SERIE DOCUMENTOS DE TRABAJO

FACULTAD ECONOMÍA Y NEGOCIOS
UNIVERSIDAD DE CHILE

DEPARTAMENTO DE ECONOMÍA

SDT 243

HETEROGENEOUS ENTREPRENEURS, CREATION AND DESTRUCTION: A SIMPLE MODEL

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Santiago, Jun. 2007

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Serie Documentos de Trabajo
N 243

Heterogeneous Entrepreneurs, Creation and Destruction: A Simple Model

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Resumen

Construimos una economía con empresarios heterogéneos en productividad, riqueza y tamaño, con un problema contractual con la banca derivado de un problema de acción escondida. Esta economía genera márgenes duales con firmas de alta productividad de empresarios pobres siendo restringidos por incentivos y firmas de baja productividad con empresarios ricos siendo restringidos por participación. Mostramos que el margen de partición compuesto por empresarios más pobres, pero de mayor productividad es más sensible a shocks, más beneficiado por mejorías institucionales en la contratación, más beneficiado por la apertura financiera, por el desarrollo financiero y por la provisión de apoyo técnico por parte del gobierno.

Palabras Clave:

Productividad, empresarios.

Abstract

We construct an economy with heterogeneous entrepreneurs in productivity, wealth and also size, with contracting originating from a hidden action problem. This economy generates dual margins with high productivity firms owned by poor entrepreneurs being incentive constrained and low productivity firms owned by richer entrepreneurs being participation constrained. We show that the incentive constrained margin composed of poorer but productive entrepreneurs will be more sensitive to shocks in this economy, more benefited by institutional improvements to contracting, financial opening, financial development and the provision of technical support by the government.

Key Words:

Productivity, entrepreneurs.

Heterogeneous Entrepreneurs, Creation and Destruction: A Simple Model*

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4th June 2007

Abstract

We construct an economy with heterogeneous entrepreneurs in productivity, wealth and also size, with contracting originating from a hidden action problem. This economy generates dual margins with high productivity firms owned by poor entrepreneurs being incentive constrained and low productivity firms owned by richer entrepreneurs being participation constrained. We show that the incentive constrained margin composed of poorer but productive entrepreneurs will be more sensitive to shocks in this economy, more benefited by institutional improvements to contracting, financial opening, financial development and the provision of technical support by the government.

*Comments and advice of the following economists are gratefully acknowledged: Andrea Repetto, Felipe Zurita, Francisco Gallego, Daron Acemoglu, Roberto Rigobon, Guido Lorenzoni, Olivier Blanchard, Daniel Paravisini and Vernica Rappoport. The usual disclaimer applies.

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1 Introduction

The view that most economies have a sector of fragile entrepreneurs that own small firms or have small entrepreneurial plans, and require the support of the governments to exist, has appeared frequently in development economics. These small enterprises are seen as an important mechanism for social mobility and equal opportunity, and their failure and fragility is usually seen as a social defect of a market economy. The theoretical underpinning for such a failure was first laid out by (**?**) and has since become mainstream economic theory, with important practical applications such as the microlending experiences.

On the other hand, the existence of a dynamic churning process, were small firms try out new ideas, and are either selected to survive or, when failing, scraped and their resources recycled into new projects, is seen by macroeconomists as a virtue of a dynamic and modern economy. This view has been with us since the inspiring work of (**?**), and it has had a recent expression in the work of (**?**) and (**?**). The failure of an economy to churn resources to truly productive projects is seen by them as an efficiency defect of a modern market economy: what they call economic sclerosis.

Both views are perfectly compatible and probably coexist in a world of heterogeneous entrepreneurs. We will argue that both views are different faces of the same problem: the existence of a sensitive margin of fragile but very productive entrepreneurs. In this paper we will understand "fragility" not only as the possibility of a firm to go bankrupt or to close, but also as the possibility that a competent entrepreneur will not be able to implement a productive and socially desirable project. We will show how a standard limited liability hidden action model can deliver socially and economically meaningful multiple margins that react heterogeneously to shocks and policy innovations.

2 A Simple Model of Creation and Destruction in the Presence of Heterogeneity

In this section we set up an economy with a typical hidden action information asymmetry and heterogeneous entrepreneurs in wealth and in the quality of their entrepreneurial ideas. We show that in an economy such as this, the presence of an information asymmetry will generate dual margins with differential sensitivity to shocks. We show that shocks tend to impact most fiercely on a margin composed of poor entrepreneurs with high quality projects, and less on a margin of rich entrepreneurs with low quality projects. The consequence of the model is that the same market failure that scleroses the economy, minimizes the social and distributive impact of entrepreneurship, by creating a fragile margin of poor but highly productive entrepreneurs.

The most relevant methodological aspect of the model is the mapping of the econ-

omy into a partition representing types of entrepreneurs that are subject to different contracts and interest rates. Also, the margins of these partitions may be constituted either by incentive constrained or participation constrained firms. This difference in the nature of the active constraint determines differential sensibilities of these margins to different shocks. The same basic methodological mechanism can be found in a corporate structure model by (**?**).

The logic of our model is best understood comparing two entrepreneurs: a rich entrepreneur with a low quality project and almost enough resources to finance it himself, and a poor entrepreneur with a high quality project and almost no financial resources. From the bank's perspective the rich entrepreneur can be trusted to exert effort since his incentives are almost totally aligned with the project. The big question in lending to him will be how high an interest rate can be charged without the entrepreneur abandoning the project for the outside option. Hence, marginal rich entrepreneurs will be *participation constrained*. On the other hand, enforcing effort is the central issue when lending to the poor entrepreneurs. Hence, the bank will demand higher productivity levels and/or higher interest rates to lend to theses entrepreneurs. They will be *incentive constrained*. In this way, a heterogeneous economy will present dual margins.

The *incentive constrained* margin will be more sensitive to shocks and institutional innovations. The intuitive reason is that entrepreneurs on this margin rely on their ability to extract productivity from the economy to obtain credit from the banks. A deterioration of this ability will be very serious for them. On the other hand the wealthy *participation constrained* entrepreneurs rely mostly on their resources to credibly promise effort. Hence, they will be less sensitive to shocks and institutional innovations.

We explore two extensions an generalizations of the model. First, we find that in this context most risk sources will end up depleting the economy of wealthy entrepreneurs with relatively low productivity projects and increasing interest rates for poor entrepreneurs with relatively high productivity projects. The logic behind this is that *participation constrained* entrepreneurs will be comparing projects inside the risky economy, with projects outside it, while *incentive constrained* entrepreneurs will be in effect comparing two alternative projects inside the risky economy (the one with shirking and the one without it).

Second, we find that a model in which we add an additional source of heterogeneity: scale or size, does replicate the stylized fact that fragility is mostly seen among small firms. The intuition is that as long as effort is not completely proportional to size (as long as there is a fixed cost), scale will substitute for productivity. Hence, larger firms will be less sensitive to shocks and we will observe lower interest rates and more productive projects with increased scale.

This model is clearly connected to the volatility and credit market incompleteness literature pioneered by (**?**) and further developed in papers such as (**?**), (**?**) and

(**?**). In any case, the objective of most of this literature is to show how the market failure increases macroeconomic volatility. In this section we take this for granted and explore the distributive and efficiency aspects of macroeconomic fluctuations in economies with incomplete markets of the same sort but with economic and socially meaningful heterogeneity.

The model is also related somewhat with the credit monetary transmission literature studied empirically by papers such as Gertler and Gilchrist (1994) and (**?**)), also surveyed and documented by (**?**). Again this literature seeks usually to quantify the importance of these constraints and uses heterogeneity in size as an instrument that approximates financial constraints. Towards the end of the paper we show that using size as an instrument in this way is consistent with our model. However, we wish to explain the heterogeneous reactions by understanding how the characteristics of firms and entrepreneurs affect contracting rather than use the characteristics as an instrument.

The model is also somewhat related to the liquidity shortages and financial crises literature developed by (**?**) and (**?**) for balance of payments crises in emerging markets, and by (**?**) and (**?**) in a more general financial context. This literature emphasizes financial transmission of shocks towards firms with illiquid assets, and particularly the effect of illiquidity on financial prices such as the exchange rate or the stock market. One could extend the conclusions of this literature to construct a plausible explanation for the higher sensitivity to shocks and the fragility of smaller or poorer entrepreneurs. We do not use these types of models in our paper, our financial market is extremely simplistic, but we view our explanation as complementary rather than competing with this one.

The section is organized in the following way: subsections 2.1 to 2.1.4 present the model and develops the main results of the paper, the mapping of the economy into different types of contracts and the differential sensitivities of different margins to shocks and institutional innovations. Subsection 2.2 explores the effects of international recessions, international interest rate hikes, capital controls and financial liberalizations, financial development, improvements to the contracting environment, microfinance and technical support. Section 2.3 presents extensions to the main model, the introduction of risk and the introduction of size heterogeneity.

2.1 Dual Economy with Heterogeneity

2.1.1 Setup of the Economy

Consider the following static hidden action problem in the midst of competitive markets (for banking and output) and risk neutral individual agents (firms and banks). Firms live only to attempt the execution of a single entrepreneurial project, and banks live only to provide intermediated finance for these projects. There is a single good

in this economy that will serve as a numeraire. The output market is a spot market for this single good. The banking market is an intertemporal market for this good that lasts for one period.

Entrepreneurs can decide not to contract with the bank or the bank may decide that it is not willing to lend. In that case, the entrepreneur can deposit it's wealth outside of the economy at the risk free interest rate r. The banking sector is competitive and exogenous to this economy and demands the same risk free gross interest rate of r for riskless loans.

There is a continuum of agents with mass 1 that are heterogeneous in two dimensions: the proportion of internal funds ($0 \leq k \leq 1$) as a percent of the investment required for each project, and the quality of the project or productivity ($0 \leq \pi \leq 1$) as a percent of the aggregate productivity of the economy (W). Both k and π are observable to the banks and W will be assumed to be exogenous.

All entrepreneurial projects will require an investment of 1 and will be implemented if the agent is able to raise from the banks the $1-k$ that he lacks. Banks are risk neutral, face no quantum credit constraints but face limited liability when lending to entrepreneurs in this economy since agents have no wealth other than k. All projects have the following identical production function: with probability p_s it will render πW and with probability $1-p_s$ it will fail and render 0. Output is also sold in an exogenous competitive environment where the production of a firm of productivity π is sold for πW. The probability of success of the project will depend on an universally observable effort decision of cost c made by the entrepreneur. If the agent makes an effort he will have a success probability of p_h for his project, otherwise the success probability will be p_l which is lower. Hence expected gross output by a project of quality π will be

$$E[y] = \left\{ \begin{array}{ll} p_h \pi W - c & \text{if high effort is exerted} \\ p_l \pi W & \text{if low effort is exerted} \end{array} \right\} \tag{1}$$

2.1.2 Perfect Information

Call $A = \{\{H, L, O\}_i\} \subset \Re^2$ the set of all possible partitions of space $B = [0,1] \times [0,1]$ such that $H \cup L \cup O = B$ and $H \cap L \cap O = \emptyset$. Sets H, L and O describe collections of entrepreneurs described by a pair (π, k). H is the set of entrepreneurs that are financed to execute their project with a high level of effort, a set of firms that we will call Tier 1 from now on. L, is the set of entrepreneurs that are financed to execute their project with a low level of effort, and we will call Tier 2 henceforth. Finally set O, is the set of entrepreneurs that decide to invest outside the economy. Call $R = (R_h, R_l) \in \Re^2_+$ a vector describing the interest rate structure of this economy. Equilibrium in this economy is described by $(P, R) \in A \times \Re^2_+$ such that all entrepreneurs maximize their expected profits and banks comply with the zero profit free entry condition.

Hence, set $S \in \{H, L\}$ is the subset of entrepreneurs that, producing inside the economy, maximize their expected positive profits

$$U(s) = p_s(\pi W - (1-k)R_s) - kr - c_s \tag{2}$$

by choosing $s \in \{h, l\}$, where $\{c_h, c_l\} = \{c, 0\}$. Set O, on the other hand, is the subset of entrepreneurs that obtain negative results in (2) for any choice of s and choose to produce outside of the economy to obtain a risk free return of kr, and is characterized in the definition of A.

Scalars $\{R_s\}$ are the interest rates that are charged by the banks to entrepreneurs belonging to each set S. Under perfect information the level of effort exerted is universally observable, so banks will finance all projects that accept to exert effort S and pay rate R_s such that zero expected profit condition

$$R_s p_s = r \tag{3}$$

is satisfied for each penny loaned. Hence, equation (3) fully characterizes equilibrium interest rates in this economy.

We can characterize the first best perfect information equilibrium in the following way: firms can only be expected to sign the high effort contract if they have drawn π larger than:

$$\pi_{pch} = \frac{(r+c)p_h^{-1}}{W} \tag{4}$$

so we will refer to threshold (4) as the *high effort participation compatibility constraint* or PCH henceforth. On the other hand, firms can only be expected to sign the low effort contract if they have drawn π larger than:

$$\pi_{pcl} = \frac{rp_l^{-1}}{W} \tag{5}$$

so we will refer to threshold (5) as the *low effort participation compatibility constraint* or PCL henceforth. Firms can only be expected to prefer the low effort contract to the high effort one if they have drawn π smaller than:

$$\pi_{bl} = \frac{c(p_h - p_l)^{-1}}{W} \tag{6}$$

so we will refer to threshold (5) as the *low effort boundary* or BL henceforth.

Like all other results and proofs in section 2, equations (4)-(6) are derived and developed in A-1, and are simply the development of (2) subject to (3).

Sets H are the lightly shaded areas in Figure 1 and sets L are the darker shaded areas. Under perfect information, firms are solely discriminated according to their productivity and the level of internal finance or the capital owned by the entrepreneur is totaly irrelevant.

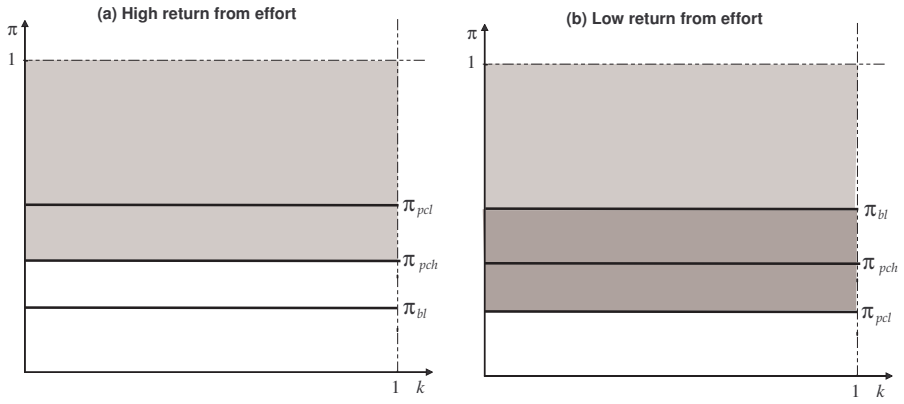

Figure 1: First Best Economy

We can have two types of economy. Panel (a) of Figure 1 represents the case where $\pi_{pcl} \geq \pi_{pch} \geq \pi_{bl}$, which holds as long as $\frac{p_h - p_l}{p_l} \geq \frac{c}{r}$. Panel (b) represents the case where $\pi_{bl} \geq \pi_{pch} \geq \pi_{pcl}$, which holds as long as $\frac{p_h - p_l}{p_l} \leq \frac{c}{r}$. Economies (a) and (b) both preserve their first best character and have no discrimination among firms by any criteria other than productivity. Economy (a) has only one tier of firms that all exert high effort and, from (3), pay rp_h^{-1} for every penny they are loaned. On the other hand economy (b) has two tiers of firms. In addition to the tier 1 firms of economy (a), there is a second tier of firms that will produce inside the economy but not exert effort and, from (3), pay rp_l^{-1} for every penny they are loaned.

Ratio $\frac{p_h - p_l}{p_l}$ is a measure of the relative gain in success probability that is derived from effort, since it compares the increase in success probabilities derived from effort with the success probability that the project will have if no effort is exerted. Ratio $\frac{c}{r}$ is a measure of the relative cost of effort since it compares the cost of effort with the alternative use of those resources. As long as the gain derived from effort is higher than the cost, we will be in economy (a) with only one tier. When the gain is lower than the cost we can have a two tier (b) economy, where only very high productivity firms, that have, in a sense, a lot of productivity to protect with their effort will choose high effort finance. Also, among the Tier 2 firms there will be two types. Firms in the $\pi_{pch} - \pi_{pcl}$ range do not derive positive expected returns from effort, but can make a profit executing without effort. On the other hand firms in the $\pi_{bl} - \pi_{pch}$ range expect to derive profits from a low or high effort execution of their project. However, since their productivity is low, when they compare the expected gain from effort with the cost, they prefer low effort finance.

During the next subsections we will assume that effort is relatively important, so that $\frac{p_h - p_l}{p_l} \geq \frac{c}{r}$ and that the first best version of the economy that we distort with a market failure will be the one tier economy represented in panel (a) of Figure 1.

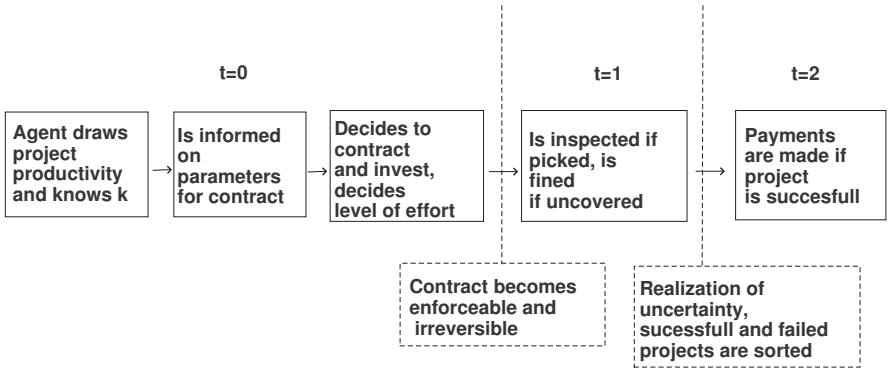

Figure 2: Timing of Events

2.1.3 Setup of the Market Failure and the Contract

There are three features that make this a plausible model for an emerging economy. First, it is an economy that depends entirely on international capital markets to finance the implementation of it's most profitable projects (mass 1 of entrepreneurs has $k < 1$). Second it is a financially open economy that faces an exogenous risk free rate (r). Third, there is relevant multidimensional heterogeneity including entrepreneurs with good ideas but very little capital (small k and high π). Two other characteristics of emerging markets that are useful to introduce are relatively high information costs and costly monitoring. These two characteristics will serve us to set up the market failure of this paper.

The hidden action problem comes from making effort non observable. Faced with this asymmetric information problem, we will assume that the exogenous banking sector will be able to make use of a costless inspection technology. With probability q it will inspect a firm expected to be expending effort, and will fine it by the amount f if it finds it not to be doing so. Both q and f are exogenous to this economy and are costlessly applied to each marginal firm. The contract signed by the entrepreneur is characterized by $\{k, R_s, q, f, s\}; s \in \{h, l\}$. The contract specifies the amount of the loan $(1-k)$, the rate of the loan $(R_s = rp_s^{-1}, s \in \{h, l\})$, the level of effort expected (h or l), the probability of inspection q and the level of the fine f.

The sequencing of events is presented in figure 2, and unfolds in the following way: an agent of capital k draws a project of productivity π, informs himself of the parameters of the contract available in the exogenous banking sector, decides to contract or not (an how) and the level of effort to exert. Once the contract is signed, the firm is subjected to the inspection draw and fined if uncovered to be shirking under a high effort contract. Then, uncertainty is realized, and successful and failed projects are sorted out. Successful projects repay the loan, failed projects do not pay

since there is limited liability.

2.1.4 Hidden Action

Equilibrium in this economy again is described by a pair $(P, R) \in A \times \Re_+^2$ just as in section 2.2, in which all entrepreneurs maximize their expected profits and banks comply with the zero profit free entry condition. Just as in the first best economy, entrepreneurs choose in what subset of the partition they prefer to be. However, since effort is not observable any more, they must choose considering the whole of the contract described in section 2.3. So the entrepreneurs problem is modified slightly from (2), it is now to maximize:

$$U(s,z) = p_s(\pi W - (1-k)R_z) - kr - c_s - qf(z,s) \tag{7}$$

by choosing actual effort $s \in \{h,l\}$ and promised effort $z \in \{h,l\}$, where $\{c_h, c_l\} = \{c, 0\}$ as before, but now also

$$f(z,s) = \left\{ \begin{array}{ll} f & \text{if } z \neq s \\ 0 & \text{if } z = s \end{array} \right\} \tag{8}$$

and since we have assumed the inspection technology of the banks to be free (very cheap or just rationed) banks will lend so that the zero profit condition (3) is satisfied for every penny lent.

We can characterize the imperfect information equilibrium equilibrium of this economy in the following way. Firms owned by entrepreneurs with capital k can credibly promise to exert effort if they have drawn π larger than:

$$\pi_{ich} = \frac{(1-k)rp_h^{-1} + (c-fq)(p_h - p_l)^{-1}}{W} \tag{9}$$

so we will refer to constraint (9) as the *high effort incentive compatibility constraint* or ICH henceforth. It is important to notice that equation (9) has a negative slope on the $\{\pi, k\}$ plane. Set H of Tier 1 firms will be bounded from below by the upper envelope of equations (9) and (4). Set L of Tier 2 firms (if existent) will be bounded from below by (5) and from above by the lower envelope of the lower bound of the aforementioned Tier 1 set.

Figure 3 shows the effect of the market failure on this economy. Consider panel (a), two notorious things happen as a result of non observable effort. First, a whole section of the economy (triangle ACD) looses it's access to high effort interest rates. The losers tend to be highly productive firms that are the property of poorer entrepreneurs. The most productive among these losers (subtriangle ABE), manage to stay eligible for low effort interest rates. These firms stop exerting effort but continue to produce, albeit with more expensive financing. Compared to the first best economy of Subsection 2.2, there are two sources of welfare loss: the entrepreneurs of rectangular

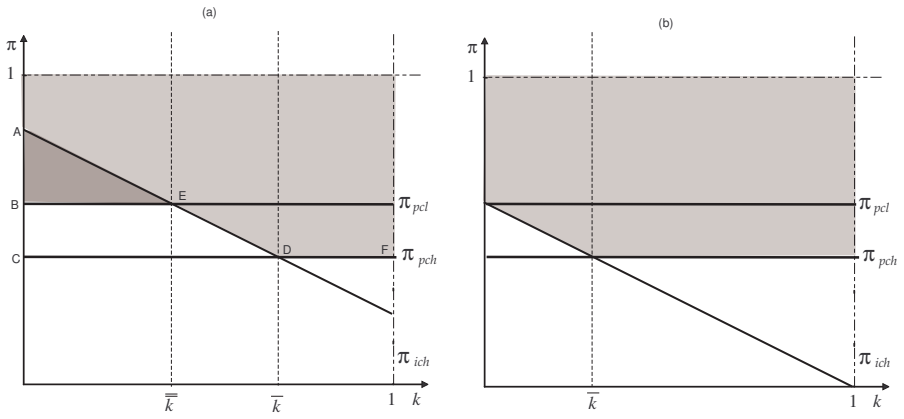

Figure 3: Dual Margins and Dual Economies

trapeze BCDE seize to produce inside the economy and the fall in expected profits for entrepreneurs of triangle ABE.

The information asymmetry has created an economy with a triple margin and dual tiers of firms. Margin DF is a participation constrained margin conformed of the least productive active entrepreneurs of the economy. These entrepreneurs are relatively wealthy and are able to secure low cost finance for their projects. Since they have abundant internal resources to invest in the project, their incentives are aligned with the project and the ICH constraint is not binding (they are betting mostly with their own money). Margin DE is a incentive constrained margin of less wealthier entrepreneurs that compensate this handicap with an increasing level of productivity. Margin BE is a participation constrained margin conformed of the most productive marginal entrepreneurs of this economy. These entrepreneurs are relatively poor and are able to secure only high cost finance for their projects. Finally, margin EA is an internal margin of this economy, a frontier between Tier 1 and Tier 2 projects (and consequently high or low interest rates). It is a incentive constrained margin composed of projects of the highest quality owned by the poorest entrepreneurs. Generally speaking, however, this economy could look like the one in panel (b) where there is no Tier 2 and the economy only has a dual margin. In subsection 2.4 below, we discuss what can move an economy from looking like (a) to looking like (b).

Notice that the intersection of the ICH and the PCH occurs at:

$$\bar{k} = \left(\frac{c}{r}\right)\left(\frac{p_l}{p_h - p_l}\right) - \left(\frac{fq}{r}\right)\left(\frac{p_h}{p_h - p_l}\right) \tag{10}$$

so the sufficient condition for this economy to have a dual margin is that the relative benefit of effort $\frac{p_h - p_l}{p_l}$ is larger than the cost of effort $\frac{c}{r}$. This is the same condition that we needed in section 2.2 to have a first best economy with only one tier like the

one represented in panel (a) of Figure 1. Hence, under these conditions the existence of a second tier and the existence of a dual margin can both be attributed to the market failure that results in the imposition of the ICH constraint. More generally, if the effect of effort were not so large, and our first best baseline were panel (b) of Figure 1, $\bar{k}$ could be larger than 1 or not. In any case, the effect of the market failure would be to create a dual margin between tiers and to deny low cost finance to a triangle of highly productive yet relatively poor entrepreneurs.

Also note that introducing a more general effort function does not alter qualitatively the model, but does change the geometry and reduce it's elegance. Consider a function $c(\pi)$ such that more productive projects require more effort. Equations (4)-(6) will not be affected in their forms (they will continue to be flat) but may be affected in their sensitivity to parameters. On the other hand, equation (9) will cease to be a line. If the effort function is concave (which is the most reasonable assumption), then the ICH will be convex, so that incentive compatibility does not relax very much with grater k, but does so with greater π. Conversely the ICH will be concave if the effort function is convex. So, the model is not altered substantially, as it will not if we introduce more general inspection effectiveness functions $f(\pi)$.[1]

In the following subsections, we will do comparative statics on the three margin two tier economy of panel (a) in Figure (3).

2.2 Comparative Statics of the Dual Economy Model

2.2.1 Capital Flows and Capital Controls

We can represent capital controls in this economy as taxes either on capital inflows or outflows. A tax on inward capital flows will also be equivalent to a shock to the idiosyncratic rates that local firms face in international capital markets.[2] Conversely we can represent a capital control on outward flows as a tax that diminishes the profitability of investing in the risk free outside option. So the entrepreneurs problem is again modified slightly from (7), it is now to maximize:

$$U(s,z) = p_s\left(\pi W - (1-k)(1+t_{in})R_z\right) - kr(1-t_{out}) - c_s - qf(z,s) \qquad (11)$$

by choosing actual effort $s \in \{h,l\}$ and promised effort $z \in \{h,l\}$, where $\{c_h, c_l\} = \{c, 0\}$ and

[1] The slope of the ICH from equation (9) with a general function $c(\pi)$ rather than c is: $\frac{\partial \pi}{\partial k} = -\frac{rp_h^{-1}}{W - \frac{\partial C}{\partial \pi}}$ It is negative, and becomes less negative with π if the effort cost function is concave.

[2] A shock could be a diminishment in the international taste for assets in this economy, such as shifts to quality in international capital markets that drives lenders away from emerging markets, or an expected devaluation of the currency for some reason exogenous to our model.

$$f(z,s) = \left\{ \begin{array}{ll} f & \text{if } z \neq s \\ 0 & \text{if } z = s \end{array} \right\} \tag{12}$$

where t_{out} is the tax on capital outflows and t_{in} is the tax on capital inflows. Banks will continue to lend so that the zero profit condition (3) is satisfied for every penny lent.

The new thresholds are

$$\begin{aligned} \pi_{ich} &= \frac{(1-k)(1+t_{in})rp_h^{-1} + (c-fq)(p_h - p_l)^{-1}}{W} \\ \pi_{pch} &= \frac{((1-t_{out})r + c - rk(t_{out}+t_{in}))p_h^{-1}}{W} \\ \pi_{pcl} &= \frac{((1-t_{out})r - rk(t_{out}+t_{in}))p_l^{-1}}{W} \end{aligned} \tag{13}$$

and are derived in A-1.

An ad-valorem tax of rate t either on outward or inward capital flows will drive a wedge between the passive and active interest rates faced by this economy.[3] Geometrically this will cause both participation constraints to acquire a slope since the alternative cost of funds will now be different from the marginal cost of credit that firms face.

The inflow tax will be increasingly costly for poorer entrepreneurs, since a larger proportion of their investment will be financed with credit. The outflow tax (levied only on residents), on the other hand, will usher investment into the economy by reducing the returns of the outside option. This effect will be stronger for richer entrepreneurs that actually have funds to invest in the outside option. Moreover, the PCL constraint will become steeper than the PCH constraint for the same tax, since the tax is not affecting the cost of effort which is an important part of the high effort participation choice but is not part of the low effort participation choice. On the other hand, the effect on the participation constraint is totally different in one case or the other. The incentive constraint compares two options for executing the project inside the economy considering the cost of effort. The tax on capital inflow makes the return from effort lower. The tax on capital outflow does not affect this constraint, rather, it affects the constraints that compare an option inside the economy with an option outside the economy. Hence, the ICH constraint will become more restrictive in the case of a tax on inflows and will not change in the case of a tax on outflows.

[3]Here, for purely expositional purposes we assume that the a capital flow that has been charged when coming in is not charged when it goes back out (when credit is settled). Moreover, conceptually a tax on capital outflows is a tax on all capital flows that are relevant for this economy. So the capital outflow tax that we present is a capital outflow tax on residents in the economy.

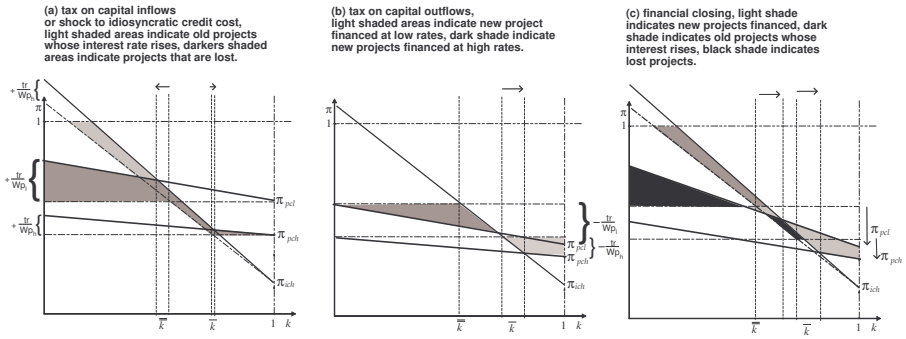

Figure 4: Capital Controls, Taxes on Capital Flows or Shocks, and Closing the Economy Financially

Figure (4) shows the effects of taxes on capital flows on this economy. Panel (a) shows the effect of a tax on capital inflows. The lighter shaded area represents Tier 1 firms that are downgraded to Tier 2 and consequently charged a higher interest rate. The darker shaded area indicates firms that are no longer financed. As a result of the pivoting of the constraints that was discussed in the previous paragraph, the majority of the adverse effects are on poorer entrepreneurs with projects of relatively high productivity.

Panel (b) shows the effect of a tax on capital outflows. The darker shaded area indicates new projects that are financed as Tier 2, and the lighter shaded area indicates new projects that are financed as Tier 1. As a result of the pivoting of the constraints that was discussed in the previous paragraph, new low quality projects are financed among relatively rich entrepreneurs, although they would have preferred to invest abroad.

Finally panel (c) indicates an economy that closes financially, that is, that levies a tax on all capital outflows or, equivalently, a tax on inflows from non-residents and a tax on outflows from residents. The light shaded areas indicate new firms that come into existence as a result of the financial closing of the economy. The darker shaded area indicates Tier 1 firms that are downgraded to Tier 2, and the black areas indicate firms that loose access to finance. The result of the financial closure of this economy, is a transfer of finance from high quality projects owned by poor entrepreneurs to low quality projects owned by rich entrepreneurs, and an aggregate loss of welfare.

The effect of capital flow taxes in this model can be interpreted to represent the effects of any form of imperfect or costly access to international lending by domestic entrepreneurs. Quantum restrictions such as capital controls and quantum total lending constraints faced by countries can be interpreted as a convex tax function $t(L)$ that is a function of total lending to the economy L.[4] In this case, equilibrium

[4]A microfoundation of how a country can end up sharing assets when dealing with international

continues to be described by the partition of entrepreneurs into three sets, plus an equilibrium tax rate, but it is no longer constrained efficient due to the assumed externality. A development of policy choices faced by the government of such an economy is developed in section 3.2 below.

In the following subsection, I will do comparative statics on a three margin two tier economy, such as the one I use in Figure (4) that faces some form of imperfect access to international lending. Hence, participation constraints will have a slope, as we have derived in this subsection.

2.2.2 Financial Policies and Development

Financial development can be represented in this model by an increase in the enforceability of the promise to expend high effort. Hence, it can be represented by an increase in the expected fine fq. This increase affects only the ICH constraint as is clear from inspecting equation array (13).

Panel (a) of Figure 5 shows the increase in the expected fine as a parallel relaxation of the ICH constraint. The light shaded area is the set firms that are upgraded from Tier 2 to Tier 1 status, the dark shaded area is the set of new projects that become eligible for finance. Three things are important to notice. First, the effect is concentrated among poorer entrepreneurs with higher quality projects. The reason for this is that entrepreneurs on this margin are "working with other peoples money" so their main problem is contracting and promising effort. Second, all the new projects are financed as Tier 1, since low effort contracting is not affected. Third, a negative shock to financial contracting, such as reduced enforceability, or even a credibility shock were international banks adjust downwards their belief about q, will have the same effects in reverse: concentrating it's effect on incentive constrained, poor, highly productive entrepreneurs. In the limit, by implementing policies that increase enforceability from abroad and transparency, economies will transit from the three margin two tier configuration of panel (a) of Figure 3 to the two margin one tier configuration of panel (b).

An alternative to increasing fq for the whole economy is to lower the technological cost of effort c. This is a very common type of policy towards poorer and smaller entrepreneurs. The effect of a subsidy to c is shown in panel (b) of Figure (5) and it's magnitudes are straightforward from equation array (13). Geometrically it is a parallel shift downward of all the constraints, although the effect will be different for each of them. There will be a large area of new firms financed as a result of the policy, an these firms will be of all qualities and types of owners. The effect, however, will be larger for poorer entrepreneurs for two reasons. First, there will be an area of poor but productive Tier 2 entrepreneurs that will be upgraded to Tier 1. Second,

credit markets can be found in the series of papers by Caballero and Krishnamurthy quoted in the References.

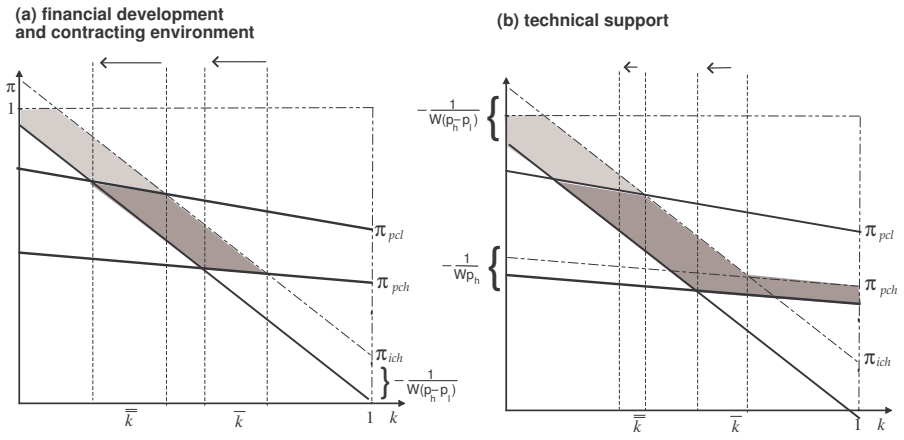

Figure 5: Financial Development and technical support

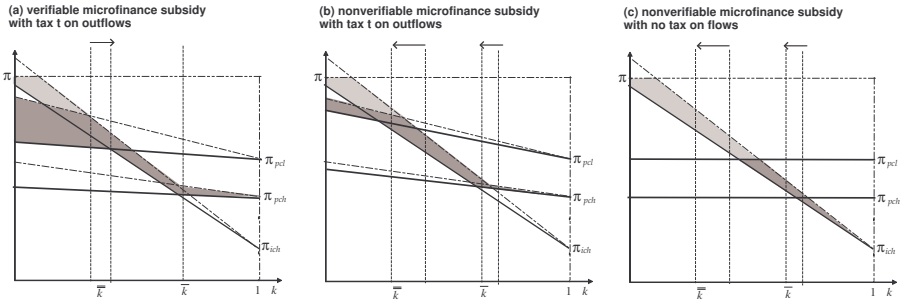

Figure 6: Microfinance

the effect of the reduced technological cost of effort on the ICH constraint will be larger than on the PCH constraint since promising effort is much more critical for incentive constrained entrepreneurs than for participation constrained entrepreneurs. Hence, even if technical support is not focalized on poorer entrepreneurs, it will end up having a larger effect on them since it is they who are on the incentive constrained margin.

Notice that a development policy that provides technical support to the projects, that is, that increases π for all firms (or for all that request it) will shift all margins down in the same distance, and hence relax the constraints of entrepreneurs of all k and π. This policy can, of course, be focalized on poorer entrepreneurs with higher quality projects. However, the policy of providing technical support to the effort associated with the contracting problem will naturally be focalized on poorer entrepreneurs that own higher quality projects.

Microfinance policies, usually consisting of small and cheap loans to poor entrepreneurs (usually to entrepreneurial cooperatives) can be introduced in a simple way into this model by assuming that the development agency gives a gift of $\alpha(1-k)$ to every entrepreneur of capital k. Unfortunately, the development agency will face a hidden action problem in verifying that entrepreneurs actually end up investing in the project and do not deposit the funds (incremented with the subsidy) in the risk free outside option. This turns out to be critical when evaluating the effects of this policy. Panel (a) of Figure 5 shows the effect on this economy of microfinance policy α when investment by the entrepreneur is verifiable by the development agency. All three constraints pivot in the same way that they would if the outflow tax were lowered. Moreover, the effect on the PCL constraint is larger than the effect on the PCH and ICH constraints (which are equal), once again because an increase in capital directly targets the whole problem that a low effort entrepreneur faces, while the high effort entrepreneurs problem with the cost of effort is not being helped directly.

In panel (a) we can see very high creation of high quality firms among poorer entrepreneurs. In panel (b) we make it impossible for the development agency to verify if the subsidized entrepreneur actually invested or just deposited its increased funds at rate r. The effect on participation constraint almost disappears (see A-1). In fact, in panel (b) at $k = 0$, the effect on both participation constraints will be smaller than in panel (a) by the proportion $t/(1+t)$. Panel (c) shows how the effect completely disappears when $t \to 0$ if investment is not verifiable by the subsidiser.

In the following subsection, we will do comparative statics on the simpler economy with no taxes to capital flows.

2.2.3 International Shocks

International recessions for this economy are a general fall in the aggregate gross productivity W from which firms extract with varying degrees of efficiency. Panel (a) of Figure (7) shows a the effect. As proven in A-1, the effect will be larger for marginal firms of higher productivity (the darker shade in the figure). Intuitively this is due to the fact that these firms depend almost completely on their capability of extracting from aggregate productivity to convince banks to lend to them. Moreover, they depend on their capacity to extract productivity to convince themselves to participate, so even the PCL constraint will contract significantly. In this case, there will be a relatively large mass of the most productive firms owned by the poorest entrepreneurs that will be downgraded from Tier 1 to Tier 2.

An international increase in interest rates is similar to the international recession in the way it moves the constraints, but different in the relative size of the adjustments of each margin. In this case (see A-1), the PCL margin adjust the most since it is these firms that are exclusively comparing productivity with outside options (with no role for effort). The ICH constraint will pivot on $k = 1$ since firms that finance

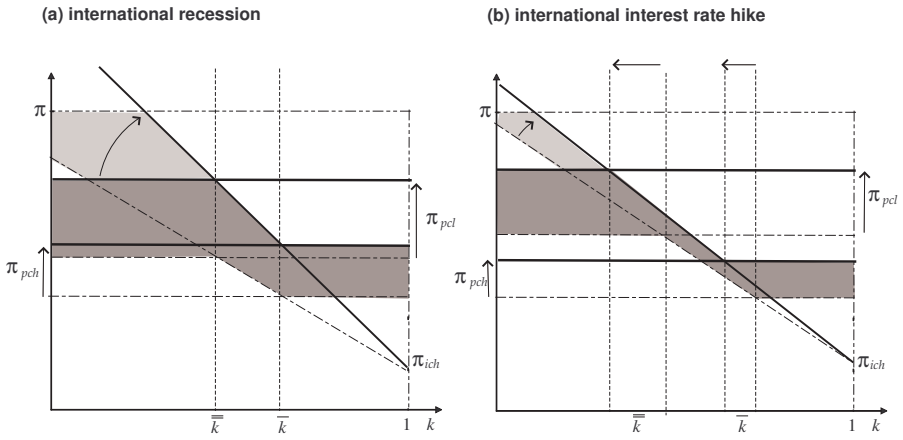

Figure 7: Capital Controls, Taxes on Capital Flows or Shocks, and Closing the Economy Financially

themselves with internal funds will not feel that the financial cost of effort has changed very much when the interest rate increases. Finally, as $k \to 0$ the effect on the ICH and PCH constraints becomes the same, since the absence of internal funds becomes the overriding problem in both deciding to participate and convincing creditors to loan at the low rate. In this case, once again, the most affected projects are high quality ones owned by poorer entrepreneurs. A nuance of this last shock is that projects of middle income entrepreneurs are affected less than those of richer entrepreneurs. This happens because the richest entrepreneurs are participation constrained and will move their funds to the outside riskless option very quickly. Middle income entrepreneurs don't have enough funds to make that option profitable. On the other hand, credit is not such a large proportion of their funding as for poorer entrepreneurs.

2.2.4 Summary

I have shown how a very simple static economy with heterogeneous entrepreneurs can have firms with dual access to credit (two tiers of financed firms and non financed firms), although projects are identical. I have shown how this can depend on the existence of a simple asymmetric information problem of the hidden action variety with the absence of externalities. I have shown how this economy can also display dual or even triple margins, and how these margins have different sensitivities to a variety of shocks and policies. In all cases, the effects of shocks (positive and negative) as well as different varieties of development policies are larger on the margin composed of poorer entrepreneurs. The central feature of this paper is that these poorer and more sensitive entrepreneurs happen to own the most productive marginal projects

in the economy.

2.3 Extensions and Generalizations, Risk and Size

2.3.1 Risk

Up to this point we have assumed that the entrepreneurs of this economy are risk neutral. Also, the banks do not face any risk since they hold portfolios composed of a continuum of identical entrepreneurial projects. In this section we will introduce different types of risk, and analyze how it affects the mapping of this economy into different credit contracts zones.

Call $\nu_x \sim \nu(\epsilon, x)$ any mean preserving shock that affects a parameter x of the economy with a standard deviation of ϵ. In order to remember them we can label the shocks in the following way: call ν_w domestic productivity risk, ν_r international credit risk, and ν_c domestic institutional risk. Two of these types of risk are domestically generated, and one is imported, but all are exogenous to the problem. Assume that entrepreneurs have a concave utility function $U(.)$. As a result the entrepreneurs problem is modified slightly from program (7). Now each entrepreneur has to maximize expected utility:

$$EU(s,z) = EU\left(\begin{array}{l} p_s\left(\pi(W+\nu_w) - (1-k)(r+\nu_r)p_z^{-1}\right) \\ -k(r+\nu_r) - c_s - qf(z,s) \end{array} \right) \tag{14}$$

by choosing actual effort $s \in \{h, l\}$ and promised effort $z \in \{h, l\}$, only now $\{c_h, c_l\} = \{c + \nu_c, 0\}$, and as before

$$f(z,s) = \left\{ \begin{array}{ll} f & \text{if } z \neq s \\ 0 & \text{if } z = s \end{array} \right\} \tag{15}$$

and, we are already assuming in equation (14), that the zero profit condition (3) is satisfied for every penny lent.[5] For the sake of simplicity, we have assumed that the cost of effort c_s is potentially subject to risk, and not the monitoring probability q or the fine function $f(z,s)$, since introducing the mean preserving shock to any of them delivers the same results.

We can characterize the imperfect information equilibrium equilibrium of this economy in a similar way to that of section 2.4. Firms owned by entrepreneurs with capital k and productivity π can credibly promise to exert effort if:

$$EU(l,h) \leq EU(h,h) \tag{16}$$

which is our incentive compatibility constraint. Firms will be willing to participate in high effort contracts only if:

[5] Instead of the term R_z that we had in equation (7) we have the explicit interest rate when the zero profit constraint is satisfied: rp_z^{-1}

$$0 \leq EU(h,h) \tag{17}$$

which is our high effort participation constraint PCH and they will be willing to participate in low effort contracts only if:

$$0 \leq EU(l,l) \tag{18}$$

which is our low effort participation constraint PCL.

Since all shocks are mean preserving and independent and the utility function is concave, all expected utilities will fall as a result of an amplification of any source of risk. The transition from an economy with no risk to one with risk will be a particular case of the amplification of ϵ. Hence, from equations (17) and(18) we see that both the PCL and the PCH will tighten as a result of an increase in any of the three types of risk sources, with the exception of domestic institutional risk ν_c that will not have an effect on the PCL constraint, since the enforcement of effort is not relevant for this type of contract.

It is interesting to note from equation (14) that the domestic productivity shock is amplified by π and p_s. This will mean two things. First, that the constraint will tighten more for higher productivity marginal firms. Second, that the PCH constraint will tighten more than the PCL constraint, since the shock has a grater amplification. The intuition is simple: entrepreneurs that would have contracted high effort credit contracts have more riding on productivity than others. Uncertainty on these return will discourage them from signing these contracts and committing to high effort. Also note that the international credit risk ν_r will have the same effect on both EU(h,h) and EU(l,l), and will have the same effect on entrepreneurs of all productivity levels and wealth. These effects are represented in Figure 8. Proofs can be found in A-2.

To see the effect of risk on the ICH constraint we must see if EU(l,h) or EU(h,h) tightens more with an the amplifications of ϵ. The simplest case is the amplification of domestic institutional risk (ν_c), since this random variable is in EU(h,h) (which will fall) but not in EU(l,h). Hence, an increase in domestic institutional risk will unambiguously tighten the ICH constraint, making it less possible to credibly commit not to shirk.

The amplification of international credit risk (ν_r) generates different effects on EU(h,h) and EU(l,h). In A-2, we prove that the ICH will pivot and tighten less than the PCH and PCL constraints. The intuition is similar to the one that is behind the existence of multiple margins in this economy. International credit risk has two effects on contracting: on the zero profit interest rate that is charged by banks, and on the returns on the outside option that is faced by entrepreneurs. Since entrepreneurs need to complete the investment of 1, they will either be paying an interest rate for whatever capital they borrow, or foregoing the interest rate that they would earn on the capital that they already have. Hence, the amount of capital that they have does

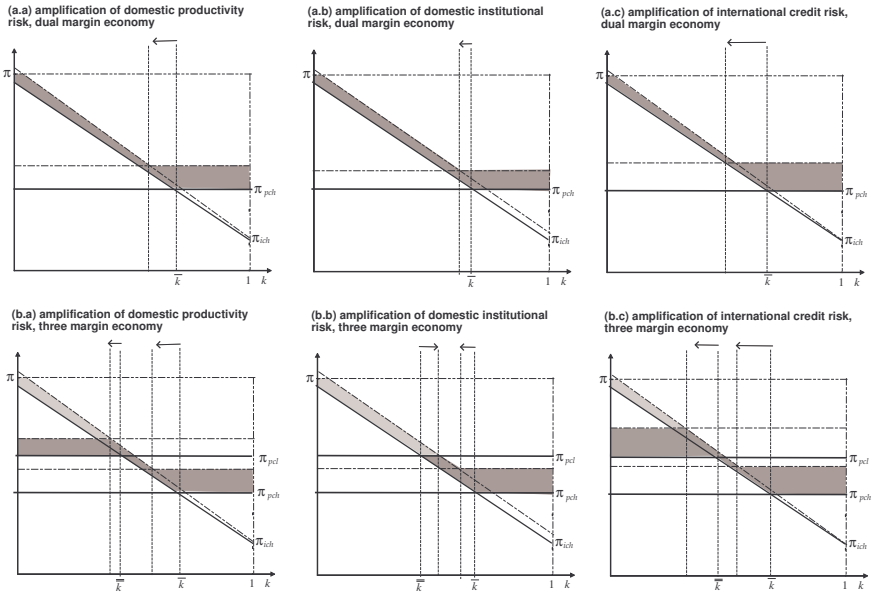

Figure 8: Effects of Risk Amplification

not amplify the effect of the increase in risk. On the other hand if the entrepreneur contracts on effort and then shirks, the interest rate on the outside option and the opportunity cost are different. Moreover, since the benefits of shirking dilute away as $k \to 1$, so will the effect of this wage and, hence, the effect of risk on the ICH constraint.

The amplification of domestic productivity risk (ν_w) will have a smaller effect on EU(l,h) than on EU(h,h), for the marginal firm, since it is amplified by the technological success probability that is actually chosen. Hence, again the ICH will contract, in this case, amplified by productivity π, so it's slope will increase.

In Figure 8 we put together the effects on the three types of constraints for two types of economy. The top three panels show the effects on a dual margin economy such as the one we presented in panel (b) of Figure 3, the bottom three panels show the effects on a triple margin economy such as that of panel (a) of Figure 3. It is important to note that the effects of heterogeneous k or π on the amplification of shocks is, in this case, circumscribed to the ICH constraint since we are asuming an economy with no taxes or distortions that make the participation constraint slopes, such as those studied in the last subsections of section 2.

As we can see, the effect of a domestic institutional risk amplification is identical to that of a international credit risk amplification (panels a.a, a.c, b.a and b.c of Figure 8). In both cases, the effect is concentrated on the participation constrained margin composed of low productivity yet relatively wealthy entrepreneurs. This is especially

true in the dual margin economy, where the effect on poorer and higher productivity entrepreneurs is circumscribed to the tightening of the ICH constraint. We prove in the A-2 that the contraction of ICH will be less than the contraction of PCH for any level of k. There is a powerful intuition behind this result and it is the following: the ICH curve is the result of comparing two projects inside the economy, the PCH curve, on the other hand is the result of comparing a project within the economy with a project outside of it. In the case of the domestic productivity shock it is very clear why it makes the domestic economy relatively more risky than the outside option, and hence tightens the PCH constraint more. In the case of the international credit risk it also ends up making the domestic economy more risky. Why? Well, because entrepreneurs that invest inside the economy will save interest rate payments on the amount of k, while if they stay outside they will receive payments for k. It makes no difference. What changes? Well, if they stay inside the economy, the international credit risk becomes risk on the margin that they keep on the productivity of their borrowed capital

In the case of the triple margin economy this changes slightly since we now have to add the effect of the tightening of the ICL constraint on the Tier 2 firms. Now, part of the highest productivity IC constrained firms suffer only an interest rate increase (light shaded areas). Still the effect on richer entrepreneurs is greater in the case of an amplification of domestic productivity risk ν_w because, as shown in A-2, the contraction of the PCH constraint is larger than that of the PCL constraint. In the case, of international credit risk, they are the same.

Consider the amplification of domestic institutional risk ν_c, that is, uncertainty on the cost of shirking. In this case, as shown in A-2, the ICH constraint will tighten less than PCH. The intuition, as above, is that the PCH compares the project inside the economy with one outside, while the ICH compares two projects inside. An amplification of the risks involved in being inside the economy will affect more entrepreneurs that are participation constrained.

This general intuition behind this section has two implications. First, riskier economies will be predominantly drained of wealthy, but relatively low quality entrepreneurs. Second, average interest rates charged to these economies will, naturally, increase, but will not across the whole economy, but rather across the incentive constrained margin of poorer and productive entrepreneurs.

Finally, it must be pointed out that in this economy all entrepreneurs have the same residual risk aversion as far as the model is concerned. In reality, it might be the case that richer entrepreneurs have several projects, that they are able to hedge against each other, while poorer entrepreneurs only have one. In the extreme, a rich entrepreneur, could own a continuum of projects and face no risk at all. This could make the effects of this economy less realistic.

2.3.2 Size

In this section we outline a model of an identical economy to which we add an additional source of heterogeneity: size, understood as the scale of the project relative to the capital owned by the entrepreneur. It's interesting to understand how the size of the projects would enter into our model since, as we have shown in subsection 1.1, most empirical evidence on heterogenous reactions to shocks is constructed using size as a proxy of firm fragility (even most of our own). The model of this paper, up to now, does not consider heterogeneity in size. Moreover, all projects required an investment of 1, but had different returns. In this section we will show that including this source of heterogeneity, maintains the characteristics of the model and generates an economy in which size is correlated with fragility on the margin.

The model can be adapted with great ease. Up to this point all projects return πW as a result of an investment of 1. Now, we will assume that all projects will return $\phi\pi W$ as a result of an investment of $0 \leq \phi \leq 1$, so this parameter will be our indicator of both the scale of a project and the required investment. Before writing out the program to be solved in characterizing the contracts that will finance each project, we must note a nuance of this section: that in this economy there will be an area of entrepreneurs that will not require external finance to execute their projects. That is, all entrepreneurs for which $\phi \leq k$ will finance their projects directly and even invest their remaining capital in their outside option k. The constraints will, therefore, be relevant for the space of entrepreneurs that do not have enough capital to finance their projects on their own.

Now we must redefine $A = \{\{H, L, O, S\}_i\} \subset \Re^3$ the set of all possible partitions of space $B = [0,1] \times [0,1] \times [0,1]$ such that $H \cup L \cup O \cap S = B$ and $H \cap L \cap O \cap S = \emptyset$. Sets H, L O and S describe collections of entrepreneurs described by a pair (π, k). H is the set of entrepreneurs that are financed to execute their project with a high level of effort, a set of firms that we will call Tier 1 from now on. L, is the set of entrepreneurs that are financed to execute their project with a low level of effort, and we will call Tier 2 henceforth, set O, is the set of entrepreneurs that decide to invest outside the economy, and finally, set S is the set of entrepreneurs for which $\phi \leq k$, that are able to invest in the project without external finance. Call $R = (R_h, R_l) \in \Re^2_+$ a vector describing the interest rate structure of this economy. Equilibrium in this economy is described by $(P, R) \in A \times \Re^2_+$ such that all entrepreneurs maximize their expected profits and banks comply with the zero profit free entry condition.

The entrepreneurs problem is modified slightly from (7), it is now to maximize:

$$U(s, z) = p_s \left(\phi\pi W - (\phi - k) r p_z^{-1}\right) - kr - c_s - q f(z, s) \tag{19}$$

if $\phi \leq k$. The maximization is done by choosing actual effort $s \in \{h, l\}$ and promised effort $z \in \{h, l\}$. To be as general as possible we will assume that the cost of effort is proportional to the size of the project $\{c_h, c_l\} = \{c\phi, 0\}$ and also, that the fine

(of enforced) is proportional to the size of the project, although this is maybe less intuitive.

$$f(z,s) = \left\{ \begin{array}{ll} f\phi & \text{if } z \neq s \\ 0 & \text{if } z = s \end{array} \right\} \tag{20}$$

and we are already assuming in equation (19) that banks will lend so that the zero profit condition (3) is satisfied for every penny lent. If, on the other hand $\phi \geq k$, the entrepreneur will always execute the project as long as $\phi W \geq r$, or otherwise will simply invest abroad.[6]

The new thresholds will be

$$\begin{aligned} \pi_{ich} &= \frac{(1-k\phi^{-1})rp_h^{-1} + (c-fq)(p_h-p_l)^{-1}}{W} \\ \pi_{pch} &= \frac{(r+c)p_h^{-1}}{W} \\ \pi_{pcl} &= \frac{rp_l^{-1}}{W} \end{aligned} \tag{21}$$

and it is especially important to note that size does not affect any of the participation constraints resulting from the model. The reason is that the participation constraints result from the comparison of the use of capital inside and outside of the economy. The scale of the project on the other hand has to be dealt with using credit, and hence is constrained by incentive problems. In the case of the ICH constraint scale is critical, since a larger project will increase the return from effort, and hence make promises more credible.

Figure 9 shows a slice of the partition at size *phi*. In addition to the areas with the two tiers of firms and the area of entrepreneurs that are not able to finance the project, there is now an area of entrepreneurs (lightest shade) that are able to self finance their projects.

If we assumed the cost of effort and fines not to be proportional to the size of the project, Figure 9 would not change very much (although it's comparative statics will). If $\{c_h, c_l\} = \{c, 0\}$ and also, the fine of equation (20) is back to

$$f(z,s) = \left\{ \begin{array}{ll} f & \text{if } z \neq s \\ 0 & \text{if } z = s \end{array} \right\} \tag{22}$$

then, the constraints become

[6] In our model internal funds are always cheaper than external funds. The cost of internal funds is r while the cost of external funds is rp_s^{-1} depending on what level of effort s was promised.

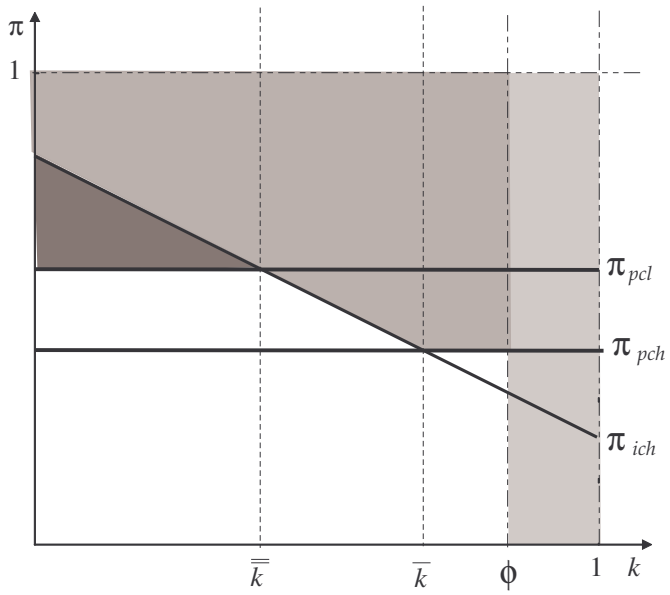

Figure 9: Slice of B at Size ϕ

$$\begin{aligned} \pi_{ich} &= \frac{(1-k\phi^{-1})rp_h^{-1} + (c-fq)\phi^{-1}(p_h-p_l)^{-1}}{W} \\ \pi_{pch} &= \frac{(r+c\phi^{-1})p_h^{-1}}{W} \\ \pi_{pcl} &= \frac{rp_l^{-1}}{W} \end{aligned} \tag{23}$$

and the PCH becomes sensitive to size since effort acts as a fixed cost that is being compared to a benefit that is proportional to size. On the other hand PCL remains insensitive to size. For robustness, we will analyze both cases.

In Figure 10 we show the change in the constraints for two sizes $\phi_h \geq \phi_l$. The top panels illustrate the effects on the two tier economy of panel (b) in Figure 3, and the bottom two panels illustrate the effects on the three tier economy of panel (a) in Figure 3 Panels (a.a) and (b.a) show the case of equation array (21) with total proportionality to size in the costs of effort and fines. Panels (a.b) and (b.b), on the other hand, shows the case of equation array (23) where there is no such proportionality. In all cases, the increase in size of the project has the trivial effect of leaving out a group of marginal low productivity projects owned by wealthy entrepreneurs. They are not eligible for contracting and they cannot finance this size of project, so they will be investing abroad.

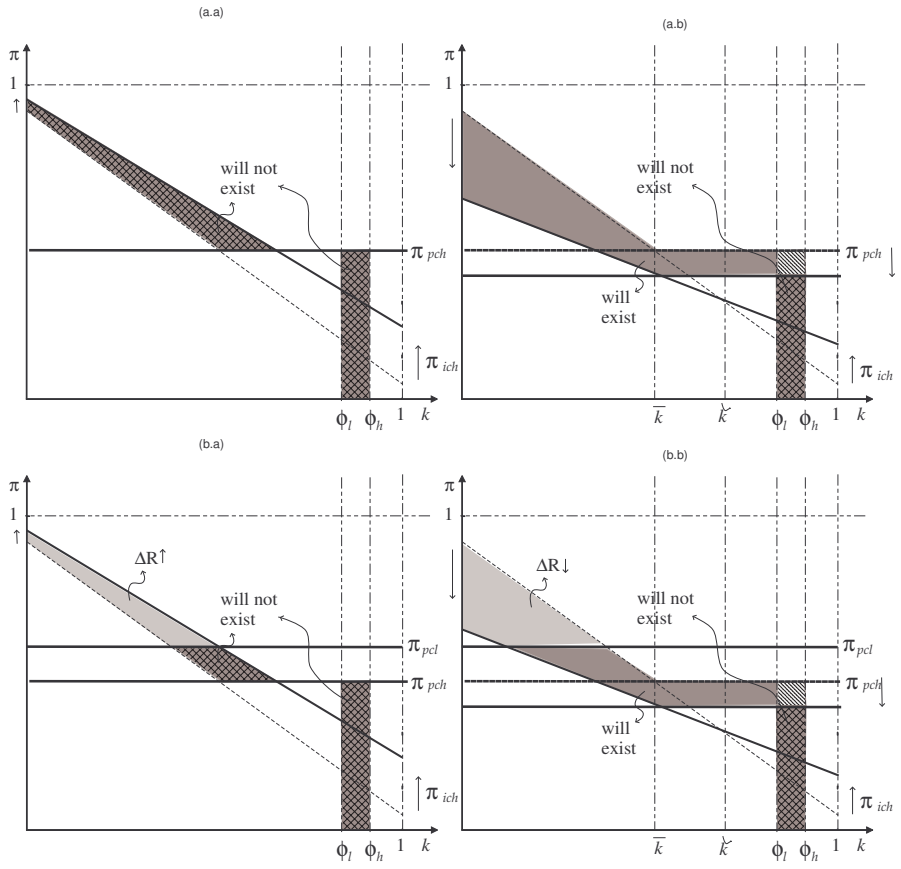

Figure 10: Effect of Size ϕ

When effort costs and fines are proportional to size, the only effect of size will be that the ICH constraint will tighten for larger projects. As the size of the project increases, it becomes less and less credible for the entrepreneur to pledge effort, since, the moral hazard problem becomes more and more important. The entrepreneur's capital is a lower proportion of the investment and enforcing effort is more and more difficult. Moreover, this effect of size is more important when capital is the main source of incentive compatibility (larger k) and less when productivity is the source of incentive compatibility (larger π) since productivity will be high for the whole project no matter it's size. Hence, the ICH constraint become more elastic for slices of B at higher sizes ϕ. In this case, then, less projects are financed at higher scales, and the entrepreneurs that loose credit to scale are mostly poor and own highly productive projects. In the case of the triple margin (panel b.a.) an important portion of these entrepreneurs will be offered credit at Tier 2 interest rates. Hence, in this economy we should not expect size to be correlated with either productivity or wealth, and moreover, we should expect size to be correlated with higher interest rates. All of which is, of course, not realistic.

When effort costs and fines are not totally proportional to size, the model starts giving more realistic stylized predictions. Two thing change. First, the PCH curve is now relaxed by the growing scale of projects. The reason is that the cost of effort starts to act as a fixed cost that is diluted with size. Secondly, the ICH constraint pivots around a fixed point $\breve{k}$ (see A-2 for proofs) as the amplification of moral hazard that we described in the previous paragraph due to the decreasing of the proportion of investment financed by the entrepreneur interacts with another effect. In this case moral hazard is loosened due to the dilution of the cost of effort. Hence, for poorer entrepreneurs that rely on productivity to commit credibly to effort, the ICH constraint will relax. On the other hand, less productive IC constrained marginal entrepreneurs, that rely on their wealth to overcome moral hazard, the ICH constraint will tighten. Fortunately for them we can prove (see A-2) that all of the entrepreneurs for which the ICH constraint tightens will be in fact participation constrained, and since the PCH constraint relaxes, they will, in fact be benefited by size. Hence, in this other economy we should expect size to be correlated with both higher average productivity, less financial constraints (just like (**?**) assumed) and lower interest rates. But it is important to note that we rely on a non proportionality of effort and fines, hence, in the end, on a fixed cost.

One interesting question that one could ask is the following: are the heterogeneous effect of shocks that we document in subsections 2.5-2.7, increased or decreased in intensity by size? Or, in other words, we have already shown that productive firms owned by financially dependent entrepreneurs are relatively more sensitive to shocks. Will they also be more sensitive if they are small? For a sufficiently clear answer we will focus on the dual margin economy (because it has the simplest geometry), and on the international shocks of subsection 2.7, although the conclusions are generalizable

Table 1: Summary of Effects of International Shocks

Constraint	ICH	PCH	PCL
W falls	$\left\vert\frac{\partial \pi_{ich}}{\partial W}\right\vert = \frac{\pi_{ich}}{W}$	$\left\vert\frac{\partial \pi_{pch}}{\partial W}\right\vert = \frac{\pi_{pch}}{W}$	$\left\vert\frac{\partial \pi_{pcl}}{\partial W}\right\vert = \frac{\pi_{pcl}}{W}$
r increases	$\frac{\partial \pi_{ich}}{\partial r} = \frac{(1-k\phi^{-1})p_h^{-1}}{W}$	$\frac{\partial \pi_{ich}}{\partial r} = \frac{p_h^{-1}}{W}$	$\frac{\partial \pi_{ich}}{\partial r} = \frac{p_l^{-1}}{W}$

to all shocks of section 2. Also, we will constraint ourselves to an economy with non proportional costs of effort and fines which is described by equation array (23), since it has delivered the most realistic stylized predictions. The partial derivatives of these constraints with respect to adverse shocks to gross productivity W (that we use to represent the world economy) and the international interest rate r are:

In panels (a.b) and (b.b) of Figure 7 we have illustrated how the ICH constraint will be looser for poor but productive entrepreneurs if the scale of their projects is large. We can infer from Table 3.2.1 that the effect of a world recession will also be smaller (since π_{ich} is lower). So, these entrepreneurs will be relatively insulated by their scale from the effect of the global recession on moral hazard. In fact, this will also be true for participation constrained entrepreneurs on the PCH margin, since, as we have shown above, the PCH will also be more relaxed for large firms. In contrast, the effect of the interest rate shock on the PCH and PCL is the same for all sizes. On the other hand, the effect on the ICH constraint is smaller again since:

$$\frac{\partial^2 \pi_{ich}}{\partial r \partial \phi} = -\frac{k p_h^{-1}}{W \phi^2} \tag{24}$$

In summary we find that it is consistent with this model, to use size as a proxy for productivity or financial access as the literature has. Also, we find that scale will help to ameliorate the heterogeneous effects of different shocks.

3 Conclusions

We have studied a model of an economy with heterogeneous entrepreneurs in productivity, wealth and also size. We have shown that in such an economy, with contracting originating from a hidden action problem, there will be at least a dual margin with high productivity firms owned by poor entrepreneurs being incentive constrained and low productivity firms owned by richer entrepreneurs being participation constrained. The constraint will be tighter for poorer and more productive entrepreneurs. We have shown that this setup can also lead to an economy with multiple margins and differential interest rates in contracts. Again, poorer entrepreneurs will be the ones to face the higher interest rates.

We show that this participation constrained margin composed of poorer but productive entrepreneurs will be more sensitive to shocks in this economy, more benefited by institutional improvements to contracting, financial opening, financial development and the provision of technical support by the government. This can help explain why in a developed country such as Chile, small firms and financially dependent firms tend to be more volatile and generally sensitive to shocks.

We show that these differences in sensitivity effects are amplified for smaller scale projects and that it does seem reasonable to use size as a proxy for fragility in a model such as the one presented in this paper. Finally, we show that risk generally affects more the participation constrained margin composed of wealthier entrepreneurs that will flee. We also show that in riskier economies interest rates will be higher among poor entrepreneurs with relatively high productivity.

Appendix

A-1 Proofs for Heterogenous Productivity Model

A-1.1 Derivation of Equations (4)-(9)

First notice that the cost of effort in the high effort incentive compatibility constraint is $c - fq$ which is the technological cost of effort minus the expected fine from shirking. In both the high effort participation constraints and in the low effort incentive compatibility constraint the cost of effort is only c, since in those constraints we are not considering the possibility of cheating.

Incentive compatibility to exert effort is satisfied if the firm is willing to expend effort at const $c - fq$ to increase the probability of success from p_l to p_h, given that the firm has signed a contract on the zero profit rate for high effort rp_h^{-1}, hence:

$$\left(\pi W - (1-k)\, rp_h^{-1}\right) p_l \leq \left(\pi W - (1-k)\, rp_h^{-1}\right) p_h - (c - fq) \tag{25}$$

another way to see this condition is to say that R_{ich}, the maximum interest rate at which a hypothetical bank would be willing to lend to project $\{k, \pi\}$, believing that the incentives of the entrepreneur are aligned with high effort is:

$$R_{ich} = \frac{\pi W - (c - fq)(p_h - p_l)^{-1}}{1-k} \tag{26}$$

or that, at zero profit interest rate rp_h^{-1} for high effort, the minimum productivity at which the bank will believe effort is expended is:

$$\pi_{ich} = \frac{(1-k) rp_h^{-1} + (c - fq)(p_h - p_l)^{-1}}{W} \tag{27}$$

which is our equation (9).

Participation compatibility within a high effort contract is satisfied if the net result of expending effort and reaping the higher expected value is larger than the outside option of depositing in the riskless interest rate, hence:

$$rk \leq \left(\pi W - (1-k)\, rp_h^{-1}\right) p_h - c \tag{28}$$

another way to see this condition is to say that R_{pch}, the maximum interest rate at which a hypothetical entrepreneur owning project $\{k, \pi\}$, would be willing to engage in a high effort contract is:

$$R_{pch} = \frac{\pi W - (\delta k + c) p_h^{-1}}{1-k} \tag{29}$$

or that, at zero profit interest rate rp_h^{-1} for high effort, the minimum productivity at which the bank will believe effort is expended is:

$$\pi_{pch} = \frac{(r+c)p_h^{-1}}{W} \tag{30}$$

which is our equation (4).

Participation compatibility within a low effort contract is satisfied if the result of executing the project with no effort is larger than the outside option of depositing in the riskless interest rate, hence:

$$rk \leq \left(\pi W - (1-k)\, r p_l^{-1}\right) p_l \tag{31}$$

another way to see this condition is to say that R_{pcl}, the maximum interest rate at which a hypothetical entrepreneur owning project $\{k, \pi\}$, would be willing to engage in a low effort contract is:

$$R_{pcl} = \frac{\pi W - rkp_l^{-1}}{1-k} \tag{32}$$

or that, at zero profit interest rate rp_h^{-1} for low effort, the minimum productivity at which the bank will believe effort is expended is:

$$\pi_{pcl} = \frac{rp_h^{-1}}{W} \tag{33}$$

which is our equation (5).

Incentive compatibility within a low effort contract is satisfied if the result of executing a low effort project is greater than executing a high effort project:

$$\left(\pi W - (1-k)\, r p_h^{-1}\right) p_h - c \leq \left(\pi W - (1-k)\, r p_l^{-1}\right) p_l \tag{34}$$

another way to see this condition is to say that R_{bl}, the minimum interest rate at which a hypothetical entrepreneur owning project $\{k, \pi\}$, would be willing to prefer a low effort contract to a high effort contract is:

$$R_{bl} = \frac{\pi W - c(p_h - p_l)^{-1}}{1-k} \tag{35}$$

or that, at zero profit interest rates for high and low effort, the maximum productivity at which an entrepreneur will prefer a low effort contract to a high effort contract is:

$$\pi_{bl} = \frac{c(p_h - p_l)^{-1}}{W} \tag{36}$$

which is our equation (6).

A-1.2 Derivation of $\bar{k}$ and $\bar{\bar{k}}$ with related proofs.

First, from equations (4) and (6) we can see that a sufficient condition for low effort contracts to never be preferred over high effort contracts is:

$$c(p_h - p_l)^{-1} \leq (r + c)p_h^{-1} \tag{37}$$

which ends up implying

$$\frac{c}{r} \leq \frac{p_h - pl}{p_l} \tag{38}$$

or that the financial cost of effort is lower than the financial gain from effort.

Second, from equations (4) and (5) we can see that a sufficient condition for there to never be a two tier economy in the absence of information asymmetry is that:

$$rp_l^{-1} \leq (r + c)p_h^{-1} \tag{39}$$

which ends up implying (38).

Third, from equations (4) and (9) we derive equation (10) retyped here:

$$\bar{k} = \left(\frac{c}{r}\right)\left(\frac{p_l}{p_h - p_l}\right) - \left(\frac{fq}{r}\right)\left(\frac{p_h}{p_h - p_l}\right)$$

so that for there to be a dual margin economy condition (38) must be satisfied.

Fourth, although it is redundant given our three previous steps, it is interesting to note that from equations (5) and (6) we can see that a sufficient condition for the BL roof to be lower than the PCL floor is:

$$c(p_h - p_l)^{-1} \leq rp_l^{-1} \tag{40}$$

which implies condition (38).

So summarizing, either $\pi_{pch} \geq \pi_{pcl} \geq \pi_{bl}$ and $\bar{k} \geq 1$, and/or $\pi_{bl} \geq \pi_{pch} \geq \pi_{pcl}$ and $\bar{k} \leq 1$. Hence if condition (38) is satisfied we will have an economy such as panel (a) in Figure A-1.1, with the possibilities of a dual and tripe margin economies and tiers of firms. If not, we will have an economy such as panel (c) in Figure A-1.1, with a unique participation constrained low effort margin, two tiers, and the potential for a dual tier margin. Finally, panel (b) shows the case where the financial cost of effort is exactly equal to the financial effect of effort. In this case there is no dual margin, but rather a tier margin with a slope where it becomes easier for richer entrepreneurs to qualify for low interest rates.

In panels (a) and (b), the kink in the tier margin $\bar{\bar{k}}$ is the intersection of (5) and (9) and is

$$\bar{\bar{k}} = 1 + \frac{cp_h}{r(p_h - p_l)} - \frac{p_h}{p_l} \tag{41}$$

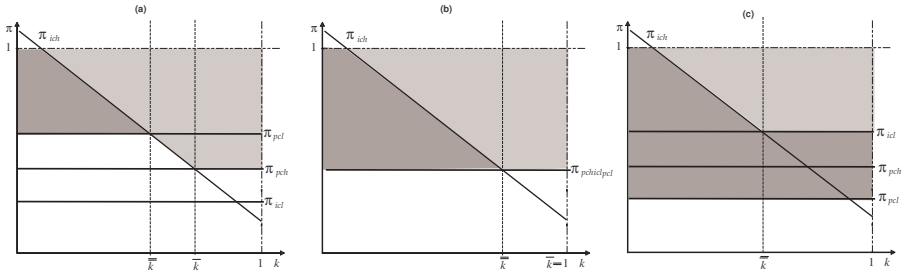

Figure A-1.1: Alternative Levels of Relative Financial Relevance of Effort

and in panels (b) and (c) the tier margin $\bar{\bar{k}}$ is the intersection of (6) and (9) and is

$$\bar{\bar{k}} = 1 - \frac{fqp_h}{r(p_h - p_l)} \tag{42}$$

so that as $fq \to 0$, and condition (38) is satisfied as an equality, $\bar{\bar{k}}$ in both (41) and (42) converges to 1.

A-1.3 Derivation of effect of taxes and controls on capital flows.

Consider an ad-valorem tax on capital outflows for residents of t_{out} and an ad-valorem tax on capital inflows of t_{in} for non-residents. The ICH equation of (25) is recast as

$$\left(\pi W - (1-k)\,(1+t_{in})rp_h^{-1}\right)p_l \leq \left(\pi W - (1-k)\,(1+t_{in})rp_h^{-1}\right)p_h - (c - fq) \tag{43}$$

which means that (9) is now

$$\pi_{ich} = \frac{(1-k)(1+t_{in})rp_h^{-1} + (c-fq)(p_h - p_l)^{-1}}{W} \tag{44}$$

and is not affected by the outflow tax, since the ICH constraint compares two options that are realized inside the economy. At $k = 0$ equation (44) is identical to equation (9), but not at $k = 1$, hence the pivoting movement shown in Figure 4. The slope of (44) is $-(1+t_{in})r(p_hW)^{-1}$.

The PCH equation of (30) is recast as

$$(1-t_{out})rk \leq \left(\pi W - (1-k)\,(1+t_{in})rp_h^{-1}\right)p_h - c \tag{45}$$

which means that (4) is now

$$\pi_{pch} = \frac{((1-t_{out})r + c - rk(t_{out}+t_{in}))p_h^{-1}}{W} \tag{46}$$

and is affected by both taxes since it compares options inside and outside of the economy. The slope of (46) is $-(t_{out}+t_{in})r(p_hW)^{-1}$. Hence, both ad-valorem taxes affect the slope of the constraint in the same way, what changes is the point on which the constraint pivots. In the case of the outflow tax it pivots on $k=1$, in the case of the inflow tax it pivots on $k=0$.

The PCL equation of (34) is recast as

$$(1-t_{out})rk \leq \left(\pi W-(1-k)\,(1+t_{in})rp_l^{-1}\right)p_l \tag{47}$$

which means that (5) is now

$$\pi_{pcl}=\frac{((1-t_{out})r-rk(t_{out}+t_{in}))p_l^{-1}}{W} \tag{48}$$

and is also affected by both taxes since it compares options inside and outside of the economy. The slope of (48) is $-(t_{out}+t_{in})r(p_lW)^{-1}$. The PCL constraint is affected in the same way that the PCH constraint only amplified by the fact that $p_l^{-1}>p_h^{-1}$.

A-1.4 Derivation of effect of microfinance policy α.

Consider an economy with a tax on capital inflows t. The ICH constraint with a microfinance policy consisting of a gift of $\alpha(1-k)$ to every entrepreneur of capital k is

$$(c-fq)\leq\left(\pi W-(1-k)\,(1+t)(1-\alpha)rp_h^{-1}\right)(p_h-p_l) \tag{49}$$

which means that (9) is now

$$\pi_{ich}=\frac{(1-k)(1+t)(1-\alpha)rp_h^{-1}+(c-fq)(p_h-p_l)^{-1}}{W} \tag{50}$$

and a marginal increase in α relaxes the constraint in

$$\frac{\partial\pi}{\partial\alpha}=-\frac{(1-k)(1+t)rp_h^{-1}}{W} \tag{51}$$

so, the effect is larger for lower k.

The PCH constraint in this economy is now

$$rk\leq\left(\pi W-(1-k)\,(1+t)(1-\alpha)rp_h^{-1}\right)p_h-c \tag{52}$$

if the development agency is capable of verifying that the entrepreneur will use the subsidy to complement his own funds in implementing the project. Otherwise, the PCH constraint will be

$$r(k+k(1-\alpha))\leq\left(\pi W-(1-k)\,(1+t)(1-\alpha)rp_h^{-1}\right)p_h-c \tag{53}$$

and the PCL will be either

$$rk \leq \left(\pi W - (1-k)\,(1+t)(1-\alpha)rp_l^{-1}\right) p_l \tag{54}$$

or

$$r(k+k(1-\alpha)) \leq \left(\pi W - (1-k)\,(1+t)(1-\alpha)rp_l^{-1}\right) p_l \tag{55}$$

which means that the effect of a marginal increase in α on the PCS $S \in \{H, L\}$ constraint is now either

$$\frac{\partial \pi}{\partial \alpha} = -\frac{(1-k)(1+t)rp_s^{-1}}{W} \tag{56}$$

if investment is verifiable to the development agency, or

$$\frac{\partial \pi}{\partial \alpha} = -\frac{(1-k)trp_s^{-1}}{W} \tag{57}$$

if it is not. Hence, the effect is lower and the development agency will have to either expend resources verifying investment or make gifts to fewer entrepreneurs if it cannot verify the investment. Moreover, if there are no taxes or distorted access to international banking markets there is no effect on the PC constraints in the nonverifiable case, and the only effects on this economy are those created by the relaxation of the ICH constraint. Note that in this case (t=0) both participation constraints acquire positive slope when $\alpha > 0$.

A-1.5 Proofs and derivations of effects of international shocks.

Notice that all constraints have the following general form

$$\pi_x = \frac{\theta_x}{W}; \forall x \in \{ich, pch, pcl, icl\} \tag{58}$$

which implies that

$$\frac{\partial \pi_x}{\partial W} = -\frac{\pi_x}{W} \tag{59}$$

since all firms in this economy share W, any shock to W will have a larger effect on the margin composed by firms of higher productivity π. Since, in the three margin two tier economy PCL is higher than PCH, we show in panel (a) of Figure (7) a larger contraction (upward shift) of PCL. Moreover, ICH will contract more than constraint PCX whenever it is above and less when it below. We know then, that ICH will shift upward in the same distance than PCH at $\bar{k}$ and in the same distance as PCL at $\bar{\bar{k}}$. These two points indicate the new position of ICH.

From equations (4), (5) and (9) we can derive the comparative static effects of the interest rate hike as

$$\begin{aligned} \frac{\partial \pi_{ich}}{\partial r} &= \frac{(1-k)p_h^{-1}}{W} \\ \frac{\partial \pi_{pch}}{\partial r} &= \frac{p_h^{-1}}{W} \\ \frac{\partial \pi_{pcl}}{\partial r} &= \frac{p_l^{-1}}{W} \end{aligned} \tag{60}$$

so that both PC constraints will move up in parallel to their original positions, the PCL constraint will move more than the PCH constraint and the ICH constraint will move the same as the PCH constraint at $k = 0$ and will pivot on $k = 1$.

A-2 Proofs for Extensions and Generalizations

A-2.1 Effect on ICH, PCH and PCL of amplifications of risk.

We can rewrite equation (14), by grouping the deterministic and random terms in the following way:

$$EU(s,z) = EU \begin{pmatrix} p_s\left(\pi(W-(1-k)rp_z^{-1}) - kr - qf(z,s)\right) \\ -c_s + p_s\pi\nu_w - (p_s(1-k)p_z^{-1} + k)\nu_r \end{pmatrix} \tag{61}$$

Notice that when $s = z$ this reduces to:

$$EU(s,s) = EU \begin{pmatrix} p_s\left(\pi(W-(1-k)rp_z^{-1}) - kr \right. \\ -c_s + p_s\pi\nu_w - \nu_r \end{pmatrix} \tag{62}$$

, so we can see that the effect of international credit risk ν_r is the same on both PC constraints, no matter what the choice of effort s is, and the same for marginal firms of any productivity level π that are the property of entrepreneurs of any wealth k. Now, an identical amplification of risk will have a lower effect if it is applied on a higher initial level of expected utility. So, it is important to remember that the base expected utility levels on the PCL and PCH curves are the same, that is, zero. For this reason, the effect on both constraints of the amplification of international credit risk will be the same. In the case of domestic institutional risk ν_c, only the PCH tightens since, since $c_l = 0$. The effect of domestic productivity risk will be larger on the PCH constraint since it is amplified by p_h rather than p_l. In the case we study in section 3.1, there is no slope on the PC constraints, so all marginal firms will have the same productivity π, and the constraints will shift in a parallel way and remain horizontal.

Now consider the case where $s \neq z$, which only makes sense when the entrepreneurs promises more effort than he is willing to deliver, hence $s = l$ and $z = h$. In

this case, the term that accompanies the international credit risk random variable ν_r is $(p_l p_h^{-1}(1-k)+k)$ which is smaller than 1. Hence the effect of interest rate risk on EU(h,l) is smaller than the effect either on EU(h,h) or EU(l,l) (remember, they are the same). The difference between these amplifications will increase as the technological importance of effort increases. The implication is that the ICH constraint tightens (since EU(h,h) falls more than EU(h,l)), but less than the PCH or PCL constraints.

Another difference is that these constraints are not amplified by the level of k while the effect on EU(h,l) is. This means that for higher k, EU(h,l) falls more relative to EU(h,h) and hence the PCH constraint will tighten less. Since $\lim_{k\to 1}(p_l p_h^{-1}(1-k)+k) = 1$ the ICH constraint will, in fact, pivot on $k = 1$ as the interest rate risk amplifies, while the PCH and PCL constraints will tighten in a parallel way.

Consider now the effect of the amplification of the domestic productivity shock ν_w. The amplified risk will be larger if high effort is chosen, also if productivity of the firms is higher. Hence, all three constraints will increase their slope, the PCH constraint will tighten more than the PCL, and the ICH constraint will tighten less than the PCH constraint since the fall in EU(h,h) is, in this case, ameliorated by the fall in EU(l,h).

A-2.2 Effect of Size on ICH with non proportional effort and fines.

Consider the ICH constraint of equation array (23). The effect on π_{ich} of an increase in size ϕ is:

$$\frac{\partial \pi_{ich}}{\partial \phi} = \frac{r p_h^{-1} k - (c - fq)(p_h - p_l)^{-1}}{W\phi^2} \tag{63}$$

it is only positive if $k \geq \left(\frac{c-fq}{r}\right)\left(\frac{p_h}{p_h-p_l}\right)$, a point that we have labeled $\breve{k}$ and does not change with ϕ. Hence, above $\breve{k}$ the constraint will tighten, and below it will loosen. Now compare $\breve{k}$ to $\bar{k}$ of equation (10) retyped here:

$$\bar{k} = \left(\frac{c}{r}\right)\left(\frac{p_l}{p_h - p_l}\right) - \left(\frac{fq}{r}\right)\left(\frac{p_h}{p_h - p_l}\right)$$

hence, the ICH constraint will only tighten with size for entrepreneurs that are participation constrained.