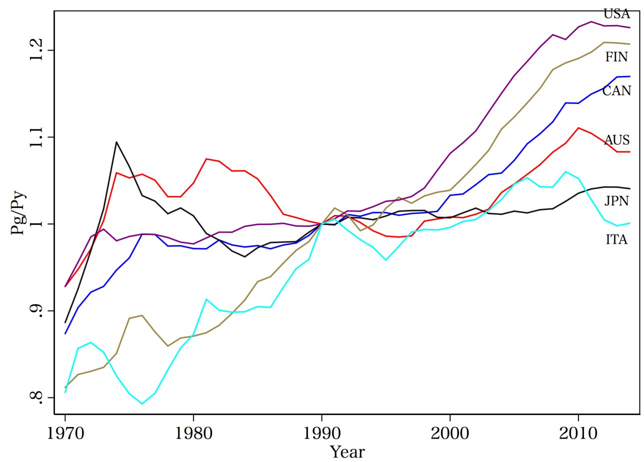

Figure 1: Pg/Py series for OECD countries

This figure plots the series of Pg/Py base 1990 for some OECD countries. All countries have an increasing ratio over time, but with a ratio stabilization in recent years. Source: Own elaboration based AMECO and World Bank, Engel & Wagner (2016)

3 Literature

The importance of non-tradable spending by the government has been well documented in the literature (Edwards, 1989). In particular, it has been reported that about 94-98% of this corresponds to non-tradable expenditure on average from 1995 to 2011 (see Appendix A.1 and A.2). In this line, several authors began to study the real macroeconomic effects of fiscal shocks, particularly on the real exchange rate (RER) and prices of non-tradable goods. The conclusion is that the consequences of an increase in government spending lead to currency appreciation, just as predicted by the IS-LM-BoP model. Within this literature, Froot & Rogoff (1991) and De Gregorio, Giovannini & Wolf (1994) explain theoretically and empirically that higher government spending leads to this appreciation and an additional effect of inflation on non-tradable. Galstyan & Lane (2009) studied the same effect, argued that the composition of government spending is crucial and that only expenditure related to purely non-tradable components appreciate the RER. Balassa-Samuelson (1964) essentially rescued a similar message, the productivity differential is an exogenous force, causing the disproportionate increase in the price of non-tradable prices. In the same line, Baumol (1967) decades ago explains how cities and their spending on non-tradable can precisely show financial challenges since expense are not associated with productivity growth, such as education or security. This phenomenon denominated as Baumol's cost disease has been revitalized in recent decades. For example, Nordhaus (2006) using US industry data for the period 1948-2001 investigates this fact finding that technologically stagnant sectors have rising relative prices.

As we have already mentioned, one of the main stylized facts of our paper corresponds precisely to this differential price effect and has been documented by Engel & Wagner (2016). However, unlike the macroeconomic literature that focuses on government spending as a cause of RER appreciation, we want to show is that appreciation, connected with Balassa-Samuelson and Baumol's cost disease, can generate a problem for governments when they want to finance sustainable such expenditure. Moreover, it is also important to answer if the tax system can provide resources with an increasing expenditure. This latter denominated as exposure comes from the literature of Olivera-Tanzi effect (1967;1978) which government tax revenues had a net exposure to inflation due of inflation erosion revenues of tax collection when inflation rises exponentially from one period to another. In the same line, the exposure is related to the original sin literature that is defined as a situation in which countries can not borrow abroad in their currency. Eichengreen, Hausmann & Panizza (2007) argued that in the presence of high levels of original sin, domestic investments will have a currency mismatch (projects that generate national currency will be financed with a foreign exchange). As it is possible to argue, in our framework this exposure of tax revenues respect to expenditures will be motivated by the price differential that exists between the non-tradable and tradable of the economy. Obviously, this exposure implies a financial management (Brigham

& Ehrhardt, 2013) by the government to satisfy sustainability. Therefore, there is a direct relationship between price risk and the Sovereign Wealth Fund (SWF). SWF might seem to be an excellent opportunity for countries with high variance in public revenues to ensure steady cash flows and provide resources for long-term investments. For example, countries relying on commodity trade that occasionally encounters windfalls of natural resources (Berstein, Lerner & Schoar, 2013). However, the motivation for SWF in our model for government savings is not risking sharing by reducing consumption volatility due to uncertain shocks (Engel, Neilson & Valdes, 2013), but of an inherent risk of the economy given the increase in the price differential between non-tradable goods in tradable.

The literature related to Baumol's cost disease provided useful theoretical models to understand the mechanism behind price differentials linking with the presence of taxes and government expenditures. Specifically, this focuses on the basic elements that a model must have, highlighting the presence of two sectors on the side of firms and different forms of preferences that explain this dynamic (Van der Ploeg, 2007). On the other hand, from the perspective of the tax structure and its impact the behavior of economic agents Andersen & Kreiner (2015) provide a model that permits understand how welfare state leave to policymakers with a trilemma; increase taxes (and hence tax distortions), cut spending or redistribute less. In a similar way, Mann (2014) investigates Baumol's cost disease in the presence of distortionary taxation. If the government takes over the provision of low productivity sector, then the public sector will continue to grow, and the tax rate will be pushed to the top of the Laffer's curve over time. These elements allow us to investigate what happens concerning fiscal sustainability and optimal government behavior in a theoretical form.

From an empirical point of view, there is a vast literature of magnitudes of tax buoyancy or Wagner's Law; however, these are always seen in isolation without understanding that there is a close relationship between them. For revenue side, and as we already mentioned, one of the most used corresponds to the tax buoyancy (Haughton, 1998), which has been applied to developed and developing countries (Leuthold & N'Guessan, 1986) and which explain how tax revenues vary with changes in GDP. Following Mansfield (1972), for a tax system, the response of tax revenue to the evolution in income has often been singled out as a vital ingredient. There are a lot of evidence and methodologies of estimation, including different levels as panels, country, and state. For example, Mansfield (1972) estimate elasticity and buoyancy for total and disaggregated level tax revenues for Paraguay. This similar estimation there is for Ivory Cost (Leuthold & N'Guessan, 1986), India (Upender, 2008), Trinidad and Tobago (Cotton, 2012) and Zimbabwe (Bonga et al., 2015). Muhammad & Ahmed (2010) studied the determinants of tax buoyancy, estimating this for 25 countries including Chile, Brazil, and Mexico. Similar, Belinga et al. (2014) explore the way of this indicator allows bring down fiscal deficits through economic growth for 34 OECD countries with Error Correction Model (ECM), differentiating short and long term. The buoyancy of one would imply that one percent of GDP would increase tax revenue also by one percent, thus leaving the tax-to-GDP ratio unchanged. A tax buoyancy that exceeding one, however, would increase tax revenue by more than GDP and potentially lead reductions in the deficit ratio. In essence, the tax buoyancy is like an elasticity of government revenues via taxes to economic growth but with a constant tax rate. Estimations of this measure have a range between 0.9% to 1.1% based on previous literature, showing that there would be no long-term problems on the fiscal deficit. Again, the main fact is that this literature forgets what happens with government expenditure and prices. In another hand, in public spending side, we have Wagner's law (Wagner, 1911) that states that government grows because there is an increasing demand for public goods (Magazzino, 2012). Besides, the last was empirically tested for several countries using time series and cross section data. Wagner's Law can be divided into two groups, based on the different types of the econometric methodology they apply. Early studies which are performed until the mid-1990s assume stationary data series and apply simple OLS regressions to test alternative versions of the law (Ram, 1987; Courakis et al., 1993). In another hand, cointegration-based studies are performed from the mid-1990s and on. This test for cointegration between government expenditure and national income (Henrekson, 1993; Murthy, 1994; Ahsan et al., 1996; Biswal et al., 1999; Kolluri et al., 2000; Islam, 2001; Al-Faris, 2002; Burney, 2002; Wahab, 2004). The empirical studies have produced mixed and sometimes contradictory results due of different methodologies. However, some authors show evidence in favor of Wagner's law for UK (Oxley, 1994; Chow et al., 2002), China (Cotsomitis et al., 1996), Ghana (Ansari et al., 1997), Iraq (Asseery et al., 1999), Denmark, Germany, Italy, Norway, Sweden (Thornton, 1999), Saudi Arabia (Albatel, 2002), Kuwait (Burney, 2002), Finland (Karagianni et al., 2002) and New Zealand (Kumar et al., 2009). Moreover, evidence contrary for Kenya, South Africa (Ansari et al., 1997), Greece (Clethsos & Kollias, 1997), and Turkey (Demirbas, 1999). The disadvantage is that a comparable measure of elasticity is not reported as tax buoyancy, creating a difficulty in making comparisons. However, Akitoby et al. (2006) provide recent estimations for elasticities for short and long term in real terms. For long-term estimations, total expenditure has an elasticity near to one, similar for current expenditure. In another hand, short-term estimations have more heterogeneity for different definitions of fiscal spending, even with elasticities higher than two. These are relative high due that not incorporate price effect.

Finally, it is possible to observe that the literature looks at these topics in isolation, not taking into account the various factors that may affect the fiscal sustainability. That is why the final objective is to build both a theoretical model and a general framework that allows understand the impact of the differential between prices on a budget of government and explain why these elements are crucial to understanding fiscal sustainability.

4 The model

This paper develops a simplified two-period model that allows us to understand the impact of a price differential for the optimal decision of the government and its fiscal sustainability in different tax structures. The general idea is that this model is sufficient to shows the main facts that occur when studying this phenomenon and that any changes in some assumptions do not diametrically impact the results obtained[2]. In specific, we set a traditional equilibrium model where firms, household, and government optimize[3]. For production side, we follow Baumol (1967) and Van der Ploeg (2007). There is a two-sector production structure with manufacture (traded) and services (non-traded) sectors, which use labor as the only input. Household optimizes intertemporally and internalizes the tax structure in their decisions. In particular, we set three kinds of taxes separately; services, manufacture, and income tax rate. For the fiscal sector, the government chooses the optimal tax rate using equilibrium conditions and optimal decisions of agents of the economy with a criterion that maximizes the present value of welfare (and thus allows it to save or borrow). The intention of this shows the differences that exist on the decisions of a policy of the government in different tax structures, in particular, to study optimal fiscal saving rule in each type of tax structure to maintain fiscal sustainability.

The model allows us to obtain meaningful conclusions about the differential of prices between sectors, household preferences and government expenditure for fiscal decisions. On the one hand, the differential in productivity increases the costs to finance services (non-traded) due of Balassa-Samuelson effect (or Baumol's cost disease) and on the other hand, given the preferences, households demand services and manufactured goods through time. In this line, the government reduces the distortions choosing optimal tax rate depending on tax structure facing budget restrictions. On the expenditure side, it is clear that there would be an increase in costs to finance (denominated Wagner's Law). However, on the revenue side, the effect is uncertain and will depend on what is being taxed. Therefore, to maintain fiscal sustainability and also to choose optimally, the government should assume different efforts based on the tax rate of each context. This effort will

[2]For example, this model could be of over-lapping generations or well with more complex production functions and preferences.

[3]This model can be understood as an open economy with a perfect capital market where the interest rate is given, and household has no liquidity constraints.

allow understanding in which cases the government will be harder to satisfy the fiscal sustainability according to the optimal tax rate and fiscal saving rule. Sometimes the government will have to save to be able to pay its future commitments and at other times to borrow to be able to pay it today. The intention of this model is to show precisely this fact, the optimal saving rule for finance this mismatch will depend on exposure and non-tradable goods price growth (source of risk or shock) having a direct impact on fiscal policy decisions.

4.1 Firms

We considered a two-sector economy with a manufactured good (traded) Y_t^M and services (non-traded) Y_t^S for each period $t = 0$ and $t = 1$. The production functions are given as

$$\begin{aligned} Y_t^M &= A_0^M L_t^M \ (1 + g_M)^t \\ Y_t^S &= A_0^S L_t^S \ (1 + g_S)^t \end{aligned} \tag{1.1}$$

Where L_t^i and A_0^i denotes labor and initial productivity in each sector $i = M, S$, respectively. For simplicity, it is assumed that $A_0^M = A_0^S = A_0$ and normalize the price of manufactured good p_t^M to one. So we denoted the relative price of service respect to manufactured goods as $p_t = p_t^S$. The wage is w_t, equal between sectors. Also, labor is completely mobile between sectors and firms face perfect competition in input and output market. According to Baumol (1967), we refer to services as a stagnant sector and manufactured goods as the growing sector. Service has a relatively low growth rate in labor productivity, g_S, and manufacture sector with higher and persistent growth rate in labor productivity, $g_M > g_S$[4], for this it is assumed that $g_S = 0$. In this way, it is possible to find the price and the equilibrium wage through the maximization of profits,

$$p_t = (1 + g_M)^t \tag{1.2}$$

[4]This condition is known as *Baumol's growth*.

$$w_t = A_0(1 + g_M)^t \tag{1.3}$$

Equation (1.2) reflects that relative price of services respect to manufactured goods in each period t which depends on the growth rate of productivity in manufacture sector being increased over time. To maintain equilibrium is necessary that relative price of services respect to manufacture sector serve as a mechanism of balance between two kinds of goods. Finally, the demand for labor in each area is expressed in equation (1.3) due of firms have constant returns to scale[5] given a perfect elasticity shape. Therefore, the equilibrium will be determinate by labor supply which we will assume exogenous (for more details Appendix A.3.1).

4.2 Preferences

For the household problem, we set a standard intertemporal problem to find optimal demands of manufactured goods M_t and services S_t. For this is maximized the present value of utility that assumes as Cobb-Douglas with α share for manufacture goods and $1-\alpha$ for services given by

$$U = \log(M_0^{\alpha} S_0^{1-\alpha}) + \beta \log(M_1^{\alpha} S_1^{1-\alpha}) \tag{1.4}$$

Where β represent intertemporal discount factor. The idea is that household maximize (1.4) subject to intertemporal budget constraint (1.5) which depends on source of income given by wage w_t and indirectly of financial asset a_t. Obviously, the expenditure (or income) depends on the type of tax that faces the household, which may be the consumption of services τ_t^S, manufactured goods τ_t^M and revenue τ_t^w. In a general form, we can express this as

$$w_0(1-\tau_0^w) + \frac{w_1(1-\tau_1^w)}{1+r} = (1+\tau_0^M)M_0 + \frac{(1+\tau_1^M)M_1}{1+r} + (1+\tau_0^S)p_0 S_0 + \frac{(1+\tau_1^S)p_1 S_1}{1+r} \tag{1.5}$$

One important issue of the Cobb-Douglas is the property that demands S_t and M_t only depends on the current relative price and tax rate due to the condition of optimality of the household over the allocation of goods. For

[5] This assumption is not relevant to the main results of the model.

example, an increase in the tax rate in period 1 is automatically offset by a fall in demand in that period 1. So it will not affect the demand of period 0 through Euler equation. This property is contrary to a CES function since an increase in the tax rate will not be offset completely affecting the demand of the other period through the Euler equation.

4.3 Government

For the government, it is assumed a welfare maximizing behavior (as a centralized economy). Therefore, this maximize present value of household utility expressed in (1.4) subject to fiscal budget restriction and optimal demands M_t and S_t. Obviously, contrary of household, the revenues side of the budget constraint depends on the type of tax rate and respective base which may be the consumption of services S_t, manufactured goods M_t or income w_t. For expenditures, it is assumed that government only spend in non-tradable goods G_t being exogenous and constant over time[6]. In a general form, we can express this as

$$\tau_0^S S_0 + \frac{\tau_1^S S_1}{1+r} + \tau_0^M M_0 + \frac{\tau_1^M M_1}{1+r} + \tau_0^w w_0 + \frac{\tau_1^w w_1}{1+r} = p_0 G + \frac{p_1 G}{1+r} \tag{1.6}$$

The decision variable of government depends on the tax structure, being τ_t^S, τ_t^M or τ_t^w in each case. The government will differ in its tax rate response depending on the tax structure and the decisions of the agents of the economy, having an impact on the way in which the intertemporal restriction of the government is satisfied. As already mentioned, the objective is to obtain an optimal fiscal saving rule based on two key elements, the exposure of revenues and expenditure, and the growth rate of the relative price of non-tradable goods. The main message of this condition implies that government saves when price growth (which is an inherent risk in the economy due of happens with probability one). That is if there is no price differential the government will not save. Besides, we will obtain the different results

[6]According to NOIT (National Output and Input Table) governments are biased in nontradable spending in a 94-98% of total expenditure on average, depending on the definition of non-tradable.

under the different tax structures comparing the optimal tax rate and the fiscal saving rule to determine the characteristics of each of them.

4.4 Equilibrium

In equilibrium, all agents of economy maximize subject to individual resources constraint. Besides, the economy is restricted by resource constraint, i.e. the demand for good should be equal to supply in each period (for more details of equilibrium and equations see Appendix A.3.2),

Definition 1. *In this economy, there is an equilibrium when all agents of the economy maximize satisfying the resources constraints of the economy.*

Also, as our final objective is to study the optimal behavior of government when there is a price differential in economy, is that we define an equilibrium solution,

Definition 2. *In this economy, there is an equilibrium solution when the government internalizes all optimality conditions provided by firms and household to find a tax rate and saving that satisfies the present value of the fiscal constraint.*

With these definitions, we can be identified our economy perfectly. Obviously, as mentioned earlier, different tax structure will change the budget constraint of household, optimal demands for goods and government problem. In sections 4.5, 4.6 and 4.7 will be analyzed and discuss the different tax structures for the optimal fiscal saving rule for government to satisfy fiscal sustainability.

4.5 Tax on services goods

As mentioned above, the budget constraint for household depends on tax structure. In a first case, it is assumed only a services goods τ_t^S. Therefore, we impose $\tau_t^w = 0$ and $\tau_t^M = 0$ in the budget constraint (1.5). In this way, household maximizes (1.4) restricted to (1.5) obtaining the optimal allocation of consumption between goods (S_t and M_t) in each period $t = 0$ and

$t = 1$. With these demands the government can solve its problem choosing optimal τ_t^S maximizing present value of welfare, which assume similar to utility for household expressed by equation (1.4), subject to intertemporal fiscal constraint (1.6) (with $\tau_t^w = 0$ and $\tau_t^M = 0$) and optimal demand for S_0 and S_1. The result of this problem implies that in the optimum it must be satisfying,

$$p_0(1 + \tau_0^S) = p_1(1 + \tau_1^S) \tag{1.7}$$

i.e. the optimal tax rate for government depends on price dynamics in two periods. Using Definition 1 and Definition 2 the optimal tax rate is given by

$$\tau_0^S = \frac{G(1 + g_M)(2 + r) + (1 - \alpha)A_0 g_M}{(1 - \alpha)A_0[(1 + g_M)(1 + r) + 1] - G(1 + g_M)(2 + r)} \tag{1.8}$$

It is possible to observe that τ_0^S depends only on exogenous parameters of the model. A_0, G, α, g_M and r which represents the maximum production level (in hours) per worker, hours spend for government in the non-tradable sector, the share of manufactures in consumption, productivity growth in manufacture sector and interest rate respectively. Beyond the determinants, we are interested in comparing this tax rate with those of the other tax structures. On the other hand, τ_1^S is given

$$\tau_1^S = \frac{G(1 + g_M)(2 + r) - (1 - \alpha)A_0 g_M(1 + r)}{(1 - \alpha)A_0[(1 + g_M)(1 + r) + 1] - G(1 + g_M)(2 + r)} \tag{1.9}$$

Therefore, is possible to determinate that $\tau_0^S > \tau_1^S$. The intuition behind this is that the government imposes a higher tax rate in the first period to smooth household consumption. The tax rate allows that the use of services in $t = 1$ be the same of $t = 0$, despite the increase in the relative price. This condition will only be possible if the tax rate is higher in $t = 0$ (for details see Appendix A.3.3). In next sections will be studied what happens to the optimal saving rule of the government under this tax structure to compare and discuss it with the other cases.

4.6 Tax on manufactured goods

In this case, it is assumed only a manufacture tax goods τ_t^M. Therefore, we impose $\tau_t^w = 0$ and $\tau_t^S = 0$ in the budget constraint (1.5). Similar to Section 4.5 household maximizes and government choose optimal τ_t^M maximizing present value of welfare, subject to intertemporal fiscal constraint (1.6) (with $\tau_t^w = 0$ and $\tau_t^S = 0$) and optimal demand for M_0 and M_1. The result of this problem implies that in the optimum it must be satisfying,

$$\tau_0^M = \tau_1^M \tag{1.10}$$

In this case, the optimal tax is constant over time similar to a model proposed by Barro (1979) with smoothing taxes. The intuition of this results is due to demands for manufactured goods M_0 and M_1, which depends on the current tax rate. For the government smooth the consumption, the only possible solution corresponds to set the same tax rate in both periods. Contrary to the tax structure with service tax rate, the relative price does not directly affect the optimality condition of the government. Thus, with Definition 1 and Definition 2 government's optimal tax rate

$$\tau_t^M = \frac{G}{\alpha A_0 - G} \tag{1.11}$$

Similar to service tax structure, τ_t^M depends only on exogenous parameters of the model (for details see Appendix A.3.4). The fact that the tax rate is equal to both periods does not imply that the optimal saving will be zero since this will depend on government spending.

Studying a particular case, what would happen if the government omitted the optimum dynamics of the tax rate over time? "Unaware" government would only try to satisfy its restriction in a period by period choosing the respective tax rate. This assumption could be an adequate assuming that the government is more concerned about keeping its promises today and not necessarily those of tomorrow. For this the tax rate,

$$\tilde{\tau}_t^M = \frac{G}{\alpha A_0 \gamma_t - G} \tag{1.12}$$

Where $\gamma_0 = (2+r+g_M)/(2+r) > 1$ and $\gamma_1 = (2+r+g_M)/(2+r)(1+r) < 1$. This solution implies that the government chooses a lower tax rate in period 0 but a higher tax rate in period 1 respect to optimal solution (1.11), making a greater effort in the future and at the same time distorting the decisions of the household. In the next sections, we will study what happens to the tax revenue collected and the optimal fiscal savings of the government for this particular case.

4.7 Tax on income

Finally, we will study the case where $\tau_t^S = 0$ and $\tau_t^M = 0$. in the budget constraint (1.5). Similar to Section 4.5 and 4.6 household maximizes and government choose optimal τ_t^w maximizing present value of welfare, subject to intertemporal fiscal constraint (1.6) (with $\tau_t^S = 0$ and $\tau_t^M = 0$) and optimal demands. The result of this problem implies that in the optimum there are multiple solutions for τ_0^w and τ_1^w. This condition happens, unlike other tax structures, because the tax base of the income tax is the same wage which does not depend directly on the tax rate. The latter allows the government to have multiple tax options to satisfy with Definition 1 and Definition 2. In another hand, this result can be understood from the household since the conditions of optimality are not affected by the tax rate of wage. In this line, if used one of the possible solutions $\tau_0^w = \tau_1^w$,

$$\tau_t^w = \frac{G}{A_0} \tag{1.13}$$

Note that this is a particular solution of multiple solutions with income tax rate (for details see Appendix A.3.5). However, this latter permits a simple comparison with other tax structure.

4.8 Fiscal saving rule

In this section, we compare and discuss different results obtained from last sections with a particular focus on tax rate dynamics, tax revenues, government expenditure and optimal fiscal saving rule to understand the results on differents tax structures and find a generalized expression for the optimal behavior of government.

Optimal tax rate dynamics

In sections 4.5, 4.6 and 4.7 we obtain the optimal tax rate dynamics for the three tax structures defined, tax on service goods τ_t^S, manufactured goods τ_t^M and income τ_t^w. First, for services goods tax structure it is noted a decreasing dynamics. The government must burden higher tax rate in period 0 to satisfy optimal conditions for household and which involve smoothing their consumption. The government internalizes this fact when choosing the optimal tax rate τ_t^S through equations (1.8) and (1.9) . This solution implies,

$$\tau_0^S > \tau_1^S \tag{1.14}$$

In the other hand, tax on manufactured goods implies a constant tax rate τ_t^M to satisfy household and government conditions. This expressed by equation (1.11). Besides, for "unaware" government we observe an increasing dynamics. So, if we compare both tax structure based on the optimal tax rate (for proofs see Appendix A.3.6),

$$\tau_t^M > \tau_0^S > \tau_1^S \tag{1.15}$$

Therefore, the government should make more efforts when it has a tax structure based on manufacturing rather than services.

Lemma 1. *Under this economy and in equilibrium, the tax rate on tradable goods will be higher than non-tradable goods for each period.*

Lemma 1 explains that for the government satisfies its fiscal constraints (net present value of revenues and expenditures) and conditions that optimize the decisions of agents of the economy it must impose a higher tax rate on tradable goods (manufactured goods) than non-tradable goods (services). The intuition of this is the sources of revenues that the government perceives. On the one hand, in the service tax structure, the sources of income for the government are similar to the sources of expenditure since the dynamics of the relative price of goods goes in the same direction through the tax rate (which allows smoothing consumption). On the other hand, in the manufacturing structure, revenues sources are not able to finance expenditure (which is increasing due to relative price increases) if not for a higher tax rate in both periods. Finally, it can be mentioned that the solution used by a structure of income (1.13) implies even less effort for the government than the service case (1.8) and (1.9). The latter is intuitive, since being the source of income the same wage that permits finance spending through multiple solutions[7]. Besides, "unaware" government solution implies a tax rate between optimal manufacture rate. Figure 2 depicts the differences between tax structures,

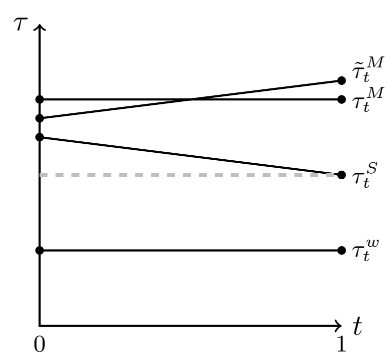

Figure 2: Tax rate dynamics: τ_t^S, τ_t^M, τ_t^w, $\tilde{\tau}_t^M$

Optimal fiscal revenues, expenditures and saving rule

Now, we will be described that occurs with revenues, spending and optimal savings in each of the tax structures studied. First, fiscal revenues depend

[7]Because this is a particular solution, a generalized proposition can not be described.

on two elements, the tax rate which is chosen by government and tax base that depends on the demand of goods or wage. For this, it is necessary the optimal fiscal revenues, which strictly represent the present value of the income collected by the government expressed by the left side of equation (1.6). Obviously, as the current value of these will be the same for each tax structure[8] will be studied the tax revenue of each period separately. In this way for tax on non-tradable goods,

$$\tau_0^S S_0 = \left(\frac{2+r+g_M}{2+r}\right) \frac{G(1+g_M)(2+r)+(1-\alpha)A_0 g_M}{(1+g_M)(2+r)} \tag{1.16}$$

$$\tau_1^S S_1 = \left(\frac{2+r+g_M}{2+r}\right) \frac{G(1+g_M)(2+r)-(1-\alpha)A_0 g_M(1+r)}{(1+g_M)(2+r)} \tag{1.17}$$

If we compare these revenues for both periods, it is possible to determine that,

$$\tau_0^S S_0 > \tau_1^S S_1 \tag{1.18}$$

Similar to what happens with the tax rate case, the service tax revenue of period 0 is greater than period 1, i.e. in period 0, the government saved enough not to have to raise more revenues in period 1. For tradable goods sector and using conditions provided by Definition 1 and 2,

$$\tau_t^M M_t = \left(\frac{2+r+g_M}{2+r}\right) G \tag{1.19}$$

The revenues collected from manufacturing taxes is equal in both periods, similar to the optimal tax rate. Finally, for particular solution of income tax revenues,

$$\tau_0^w w_0 = G \tag{1.20}$$

$$\tau_1^w w_1 = (1+g_M)G \tag{1.21}$$

This last solution is the same for "unaware" government. Therefore, if

[8]This can be easily demonstrated since in equilibrium the present value of fiscal revenues must be equal to that of present value of expenditures, which does not vary for each type of tax.

we compare all tax structures,

$$\tau_0^S S_0 > \tau_1^w w_1 > \tau_t^M M_t > \tau_0^w w_0 > \tau_1^S S_1 \tag{1.22}$$

Note that $\tau_t^w w_t = \tilde{\tau}_t^M M_t$. Service revenues are higher in period 0 due of government's ability to raise revenues through demand for services despite a lower tax rate. However, this income capacity is lower for period 1 because government must smooth the consumption of the household. In another hand, manufactured goods tax structure implies a permanent revenue collection due to the higher tax rate imposed by the government in both periods. For income tax structure the government collect more income in the period 1 due of wage dynamics that depends on productivity rate g_M (for details and proofs see Appendix A.3.7). Figure 3 shows the differences of tax revenues,

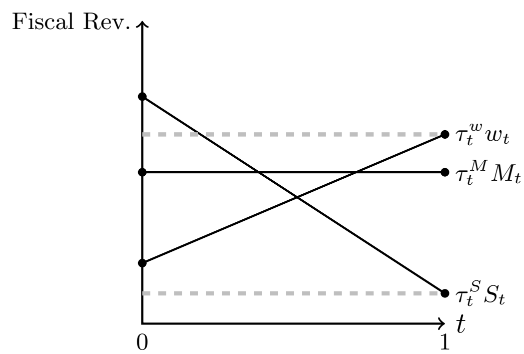

Figure 3: Tax revenue dynamics: $\tau_t^S S_t$, $\tau_t^M M_t$, $\tilde{\tau}_t^M M_t$, $\tau_t^w w_t$.

Second, the dynamics of government spending is the same for each tax structure since the relative price p_t is given by the optimization of firms and expenditure G is exogenous and constant. Thus,

$$p_0 G_0 = G \tag{1.23}$$

$$p_1 G_1 = (1 + g_M) G \tag{1.24}$$

The only element that makes non-tradable consumption of government more expensive is relative price growth g_M. Finally, with the optimal tax revenues of each type of tax structure expressed in the equations (1.16), (1.19) and (1.20), and the expenditure (1.23) it is possible to obtain the optimal fiscal saving rule in each case that government should incur in period 0 in order to satisfy intertemporal fiscal restriction[9]. For this,

Definition 3. *Optimal fiscal saving rule (FSR) occurs when the government chooses the tax rate that maximizes the welfare of the economy and satisfies the intertemporal restriction of revenues and expenditure..*

Definition 3 describes that there will be optimal fiscal saving when government analyzes the fiscal constraint in period $t = 0$ to understand the optimal response concerning resources which should save for period $t = 1$. Therefore, the optimal fiscal saving rule for non-tradable tax structure,

$$FSR^S = \left(\frac{g_M}{2+r}\right) G + \left(\frac{2+r+g_M}{2+r}\right) \frac{(1-\alpha)A_0 g_M}{(1+g_M)(2+r)} \tag{1.25}$$

Similar for tradable tax structure and using Definition 3,

$$FSR^M = \left(\frac{g_M}{2+r}\right) G \tag{1.26}$$

Therefore, if both rules are compared

$$FSR^S > FSR^M \tag{1.27}$$

Lemma 2. *Under this economy and in equilibrium, the FSR on tradable goods will be lower than non-tradable goods.*

Lemma 2 explains that government should save more on a structure based on services (non-tradable) than on manufactured goods (tradable)

[9]This is derived from the budget constraint of period 0 that generally implies:

$$\text{Revenues} - \text{Expenditures} = -b_0$$

Where b_0 is defined as debt. Therefore, if revenues are higher than expenditures government should save.

structure. This result comes from the general equilibrium of this economy. On the one hand, in the service structure, the government chooses a relatively low tax rate on manufacturing, but the tax base for demand for services is relatively high relative to manufactured goods, allowing the government to collect more tax revenues. The intuition of this comes from the smoothing of the consumption of household since to be able to consume in the same form in both periods and to impose a low rate of tax the government must collect relatively more revenues than in the other structures. On the other hand, for the case of manufactures occurs the opposite. For household consume softly, and the government imposes a high rate, it should collect less revenue than the service structure. However, this less revenue is constant through time. This condition is contrary to the service case in which the relative price plays a major role in the government's decision (for details and proofs see Appendix A.3.8).

Lemma 3. *The FSR of government in service and manufactures tax structure depends on the growth of rate of relative price and exposure or mismatch.*

As can be seen from the equations (1.25) and (1.26), the optimal fiscal savings rules of both cases studied depend on the growth rate of the relative price g_M and the potential mismatch or exposure that the government has. This last depends on the expenditure incurred G, interest rate r, the preferences α and the maximum production level per worker A_0. In this way, it is possible to generalize the optimal saving rule given by Lemma 3,

$$FSR = \underbrace{g_M}_{\text{Shock}} \times \underbrace{f(\alpha, r, G, A_0)}_{\text{Exposure}} \tag{1.28}$$

An interesting stylized fact is that the relative price will grow by g_M, i.e. its dynamics has no uncertainty as a classic shock, and therefore presents itself as an inherent risk of this economy. The latter can be linked to the literature of SWF, which is based on that the source of savings is given by precautionary elements of the government (i.e. a random event provided by a shock). However, as explained, this comes from an inherent in our model, as is the differential price growth that faces the government. In this sense, the type of “SWF” changes nature (or rationale) due to the exposure that the government has. Therefore, we can rename this saving as “Sustainability Fund” that permits face this mismatch. Besides, equation (1.28) can be re-written as

$$FSR = \underbrace{\frac{\Delta p}{p}}_{\text{Shock}} \times \underbrace{\left[\frac{\Delta R}{\Delta p} - \frac{\Delta p G}{\Delta p}\right]}_{\text{Exposure}} \tag{1.29}$$

As we have already mentioned, the FSR depends on the growth rate of price and exposure. The latter, in turn, will depend on the reaction of fiscal revenues and expenditures on price, which we can approximate as “elasticities”. Hence its name of exposure. For example, in manufacture tax structure the growth rate of price is g_M, the elasticity of fiscal revenues $G/(2+r)$ and elasticity of expenditure zero[10]. These elements give us a significant result on the effect of the price differential for the optimal decision of the government.

Corollary 1. *The optimal saving rule of government is zero in service and manufactures tax structure if the growth of rate of relative price is zero.*

Corollary 1 can be concluded directly from the equation (1.28) or (1.29) provided by Lemma 3. If in the economy there is no risk that the relative price increases at a certain rate, the FSR in the service and manufactured goods structure will be zero independent of the exposure that exists. Therefore, it is possible to conclude that a mismatch will have an effect on the optimal decision of the government as long as there is an inherent risk in the economy, which is when the price of services is relatively higher than that of manufactured goods. Besides, it can be observed that for the case of the income tax structure optimal saving will always be zero, regardless of the relative price dynamics that exist. Similarly for the case of “unaware” government but with a suboptimal solution. This fact is precisely because of the sources of income that the government has since in each period it can supply all of the expenditure without the need to save.

[10]In our model, the elasticity of spending in all tax structures is zero.

Lemma 4. *The FSR of government in income tax structure is zero independent of growth of rate of relative price.*

Finally, the case of "unaware" government allows us to compare what would happen if the government is not able to include all the relationships of the economy among the agents. The result of this implies that in the period 0 the fiscal revenues (1.20) satisfy the expenditure (1.23), however, this can not maintain this balance, since in period 1 should increase the tax rate and fiscal income to satisfy the restriction of that period, generating inefficiency for agents decisions. Again, this reflects the importance of how the price differential in this economy directly affects the optimal decision of the government, on the one hand, because the different tax structures imply different decisions and on the other, with an "unaware" government that can not sustain its spending. Figure 4 depicts the dynamics of deficits, i.e. the FSR for optimal decisions and deficit for "unaware" government,

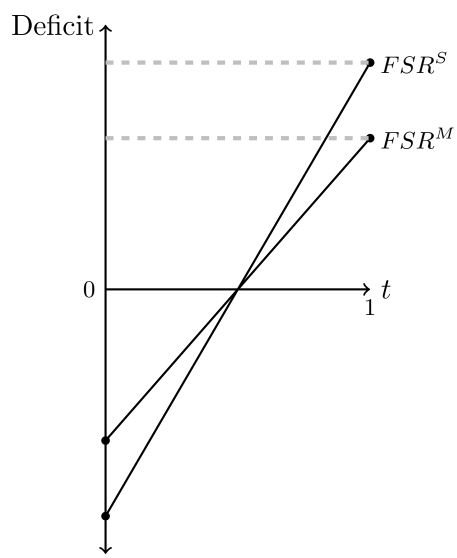

Figure 4: Fiscal deficit dynamics for optimal FSR

The main message of the model is to explain that the relative price dynamics (which is endogenous to the model) corresponds to a risk variable (shock) to the government for its optimal decision of tax rate and saving. Also, the model shows that the optimal decision of the government implies a higher tax rate for the tradable structure, but FSR is greater in the structure of non-tradable due to the intensity of consumption, allowing to save more for the future without generating distortions in the optimal decisions of the agents. The FSR or "Sustainability Fund" can be summed up in the risk that the economy is confronted through the price growth rate and the exposure that the fiscal revenues and expenses have at this price. So the exposure would be much greater for the case of non-tradable due of the direct effect on optimality conditions of household generating a greater provision for the mismatch. An interesting fact is if this dynamic did not exist (i.e., assuming a $g_M = 0$) would imply a similar behavior under the structures studied with a constant tax rate and "Sustainability Fund" equal to zero.

5 Empirical Analysis

5.1 Simple model

As explained in the literature, fiscal sustainability has had an isolated perspective of certain processes occurring in the economy, as economic growth not only affects the income and expenses that perceive and realizes the government, but also the prices that are facing. Thus, to simplify the intuitions outlined above and understand the mechanisms in a reduced form we use the definition according to Burnside (2005), in the long term:

$$\Delta B + T = G + rB \tag{2.1}$$

Where B is government debt, T tax revenues, r interest rate and G spending. So, if it extends this differentiating by the existence of prices for government Pg and the economy Py:

$$Py \cdot \Delta B + Py \cdot T = Pg \cdot G + Py \cdot rB \tag{2.2}$$

Also, assuming that the public debt is zero ($B = 0$) since it does not depend on economic growth in a direct way[11]. The equation (2.2) in logarithm implies:

$$\log T = \log p + \log G \tag{2.3}$$

Where $p = Pg/Py$. Therefore, differentiating (2.3) with respect to economic growth γ:

$$\underbrace{\frac{T'(\gamma)}{T}}_{\text{Tax Buoyancy}} = \underbrace{\frac{p'(\gamma)}{p}}_{\text{Balassa-Samuelson}} + \underbrace{\frac{G'(\gamma)}{G}}_{\text{Wagner's law}} \tag{2.4}$$

Equation (2.4) is a reduce form of equation (1.29) due of this expresses the exposure of fiscal revenues and expenditures. When there is economic growth, affect not only tax revenues but also spending and prices. In this sense, to ensure fiscal sustainability care about not only the quantity effect but also the differential effect of government prices, which is affected by the Balassa-Samuelson effect. An essential element from the practical point of view is that being the equation (2.4) a reduced version implies that the estimated magnitude will be necessary but not sufficient to determine the fiscal sustainability.

5.2 Data

This paper used annual panel data of tax revenues, prices and spending of 28[12] high-middle income countries (Appendix A.4) between 1980-2014. For taxes were used Government Financial Statistic (GFS) database provided by IMF, these to obtain the total tax revenues and their components in disaggregated level as income, consumption and property tax. Besides, corporate and VAT tax rates were collected from different sources as OECD, Eurostat and tax offices of each country. On the side of prices, the main data on government deflators Pg and GDP deflators Py come from the Annual Macro Economic Database (AMECO) compiled by the European Commis-

[11]While this is related to the literature, considering this fact is a potential limitation of subsequent analysis and results.

[12]The panel is unbalanced.

sion. Complemented it with nominal and real GDP data from World Bank's World Development Indicators (WDI), due that price of the economy (government) is the ratio between the expenditure of consumers (general government)[13] in current and constant prices. For the combination of these two data there was a long process of cross-validation, i.e. statistical and random review of AMECO, WDI, Central Banks and statistical offices data of each country[14]. Finally, and as stated above, with WDI data obtain government spending, GDP at constant prices and size of government (as % of GDP). The definition of a real variable is simply the nominal variable deflated by their respective price (See Appendix A.5 for summary of variables).

Table 1: Descriptive Statistics: Main Variables (1980-2014)

Variable	**Mean**	**Std. Dev.**	**Observations**	**Countries**	**T**
Growth Pg	0.048	0.051	853	28	29
Growth Py	0.043	0.047	853	28	29
Growth p	0.005	0.017	853	28	29
Growth T	0.024	0.052	692	28	25
Growth G	0.021	0.022	855	28	29
Growth Y	0.025	0.027	858	28	30
G/Y	0.189	0.042	861	28	30

This table displays the descriptive statistics for the main variables in the study for the baseline sample for 1980 onwards. The base year for Pg, Py and Pg/Py is 2010. We use notation Pg/Py = p. Pg (4.8%) growth faster than Py (4.3%) on average each year (statistically significant). Also, Pg/Py has grown in 0.5% on average. Tax revenue (T) and expenditure (G) in real terms grown 2.4% and 2.1% (this difference is not statistical significant), respectively. The size of government is 19% of GDP (G/Y). Y: Real GDP. T: Average length of time series.

Table 1 shows summary statistics for the sample concerning growth for the most relevant variables. The mean of annual growth of price government Pg is 4.8% and is higher (statistically) than the economy Py with an average of 4.3%. Besides, relatively speaking we see that there is a positive price growth of government over the economy of a 0.5% annually on average.

[13]This corresponds to the expenditure of government of national accounts, i.e. not include transfers.

[14]The data is the same used in Engel & Wagner (2016).

About tax revenue sources T and spending G in real terms we see that they have a growth of 2.4% and 2.1%, respectively. However, we can not say they are statistically different. Finally, GDP growth is 2.5% annually, and the size of government is 19% of GDP on average for the countries studied.

Table 2: Descriptive Statistics: Tax Variables (1980-2014)

Variable	**Mean**	**Std. Dev.**	**Observations**	**Countries**	**T**
Growth IT real	0.025	0.084	559	28	20
Growth PIT real	0.018	0.089	515	28	18
Growth CIT real	0.023	0.177	422	27	16
Growth CONSUMIT real	0.021	0.049	449	27	17
Growth VAT real	0.025	0.065	417	27	16
Growth TRADE real	-0.057	0.651	420	26	16
Growth PROPIT	0.037	0.375	451	28	30
Corporate Tax Rate	0.341	0.104	819	28	30
VAT Tax Rate	0.224	0.133	743	28	30

This table displays the descriptive statistics for tax variables in the study for the baseline sample for 1980 onwards. T: Average length of time series. IT growth is 2.5%, very similar to that of total revenues for the sample. PIT have grown less than corporate (CIT), not significant. CONSUMIT growth stands at 2.1 %. However, the increase of VAT is 2.5% while TRADE decreased in -5.7%. PROPIT growth is higher than others with 3.7%. The corporate tax rate and VAT on average are 34.1% and 22.4% respectively.

Table 2 shows more detailed descriptive statistics for tax variables. In particular, it notes that there is heterogeneity in how tax funding sources have grown. In the case of income tax (IT), we see that the average annual growth was 2.5%, very similar to that of total revenues for the sample. Also, it is noted that the personal tax (PIT) have grown less than corporate (CIT). However, this difference is not significant. For consumption taxes (CONSUMIT) the average growth stands at 2.1 %. However, this show heterogeneity since the increase of VAT is 2.5% in average while trade taxes (TRADE) decreased strongly over time (-5.7%). The growth of property taxes (PROPIT) attracts attention because this is higher than others with 3.7% annually on average. Finally, the corporate tax rate and VAT on average are 34.1% and 22.4% respectively for the sample.

5.3 Empirical strategy

To test the effects of tax revenues, expense and prices in fiscal sustainability will use an econometric specification that follows an SUR system (Seemingly Unrelated Regression) connecting the three variables of interest to economic growth simultaneously. In simple words, each equation of this system represent the reaction of interest variables respect to economic growth, so each $\beta_i^j \; \forall j = \rho, G, T$ estimated represent an elasticity. Following the scheme of Belinga et al. (2004), each equation represents an ADL model (Autoregressive Distributed Lag) with an optimal lag of one for the study variables (relative prices, expenditure, and revenues) and economic growth. This framework is consistent with using the first difference in each equation[15]. Therefore:

$$
\begin{aligned}
\Delta \log p_{it} &= \alpha_{1_i} + \lambda_{1_t} + \beta_i^{\rho} \Delta \log Y_{it} + x'_{it}\gamma_{1_{it}} + \varepsilon_{it} \\
\Delta \log G_{it} &= \alpha_{2_i} + \lambda_{2_t} + \beta_i^{G} \Delta \log Y_{it} + x'_{it}\gamma_{2_{it}} + \vartheta_{it} \\
\Delta \log T_{it} &= \alpha_{3_i} + \lambda_{3_t} + \beta_i^{T} \Delta \log Y_{it} + x'_{it}\gamma_{3_{it}} + \varphi_{it}
\end{aligned}
$$

Where p_{it} represent the ratio Pg/Py, G_{it} government expenditure in real terms, T_{it} revenue in real terms and Y_{it} the GDP for each country i and year t. α_{j_i} and λ_{j_t} correspond fixes effect by countries and years, respectively, for each equation $j = 1, 2, 3$. ε_{it}, ϑ_{it} and φ_{it} are errors correlated with each other. Finally, x'_{it} correspond a control vector as the size of government and tax rates. Thus, to test the hypothesis is utilized the expression (2.4) approaching it with elasticities estimated in empirical strategy described

$$
\underbrace{\beta^T}_{\text{Tax Buoyancy}} = \underbrace{\beta^{\rho}}_{\text{Balassa-Samuelson}} + \underbrace{\beta^G}_{\text{Wagner's law}} \tag{2.5}
$$

[15] Also; we use unit root and cointegration test for panel data to demonstrate the use of stationary data.

Therefore, denoting,

$$\beta^M = \underbrace{\beta^T - \beta^\rho - \beta^G}_{\text{Exposure or mismatch}} \tag{2.6}$$

There are three cases if $\beta^M < 0$ there will be a mismatch (exposure) of total taxes, i.e. economic growth will not be sufficient to ensure fiscal sustainability. In another hand, if $\beta^M = 0$ the total taxes will be hedged against economic growth. Finally, if $\beta^M > 0$ the total taxes will be over-hedged. To test $\beta^M > 0$, it is performed an F test.

5.4 Results

Basic results

Table 3 shows the results of the effect of economic growth on prices, expenditure and revenue under different specifications between 1980 and 2014. A mismatch is not observed. Therefore, they would at least cover by economic growth. If these results are observed in detail, it is noted that estimation of the different elasticities, which have many relationships with the estimations in the literature. In the case of prices, these have a range of 0.29% - 0.44% very close to Engel & Wagner (2016), even showing that this effect could be stronger. The same applies to tax revenues which range from 0.9 % - 1.05 % similar to estimated by Belinga et al.(2004) for a sample of 1965 - 2012. In the case of spending, there is not elasticity to compare directly, but are similar to existing estimations. Another important issue is how tax controls or the size of government influence coefficients β^ρ, β^G and β^T. On the side of taxes, columns (2), (3) and (4) shows that these contribute significantly to the reduction of the over-hedging β^M from 0.42 to 0.33. The consumption tax provides more than corporate. However, on the side of government size, it is observed a greater relative effect allowing at least observe complete hedge, as can be seen in column (5) with a non-significant coefficient of 0.113. The full effect will not change much by controlling both taxes and government size (column 6).

On the other hand, Table 4 shows the importance of mismatch in the last 20 years. In particular, the specification with all controls (column 6) shows a mismatch of 0.23%. Doing the same analysis of the results from 1980 - 2014, it can be seen that tax rates have less importance in the coefficients than the size of government, this is consistent with the literature, which by robustness analysis shows that controlling for tax rates does not change the main results. Therefore, economic growth does not necessarily ensure fiscal sustainability. These insights are maintained for subsequent years with an increase of mismatch (Appendix A.6 and A.7). The principal mechanism to evaluate this change in the mismatch is how the tax buoyancy has lost strength over the past 20 years since the coefficient has fallen from 0.9% - 1.05 % to 0.54% - 0.68%, remaining relatively constant for the case of prices and expenses. One potential explanation for this phenomenon is the economic instability that occurs in the world economy from 2000 onwards, which produces a term of the period known as Great Moderation (Bernanke, 2004). Clearly, the last being related to the global financial crisis which led a lower tax revenue for a long time (Belinga et al., 2004). In this line, Table 5 shows the evolution of fiscal mismatch for different samples from 1995 to 2005. As noted, the mismatch increases over time without losing significance, indicating the importance of this phenomenon in the last 20 years. Moreover, this intuition is checked if we look at these same results without the periods of crisis (Appendix A.8) where tax buoyancy as Wagner's law increase. However, the mismatch still a relevant and significant coefficient.

Heterogeneous effects: component of taxes

Table 6 and Table 7 display the results of the mismatch for income (IT) and consumption (CONSUMIT) taxes for specification with all controls. There is a clear difference in effect for both types of taxes. On the one hand, income taxes (Table 6) does not present a mismatch, at least more would be covered by economic growth (column 1). However, if it is looked at a disaggregate level, the income taxes have a great difference in their composition, as for corporate tax (CIT) there is a large over-hedging (column

Table 3: Basic Results for Sample: 1980 - 2014

Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
β^{ρ}	0.287***	0.291***	0.331***	0.333***	0.439***	0.441***
	(0.0302)	(0.0298)	(0.0332)	(0.0334)	(0.0265)	(0.0290)
β^{G}	0.344***	0.359***	0.377***	0.375***	0.344***	0.375***
	(0.0397)	(0.0405)	(0.0449)	(0.0454)	(0.0397)	(0.0454)
β^{T}	1.054***	1.041***	1.041***	1.040***	0.896***	0.911***
	(0.0957)	(0.0969)	(0.109)	(0.109)	(0.0984)	(0.110)
β^{M}	**0.423*** **	**0.391*** **	**0.333*** **	**0.333*** **	**0.113**	**0.095**
Observations	689	664	582	572	689	572
Countries	28	28	28	28	28	28
Controls						
Corporate Tax Rate	NO	YES	NO	YES	NO	YES
Consumption Tax Rate	NO	NO	YES	YES	NO	YES
Δ G/Y	NO	NO	NO	NO	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG does not contain control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according to of sign interpretation. If this is negative, we want to reject a positive β^M and vice-versa.

Table 4: Basic Results for Sample: 1995 - 2014

Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
β^{ρ}	0.301***	0.301***	0.309***	0.309***	0.391***	0.392***
	(0.0374)	(0.0374)	(0.0384)	(0.0384)	(0.0318)	(0.0328)
β^{G}	0.375***	0.372***	0.389***	0.385***	0.375***	0.385***
	(0.0514)	(0.0516)	(0.0529)	(0.0531)	(0.0514)	(0.0531)
β^{T}	0.657***	0.665***	0.678***	0.663***	0.538***	0.544***
	(0.131)	(0.132)	(0.134)	(0.135)	(0.132)	(0.135)
β^{M}	**-0.019**	**-0.008**	**-0.044**	**-0.031**	**-0.228** **	**-0.233** **
Observations	458	455	436	433	458	433
Countries	28	28	28	28	28	28
Controls						
Corporate Tax Rate	NO	YES	NO	YES	NO	YES
Consumption Tax Rate	NO	NO	YES	YES	NO	YES
Δ G/Y	NO	NO	NO	NO	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG does not contain control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according to of sign interpretation. If this is negative, we want to reject a positive β^M and vice-versa.

Table 5: Evolution of Tax Mismatch

Sample	Estimated β^M	Observations
1995-2014	-0.23**	433
1996-2014	-0.21**	420
1997-2014	-0.24**	406
1998-2014	-0.23**	388
1999-2014	-0.23**	372
2000-2014	-0.31**	356
2001-2014	-0.28**	339
2002-2014	-0.29**	318
2003-2014	-0.28**	295
2004-2014	-0.32**	272
2005-2014	-0.36**	248

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. β^M includes all controls.

2), due to the significant tax buoyancy which allows offsetting the effect of prices and spending. This estimation does not happen in the case of personal taxes (PIT), which may be exposed (column 3), although its coefficient is not significant.

On the other hand, consumption taxes (CONSUMIT) (Table 7) are in an opposite situation, at least for all controls (column 1) with a mismatch of taxes of 0.23%. In the case of the more disaggregated level there is a clear mismatching for VAT and over-hedging for trade tax (TRADE), but can not say anything because these coefficients are not significant. In the case of property tax, these results are not reported because they have no relationship with economic growth and the coefficient is β^M not significant.

Therefore, there is evidence that certain taxes are more exposed than others. Relatively speaking, consumption tax (CONSUMIT) would be more exposed. Instead, corporate tax (CIT) would not have problems to finance spending. The main message is some tax structures are naturally more hedged against economic growth. Also, this evidence shows consistency among the results of the model, since a more intensive structure in non-tradable goods generates greater resources as it happens with the case of

Table 6: Income Tax Results for Sample: 1995 - 2014

Coefficient	(1) Income Tax	(2) Personal Tax	(3) Corporate Tax
β^{ρ}	0.379***	0.382***	0.372***
	(0.0319)	(0.0322)	(0.0328)
β^{G}	0.382***	0.386***	0.380***
	(0.0524)	(0.0551)	(0.0577)
β^{T}_{income}	0.742***	0.566**	2.438***
	(0.227)	(0.226)	(0.510)
β^{M}	**-0.018**	**-0.201**	**1.69*** **
Observations	434	401	375
Countries	28	28	27
Controls			
Corporate Tax Rate	YES	YES	YES
Δ G/Y	YES	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG does not contain control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according to of sign interpretation. If this is negative, we want to reject a positive β^M and vice-versa.

Table 7: Consumption Tax Results for Sample: 1995 - 2014

Coefficient	(1) Total Consumption Tax	(2) VAT Tax	(3) Trade Tax
β^{ρ}	0.383***	0.383***	0.347***
	(0.0336)	(0.0336)	(0.0425)
β^{G}	0.391***	0.391***	0.400***
	(0.0550)	(0.0550)	(0.0683)
$\beta^{T}_{consumption}$	0.546***	0.546***	0.871
	(0.130)	(0.130)	(3.087)
β^{M}	**-0.227***	**-0.168**	**0.123**
Observations	392	392	259
Countries	27	27	26
Controls			
VAT Tax Rate	YES	YES	YES
Δ G/Y	YES	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG does not contain control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according to of sign interpretation. If this is negative, we want to reject a positive β^M and vice-versa.

corporate taxes. In contrast, taxes closer to tradable goods such as consumer goods have a potential problem to finance intensive non-tradable expenditure.

6 Policy discussion

With the theoretical model developed and the results of the empirical evidence, it is possible to conclude that Balassa-Samuelson effect or Baumol's cost disease is a fundamental element for government decisions. Therefore, this stylized fact should be incorporated into the analysis related to sustainability and fiscal policy in practice. In this way, from public policy should be considered in the following dimensions.

First, the government assumes an in-kind defined benefit for the future when spending in non-tradable goods, like promising to have one teacher forever, then the expenditure side of that promise has an exposure of P_g, in the sense that systematic increases in P_g make more expensive to finance that teacher. One possibility is that this exposure is fully hedged in real terms by the fundamentals of tax revenue, but this would only be possible if taxes go to the same factors that appreciate with P_g. Therefore, there will be more favorable sources of tax revenue to face this inherent tendency in the economy. As it was studied in the theoretical model of section 3, the optimal decision of the government implies a higher tax rate for the tradable system, but provision is greater in the structure of non-tradable due to the intensity of consumption in this sector. The difficulty of classifying a type of tax in one of these two areas makes it harder to connect these empirical results. However, we show that corporate taxes allow better financing of the expenditures being closer to a non-tradable structure. In other hand, taxes on consumption would be exposed since this has more tradable components. The main message here is the ability of the government through tools such as the tax base and the tax rate to incorporate price dynamics in fiscal policy for its analysis and thus generate a tax hedging.

Second, given the difficulty of conducting discretionary changes in tax rates or tax base in the short term, the government has other options to face this exposure due to differential price increases. This decision is precisely through the fiscal saving that can be exercised, which unlike the precautionary saving of the household this is not given by the uncertainty shock but by an inherent and certain variable of the economy, i.e. the growth rate of P_g. Moreover, it may be thought that fiscal rules towards "Sustainability Fund" are a useful tool that allows an automatically balance to potential exposures. In this sense, the type of "SWF" changes nature (rationale) due to the exposure that the government has, i.e. a certain price increasing. This provision is in the line of corporate hedging, where companies are insured against changes in prices or currencies.

Finally, it is clear that measures of fiscal sustainability may also need to include hedging of their stream of expenditures and revenues against the potential increase in non-traded prices. An example of this is in the measure of tax buoyancy, which holds that an elasticity of one is sufficient to maintain the fiscal balance regarding government expenditure. However, it is clear that this will not be the case if it is included the elements studied which would even involve levels of 1.2-1.3 concerning elasticity as the minimum. Similarly, it occurs with other indicators such as the debt-to-GDP ratio (Blanchard et al., 1991) since the fact that the economy grows not only serves to keep the debt constant as this same will affect the price differential by increasing it. Mathematically,

$$b = \frac{-d}{(r - \gamma)} \tag{3.1}$$

Equation (3.1) relates debt (b) to the primary deficit (d) and interest and growth rates in steady state. If we have a constant primary surplus, economic growth will help since a higher debt-to-GDP ratio can be achieved. However, this equation does not include directly the effect of the price differential which directly impacts $-d$. For this,

$$-d = \left[\frac{T(\gamma)}{Y(\gamma)} - \frac{pG(\gamma)}{Y(\gamma)}\right] \tag{3.2}$$

Equation (3.2) explains that economic growth does not necessarily imply a higher growth rate of the debt-to-GDP ratio in the steady state since when the economy grows, it also affects revenues and expenditures, which may even reduce the effect of economic growth if the expenditure-to-GDP ratio is higher than revenues-to-GDP ratio.

7 Conclusions

This paper intends to show that it is important to consider how differential price effect affects fiscal sustainability. For this and with the intention of studying underlying mechanism it is built a novel and simple model that explain the optimal response of the government in different tax structure context. The main message is that Balassa-Samuelson effect (or Baumol's cost disease) corresponds a key variable for the optimal provision, which will depend on exposure and non-tradable goods price growth. Besides, the model shows that countries intensive in tradable tax structure imply a higher tax rate, but those intensive in non-tradable goods have a greater optimal fiscal saving rule due to the intensity of consumption, allowing to save more for the future without generating distortions in the optimal decisions of the agents. Moreover, without this growth there is no mismatch, giving much relevance to the analysis of this phenomenon in fiscal terms. In another hand, in an empirical approach, it was found that there is a mismatch at least in the last 20 years for a panel of 28 high-middle income countries. In particular, it is noted that an increase in 1% in GDP growth implies a 0.23% in the mismatch in average between 1995- 2014. Moreover, this effect is higher through the years with ranges between 0.21% and 0.36%. While one of the possible explanations is the economic instability resulting from the financial crisis, the results remain relevant and significant if we eliminate those years, with a mismatch between 0.19% and 0.37%. These elements allow us to study that an elasticity of one, as predicted by indicators such as tax buoyancy, is not sufficient and would be needed between 1.2-1.3 regarding elasticity as the minimum. Also, there is evidence that certain taxes have more exposed than others. A consumption tax would be more exposed. Instead, corporate taxes would not have problems to finance spending, consistent with that proposed in the model.

The fact that price differential is significant for government decisions from a theoretical and empirical point of view makes it possible to understand that it must also have a substantial impact on the indicators and models used to maintain fiscal sustainability in practice. For example, there will be more favorable sources of tax revenue to face this inherent tendency in the economy. Government's fiscal policy decision concerning tax rate and tax base may be an appropriate option. In another hand, fiscal rules towards "Sustainability Funds" are a useful tool that allows an automatically balance to potential exposures changing the nature of typical SWF. Moreover, measures of fiscal sustainability may also need to include hedging of their stream of expenditures and revenues against the potential increase in non-traded prices as tax buoyancy and debt-to-GDP ratio. Economic growth does not necessarily imply a higher growth rate of the debt-to-GDP ratio in the steady state since when the economy grows, it also affects revenues and expenditures, which may even reduce the effect of economic growth if the expenditure-to-GDP ratio is higher than revenues-to-GDP ratio.

Finally, it is important to mention that future research is extensive since this paper aims to be a first theoretical and empirical approximation on the price differential and its effects concerning fiscal sustainability. The model is still a simple, nevertheless, allows to understand the key findings and facts in the decision that must take the government. However, we want to extend this model to a complete tax structure, not analyzing each tax separately but through a government that chooses the optimal share of tax under the structures to understand what type of tax allows to better finance this government expense in non-tradable. Also, it is important to mention that this paper has limitations. Our theoretical analysis does not include the effect of income distribution, understanding that the government's exposure and its optimal outcome affects it. The model only cares about the price effect across the production side, not considering the interrelations that may exist from the side of government or preferences. In another hand, there is an inability to connect the theoretical results directly with the empirical results or potential interrelation that may exist between tax structure. Another element is the definition we use for fiscal spending, which does not include transfers. The latter is important from the fiscal point of view, specifically sustainability. The latter will be incorporated into future versions. For this reason, it is that an extensive line of future research.

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A Appendix

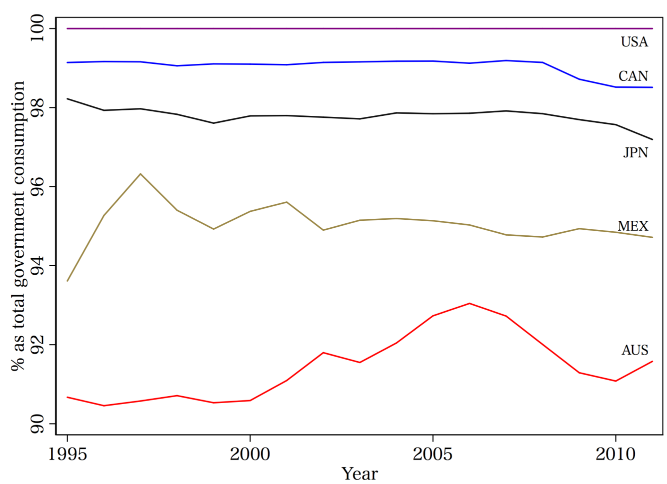
A.1 Share of Non-Tradables of Government as % of total expenditure of government (1995-2011)

Variable	Mean	Std. Dev.	Observations	Countries
Share of NT (Traditional)	98.53	2.33	680	40
Share of NT (De Gregorio et al. 1994)	94.02	6.59	680	40

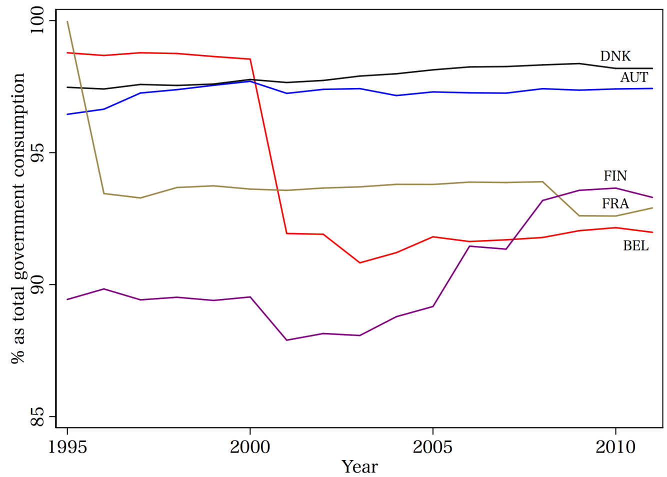
This table displays the descriptive statistics for government expenditure with NIOT database with two definitions of non-tradable goods from 1995 to 2011. The traditional measure was defining all service industries as non-traded. Instead, De Gregorio et al. (1994) define tradable industry if the average export to value added ratio is greater than 10 percent. For this, we used information provided by Mano & Castillo (2015). The share of Non-Tradable (NT) represents the expenditure in non-tradable as a percentage of the total spending of government. EU countries: Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Sweden, and United Kingdom. Others: Canada, United States, Brazil, Mexico, China, India, Japan, South Korea, Australia, Taiwan, Turkey, Indonesia, and Russia.

A.2 Evolution of Share of Non-Tradables: Selected Countries.

Measure: De Gregorio et al. (1994)



Source: Own elaboration based on NIOT database



Source: Own elaboration based on NIOT database.

A.3 The model

A.3.1 Firms

We considered a two-sector economy with a manufactured good (traded) Y_t^M and services (non-traded) Y_t^S for each period $t = 0$ and $t = 1$. The production functions are given as

$$Y_t^M = A_0^M L_t^M \, (1 + g_M)^t \tag{1}$$

$$Y_t^S = A_0^S L_t^S \, (1 + g_S)^t \tag{2}$$

For simplicity, it is assumed that $A_0^M = A_0^S = A_0$ and normalize the price of manufactured good p_t^M to one. So we denoted the relative price of service respect to manufactured as $p_t = p_t^S$. The wage is w_t. Assuming that $g_S = 0$. The profit maximizations of firms are

$$\pi_t^M = A_0 L_t^M \, (1 + g_M)^t - w_t L_t^M$$

$$\pi_t^S = p_t A_0 L_t^S - w_t L_t^S$$

The first order conditions

$$\frac{\partial \pi_t^M}{\partial L_t^M} = A_0 \ (1+g_M)^t - w_t = 0 \tag{3}$$

$$\frac{\partial \pi_t^M}{\partial L_t^M} = p_t A_0 - w_t = 0 \tag{4}$$

Dividing (3) and (4) we obtain equilibrium price

$$p_t = (1+g_M)^t \tag{5}$$

Equation (5) reflects that relative price of services respects to manufactured goods in each period t. Also, the equilibrium wage is the same in both sectors (for equations (3) or (4))

$$w_t = A_0(1+g_M)^t \tag{6}$$

To maintain equilibrium is necessary that relative price of services respect to manufactured goods serve as a mechanism of balance between two sectors. Also, it is possible to observe that the wage increases through the time similar to relative price. Finally, the demand for labor in each area is expressed in equation (6) due of firms have constant returns to scale given a perfect elasticity. Therefore, the equilibrium will be determinate by labor supply which we will assume exogenously. These are denoted by

$$L_t^M = \bar{L}^M \tag{7}$$

$$L_t^S = \bar{L}^S \tag{8}$$

With demand and supply of labor, we can obtain the equilibrium labor in each sector and period. Note that in equilibrium labor will be the same for both sectors and in two periods.

A.3.2 Equilibrium

In equilibrium, firms solve the relative price equilibrium provided by equation (5). Due to constant returns to scale in production functions the demand for labor is perfectly elastic, implying an equilibrium wage given by equation (6). For optimal Y_t^M and Y_t^S we need to impose the supply given by equations (7) and (8). On the other hand, household maximizes the present value of utility subject to the intertemporal budget constraint which depends on equilibrium price and wage of firms problem given by equations (5) and (6) respectively. The optimal demands of M_t and S_t depend on preferences, income stream, relative price and respective tax rate. Government maximizes the present value of welfare subject to intertemporal fiscal constraint, which depending on the tax structure, depends on optimal demands M_t, S_t or equilibrium wage (6). Besides, the intertemporal fiscal constraint includes equilibrium price (5) and respective tax rate τ_t^S, τ_t^M or τ_t^I. The final decision variable corresponds to the tax rate, which with all the equilibrium conditions described above will depend on exogenous parameters of the model. Finally, the economy is restricted by resource constraint, i.e. the demand for good should be equal to supply in each period,

$$Y_t^M = M_t \tag{9}$$

$$Y_t^S = S_t + G_t \tag{10}$$

With these conditions can be identified our economy entirely. Obviously, as mentioned earlier, different tax structure will change the budget constraint of household, optimal demands for goods and government problem.

A.3.3 Service tax structure

The problem of household is,

$$\begin{aligned}
max \quad & U = \log(M_0^{\alpha} S_0^{1-\alpha}) + \beta \, \log(M_1^{\alpha} S_1^{1-\alpha}) \\
s.t \quad & \\
& w_0 + \frac{w_1}{1+r} = M_0 + \frac{M_1}{1+r} + (1+\tau_0^S)p_0 S_0 + \frac{(1+\tau_1^S)p_1 S_1}{1+r}
\end{aligned}$$

Obviously, indirectly in this restriction are the financial assets of the household. For simplicity, it is assumed that the initial asset $a_0 = 0$. Besides, in an intertemporal problem, it is necessary to include one more restriction which implies that household will not have assets in the last period

$$\lim_{t\to\infty} \frac{a_{t+1}}{(1+r)^t} \geq 0 \tag{11}$$

Equation (11) implies that $a_2 = 0$. Thus, to solve this problem,

$$\mathcal{L} = U + \lambda \left[w_0 + \frac{w_1}{1+r} - M_0 - \frac{M_1}{1+r} - (1+\tau_0^S)p_0 S_0 - \frac{(1+\tau_1^S)p_1 S_1}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{M_0} = \frac{1}{M_0^{\alpha} S_0^{1-\alpha}} \alpha M_0^{\alpha-1} S_0^{1-\alpha} - \lambda = 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{S_0} = \frac{1}{M_0^{\alpha} S_0^{1-\alpha}} (1-\alpha) M_0^{\alpha} S_0^{-\alpha} - \lambda (1+\tau_0^S) p_0 = 0 \tag{13}$$

$$\frac{\partial \mathcal{L}}{M_1} = \beta \, \frac{1}{M_1^{\alpha} S_1^{1-\alpha}} \alpha M_1^{\alpha-1} S_1^{1-\alpha} - \lambda \frac{1}{1+r} = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{S_1} = \beta \, \frac{1}{M_1^{\alpha} S_1^{1-\alpha}} (1-\alpha) M_1^{\alpha} S_1^{-\alpha} - \lambda \frac{(1+\tau_1^S) p_1}{1+r} = 0 \tag{15}$$

Using (12) and (13) or (14) and (15),

$$\frac{\alpha S_0}{(1-\alpha) M_0} = \frac{1}{(1+\tau_0^S) p_0} \tag{16}$$

$$\frac{\alpha S_1}{(1-\alpha)M_1} = \frac{1}{(1+\tau_1^S)p_1} \tag{17}$$

Equation (16) and (17) represent the optimal allocation of consumption for households for each period. Now, using (13) and (15)

$$\frac{1}{\beta}\frac{S_1}{S_0} = (1+r)\frac{(1+\tau_0^S)p_0}{(1+\tau_1^S)p_1}$$

With $\beta = 1/1+r$,

$$(1+\tau_0^S)p_0 S_0 = (1+\tau_1^S)p_1 S_1 \tag{18}$$

Similar using (12) and (14),

$$M_0 = M_1 \tag{19}$$

(18) and (19) are Euler equations. Now using budget constraint of household,

$$w_0 + \frac{w_1}{1+r} = M_0 + \frac{M_1}{1+r} + (1+\tau_0^S)p_0 S_0 + \frac{(1+\tau_1^S)p_1 S_1}{1+r}$$

Using (18) and (19),

$$w_0 + \frac{w_1}{1+r} = M_0 + \frac{M_0}{1+r} + (1+\tau_0^S)p_0 S_0 + \frac{(1+\tau_0^S)p_0 S_0}{1+r}$$

Replacing with equation (16),

$$w_0 + \frac{w_1}{1+r} = \frac{2+r}{1+r}\left[\frac{(1+\tau_0^S)p_0 S_0}{1-\alpha}\right]$$

Re-written $W = w_1 + w_2/(1+r)$. The optimal demand of M_0,

$$S_0 = \frac{(1-\alpha)W(1+r)}{(1+\tau_0^S)p_0(2+r)} \tag{20}$$

Using (19) the optimal demand of M_0,

$$M_0 = \frac{\alpha W(1+r)}{(2+r)} \tag{21}$$

For optimal S_1 and M_1 we use (18) and (19),

$$S_1 = \frac{(1-\alpha)W(1+r)}{(1+\tau_1^S)p_1(2+r)} \tag{22}$$

$$M_1 = \frac{\alpha W(1+r)}{(2+r)} \tag{23}$$

The optimal demands depend on the present value of income W. Besides, it can be observed that demands for S_0 and S_1 only depends on current relative price and tax rate due to the condition of optimality of the household over the allocation of goods (16). For example, an increase in the relative price in period 1 is automatically offset by a fall in demand in that period 1. So it will not affect the demand of the other period according to Euler equation (18). This condition is similar to tax rate. In another hand, the problem of government is maximizing utility and choose optimal τ_S. For this, result the government internalizes all the relations of the economy (expressed in our system).

$$\begin{aligned}
max \qquad & W = \log(M_0^\alpha S_0^{1-\alpha}) + \beta \log(M_1^\alpha S_1^{1-\alpha}) \\
s.t \qquad & \\
& \tau_0^S S_0 + \frac{\tau_1^S S_1}{1+r} = p_0 G + \frac{p_1 G}{1+r} \\
& \\
& (20), \quad (21), \quad (22) \;\; \& \;\; (23)
\end{aligned}$$

For simplicity, we denote $G_t = G$ due is constant and exogenous. Besides, and similar to household, indirectly in this restriction are debt for government denoted by b_t which assume $b_0 = 0$ for period $t = 0$ and $b_2 = 0$

for period $t = 1$ due of debt need to be pay in last period,

$$\lim_{t\to\infty} \frac{b_{t+1}}{(1+r)^t} \geq 0 \tag{24}$$

To solve this problem,

$$\mathcal{L} = W - \lambda \left[\tau_0^S S_0 + \frac{\tau_1^S S_1}{1+r} - p_0 G - \frac{p_1 G}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{\tau_0} = -(1-\alpha)\frac{(1+\tau_0^S)}{(1+\tau_0^S)^2} - \lambda \frac{(1-\alpha)W(1+r)}{(1+\tau_0^S)^2 p_0 (2+r)} = 0 \tag{25}$$

$$\frac{\partial \mathcal{L}}{\tau_1} = -\beta\ (1-\alpha)\frac{(1+\tau_1^S)}{(1+\tau_1^S)^2} - \frac{\lambda}{1+r}\frac{(1-\alpha)W(1+r)}{(1+\tau_1^S)^2 p_1 (2+r)} = 0 \tag{26}$$

Dividing (25) and (26),

$$\frac{1}{\beta}\frac{(1+\tau_1^S)}{(1+\tau_0^S)} = \frac{1}{1+r}\frac{(1+\tau_1^S)^2 p_1}{(1+\tau_0^S)^2 p_0}$$

$$p_0(1+\tau_0^S) = p_1(1+\tau_1^S) \tag{27}$$

Using the net present value of government budget constraint and optimal demands (20) and (22),

$$\tau_0^S S_0 + \frac{\tau_1^S S_1}{1+r} = p_0 G + \frac{p_1 G}{1+r}$$

First, we know that

$$p_t = (1+g_M)^t$$

So,

$$p_0 = 1$$

$$p_1 = (1+g_M)$$

Using this,

$$\tau_0^S S_0 + \frac{\tau_1^S S_1}{1+r} = G\left(\frac{2+r+g_M}{1+r}\right)$$

Now using (20) and (22),

$$\tau_0^S \frac{(1-\alpha)W(1+r)}{(1+\tau_0^S)p_0(2+r)} + \tau_1^S \frac{(1-\alpha)W(1+r)}{(1+\tau_1^S)p_1(2+r)(1+r)} = G\left(\frac{2+r+g_M}{1+r}\right)$$

With (27),

$$\left(\tau_0^S + \frac{\tau_1^S}{1+r}\right)\frac{(1-\alpha)W(1+r)}{(1+\tau_0^S)p_0(2+r)} = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\left(\frac{(1+g_M)(1+r)\tau_0^S + \tau_0^S - g_M}{(1+g_M)(1+r)}\right)\frac{(1-\alpha)W(1+r)}{(1+\tau_0^S)p_0(2+r)} = G\left(\frac{2+r+g_M}{1+r}\right)$$

Note that W,

$$W = w_0 + \frac{w_1}{1+r}$$

Where,

$$w_0 = A_0$$

$$w_1 = A_0(1+g_M)$$

Thus,

$$W = A_0\left(\frac{2+r+g_M}{1+r}\right)$$

Using this fact,

$$\left(\frac{(1+g_M)(1+r)\tau_0^S + \tau_0^S - g_M}{(1+g_M)(1+r)}\right)\frac{(1-\alpha)A_0(1+r)}{(1+\tau_0^S)(2+r)} = G$$

So the optimal tax rate

$$\tau_0^S = \frac{G(1+g_M)(2+r) + (1-\alpha)A_0 g_M}{(1-\alpha)A_0[(1+g_M)(1+r)+1] - G(1+g_M)(2+r)} \tag{28}$$

$$\tau_1^S = \frac{G(1+g_M)(2+r) - (1-\alpha)A_0 g_M(1+r)}{(1-\alpha)A_0[(1+g_M)(1+r)+1] - G(1+g_M)(2+r)} \tag{29}$$

A.3.4 Manufactured goods tax structure

The problem of household is,

$$
\begin{aligned}
max \quad & U = \log(M_0^{\alpha} S_0^{1-\alpha}) + \beta \, \log(M_1^{\alpha} S_1^{1-\alpha}) \\
s.t \quad & \\
& w_0 + \frac{w_1}{1+r} = (1+\tau_0^M)M_0 + \frac{(1+\tau_1^M)M_1}{1+r} + p_0 S_0 + \frac{p_1 S_1}{1+r}
\end{aligned}
$$

To solve this problem, we maximize utility subject to intertemporal budget constraint and equation (11),

$$\mathcal{L} = U + \lambda \left[w_0 + \frac{w_1}{1+r} - (1+\tau_0^M)M_0 - \frac{(1+\tau_1^M)M_1}{1+r} - p_0 S_0 - \frac{p_1 S_1}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{M_0} = \frac{1}{M_0^{\alpha} S_0^{1-\alpha}} \alpha M_0^{\alpha-1} S_0^{1-\alpha} - \lambda(1+\tau_0^M) = 0 \tag{30}$$

$$\frac{\partial \mathcal{L}}{S_0} = \frac{1}{M_0^{\alpha} S_0^{1-\alpha}} (1-\alpha) M_0^{\alpha} S_0^{-\alpha} - \lambda p_0 = 0 \tag{31}$$

$$\frac{\partial \mathcal{L}}{M_1} = \beta \, \frac{1}{M_1^{\alpha} S_1^{1-\alpha}} \alpha M_1^{\alpha-1} S_1^{1-\alpha} - \lambda \frac{(1+\tau_1^M)}{1+r} = 0 \tag{32}$$

$$\frac{\partial \mathcal{L}}{S_1} = \beta \, \frac{1}{M_1^{\alpha} S_1^{1-\alpha}} (1-\alpha) M_1^{\alpha} S_1^{-\alpha} - \lambda \frac{p_1}{1+r} = 0 \tag{33}$$

Using (30) and (31) or (32) and (33),

$$\frac{\alpha S_0}{(1-\alpha)M_0} = \frac{(1+\tau_0^M)}{p_0} \tag{34}$$

$$\frac{\alpha S_1}{(1-\alpha)M_1} = \frac{(1+\tau_1^M)}{p_1} \tag{35}$$

Equation (34) and (35) represent the optimal allocation of consumption for households for each period. Now, using (30) and (32)

$$\frac{1}{\beta}\frac{M_1}{M_0} = (1+r)\frac{(1+\tau_0^M)}{(1+\tau_1^M)}$$

With $\beta = 1/1+r$,

$$M_0(1+\tau_0^M) = M_1(1+\tau_1^M) \tag{36}$$

Similar using (31) and (33),

$$p_0 S_0 = p_1 S_1 \tag{37}$$

(36) and (37) are Euler equations. Now using budget constraint of household,

$$w_0 + \frac{w_1}{1+r} = (1+\tau_0^M)M_0 + \frac{(1+\tau_1^M)M_1}{1+r} + p_0 S_0 + \frac{p_1 S_1}{1+r}$$

Using (36) and (37),

$$w_0 + \frac{w_1}{1+r} = (1+\tau_0^M)M_0 + \frac{(1+\tau_0^M)M_0}{1+r} + p_0 S_0 + \frac{p_0 S_0}{1+r}$$

$$w_0 + \frac{w_1}{1+r} = \frac{2+r}{1+r}\left[(1+\tau_0^M)M_0 + p_0 S_0\right]$$

Replacing with equation (34),

$$w_0 + \frac{w_1}{1+r} = \frac{2+r}{1+r}\left[\frac{(1+\tau_0^M)M_0}{\alpha}\right]$$

Re-written $W = w_1 + w_2/(1+r)$. The optimal demand of M_0,

$$M_0 = \frac{\alpha W(1+r)}{(1+\tau_0^M)(2+r)} \tag{38}$$

Using (34) the optimal demand of S_0,

$$S_0 = \frac{(1-\alpha)W(1+r)}{p_0(2+r)} \tag{39}$$

For optimal M_1 and S_1 we use (36) and (37),

$$M_1 = \frac{\alpha W(1+r)}{(1+\tau_1^M)(2+r)} \tag{40}$$

$$S_1 = \frac{(1-\alpha)W(1+r)}{p_1(2+r)} \tag{41}$$

In another hand, the problem of government is maximizing utility and choose optimal τ_M. For this, the government internalizes all the relations of the economy (expressed in our system). Besides, we include condition (24),

$$\begin{aligned}
max \qquad & W = \log(M_0^{\alpha} S_0^{1-\alpha}) + \beta \log(M_1^{\alpha} S_1^{1-\alpha}) \\
s.t \qquad & \\
& \tau_0^M M_0 + \frac{\tau_1^M M_1}{1+r} = p_0 G + \frac{p_1 G}{1+r} \\
& \\
& (38), \quad (39), \quad (40) \;\; \& \;\; (41)
\end{aligned}$$

To solve this problem,

$$\mathcal{L} = W - \lambda \left[\tau_0^M M_0 + \frac{\tau_1^M M_1}{1+r} - p_0 G - \frac{p_1 G}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{\tau_0} = -\alpha \frac{(1+\tau_0^M)}{(1+\tau_0^M)^2} - \lambda \frac{\alpha W(1+r)}{(1+\tau_0^M)^2(2+r)} = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}}{\tau_1} = -\beta \; \alpha \frac{(1+\tau_1^M)}{(1+\tau_1^M)^2} - \frac{\lambda}{1+r} \frac{\alpha W(1+r)}{(1+\tau_1^M)^2(2+r)} = 0 \tag{43}$$

Dividing (42) and (43),

$$\frac{1}{\beta}\frac{(1+\tau_1^M)}{(1+\tau_0^M)} = \frac{1}{1+r}\frac{(1+\tau_1^M)^2}{(1+\tau_1^M)^2}$$

$$(1+\tau_0^M) = (1+\tau_1^M)$$

$$\tau_0^M = \tau_1^M \tag{44}$$

Using the net present value of government budget constraint,

$$\tau_0^M M_0 + \frac{\tau_1^M M_1}{1+r} = G\left(\frac{2+r+g_M}{1+r}\right)$$

Now using (38) and (40),

$$\tau_0^M \frac{\alpha W(1+r)}{(1+\tau_0^M)(2+r)} + \tau_1^M \frac{\alpha W(1+r)}{(1+\tau_0^M)(2+r)(1+r)} = G\left(\frac{2+r+g_M}{1+r}\right)$$

With (44),

$$\tau_0^M \frac{\alpha W(1+r)}{(1+\tau_0^M)(2+r)}\left(\frac{2+r}{1+r}\right) = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\tau_0^M \frac{\alpha W}{(1+\tau_0^M)} = G\left(\frac{2+r+g_M}{1+r}\right)$$

Thus,

$$\tau_0^M \frac{\alpha A_0}{(1+\tau_0^M)}\left(\frac{2+r+g_M}{1+r}\right) = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\tau_0^M \frac{\alpha A_0}{(1+\tau_0^M)} = G$$

So the optimal tax rate

$$\tau_0^M = \tau_1^M = \frac{G}{\alpha A_0 - G} \tag{45}$$

For the case of "unaware" government, we use (38). So in the period 0

$$\tau_0^M \frac{\alpha W(1+r)}{(1+\tau_0^M)(2+r)} = G$$

Therefore, the tax rate for each period,

$$\tilde{\tau}_0^M = \frac{G}{\alpha A_0 \gamma_t - G} \tag{46}$$

With $\gamma_0 = (2+r+g_M)/(2+r) > 1$ and $\gamma_1 = (2+r+g_M)/(2+r)(1+r) < 1$.

A.3.5 Income tax structure

The problem for household is,

$$\begin{aligned}
max \quad & U = \log(M_0^\alpha S_0^{1-\alpha}) + \beta \log(M_1^\alpha S_1^{1-\alpha}) \\
s.t \quad & \\
& w_0(1-\tau_0^w) + \frac{w_1(1-\tau_1^w)}{1+r} = M_0 + \frac{M_1}{1+r} + p_0 S_0 + \frac{p_1 S_1}{1+r}
\end{aligned}$$

To solve this problem and with restriction of equation (11),

$$\mathcal{L} = U + \lambda \left[w_0(1-\tau_0^w) + \frac{w_1(1-\tau_1^w)}{1+r} - M_0 - \frac{M_1}{1+r} - p_0 S_0 - \frac{p_1 S_1}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{M_0} = \frac{1}{M_0^\alpha S_0^{1-\alpha}} \alpha M_0^{\alpha-1} S_0^{1-\alpha} - \lambda = 0 \tag{47}$$

$$\frac{\partial \mathcal{L}}{S_0} = \frac{1}{M_0^\alpha S_0^{1-\alpha}} (1-\alpha) M_0^\alpha S_0^{-\alpha} - \lambda p_0 = 0 \tag{48}$$

$$\frac{\partial \mathcal{L}}{M_1} = \beta \ \frac{1}{M_1^\alpha S_1^{1-\alpha}} \alpha M_1^{\alpha-1} S_1^{1-\alpha} - \lambda \frac{1}{1+r} = 0 \tag{49}$$

$$\frac{\partial \mathcal{L}}{S_1} = \beta \ \frac{1}{M_1^\alpha S_1^{1-\alpha}} (1-\alpha) M_1^\alpha S_1^{-\alpha} - \lambda \frac{p_1}{1+r} = 0 \tag{50}$$

Using (47) and (48) or (49) and (50),

$$\frac{\alpha S_0}{(1-\alpha)M_0} = \frac{1}{p_0} \tag{51}$$

$$\frac{\alpha S_1}{(1-\alpha)M_1} = \frac{1}{p_1} \tag{52}$$

Equation (51) and (52) represent the optimal allocation of consumption for the household for each period. Now, using (48) and (50)

$$\frac{1}{\beta}\frac{S_1}{S_0} = (1+r)\frac{p_0}{p_1}$$

With $\beta = 1/1+r$,

$$p_1 S_1 = p_0 S_0 \tag{53}$$

Similar using (47) and (49),

$$M_0 = M_1 \tag{54}$$

(53) and (54) are Euler equations. Now using budget constraint of household,

$$w_0(1-\tau_0^w) + \frac{w_1(1-\tau_1^w)}{1+r} = M_0 + \frac{M_1}{1+r} + p_0 S_0 + \frac{p_1 S_1}{1+r}$$

Using (53) and (54),

$$w_0(1-\tau_0^w) + \frac{w_1(1-\tau_1^w)}{1+r} = M_0 + \frac{M_0}{1+r} + p_0 S_0 + \frac{p_0 S_0}{1+r}$$

Replacing with equation (51),

$$w_0(1-\tau_0) + \frac{w_1(1-\tau_1)}{1+r} = \frac{2+r}{1+r}\left[\frac{p_0 S_0}{1-\alpha}\right]$$

Re-written $\widetilde{W} = w_0(1-\tau_0) + w_1(1-\tau_1)/(1+r)$. The optimal demand of S_0,

$$S_0 = \frac{(1-\alpha)\widetilde{W}(1+r)}{p_0(2+r)} \tag{55}$$

Using (51) the optimal demand of M_0,

$$M_0 = \frac{\alpha \widetilde{W}(1+r)}{(2+r)} \tag{56}$$

For optimal S_1 and M_1 we use (53) and (54),

$$S_1 = \frac{(1-\alpha)\widetilde{W}(1+r)}{p_1(2+r)} \tag{57}$$

$$M_1 = \frac{\alpha \widetilde{W}(1+r)}{(2+r)} \tag{58}$$

The problem of government is maximizing utility and choose optimal τ^w. For this, the government internalizes all the relations of the economy (expressed in our system).

$$\begin{aligned} max \qquad & W = \log(M_0^{\alpha} S_0^{1-\alpha}) + \beta \log(M_1^{\alpha} S_1^{1-\alpha}) \\ s.t \qquad & \\ & \tau_0^w w_0 + \frac{\tau_1^w w_1}{1+r} = p_0 G + \frac{p_1 G}{1+r} \\ & \\ & (55), \quad (56), \quad (57) \quad \& \quad (58) \end{aligned}$$

To solve this problem,

$$\mathcal{L} = W - \lambda \left[\tau_0^w w_0 + \frac{\tau_1^w w_1}{1+r} - p_0 G - \frac{p_1 G}{1+r} \right]$$

F.O.C

$$\frac{\partial \mathcal{L}}{\tau_0} = -\frac{w_0}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \left(\frac{2+r}{1+r} \right) \frac{1}{\widetilde{\widetilde{W}}} \left[p_0^{1-\alpha} - \beta p_1^{1-\alpha} \right] - \lambda w_0 = 0 \tag{59}$$

$$\frac{\partial \mathcal{L}}{\tau_1} = -\frac{w_1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+r)} \left(\frac{2+r}{1+r} \right) \frac{1}{\widetilde{\widetilde{W}}} \left[p_0^{1-\alpha} - \beta p_1^{1-\alpha} \right] - \lambda \frac{w_1}{1+r} = 0 \tag{60}$$

With (59) and (60) is possible to determine that there are multiple solutions of tax rate τ^w. For example, one solution is $\tau_0^w = \tau_1^w$. Using the net present value of government budget constraint,

$$\tau_0^w w_0 + \frac{\tau_1^w w_1}{1+r} = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\tau_0^w \left(w_0 + \frac{w_1}{1+r}\right) = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\tau_0^w \left(A_0 + \frac{A_0(1+g_M)}{1+r}\right) = G\left(\frac{2+r+g_M}{1+r}\right)$$

$$\tau_0^w A_0 \left(\frac{2+r+g_M}{1+r}\right) = G\left(\frac{2+r+g_M}{1+r}\right)$$

Therefore,

$$\tau_0^w = \tau_1^w = \frac{G}{A_0} \tag{61}$$

A.3.6 Optimal tax rate

Proof. To prove that $\tau_0^S > \tau_1^S$ in optimum,

$$G(1+g_M)(2+r) + (1-\alpha)A_0 g_M > G(1+g_M)(2+r) - (1-\alpha)A_0 g_M(1+r)$$

$$(1-\alpha)A_0 g_M > -(1-\alpha)A_0 g_M(1+r)$$

$$0 > -(1+r)$$

This inequality is satisfied unrestrictedly. □

Proof. To prove that $\tau_t^M > \tau_0^S$ in optimum,

$$G(1+g_M)(2+r)(1-2\alpha) > A_0(1-\alpha)\alpha g_M$$

This inequality is satisfied unrestrictedly due of $(1-\alpha)\alpha g_M \simeq 0$. □

A.3.7 Optimal fiscal revenues

For tax on services and with equations (20) and (28):

$$\tau_0^S S_0 = \left(\frac{2+r+g_M}{2+r}\right) \frac{G(1+g_M)(2+r)+(1-\alpha)A_0 g_M}{(1+g_M)(2+r)} \tag{62}$$

Similar with (22) and (29):

$$\tau_1^S S_1 = \left(\frac{2+r+g_M}{2+r}\right) \frac{G(1+g_M)(2+r)-(1-\alpha)A_0 g_M(1+r)}{(1+g_M)(2+r)} \tag{63}$$

Proof. To prove that $\tau_0^S S_0 > \tau_1^S S_1$ in optimum,

$$G(1+g_M)(2+r)+(1-\alpha)A_0 g_M > G(1+g_M)(2+r)-(1-\alpha)A_0 g_M(1+r)$$

$$(1-\alpha)A_0 g_M > -(1-\alpha)A_0 g_M(1+r)$$

$$0 > -(1+r)$$

This inequality is satisfied unrestrictedly. □

For tax on manufactured goods and with equations (38), (40) and (45):

$$\tau_0^M M_0 = \tau_1^M M_1 = \left(\frac{2+r+g_M}{2+r}\right) G \tag{64}$$

Proof. To prove that $\tau_0^S S_0 > \tau_t^M M_t$ in optimum,

$$G(1+g_M)(2+r)+(1-\alpha)A_0 g_M > G(1+g_M)(2+r)$$

$$(1-\alpha)A_0 g_M > 0$$

This inequality is satisfied unrestrictedly. □

Proof. To prove that $\tau_t^S M_t > \tau_1^S S_1$ in optimum,

$$G(1+g_M)(2+r) > G(1+g_M)(2+r)-(1-\alpha)A_0 g_M(1+r)$$

$$0 > -(1-\alpha)A_0 g_M(1+r)$$

This inequality is satisfied unrestrictedly. □

For the case of "unaware" government with (38), (40) and (46),

$$\tilde{\tau}_0^M M_0 = G \tag{65}$$

$$\tilde{\tau}_1^M M_1 = G(1 + g_M) \tag{66}$$

For tax on income and with equation (61) and equilibrium wage (6):

$$\tau_0^w w_0 = G \tag{67}$$

$$\tau_1^w w_1 = G(1 + g_M) \tag{68}$$

Proof. To prove that $\tau_t^S M_t > \tau_0^w w_0$ in optimum,

$$\left(\frac{2 + r + g_M}{2 + r}\right) G > G$$

$$G(2 + r + g_M) > (2 + r)G$$

$$Gg_M > 0$$

This inequality is satisfied unrestrictedly. □

Proof. To prove that $\tau_1^w w_1 > \tau_t^S M_t$ in optimum,

$$G(1 + g_M) > \left(\frac{2 + r + g_M}{2 + r}\right) G$$

$$G(1 + g_M)(2 + r) > (2 + r + g_M)GG$$

$$Gg_M(1 + r) > 0$$

This inequality is satisfied unrestrictedly. □

Proof. To prove that $\tau_1^S S_1 > \tau_1^w w_1$ and $\tau_0^w w_0 > \tau_1^S S_1$ in optimum due of,

$$(1 - \alpha)A_0 > G$$

This inequality is satisfied unrestrictedly. ☐

A.3.8 Optimal FSR

For service tax structure with equation (62) and expenditure,

$$FSR^S = \tau_0^S S_0 - G = G\left(\frac{g_M}{2+r}\right) + \left(\frac{2+r+g_M}{2+r}\right)\frac{(1-\alpha)A_0 g_M}{(1+g_M)(2+r)}$$

$$FSR^S = G\left(\frac{g_M}{2+r}\right) + \left(\frac{2+r+g_M}{2+r}\right)\frac{(1-\alpha)A_0 g_M}{(1+g_M)(2+r)} \tag{69}$$

Similar, for manufactured goods tax structure, with equation (64) and expenditure,

$$FSR^M = \tau_0^M M_0 - G = G\left(\frac{g_M}{2+r}\right)$$

$$FSR^M = G\left(\frac{g_M}{2+r}\right) \tag{70}$$

Finally, for income tax structure, with equation (66),

$$\tau_0^w w_0 - G = G - G = 0$$

$$FSR^w = 0 \tag{71}$$

This solution is similar for "unaware" government.

Proof. To prove that $FSR^S > FSR^M$ in optimum,

$$\left(\frac{2+r+g_M}{2+r}\right)\frac{(1-\alpha)A_0 g_M}{(1+g_M)(2+r)} > 0$$

This inequality is satisfy unrestrictedly. ☐

A.4 Sample countries

Australia	Spain	Korea
Austria	Finland	Netherlands
Belgium	France	Norway
Canada	UK	New Zealand
Switzerland	Grecce	Poland
Czech Republic	Hungary	Portugal
Chile	Israel	Slovakia
Germany	Italy	Sweden
Denmark	Japan	USA

Restrictions: (i) status of income (World Bank definition) (ii) population of 3 million (iii) hyperinflation and (iv) socialist countries who lived transition.

A.5 Summary of Variables

Type of Variable	Measure	Source
Total taxes	Current and in % of GDP	GFS - IMF
Disaggregated taxes	Current and in % of GDP	GFS - IMF
Tax rates	Corporate and VAT	OECD, Eurostat and tax offices
Gov. expenditure	Current and real, national accounts	AMECO and WDI
GDP	Real	WDI
G/Y	%	WDI

A.6 Basic Results for Sample: 2000 - 2014

Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
β^{ρ}	0.308***	0.308***	0.321***	0.321***	0.385***	0.387***
	(0.0407)	(0.0407)	(0.0415)	(0.0415)	(0.0344)	(0.0352)
β^{G}	0.373***	0.373***	0.388***	0.388***	0.373***	0.388***
	(0.0591)	(0.0591)	(0.0603)	(0.0603)	(0.0591)	(0.0603)
β^{T}	0.534***	0.542***	0.543***	0.553***	0.446***	0.467***
	(0.142)	(0.141)	(0.143)	(0.143)	(0.143)	(0.143)
β^{M}	**-0.147**	**-0.139**	**-0.165**	**-0.155**	**-0.312****	**-0.307****
Observations	372	372	356	356	372	356
Countries	28	28	28	28	28	28
Controls:						
Corporate Tax Rate	NO	YES	NO	YES	NO	YES
Consumption Tax Rate	NO	NO	YES	YES	NO	YES
Δ G/Y	NO	NO	NO	NO	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG not include control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according of sign interpretation. If is negative we want to reject a positive β^M and viceversa.

A.7 Basic Results for Sample: 2005 - 2014

Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
β^{ρ}	0.278***	0.278***	0.293***	0.293***	0.378***	0.379***
	(0.0471)	(0.0471)	(0.0476)	(0.0476)	(0.0403)	(0.0410)
β^{G}	0.309***	0.309***	0.323***	0.323***	0.309***	0.323***
	(0.0639)	(0.0639)	(0.0650)	(0.0650)	(0.0639)	(0.0650)
β^{T}	0.427***	0.431***	0.427**	0.434***	0.337**	0.343**
	(0.164)	(0.165)	(0.167)	(0.167)	(0.166)	(0.168)
β^{M}	**-0.159**	**-0.156**	**-0.188**	**-0.181**	**-0.345****	**-0.359****
Observations	258	258	248	248	258	248
Countries	28	28	28	28	28	28
Controls:						
Corporate Tax Rate	NO	YES	NO	YES	NO	YES
Consumption Tax Rate	NO	NO	YES	YES	NO	YES
Δ G/Y	NO	NO	NO	NO	YES	YES

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. Corporate and consumptions tax control only for tax equation. Include these in the other equations do not change the results. Equation ΔG not include control Δ G/Y for spurious and nonsignificant reasons. The test for β^M was for one tail hypothesis, according of sign interpretation. If is negative we want to reject a positive β^M and viceversa.

A.8 Evolution of Tax Mismatch without 2008-2009

Sample	Estimated β^M	Observations
1995-2014	-0.19*	382
1996-2014	-0.14	369
1997-2014	-0.15	353
1998-2014	-0.13	337
1999-2014	-0.12	321
2000-2014	-0.25**	305
2001-2014	-0.22*	288
2002-2014	-0.25*	267
2003-2014	-0.25*	244
2004-2014	-0.29**	221
2005-2014	-0.37**	197

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. All regressions include country and year fixed effects. β^M includes all controls.